BMS

INSTITUTE OF TECHNOLOGY AND MANAGEMENT

Avalahalli, Doddaballapur Main Road, Bengaluru – 560064

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

NETWORK THEORY

18EC32

STUDY MATERIAL

III SEMESTER

NETWORK ANALYSIS (18EC32)

Syllabus:-

Module -1

Basic Concepts: Practical sources, Source transformations, Network reduction using Star – Delta transformation, Loop and node analysis With linearly dependent and independent sources for DC and AC networks, Concepts of super node and super mesh.

Text Books:

1. M.E. Van Valkenberg (2000), "Network analysis", Prentice Hall of India, 3rd edition, 2000, ISBN: 9780136110958.

2. Roy Choudhury, "Networks and systems", 2nd edition, New Age International Publications, 2006, ISBN: 9788122427677.

Reference Books:

1. Hayt, Kemmerly and Durbin "Engineering Circuit Analysis", TMH 7th Edition, 2010.

2. J. David Irwin /R. Mark Nelms, "Basic Engineering Circuit Analysis", John Wiley, 8th edition, 2006.

3. Charles K Alexander and Mathew N O Sadiku, "Fundamentals of Electric Circuits", Tata McGraw-Hill, 3rd Ed, 2009.

Theory Question's.

Q). Explain

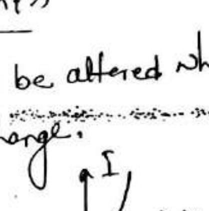
1. Unilateral and Bilateral elements
2. Independent and dependent Sources. } J/J 2014 (6m)

3. Linear and Non Linear
4. Active and passive elements
5. Lumped and distributed.
6. Ideal and practical Sources. } Dec 2012 (8m)

Solu: 1. Unilateral and Bilateral element's

Unilateral: V-I characteristic is to be altered when the direction of current is to be change.

Ex: diode, Transistor.

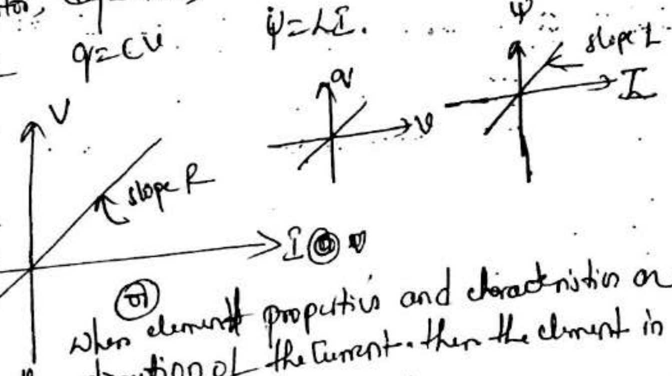


Bilateral: V-I characteristics remain's same irrespective of Current direction. (or) When element properties and characteristics are dependent on the direct of Current. then the element is called Unidirectional (or) Unilateral element's.

Ex: Resistor, Capacitor, Inductor.

$V = IR$ $\quad q = CV$ $\quad \psi = LI$.

An element is to be bilateral if it has same impedance for the different direction of same Current flow otherwise it is said to be Unilateral.



(or) When element properties and characteristics are independent on the direction of the Current. then the element is called as bi directional (or) bilateral element.

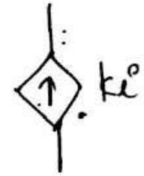
→ Independent and Dependent Sources:

Dependent Sources.

* The strength of Dependent Source is depends on one of the ckt parameters of the n/w in which it is connected.

Exp:-

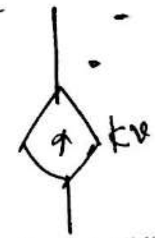
1. Current Controlled Current Source (CCCS)



(BJT)

$I_c = \beta I_B$

2. Voltage Controlled Current Source (VCCS)



→ F.E.T

$V_D = f^{\mu}(I_D)$.

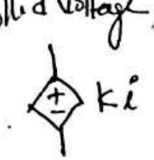
3. Voltage controlled Voltage Source

(VCVS)

(op amp)

4. Current Controlled Voltage Source (CCVS)

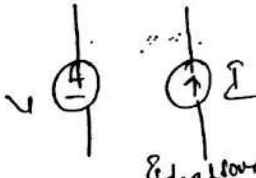
(CRO)

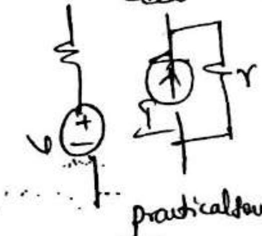


Independent Sources

* The strength of Independent Sources does not depend on the any of the ckt parameters

Exp: ideal & practical voltage + current Sources.





3. Linear and Non-Linear elements

Linear :- A two terminal element is said to be Linear if for all time 't' its characteristics is Straight Line through the origin otherwise it is Said to be non Linear.

(or)

* elements which obeys ohm's Law.
* Elements which obeys the principle of Superposition (additivity and homogeneity rule).

Ex:- R, L, C.

Non-Linear :- * A two terminal Element is said to be Non Linear if for all time 't' its characteristics is not a Straight Line that passes through origin.

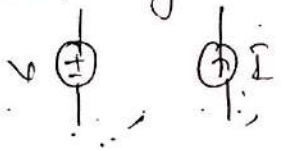
* Doesn't obeys Ohm's Law.
* fails to obeys the principle of Superposition.

Ex:- Diode, transistor, transformer etc, Independent Voltage & Current Sources.

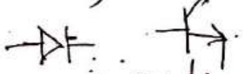
4. Active (or) passive elements

Active elements :- when element is capable of delivering energy independently for infinite time (or) when the element is having property of internal amplification (or) signal rectification then the element is called as active element.

Ex:- Independent voltage and Current Sources



V I Diode, transistor etc



passive elements :- elements which are not capable of delivering energy (or) not capable to do signal amplification (or) rectification then it is called as passive elements.

Ex:- R, L, C, bulbs, transformer etc.

5. Lumped & Distributed elements

Lumped elements :-

* ohm's Law can be applicable for Lumped (Linear) elements.
* R, L, and C can be Separable.
* In Lumped elements $\lambda >> l$.

Ex:- R, L, C.

Distributed elements :-

* KVL and KCL fails for distributed parameters Since in distributed parameters electrically, it is not possible to separate resistance, Inductance, and capacitance effect.
* but ohm's Law can be applied for Lumped and distributed parameters. Ex:- Co-axial cable, Transmission Line etc.

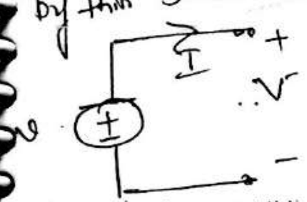
Time Varying and Time-Invariant Element's

→ An element is said to be Time-invariant if all time its characteristic's does not change with time, otherwise it is said to be time-varying.

Ex:- R, L, C ← Time Invariant element's.

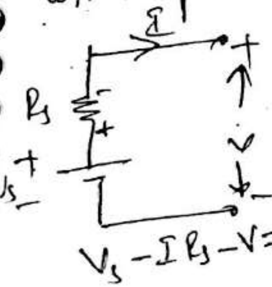
Ideal and practical Sources:-

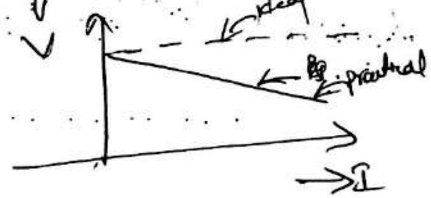
→ Ideal voltage Source:- Ideal voltage source delivers energy at the specified voltage (V), which is independent on current delivered by this source.



$R_s = 0$

→ practical Voltage Source:- practical voltage source delivers energy at specified voltage (V) which depend's on current delivered by the source.





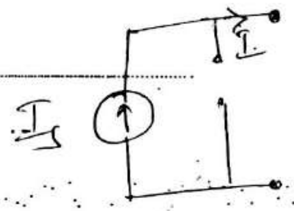
$R_s \neq 0$

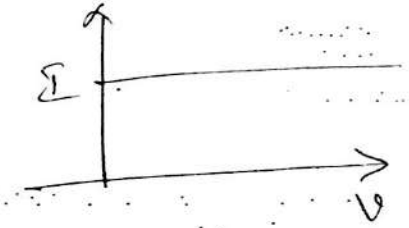
$V_s - IR_s - V = 0 \Rightarrow V = V_s - IR_s$

→ Ideal Current Source:-

Ideal current source delivers energy at specified current (I) which is independent on voltage across the source.

Internal resistance of ideal current source = ∞.

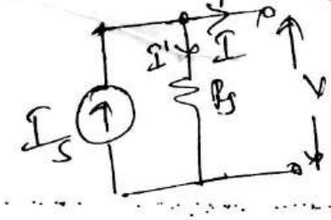




$R_{internal} = \infty\ \Omega$

→ practical Current Source:-

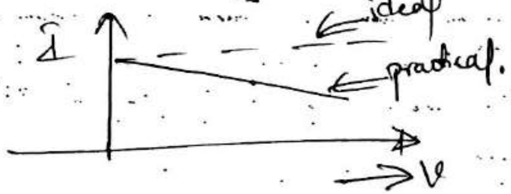
practical current source delivers energy at specified current (I) which depend's on voltage across the source.



$I_s = I' + I$

$I = I_s - I'$

$= I_s - \frac{V}{R_s}$



3. Linear and Non-Linear elements

Linear: A two terminal element is said to be Linear if for all time 't' its characteristics is Straight Line through the origin otherwise it is Said to be nonLinear.

(or) * elements which obey's ohm's Law.

* Elements which obey's the principle of Superposition (additivity and homogeneity rule).

Ex:- R, L, C.

Non-Linear:- * A two terminal Element is said to be Non Linear if for all time 't' its characteristics is not a Straight Line that pass through origin.

* Doesn't Obey's Ohm's Law.

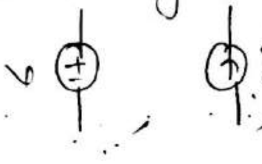
* fail's to Obey's the principle of Superposition.

Ex:- Diode, transistor, transformer etc, Independent Voltage & Current Sources.

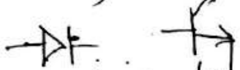
4. Active (or) passive elements

Active element:- when element is capable of delivering energy independently for infinite time (or) when the element is having property of internal amplification (or) signal rectification then the element is called as active element.

Ex:- Independent voltage and Current Sources



Diode, transistor etc



Passive element:- elements which are not capable of delivering energy (or) not capable to do Signal amplification (or) rectification then it is called as passive elements.

Ex:- R, L, C, bulbs, transformer etc.

5. Lumped & Distributed elements

Lumped elements:-

* ohm's Law can be applicable for Lumped (Linear) elements.

* R, L, and C can be Separable.

* In Lumped elements $\lambda >> l$.

Ex:- R, L, C.

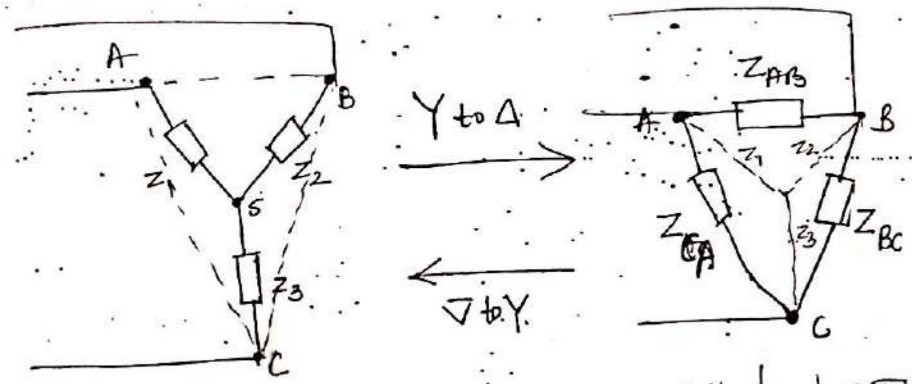
Distributed elements:-

* KVL and KCL fail's for distributed parameters Since in distributed parameters electrically, it is not possible to separate resistance, Inductance, and Capacitance effect.

* but ohm's Law can be applied for Lumped and distributed parameters. Ex:- Co-axial cable, Transmission line etc.

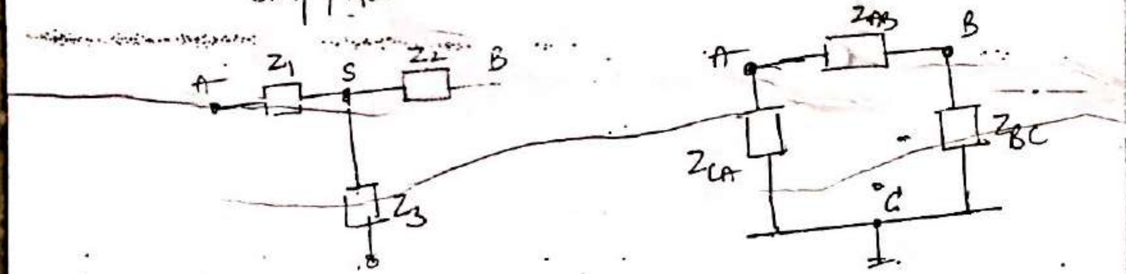
Star ↔ Delta transformations :-

* N/w reduction tool.



Star/Y N/w

Delta/mesh (or) π N/w.



a) Star to Δ Conversion :-

given $Z_1, Z_2,$ and Z_3 $\Longrightarrow$ find Z_{AB}, Z_{CA}, Z_{BC}.

$$Z_{AB} = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$$

$$= \frac{Z_1 Z_3 + Z_2 Z_3 + Z_1 Z_2}{Z_3} = \frac{\sum Z_1 Z_2}{Z_3}\ \Omega$$

|||ly $$Z_{BC} = Z_2 + Z_3 + \frac{Z_2 Z_3}{Z_1}$$

$$= \frac{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}{Z_1} = \frac{\sum Z_1 Z_2}{Z_1}\ \Omega$$

and

$$Z_{CA} = Z_1 + Z_3 + \frac{Z_1 Z_3}{Z_2}$$

$$= \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_2} = \frac{\sum Z_1 Z_2}{Z_2}\ \Omega$$

Note :- if $Z_1 = Z_2 = Z_3 = R\ \Omega$ then

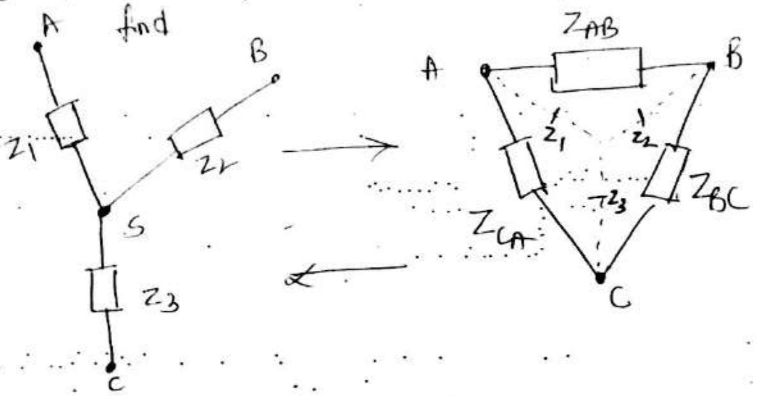
$$Z_{AB} = R + R + \frac{R^2}{R} = 3R.$$

|||ly $Z_{BC} = 3R$ and $Z_{CA} = 3R$.

In general $\boxed{Z_\Delta = 3 Z_Y}$ (or) $\boxed{Z_Y = \frac{Z_\Delta}{3}}$

(b) Delta to star N/w

given
find



$$Z_1 = \frac{Z_{AB}\cdot Z_{CA}}{Z_{AB}+Z_{BC}+Z_{CA}} = \frac{Z_{AB}\cdot Z_{CA}}{\sum Z_{AB}}$$

$$Z_2 = \frac{Z_{AB} Z_{BC}}{Z_{AB}+Z_{BC}+Z_{CA}} = \frac{Z_{AB}\cdot Z_{BC}}{\sum Z_{AB}}$$

$$Z_3 = \frac{Z_{CA}\cdot Z_{BC}}{Z_{AB}+Z_{BC}+Z_{CA}} = \frac{Z_{BC}\cdot Z_{CA}}{\sum Z_{AB}}$$

Notes:- if $Z_{AB} = Z_{BC} = Z_{CA} = R\ \Omega$

then $Z_1 = \frac{[R\cdot R]}{3R} = R/3\ \Omega$

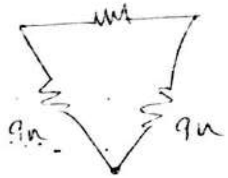
$|||^{ly}$ $Z_2 = R/3\ \Omega$ and $Z_3 = R/3\ \Omega$

∴ In general $Z_Y = \frac{Z_\Delta}{3} \Rightarrow \boxed{Z_Y = \frac{Z_\Delta}{3}}\ \Omega$

(or) $\boxed{Z_\Delta = 3 Z_Y}$

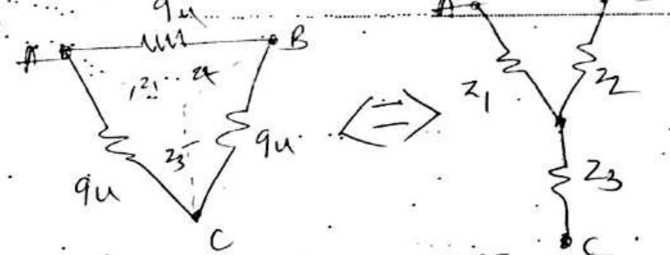
Example:

(1) Given



Convert it equivalent star N/w. 6

Sol:



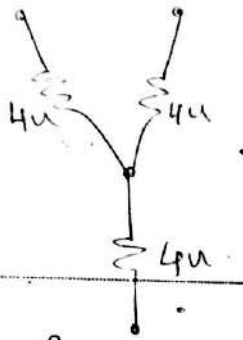
Obs: $Z_{AB} = Z_{BC} = Z_{CA} = 9\Omega = Z_\Delta$

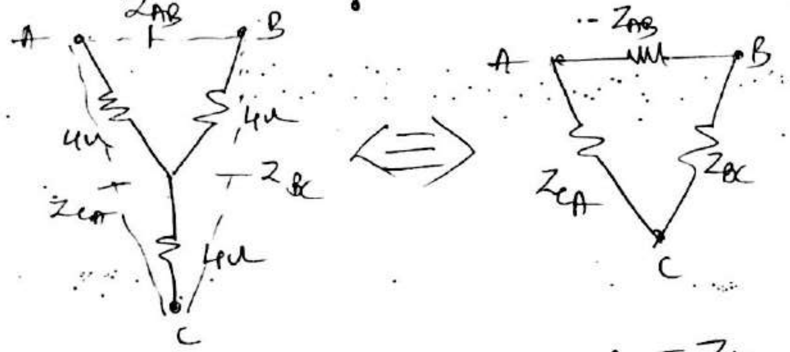
∴ $Z_Y = ?$ $\boxed{Z_Y = Z_\Delta/3}$

given $Z_Y = 9/3 = 3\Omega = Z_1 = Z_2 = Z_3$

∴

equivalent star N/w.

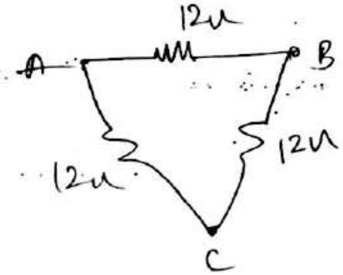
(Q2) given  find its equivalent Delta value.



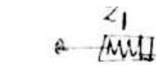
Since given $Z_1 = Z_2 = Z_3 = 4\Omega = Z_Y$

W.k.t $Z_Y = \frac{Z_\Delta}{3} \Rightarrow Z_\Delta = 3Z_Y$.

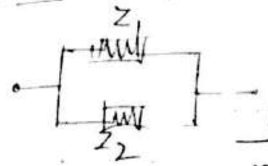
$Z_\Delta = 3(4) = 12\Omega$.



equivalent Delta value.

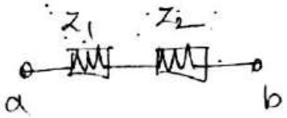
Note 1)  impedance (Z_1). Ω

admittance $Y_1 = \frac{1}{Z_1}$ ℧

(2) 

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} \leftarrow \text{impedance}$$

$$\Rightarrow \boxed{Y_{eq} = Y_1 + Y_2} \leftarrow \text{admittance}$$

(3) 

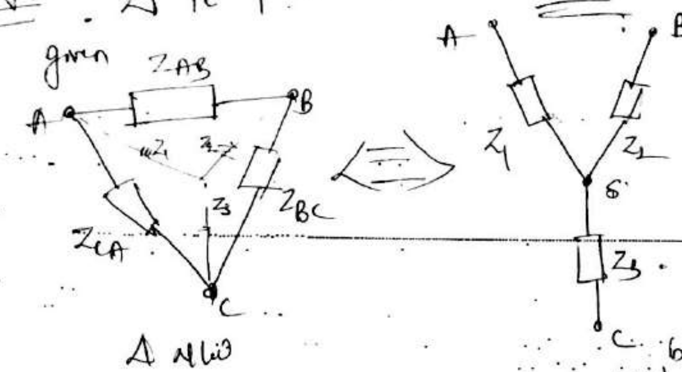
$Z_{ab} = Z_{eq} = Z_1 + Z_2$. Ω

$$\Rightarrow \frac{1}{Y_{ab}} = \frac{1}{Y_{eq}} = \frac{1}{Y_1} + \frac{1}{Y_2}$$

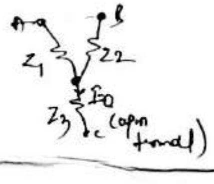
$$\frac{1}{Y_{ab}} = \frac{Y_1 + Y_2}{Y_1 Y_2}$$

$$\Rightarrow \boxed{Y_{ab} = Y_{eq} = \frac{Y_1 Y_2}{Y_1 + Y_2}} \text{ ℧.}$$

Proof: Δ to Y N/w. given / find?



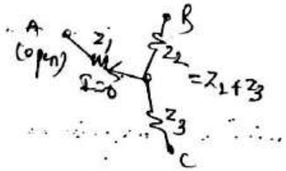
Equivalent impedance across terminal's A-B with 'C' open



$$Z_1 + Z_2 = Z_{AB} \parallel (Z_{BC} + Z_{CA})$$

$$Z_1 + Z_2 = \frac{Z_{AB}(Z_{BC} + Z_{CA})}{Z_{AB} + Z_{BC} + Z_{CA}} \longrightarrow (1)$$

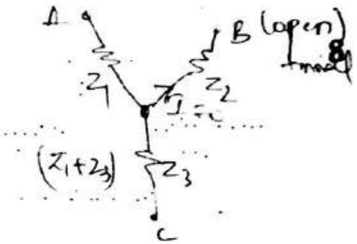
Equivalent impedance across terminal's B-C with terminal 'A' in open.



$$Z_2 + Z_3 = Z_{BC} \parallel (Z_{AB} + Z_{CA})$$

$$Z_2 + Z_3 = \frac{Z_{BC}(Z_{AB} + Z_{CA})}{Z_{AB} + Z_{BC} + Z_{CA}} \longrightarrow (2)$$

Equivalent impedance across terminal's C-A with terminal 'B' in open.



$$Z_1 + Z_3 = Z_{CA} \parallel (Z_{AB} + Z_{BC})$$

$$Z_1 + Z_3 = \frac{Z_{CA}(Z_{AB} + Z_{BC})}{Z_{AB} + Z_{BC} + Z_{CA}} \longleftarrow (3)$$

eq^n (1) $-$ eq^n (2)

$$Z_1 + \cancel{Z_2} - \cancel{Z_2} - Z_3 = \left\{\frac{Z_{AB}(Z_{BC} + Z_{CA})}{Z_{AB} + Z_{BC} + Z_{CA}}\right\} - \left\{\frac{Z_{CA}(Z_{AB} + Z_{BC})}{Z_{AB} + Z_{BC} + Z_{CA}}\right\}$$

$$= \frac{Z_{AB}(Z_{BC} + Z_{CA}) - Z_{BC}(Z_{AB} + Z_{CA})}{Z_{AB} + Z_{BC} + Z_{CA}}$$

$$= \frac{\cancel{Z_{AB}Z_{BC}} + Z_{AB}Z_{CA} - \cancel{Z_{BC}Z_{AB}} - Z_{BC}Z_{CA}}{Z_{AB} + Z_{BC} + Z_{CA}}$$

$$Z_1 - Z_3 = \frac{Z_{AB}Z_{CA} - Z_{BC}Z_{CA}}{Z_{AB} + Z_{BC} + Z_{CA}} \longleftarrow (4)$$

eq^n (3) + eq^n (4) $\quad Z_1 + \cancel{Z_3} + Z_1 - \cancel{Z_3} = \dfrac{Z_{CA}Z_{AB} + \cancel{Z_{BC}Z_{CA}} + Z_{AB}Z_{CA} - \cancel{Z_{BC}Z_{CA}}}{Z_{AB} + Z_{BC} + Z_{CA}}$

$$\cancel{\Delta} Z_1 = \frac{\cancel{\Delta} Z_{AB} Z_{CA}}{Z_{AB} Z_{BC} + Z_{AB} Z_{CA} + Z_{BC} Z_{CA}}$$

$$\therefore \boxed{Z_1 = \frac{Z_{AB} Z_{CA}}{\sum Z_{AB}}}$$

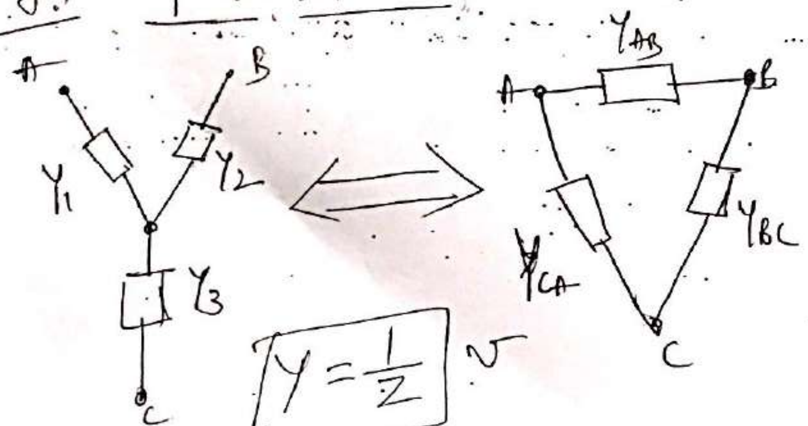
Ily, we can simplify

$$\boxed{Z_2 = \frac{Z_{AB} Z_{BC}}{\sum Z_{AB}}}$$

and

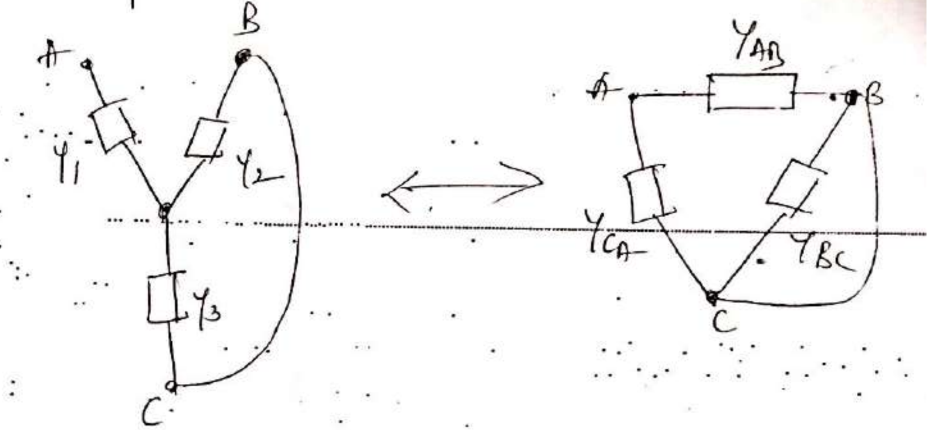
$$\boxed{Z_3 = \frac{Z_{BC} Z_{CA}}{\sum Z_{AB}}}$$

proof:- Y to Δ n/w



$$\boxed{Y = \frac{1}{Z}}$$

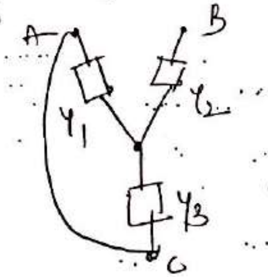
→ Equivalent admittance b/w AB with B shorted



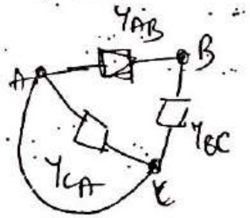
$$y_1 \parallel (y_2 + y_3) = Y_{AB} + Y_{CA}$$

$$\frac{y_1(y_2+y_3)}{y_1+y_2+y_3} = Y_{AB} + Y_{CA} \longrightarrow (1)$$

→ Equivalent admittance b/w BC with C shorted to A.

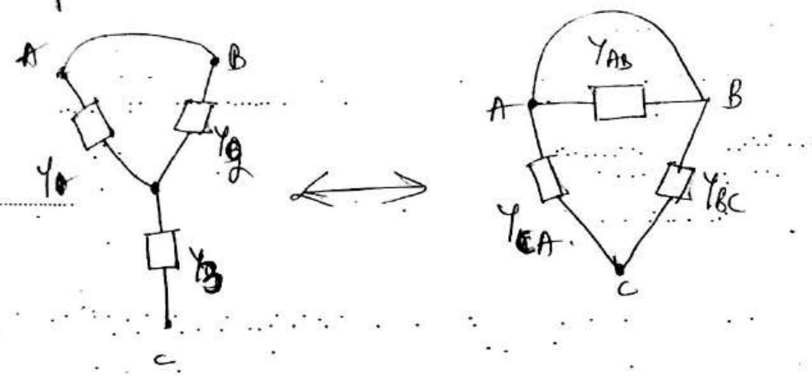


$$y_2 \parallel (y_1 + y_3) = Y_{AB} + Y_{BC}$$



$$\frac{y_2(y_1+y_3)}{y_2+y_1+y_3} = Y_{AB} + Y_{BC} \longrightarrow (2)$$

equivalent admittance b/w CA with A shorted to B.



$$(Y_1+Y_2) \,\|\, Y_3 = Y_{BC}+Y_{CA}$$

$$\frac{Y_3(Y_1+Y_2)}{Y_1+Y_2+Y_3} = Y_{BC}+Y_{CA} \leftarrow (3)$$

eq (1) − eq (2)

$$\frac{\cancel{Y_1Y_2}+Y_1Y_3-\cancel{Y_2Y_1}-Y_2Y_3}{Y_1+Y_2+Y_3} = \cancel{Y_{AB}}+Y_{CA}-\cancel{Y_{AB}}-Y_{BC}$$

$$\frac{Y_1Y_3-Y_2Y_3}{Y_1+Y_2+Y_3} = Y_{CA}-Y_{BC} \leftarrow (4)$$

eq (3) + (4)

$$\frac{Y_1Y_3+\cancel{Y_2Y_3}+Y_1Y_3-\cancel{Y_2Y_3}}{Y_1+Y_2+Y_3} = 2Y_{CA}$$

$$\frac{\cancel{2}Y_1Y_3}{(Y_1+Y_2+Y_3)} = \cancel{2}Y_{CA}$$

$$\Rightarrow Y_{CA} = \frac{Y_1Y_3}{Y_1+Y_2+Y_3}$$

$$\frac{1}{Z_{CA}} = \frac{\frac{1}{Z_1Z_3}}{\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}} = \frac{1}{(Z_1Z_3)} \times \frac{Z_1Z_2Z_3}{Z_2Z_3+Z_1Z_3+Z_1Z_2}$$

$$\frac{1}{Z_{CA}} = \frac{Z_2}{Z_2Z_3+Z_1Z_3+Z_1Z_2}$$

$$Z_{CA} = \frac{Z_2Z_3+Z_1Z_3+Z_1Z_2}{Z_2} = \frac{\sum Z_1Z_2}{Z_2}\ \Omega$$

Ily. we can solve

$$Z_{AB} = \frac{\sum Z_1Z_2}{Z_3}\ \Omega$$

and

$$Z_{BC} = \frac{\sum Z_1Z_2}{Z_1}\ \Omega.$$

8) Find the equivalent Resistance across the terminals AB of n/w shown in fig.

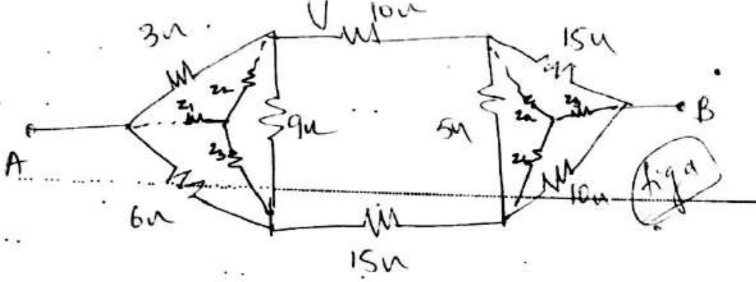
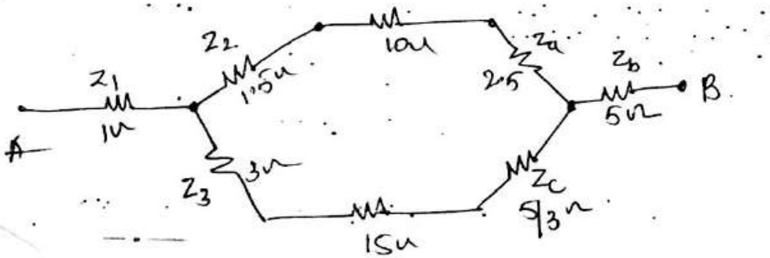


fig a

Sol:-



$$Z_1 = \frac{3\times6}{6+3+9} = 1\,\Omega$$

$$Z_2 = \frac{3\times9}{18} = 1.5\,\Omega$$

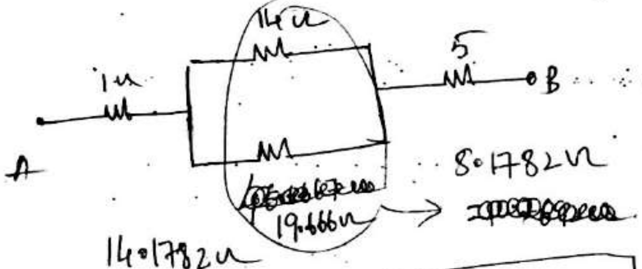
$$Z_3 = \frac{6\times9}{18} = 3\,\Omega$$

$$Z_a = \frac{5\times15}{30} = 2.5\,\Omega$$

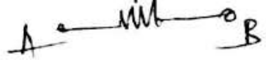
$(5+15+10)$

$$Z_b = \frac{15\times10}{30} = 5\,\Omega$$

$$Z_c = \frac{5\times10}{30} = 5/3\,\Omega$$



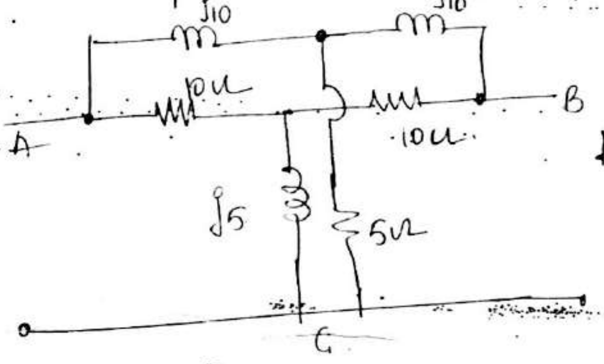
14.1782Ω

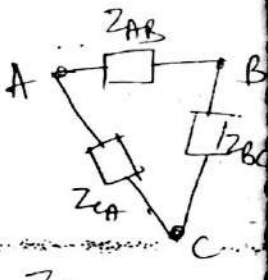


$$\boxed{R_{AB} = 14.1782\,\Omega}$$

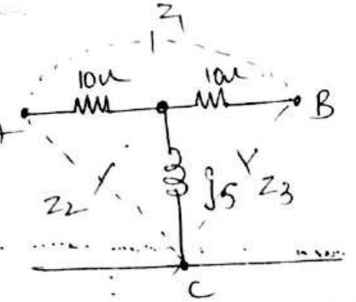
(Q2) In the n/w shown below (fig a.) Find the voltage need to be applied across AB so that the current drawn by the circuit is 1A. $\underline{V_{AB} = 14.1782 \text{ volts}}$

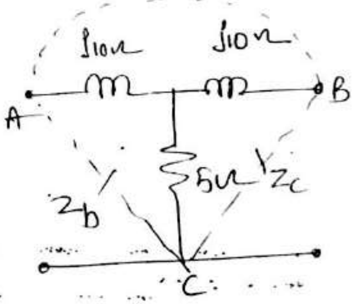
(Q3) Find the equivalent delta n/w of the n/w shown in fig





Sol:-

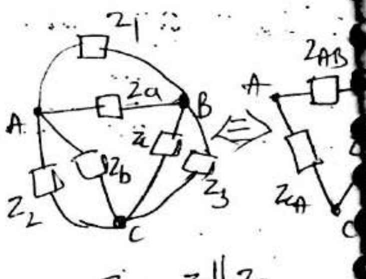




$$Z_1 = 10+10+\frac{10\times10}{j5} = (20-20j)\,\Omega$$

$$Z_2 = 10+j5+\frac{10\times j5}{10} = 10+j5+\frac{j50}{10} = (10+j10)\,\Omega$$

$$Z_3 = 10+j5+\frac{10\times j5}{10} = (10+j10)\,\Omega.$$



$$Z_{AB} = Z_1 \| Z_a$$

$$Z_{CA} = Z_2 \| Z_b$$

$$Z_{BC} = Z_c \| Z_3$$

$$Z_a = j10 + j10 + \frac{j10 \times j10}{5}$$

$$= j20 - \frac{100}{5} = (-20 + j20)\,\Omega$$

$$Z_b = j10 + 5 + \frac{j50}{j10} = (10 + j10)\,\Omega$$

$$Z_c = j10 + 5 + \frac{j50}{j10} = (10 + j10)\,\Omega$$

$Z_1 = (20 - j20)\,\Omega$

$Z_2 = (10 + j10)\,\Omega$

$Z_3 = (10 + j10)\,\Omega$

$$Z_{AB} = Z_1 \| Z_a = \frac{Z_1 Z_a}{Z_1 + Z_a} = \frac{(-20 + j20)(20 - j20)}{\cancel{-20} + \cancel{j20} + \cancel{20} - \cancel{j20}} = \frac{(\ )}{0} = \infty$$

$$\frac{1}{Z_{AB}} = \frac{1}{Z_1} + \frac{1}{Z_2} \Rightarrow \boxed{Z_{AB} = \infty}\,\Omega$$

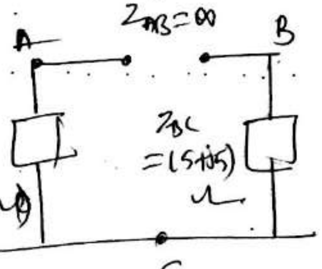
$$Z_{CA} = Z_2 \| Z_b = \frac{Z_2 Z_b}{Z_2 + Z_b} = \frac{(10 + j10)(10 + j10)}{(10 + j10) + (10 + j10)}$$

Since $Z_b = Z_2$

$$= (5 + j5)\,\Omega$$

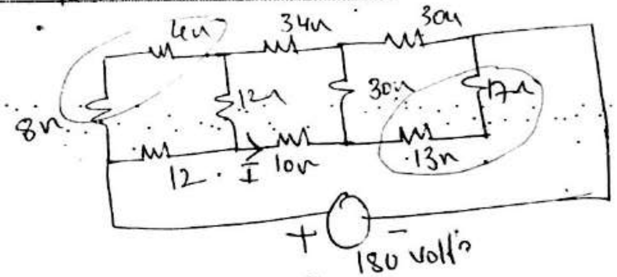
$$Z_{BC} = Z_3 \| Z_c = (10 + j10) \| (10 + j10)$$

$$= (5 + j5)\,\Omega$$

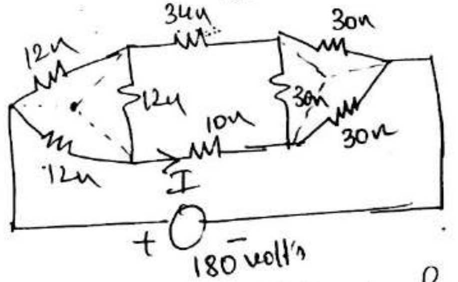


Q) A squared based pyramid is having its 8 edges each of $60\,\Omega$. Find the equivalent resistance across the diagonally opposite corners.

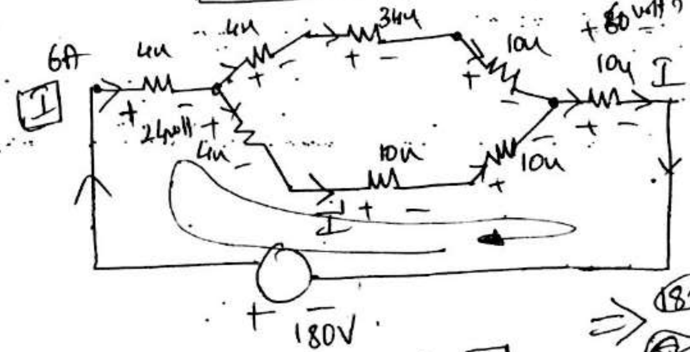
8) Using Y-Δ transformation find the current through the $10\,\Omega$ resistor in the N/W shown.



Sol:-



$$\boxed{R_\Delta = 3 R_Y} \Rightarrow R_Y = R_\Delta / 3$$



$$180 - 4I - 4I' - 10I' - 10I' - \boxed{10I} = 0$$

the Source Current $I = ?$.

$$I = \frac{180}{4 + 48 \| 24 + 10} = 6A$$

$$\Rightarrow \cancel{180 = 4I + 34I'}$$

$$\cancel{180 - 4(6) = 34I'}$$

$$\cancel{\Rightarrow I' = \frac{180-24}{34}}$$

$$180 - 24 - 24I - 6a = 0$$

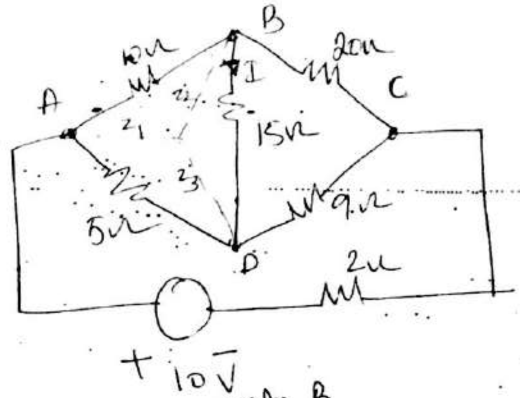
$$\boxed{Our\ I = \frac{180 - 48}{84}}$$

$$\cancel{I = 182/24}\quad I = 96/24 = 4A$$

the Current flow through 10 Ω resistor

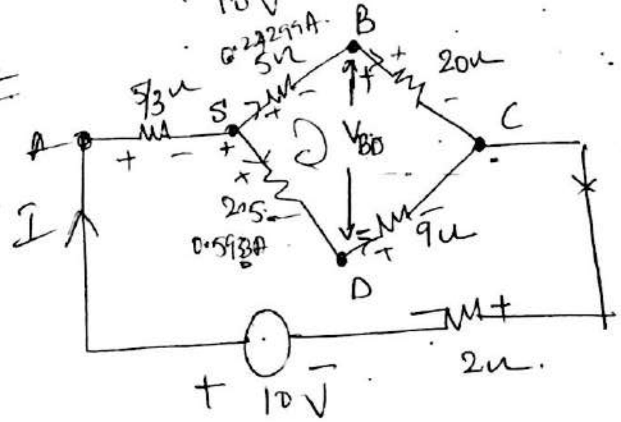
$$\boxed{I_{10\Omega} = 4\ A}$$

(8) using star: delta transformation find the Current through the branch Connected b/w BD.



10+5+15 =

Sol:



the Source Current $I = \dfrac{10}{5/3 + (25 \| 11.5) + 2} = 0.86629$

$$10 - \frac{5}{3}(I) - V_{SC} - 2(I) = 0$$

$$V_{SC} = 10 - \frac{5}{3}(0.86629) - 2(0.86629).$$

$$\boxed{V_{SC} = 6.823}\ \text{Volts}$$

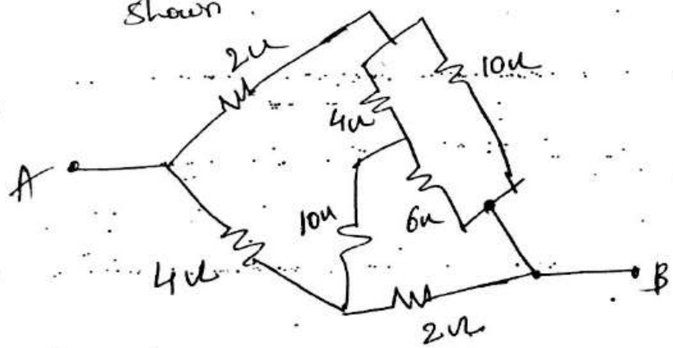
$0.5933(2.5) - 5(0.27291) - V_{BD} = 0$

$\boxed{V_{BD} = 0.1183}$ Volts

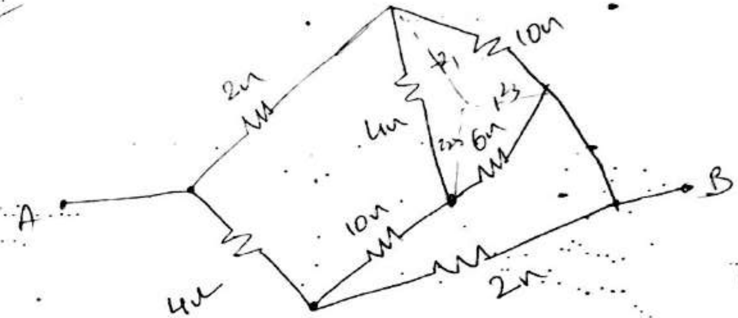
$$I = \frac{V_{BD}}{15\Omega} = \frac{0.1183}{15}$$

$\boxed{I = 7.886 \text{ mA}} = 0.0071$ Ampere

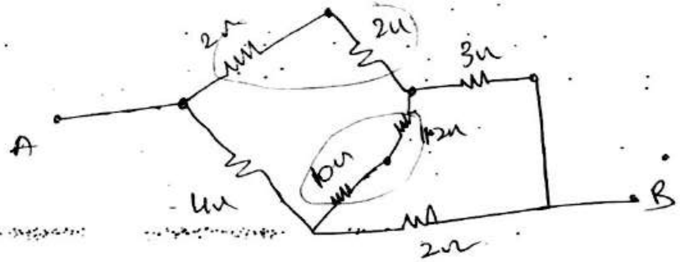
(8) Find the equivalent Resistance across AB for the ckt shown

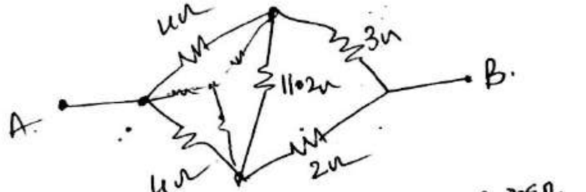


Soln:

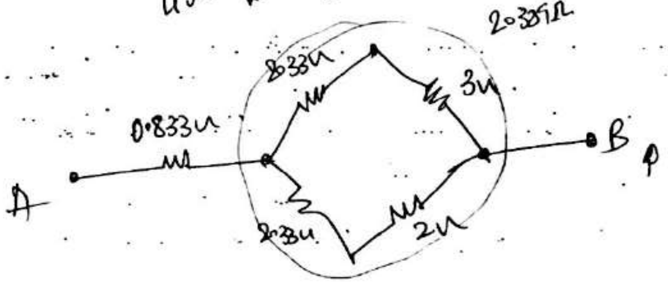


$4+10+6=20$



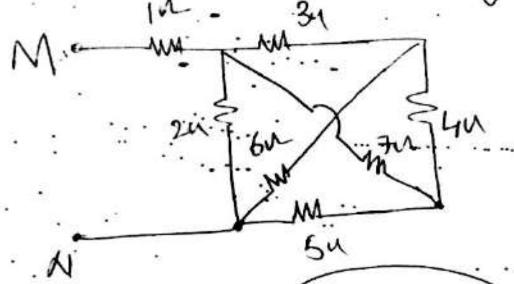


$4+4+11.2=19.2$

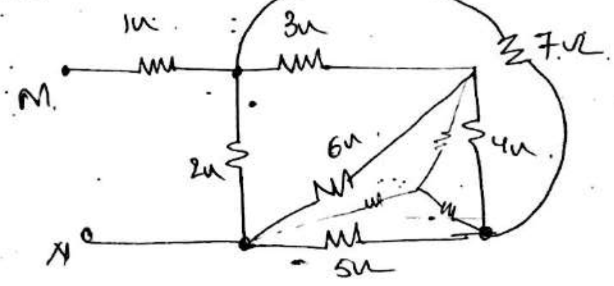


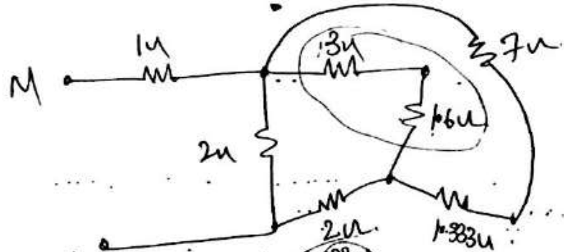
$\boxed{R_{AB} = 3.222}\ \Omega$

(8) Using star/delta transformation, determine the resistance b/w M and N of N/w shown in fig.

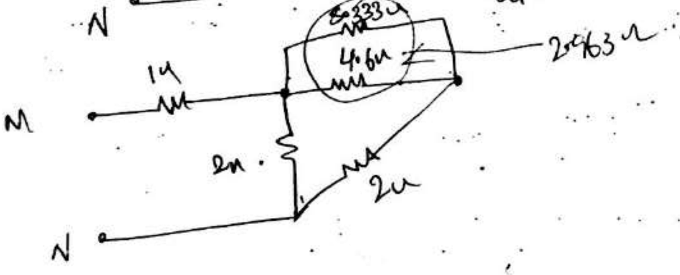


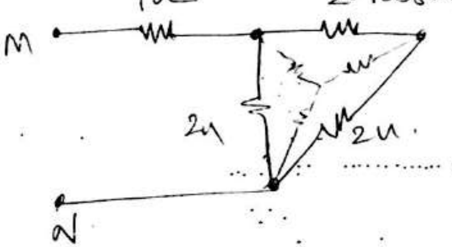
Sol:-



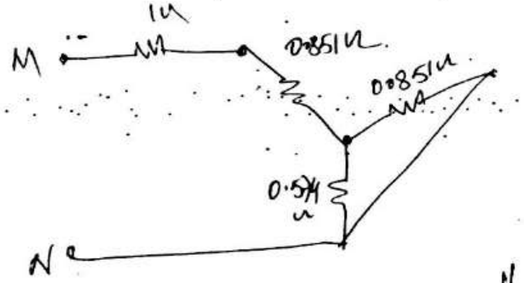


$6+4+5=15\Omega$.





$= 6.9638\,\Omega$

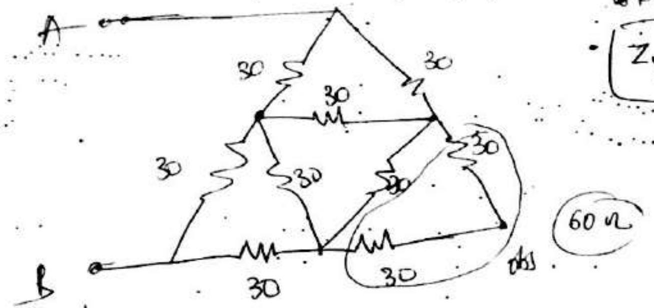


$$R_{MN} = 1 + 0.851 + (0.851) \| (8.574)$$

$$\boxed{R_{MN} = 2.1937}\,\Omega$$

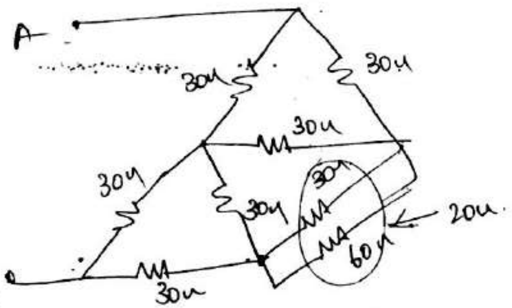
(8) obtain expressions for an equivalent set of star connected impedances to replace a set of delta connected impedances. (5M) J/J 2014.

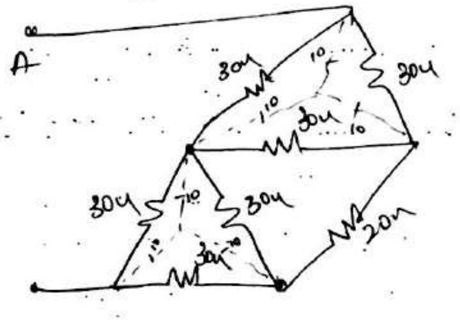
(8) Find the equivalent resistance at AB using Y-Δ transformation technique for the circuit shown in fig (SM) Jan 2014

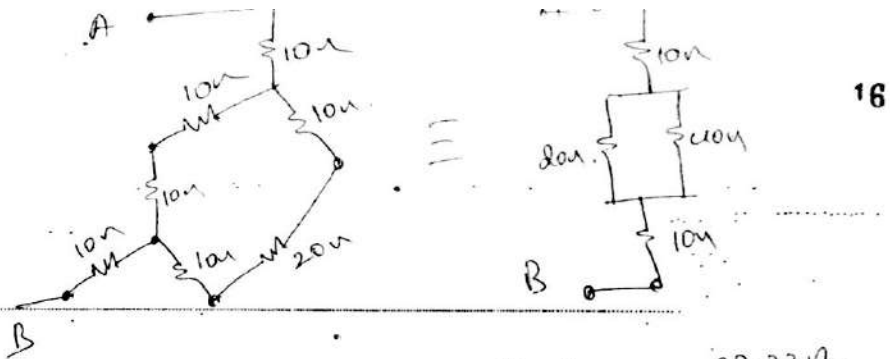


$\boxed{Z_Y = \frac{Z_\Delta}{3}}$

Soln:







$R_{AB} = 10 + (20 \| 40) + 10 = 33.33\,\Omega$

(8) Using star-delta transformation reduce the given network shown in fig (a) and determine the total current supplied by the source.

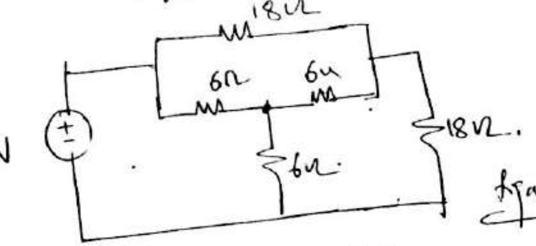
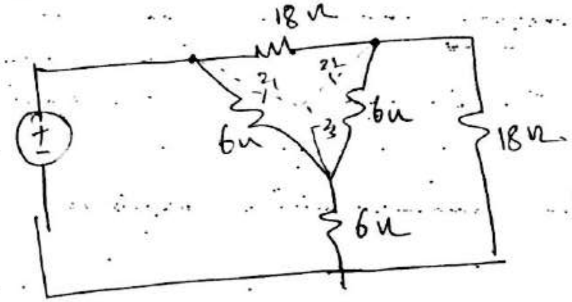
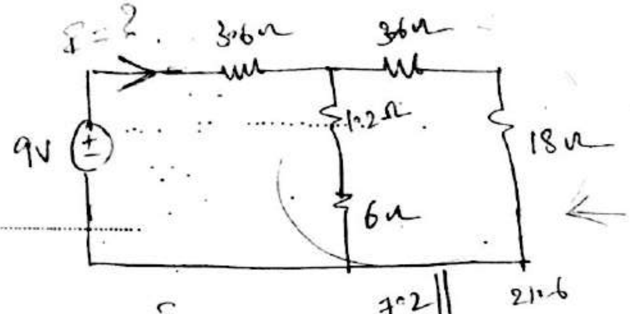


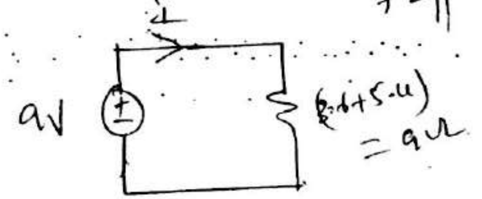
fig a

Soln:



$18+6+6 = 30\,\Omega$



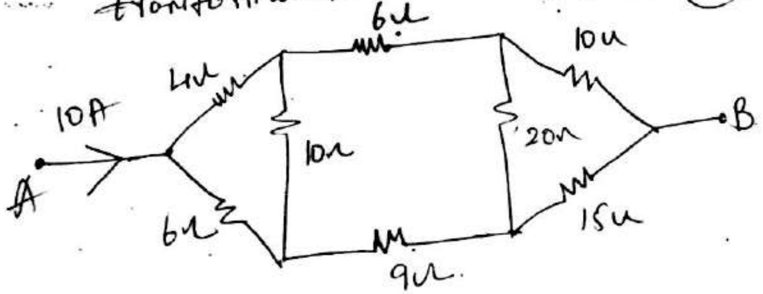


$$I = \frac{V}{R} = \frac{9}{9} = \cancel{1A}$$

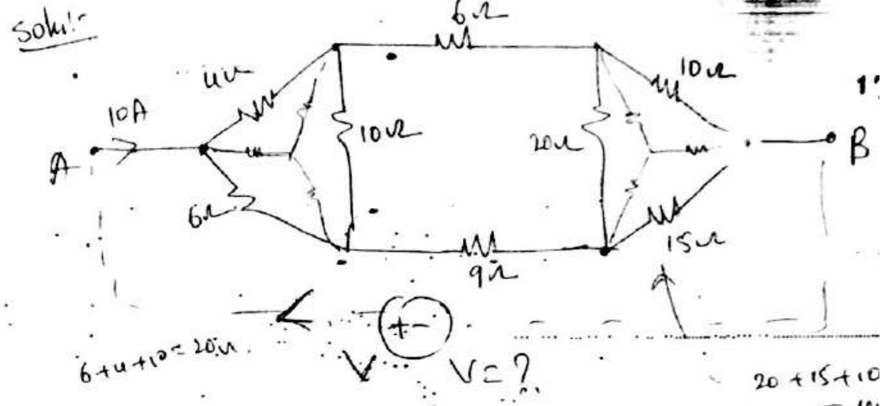
$$\boxed{I = 1\ \text{Ampere}}$$

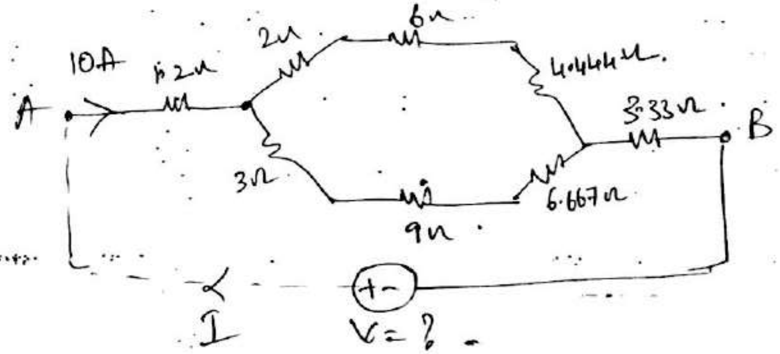
the total Current supplied by the Source is '1' Ampere

Q8) Find the voltage to be applied across AB in order to drive the Current of 10A into the Circuit using star-Delta transformation. (6m) 5/3/2013



Soln:





$$I = \frac{V}{R} \Rightarrow V = I \cdot R$$

$$V = 10\left[1.2 + (\cancel{12.444} \,\|\, 18.667) + 3.33\right]$$

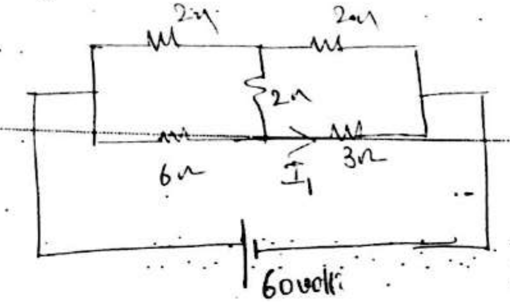
7.4667Ω

$$= 10 \times 11.9967$$

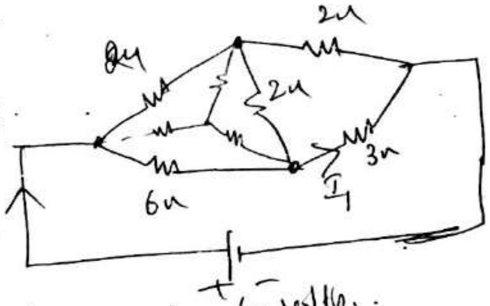
$$V = 119.967 \simeq 120\ \text{Volts}$$

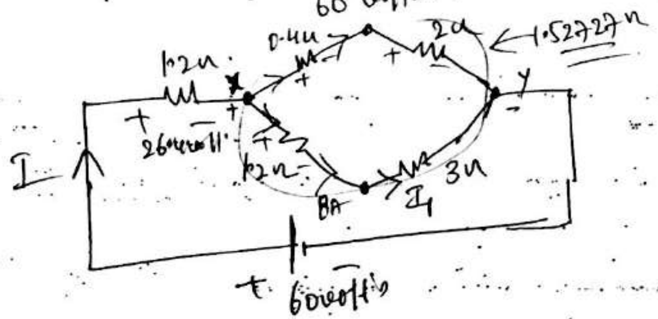
(8) Determine Current I_1 in the n/w shown in fig. using star-delta conversion. (6m)

Jun 2012



Soln:





$2+2+6=10\Omega$

$$I = \frac{V}{R} = \frac{60}{1.2+(2.4 \| 4.2)} = \frac{60}{2.0272} = 22\text{ A}$$

$$60 - 26.4 - V_{xy} = 0 \Rightarrow \boxed{V_{xy} = 33.6}\text{ volts}$$

$$I_1 = \frac{V_{xy}}{(1.2+3)} = \frac{33.6}{4.2} = 8A$$

MISSION

Provide quality and contemporary education, in the domain of Electronics and communication and related fields, which enable collaborative ventures with industries and research organizations. Emphasis laid on creating innovative teaching-learning processes that motivate self-learning. by imparting quality education embedded with discipline & national honor.

VISION

To create a rich intellectual potential implanted with multidisciplinary knowledge, human values and professional ethics among the aspirant of becoming Engineers and technologies, so as to unlock their imagination and discover their potential.

OBJECTIVES

1. To impart good technical knowledge to the students.
2. To produce Excellent Engineers in Electronics & Communication fields.
3. To fulfil the needs of the society in the various fields related to Electronics and Communication engineering.
4. To bring post-graduate program in the diverse field of electronics and communication Engineering.
5. To upgrade the facilities in Research & Development Centre of the department with the use of modern aids.
6. To organize training programs / workshops for upgrading staff performance.
7. To establish Industry - institute Interaction.
8. To publish technical papers in National / International journals and conferences.

GOALS (Short Term):

1. Modernizing the Laboratories with new software & state-of-the art hardware in tune with the latest technological developments.
2. To obtain Quality certification from an agency of reputed.
3. Teaching Aids : LCD Projector, Smart Boards.
4. Promoting Faculty Development Programmes.
5. Conducting the need based training programs for Faculty & Students.
6. To improve the pass percentage 2-5% compared to previous year.

GOALS (Long Term):

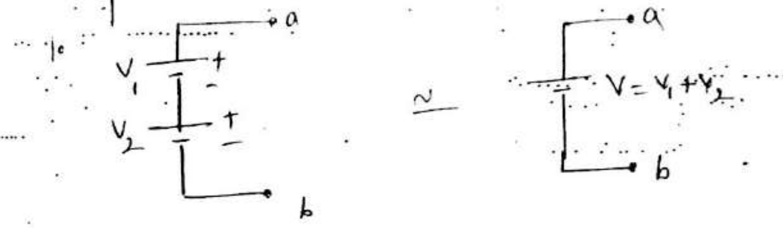
1. To start additional P.G. Programmes in Electonic and Communication engineering discipline.
2. To enter into understanding with globally renowned universities for special programmes in emerging technologies.
3. Promoting Industry - Institute interaction through projects and R & D work.

Topic:- Source transformation and Source Shifting:-

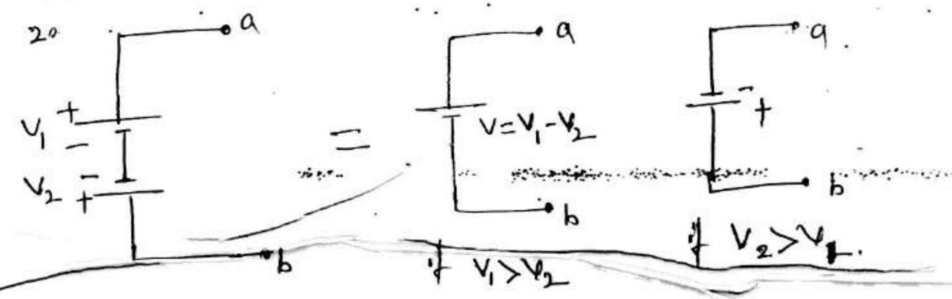
1

→ Equivalent voltage source

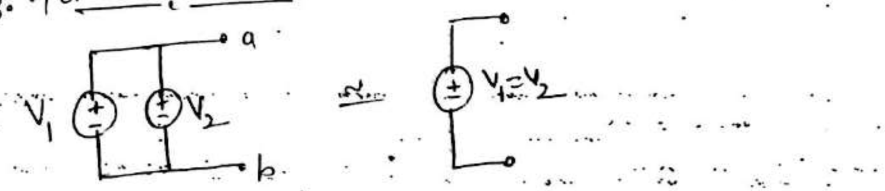
1.



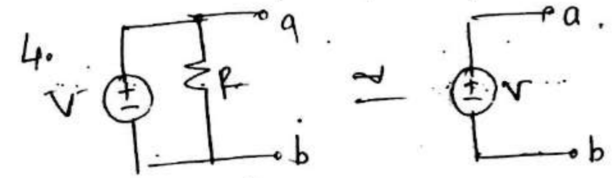
2.



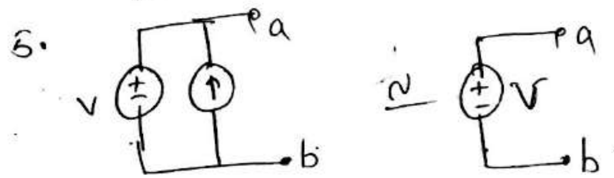
3. for ideal sources.



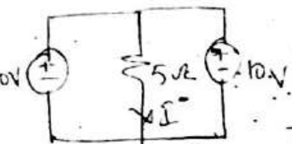
4.



5.



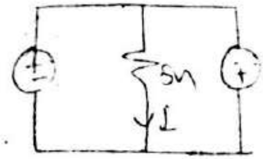
Q) Find the value of I for the ckt shown below.



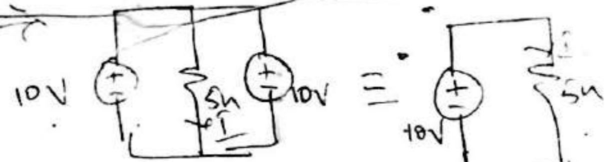
(a) 2A (b) 4A (c) 6A (d) None.

$\Longrightarrow$ Violation of KVL.

Sol:- W.k.t voltage across the all the parallel branches should be equal.

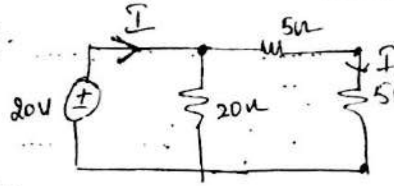


$\rightarrow$ Violation of KVL.



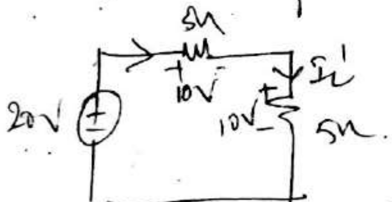
$I = 10/5 = 2A$

Example:



find. I_L and I and P_{deli} by source.

as for the ckt equivalent neglect $R = \underline{20\Omega}$

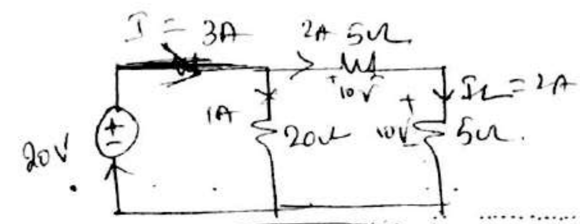


$V_L = 2 \times 5 = 10$ Volts

$I = \frac{20}{10} = 2A$

$I_L = 2A$

$+ P_{deli} = I \times V = 2 \times 20 = 40$ Watts



$V_L = 10$ Volts

$\boxed{I_L = \frac{20}{10} = 2A}$

$\boxed{I = 3A}$

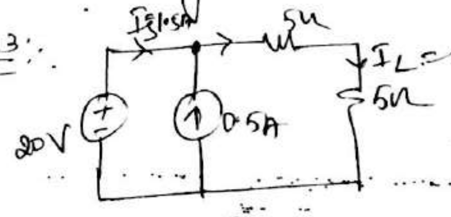
and $P_{deliver} = VI = 20 \times 3 = \underline{60 \text{ watts}}$

Note:-

(i) In the above ckt 20Ω resistance can be neglected while calculating either Load Current or Load voltage.

(ii) In the above circuit 20Ω resistance cannot be neglected while calculating Source Current or power.

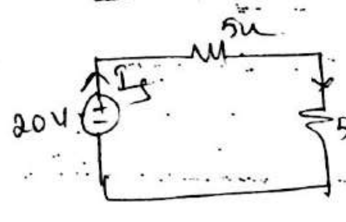
Sample 3:



$I_L = 2A$

$V_L = 10$ volts.

$\boxed{I_S = 1.5A}$ $P_{del} = 1.5 \times 20 =$



$I_L = 2A$

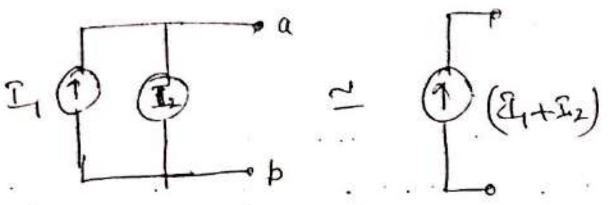
$V_L = 10$ volts

$I_S = \underline{2A}$. $P_{del} = 20 \times 2 = \underline{40W}$.

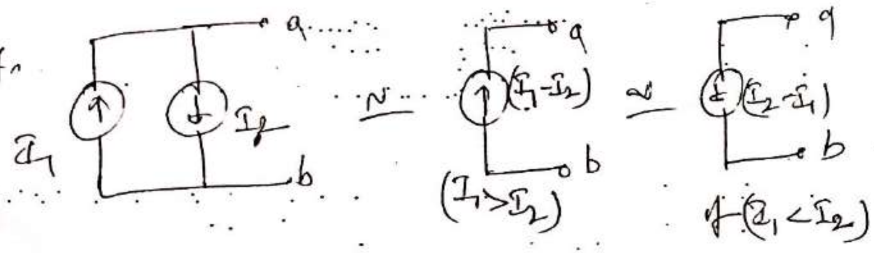
Note: 1. In the above ckt Current Source can be neglected while Calculating either Load Current or Load voltage.

2. In the above ckt Current Source cannot be neglected while Calculating either voltage source current or power.

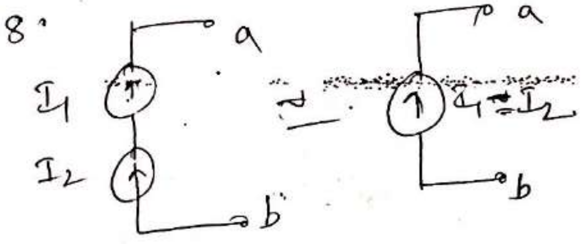
6.



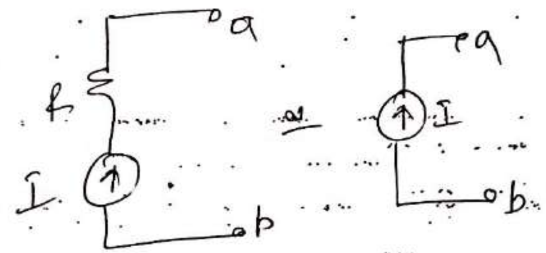
7.



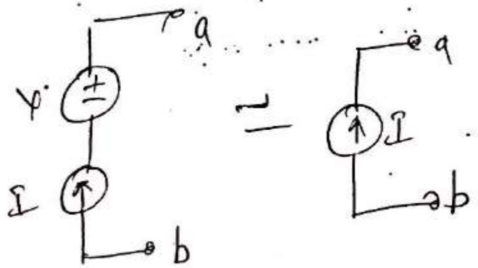
8.



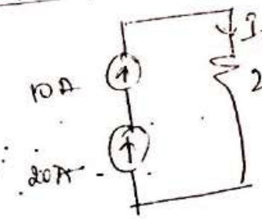
9.



10.



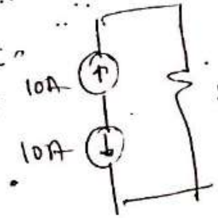
Example 1.



(a) 20A (b) 10A (c) 30A (d) none.

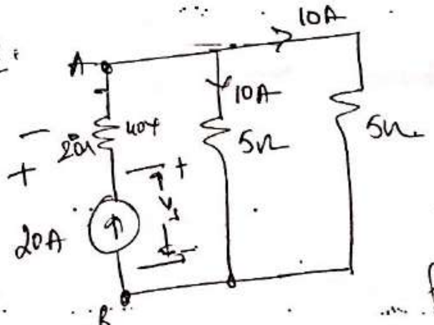
KCL Violation.

Ex2.



→ KCL violation.

Ex3.



$V_{ab} = V_s - 40$

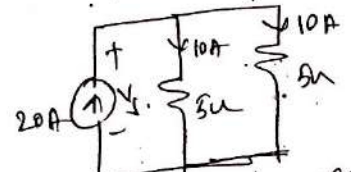
$V_{ab} = 10\times5 = 50V.$

$I_L = 10A$, $V_L = 50$ volts

$\therefore V_s = V_{ab}+40 = 50+40 = 90$ Volt

$P_s = I\times V_s = 20\times90 = 1800$ watts

if neglect "2Ω"



$V_s' = 50$ volts

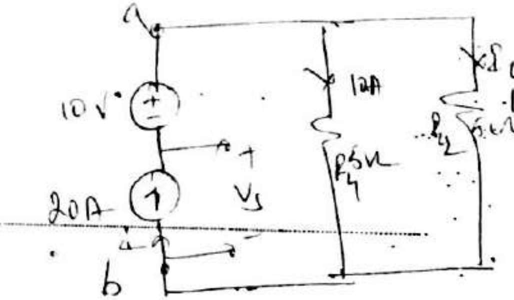
$P_s = 50\times20 = 1000$ watts.

$I_L = 10A$ + $V_L = 50$ volt

Notes: ①. In the above circuit 2Ω resistance can be neglected while calculating either Load current or Load voltage.

②. In the above circuit 2Ω resistance cannot be neglected while calculating either Source voltage or Source power.

Ex:



$V_{ab} = V_s + 10$

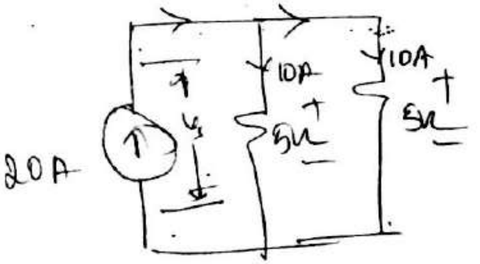
$V_{ab} = 5 \times 10 = 50$ volts

$V_s = V_{ab} - 10$
$= 50 - 10 = 40$ volts

$\boxed{V_s = 40 \text{ volts}}$

neglect 10V source.

$P_s = V_s I = 40 \times 20 = 800$ watts



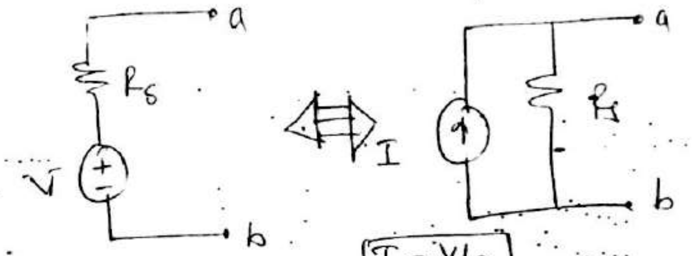
$V_s' = 5 \times 10 = 50$ volts

$P_s' = 50 \times 20 = 1000$ watts

Note 1. In the above Circuit voltage Source can be neglected while calculating either Load Current or Load voltage.

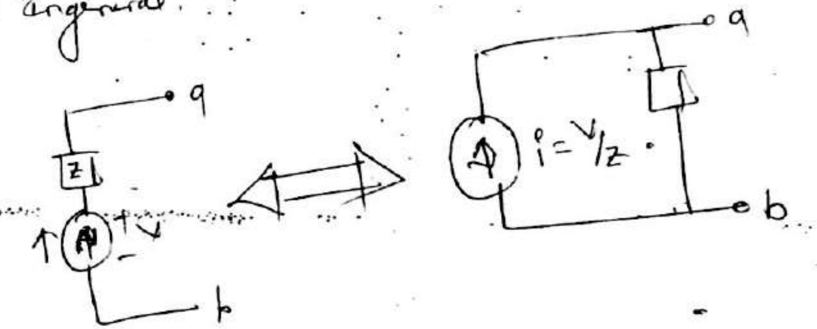
Note 2. In the above ckt voltage Source cannot be neglected while calculating the current Source voltage & Source power.

Source Transformation :-



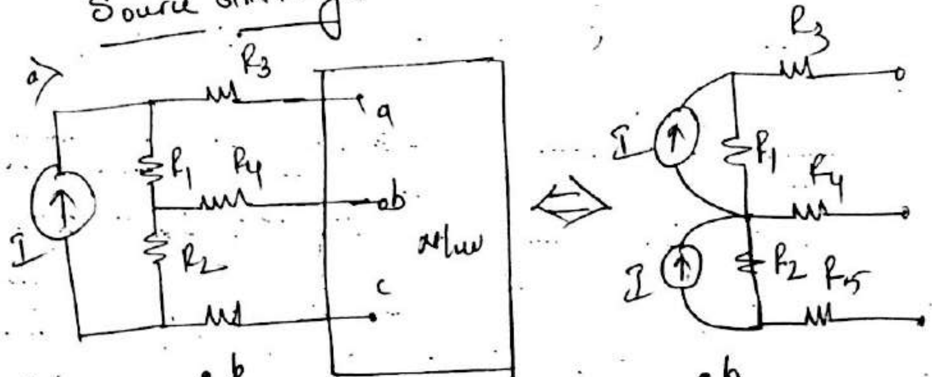
$\boxed{I = V/R_s}$

In general.

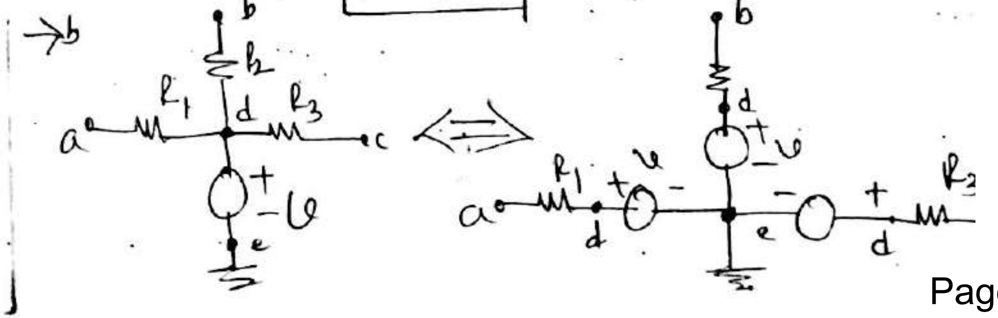


Source shifting :-

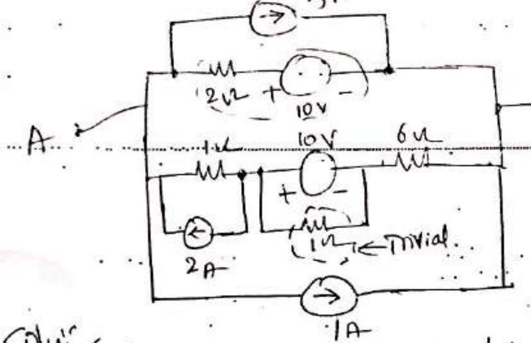
a)



b)

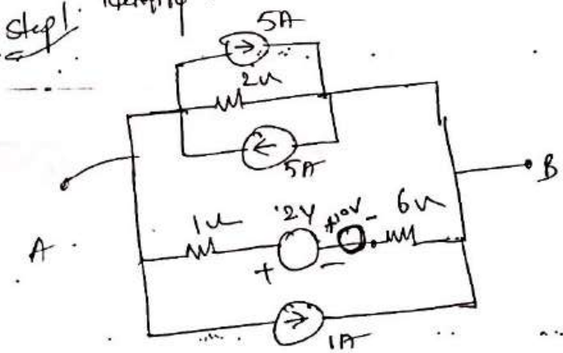


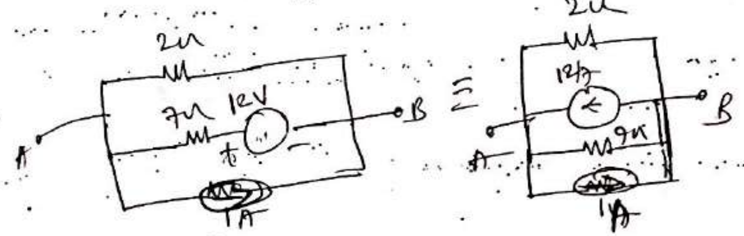
7) Reduce the fig shown into a practical Voltage Source across the terminal AB.

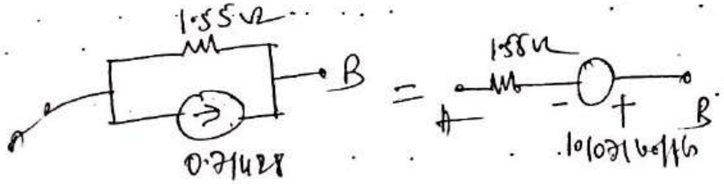


Soln:

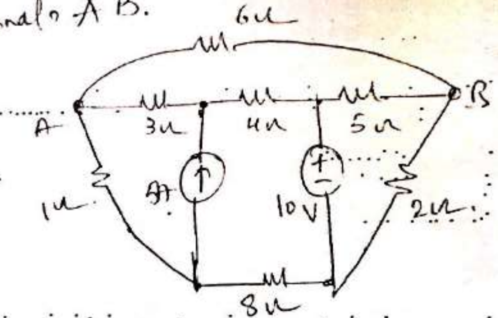
Step1 identify the Trivial element & remove if any.







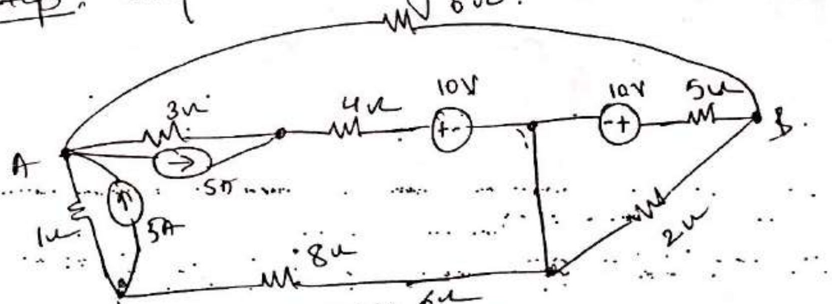
8) Reduce a N/w in to an equivalent voltage source across terminals A B.

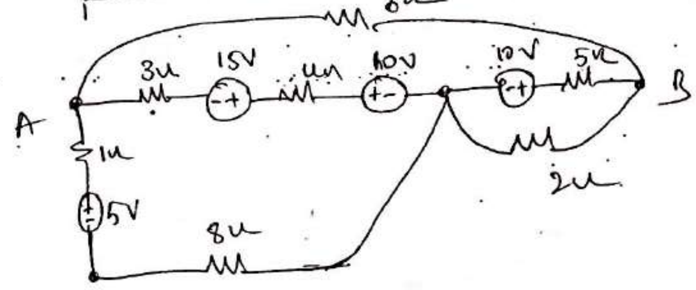


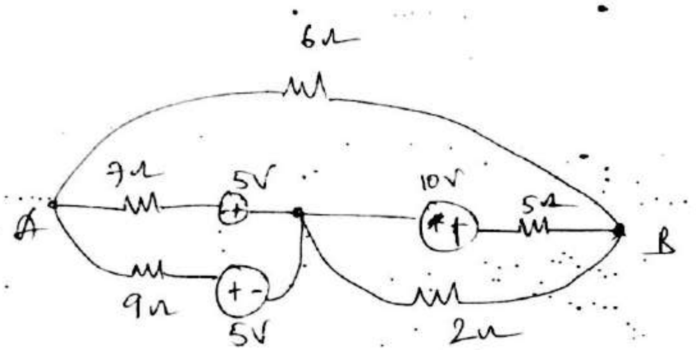
Soln: Step1:- no trivial elements present.

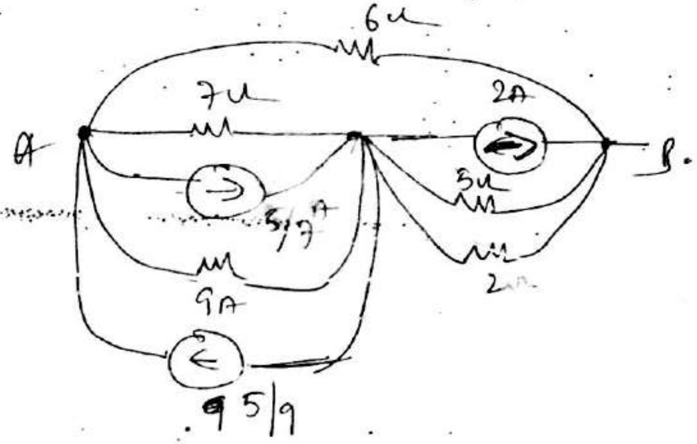
Step2:- Since all are ideal sources: source transformation cannot be applied.

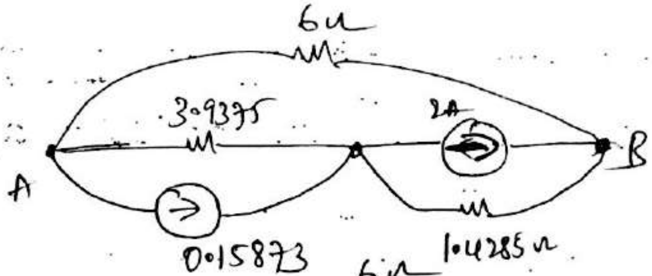
Step3. only source shifting is to be done.

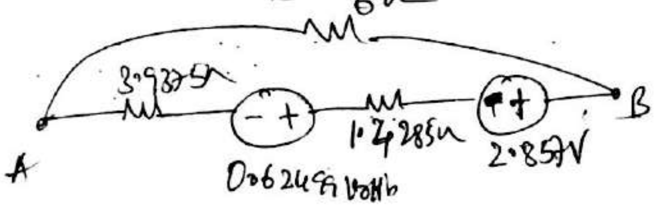


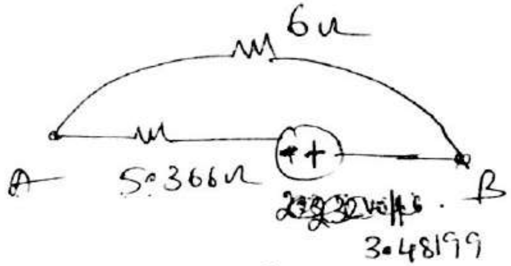


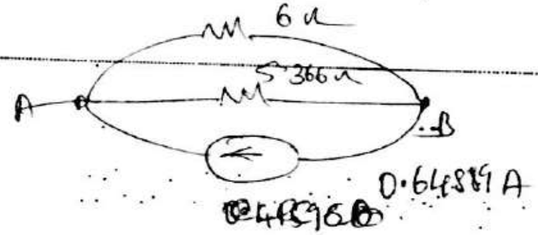

6Ω
7Ω
5V
10V
5Ω
A
B
9Ω
5V
2Ω

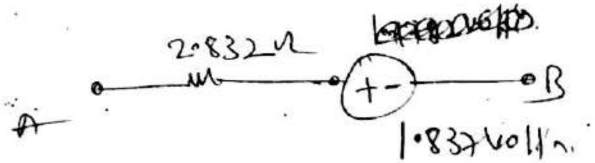

6Ω
7Ω
2A
A
B
3Ω
5/7 A
9A
2Ω
5/9


6Ω
3.9375
2A
A
B
0.15873
1.4285Ω


6Ω
3.9375Ω
A
B
0.62499 volts
1.4285Ω
2.857V

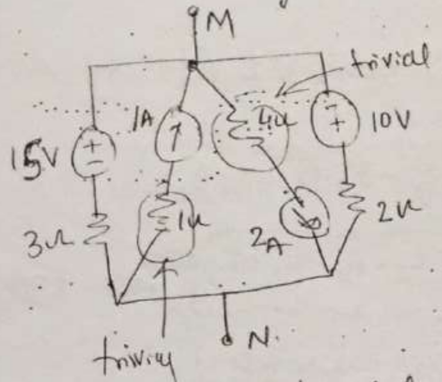

6Ω
A
B
5.366Ω
3.48199

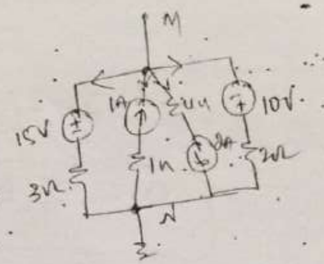

6Ω
5.366Ω
A
B
0.64889 A


2.832Ω
A
B
1.837 volts

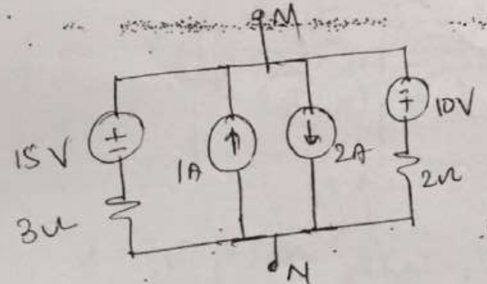
Q) For the ckt shown in fig. Find the potential difference b/w M and N using Source transformation. (4M)

DEC/Jan 2015.





sol^n step 1. identify the trivial elements & remove it. $\Sigma I_{leaving} = 0$

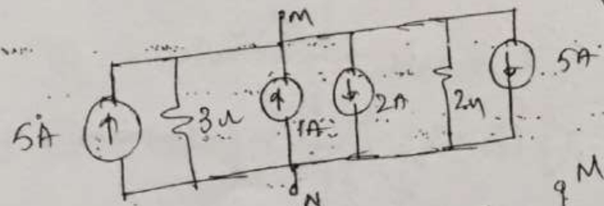


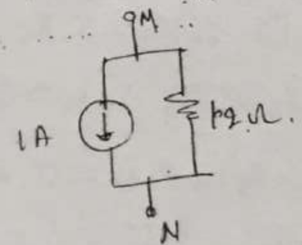
KCL @ m.

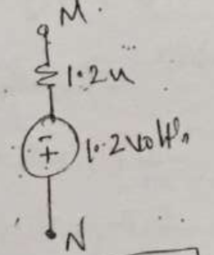
$$\frac{V_m - 15 - 0}{3} - 1 + 2 + \frac{V_m + 10 - 0}{2} = 0$$

$$\Rightarrow 3^{-1}(V_m - 15) + 2^{-1}(V_m + 10) + 1 = 0$$

$$\Rightarrow \boxed{V_m = -1.2 \text{ volts}}$$







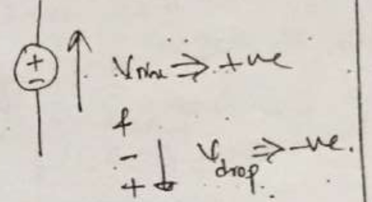
$\boxed{V_{MN} = -1.2 \text{ volts}}$

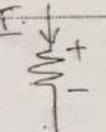
Tools

→ 1. Loop Analysis (or) Mesh Analysis ⇒ [KVL + Ohm's]

→ 2. Node voltage method [KCL + Ohm's Law].

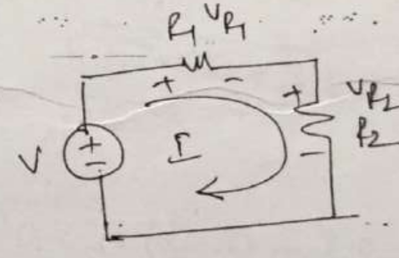
Fundamentals:





$V_{rise} = +IR$ (− to +)

$V_{drop} = -IR$ (+ to −)



→ ckt has only one loop

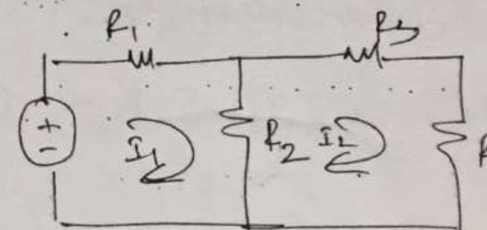
∴ Loop Current I is same as branch Current.

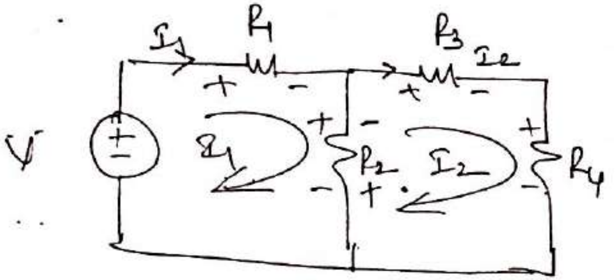
i.e. $I = I_{R_1} = I_{R_2}$

KVL

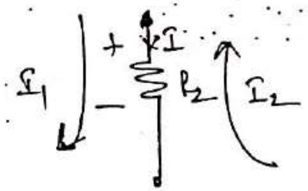
$$V - V_{R_1} - V_{R_2} = 0$$

$$V - IR_1 - IR_2 = 0 \Rightarrow \boxed{V = IR_1 + IR_2} \text{ volts.}$$





- Loop Currents are I_1 and I_2
- 'R_2' in Common for both the Loops



- first Loop

$$\boxed{I = (I_1 - I_2)}$$

- Second Loop

$$\boxed{I = (I_2 - I_1)}$$

KVL 1st Loop

$$V - I_1R_1 - (I_1 - I_2)R_2 = 0$$

$$V = I_1R_1 + (I_1 - I_2)R_2 = 0$$

$$V = I_1R_1 + I_1R_2 - I_2R_2$$

$$V = (R_1 + R_2)I_1 - I_2R_2 \leftarrow (1)$$

Second Loop :-

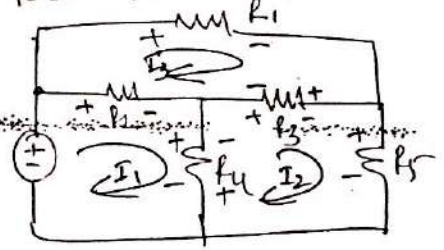
$$-(I_2 - I_1)R_2 - I_2R_3 - I_2R_4 = 0$$

$$-I_2R_2 + I_1R_2 - I_2R_3 - I_2R_4 = 0$$

$$\boxed{0 = I_1R_2 - [R_2 + R_3 + R_4]I_2} \leftarrow (2)$$

two eq two unknowns.

N/W 2



KVL Loop1

$$V - (I_1 - I_3)R_2 - R_4(I_1 - I_2) = 0$$

$$V = I_1R_2 - I_3R_2 + R_4I_1 - I_2R_4$$

$$V = I_1(R_2 + R_4) - R_4I_2 - R_2I_3 \leftarrow (1)$$

Loop2

$$-(I_2 - I_1)R_4 - [I_2 - I_3]R_3 - I_2R_5 = 0$$

$$-I_2R_4 + I_1R_4 - I_2R_3 + I_3R_3 - I_2R_5 = 0$$

$$I_1R_4 - I_2[R_3 + R_4 + R_5) + R_3I_3 = 0 \leftarrow (2)$$

Loop3

$$-I_3R_1 - R_3(I_3 - I_2) - R_2(I_3 - I_1) = 0$$

$$-I_3R_1 - R_3I_3 + I_2R_3 - R_2I_3 + R_2I_1 = 0$$

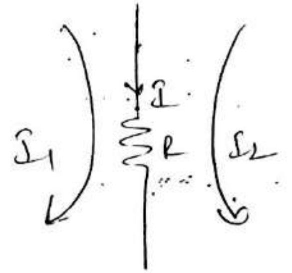
$$I_1 R_2 + I_2 R_3 - (R_1+R_2+R_3) I_3 = 0 .$$
$\leftarrow$ (3)

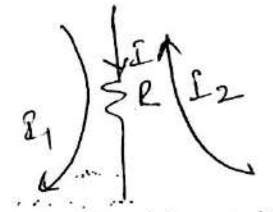
three eq^ three unknown's solve?

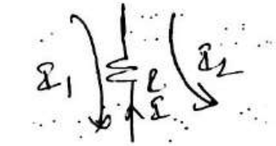
for I_1, I_2, I_3.

(Q)

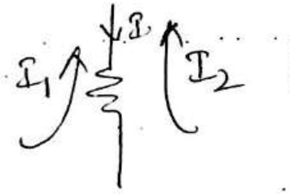
Note!

$\rightarrow$  $\boxed{I = I_1 + I_2}$

$\rightarrow$  $\boxed{I = I_1 - I_2}$

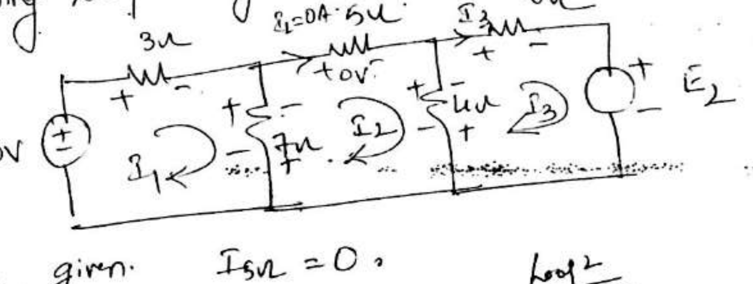
$\rightarrow$  $\boxed{I = -I_1 - I_2}$

$= -(I_1 + I_2)$

$\rightarrow$  $\boxed{I = -I_1 - I_2}$

$\rightarrow$  $\boxed{I = I_2 - I_1}$

Q) In the network shown estimate the value of E_2 such that the current through 5Ω resistor in zero. using Loop analysis (6m)



Sol^n given. $I_{5\Omega} = 0$.

KVL 1st loop

$$10 - 3I_1 - 7(I_1 - I_2) = 0$$
$$10 = 3I_1 + 7I_1 - 7I_2$$
$$10 I_1 - 7I_2 = 10$$
given $I_2 = I_{5\Omega} = 0A$
$$10 I_1 = 10 \Rightarrow \boxed{I_1 = 1A}$$

Loop 2

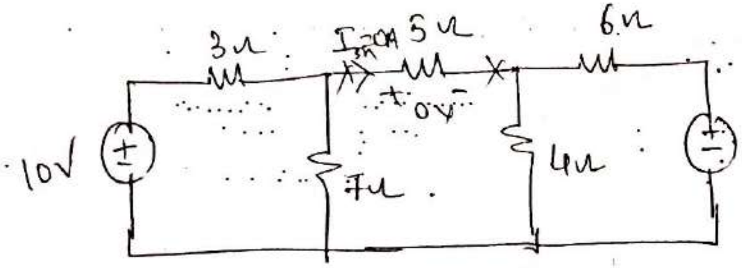
$$-7(0 - I_1) - 0 - 4(\cancel{I_2}^{0} - I_3) = 0$$
$I_1 = 1A$
$$-7(-1) - 4(-I_3) = 0$$
$$+7 = -4I_3$$
$$I_3 = 7/4 = 1.75A$$
$$= -1.75A$$

Loop 3

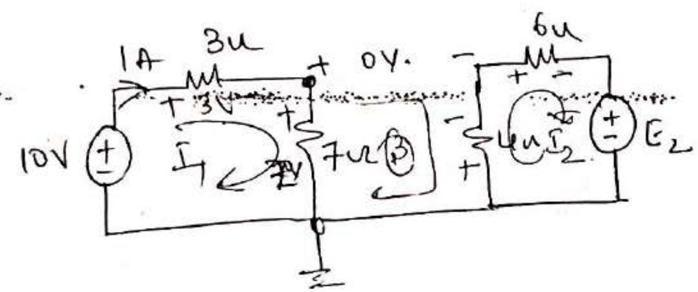
$$-4(I_3 - \cancel{I_2}^{0}) - I_3 6 - E_2 = 0$$
$$-10I_3 = E_2 \Rightarrow \boxed{E_2 = -10I_3}$$
$$\therefore E_2 = -10(-1.75) = +17.5 \text{ volts}$$

(or)

2nd method using VDR



given $\boxed{I_{5\Omega} = 0\,A}$



$I_1 = \frac{10}{3+7} = 1A$

KVL (loop 1) $(7) - 0 + 4(I_2) = 0$

$I_2 = -7/4 = -1.75\,A$

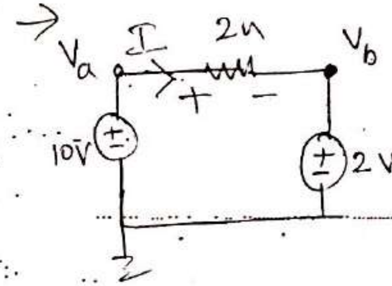
KVL loop 2

$E_2 + 6(-1.75) + 4(1.75) = 0$

$E_2 = 6 \times 1.75 + 4 \times 1.75 = 17.5$ Volts

Note:

$I = \left(\frac{V_a - V_b}{2}\right)$

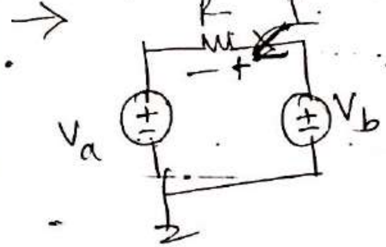


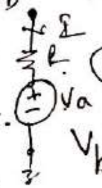
KVL

$V_a - I(R) - V_b = 0$

$I(R) = V_a - V_b$

$\Rightarrow \boxed{I = \frac{V_a - V_b}{R}}$ Ampere

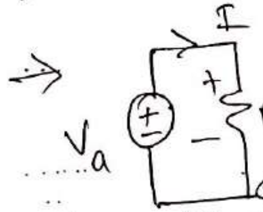




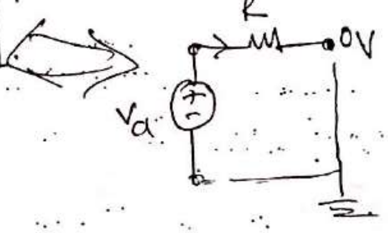
$I = \frac{V_b - V_a}{R}$ A

$V_b - IR - V_a = 0$

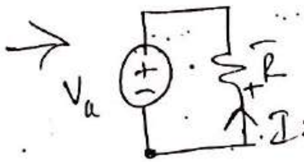
$\Rightarrow \boxed{I = \frac{V_b - V_a}{R}}$ Ampere



$V_a = IR \Rightarrow I = \frac{V_a}{R}$



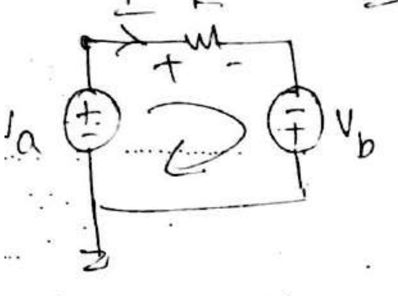
$I = \frac{V_a - 0}{R} = \frac{V_a}{R}$



(i) $V_a + IR = 0 \Rightarrow \boxed{I = \frac{-V_a}{R}}$

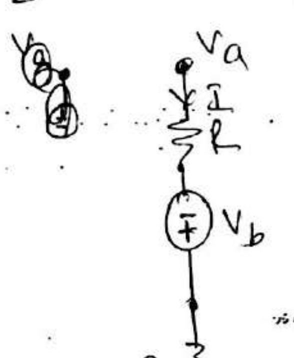
(ii) $I = \frac{0 - V_a}{R} = \frac{-V_a}{R}$ Ampere

KVL

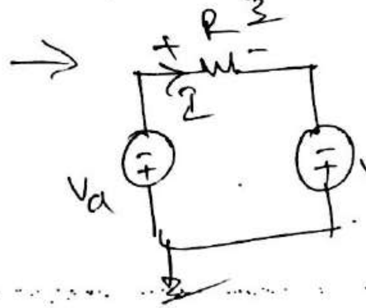


$V_a - IR + V_b = 0$

$\boxed{I = \frac{V_a + V_b}{R}}$ Ampere



$\boxed{I = \frac{V_a + V_b}{R}}$

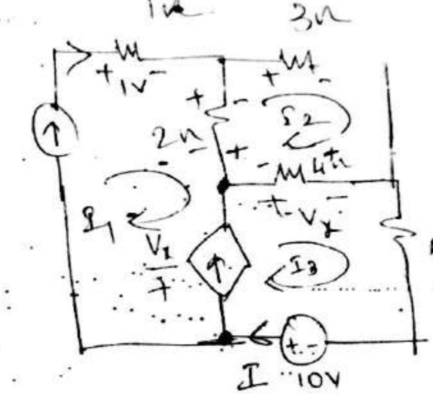


$-V_a - IR + V_b = 0$

$\boxed{I_1 = \frac{V_b - V_a}{R}}$ Ampere

$I = \frac{(-V_a) - (-V_b)}{R} = \frac{-V_a + V_b}{R} = \left(\frac{V_b - V_a}{R}\right)$ Ampere

(8)



find power supplied by 10 V Source using mesh Current analysis.

$I = ?$

$P_{del} = I \times V = 10 \times I$

$\boxed{P_{deliv} = 10I}$ W

$(I_3 - I_1) = \frac{V_x}{7} \leftarrow (1)$

$I_1 = 1A \leftarrow$

$I = \frac{V_x}{7} + 1 \leftarrow (2)$

KVL loop 2

$-2(I_2 - I_1) - 3I_2 - 4(I_2 - I_3) = 0$

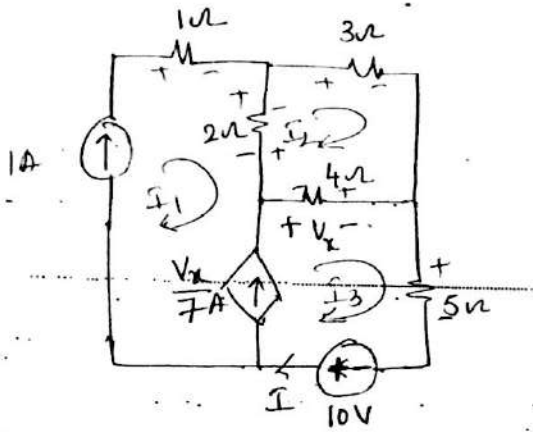
$-2I_2 + 2I_1 - 3I_2 - 4I_2 + 4I_3 = 0$

$2I_1 - 9I_2 + 4I_3 = 0 \leftarrow (3)$

$I_1 = 1A$

$V_x = (I_3 - I_2)4 \leftarrow (4)$

$I_3 - 1 = \frac{4}{7}[I_3 - I_2] \leftarrow$



$P = VI$

$\boxed{P_{10V} = 10 \times I}$ watts

Solu:

Loop 1: $I_1 = 1A \leftarrow (1)$

Loop 2: $-2(I_2 - I_1) - 3I_2 - 4(I_2 - I_3) = 0$

$-2I_2 + 2I_1 - 3I_2 - 4I_2 + 4I_3 = 0$

$2I_1 - 9I_2 + 4I_3 = 0 \leftarrow (2)$

$-9I_2 + 4I_3 = -2I_1$

$I_1 = 1$

$\boxed{-9I_2 + 4I_3 = -2} \leftarrow (a)$

Loop 3 Supermesh

$I_3 - I_1 = \frac{V_x}{7}$

but $V_x = (I_3 - I_2)4$

$I_3 - I_1 = \frac{4}{7}(I_3 - I_2)$

$I_3 - I_1 - \frac{4}{7}I_3 + \frac{4}{7}I_2 = 0$

$\frac{4}{7}I_2 + (1 - \frac{4}{7})I_3 = I_1$

$\boxed{\frac{4}{7}I_2 + \frac{3}{7}I_3 = 1} \leftarrow (b)$

Solving eqn (a) and eqn (b)

$\boxed{I_2 = 0.7906 A}$ $\boxed{I_3 = 1.279 \text{ Amperes}}$

from Fig: $\boxed{I_3 = I}$ A

$\Rightarrow \boxed{I = 1.279}$ Amperes

$P_{\text{deliverd}(10V)} = 10 \times I = V \times I$

$= 10 \times 1.279 = 12.79$ Watts

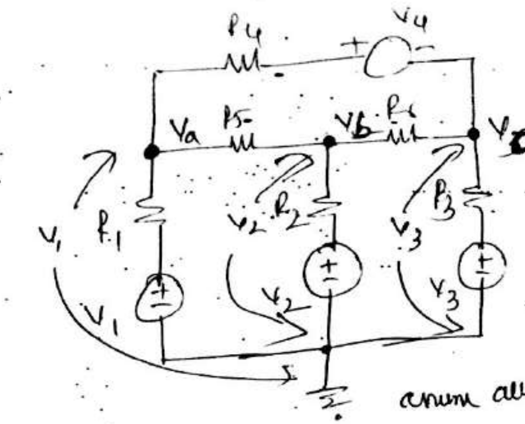
Node voltage method:

> 2

Simple node

principle Node

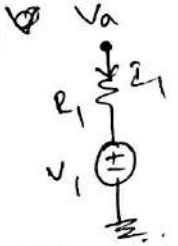
Node voltage method :-



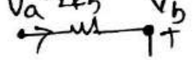
obs :- 3 principal nodes + 1 GND.

assume all the currents are leaving

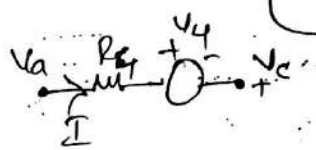
KVL @ node a



$I_1 = \left(\frac{V_a - V_1 - 0}{R_1}\right)$



$\boxed{I_2 = \frac{V_a - V_b}{R_5}}$



$\boxed{I = \frac{V_a - V_4 - V_c}{R_4} \text{ A}}$

@ node a $\left(\begin{matrix} V_a > V_b \\ V_a > V_c \end{matrix}\right)$

$$\frac{V_a - V_1}{R_1} + \frac{V_a - V_b}{R_5} + \frac{V_a - V_4 - V_c}{R_4} = 0 \quad \leftarrow (1)$$

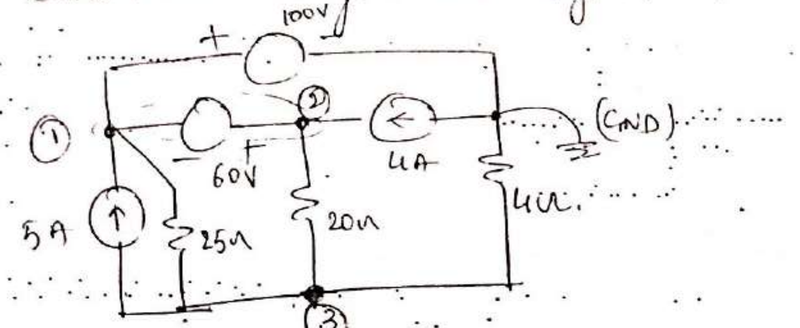
@ node b $\left(\begin{matrix} V_b > V_a \\ V_b > V_c \end{matrix}\right)$

$$\frac{V_b - V_2}{R_2} + \frac{V_b - V_a}{R_5} + \frac{V_b - V_c}{R_6} = 0 \quad \leftarrow (2)$$

@ node c $\left(\begin{matrix} V_c > V_a \\ V_c > V_b \end{matrix}\right)$

$$\frac{V_c + V_4 - V_a}{R_4} + \frac{V_c - V_b}{R_6} + \frac{V_c - V_3}{R_3} = 0 \quad \leftarrow (3)$$

(8) Find the power ~~delivered~~ absorbed by the 4Ω resistor in the Circuit shown using the Nodal analysis.



solu:- w.r.t gnd $V_1 = 100\,V \leftarrow ①$

$V_2 - V_1 = 60\,V \leftarrow ②$

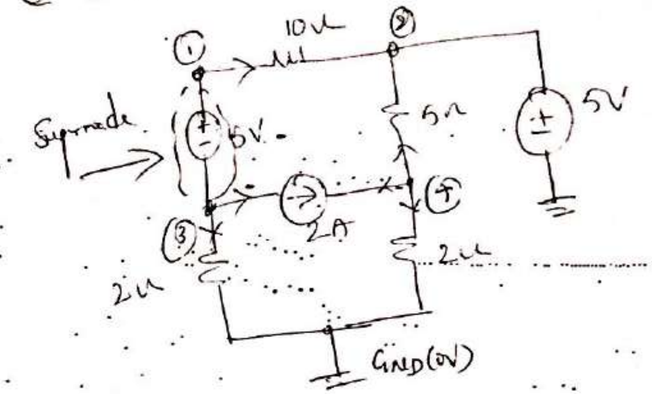
$V_2 = 100 + 60 = 160$ volts.

node 3 $\quad 5 + \frac{V_3 - V_2}{20} + \frac{V_3 - V_1}{25} + \frac{V_3}{4} = 0$.

$\Rightarrow \boxed{V_3 = 20.58}$ volts

$P_{4\Omega} = \frac{V_3^2}{R} = \frac{(20.58)^2}{4} = 105.96$ watts

(8) Determine all the node voltages.



solu:- $V_2 = 5V \leftarrow ①$

$V_1 - V_3 = 5V \leftarrow ②$

@ node (4) $\quad \frac{V_4}{2} + \frac{V_4 - V_2}{5} - 2 = 0$

$\Rightarrow \frac{V_4 - V_2}{5} + \frac{V_4}{2} = 2$

$20 = 2V_4 - 2V_2 + 5V_4$

$\Rightarrow \boxed{V_4 = 4.285}$ volts

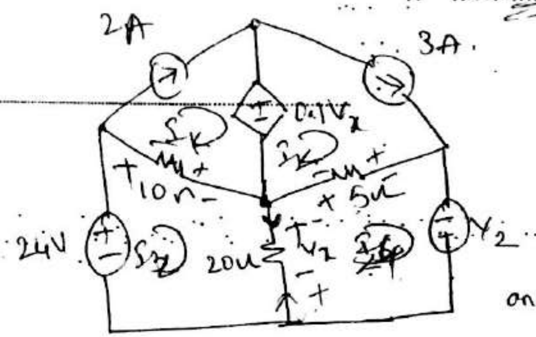
@ Supernode $\quad \frac{V_1 - V_2}{10} + \frac{V_3}{2} + 2 = 0$

$0.1V_1 - 0.1V_2 + 0.5V_3 + 2 = 0$

$V_2 = 5$ volts $\quad$ & $\boxed{V_2 = 5\text{ volts}}$

$\boxed{V_1 = 1.66\text{ volts}}$ $\quad \boxed{V_3 = -3.33\text{ volts}}$

(4) Using mesh analysis determine the value of V_2 which cause the voltage across the 20Ω resistor to be Zero.



soln:

given $V_{20\Omega} = 0$

$\Rightarrow V_{20\Omega} = V_x = 0$ volts

$I_x = I_3 - I_4$

and $V_x = I_x \times 20$

given $V_x = 0$.

$0 = [I_3 - I_4].20$

$\Rightarrow \boxed{I_3 = I_4}$ Ampere.

KVL loops

from fig Loop 1 $I_1 = 2A \leftarrow (1)$

Loop 2 $I_2 = 3A \leftarrow (2)$

Loop 3 $24 - 10(I_3 - I_1) - \cancel{V_x} 20$ (0)

$24 = 10(I_3 - I_1)$

$I_1 = 2A$

$\therefore 24 = 10 I_3 - 10(2)$

$24 + 20 = 10 I_3$

$\Rightarrow 10 I_3 = 44 \Rightarrow \boxed{I_3 = 4.4}$ $= I_4$ Ampere

KVL loop 4

$I_3 = I_4$

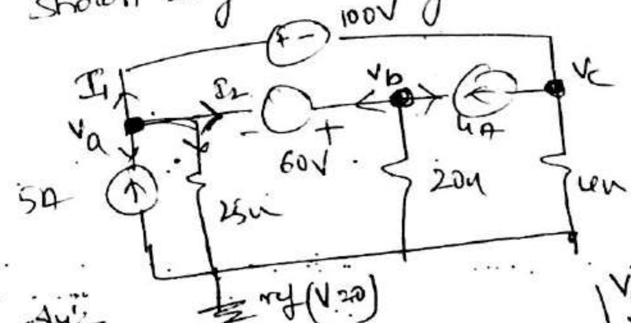
$-20(\cancel{I_4 - I_3}) - 5[I_4 - I_2] + V_2 = 0$ (0)

$V_2 = 5(I_4 - I_2)$ $\quad I_4 = I_3$

$= 5(4.4 - 3)$

$V_2 = 5 \times 1.4 = \underline{7 \text{ volts}}$

(5) Find the power ~~delivered~~ absorbed by the 4Ω resistor in the circuit shown using nodal Analysis



$P_{4\Omega} = \frac{V_c^2}{4}$ watts (absorbed)

soln:

$V_a - V_c = 100 \leftarrow (a)$

$V_b - V_a = 60 \leftarrow (b)$

$V_b - V_c = 160$ volts $\leftarrow (c)$

@a $\quad -5 + \frac{V_a}{25} + I_1 + I_2 = 0$

$\frac{V_a}{25} + I_1 + I_2 = 5 \leftarrow (1)$

@b $\quad \frac{V_b}{20} - I_2 - 4 = 0 \Rightarrow \frac{V_b}{20} - I_2 = 4 \leftarrow (2)$

@c $\quad \frac{V_c}{4} - I_1 + 4 = 0 \Rightarrow \frac{V_c}{4} - I_1 = -4 \leftarrow (3)$

$0.04\,V_a + I_1 + I_2 = 5$ | $V_a - V_c = 100$

$0.05\,V_b - I_2 = 4$ | $V_b - V_a = 60$

$0.25\,V_c - I_1 = -4$ | $V_b - V_c = 160$

$\Rightarrow 0.04\,V_a + I_1 + I_2 = 5 \leftarrow (1)$

$V_b = 60 + V_a$

$0.05(60 + V_a) - I_2 = 4$

$\Rightarrow 0.05\,V_a - I_2 = 1 \leftarrow (2)$

$V_c = V_a - 100$

$0.25\,[V_a - 100] - I_1 = -4$

$0.25\,V_a - 25 - I_1 = -4$

$\Rightarrow 0.25\,V_a - I_1 = (-4 + 25) = 21 \leftarrow (3)$

$V_a = 79.41176$ volts, $I_1 = -1.14$ Amperes

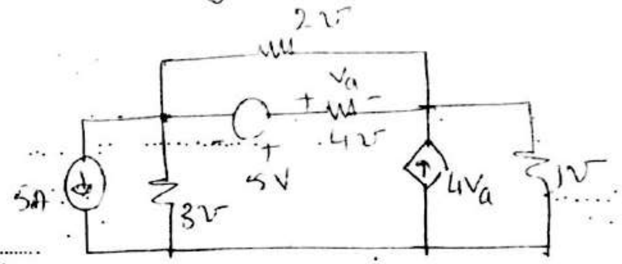
$I_2 = 2.97005$ Amperes

$\Rightarrow V_c = V_a - 100$

$V_c = 79.41176 - 100 = -20.589$ volts

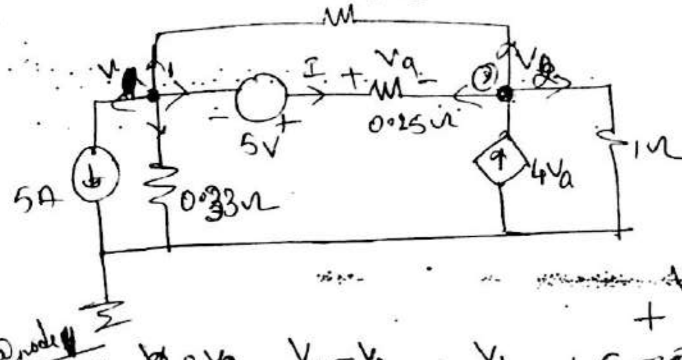
$P_{4\Omega(abs)} = \frac{V_c^2}{4} = \frac{(-20.589)^2}{4} = 105.97$ Watts

(8) Find V_a by node analysis :



16.

soln



kcl @ node 1 $\frac{V_1 - V_2}{0.5} + \frac{V_1}{0.333} + 5 + \frac{V_1 + 5 - V_2}{0.25} = 0$

@ node 2 $\frac{V_2 - V_1}{0.5} + \frac{V_2 - 5}{0.25} + \frac{V_2}{1} - 4V_a = 0$

$V_a = I\,(0.25)$

$V_a = \left(\frac{V_1 + 5 - V_2}{0.25}\right) \times 0.25$

$\boxed{V_a = V_1 - V_2 + 5}$ volts

$2V_1 - 2V_2 + 3V_1 + 5 + 4V_1 + 20 - 4V_2 = 0$

$\boxed{9V_1 - 6V_2 = -25} \leftarrow (a)$

$$2V_2 - 2V_1 + 4V_2 - 20 + V_2 - 4(V_1 - V_2 + 5) = 0$$

$$2V_2 - 2V_1 + 4V_2 - 20 + V_2 - 4V_1 + 4V_2 - 20 = 0$$

$$-6V_1 + 11V_2 - 40 = 0 \leftarrow (1)$$

$$9V_1 - 6V_2 = -25$$
$$-6V_1 + 11V_2 = 40$$

$$\boxed{V_1 = 5.8} \text{ volts} \quad \cancel{V_2 = 6.8 \text{ volts}}$$

$$9V_1 - 6V_2 = -25$$

$$\cancel{V_a = V_1 - V_2 + 5}$$
$$\cancel{= 5.8 - 6.8 + 5}$$
$$\cancel{\boxed{V_a = 4 \text{ volts}}}$$

$$3V_1 + 2(V_1 - V_2) + (V_1 + 5 - V_2)4 + 5 = 0$$
$$3V_1 + 2V_1 - 2V_2 + 4V_1 + 20 - 4V_2 + 5 = 0$$
$$9V_1 - 6V_2 = -25$$

$$2(V_2 - V_1) - 4V_a + V_2 + [V_2 - 5 - V_1]4 = 0$$

$$2V_2 - 2V_1 - 4V_a + V_2 + 4V_2 - 20 - 4V_1 = 0$$

$$\cancel{V_a = 0.25(I) = 0.25[V_1 + 5 - V_2]4}$$

$$\boxed{V_a = V_1 - V_2 + 5}$$

$$\cancel{2V_2 - 2V_1 - 4V_a}$$

$$2V_2 - 2V_1 - 4[V_1 - V_2 + 5] + V_2 + 4V_2 - 20 - 4V_1$$

$$2V_2 - 2V_1 - 4V_1 + 4V_2 - 20 + V_2 + 4V_2 - 20 - 4V_1 = 0$$

$$\boxed{-10V_1 + 11V_2 = 40} \leftarrow (2)$$

$$V_1 = -0.892 \text{ volts}$$
$$V_2 = 2.8205 \text{ volts}$$

$$V_a = V_1 - V_2 + 5$$
$$= -0.897 - 2.805 + 5$$
$$\boxed{V_a = 1.2821} \text{ volts}$$

MISSION

Provide quality and contemporary education, in the domain of Electronics and communication and related fields, which enable collaborative ventures with industries and research organizations. Emphasis laid on creating innovative teaching-learning processes that motivate self-learning. by imparting quality education embedded with discipline & national honor.

VISION

To create a rich intellectual potential implanted with multidisciplinary knowledge, human values and professional ethics among the aspirant of becoming Engineers and technologies, so as to unlock their imagination and discover their potential.

OBJECTIVES

1. To impart good technical knowledge to the students.
2. To produce Excellent Engineers in Electronics & Communication fields.
3. To fulfil the needs of the society in the various fields related to Electronics and Communication engineering.
4. To bring post-graduate program in the diverse field of electronics and communication Engineering.
5. To upgrade the facilities in Research & Development Centre of the department with the use of modern aids.
6. To organize training programs / workshops for upgrading staff performance.
7. To establish Industry-Institure Interaction.
8. To publish technical papers in National / International journals and conferences.

GOALS (Short Term) :

1. Modernizing the Laboratories with new software & state-of-the art hardware in tune with the latest technological developments.
2. To obtain Quality certification from an agency of reputed.
3. Teaching Aids : LCD Projector, Smart Boards,
4. Promoting Faculty Development Programmes.
5. Conducting the need based training programs for Faculty & Students.
6. To improve the pass percentage 2-5% compared to previous year.

GOALS (Long Term) :

1. To start additional P.G. Programmes in Electonic and Communication engineering discipline.
2. To enter into understanding with globally renowned universities for special programmes in emerging technologies.
3. Promoting Industry - Institute interaction through projects and R & D work.

Unit - III :- Network theorems - I.

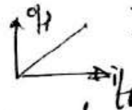
→ Superposition theorem.

→ Reciprocity theorem.

→ Millman's theorem.

To apply any theorem, the network as to full fill ~~the following~~ two properties.

1> Linearity :- An element in said to be Linear if the excitation to response characteristics is Linear.



$V = IR$

$\uparrow V \; \uparrow I/R$

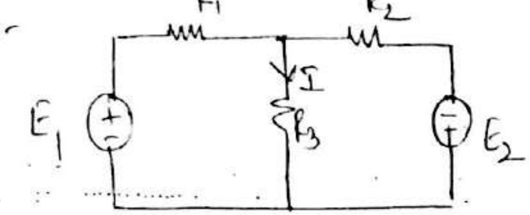
2> Bilaterals :- The response remains the same for the both the polarities of the input excitation.

I. Superposition theorem :- (SPT)

Statement :- " In Any Linear bilateral n/w having two (or) more Sources the total response in any part of the network will be equal to the algebraic Sum of responses due to each Source acting one at a time.

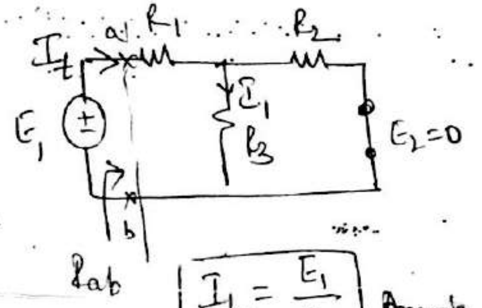
Note :- → all the ideal voltage Sources are eliminated from the n/w by shorting the Sources, all the ideal Current Sources are eliminated by opening the sources (OC) and donot disturb the dependent Source present in the n/w.

Example :-



I - total response

Step 1. with E_1 alone. $E_2 = 0V$ (short ckt)



$$R_{ab} = R_1 + R_2 \| R_3 = R_1 + \left(\frac{R_2R_3}{R_2+R_3}\right)$$

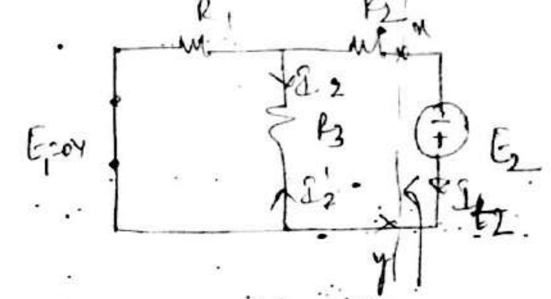
$$\boxed{I_{t_1} = \frac{E_1}{R_{ab}}} \text{ Amperes}$$

using branch current method

$$I_1 = I_{t_1} \cdot \frac{R_2}{(R_2+R_3)}$$

$$\boxed{I_1 = \left(\frac{R_2}{R_2+R_3}\right) \cdot \left(\frac{E_1}{R_1 + \left(\frac{R_2R_3}{R_2+R_3}\right)}\right)} \text{ Amperes}$$

Step 2: with E_2 alone ($E_1 = 0V$ @ short ckt).



$$\boxed{I_{t_2} = \frac{E_2}{R_{yx}}} \text{ A}$$

$$R_{yx} = R_2 + R_1 \| R_3 = R_2 + \left(\frac{R_1R_3}{R_1+R_3}\right) \Omega.$$

using branch current method

$$I_2' = I_{t_2}\left(\frac{R_1}{R_1+R_3}\right)$$

$$I_2' = \frac{E_2}{R_{yx}}\left(\frac{R_1}{R_1+R_2}\right)$$

$$I_2' = \frac{E_2}{\left(R_2 + \frac{R_1R_3}{R_1+R_3}\right)}\left(\frac{R_1}{R_1+R_2}\right) \text{ Amperes.}$$

$$I_2 = -I_2'$$

$\Rightarrow$ Total response $I = I_1 + I_2$

$$\Rightarrow \boxed{I = I_1 - I_2'} \text{ Amperes.}$$

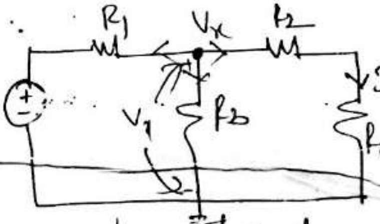
2 Reciprocity theorem:-

In any Linear bilateral Network Consisting of only one Source, the ratio of the Excitation to response remains Unchanged even after Interchanging their positions.

(a)

[In a single Source n/w the position's of the Source and responses can be interchanged]

Example:-



E - Excitation | Source | input

I - response

$$\boxed{\frac{E}{I} = \text{Constant}(k_1)}$$

kcl @ x $\quad \frac{V_x - E}{R_1} + \frac{V_x}{R_3} + \frac{V_x}{R_2+R_4} = 0$

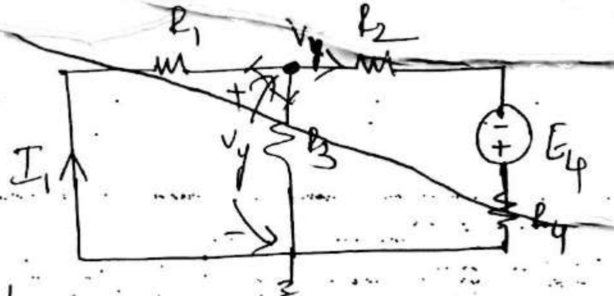
$$V_x\left[\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_2+R_4}\right] = E/R_1$$

$$V_x = \frac{E}{R_1} \times \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_2+R_4}\right)}$$

$$\boxed{I = \frac{V_x}{R_2+R_4}} \qquad \frac{E}{I} = \left[R_1\left(\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_2+R_4}\right)(R_2+R_4)\right] \leftarrow (a)$$

$$\therefore \frac{E}{I} = \frac{E}{\left(\frac{V_x}{R_2+R_4}\right)} = \frac{E}{V_x}(R_2+R_4) = k_1$$

Interchanging the positions of E and I



$$\frac{V_y}{R_1} + \frac{V_y}{R_3} + \left(\frac{V_y + E_4}{R_2+R_4}\right) = 0$$

$$V_y\left[\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_2+R_4}\right] = -\frac{E}{R_2+R_4}$$

$$V_y = \frac{-E}{(R_2+R_4)} \times \frac{1}{\left[\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_4+R_2}\right]}$$

$$I_1 = -\frac{V_y}{R_1} = -\left[\frac{-E}{R_1(R_2+R_4)\left[\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_2+R_4}\right]}\right]$$

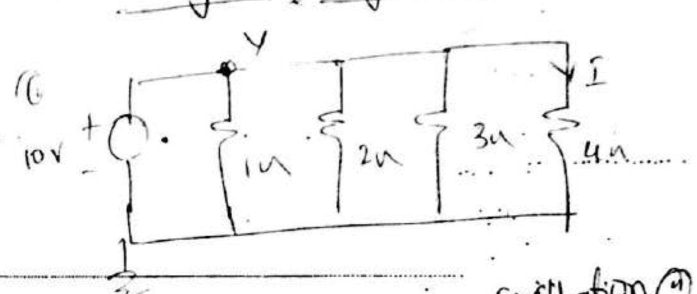
$$K_1 = \frac{E}{I_1} = (R_2+R_4)R_1\left(\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_2+R_4}\right) \leftarrow (b)$$

$$\Rightarrow K_1 = \frac{E}{I_1} = \frac{E}{I_1}$$

eqn (a) = eqn (b)

∴ Reciprocity theorem is verified.

8) Verify reciprocity theorem

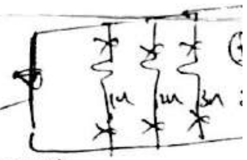


Sol: $V = 10\,V$ —— excitation (or) i/p

$I = \frac{V}{4}\,A$ —— response (or) o/p

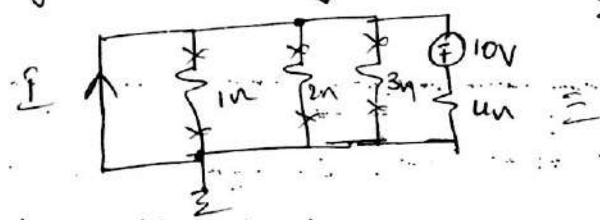
$I = \frac{10}{4} = 5/2 = 2.5$ Amperes

$\frac{I}{V} = \frac{2.5}{10} = .25\,\mho \leftarrow (a)$

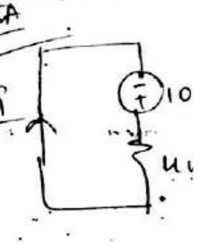


$I = \frac{10}{4} = 2.5A$

interchange the i/p & o/p



=

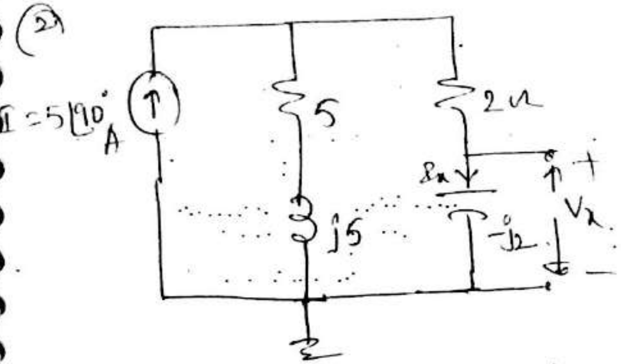


$I = \frac{10}{4} = 2.5$ Amperes

$\Rightarrow \frac{I}{V} = \frac{2.5}{10} = .25\,\mho \leftarrow (b)$

eqn (a) = eqn (b) ∴ reciprocity theorem is verified.

(2)



Soln: i/p $I = 5\angle 90^\circ$ A

o/p $V_x \leftarrow$ o/p

$V_x = I_x(-j2)$

using BCM $\quad I_x = 5\angle 90^\circ \left[\dfrac{5+j5}{(5+j5)+(2-j2)}\right]$

$= 5\angle 90^\circ \times 0.9284\angle 21.8014$

$\boxed{I_x = 4.642\angle 111.8014 \text{ Amperes}}$

ratio of o/p to i/p $\Rightarrow V_x = I_x[-j2]$

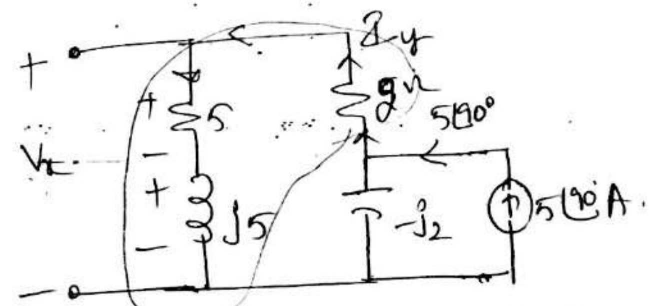
$= 4.642\angle 111.8014\,[-j2] = 9.284\angle 21.8014$ volts

o/p $\boxed{V_x = 9.284\angle 21.804 \text{ volts}}$

ratio of o/p / i/p $= \dfrac{V_x}{I} = \dfrac{9.284\angle 21.801}{5\angle 90}$

$= 1.8568\angle -68.198\ \Omega \leftarrow$ (a)

Step 2 interchange the i/p & o/p



using VDR $\quad I_y = 5\angle 90^\circ \left[\dfrac{-j2}{(7+j5)-j2)}\right]$

$= 5\angle 90^\circ \times 0.2626\angle -113.139$

$\boxed{I_y = \ldots}$ Amperes $= 1.313\angle\ldots$

$V_x = I_y[5+j5] = 1.313\angle -23.1\,[5+j5]$

$V_x = 9.284\angle 21.80$ volts

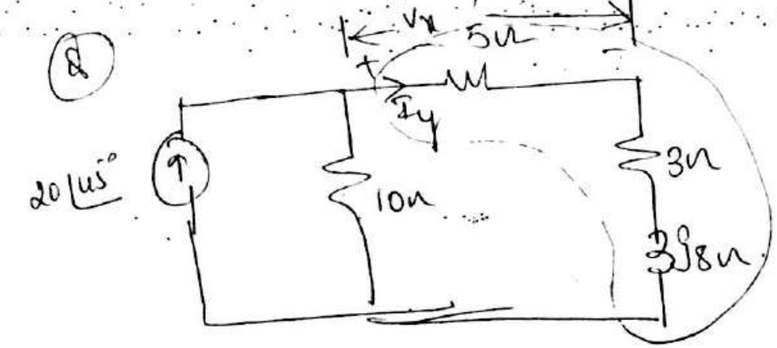
$\therefore \frac{V_x}{I} = \frac{9.284\angle 21.8014}{5\angle 90^\circ} = \cancel{9.284\angle 21.8}\ \Omega \leftarrow (b)$

$= 1.856\angle -68.198$

eqⁿ (a) = eqⁿ (b)

∴ AC Reciprocity theorem is verified.

(8)



using VDR $\quad I = 20\angle 45^\circ\ A \leftarrow$ i/p

$V_x = I_y \cdot 5$ volts $\leftarrow$ o/p

$$I_y = 20\angle 45^\circ \left[\frac{10}{10+5+3+j8}\right]$$

$$= 20\angle 45^\circ \times 0.5076\angle -23.96$$

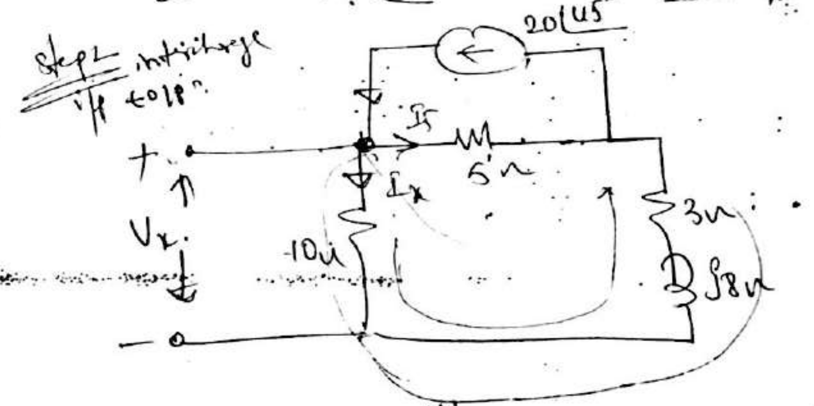
$$I_y = 10.1534\angle 21.037 \text{ Amperes}$$

$V_x = I_y \times 5 = \cancel{47.38}$

$$\boxed{V_x = 50.767\angle 21.037} \text{ volts}$$

$$\frac{V_x}{I} = \frac{50.767\angle 21.037}{20\angle 45^\circ} = 2.538\angle -23.963\ \Omega \leftarrow (a)$$

Step 2 interchange i/p to o/p.



$V_x = I_x \times 10$ volts.

using CDR $\quad I_x = 20\angle 45^\circ \left[\frac{5}{5+10+3+j8}\right]$

$$= 20\angle 45^\circ \times 0.2538\angle -23.962$$

$$I_x = 5.0767\angle 21.037 \text{ Amperes}$$

$V_x = 10 I_x = 10 \times 5.0767\angle 21.037$

$$\boxed{V_x = 50.0767\angle 21.037} \text{ volts}$$

$\frac{V_x}{I_x} = 2.538\angle -23.963 \leftarrow (b)$ eqⁿ (a) = eqⁿ (b) ∴ reciprocity theorem

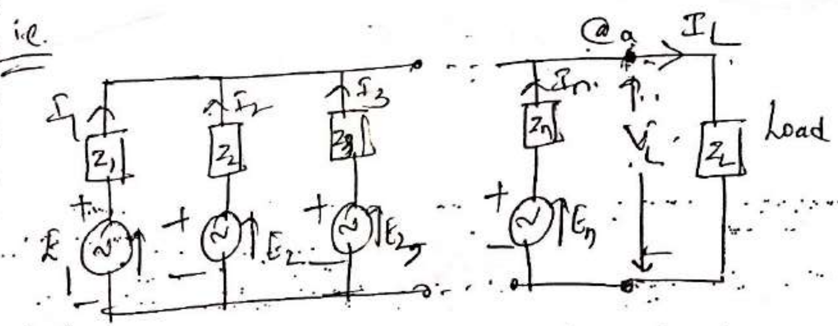
Millman's theorem (Parallel generator theorem)

Statement:-

Whenever a set of practical voltage sources working in parallel feeding into a common load, a common terminal voltage of the combination is given by

$$V = \frac{E_1Y_1 + E_2Y_2 + E_3Y_3 + \dots E_nY_n}{[Y_1 + Y_2 + Y_3 + \dots + Y_n + Y_L]}$$

i.e.



KCL @a: $I_1 + I_2 + I_3 + \dots + I_n = I_L$

$$\left(\frac{E_1 - V}{Z_1}\right) + \left(\frac{E_2 - V}{Z_2}\right) + \left(\frac{E_3 - V}{Z_3}\right) + \dots + \left(\frac{E_n - V}{Z_n}\right) = \frac{V}{Z_L}$$

$$(E_1 - V)Y_1 + (E_2 - V)Y_2 + (E_3 - V)Y_3 + \dots + (E_n - V)Y_n = VY_L$$

or

$$E_1Y_1 + E_2Y_2 + E_3Y_3 + \dots + E_nY_n - V(Y_1 + Y_2 + Y_3 + \dots + Y_n) = VY_L$$

$$E_1Y_1 + E_2Y_2 + E_3Y_3 + \dots + E_nY_n = V[Y_1 + Y_2 + Y_3 + \dots + Y_n + Y_L]$$

$$\boxed{V_L = \frac{E_1Y_1 + E_2Y_2 + \dots + E_nY_n}{[Y_1 + Y_2 + Y_3 + \dots + Y_n + Y_L]}} \text{ volts}$$

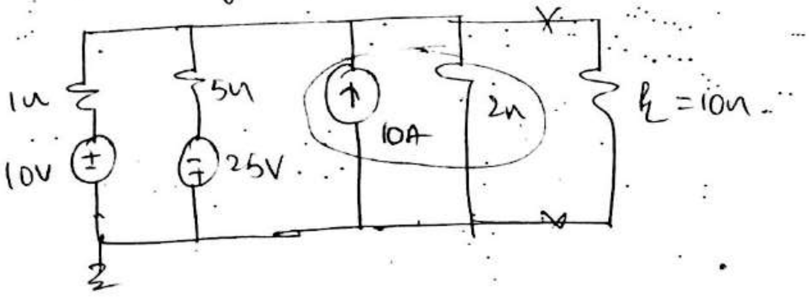
note:- if $Y_L = 0$

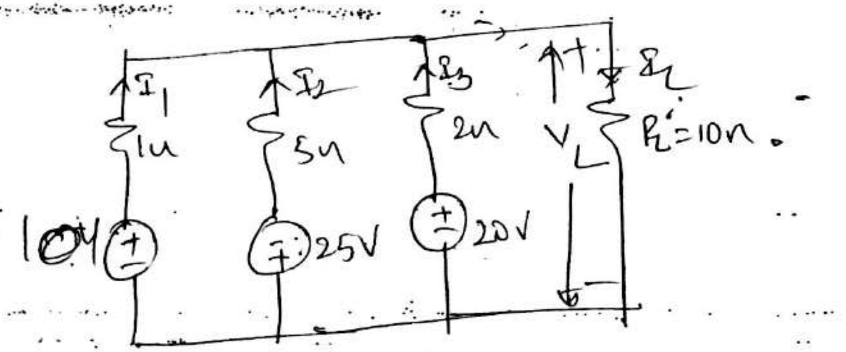
i.e. $V_{No\text{-}Load}$ with $Y_L = 0$.

$$\boxed{V_{NoLoad} = \frac{E_1Y_1 + E_2Y_2 + \dots + E_nY_n}{Y_1 + Y_2 + Y_3 + \dots + Y_n}} \quad Y_L = 0.$$

Note:- In the above case if the polarities of the source E_2 are reverse then E_2 is replaced by $-E_2$ in the expression of V_L.

(8) Find the power delivered by the load resistance R_L and current supplied by each source in the ckt shown using Millman's theorem.





$$V_L = \frac{E_1 Y_1 + E_2 Y_2 + E_3 Y_3}{Y_1 + Y_2 + Y_3 + Y_L}$$

$$= \frac{10\left(\frac{1}{1}\right) + (-25)\left(\frac{1}{5}\right) + 20\left(\frac{1}{2}\right)}{\left(\frac{1}{1}\right) + \left(\frac{1}{5}\right) + \left(\frac{1}{2}\right) + \left(\frac{1}{10}\right)} = 8.33 \text{ volts}$$

$$P_{R_L} = \frac{V^2}{R_L} = \frac{(8.33)^2}{10} = \underline{\underline{6.944 W}}$$

$$I_1 = \frac{E_1 - V}{R_1} = \frac{10 - 8.33}{1} = \underline{\underline{1.67 A}}$$

$$I_2 = \frac{E_2 - V}{R_2} = \frac{-25 - 8.33}{5} = \underline{\underline{-6.666 A}}$$

$$I_3 = \frac{E_3 - V}{R_3} = \frac{20 - 8.333}{2} = \underline{\underline{5.835 A}}$$

Check $I_L = \frac{V}{R_L} = \frac{8.333}{10} = \underline{0.833 A}$

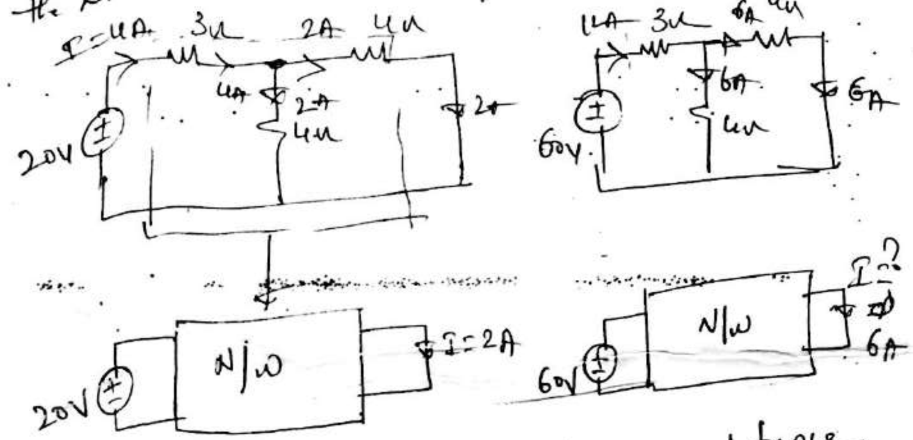
$$= I_1 + I_2 + I_3$$

$$I_L = I_1 + I_2 + I_3$$

The Homogenity principle :-

→ It is the principle obeyed by the all linear N/w's.

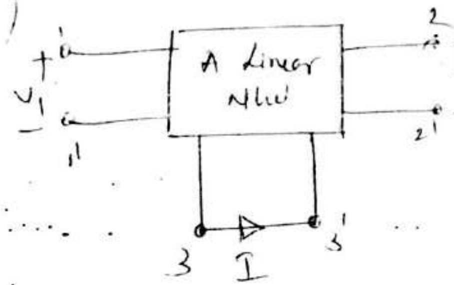
Def:- In a linear N/w if the excitation is multiplied with a Constant (K) then the response in all the other branches of the N/w are also multiplied with the Same Constant (K).



→ So, here the Excitation is ×ᵈ by 3 and hence in the responses also.

Note: When Multiple Sources are present then the SPT is applied first and Later the homogenity principle.

Q(10)



V_1	V_2	I
2V	0V	0.5A
0V	5V	-1A

If $V_1 = 10V$ and $V_2 = -5V$ then $I = ?$

Sol: Using SPT $I = k_1V_1 + k_2V_2$

$0.5 = k_1(2) \Rightarrow \boxed{k_1 = 0.25}$

$k_2(5) = -1 \Rightarrow \boxed{k_2 = -1/5}$.

$\Rightarrow I = k_1V_1 + k_2V_2$

$I = 0.25V_1 - 1/5\, V_2$

$I = 0.25(10) - 1/5(-5)$

$= \frac{10}{4} + 1 = 2.5 + 1 = 3.5$

$\boxed{I = 3.5}$ Ampere

(8) In the ckt shown

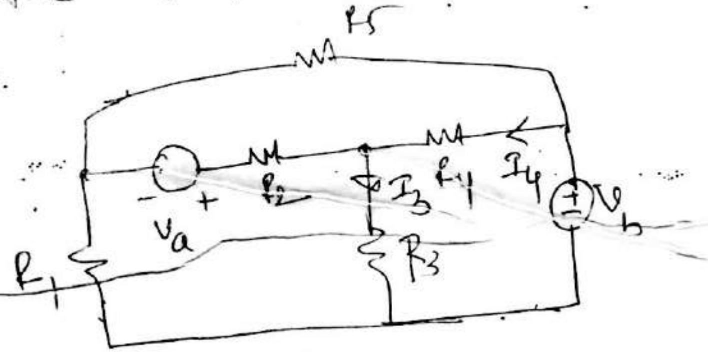
i) $I_3 = 1.5A$ when $V_a = 20V$ and $V_b = 0$. } find $I_3 = ?$
$V_a = 50V$ & $V_b = 0$

find I_3

ii) $I_4 = 2A$ when $V_a = 20V$ and $V_b = 50V$

$I_4 = -1$, when $V_a = 50V$ & $V_b = 20V$

find I_4 if $V_a = 30V$ & $V_b = 100V$



soln: using SPT

(i) $I_3 = k_1 V_a + k_2 V_b$

given $I_3 = 1.5A$; $V_a = 20$ & $V_b = 0$

$1.5 = k_1(20) + k_2(0)$

$\Rightarrow \boxed{k_1 = \frac{1.5}{20} = \frac{15}{20} = 0.075}$

$\Rightarrow V_a = 50V$; $V_b = 0V$ $I_3 = ?$

$I_3 = k_1 V_a + k_2 V_b$

$I_3 = 0.075(50) + \cancel{k_2}(0) = \underline{3.75A}$

(ii) $I_4 = k_3 V_a + k_4 V_b$

$2 = k_3(20) + k_4(50)$

$-1 = k_3(50) + k_4(20)$

$k_3 = -0.0428$, $k_4 = 0.05714$

$\Rightarrow I_4 = k_3 V_a + k_4 V_b$

$I_4 = -0.0428 V_a + 0.05714 V_b$

$= -0.0428(30) + 0.05714(100)$

$\boxed{I_4 = 4.43 A}$

Module 1: Basic Circuit Concepts

Network: Any interconnection of network or circuit elements (R, L, C, Voltage and Current sources).

Circuit: Interconnection of network or circuit elements in such a way that a closed path is formed and an electric current flows in it.

Active Circuit elements deliver the energy to the network (Voltage and Current sources)

Passive Circuit elements absorb the energy from the network (R, L and C).

Active elements:

Ideal Voltage Source is that energy source whose terminal voltage remains constant regardless of the value of the terminal current that flows. Fig.1a shows the representation of Ideal voltage source and Fig.1b, it's V-I characteristics.

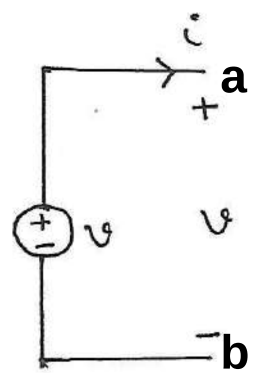


Fig.1a: Ideal Voltage source Representation

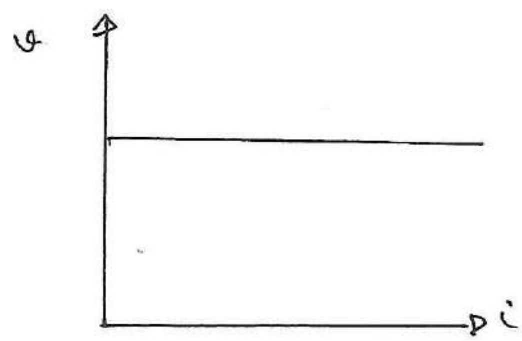


Fig. 1b: V-I characteristics

Practical Voltage source: is that energy source whose terminal voltage decreases with the increase in the current that flows through it. The practical voltage source is represented by an ideal voltage source and a series resistance called internal resistance. It is because of this resistance there will be potential drop **within the source and with the increase in terminal current or load current, the drop across resistor increases, thus**

reducing the terminal voltage. Fig.2a shows the representation of practical voltage source and Fig.2b, it's V-I characteristics.

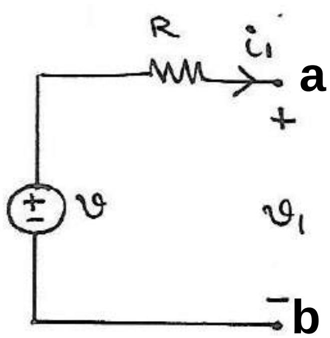


Fig. 2a: Practical Current Source

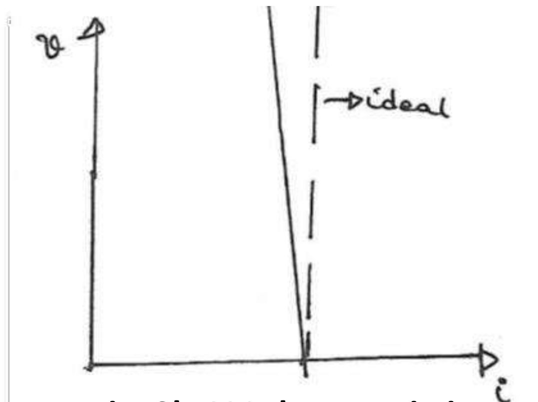


Fig. 2b: V-I characteristics

Here, $i_1 = i - v_1/R$ …… (2)

Dependent or Controlled Sources: These are the sources whose voltage/current depends on voltage or current that appears at some other location of the network. We may observe 4 types of dependent sources.

i) Voltage Controlled Voltage Source (VCVS)

ii) Voltage Controlled Current Source (VCCS)

iii) Current Controlled Voltage Source (CCVS)

iv) Current Controlled Current Source (CCCS)

Fig.3a, 3b, 3c and 3d represent the above sources in the same order as listed.

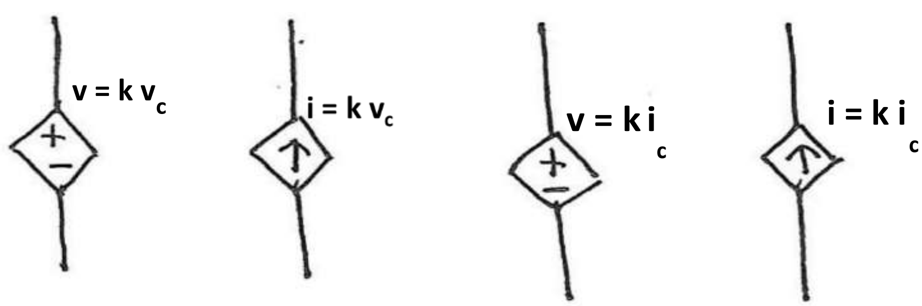


Fig. 3 a) VCVS b) VCCS c) CCVS d) CCCS

Kirchhoff's Voltage Law (KVL)

It states that algebraic sum of all branch voltages around any closed path of the network is equal to zero at all instants of time. Based on the law of conservation of energy.

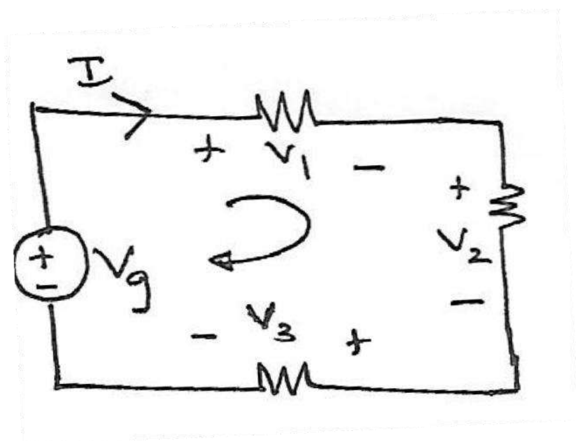


Fig. 4: Example illustrating KVL

Applying KVL clockwise, $+ V_1 + V_2 + V_3 - V_g = 0$ …… (3)

=> $V_g = V_1 + V_2 + V_3$ …… (4), indicative of energy delivered

= energy absorbed

Kirchhoff's Current Law (KCL)

The algebraic sum of branch currents that leave a node of a network is equal to zero at all instants of time. Based on the law of conservation of charge.

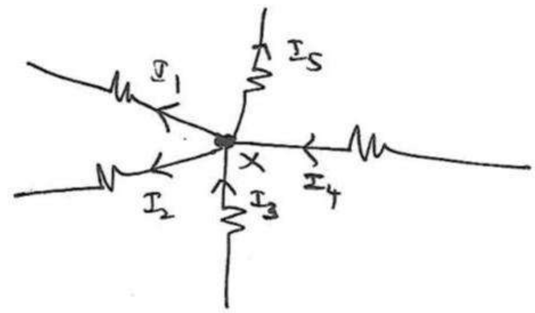


Fig. 5: Example illustrating KCL

Applying KCL at node X, $+ I_1 + I_2 - I_3 - I_4 + I_5 = 0$ ……. (5)

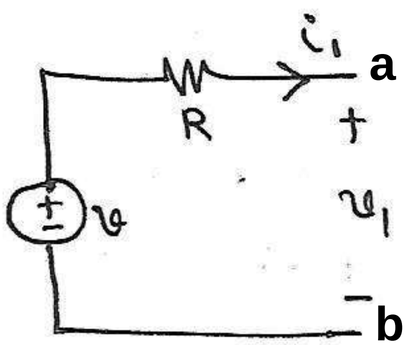
=> $I_3 + I_4 = I_1 + I_2 + I_5$ …… (6), indicative of sum of incoming currents

= sum of outgoing currents at a node.

Source Transformation

Source Transformation involves the transformation of voltage source to its equivalent current source and vice-versa.

Consider a voltage source with a series resistance R, in Fig. 6a and a current source with the same resistance R connected across, in Fig.6b.



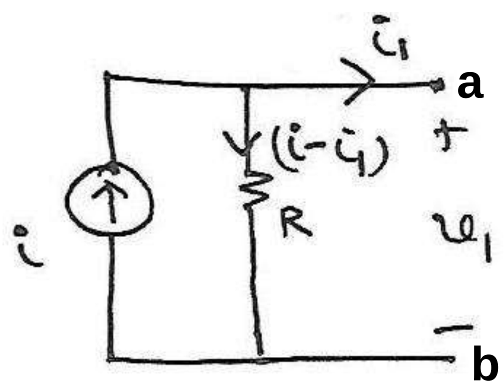


Fig.6a Voltage Source **Fig.6b Current Source**

The terminal voltage and current relationship in the case of voltage source is;

$v_1 = v - i_1 R$ …… (7)

The terminal voltage and current relationship in the case of current source is;

$i_1 = i - v_1/R$, which can be written as, $v_1 = i R - i_1 R$ …… (8)

If the voltage source above has to be equivalently transformed to or represented by, a current source then the terminal voltages and currents have to be same in both cases.

This means eqn. (7) should be equal to eqn. (8). This implies, $v = i R$ **or** $i = v / R$…(9). If eqn.(9) holds good, then the voltage source above can be equivalently transformed to or represented by, the current source shown above and vice-versa.

Problems:

1) For the network shown below in Fig.7, find the current through 2Ω resistor, using source transformation technique.

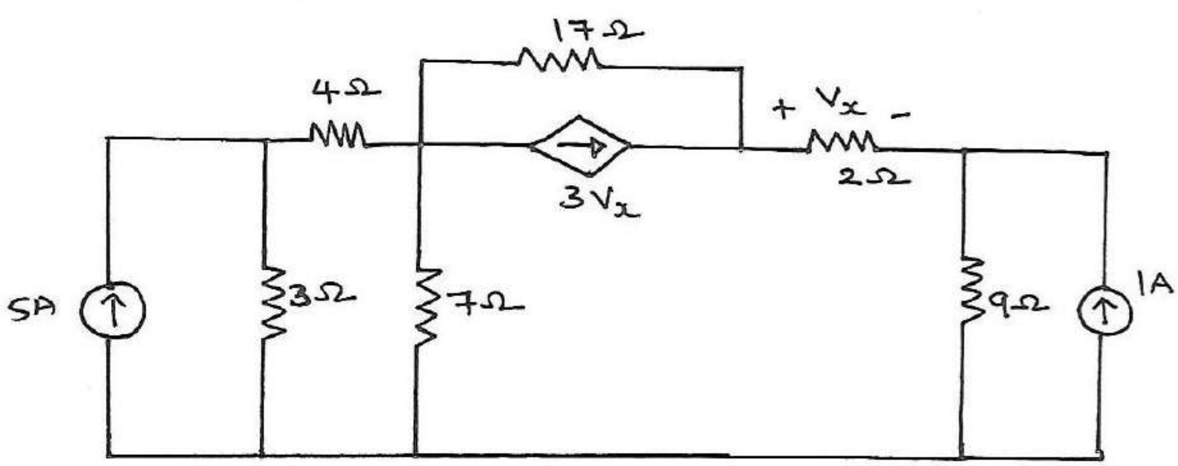
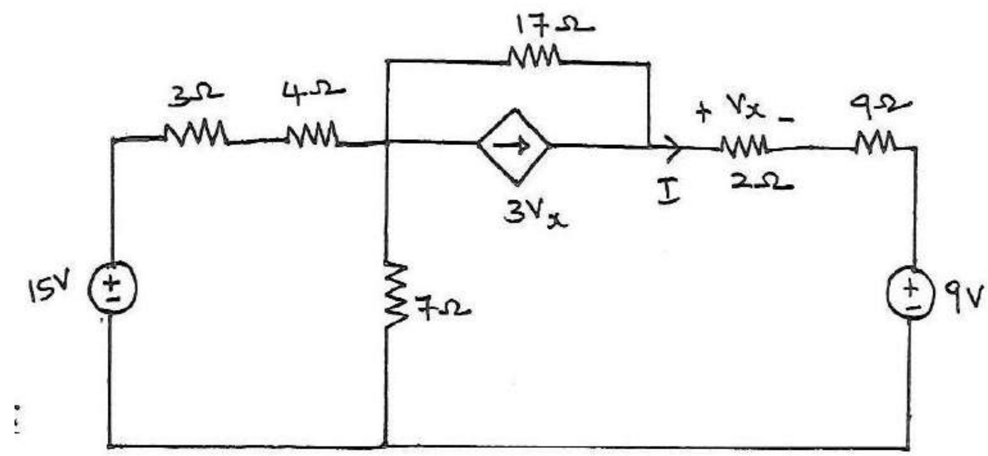
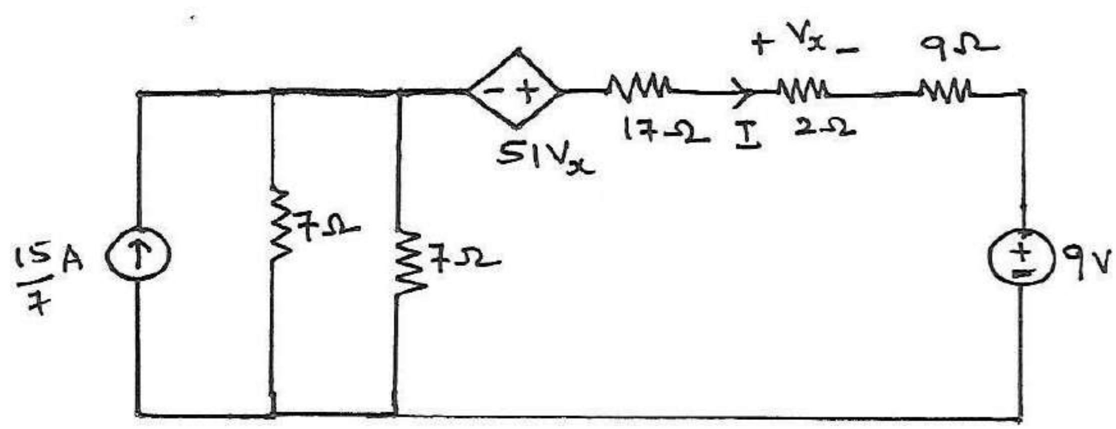


Fig.7

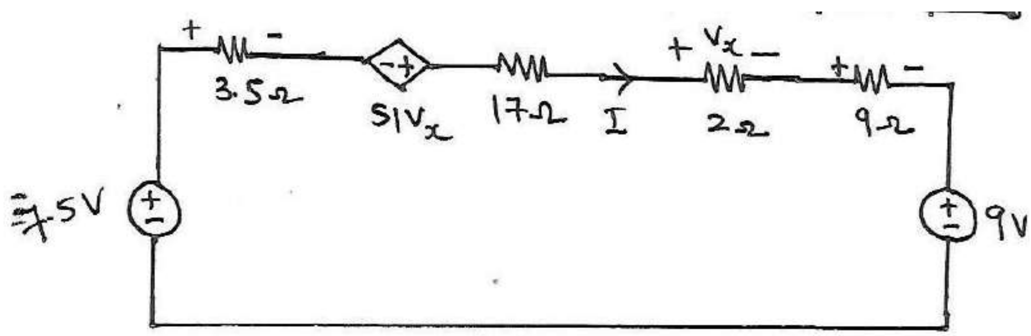
Solution: In the given circuit, Converting 5A source to voltage source so that resistor 4Ω comes in series with source resistor 3Ω and equivalent of them can be found. Also converting 1A source to voltage source, we obtain the circuit as below;



Converting 15V source above to current source and converting $3V_x$ dependent current source to dependent voltage source, we get the following;



Taking equivalent of the parallel combination of 7Ω resistors and converting 15/7 A current source to voltage source, we get as shown below;



Applying KVL to the loop above clockwise, we get;

3.5 I - 51 V_x + 17 I +2I + 9I + 9 -7.5=0

From the circuit above, V_x =2I, substitute in above eqn, then we get;

-70.5 I = -1.5

=> I = 0.02127 A = 21.27mA

2) Represent the network shown below in Fig.8, by a single voltage source in series with a resistance between the terminals A and B, using source transformation techniques

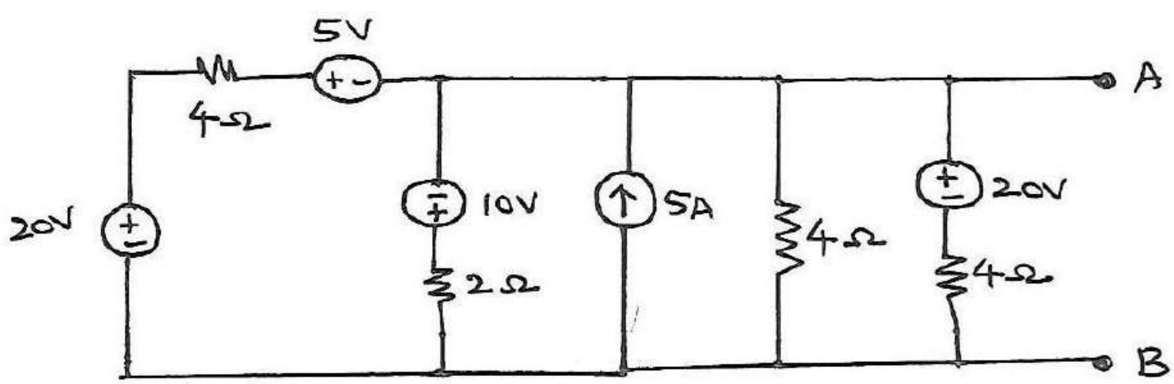
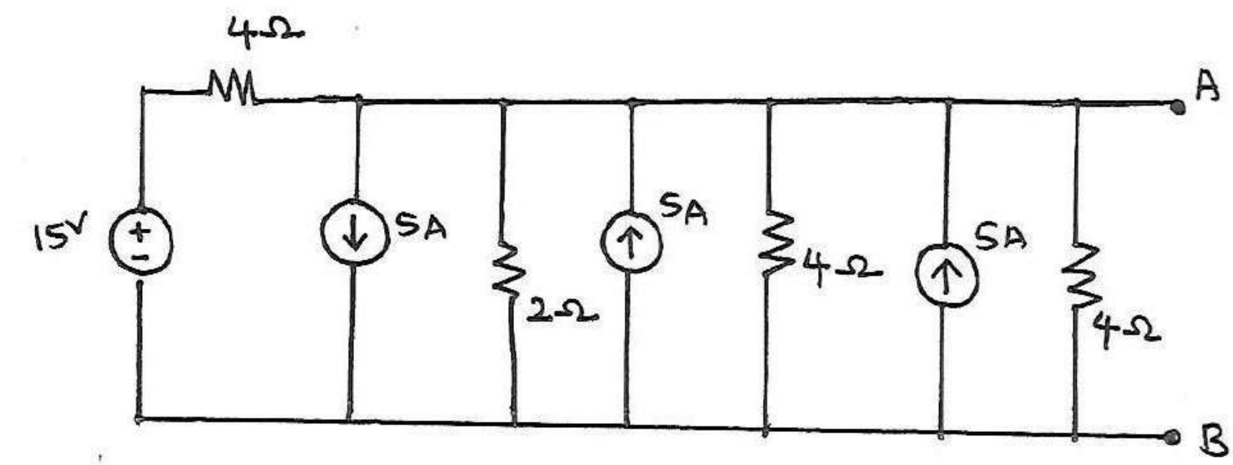


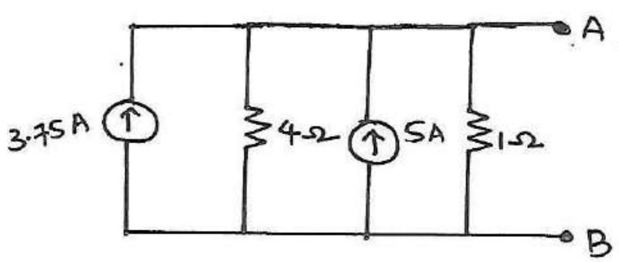
Fig.8

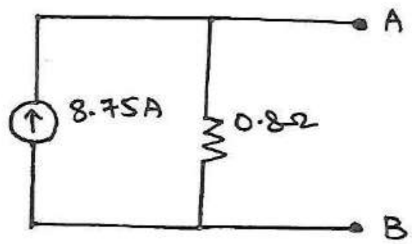
Solution: In the circuit above, 5V and 20 V sources are present in series arm and they are series opposing.

So, the sources are replaced by single voltage source which is the difference of two (as they are opposing, if series aiding then sum has to be considered). The polarity of the resulting voltage source will have same as that of higher value voltage source. Multiple current sources in parallel, can be added if they are in same direction and if they are in opposite direction, then difference is taken and resulting source will have same direction as that of higher one.

Taking source transformation, such that we get all current sources in parallel and all resistances in parallel, between the terminals. This leads to finding of equivalent current source and equivalent resistance between A-B. The source transformation leads to single voltage source in series with a resistance. These are shown below;







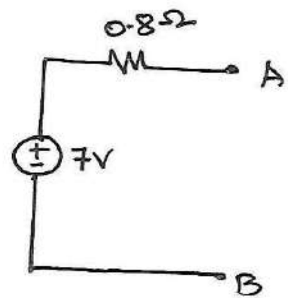


Illustration of Mesh Analysis:

3) Find the mesh currents in the network shown in fig.9

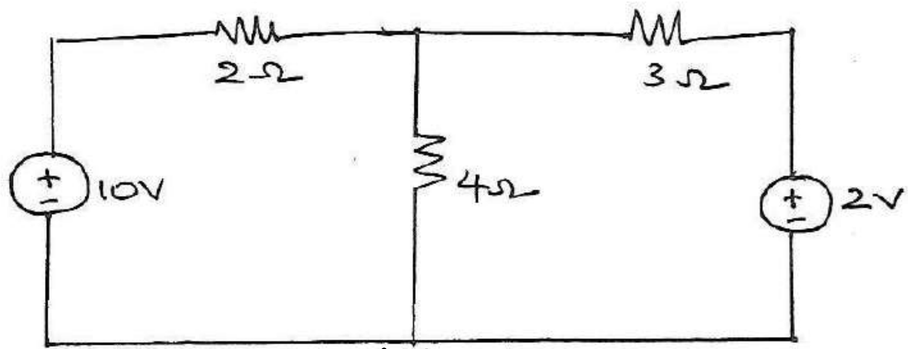


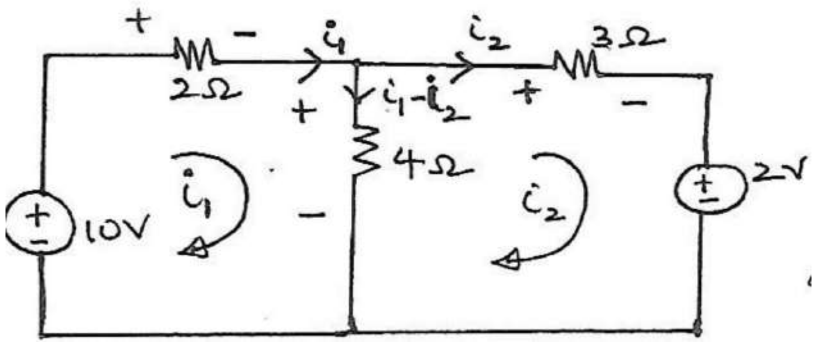
Fig.9

We identify two meshes; 10V-2Ω-4 Ω called as mesh 1 and 3Ω-2V-4 Ω called as mesh2. We consider i_1 to flow in mesh1 and i_2 to flow in mesh2. Their directions are always considered to be clockwise. If they are in opposite direction in actual, we get negative values when we calculate them, indicative of actual direction to be opposite.

10V-2Ω branch only belongs to mesh1 and so current through it is i_1 and 3Ω-2V branch only belongs to mesh2 and so current through it is always i_2. Also, 4Ω belongs to both meshes and so, the current through it will be the resultant of i_1 and i_2. These are shown below;

Next we will apply KVL to each of the meshes; As a result, In this case, we get two equations in terms of i_1 and i_2 and when we solve them we get i_1 and i_2. And when we know the mesh current values, we can find the response at any point of network.

The polarities of the potential drops across passive circuit elements are based on the directions of the current that flows through them



Applying KVL to mesh1;

$$\mathbf{+2\,i_1 + 4\,(i_1 - i_2) - 10 = 0}$$

=> $+6\,i_1 - 4\,i_2 = 10$…… (1)

Applying KVL to mesh2;

$+3\,i_2 + 2 - 4\,(i_1 - i_2) = 0$

Above equation can be rewritten as

$$\mathbf{+3\,i_2 + 2 + 4\,(i_2 - i_1) = 0}$$

=> $-4\,i_1 + 7\,i_2 = -2$ …… (2)

Also observing the bold equations above, we may say that easily the potential drops across passive circuit elements can be considered to take +ve signs. From now onwards, we will not specifically identify polarities of potential drops across **passive circuit elements**. They are considered to take positive signs. For the case of shared element, like 4Ω above, which is shared between mesh1 and mesh2, the potential drop across it , is considered to be $+4(i_1 - i_2)$, when we apply KVL to mesh1 and $+4(i_2 - i_1)$, when we apply KVL to mesh2. Now eqn1 and eqn2 above can be represented in matrix form as shown;

$$\begin{matrix} 6 & -4 \\ -4 & 7 \end{matrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{matrix} 10 \\ -2 \end{matrix}$$

Using cramer's rule;

$$\Delta = \begin{vmatrix} 6 & -4 \\ -4 & 7 \end{vmatrix} = 26$$

$$\Delta i_1 = \begin{vmatrix} 10 & -4 \\ -2 & 7 \end{vmatrix} = 62$$

$$\Delta i_1 = \begin{vmatrix} 6 & 10 \\ -4 & -2 \end{vmatrix} = 28$$

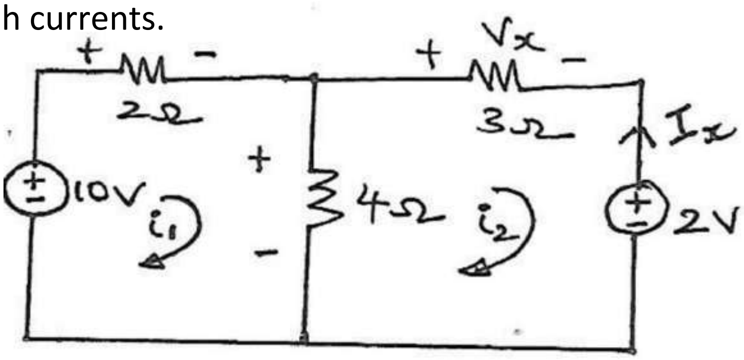
=> $i_1 = \Delta i_1 / \Delta = 2.384$ A

=> $i_2 = \Delta i_2 / \Delta = 1.076$ A

As already told, if we know the mesh current values, we can find the response at any point of network. And so, V_x and I_x identified, can be easily obtained using the mesh currents.

$I_x = -i_2 = -1.076$ A

$V_x = 3i_2 = 3.228$ A



4) Find the power delivered or absorbed by each of the sources shown in the network in Fig.10.Use meshanalysis

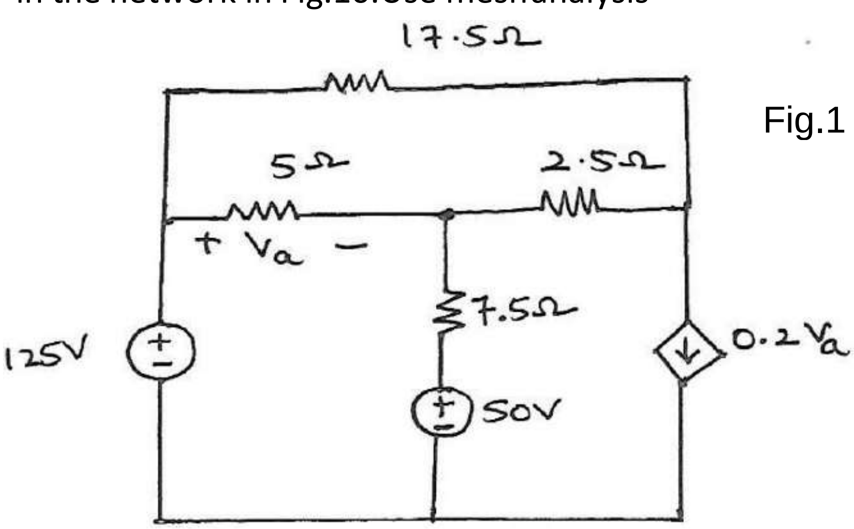
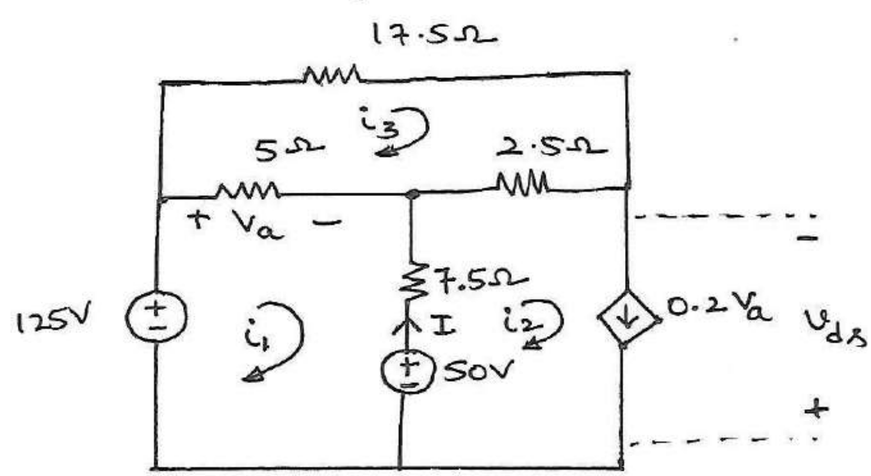


Fig.1



Solution:-

Power delivered by 125 V source, P_{125} =125 i_1

Power delivered by 50V source, P_{50}= 50 I =50 (i_2-i_1)

Power delvd. by dependent current source, P_{ds} = (0.2V_a) (v_{ds}) = (i_1-i_3) (v_{ds})

{Because V_a =5 (i_1-i_3)}

From the circuit; V_a =5 (i_1 - i_3)

Also; i_2 =0.2 V_a = $i_1 - i_3$ (it is as good as specifying the value of i_2 or we can say we have obtained equation from mesh2, so no need of applying KVL to mesh2)

Applying KVL to mesh1;

$5(i_1-i_3) + 7.5(i_1-i_2) + 50-125=0$

$12.5 i_1 - 7.5 i_2 - 5 i_3 = 75$; substituting $i_2 = i_1 - i_3$; we have;

$5 i_1 + 2.5 i_3 = 125$ …… (1)

Applying KVL to mesh3;

$17.5 i_3 + 2.5 (i_3-i_2) + 5(i_3-i_1) = 0$

$-5 i_1 - 2.5 i_2 + 25 i_3 = 0$; substituting $i_2 = i_1 - i_3$; we have;

$-7.5\ i_1 + 27.5\ i_3 = 0$ …… (2)

Solving (1) and (2), we get; i_1=13.2 A and i_3=3.6 A

So, $i_2 = i_1 - i_3$ = 13.2 -3.6 =9.6 A

P_{125} = 125 i_1= 125 (13.2) =1650 W (power delivered)

P_{50} =50 I =50 ($i_2 - i_1$) = 50 (9.6 -13.2) = -180 W, here negative value of power delivered is the indicative of the fact that power is actually absorbed by 50V source.

To find v_{ds} in the network shown, we apply KVL to the outer loop 17.5Ω→0.2V_a→125V;

+17.5 i_3 - v_{ds} -125 =0 {when **applying KVL, the potential drop across passive circuit element is taken as, + (resistance or impedance value) x (that particular current which is in alignment with KVL direction), if clockwise direction is considered, then clockwise current**)}

=> v_{ds} = - 62V

P_{ds} = (0.2 V_a)(v_{ds}) =($i_1 - i_3$) v_{ds} = - 595.2W => Dependent source absorbs power of 595.2 W

5) Find the power delivered by dependent source in the network shown in Fig.11.Use mesh analysis

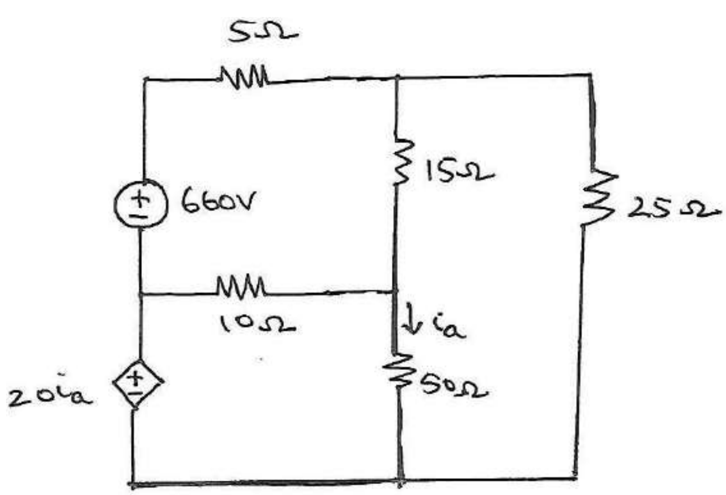
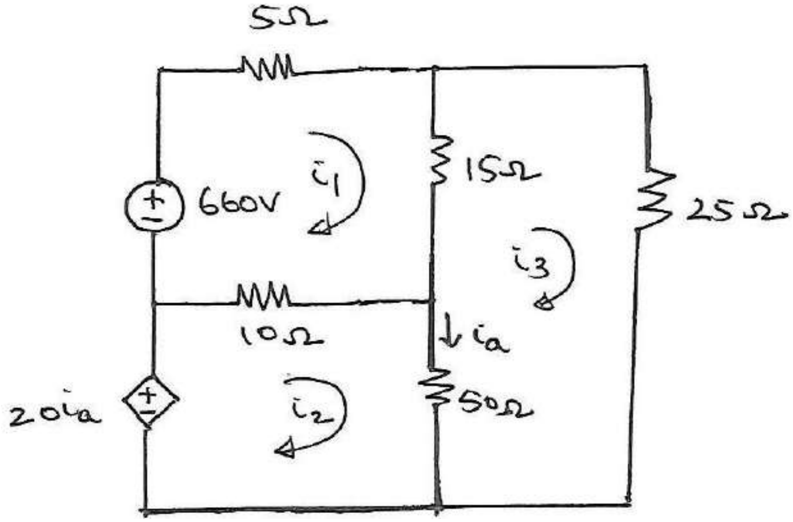


Fig.11

Solution:-



From the circuit,

$i_a = i_2 - i_3$

Power delivered by dependent source, $P_{ds} = (20\ i_a)\ (i_2) = 20\ (i_2 - i_3)\ i_2$

Apply KVL to mesh1

$\mathbf{5\ i_1 + 15\ (i_1 - i_3) + 10\ (i_1 - i_2) - 660 = 0}$

$\mathbf{30\ i_1 - 10\ i_2 - 15\ i_3 = 660 ......\ (1)}$

Apply KVL to mesh2

$\mathbf{10\ (i_2 - i_1) + 50\ (i_2 - i_3) - 20\ i_a = 0}$

$\mathbf{10\ (i_2 - i_1) + 50\ (i_2 - i_3) - 20\ (i_2 - i_3)}$

$\mathbf{-10\ i_1 + 40\ i_2 - 30\ i_3 = 0\ ......\ (2)}$

Apply KVL to mesh3

$\mathbf{25\ i_3 + 50\ (i_3 - i_2) + 15\ (i_3 - i_1) = 0}$

$\mathbf{-15\ i_1 - 50\ i_2 + 90\ i_3 = 0\ ......\ (3)}$

Solving (1), (2) and (3), we get i_2= 27 A and i_3 =22A

$P_{ds} = (20)\ (i_2 - i_3)\ i_2 = 20(5)27) = 2700W$, power delivered.

AC Circuits

These circuits consist L and C components along with R. Here we consider the excitation of the circuits by sinusoidal sources. Consider an AC circuit shown below;

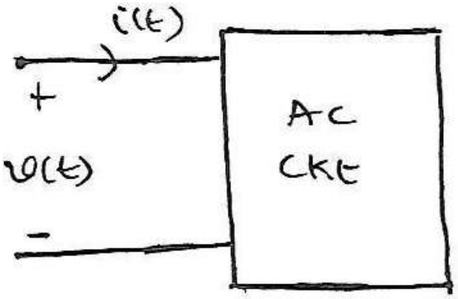


Fig.12

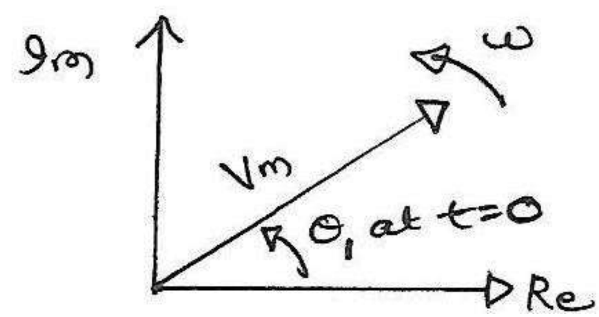


Fig.13

Let the applied voltage, $v(t) = V_m \sin(\omega t + \theta_1)$, the circuit current that flows is $i(t)$ and is given as; $i(t) = I_m \sin(\omega t + \theta_2)$. These two sinusoidal quantities can be represented by phasors; a phasor is a rotating vector in the complex plane. This is shown in Fig.13, which is a voltage phasor. The phasor has a magnitude of V_m and rotates at an angular frequency of ω with time.

The voltage phasor is given by $V_m \angle\theta_1$ (Also referred as polar form of phasor). The rectangular form is $V_m \cos\theta_1 + j V_m \sin\theta_1$.

Similarly, the current phasor is given by $I_m \angle\theta_2$ (Also referred as polar form of phasor). The rectangular form is $I_m \cos\theta_2 + j I_m \sin\theta_2$.

The ratio of voltage phasor to the current phasor is called as impedance. $Z = (V_m \angle\theta_1)/(I_m \angle\theta_2) = (V_m/I_m) \angle(\theta_1 - \theta_2) = (V_m/I_m)\angle\theta$

The impedance although a complex quantity but is not a phasor, as with respect to time, the angle of impedance do not change

- If the AC circuit above is represented equivalently by single resistance, then $Z = (V_m \angle\theta_1)/(I_m \angle\theta_1)$ {since in resistance there is no phase difference between voltage and current and so $\theta_2 = \theta_1$}.

So, $Z = (V_m/I_m)\angle 0°$

$= (V_m/I_m) \cos 0° + j (V_m/I_m) \sin 0°$

$= V_m/I_m = R.$

- If the AC circuit above is represented equivalently by single inductance, then $Z= (V_m \angle\theta_1)/ (I_m \angle(\theta_1 - 90°))$ { since in inductance, current lags the voltage in phase by 90°}

So, $Z = (V_m/I_m)\angle 90°$

$= (V_m/I_m) \cos 90° + j (V_m/I_m) \sin 90°$

$= j (V_m/I_m)$

= jωL {in inductance, the ratio of peak value of voltage to peak value of current is always the reactance which is given by ωL}. Now we can say, any inductance of L henry can be equivalently represented by impedance of jωL Ohms.

- If the AC circuit above is represented equivalently by single capacitance, then $Z= (V_m \angle\theta_1)/ (I_m \angle(\theta_1 + 90°))$ { since in capacitance, current leads the voltage in phase by 90°}

So, $Z = (V_m/I_m)\angle -90°$

$= (V_m/I_m) \cos 90° - j (V_m/I_m) \sin 90°$

$= -j (V_m/I_m)$

$= -j(1/\omega C)$

= -j/ωC {in capacitance, the ratio of peak value of voltage to peak value of current is always the reactance which is given by 1/ωc. Now we can say, any capacitance of C farad can be equivalently represented by impedance of -j/ ωC Ohms.

6) Find the current through the capacitor in the circuit shown in Fig.14. Use mesh Analysis.

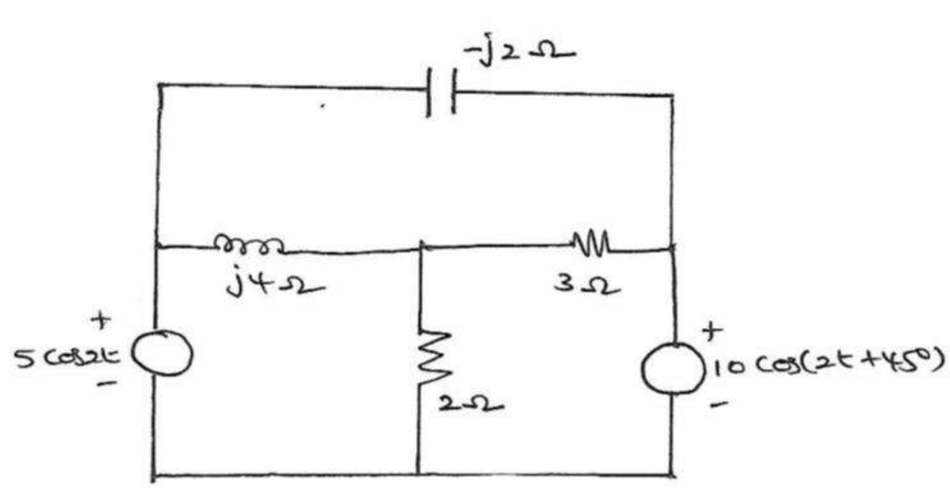
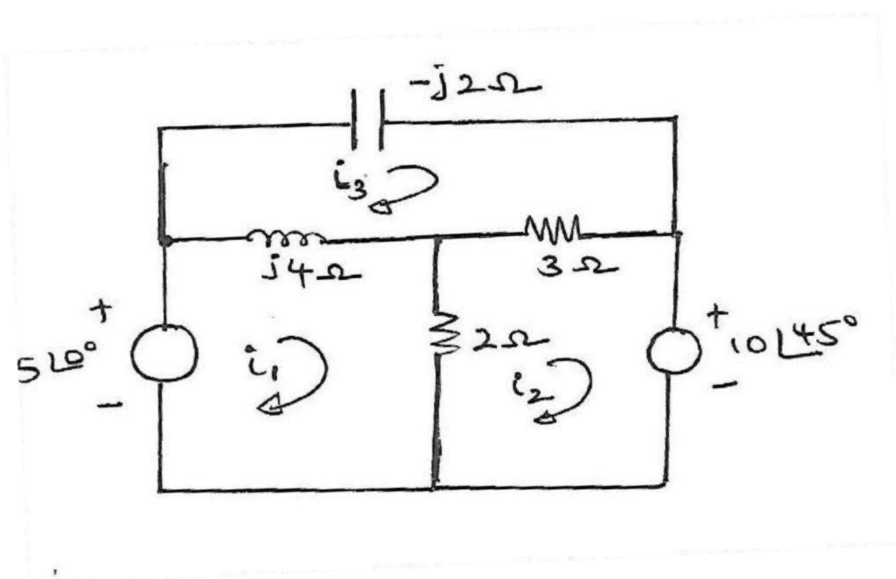


Fig.14

Solution:

The sources are represented by phasors. The mesh currents are identified. The current through the capacitor is i_3. So, i_3 needs to be found using mesh analysis.



Apply KVL to mesh1;

$j4(i_1 - i_3) + 2(i_1 - i_2) - (5\angle 0°) = 0$

$(2+j4)\, i_1 - 2\, i_2 - j4\, i_3 = 5$ ......(1)

Apply KVL to mesh2;

$3(i_2 - i_3) + (10\angle 45°) + 2(i_2 - i_1) = 0$

$-2\, i_1 + 5\, i_2 - 3\, i_3 = -(10\angle 45°) = -7.07 - j\,7.07$ ......(2)

Apply KVL to mesh3;

$-j2\, i_3 + 3(i_3 - i_2) + j4(i_3 - i_1) = 0$

$-j4\, i_1 - 3\, i_2 + (3+j2)\, i_3 = 0$ ...... (3) **Mesh equations in matrix form;**

$$\begin{pmatrix} 2+j4 & -2 & -j4 \\ -2 & 5 & -3 \\ -j4 & -3 & 3+j2 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 5 \\ -7.07 - j\,7.07 \\ 0 \end{pmatrix}$$

Using Cramer's rule to find i_3 .

$$\Delta = \begin{vmatrix} 2+j4 & -2 & -j4 \\ -2 & 5 & -3 \\ -j4 & -3 & 3+j2 \end{vmatrix}$$

$= (2+j4)[5(3+j2)-9] + 2[-2(3+j2)-(-3)(-j4)] - j4[6+j20]$

$= 40-j12$

$$\Delta i_3 = \begin{vmatrix} 2+j4 & -2 & 5 \\ -2 & 5 & -7.07-j7.07 \\ & & \end{vmatrix}$$

-j4 -3 0

$= (2 + j4)[+3(-7.07 – j\ 7.07)] + 2[+j4(-7.07-j7.07)] +5[6+ j\ 20]$

$=128.98 – j83.82$

Therefore, $i_3 = \Delta i_3 / \Delta = (128.98 – j83.82)/ (40-j12)$

$= 3.535-j1.035$

$= 3.68∟-16.31° A.$

The above result represents the phasor of capacitor current. From this we can easily write the steady state expression of capacitor current, as,

$i_3(t) = 3.68 \cos(2t -16.31°)$ A

Node analysis

Here, we identify nodes of the given network and consider one node as ground node, which is considered to be zero potential point. We then identify the voltage at each of the remaining nodes which is nothing but potential difference between a node of interest and ground node, with ground node as reference. Node analysis involves the computation of node voltages, and when once these are found, we can find the response at any point of network.

Illustration

7) Find the node voltages in the network shown in Fig.15;

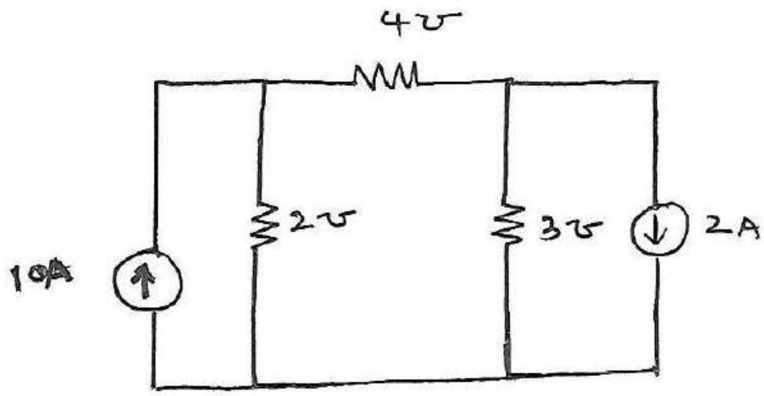
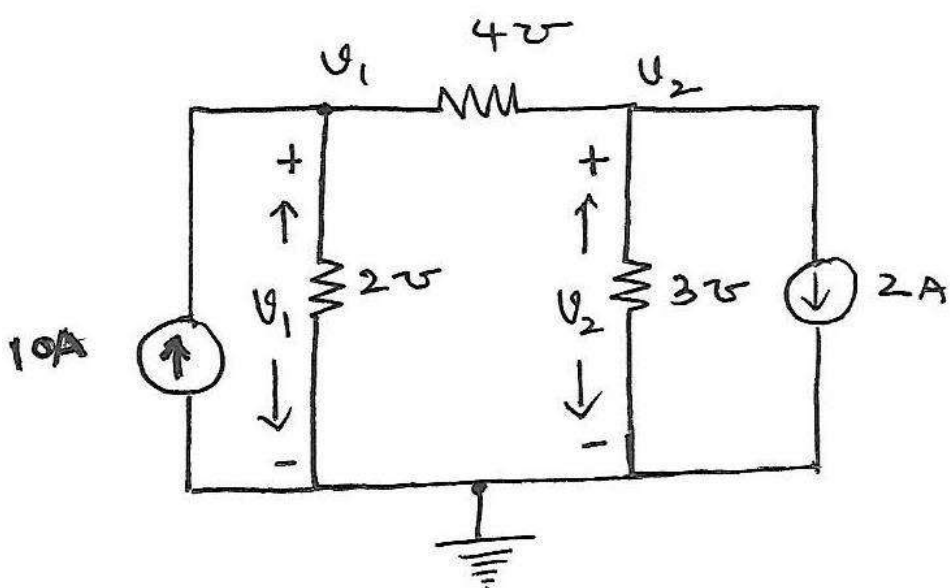


Fig.15

Solution:

There are 3 nodes in the network. The bottom node is selected as ground node. The voltage at node1 is identified as v_1 and it is the potential difference between the node1 and the ground, with ground as reference. The voltage at node2 is identified as v_2 and it is the potential difference between node2 and the ground, with ground as reference.



Recall KCL statement that "the algebraic sum of branch currents leaving a node of a network is zero at all instants of time".

Apply KCL at node1;

$-10 + 2v_1 + 4(v_1 - v_2) = 0$

$\Rightarrow 6v_1 - 4v_2 = 10$ ……(1)

Apply KCL at node2;

$+4(v_2 - v_1) + 3v_2 + 2 = 0$

$\Rightarrow -4v_1 + 7v_2 = -2$ …… (2)

Node equations in Matrix form

$$\begin{pmatrix} 6 & -4 \\ -4 & 7 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 10 \\ -2 \end{pmatrix}$$

Using Cramer's rule;

$$\Delta = \begin{vmatrix} 6 & -4 \\ -4 & 7 \end{vmatrix} = 26$$

$$\Delta v_1 = \begin{vmatrix} 10 & -4 \\ -2 & 7 \end{vmatrix} = 62$$

$$\Delta v_2 = \begin{vmatrix} 6 & 10 \\ -4 & -2 \end{vmatrix} = 28$$

$v_1 = \Delta v_1 / \Delta = 62/26$

v_1 = 2.384V

v_2= Δv_2 / Δ = 28/26

v_2 = 1.076V

Node Analysis Contd.

8) Use Node analysis to find the voltage V_x in the circuit shown in Fig. 16

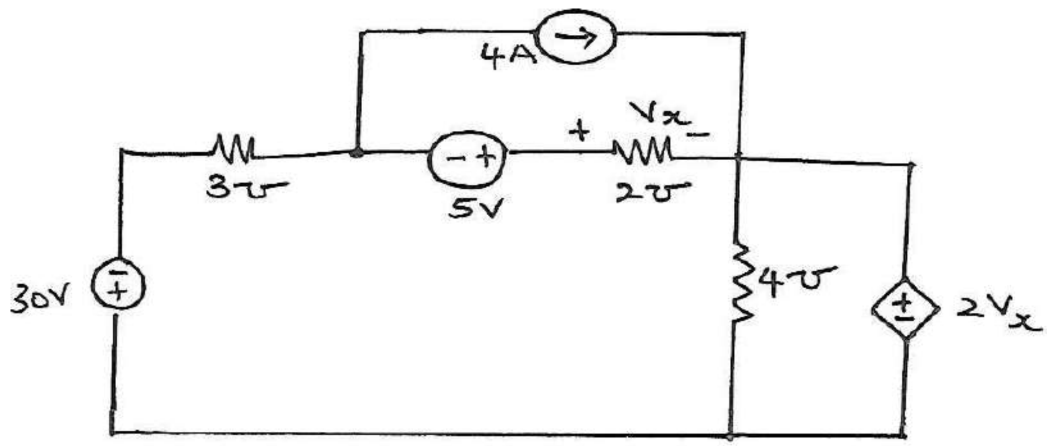
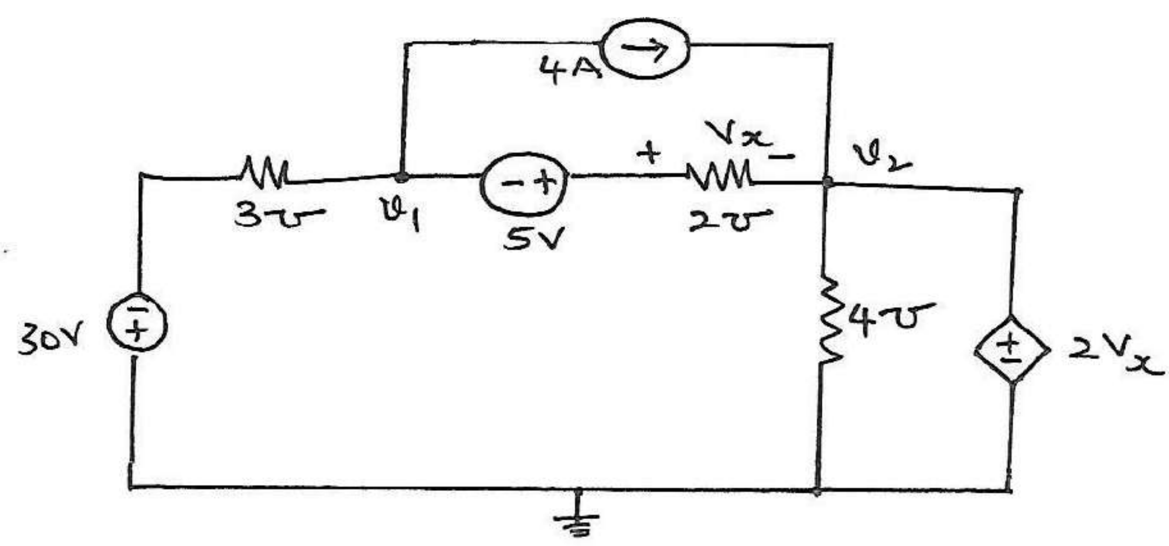


Fig.16

The ground node and other nodes with their voltages are identified as shown;



Although that point where two circuit elements join is referred as node (like 30V and 3 mho joining point above), we do not consider voltage there or apply KCL, because it will simply contribute for redundancy, as without considering the above, still the solution can be obtained. Therefore, we consider voltages or apply KCL to those nodes where three or more circuit elements join.

From the circuit; $V_x = v_1 + 5 - v_2$ and $v2 = 2V_x$

$v_2 = 2\,(v_1 + 5 - v_2)$

- $\Rightarrow 2\,v_1 - 3\,v_2 = -10$ …… (1), now we have an equation expressing v_2 or an equation associated with node 2. So no need of applying KCL at node2.

- $\Rightarrow$ Apply KCL at node1;

$3\,(v_1 - (-30)) + 4 + 2(\,v_1 + 5 - v_2) = 0$

- $\Rightarrow 5\,v_1 - 2\,v_2 = -104$ .... (2)

- $\Rightarrow$ Solving (1) and (2), we get;

- $\Rightarrow v_1 = -26.545V$ and $v_2 = -14.363V$

- $\Rightarrow$ Therefore, $V_x = v_1 + 5 - v_2$

- $\Rightarrow -26.545 + 5 + 14.363 = -7.182$ V.

9) Find the power delivered by dependent source using node analysis in the circuit shown in Fig. 17.

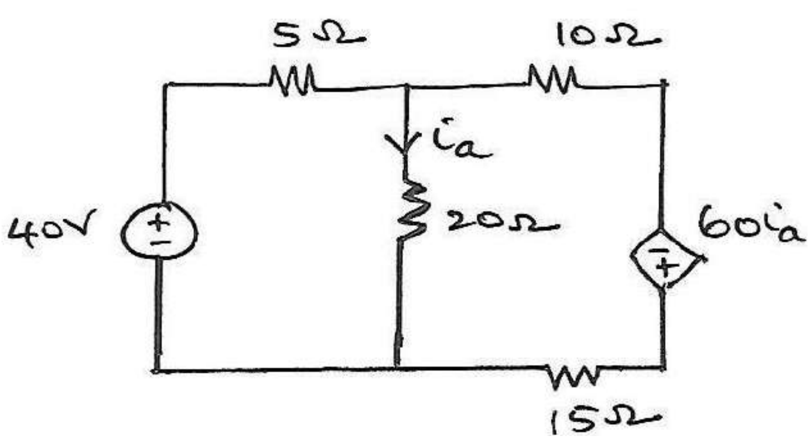
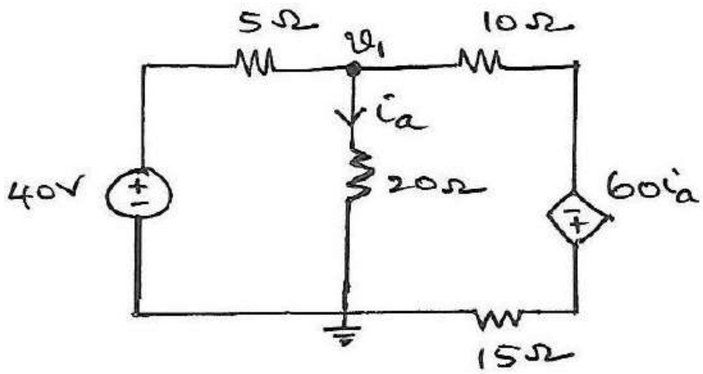


Fig.17

Solution: Identify ground node and other node with its voltage as shown;



From the circuit;

$i_a = v_1/20$ and

P_{ds} = (60 i_a) x (current that comes out of +ve polarity of $60i_a$)

= (60 i_a) [(v_1-(-60i_a))/(10 +15)]

= (60 i_a) (v_1 + 60 i_a)/25

10) Find the current i_1 in the network shown in Fig. 18. Use node Analysis.

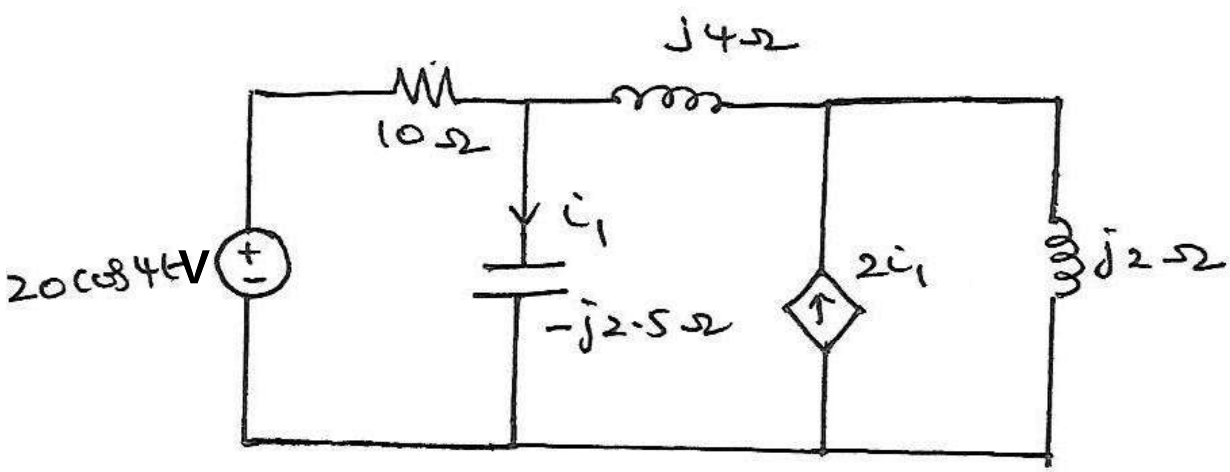
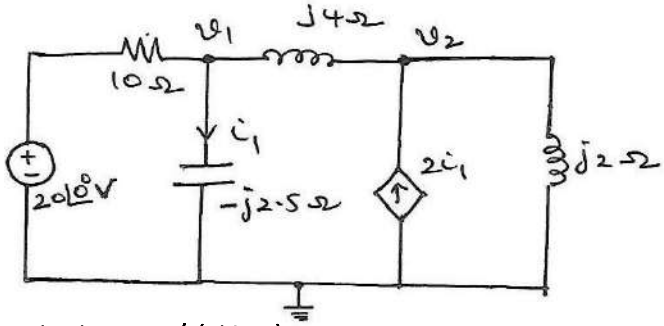


Fig.18

Identify ground node and other node voltages as shown. Also writing source using phasor representation.



From the circuit; $i_1 = v_1 / (-j2.5)$

Apply KCL at node1;

$v_1/ (-j2.5) + (v_1 - (20\angle 0°))/10 + (v_1 - v_2) / j4 = 0$

$\Rightarrow j\, 0.4\, v_1 + 0.1\, v_1 - j\, 0.25 v_1 + j0.25 v_2 = 2$

$\Rightarrow (0.1 + j0.15)\, v_1 + j\, 0.25\, v_2 = 2$ ……(1)

Apply KCL at node 2;

$-2i_1 + v_2 /j2 + (v_2 - v_1)/j4 = 0$

$\Rightarrow -2(v_1 /(-j2.5)) + v_2 /j2 + (v_2 - v_1)/j4 =0$

$\Rightarrow -j0.8\, v_1 - j\, 0.5\, v_2 - j0.25 v_2 + j0.25 v_1 = 0$

$\Rightarrow -j0.55\, v_1 - j\, 0.75\, v_2 = 0$........... (2)

Using Cramer's rule;

$$\Delta = \begin{vmatrix} 0.1 + j\, 0.15 & j\, 0.25 \\ -j0.55 & -j0.75 \end{vmatrix} \begin{aligned} &= (0.1 + j0.15)(-j0.75) - 0.25(0.55) \\ &= -0.025 - j0.075 \end{aligned}$$

$$\Delta v_1 = \begin{vmatrix} 2 & j\, 0.25 \\ 0 & -j0.75 \end{vmatrix} = -j\, 1.5$$

$v_1 = \Delta v_1/\Delta = (-j1.5) / (-0.025-j0.075) = 18 + j6 = 18.97\angle 18.43°$V

Therefore, $i_1 = v_1 /(-j2.5) = -2.4 + j7.2 = 7.58 \angle 108.43°$ A.

$i_1(t) = 7.58 \cos (4t + 108.43 °)$ A

Concept of Supermesh:

Supermesh concept is considered whenever a current source appears in common to two meshes.

Consider the Network Below;

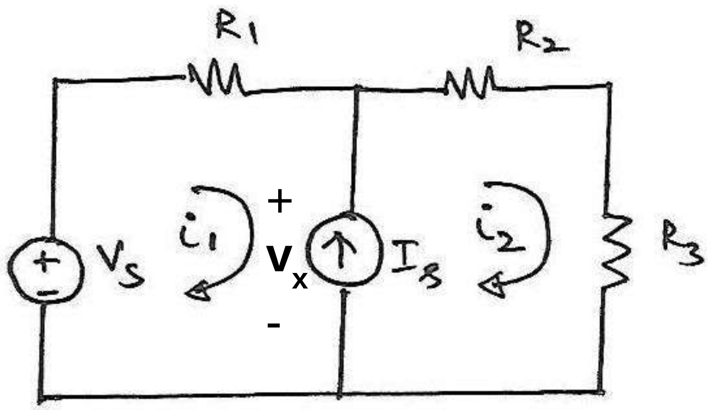


Fig.19

To know the advantage of applying supermesh concept; first consider usual way;

Applying KVL to mesh 1;

$R_1 i_1 + v_x - V_s = 0$

$R_1 i_1 + v_x = V_s$.....(1)

Applying KVL to mesh 2;

$(R_2 + R_3)i_2 - v_x = 0$

$v_x = (R_2 + R_3)i_2$ .....(2)

Substituting (2) in (1), we get;

$R_1 i_1 + (R_2 + R_3)i_2 = V_s$ .....(3)

Also from the circuit;

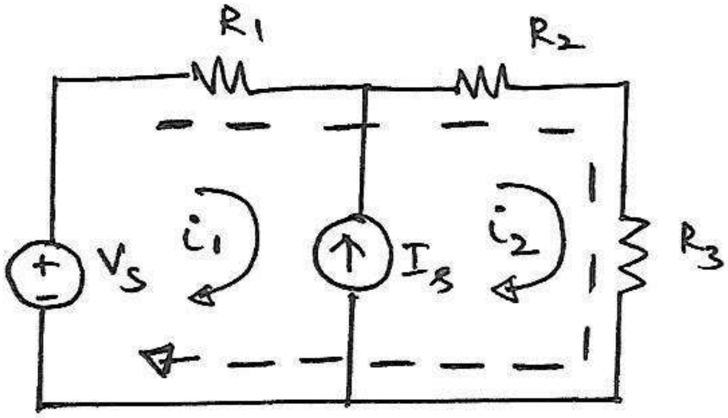
$i_2 - i_1 = I_s$

$\Rightarrow i_2 = I_s + i_1$ ......(4)

$\Rightarrow$ Substituting (4) in (3) we get, i_1;

$\Rightarrow$ Substituting i_1 in (4), we get i_2.

Applying the concept of supermesh;



Here, after identifying a current source common to two meshes; we first write constraint equation which relates corresponding mesh currents and the current source value.

$i_2 - i_1 = I_s$

Or $i_2 = I_s + i_1$..... (1)

We then club those two meshes and call it as supermesh; shown by dashed lines in the figure; Now we apply KVL to supermesh;

$R_1 i_1 + R_1 i_2 + R_3 i_2 - V_s = 0$

$R_1 i_1 + (R_1 + R_3)\, i_2 = V_s$ ......(2), this equation is exactly the same as (3) in previous case. In this case, it was easily obtained thus reducing the steps. Now, substituting (1) in (2), we get i_1. Then substituting i_1 in (1) we get i_2. Therefore, mesh currents were easily obtained using supermesh concept.

11) Use mesh analysis to find V_x in the circuit shown in fig. 20

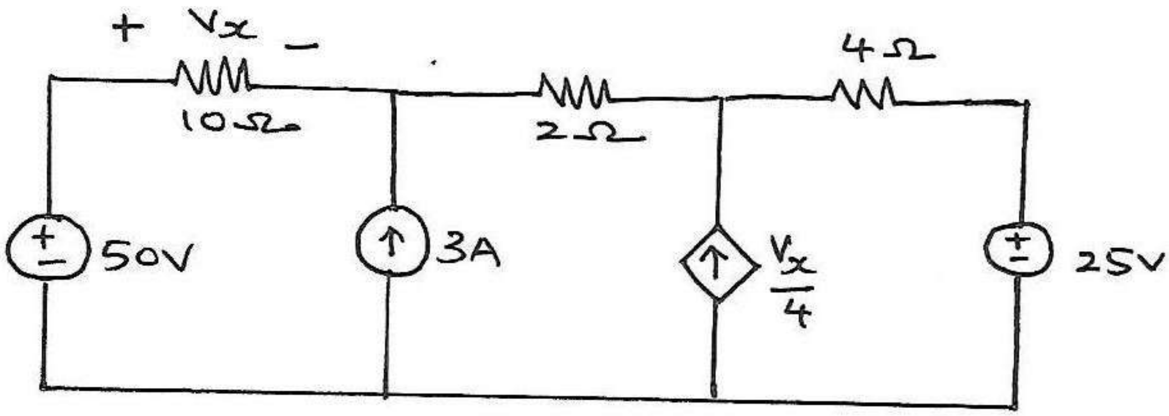
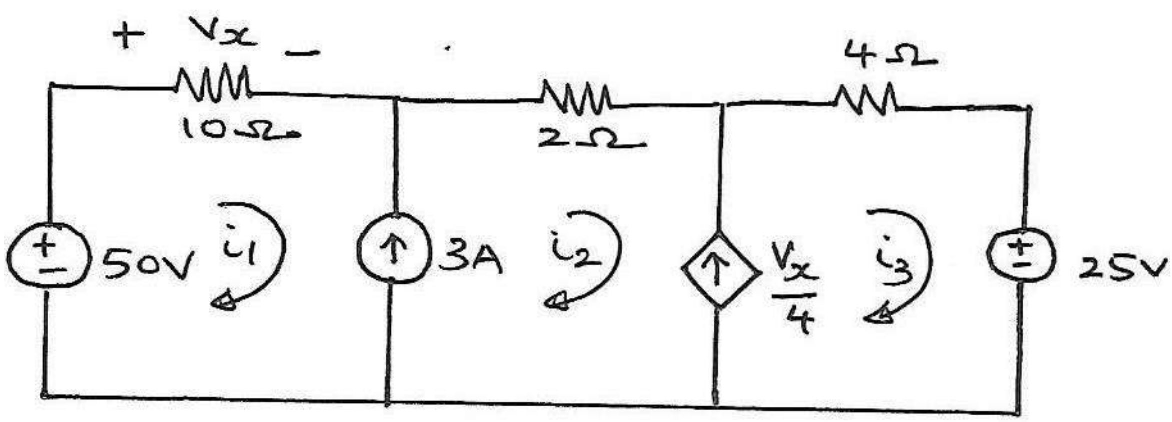


Fig.20



Solution: From the circuit; $V_x = 10i_1$

Identifying 3A and V_x /4 current sources appearing in common to mesh-1&2 and mesh-2&3 respectively; the constraint equations are written as; $i_2 - i_1 = 3$

=> $i_2 = 3 + i_1$ Also $i_3 - i_2 = V_x/4$,

wkt, $V_x = 10 i_1$

Substituting in above equation we get $i_3 - i_2 = 10 i_1/4$, wkt $i_2 = 3+i_1$ substituting this => $4 i_3 - 4(3+i_1) - 10i_1 = 0$

$-14 i_1 + 4 i_3 = 12$ …….(1)

Apply KVL to supermesh

formed by 10Ω→2 Ω→4Ω→25V→50V→10Ω

$10\ i_1 + 2\ i_2 + 4\ i_3 + 25 - 50 = 0$

$\Rightarrow 10\ i_1 + 2\ i_2 + 4\ i_3 = 25$

$\Rightarrow 10\ i_1 + 2\ (3+i_1) + 4\ i_3 = 25$

$\Rightarrow 12\ i_1 + 4\ i_3 = 19$ .... (2)

$\Rightarrow$ Solving (1) and (2), we get $i_1 = 0.2692$ A and $i_3 = 3.9423$ A

$\Rightarrow i_2 = 3 + i_1 = 3.2692$ A.

$\Rightarrow V_x = 10\ i_1 = 2.692$V

12) Find v_x in the circuit shown in fig. 21, using mesh analysis;

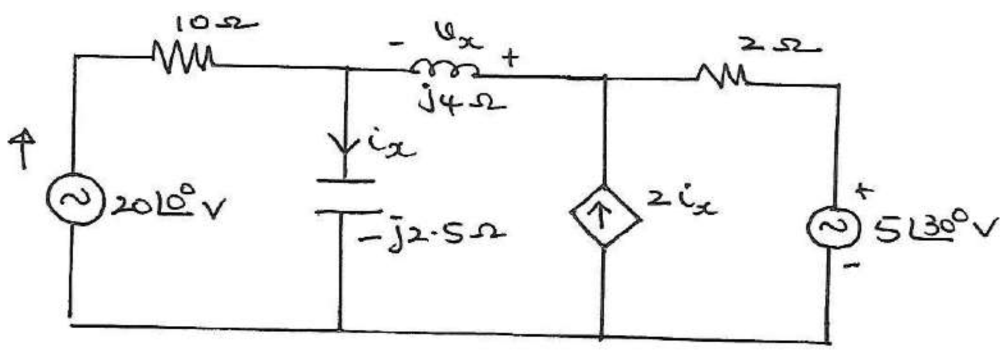
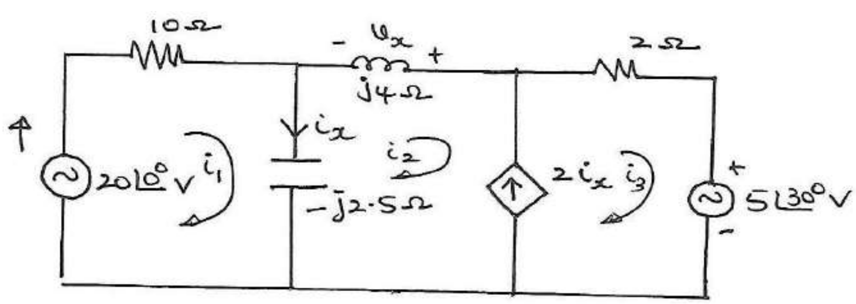


Fig.21

Solution:-



From the circuit; $v_x = -\ j4\ i_2$

$i_x = i_1 - i_2$

$i_3 - i_2 = 2\, i_x$ (current source $2i_x$ appears in common to two meshes)

$i_3 - i_2 = 2(\, i_1 - i_2)$

$i_3 = 2i_1 - i_2$

Apply KVL to mesh 1;

$10\, i_1 - j\, 2.5(i_1 - i_2) - (20\angle 0°) = 0$

$(10 - j2.5)\, i_1 + j\, 2.5\, i_2 = 20$ ....(1)

Apply KVL to supermesh formed by

$j4\Omega \rightarrow 2\Omega \rightarrow 5\angle 30° \rightarrow -j2.5\,\Omega \rightarrow j4\Omega$, we have,

$j4\, i_2 + 2\, i_3 + (5\angle 30°) - j\, 2.5\, (i_2 - i_1) = 0$

wkt $i_3 = 2i_1 - i_2$, subs in above eqn;

$j4\, i_2 + 2\, (\, 2i_1 - i_2) + (5\angle 30°) - j2.5\, (i_2 - i_1) = 0$

$(4 + j2.5)\, i_1 + (-2 + j1.5)\, i_2 = -(5\angle 30°) = -\, 4.33 - j2.5$ ....(2)

Using cramer's rule;

$$\Delta = \begin{vmatrix} 10 - j2.5 & j2.5 \\ 4 + j2.5 & -2 + j1.5 \end{vmatrix} = (10-j2.5)(-2 + j1.5) - j2.5(4+j2.5) = -10 + j10$$

$$\Delta i_2 = \begin{vmatrix} 10-j2.5 & 20 \\ 4+j2.5 & -4.33-j2.5 \end{vmatrix} = (10-j2.5)(-4.33-j2.5) - 20(4 + j2.5) = -129.55 - j64.175$$

$i_2 = \Delta i_2 / \Delta = (-129.55 - j64.175) / (-10 + j10)$

$i_2 = 3.268 + j9.686$

$i_2 = 10.22 \angle 71.35° A$

Therefore, $v_x = -j4\ i_2 = 38.74 - j13.07 = 40.89 \angle -18.64° V$

<u>Concept of Supernode:</u>

Supernode concept is applied whenever a voltage source appears in common to two nodes.

Consider the network below;

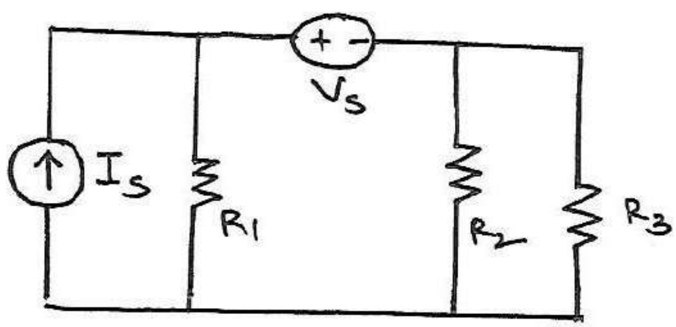
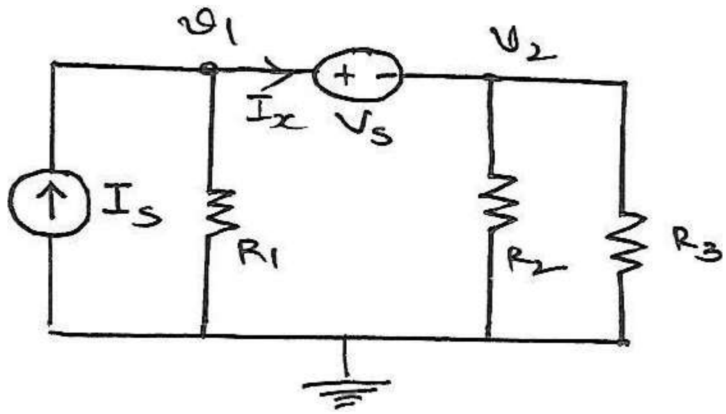


Fig.22

To illustrate the advantage of supernode concept; we first find the node voltages of the network by the usual way;



Apply KCL at node 1;

$v_1/R_1 - I_S + I_X = 0$

$v_1/ R_1 + I_X = I_S$ ……. (1)

Apply KCL at node 2;

$v_2 / R_2 + v_2 /R_3 – I_X =0$

$v_2/R_2 + v_2 /R_3 = I_X$ .....(2)

Subs (2) in (1), we get;

$v_1/R_1 + v_2/R_2 + v_2/R_3 = I_S$ ...... (3)

Also from the circuit; $v_1- v_2 = V_S$

$=> v_1 = V_S + v_2$ .... (4)

Substituting (4) in (3) will give the value of v_2

Substituting the value of v_2 in (4) will give the value of v_1.

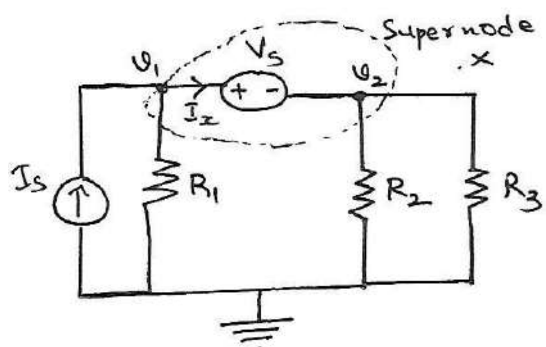
Applying the concept of supernode;

After identifying the voltage source appearing in common to two nodes;

We first write constraint equation; which relates the voltage source value with the corresponding node voltages; here it is; $v_1 – v_2 = V_S$

$v_1= v_2 +V_S$ ...... (1)

After this, we club the corresponding nodes to become one node and call it as a supernode. Then we apply KCL to supernode. Here, we apply KCL at supernode X as shown;



$v_1/R_1 – I_S + v_2/R_2 + v_2/R_3 =0$

$v_1/R_1 + v_2/R_2 + v_2/R_3 = I_S$ ......(2)

The above equation is same as eqn 3 in previous method, but the above equation was easily obtained in just one step. Therefore, when a voltage source is appearing in common to two nodes, it is always advantageous to consider the concept of supermesh.

Now, substituting (1) in (2), we get v_2.

Substituting v_2 in (2) we get v_1.

13) Find i_a and v_a in the network shown in fig. 23 using node analysis.

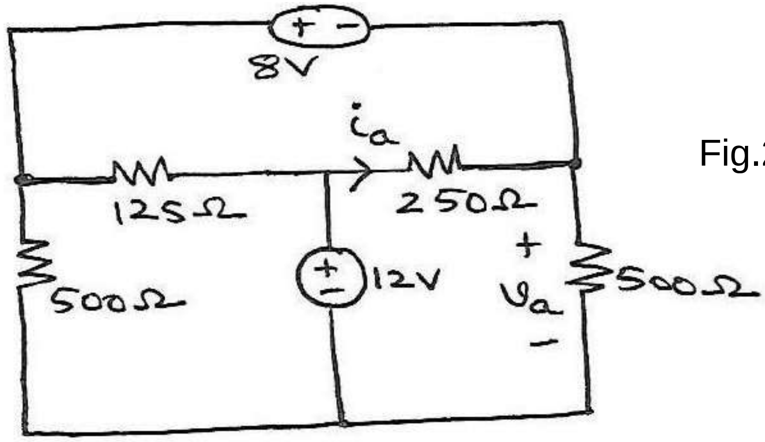
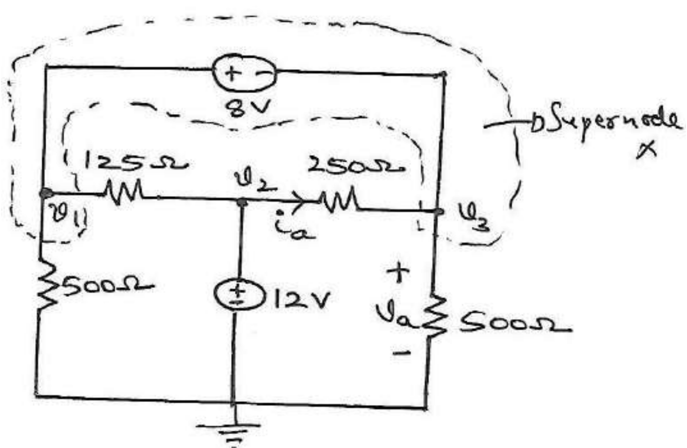


Fig.2

Solution:-



From the circuit;

$i_a = (v_2 - v_3)/250$

$v_a = v_3$

Also; v_2 =12 V

$v_1 - v_3 = 8$

$\Rightarrow v_1 = 8 + v_3$

Apply KCL at supernode X;

$V1/500 + (v_1 - v_2)/125 + (v_3 - v_2)/250 + v_3/500 = 0$

$v_1 + 4v_1 - 4v_2 + 2v_3 - 2v_2 + v_3 = 0$

$5v_1 - 6v_2 + 3v_3 = 0$

Substituting $v_1 = 8 + v_3$ in above equation, we get; $5(8+v_3) - 6v_2 + 3v_3 = 0$

$-6v_2 + 8v_3 = -40$

Wkt $v_2 = 12$ V

Therefore, $v_3 = (-40+6(12))/8 = 4V$

Now, $i_a = (v_2 - v_3)/250 = 0.032 = 32$ mA.

$v_a = v_3 = 4V$.

14) Find all the node voltages in the network shown in fig.24

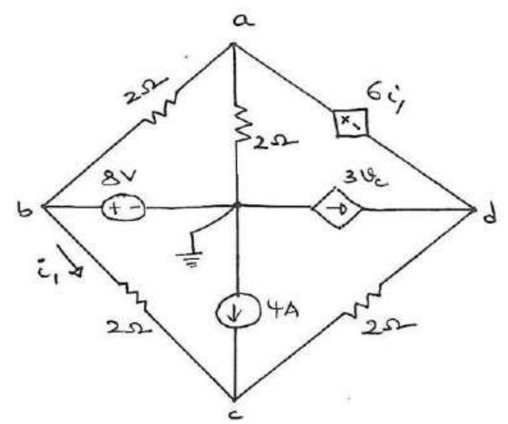
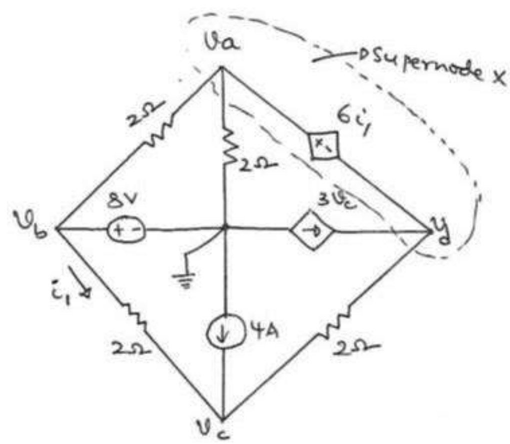


Fig.2

Solution:



From the circuit;

$v_b = 8$ V

Also, $v_a - v_d = 6\ i_1$

$i_1= (v_b-v_c)/2$ subs in above eqn. we get;

$v_a - v_d = 6\ (v_b - v_c)\ /2$

$\Rightarrow 2v_a - 2v_d = 6\ v_b - 6\ v_c$

$\Rightarrow 2v_a + 6v_c - 2v_d = 6\ v_b = 6(8) = 48$ ....(1)

Apply KCL at supernode X as shown;

$(v_a - v_b)/2 + v_a/2 - 3v_c + (v_d - v_c)/2 = 0$

$(v_a - 8)/2 + v_a/2 - 3v_c + (v_d - v_c)/2 = 0$

$\Rightarrow v_a - 8 + v_a - 6\ v_c + v_d - v_c = 0$

$\Rightarrow 2v_a - 7v_c + v_d = 8$..............(2)

Apply KCL at node C

$-4 + (v_c - v_d)/2 + (v_c - v_b)/2 = 0$

$\Rightarrow -8 + v_c - v_d + v_c - v_b = 0$

$\Rightarrow 2v_c - v_d = v_b + 8 = 16$........... (3)

Solving (1),(2) and (3), we get; $v_a = 9.142V$, $v_c = -1.142\ V$, $v_d = -18.28V$

and $v_b = 8V$ (given)

Star- delta (Δ) and delta (Δ) to star transformations

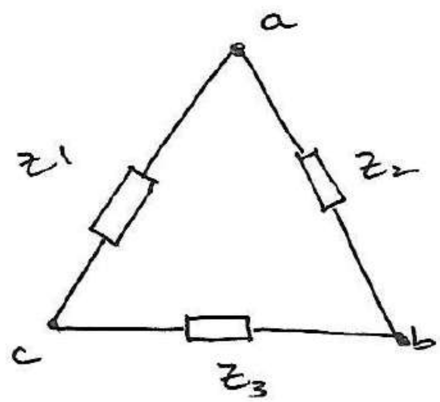


Fig25a delta arrangement

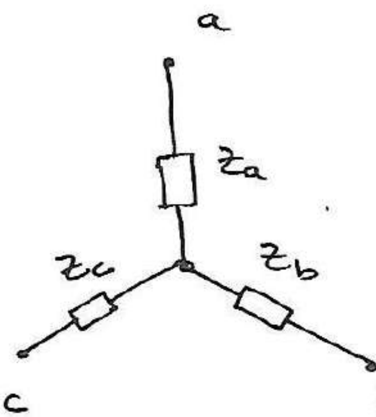


Fig.25bStar arrangement

(The positions of Z_1, Z_2 and Z_3 should be noted. Z_1 will appear between a and c; from there, going clockwise we see Z_2 and Z_3. The positions of Z_a, Z_b and Z_c should be noted. Z_a connected to vertex-a and centroid. Z_b connected to vertex-b and centroid. Z_c connected to vertex-c and centroid.)

Consider the above arrangements are equivalent; then;

$Z_{ac} = Z_1(Z_2+ Z_3) / (Z_1+Z_2+ Z_3) = Z_a + Z_c$..........(1)

Also,

$Z_{ab} = Z_2(Z_3+ Z_1) / (Z_1+Z_2+ Z_3) = Z_a + Z_b$ .... (2)

$Z_{bc} = Z_{3(} Z_1+ Z_2) / (Z_1+Z_2+ Z_3) = Z_b + Z_c$ .... (3)

Eqn. (1) –Eqn.(3)

$(Z_1Z_2 – Z_2Z_3)/(Z_1+Z_2+Z_3) = Z_a - Z_b$ .... (4)

Solving (2) and (4), we get, $Z_a = Z_1 Z_2 /(Z_1+Z_2 +Z_3)$........ (5)

Substituting (5) in (2), solving for Z_a, we get;

$Z_b = Z_2 Z_3 /(Z_1 +Z_2 +Z_3)$ ....(6)

Substituting (5) in (1), solving for Z_c, we get;

$Z_c = Z_1 Z_3 /(Z_1 + Z_2 + Z_3)$………(7)

Consider

$Z_a Z_b + Z_b Z_c + Z_a Z_c = (Z_1 Z_2^2 Z_3 + Z_1 Z_2 Z_3^2 + Z_1^2 Z_2 Z_3) / (Z_1 + Z_2 + Z_3)^2$

$Z_a Z_b + Z_b Z_c + Z_a Z_c = Z_1 Z_2 Z_3 / (Z_1 + Z_2 + Z_3)$ ………(8)

Eqn(8) / Z_b gives

$Z_1 = (Z_a Z_b + Z_b Z_c + Z_a Z_c) / Z_b$ .... (9)

Eqn(8) / Z_c gives

$Z_2 = (Z_a Z_b + Z_b Z_c + Z_a Z_c) / Z_c$ .... (10)

Eqn(8) / Z_a gives

$Z_3 = (Z_a Z_b + Z_b Z_c + Z_a Z_c) / Z_a$ .... (11)

15) Reduce the network shown in fig.26 to a single resistor between terminals a-b.

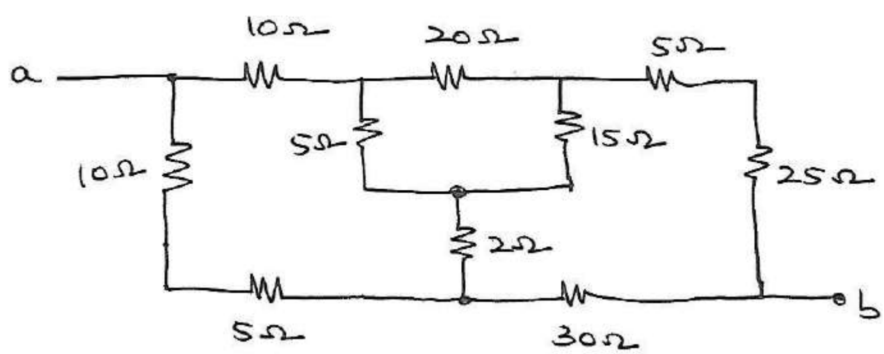
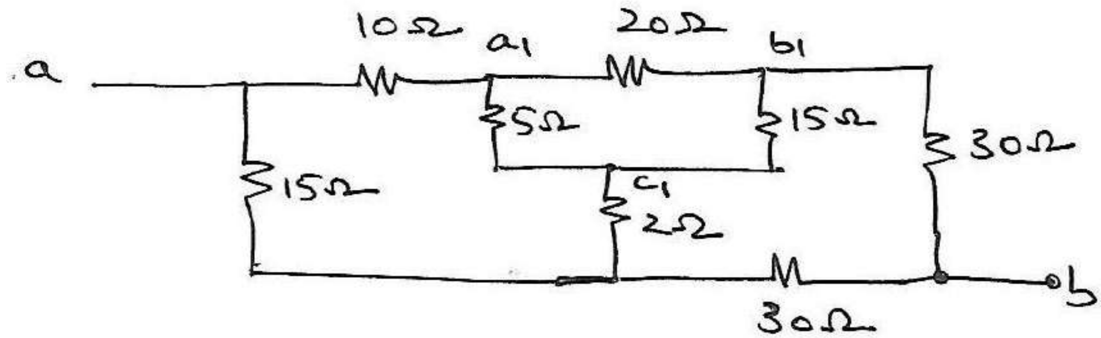


Fig.26

Solution:-



From the network above, we observe, 10Ω and 5Ω are in series and also 5Ω and 25Ω are in series. Therefore they are equivalently replaced by

15 Ω and 30 Ω as shown.

Identifying delta between the vertices a1-b1-c1;

We have $R_1 \rightarrow R_2 \rightarrow R_3$

as, 5Ω→20Ω→15Ω

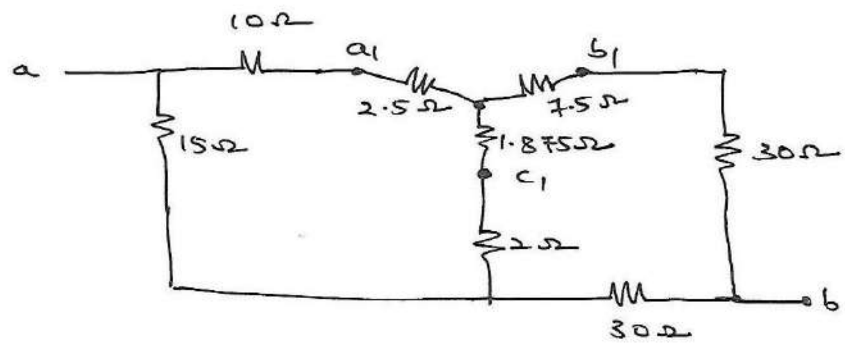
Corresponding star will have;

$R_a = R_1 R_2 /(R_1 + R_2 + R_3)$ = 100/40 =2.5 Ω (resistance connected to vertex a1)

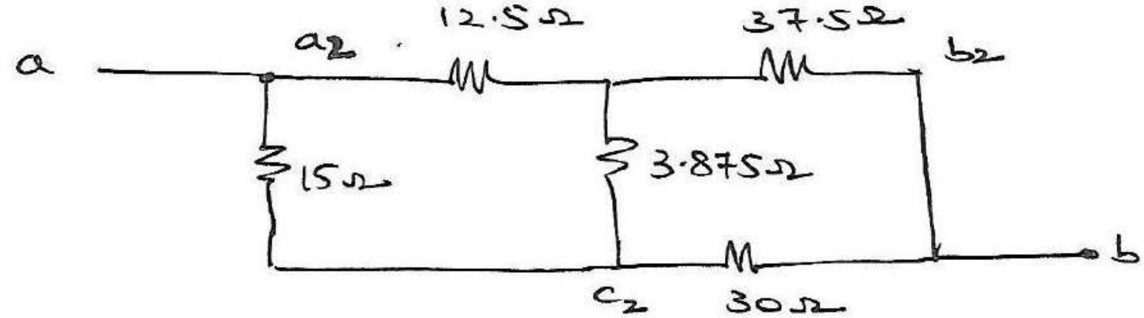
$R_b = R_2 R_3 /(R_1 + R_2 + R_3)$ = 300/40 =7.5 Ω (resistance connected to vertex b1)

$R_c = R_1 R_3 /(R_1 + R_2 + R_3)$ = 75/40 =1.875 Ω (resistance connected to vertex c1)

After replacing delta elements by corresponding star elements;



10Ω and 2.5 Ω appear in series. 30Ω and 7.5Ω appear in series. 2Ω and 1.875Ω appear in series. They are replaced by their equivalent resistances.



Identifying star between the vertices a2-b2-c2;

We have $R_a \rightarrow R_b \rightarrow R_c$

as, 12.5Ω→37.5Ω→3.875Ω

Corresponding delta will have;

$R_1 = (R_a R_b + R_b R_c + R_a R_c)/R_b$

$= [(12.5)(37.5) + (37.5)(3.875) + (3.875)(12.5)]/37.5$

$=662.5/37.5= 17.66\ \Omega$ (resistance connected b/n vertex a2 and c2)

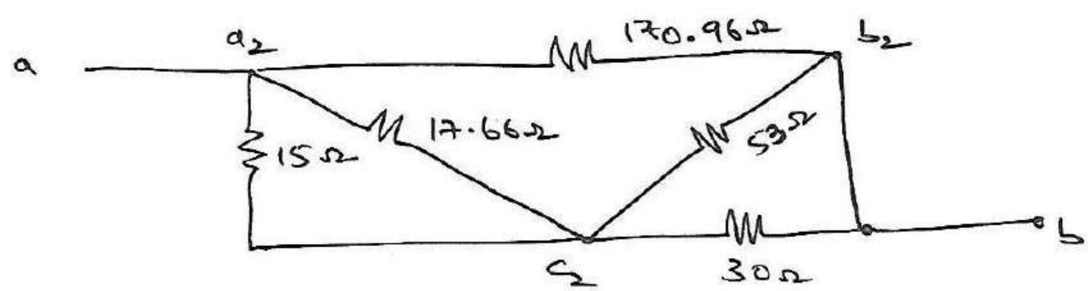
$R_2 = (R_a R_b + R_b R_c + R_a R_c)/R_c$

$=662.5/3.875= 170.96\ \Omega$ (resistance connected b/n vertex a2 and b2)

$R_3 = (R_a R_b + R_b R_c + R_a R_c)/R_a$

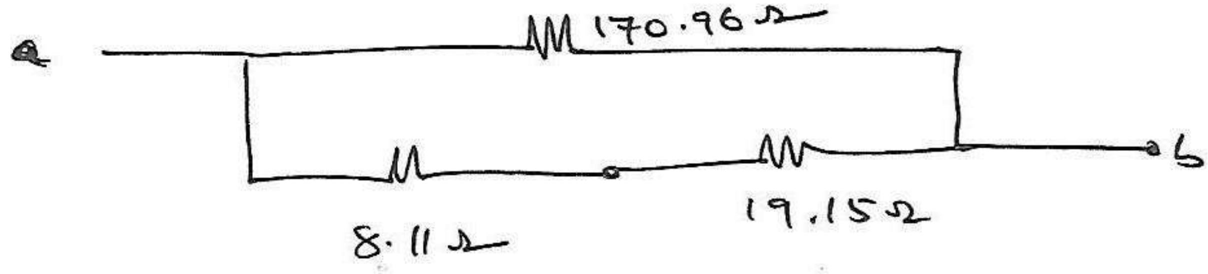
$=662.5/12.5= 53\ \Omega$ (resistance connected b/n vertex b2 and c2)

After replacing star elements by corresponding delta elements;

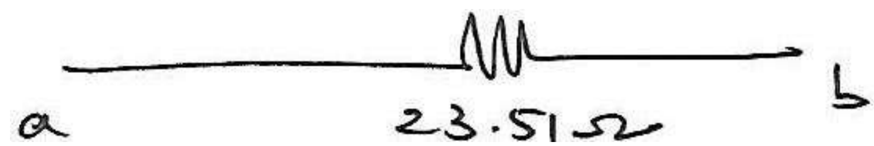


15||17.66 = 8.11Ω

53||30 =19.15Ω



Therefore, R_{ab} = (19.15 +8.11)||170.96 = 23.51Ω



Q16) Find the current I in the network shown in fig.27, by reducing the network to contain a source and and a single series impedance.

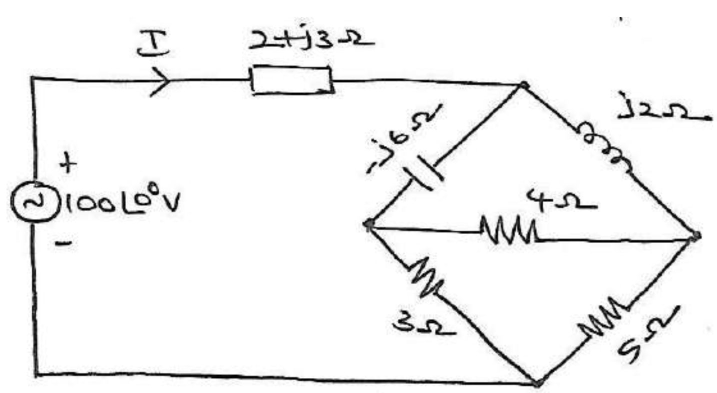
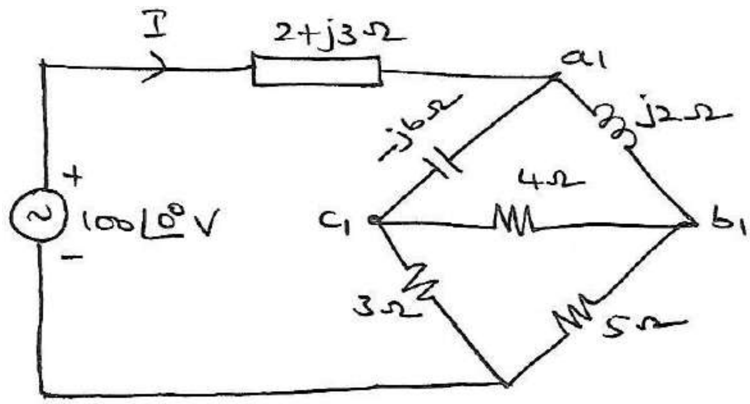


Fig.27

Solution:-



Identifying delta between the vertices a1-b1-c1;

We have $Z_1 \rightarrow Z_2 \rightarrow Z_3$

as, -j6Ω→j2Ω→4Ω

Corresponding star will have;

$Z_a = Z_1 Z_2 /(Z_1 + Z_2 + Z_3) = (-j6)(j2)/(4-j4) = 1.5 + j1.5\Omega$

(Impedance connected to vertex a1)

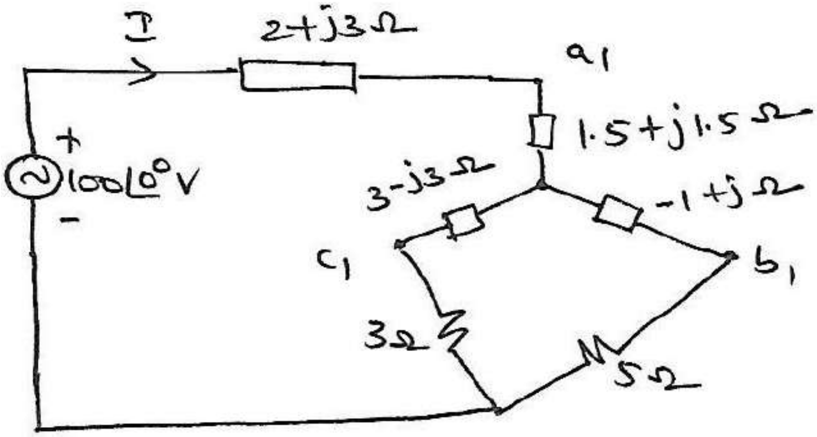
$Z_b = Z_2 Z_3 /(Z_1 + Z_2 + Z_3) = (j2)(4)/(4-j4) = -1 + j\ \Omega$

(Impedance connected to vertex b1)

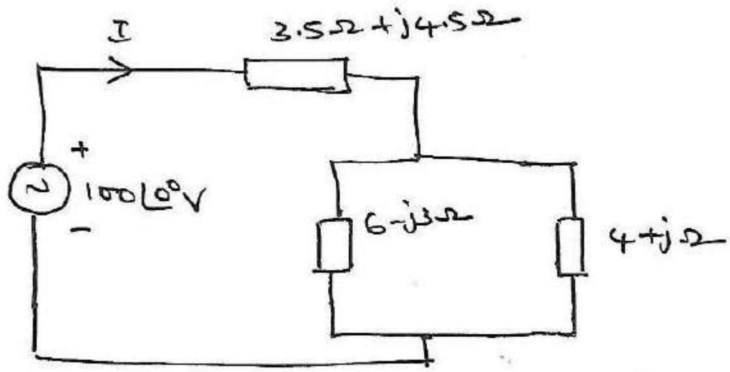
$Z_c = Z_1 Z_3 /(Z_1 + Z_2 + Z_3) = (-j6)(4)/(4-j4) = 3-j3\ \Omega$

(Impedance connected to vertex c1)

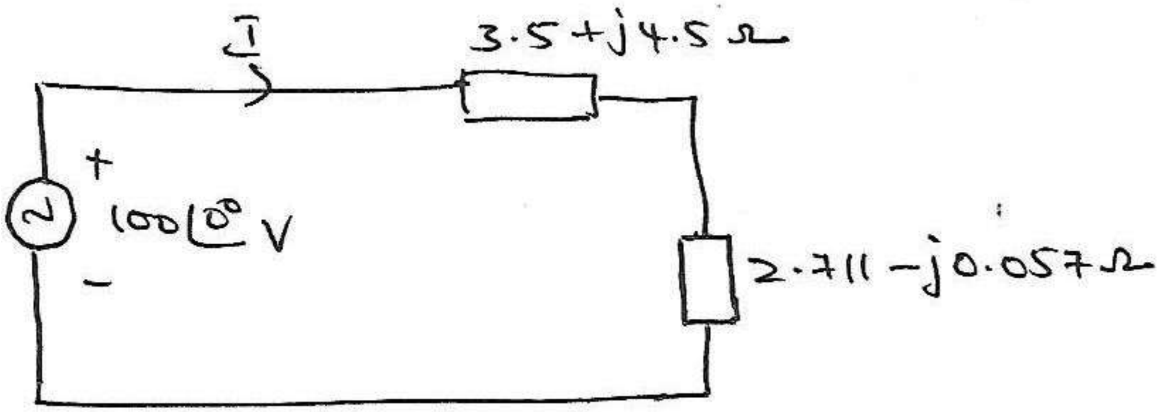
After replacing delta elements by corresponding star elements;



The series impedances are replaced by equivalent impedances



(6-j3) // (4+j) = 2.711 - j 0.057Ω



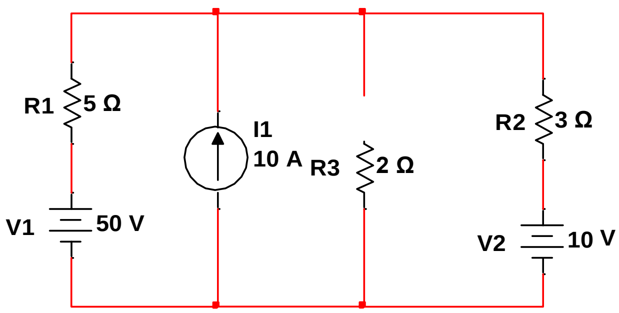
The single series impedance value , Z = (3.5 + j4.5) + (2.711- j 0.057)

$$Z = 6.211 + j\,4.443\ \Omega$$

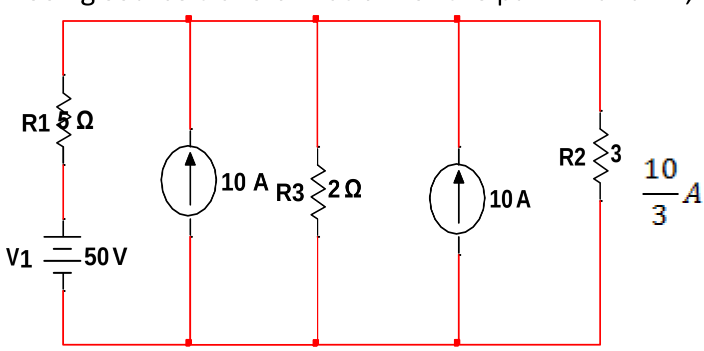
Therefore, $I = 100/Z = 100/(6.211 + j4.443) = 13.09\angle{-35.57°}$ A

Additional Problems and Solutions

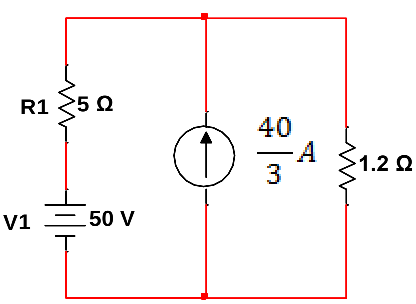
1) Using source transform, find the power delivered by the 50V source in the circuit shown:-



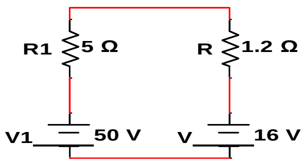
Solution: - Using source transformation for the pair V2 and R2, we get,



Adding the parallel current sources and obtaining equivalent resistance of R3 and R2, we have,



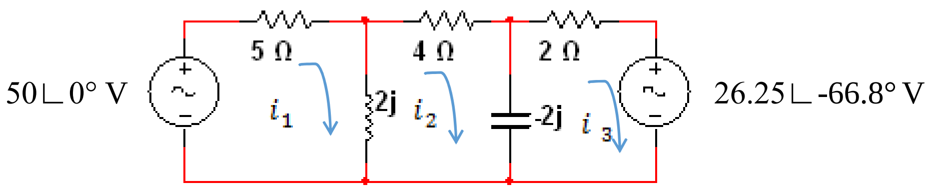
Converting the current source back to voltage source,



If I is the current in the circuit, $I = \frac{50-16}{6.2} = 5.48A$

Therefore Power delivered by 50V source is $P = I \times 50 = 5.48 \times 50 = 274.19W$.

2) Find the current through 4Ω in the network shown:



Solution: - Applying KVL to mesh 1 (mesh with i_1)

$$5i_1 + 2j(i_1 - i_2) - 50 = 0$$
$$\Rightarrow (5+2j)i_1 - (2j)i_2 = 50$$

Applying KVL to mesh 2

$$4i_2 - 2j(i_2 - i_3) + 2j(i_2 - i_1) = 0$$
$$\Rightarrow (-2j)i_1 + (4)i_2 + 2j(i_2 - i_1) = 0$$

Applying KVL to mesh 3

$(2j)i_3 + (26.25$ - $-(2j)(\ i_3 - i_2) = 0$

$\Rightarrow (2-2j)i_3 + (2j)i_2 = (26.25$ - = -10.39 + (24.12) j

Matrix form

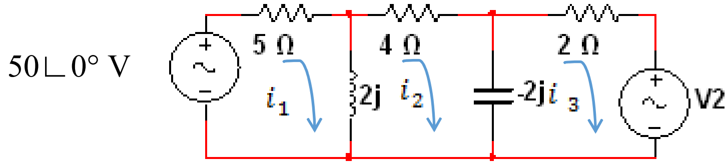
$$\begin{bmatrix} 5+2j & -2j & 0 \\ -2j & 4 & j-2 \\ 0 & 2j & 2-2j \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \\ -10.39+24.12j \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 5+2j & -2j & 0 \\ -2j & 4 & 2j \\ 0 & 2j & 2-2j \end{vmatrix} = 84 - 24j$$

$$\Delta i_2 = \begin{vmatrix} 5+2j & 50 & 0 \\ -2j & 0 & 2j \\ 0 & -10.39+24.12j & 2-2j \end{vmatrix} = 399.64 + 400.38j$$

$$i_2 = \frac{\Delta i_2}{\Delta} = 6.47 \quad \text{A}$$

3) Find the value of V2 if the current through 4Ω is zero.



Solution: - Given i_2=0

Applying KVL to mesh 3 (mesh with i_3), we get

$2i_3 + V2 - 2j(i_3) = 0$

$\Rightarrow \text{V2} = (-2 + 2\text{j})i_3$

Applying KVL to mesh 2,

$4i_2 - 2j(i_2 - i_3) + 2j(i_2 - i_1) = 0$

$\Rightarrow i_3 = i_1$

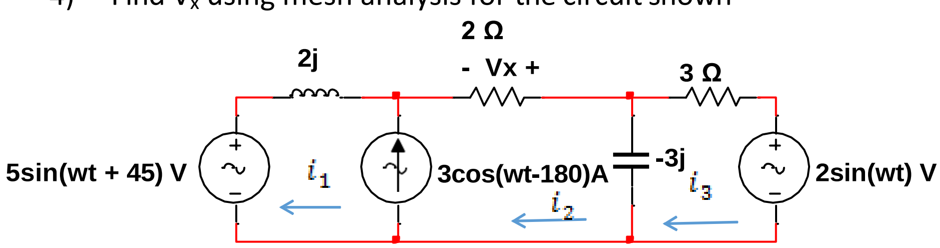
Applying KVL to mesh 1,

$5i_1 + 2j(i_1) = 50$

$\Rightarrow i_1 = 9.28\angle - 21.8°\ \text{A} = i_3$

Therefore, **V2 =** $\boldsymbol{i_3(-2 + 2j) = 26.26\angle 113.19°}$ **V**

4) Find V_x using mesh analysis for the circuit shown



Solution: - From the circuit $V_x = -2i_2$

Applying concept of super mesh, $i_2 - i_1 = 3\angle - 90°$

Therefore, $i_1 = -3\angle - 90° + i_2$

Remove the arm of the current source and apply kvl,

$(2j)i_1 - V_x - (3j)(i_2 - i_3) - 5\angle 45° = 0$

$\Rightarrow (2 - j)i_2 + (3j)i_3 = 9.535 + j3.535$

Applying KVL to mesh with i_3

$$(3-3j)i_3 + (3j)i_2 = -2$$

Therefore $\Delta = \begin{vmatrix} 2-j & j3 \\ j3 & 3-j3 \end{vmatrix} = 12-9j$

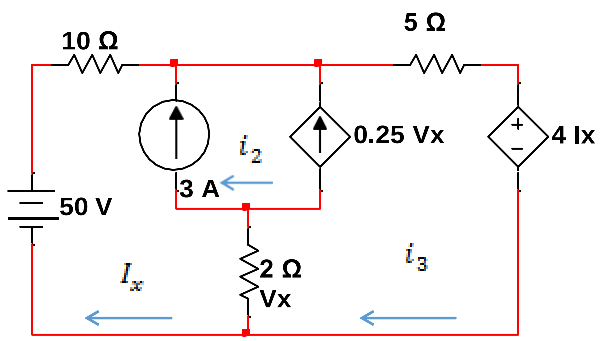
$$\Delta i_2 = \begin{vmatrix} 9.535+j3.535 & j3 \\ -2 & 3-j3 \end{vmatrix} = 39.21 - j12$$

$$i_2 = \frac{\Delta i_2}{\Delta} = 2.73\angle 19.85^\circ \text{ A}$$

$$V_x = -2(2.73\angle 19.85^\circ)\text{v}$$

Therefore, $\boldsymbol{V_x = 5.49\angle -160.15^\circ}$ **V**

5) Find V_x and I_x in the circuit shown using mesh analysis



Solution: - From the circuit $V_x = 2(I_x - i_3)\ldots\ldots(1)$

Also from the circuit $i_2 - I_x = 3 \ldots\ldots\ldots(2)$; $i_3 - i_2 = 0.25\, V_x \ldots\ldots\ldots(3)$

Substituting equations 1 and 2 in 3, we get

$6(i_3 - I_x) = 12 \Rightarrow i_3 - I_x = 2$……(4)

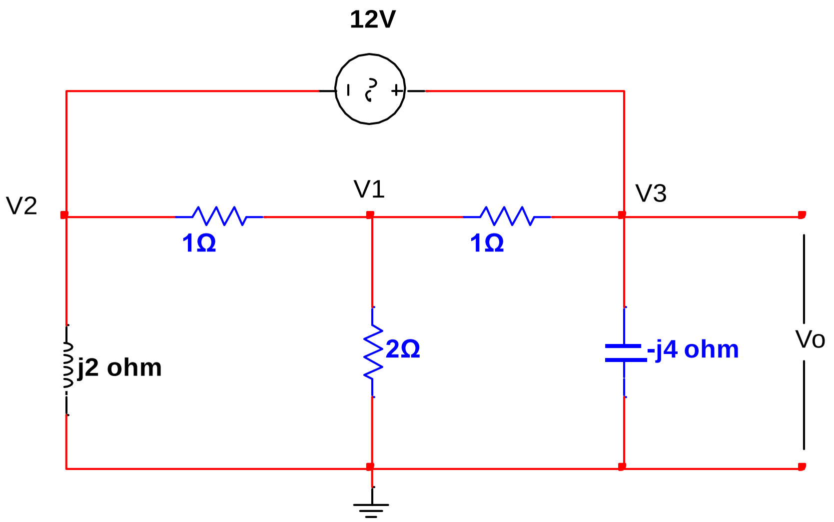
Removing the arm containing common current source and applying KVL, we get

$14I_x + 5i_3 = 50 \ldots\ldots(5)$

Solving equations 4 and 5, we get $\boldsymbol{I_x = 2.1A}$

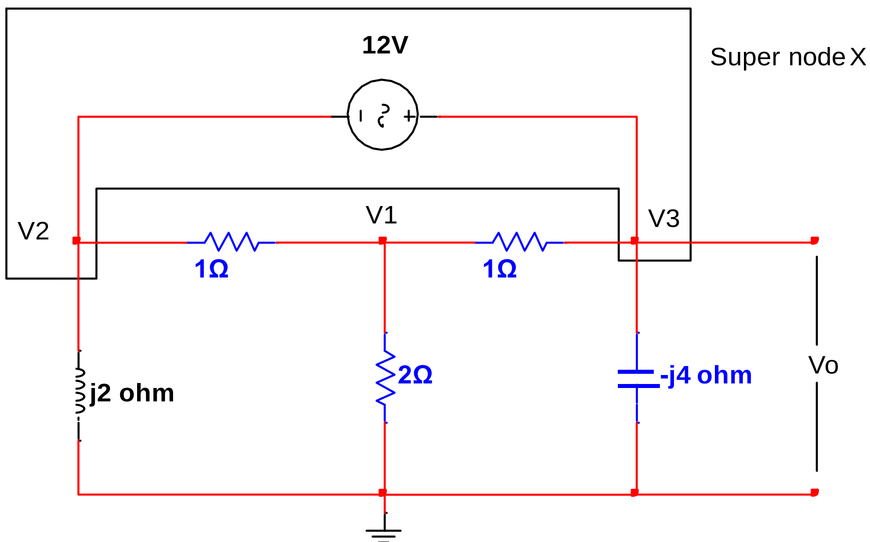
Therefore, $\boldsymbol{V_x = -4V}$.

6) Use node analysis to find V_0 in the circuit shown below



From the circuit,

$$V_0 = V_3; V_0 - V_2 = 12V \text{ ----- (1);}$$



Applying KCL to super node X,

$$\Rightarrow \frac{V_2}{j2} + \frac{V_2 - V_1}{1} + \frac{V_0 - V_1}{1} + \frac{V_0}{-j4} = 0$$

$$\Rightarrow \frac{-jV_2}{2} + V_2 - V_1 + V_0 - V_1 + \frac{jV_0}{4} = 0$$

$$\Rightarrow -2jV_2 + 4V_2 - 4V_1 + 4V_0 - 4V_1 + jV_0 = 0$$

⇨ $(4+j)V_0 - 8V_1 + (4-j2)(V_0 - 12) = 0$ (From (1))

⇨ $4V_0 + jV_0 - 8V_1 + 4V_0 - j2V_0 - 48 + j24 = 0$

⇨ $(8-j)V_0 - 8V_1 = 48 - j24$ ---------(2)

Applying KCL at V_1,

⇨ $\frac{V_1}{2} + \frac{V_1 - V_0}{1} + \frac{V_1 - V_2}{1} = 0$

⇨ $V_1 + 2V_2 - 2V_0 + 2V_1 - 2V_2 = 0$

⇨ $-2V_0 + 5V_1 - 2V_2 = 0$

⇨ $-2V_0 + 5V_1 - 2(V_0 - 12) = 0$

⇨ $-4V_0 + 5V_1 = -24$ -------------(3)

Using Cramer's rule,

$\Delta = \begin{vmatrix} 8-j & -8 \\ -4 & 5 \end{vmatrix}$

$\Delta = 5(8-j) - 32$

$\Delta = -5j + 8$

$\Delta V_0 = \begin{vmatrix} 48-j24 & -8 \\ -24 & 5 \end{vmatrix}$

$\Delta V_0 = (48 - j24)5 - 192$

Δ

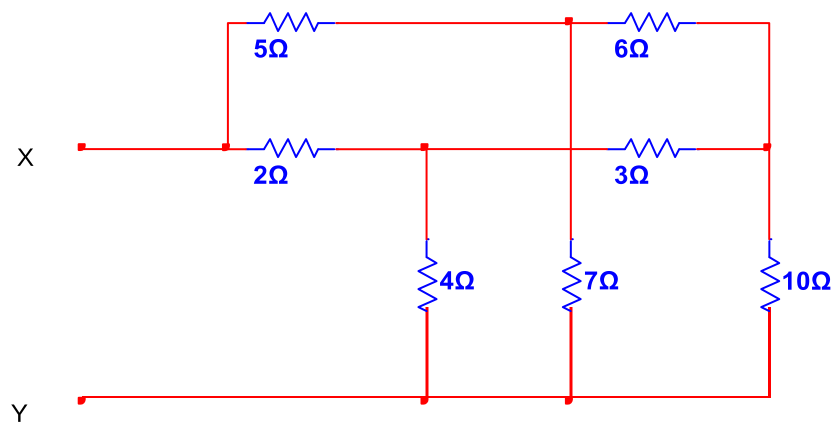
$V_0 = \frac{\Delta V_0}{\Delta}$

$V_0 = -j120 + 48$

$\therefore\ V_0 = 13.69V\ @ - 36.19°$

W.K.T,

7) Find the equivalent resistance between the terminals X and Y



Solution:-

Star 1:- $R_a = 2;\ R_b = 3;\ R_c = 4;$

Corresponding Delta will have,

$$R_1 = \frac{R_aR_b + R_bR_c + R_aR_c}{R_b}$$

$\therefore\ R_1 = 8.66\Omega$

Similarly,

$R_2 = \frac{26}{4}$ = 6.5 Ω

$R_3 = \frac{26}{3}$ = 13 Ω

Now consider star 2:- $R_a = 5;\ R_b = 6;\ R_c = 7;$

Corresponding Delta will have,

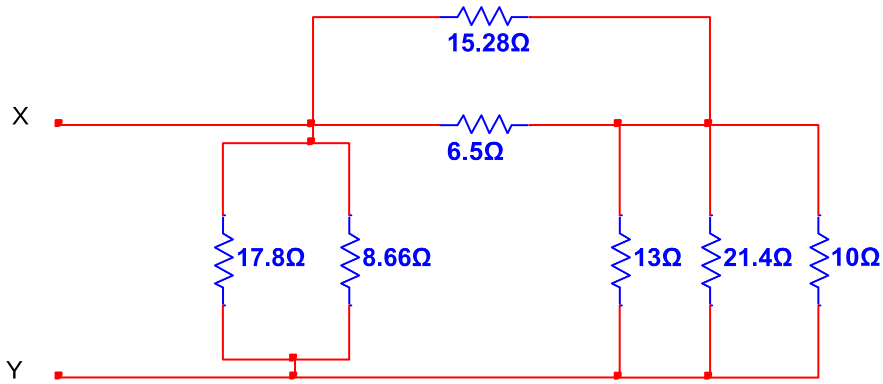
$$R_1 = \frac{R_aR_b + R_bR_c + R_aR_c}{R_b}$$

$\therefore\ R_1 = 17.8\Omega$

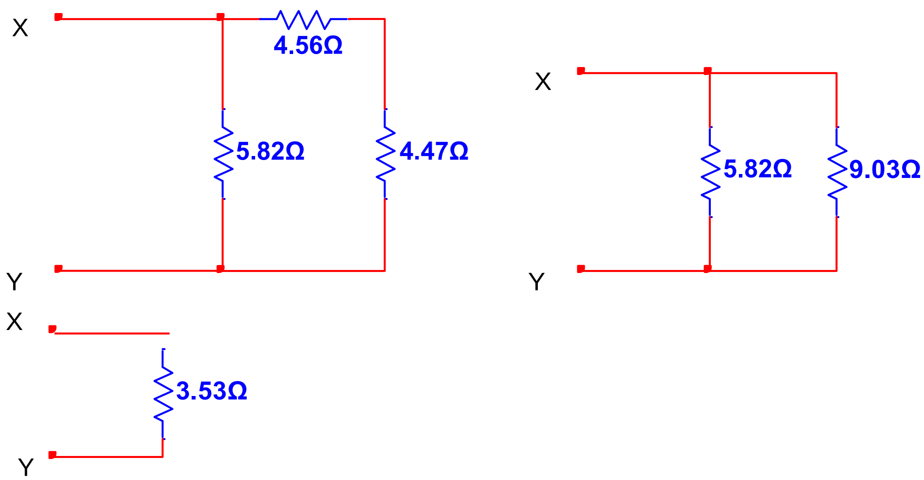
Similarly,

$R_2 = \frac{107}{7}$ = 15.28 Ω

$R_3 = \frac{107}{5}$ = 21.4 Ω

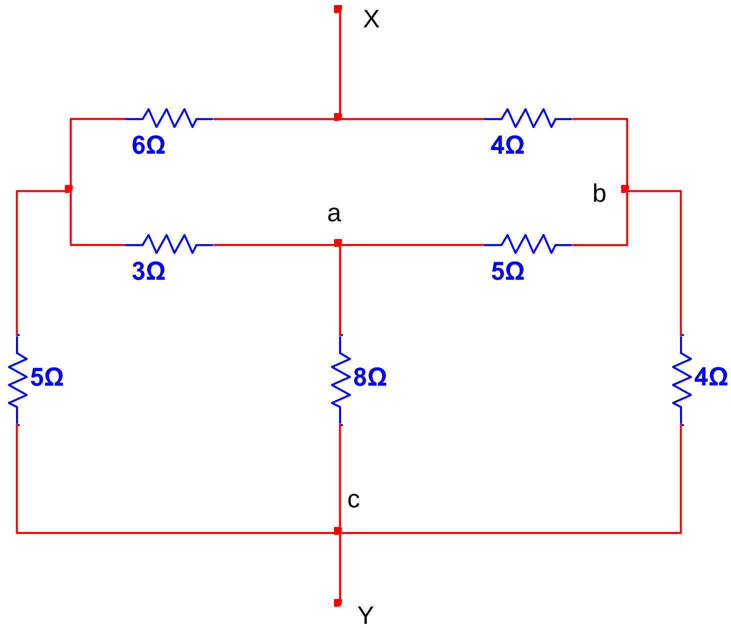


This circuit can be reduced now using parallel and series combination of resistors as show below.



Therefore the equivalent resistance between X & Y = 3.53 Ω

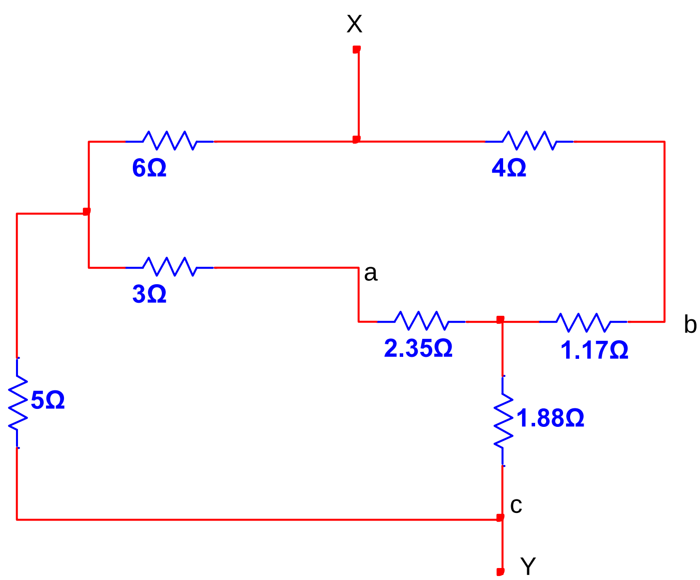
8) Determine the equivalent resistance between the terminals X & Y



Solution:

Consider the Delta $R_1 = 8;\ R_2 = 5;\ R_3 = 4;$

It can be replaced with the circuit shown below



Where, $R_a = \frac{R_1 R_2}{R_1+R_2+R_3}$

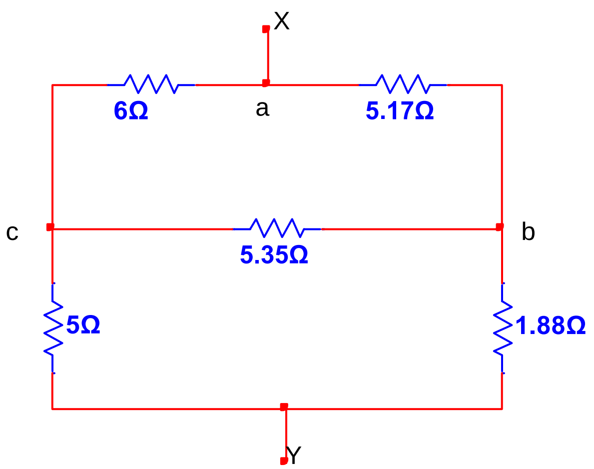
$\therefore R_a = 2.35\Omega$

Similarly,

$R_b = 1.17\Omega$

$R_c = 1.88\Omega$

The above circuit can be written as,

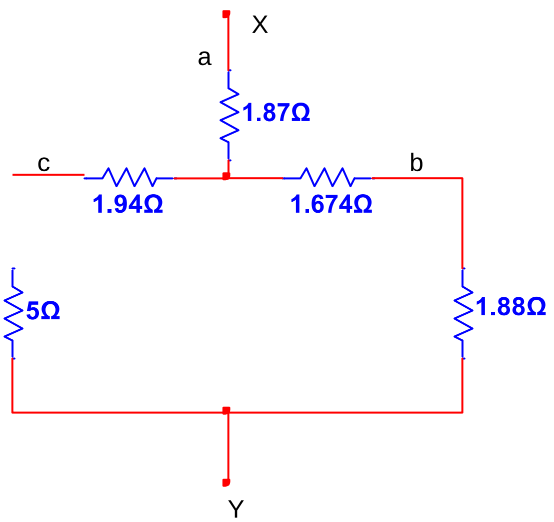


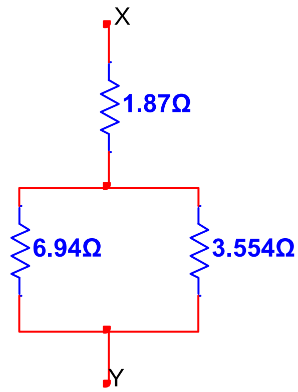
Consider the Delta, $R_1 = 6;\ R_2 = 5.17;\ R_3 = 5.35;$

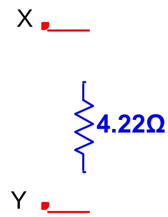
$\therefore R_a = 1.877\Omega$

$\therefore R_b = 1.674\Omega$

$\therefore R_c = 1.94\Omega$



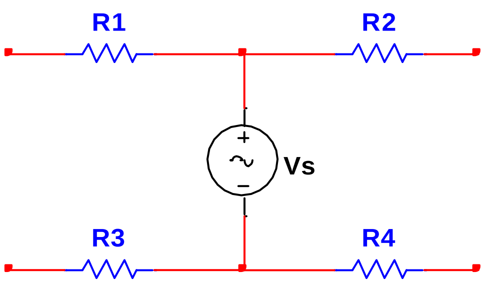




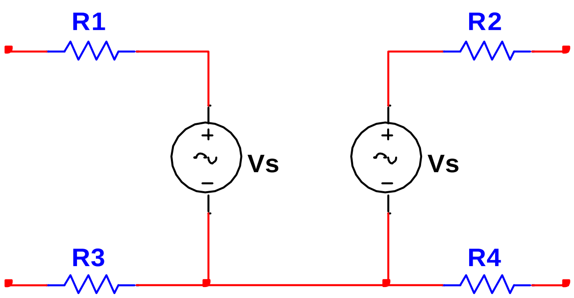
Therefore the equivalent resistance between X & Y = 4.22 Ω

Source Shifting:

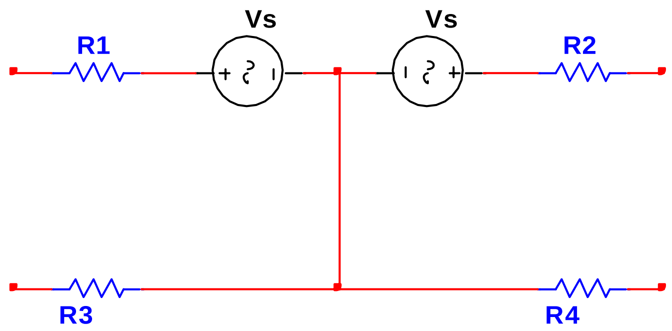
(i) Voltage Source Shifting:-



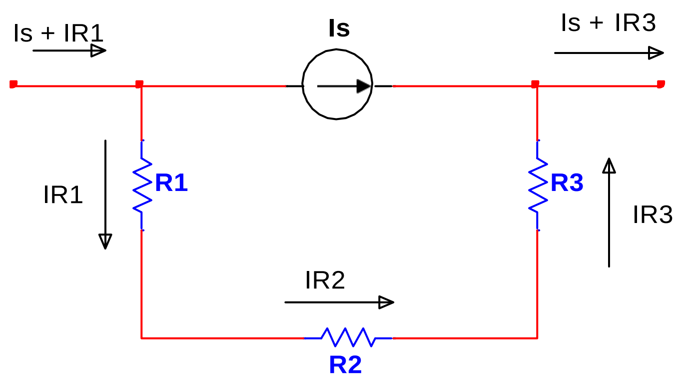
The above circuit can be written as,



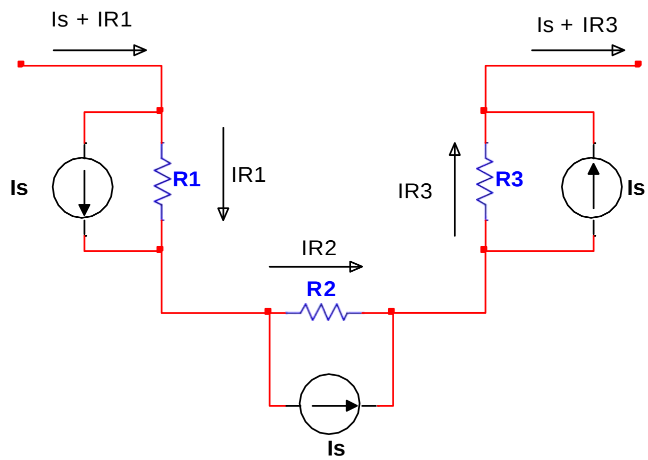
Which is equivalent to,



(ii) Current Source Shifting:-

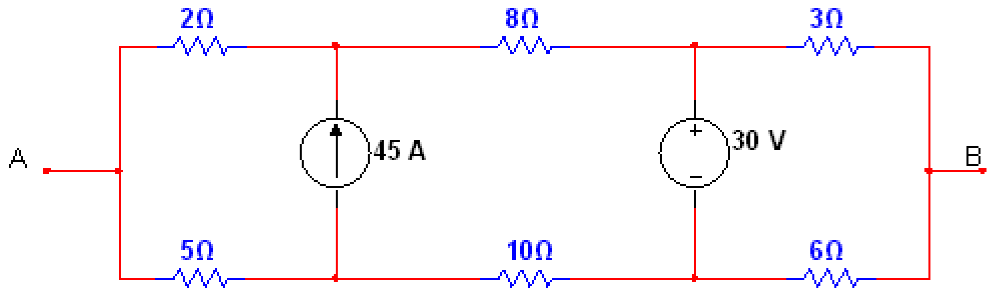


The above circuit can be redrawn as,



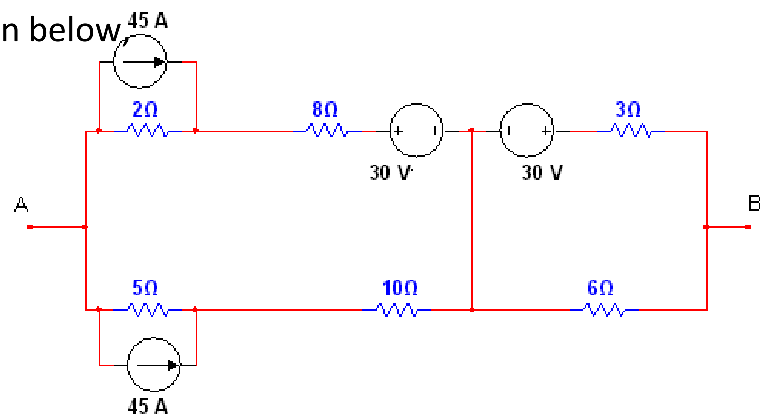
Problems on Source Shifting & Source Transformation:-

1) Reduce the network shown to a single voltage source in series with a resistance using source shifting and source transformation.

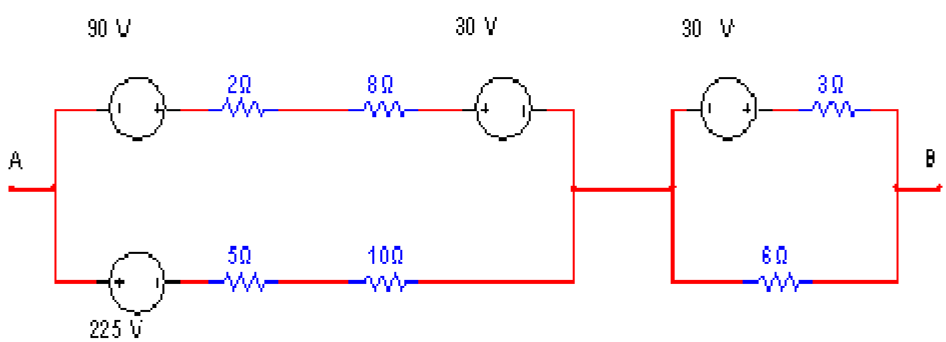


Solution:-

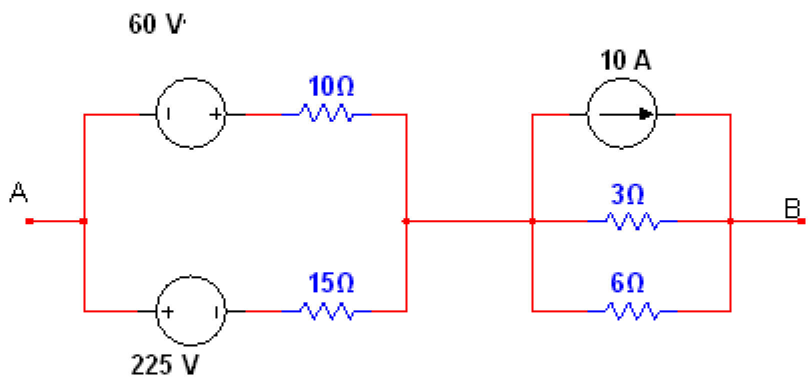
Use Source shifting property on both the sources and rewrite the circuit a shown below,



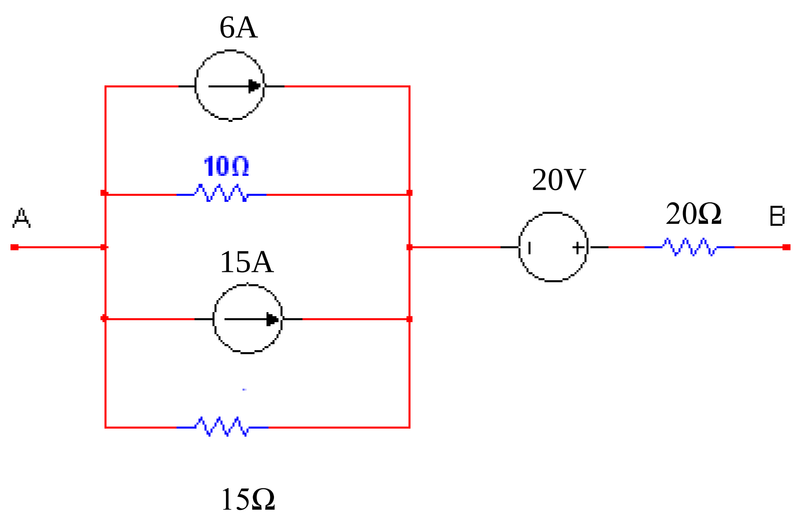
Now using Source transformation we get,

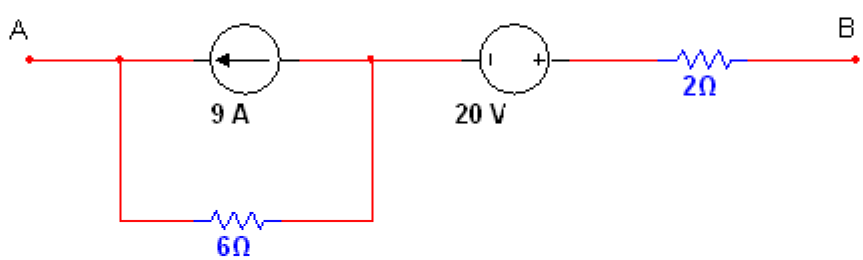


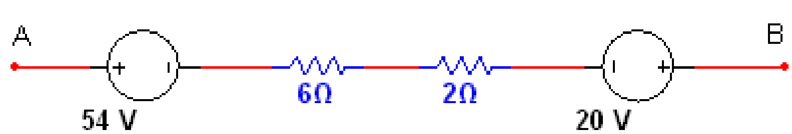
After simplifying the above circuit and applying Source transformation again, we get,

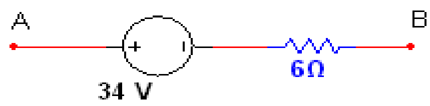


Which can be further simplified using Source transformation yet again,

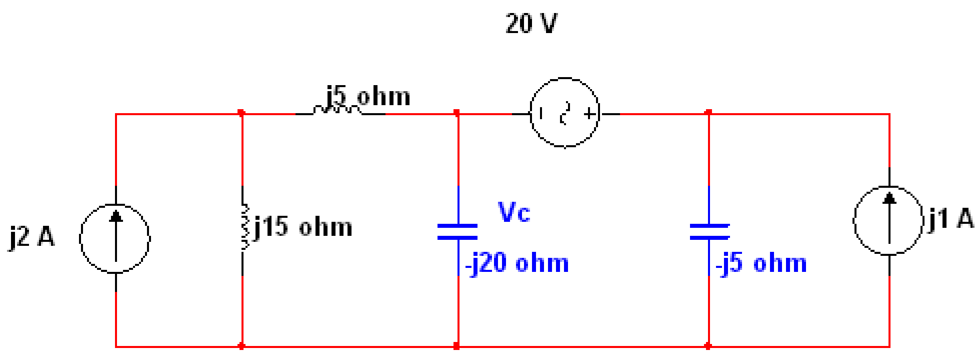




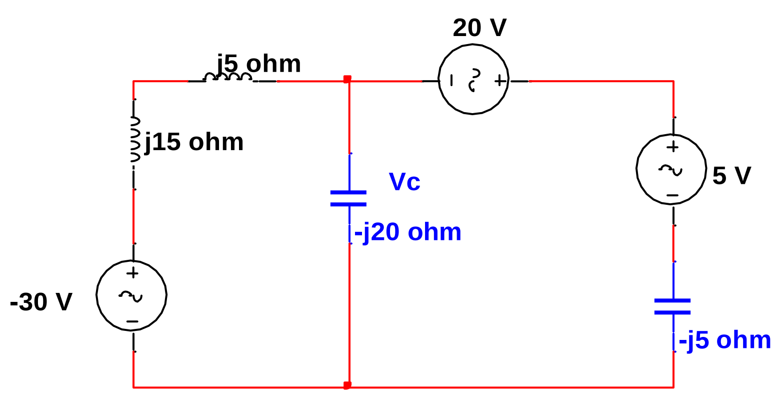


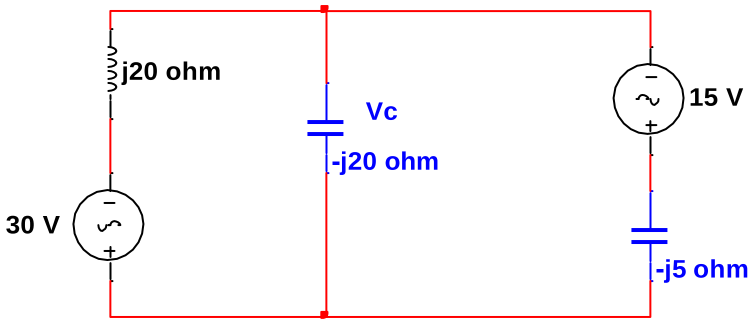


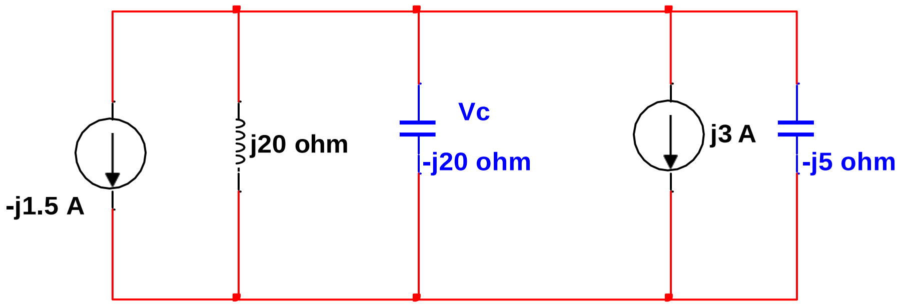
2) Find the voltage across the capacitor of 20Ω reactance of the network.

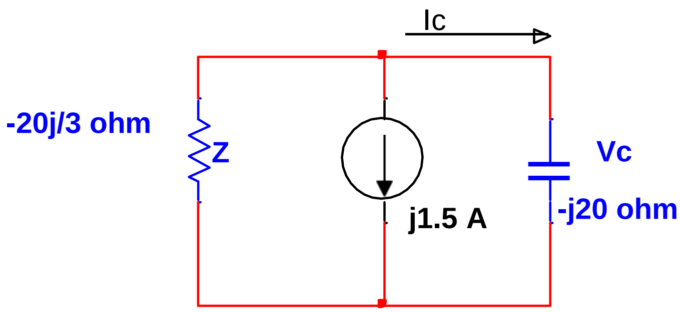


Solution:- Using Source Transformation,




j20 ohm
Vc
-j20 ohm
15 V
30 V
-j5 ohm


j20 ohm
Vc
-j20 ohm
j3 A
-j5 ohm
-j1.5 A


Ic
-20j/3 ohm
Z
Vc
j1.5 A
-j20 ohm

From the above circuit,

$$I_c = \frac{(-j1.5)(-j6.67)}{(-j26.67)}$$

$$\therefore\ I_c = -j(0.375)\ A$$

$$\therefore\ V_c = I_c(-j20)$$

$$\therefore\ V_c = \ -7.5\ V$$

NETWORK ANALYSIS (18EC32)

Syllabus:-

Module -2

Network Theorems: Superposition, Millman‘s theorems, Thevinin‘s and Norton‘s theorems, Maximum Power transfer theorem

Unit 4 → Thevinin's theorem
→ Norton's theorem.
→ Maximum power transfer theorem.

Thevinin theorem:-

→ Any Linear, active, bilateral n/w with two output terminals AB as shown in fig1. can be represented by an equivalent voltage source V_{th} in series with an equivalent impedance Z_{th} b/w the terminals A-B as shown in fig2.

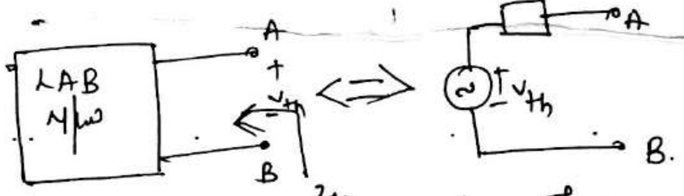


fig1. fig2. Thevinin equivalent N/w.

where V_{th} is the Thevinin's Voltage which is the open ckt voltage measured across the terminals AB. and Z_{th} is the thevinin's equivalent impedance which is the total impedance measured across the open ckted terminals AB with all the internal sources set to Zero [voltage source (SC) Current source (OC)].

Norton theorem:-

Any Linear, bilateral, active n/w with two output terminal A and B as shown in fig. Can be represented by an equivalent Current Source I_N in parallel with an equivalent impedance Z_N b/w the terminals AB as in fig

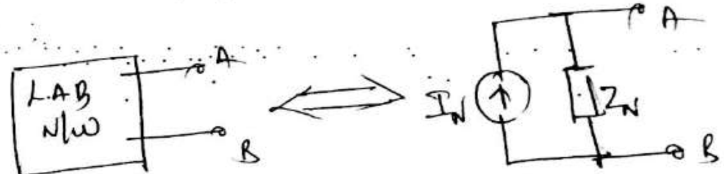
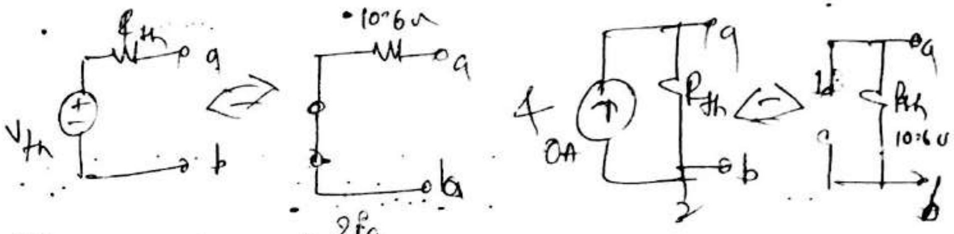


fig: Norton's equivalent ckt.

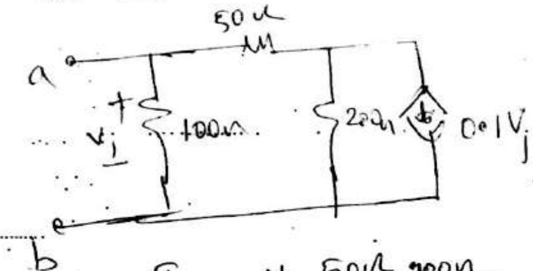
where I_N is an Norton's equivalent Current which is the short circuit current through AB, and Z_N is Norton's impedance (Z_{th}) which is the equivalent impedance measured across the open ckted terminals with all the internal sources said to be zero. [VS → SC, CS → OC]

$V_{th} = 0V + I_N = 0A$

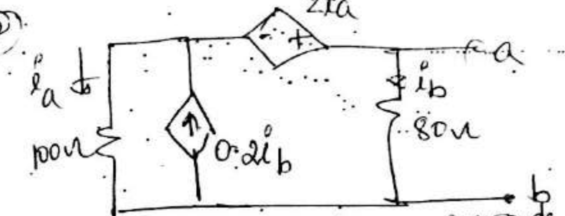


(Q) Case 3

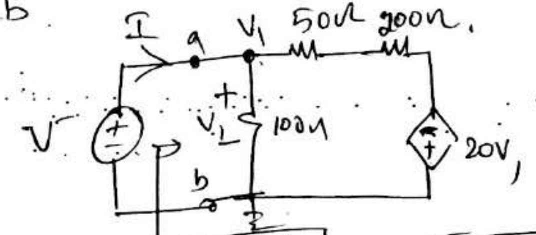
Det the Thevenin and Norton equivalent across AB.



(2)



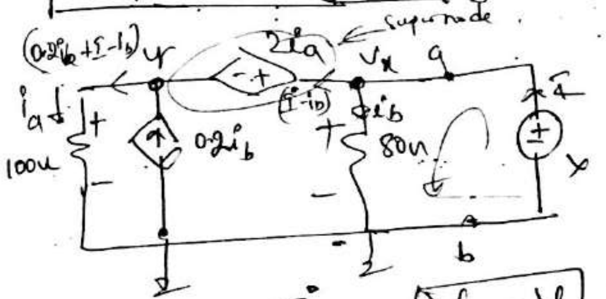
soln:-



KCL @ a

$$\boxed{R_{th} = \frac{V}{I}} \qquad \boxed{V_1 = V}$$

soln:-



$i_a = I - 0.8i_b$

$I = i_a + 0.8 i_b$

$V = 80 i_b$

$V = 80[$

Nodal

$$-I + \frac{V}{100} + \frac{V + 20V_1}{250} = 0$$

$$\boxed{V = V_1}$$

$$\frac{V}{100} + \frac{V}{250} + \frac{20V}{250} = I$$

$$\Rightarrow \frac{V}{I} = R_{th} = \frac{1}{\frac{1}{100} + \frac{21}{250}} = 10.6\Omega$$

V_x = volts $\quad V_x \in$ volts $\quad \boxed{V_a = V}$ volts

$V_{th} = 0V; \; I_N = 0A \quad R_{th} = \frac{V}{I}$

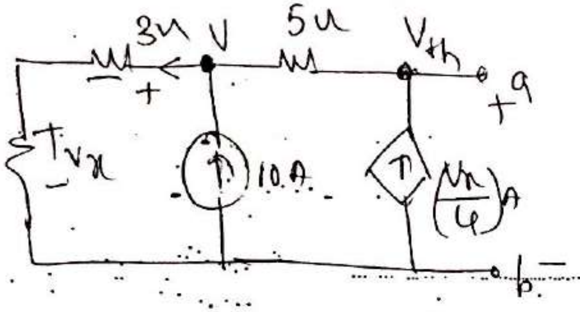
$V_x - V_y = 2i_a$ and $i_a = \frac{V_y}{100}$

$$V_x - V_y = 2\left(\frac{V_y}{100}\right) \leftarrow (1)$$

$$\boxed{V = \left[\frac{1}{50} + 1\right] V_y = \frac{51}{50} V_y}$$

$100 i_a + 2i_a - 80 i_b = 0 \Rightarrow 102 i_a = 80 i_b$.

Step 1: find V_{th}.



$V_x = V_{2\Omega} = I_{2\Omega} \cdot 2 = \left(\frac{V-0}{3+2}\right) \cdot 2$

$\boxed{V_x = \frac{2}{5} V}$ volts

⇒ Nodal

$\frac{V}{5} - 10 + \frac{V - V_{th}}{5} = 0$

$2V - V_{th} = 50 \longrightarrow (1)$

⇒ Nodal $\frac{V_{th} - V}{5} - \frac{V_x}{4} = 0$

$\frac{V_{th} - V}{5} - \frac{2V}{5 \times 4} = 0$

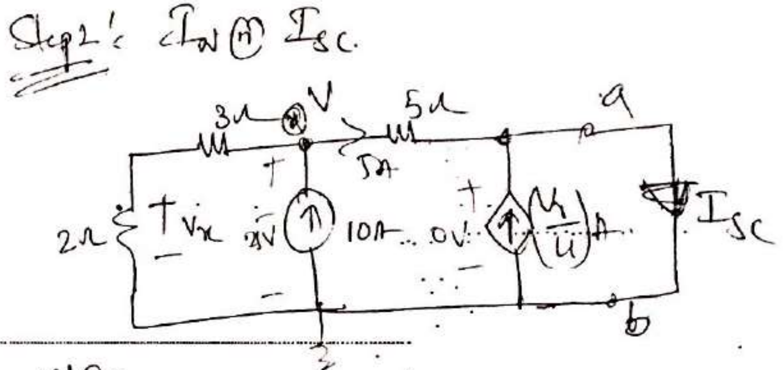
$5^{-1} V_{th} - 5^{-1} V - 10^{-1} V = 0$

$\boxed{V = \frac{2}{3} V_{th}} \longrightarrow (2)$

$V - \frac{2}{3} V_{th} = 0 \longrightarrow (2a)$

solving (1) & (2a) $\boxed{V_{th} = 150}$ volts

Step 2: I_N (or) I_{sc}



KCL@a $\frac{V}{5} - 10 + \frac{V}{5} = 0$

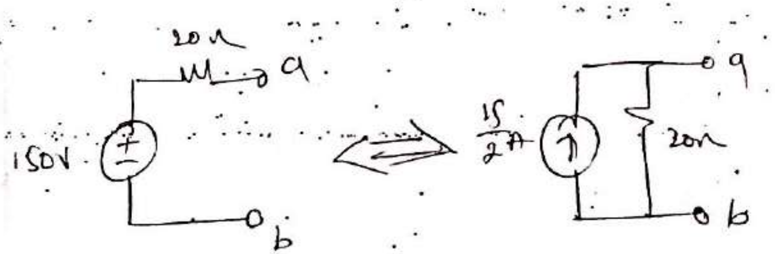
$\boxed{V = 25}$ volts $\boxed{V_x = 10}$ volts

$\cancel{I_{sc} = \frac{25}{5} = 5A}$

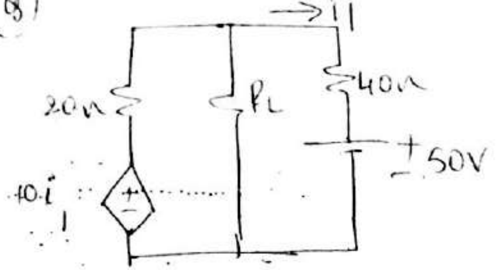
$I_{sc} = 5 + \frac{V_x}{4} = 5 + \frac{10}{4}$

$= 15/2$ A

$R_{th} = \frac{V_{th}}{I_{sc}} = \frac{150}{(15/2)} = 20\,\Omega$



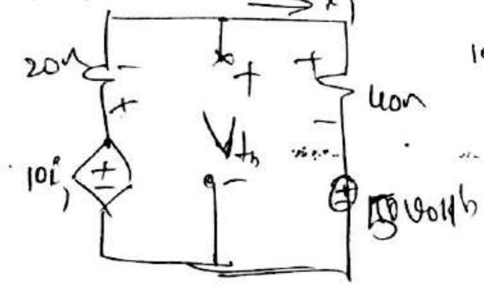
(8)



Det the thevenins + Nortons equivalent across the R_L.

Solution: $R_{th} = \frac{V_{th}}{I_N} = \frac{V_{th}}{I_{sc}}$

Step 1: for $V_{th} = ?$



$10i_1 - 20i_1 - 40i_1 + 50 = 0$

$-50 i_1 = 50$

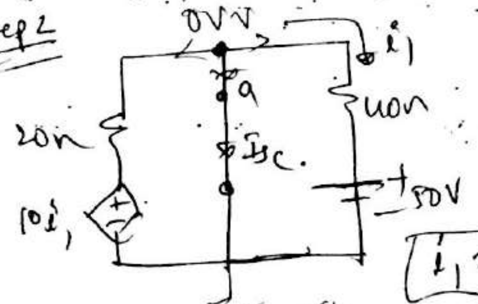
$\cancel{i_1 = 3A}$

$\boxed{i_1 = -1A}$

$V_{th} = 40(-1) + 50$

$\boxed{V_{th} = 10}$ volts

Step 2



$\frac{V - 10i_1}{2} + I_{sc} + \frac{V-50}{40} = 0$

$i_1 = \frac{V-50}{40}$

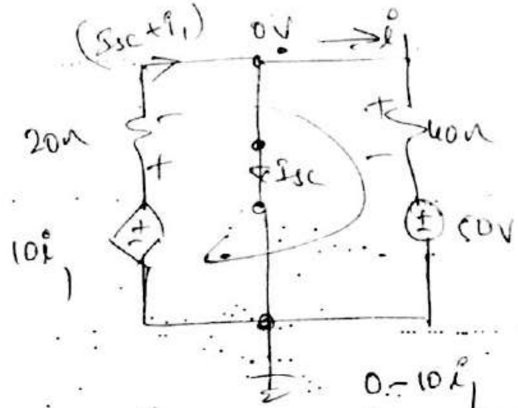
$V = 0$ volts

$\boxed{i_1 = -5/4\,A}$

$\frac{0+5/4}{20} = \frac{5}{80}A$

$\frac{5}{80} + I_{sc} - 5/4 = 0$

$I_{sc} = 5/4 - 5/80$



$\frac{0-10i_1}{20} + I_{sc} + i_1 = 0$

$I_{sc} =$

$i_1 = \frac{0-50}{40} = -5/4\,A$

$10i_1 - 20(I_{sc}+i_1) - 40(i_1) - 50 = 0$

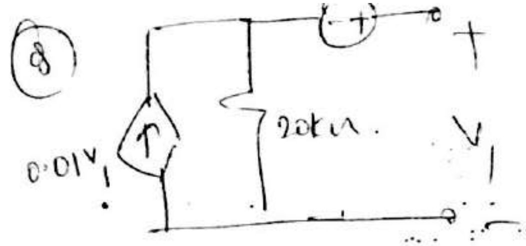
$10(-5/4) - 20(I_{sc} - 5/4) - 40(-5/4) - 50 = 0$

$-\frac{50}{4} - 20I_{sc} + \frac{100}{4} + \frac{200}{4} - 50 = 0$

$\boxed{I_{sc} = 5/8 = 0.625}$ Ampere

$R_{th} = \frac{V_{th}}{I_{sc}} = \frac{10}{5/8} = \frac{80}{5} = 16\Omega$

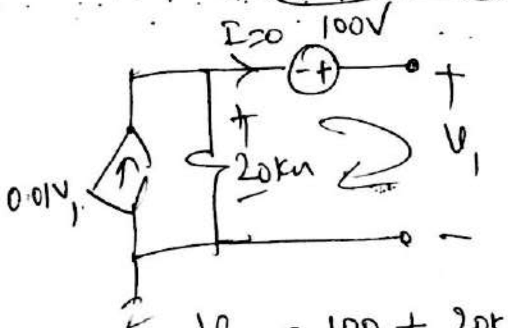
(8)



Sol^n:

$V_{th} = V_1 = ?$ $I_N = I_{sc} = ?$

$$\boxed{R_{th} = \frac{V_{th}}{I_{sc}}}\ \Omega$$

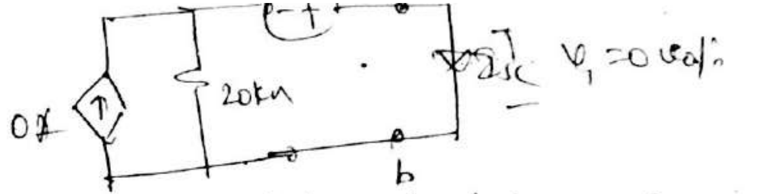


$$V_1 = 100 + 20k(0.01 V_1)$$

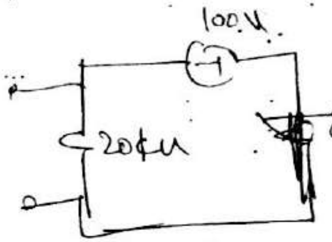
$$V_1 - 20k(0.01)V_1 = 100$$

$$V_1 = \frac{100}{[1 - 20k(0.01)]} = -0.502511 \text{ volts}$$

$$V_1 = V_{th} = -502.512 \text{ m volts}$$



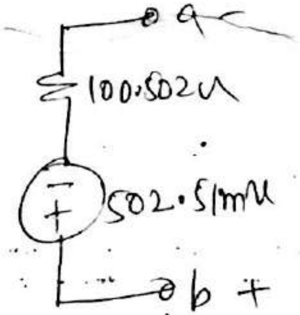
$V_1 = 0$ volts



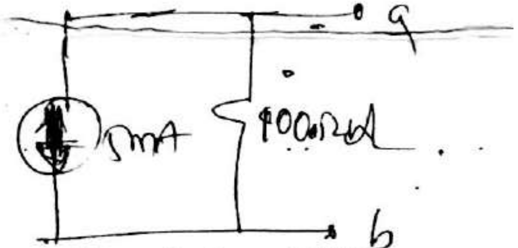
$$I_{sc} = \frac{100}{20k}$$

$$\boxed{I_{sc} = 5mA}$$

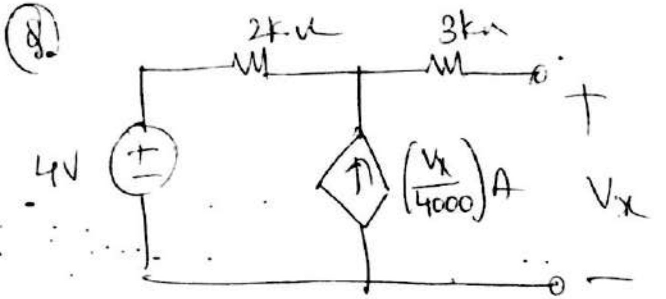
$$R_{th} = \frac{V_{th}}{I_{sc}} = \left|\frac{-502.512m}{5m}\right| = 100.502\Omega$$



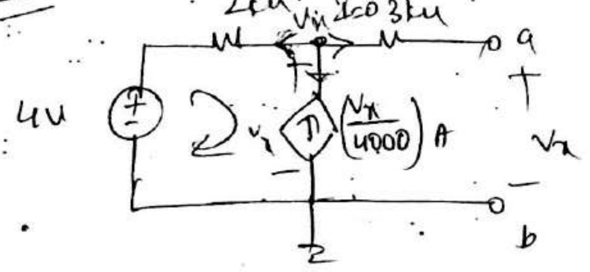
=



(Q)

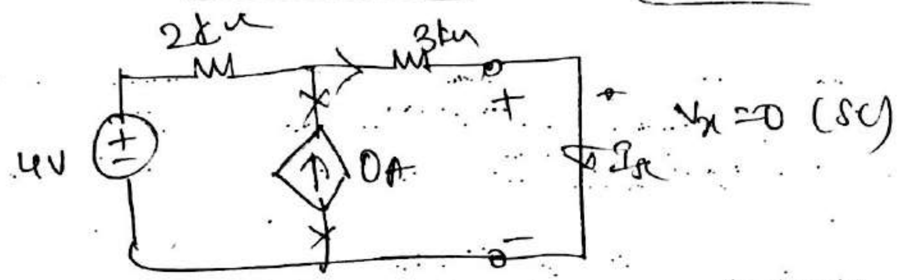


solution



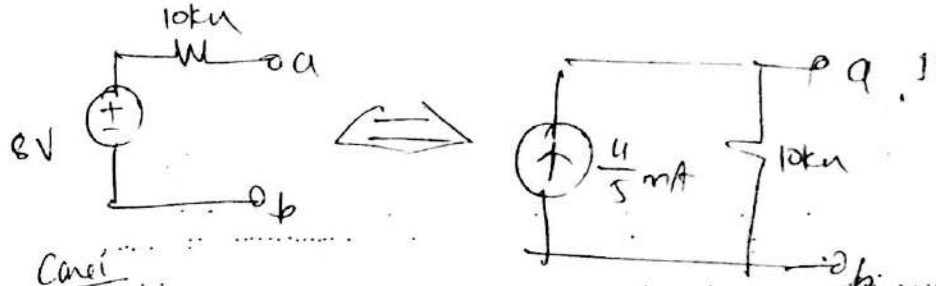
$$\frac{V_x - 4}{2k} - \frac{V_x}{4000} = 0$$

$$\boxed{V_x = 8 \text{ volts}} \Rightarrow V_{th} = 8 \text{ volts}$$



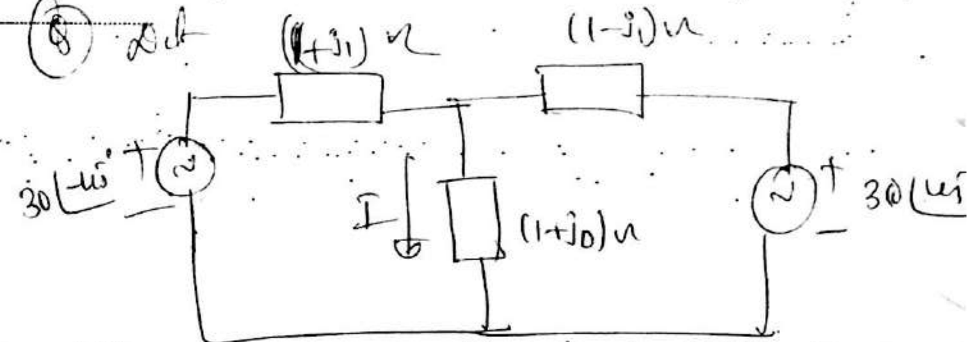
$$I_{sc} = \frac{4}{2k+3k} = \frac{4}{5k} = \frac{4}{5} mA$$

$$R_{th} = \frac{V_{th}}{I_N} = \frac{8}{(4/5m)} = 10k\Omega$$



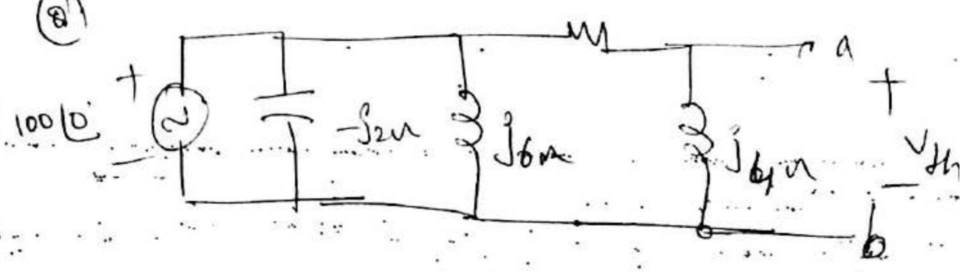
Case

(Q) Sol



$$\boxed{V_{th} = I_{sc} = 0} \quad \boxed{Z_{th} = (1+j0)\Omega}$$

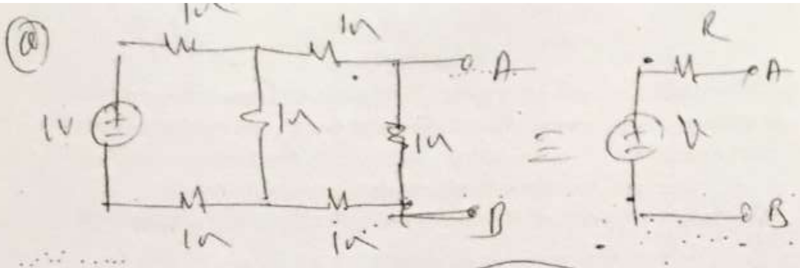
(Q)



$$V_{th} = j16\,(3 - j4) \text{ volts}$$

$$Z_{th} = (3 \,||\, (j4))$$

(a)



$\boxed{V = \frac{1}{11} \text{ volt}}$ $\boxed{R_{th} = 8/11}\ \Omega$

MISSION

Provide quality and contemporary education, in the domain of Electronics and communication and related fields, which enable collaborative ventures with industries and research organizations. Emphasis laid on creating innovative teaching-learning processes that motivate self-learning. by imparting quality education embedded with discipline & national honor.

VISION

To create a rich intellectual potential implanted with multidisciplinary knowledge, human values and professional ethics among the aspirant of becoming Engineers and technologies, so as to unlock their imagination and discover their potential.

OBJECTIVES

1. To impart good technical knowledge to the students.
2. To produce Excellent Engineers in Electronics & Communication fields.
3. To fulfil the needs of the society in the various fields related to Electronics and Communication engineering.
4. To bring post-graduate program in the diverse field of electronics and communication Engineering.
5. To upgrade the facilities in Research & Development Centre of the department with the use of modern aids.
6. To organize training programs / workshops for upgrading staff performance.
7. To establish Industry-Institure Interaction.
8. To publish technical papers in National / International journals and conferences.

GOALS (Short Term) :

1. Modernizing the Laboratories with new software & state-of-the art hardware in tune with the latest technological developments.
2. To obtain Quality certification from an agency of reputed.
3. Teaching Aids : LCD Projector, Smart Boards.
4. Promoting Faculty Development Programmes.
5. Conducting the need based training programs for Faculty & Students.
6. To improve the pass percentage 2-5% compared to previous year.

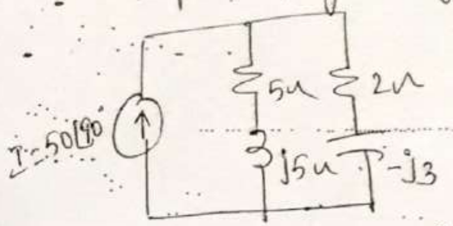
GOALS (Long Term) :

1. To start additional P.G. Programmes in Electonic and Communication engineering discipline.
2. To enter into understanding with globally renowned universities for special programmes in emerging technologies.
3. Promoting Industry - Institute interaction through projects and R & D work.

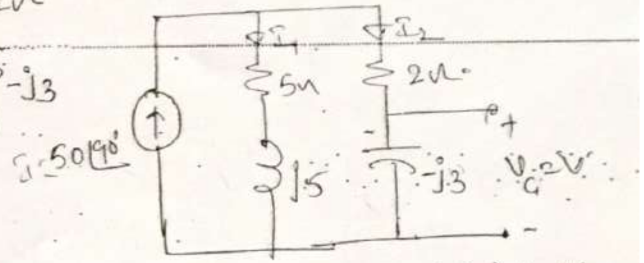
I. Problems on Reciprocity theorem:-

① Verify reciprocity theorem for the n/w shown in fig. Response being voltage across the capacitor. 1

6m. Dec/Jan 2015



Soln:-



and $V = V_C = I_2(-j3)$ volts.

using BCM

$$I_2 = \frac{I \times Z_1}{(Z_1 + Z_2)}$$

$$I_2 = \frac{50\angle 90^\circ\,[5+j5]}{(5+j5+2-j3)} = 48.564\angle 119.05 \text{ Amperes.}$$

$$= \underline{\underline{48.564\angle 119.05}}$$

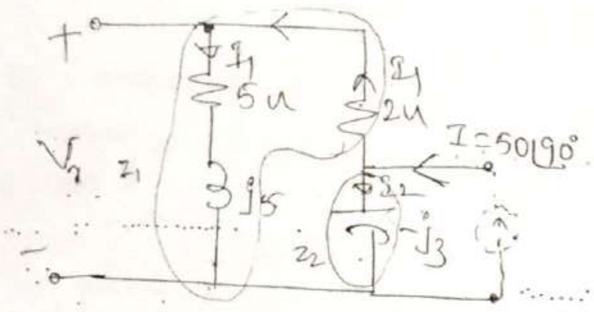
$$V = V_C = I_2(-j3) = 48.564\angle 119.65 \times (-j3)$$

$$\boxed{V = 145.69\angle 29.05} \text{ volts} = 145.69\angle 29.05 \text{ volts}$$

the ratio of $\frac{I}{V} = \frac{50\angle 90^\circ}{145.69\angle 29.05} = 0.34431\angle 60.94$ ← ①.

Now interchange the source & response.

ie



using BCM $\quad I_1 = \dfrac{I \cdot Z_2}{Z_1 + Z_2}$

$Z_1 = (2 + 5 + j5) = (7 + j5)\Omega$

$Z_2 = -j3\,\Omega$

$$I_1 = \frac{50\angle 90° \,[-j3]}{[7 + j5 - j3]} = 20.604\angle -15.945 \text{ Amperes}$$

$V_1 = I_1 \cdot [5 + j5] = 20.604\angle -15.945 \times (5 + j5)$

$V = 145\angle 29.05$ volts

$$\frac{I}{V} = \frac{50\angle 90°}{145\angle 29.05} = 0.34431\angle 60.94 \leftarrow (2)$$

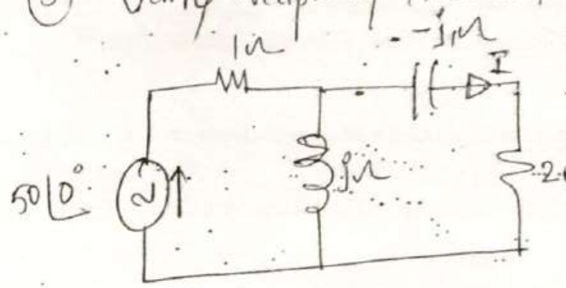
eqⁿ (1) = eqⁿ (2)

∴ Reciprocity theorem is verified.

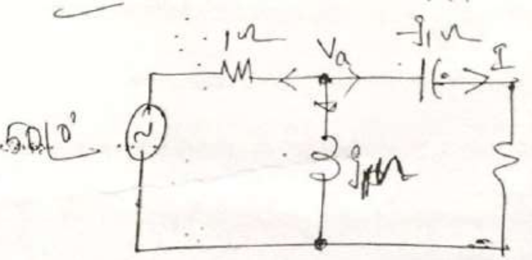
(2) State and prove reciprocity theorem (6m) Dec 14 / Jan 15.
· Sk... ref. notes.

2

(3) Verify reciprocity theorem for the ckt shown in fig.



soln: $v = 50\angle 0°$ ← i/p $\quad I$ ← o/p (response).



$I = \dfrac{V_a}{(2 - j1)}$ Amperes

KCL @ a $\quad \dfrac{V_a - 50\angle 0°}{1} + \dfrac{V_a}{(j1)} + \dfrac{V_a}{(2 - j1)} = 0$

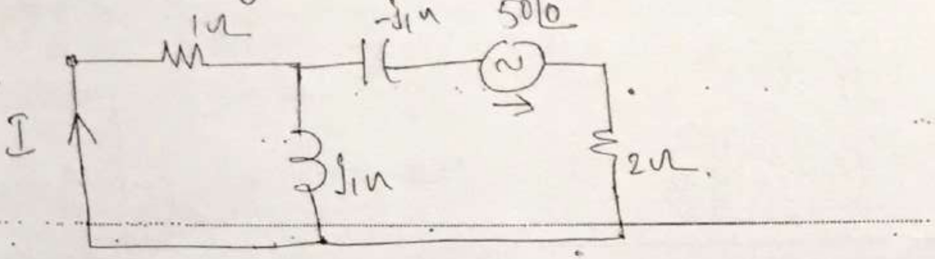
$$V_a\left[1 + (j1)^{-1} + (2 - j1)^{-1}\right] = 50\angle 0°$$

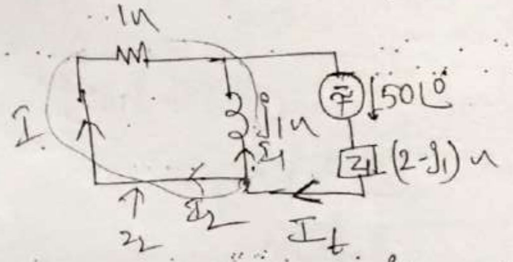
$$V_a = \frac{50\angle 0°}{1.6124\angle -29.74} = 31.008\angle 29.744$$

$\boxed{V_a = 31.008\angle 29.744}$ volts and $\dfrac{50\angle 0°}{13.867\angle 56.30} = \dfrac{V}{I} = 3.605\angle -56.30$

$$\Rightarrow I = \frac{V_a}{(2 - j1)} = \frac{31.008\angle 29.744}{(2 - j1)} = 13.867\angle 56.30 \text{ Amperes}$$

· now interchange the i/p and o/p's.





$$Z_2 = 1 \parallel j_1 = \frac{j_1}{(1+j_1)} = (0.5+j0.5)\,\Omega$$

$$Z_T = Z_1 + Z_2 = (2-j_1) + (0.5+j0.5) = \underline{\underline{(2.5-j0.5)\,\Omega}}$$

$$I = \frac{50\angle 0^\circ}{Z_T} = \frac{50\angle 0^\circ}{(2.5-j0.5)} = 19.6116\angle 11.309 \text{ Ampere's}$$

$I_2 = I = ?$

using $\underline{\underline{\text{BCM}}}$ $\quad I = \dfrac{I_t \cdot (j_1)}{(1+j_1)} = \dfrac{19.6116\angle 11.309 \times (j_1)}{(1+j_1)}$

$$I = 13.867\angle 56.309 \text{ Ampere's}$$

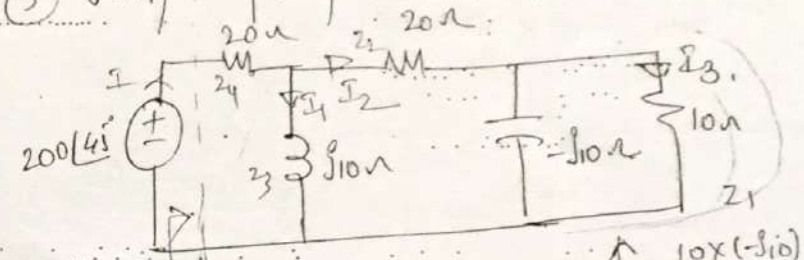
$\Rightarrow \dfrac{V}{I} = \dfrac{50\angle 0^\circ}{13.867\angle 56.309} = 3.605\angle -56.309 \leftarrow (2)$

eq^n (1) = eq^n (2) $\therefore$ reciprocity theorem is verified.

(4) State and prove reciprocity theorem. (5m). Jan 2014 (5m)

Sol^n - repeated question.

(5) Verify reciprocity theorem for the n/w of fig. with response I_3.



$$Z_1 = \frac{10 \times (-j10)}{10-j10} = (5-j5)\,\Omega$$

Sol^n:

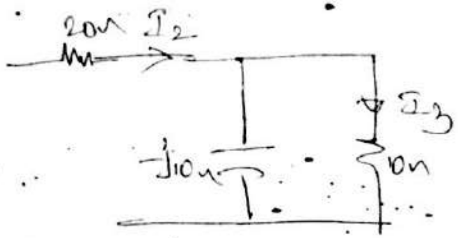
Z_t $\quad Z_t = 20$ $\quad (Z_1+Z_2) = 20 + (5-j5) = \underline{(25-j5)\,\Omega}$

$$Z_3 \parallel (Z_2+Z_1) = \frac{(25-j5) \times j10}{25-j5+j10} = (3.8461+j9.23)\,\Omega$$

$$Z_t = Z_4 + [Z_3 \parallel (Z_2+Z_1)] = 20 + (3.8461+j9.23) = (23.846+j9.23)\,\Omega$$

$$I = \frac{V}{Z_t} = \frac{200\angle 45^\circ}{(23.846+j9.23)} = 7.82\angle 23.83 \text{ Ampere's}$$

$$I_2 = \frac{I\,[Z_3]}{[Z_3 + (Z_2+Z_1)]} = \frac{7.82\angle 23.83\,(j10)}{[j10+25-j5]} = \underline{\underline{3.067\angle 102.52}} \text{ Ampere}$$

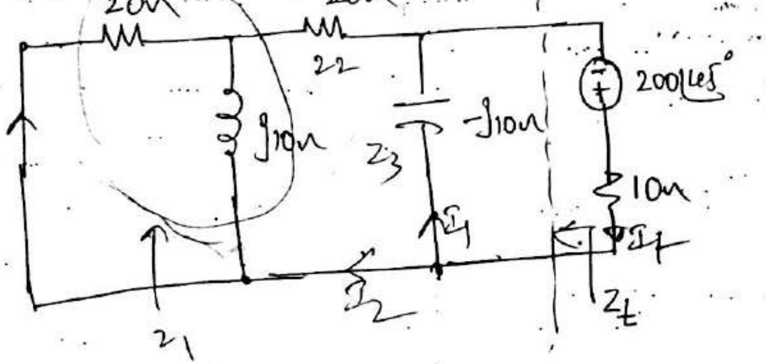


$$I_3 = \frac{I_2(-j10)}{(-j10+10)} = \frac{3.067\angle 102.52\,(-j10)}{(10-j10)}$$

$$\boxed{I_3 = 2.169\angle 57.52} \text{ Amperes}$$

$$\frac{V}{I_3} = \frac{200\angle 45^\circ}{2.169\angle 57.52} = 92.195\angle -12.52 \longleftarrow \text{(i)}$$

Now interchange the i/p and o/p.



$$Z_1 = 20 \,\|\, j10 = (4+j8)\,\Omega.$$

$$Z_1 + Z_2 = 4 + j8 + 20 = (24+j8)\,\Omega.$$

$$(Z_1+Z_2)\,\|\,Z_3 = Z_t = (24+j8)\,\|\,(-j10) = (4.1379 - j9.6$$

$$I_t = \frac{200\angle 45^\circ}{(10+Z_t)} = \frac{200\angle 45^\circ}{(10+4.1379-j9.65)}$$

$$\boxed{I_t = 11.682\angle 79.33} \text{ Amperes}$$

$$I_2 = \frac{I_t(Z_3)}{(Z_1+Z_2+Z_3)} = \frac{11.682\angle 79.33\,(-j10)}{24+j8+(-j10)}$$

$$\boxed{I_2 = 4.8507\angle 174.09} \text{ Amperes}$$

$$I_3 = \frac{I_2(j10)}{20+j10} = \frac{4.8507\angle 174.09\,(j10)}{20+j10} = \ldots$$

$$I_3 = 2.169\angle -122.47 \text{ Amperes}$$
$$= 2.169\angle 57.52$$

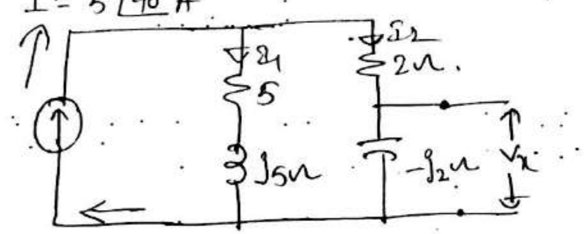
$$\frac{V}{I_3} = \frac{200\angle 45^\circ}{2.169\angle 57.52} = 92.195\angle -12.52 \longleftarrow \text{(ii)}$$

eq (i) = eq (ii) ∴ Reciprocity theorem is verified.

In the single Superposition current source shown in fig. the voltage V_x, interchange the current source and resulting voltage V_x, in the reciprocity theorem verified? (6m)

J/J 2013.

$I = 5\angle 90^\circ A$



Sol: i/p $I = 5\angle 90^\circ A$ o/p $V_x = V_x$

Repeated question lly to Q.no.1 Dec-Jan 2015

$$I_2 = \frac{5\angle 90^\circ\,[5+j5]}{[5+j5]+[2-j2]} = \cancel{1.2674\angle 135} \; 4.642\angle 111.80 \text{ Ampere}$$

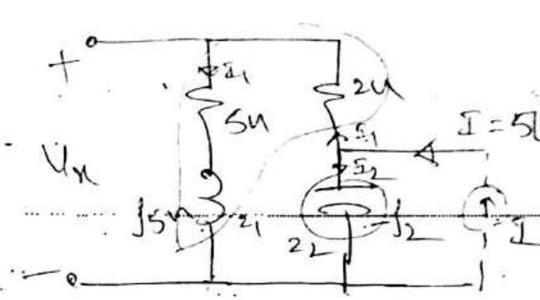
$$V_x = I_2(-j2) = \cancel{1.2674\angle 135} \; 4.642\angle 111.8 \,(-j2)$$

$\boxed{V_x = \cancel{3.5355\angle 45^\circ}}$ volts $9.284\angle 21.8$ volts

$$\frac{I}{V_x} = \frac{5\angle 90^\circ}{\cancel{3.5355\angle 45} \; 9.284\angle 21.8} = \cancel{1.4142\angle 45^\circ} \leftarrow (1)$$

$$= 0.5385\angle 68.19 \leftarrow (1)$$

Now interchange the i/p and o/p's.



$V_x = I_1(5+j5)$ volts.

$I_1 = ?$

$$I_1 = \frac{I Z_2}{Z_1+Z_2} = \frac{5\angle 90^\circ\,[-j2]}{(7+j5)-j2}$$

$\boxed{I_1 = 1.313\angle -23.198}$ Ampere

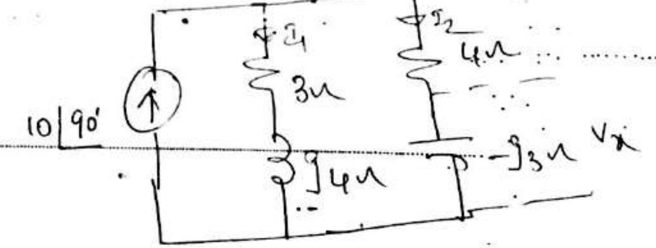
$$V_x = I_1(5+j5) = 1.313\angle -23.198\,(5+j5)$$

$$V_x = 9.284\angle 21.8014 \text{ Ampere}$$

$$\frac{I}{V_x} = \frac{5\angle 90^\circ}{9.284\angle 21.8014} = 0.5385\angle 68.19 \leftarrow (2)$$

eq (1) = eq (2) ∴ Reciprocity theorem is verified.

(7) Det Voltage V_x in the n/w shown in fig. hence Verify Reciprocity theorem. (6m) Jun 2012.



Soln: $I = 10\angle 90°$ A ← i/p $V_x = I_2(-j3)$ ← o/p. volt

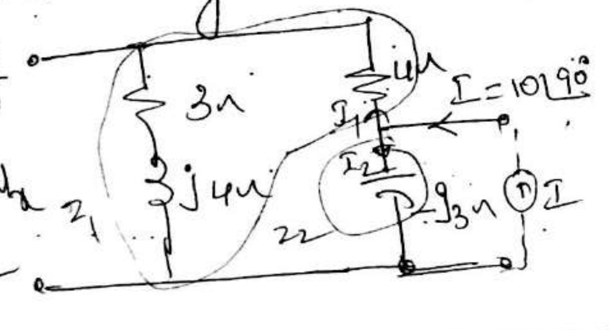
$$I_2 = \frac{10\angle 90° \,[3+j4]}{[3+j4]+[4-j3]} = 7.071\angle 135° \text{ Ampere}$$

$$V_x = I_2(-j3) = 7.071\angle 135° \,(-j3)$$

$$\boxed{V_x = 21.213\angle 45°} \text{ volts}$$

$$\frac{I}{V_x} = \frac{10\angle 90°}{21.213\angle 45°} = 0.4714\angle 45° \leftarrow (1)$$

Now interchange the i/p and o/p.



$$I_1 = \frac{I \cdot Z_2}{Z_1+Z_2} = \frac{10\angle 90° \,[-j3]}{[4+3+j4]+[-j3]}$$

$$\underline{I_1 = 4.2426\angle -8.130 \text{ Ampere}}$$

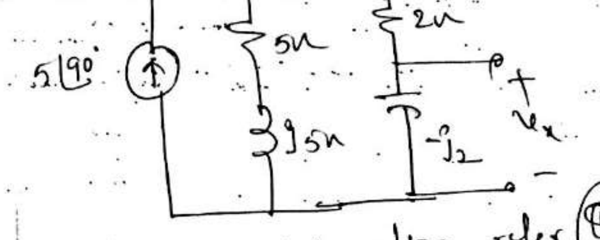
$$V_x = I_1(3+j4) = 4.2426\angle -8.130\,[3+j4]$$

$$\boxed{V_x = 21.213\angle 45°} \text{ volts}$$

$$\frac{I}{V_x} = \frac{10\angle 90°}{21.23\angle 45°} = 0.4714\angle 45° \leftarrow (2)$$

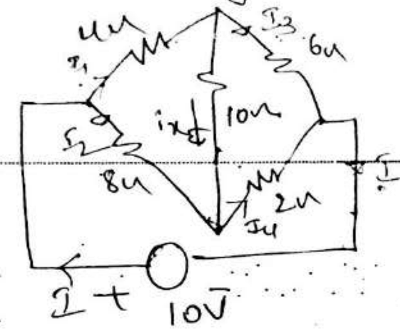
eqn (1) = eqn (2) i.e. Reciprocity theorem is Verified.

(8) Verify reciprocity theorem for the ckt. shown in fig. Dec 2012. (6m)



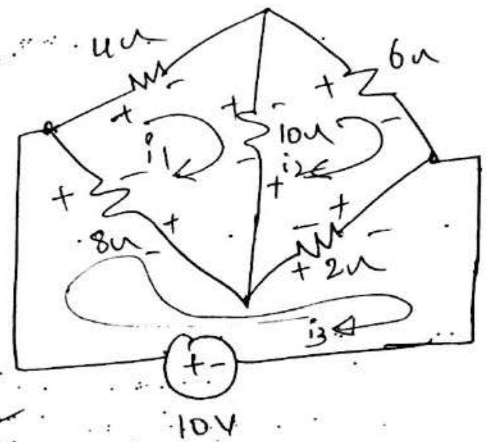
Soln:- repeated question. refer (8i).

(9) Find i_x and hence verify reciprocity theorem for the n/w shown in fig. (8m) J/J 2014.



Sol: i/p $v = 10$ volts o/p $i_x = ?$

(or)



KVL

Loop 1

$$-4i_1 - 10(i_1 - i_2) - 8(i_1 - i_3) = 0$$

$$-4i_1 - 10i_1 + 10i_2 - 8i_1 + 8i_3 = 0$$

$$-22i_1 + 10i_2 + 8i_3 = 0 \leftarrow (1)$$

Loop 2

$$-10(i_2 - i_1) - 6i_2 - 2(i_2 - i_3) = 0$$

$$-10i_2 + 10i_1 - 6i_2 - 2i_2 + 2i_3 = 0$$

$$10i_1 - 18i_2 + 2i_3 = 0 \leftarrow (2)$$

Loop 3

$$10 - 8(i_3 - i_1) - 2(i_3 - i_2) = 0$$

$$10 = 8(i_3 - i_1) + 2(i_3 - i_2)$$

$$10 = 8i_3 - 8i_1 + 2i_3 - 2i_2$$

$$-8i_1 - 2i_2 + 10i_3 = 10 \leftarrow (3)$$

solving eq (1), 2 (3) $i_1 = 1.171$ Amperes

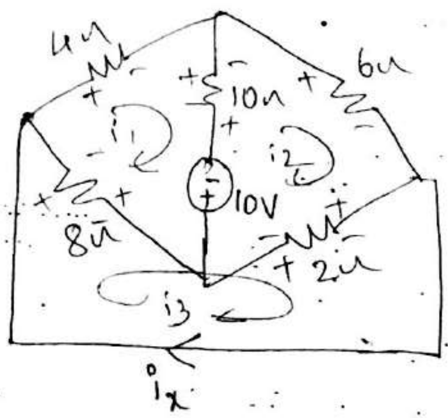
$i_2 = 0.8857$ Amperes

$i_3 = 2.1142$ Amperes

o/p $i_x = (i_1 - i_2) = 1.171 - 0.8857 = 0.2853$ Amperes

$$\frac{v}{i_x} = \frac{10}{0.2853} = 35.05 \leftarrow (1)$$

Now interchange the position of i/p and o/p.



KVL

Loop 1

$$-4i_1 - 10(i_1 - i_2) + 10 - 8(i_1 - i_3) = 0$$

$$-4i_1 - 10i_1 + 10i_2 + 10 - 8i_1 + 8i_3 = 0$$

$$-22i_1 + 10i_2 + 8i_3 = -10 \leftarrow (1)$$

Loop 2

$$-10 - 10(i_2 - i_1) - 6i_2 - 2(i_2 - i_3) = 0$$

$$-10 - 10i_2 + 10i_1 - 6i_2 - 2i_2 + 2i_3 = 0$$

$$10i_1 - 18i_2 + 2i_3 = 10 \leftarrow (2)$$

Loop 3

$$-8(i_3 - i_1) - 2(i_3 - i_2) = 0$$

$$-8i_3 + 8i_1 - 2i_3 + 2i_2 = 0$$

$$8i_1 + 2i_2 - 10i_3 = 0 \leftarrow (3)$$

Solving eqn (1), (2) (3)

$i_1 = 0.428A$, $i_2 = -0.2857A$, $i_3 = 0.2857A$

$i_3 = i_x = 0.2853$ Amperes

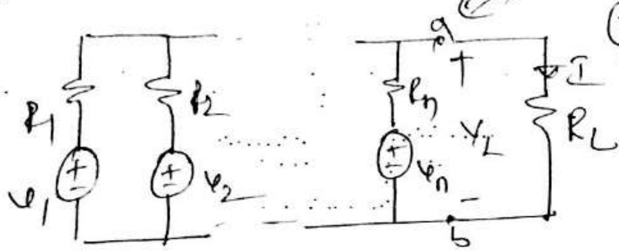
$$\frac{V}{i_3} = \frac{10}{0.2853} = 35.05 \leftarrow (2)$$

eqn (1) = eqn (2) ∴ reciprocity theorem is verified

(10) State and explain the reciprocity theorem. (5m) Jan 2013

Soln: repeated question.

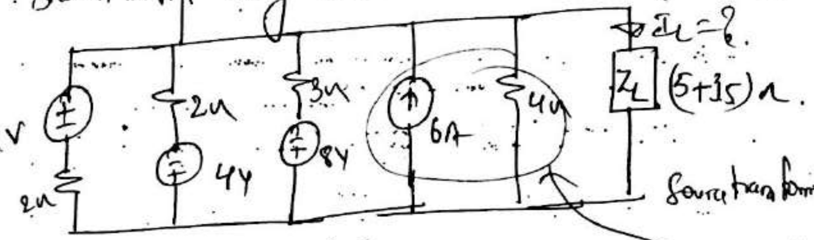
Millman's theorem :-



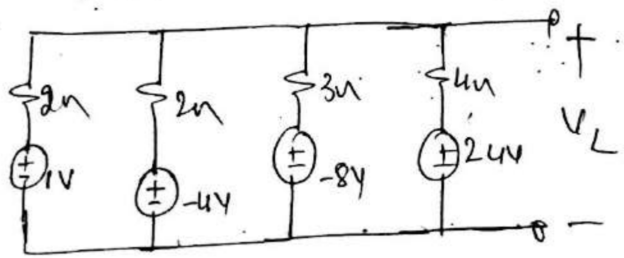
$$V_L = \frac{V_1Y_1 + V_2Y_2 + V_3Y_3 + \cdots + V_nY_n}{Y_1 + Y_2 + Y_3 + \cdots + Y_n} \text{ volts.}$$

$$\boxed{R_{Lin} = \frac{1}{Y_1 + Y_2 + Y_3 + \cdots + Y_n}} \ \Omega$$

Q1) . Find the Current through the Load impedance Z_L for the ckt shown in fig. using millman's theorem. 6m. (Jan 2015)



Soln



$$I_L = \frac{V_L}{(Z_L + R_{Lin})} \text{ Ampere}$$

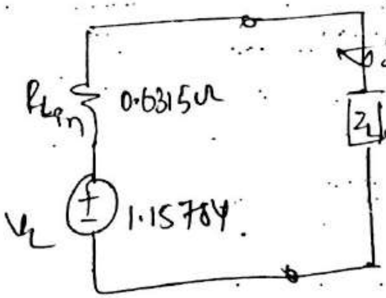
$$Y_L = \frac{V_1Y_1 + V_2Y_2 + V_3Y_3 + V_4Y_4}{[Y_1 + Y_2 + Y_3 + Y_4]}$$

$$V_L = \frac{1(2)^{-1} + (-4)(2)^{-1} + (-8)(3)^{-1} + (24)(4)^{-1}}{2^{-1} + 2^{-1} + 3^{-1} + 4^{-1}}$$

$$\boxed{V_L = 1.15789} \text{ volts}$$

$$R_L = \frac{1}{Y_1 + Y_2 + Y_3 + Y_4} = \frac{1}{(2^{-1} + 2^{-1} + 3^{-1} + 4^{-1})}$$

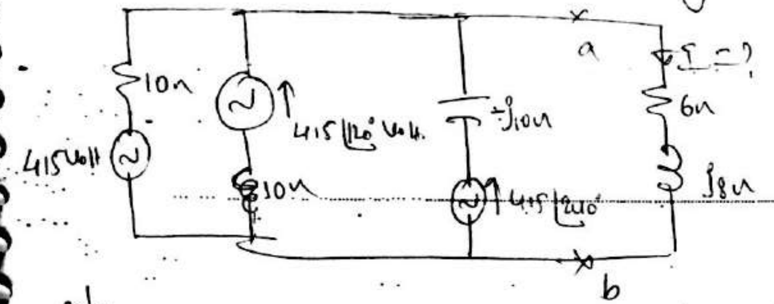
$$\boxed{R_L = 0.6315} \ \Omega$$



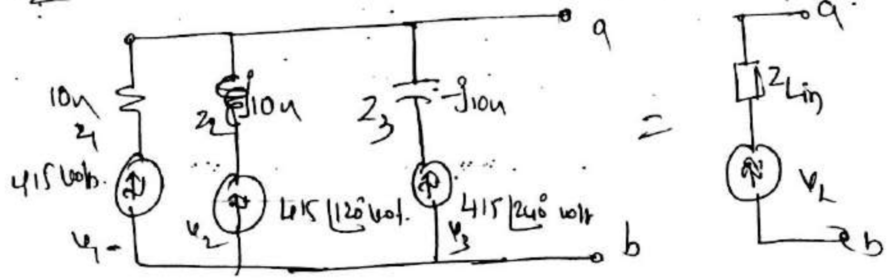
$$I_L = \frac{V_L}{(R_{Lin} + Z_L)} = \frac{1.1578}{(0.6315 + 5 + j5)}$$

$$\boxed{I_L = 0.1537 \angle -41.6} \text{ Ampere}$$

(3) For the ckt shown in fig. find Current "I" using Millman's theorem. (6m) Jan 2015



Sol^n

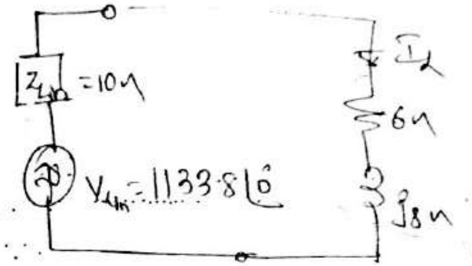


$$V_L = \frac{V_1Y_1 + V_2Y_2 + V_3Y_3}{Y_1 + Y_2 + Y_3}$$

$$V_L = \frac{415(10)^{-1} + 415\angle 120^\circ (j10)^{-1} + 415\angle 240^\circ (-j10)^{-1}}{(10)^{-1} + (j10)^{-1} + (-j10)^{-1}}$$

$$V_L = 1133.80\angle 0^\circ \text{ volts}$$

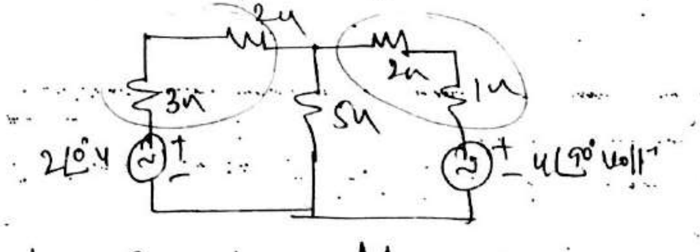
$$Z_L = \frac{1}{Y_1 + Y_2 + Y_3} = \frac{1}{10^{-1} + (j10)^{-1} + (-j10)^{-1}} = 10\,\Omega$$



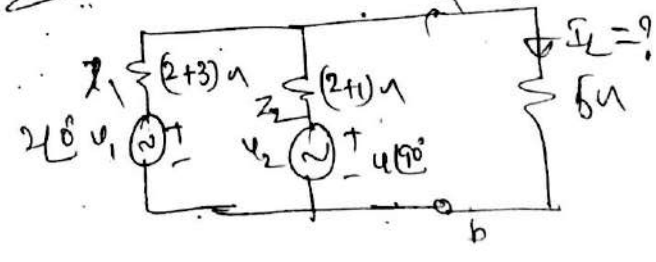
$$I_L = \frac{V_{Lin}}{Z_{Lin} + Z_L} = \frac{1133.8\angle 0}{10 + 6 + j8}$$

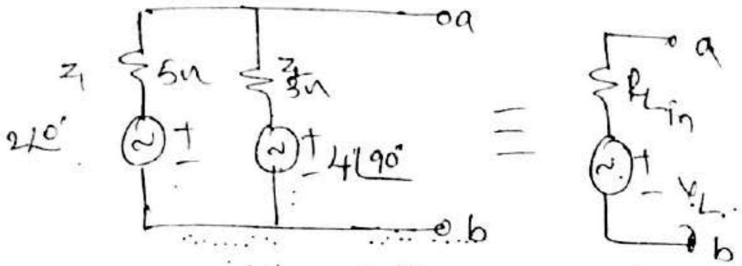
$$\boxed{I_L = 63.381\angle -26.6^\circ} \text{ Ampere}$$

(4) Find the power delivered by the 5Ω resistor in the circuit shown in fig. and find the Current supplied by each source. Use Millman's principle. (8m)



Sol^n Re-write the ckt





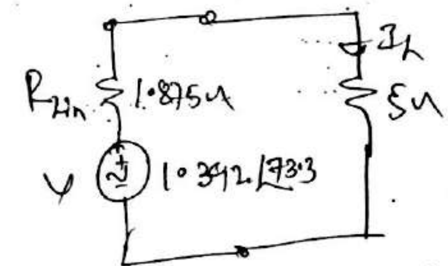
$$V_L = \frac{V_1 Y_1 + V_2 Y_2}{Y_1 + Y_2}$$

$$V_L = \frac{2\angle 0^\circ (5)^{-1} + 4\angle 90^\circ (3)^{-1}}{5^{-1} + 3^{-1}}$$

$$\boxed{V_L = 1.3920\angle 73.30} \text{ volts}$$

$$R_{L_{in}} = \frac{1}{Y_1 + Y_2} = \frac{1}{5^{-1} + 3^{-1}}$$

$$\boxed{R_{L_{in}} = 1.875}\ \Omega$$



$$I_L = \frac{V}{R_{Lin} + R_L} = \frac{1.392\angle 73.30^\circ}{(1.875 + 5)}$$

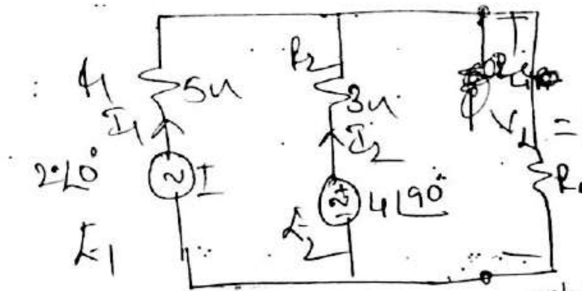
$$\boxed{I_L = 0.202\angle 73.3} \text{ Ampere}$$

$$P_{L\,abs} = |I_L|^2 \times R_L$$

$$= \left[(0.202)^2 \times 5\right] = (0.202)^2 \times 5$$

$$\boxed{P_{L(abs)} = 0.2049} \text{ Watts}$$

I_1, I_2 without Load 5Ω



V_L without Load $= 1.3920\angle 73.3$ volts

$$I_1 = \frac{2\angle 0^\circ - V_L}{5}$$

$$I_1 = \frac{2\angle 0^\circ - 1.392\angle 73.3}{5} = 0.416\angle -39.0804 \text{ A}$$ (no load)

$$I_2 = \frac{4\angle 90^\circ - 1.3920\angle 73.3}{3} = 0.8988\angle 98.53 \text{ A}$$ (no load)

V_L with Load

I_1, I_2 with load 5Ω

(d) $$V_L = \frac{V_1 Y_1 + V_2 Y_2}{Y_1 + Y_2 + Y_L} = \frac{2\angle 0^\circ (5^{-1}) + 4\angle 90^\circ (3^{-1})}{5^{-1} + 3^{-1} + 5^{-1}}$$

$= 1.8982\angle 73.30$ Volts

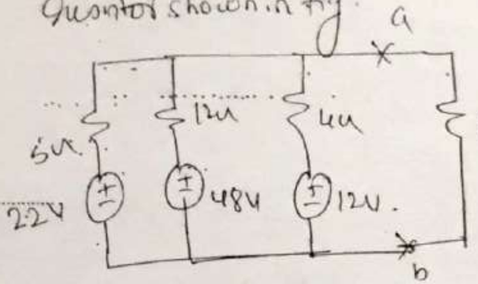
$$I_1 = \frac{E_1 - V_{L(\text{with load})}}{R_1} = \frac{2\angle 0^\circ - 1.8982\angle 73.30}{5} = 0.4465\angle -51.33 \text{ Ampere}$$

$$I_2 = \frac{E_2 - V_{L(\text{with load})}}{R_2} = \frac{4\angle 90^\circ - 1.8982\angle 73.3}{3} = 0.0749$$

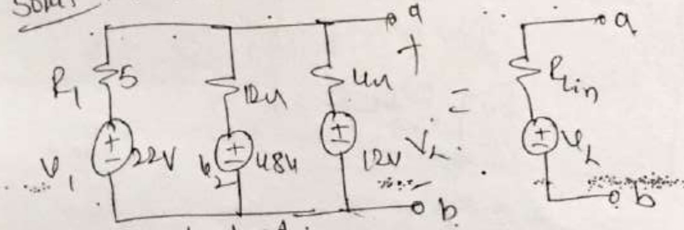
(5) Using millman's theorem find the current through 10Ω resistor shown in fig.

Jan 2014

(7m)



Solu:-



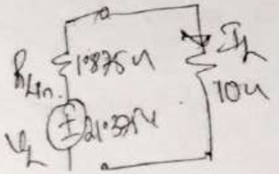
V_L without load.

$$V_L = \frac{V_1 Y_1 + V_2 Y_2 + V_3 Y_3}{Y_1 + Y_2 + Y_3}$$

$$V_L = \frac{22(5^{-1}) + (48)(12^{-1}) + 12(4^{-1})}{5^{-1} + 12^{-1} + 4^{-1}}$$

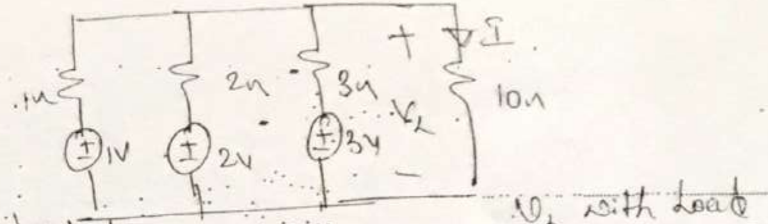
$\boxed{V_L = 21.375}$ volts

$$R_L = \frac{1}{5^{-1}+12^{-1}+4^{-1}} = \frac{1}{Y_1+Y_2+Y_3} = 1.875\,\Omega$$



$$I_L = \frac{V_L}{R_{in}+10} = \frac{21.375}{1.875+10} = 1.8\,A$$

(6) Find the load current I in the ckt of fig by using Millman's theorem. (6m) J/J 2013. 13



V_L with load

2nd Method

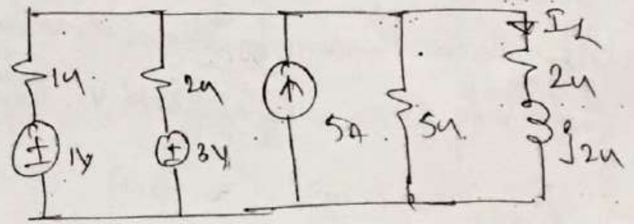
Solu: $V_L = \dfrac{V_1 Y_1 + V_2 Y_2 + V_3 Y_3}{Y_1 + Y_2 + Y_3 + Y_L}$

$$V_L = \frac{1(1)^{-1} + 2(2^{-1}) + 3(3^{-1})}{1^{-1} + 2^{-1} + 3^{-1} + 10^{-1}} = 1.5517 \text{ volts}$$

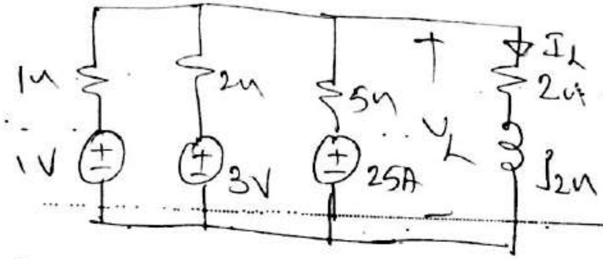
$$I = \frac{V_L}{10} = \frac{1.5517}{10} = 0.15517 \text{ Amperes}$$

(6) State and explain millman's theorem. (04m) J/J 2013.

(7) Det. Load Current I_L in the ckt shown in fig using millman's theorem. (6m) June 2012.



Sol:- Convert Current Source to practical voltage source.



V_L with load is given by

$$V_L = \frac{V_1 Y_1 + V_2 Y_2 + V_3 Y_3}{Y_1 + Y_2 + Y_3 + Y_L}$$

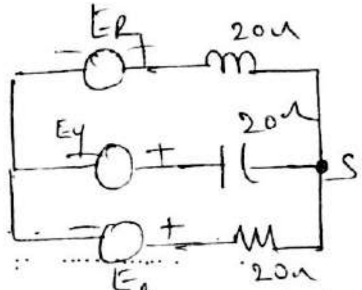
$$= \frac{1(1)^{-1} + 3\times 2^{-1} + 25\times 5^{-1}}{1^{-1} + 2^{-1} + 5^{-1} + (2+j2)^{-1}}$$

$$= \underline{\underline{3.8149 \angle 7.305^\circ}} \text{ volts}$$

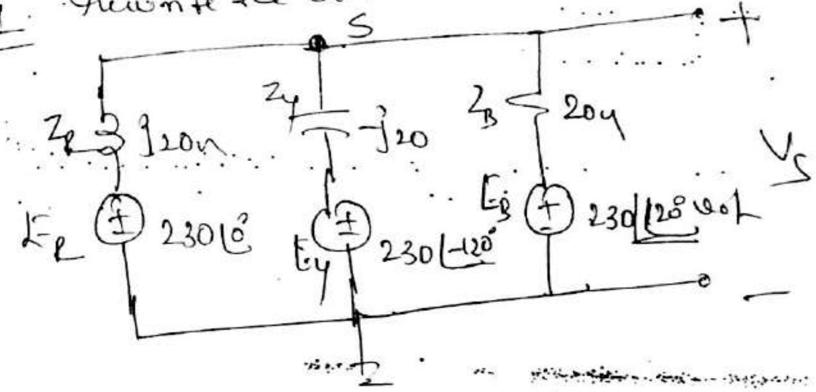
$$I_L = \frac{V_L}{(2+j2)} = \frac{3.8149\angle 7.305^\circ}{(2+j2)} = \underline{1.3487\angle -37.69} \text{ Amperes}$$

(8) Use millman theorem to determine the voltage V_s of the ckt shown in fig. given $E_R = 230\angle 0^\circ$ V, $E_Y = 230\angle -120^\circ$ V

$E_B = 230\angle 120^\circ$ V (6m)



Sol: Rewrite the ckt

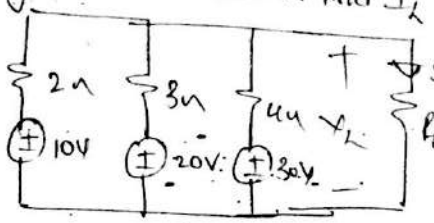


$$V_S = \frac{E_R Y_R + E_Y Y_Y + E_B Y_B}{Y_R + Y_Y + Y_B}$$

$$V_S = \frac{230\angle 0^\circ (j20)^{-1} + 230\angle -120^\circ (-j20)^{-1} + 230\angle 120^\circ}{(j20)^{-1} + (-j20)^{-1} + (20)^{-1}}$$

$$\boxed{V_S = 168.371\angle -60^\circ} \text{ volts}$$

(9) Using millman's theorem find I_L through R_L for the below



(6m) J/J 2014.

Soln: V_L with Load

$$V_L = \frac{V_1Y_1 + V_2Y_2 + V_3Y_3}{Y_1 + Y_2 + Y_3 + Y_L}$$

$$V_L = \frac{10\times2^{-1} + 20\times3^{-1} + 30\times4^{-1}}{2^{-1} + 3^{-1} + 4^{-1} + 10^{-1}}$$

$\boxed{V_L = 16.197}$ volts.

$$I_L = \frac{V_L}{10} = 1.6197 \text{ Amperes.}$$

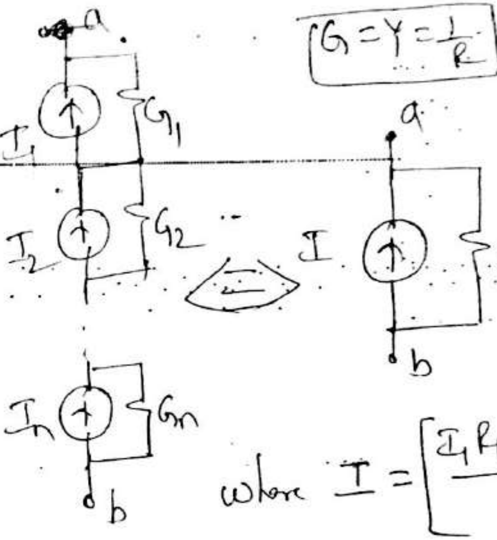
(10) State and prove millman's theorem for Current Sources in Series. (6m) Jan 2013.

Statement: If 'n' Practical Current Sources having internal Conductance (or) admittance which can be replaced by a Single Current Source I in parallel with an equivalent conductance (or) admittance.

i.e

$G_1 = Y_1 = \frac{1}{R_1}$; ... $G_n = \frac{1}{R_n}$

$\boxed{G = Y = \frac{1}{R}}$ ℧

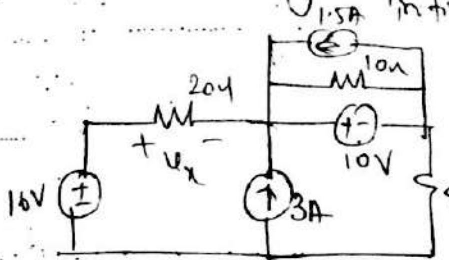


where $I = \left[\frac{I_1R_1 + I_2R_2 + \cdots + I_nR_n}{R_1 + R_2 + R_3 + \cdots + R_n}\right]$ A

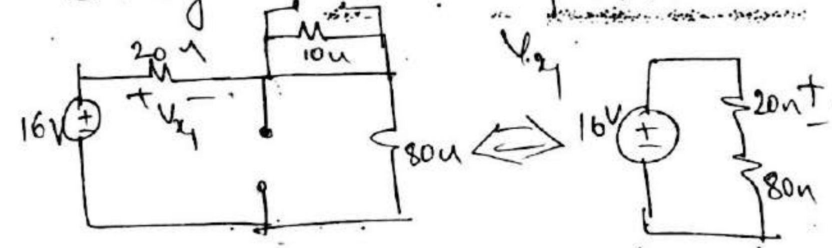
and $G = \frac{1}{R_1 + R_2 + R_3 + \cdots + R_n}$ ℧

3 Superposition theorem:-

(1) Find 'V_x' using Superposition theorem for the n/w shown in fig. (8M) Jun-2015.

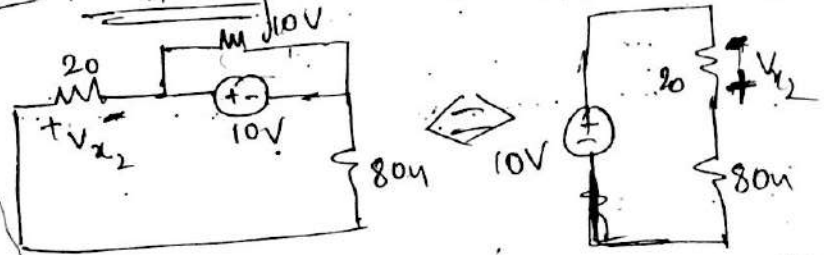


Sol:- Step 1. 16 V acting alone make other source to zero i.e open ckt all Current Source & Short ckt the voltage sources. using V.D.R



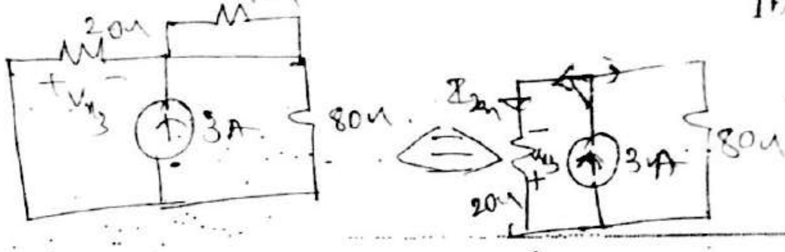
$$V_{x_1} = \frac{16}{(20+80)} \times 20 = 3.2 \text{ volts}$$

Step 2 10 volts acting alone



$$V_{x_2} = -\frac{10}{(20+80)} \times 20 = -2 \text{ volts}$$

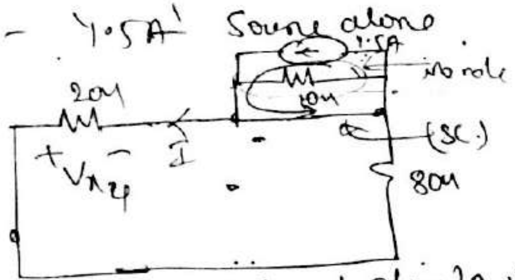
Step 3 :- 3A Source alone.



$$I_{20\Omega} = \frac{3 \times 80}{(20+80)} = 2.4 \text{ Amperes}$$

$$V_{x_3} = -20 \times 2.4 = -48 \text{ Volts}$$

Step 4 :- '1.5A' Source alone



due to short ckt no current flows in 20Ω resistor

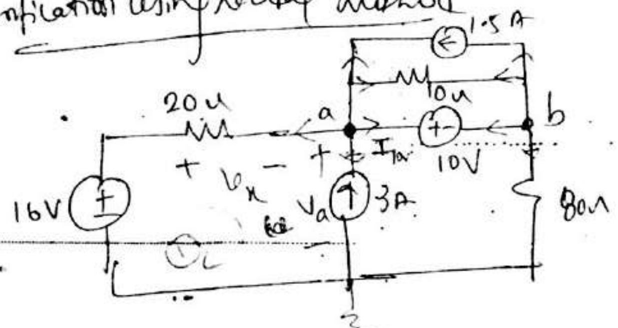
∴ $V_{x_4} = 0$ volts

using SPT

$$V_x = V_{x_1} + V_{x_2} + V_{x_3} + V_{x_4}$$

$$= 3.2 - 2 - 48 + 0 = -46.8 \text{ volts}$$

Verification using nodal method



KCL @ a

$$\frac{V_a-16}{20}+\frac{V_a-V_b}{10}-1.5-3+I_{10}=0$$

$\boxed{V_a - V_b = 10}$ volts ← (a)

$V_b - V_a = -10$

KCL at "b"

$$\frac{V_b - V_a}{10} + 1.5 + \frac{V_b}{80} - I_{10} = 0$$

$20^{-1}V_a - \left(\frac{16}{20}\right) + 1 - 4.5 + I_{10} = 0$

$20^{-1}V_a + I_{10} = 4.3$ ← (1)

$-1 + 1.5 + \frac{V_b}{80}\ 80^{-1}V_b - I_{10} = 0$

$80^{-1}V_b - I_{10} = -0.5$ ← (2)

eqⁿ (1) + eqⁿ (2) $\quad 20^{-1}V_a + 80^{-1}V_b = 3.8$ ← (b)

Solving eqⁿ (a) and (b).

$V_a - V_b = 10$

$20^{-1}V_a + 80^{-1}V_b = 3.8$

$V_a = 62.8$ volts

$V_b = 52.8$ volts

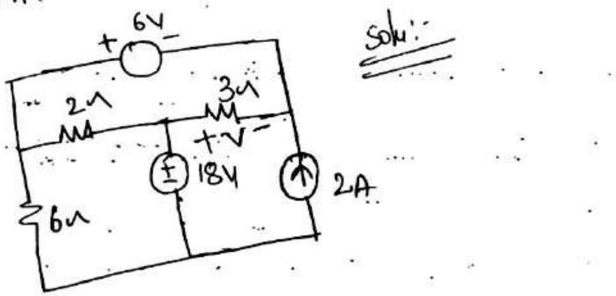
KVL loop 1

$16 - V_x - V_a = 0.$

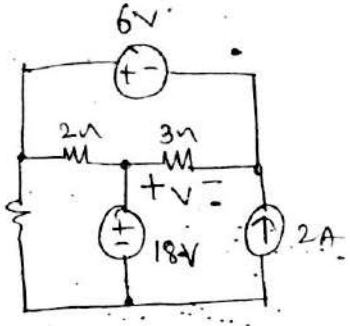
$V_x = 16 - V_a = 16 - 62.8 = -46.8$ volts

$\boxed{V_x = -46.8}$ volts hence verified

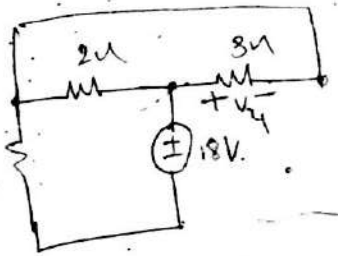
(2) Find the voltage 'V' across 3Ω resistor using superposition theorem for the circuit shown in the fig.

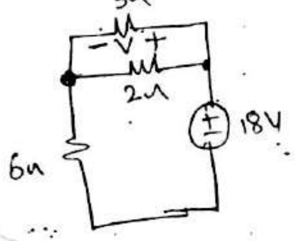


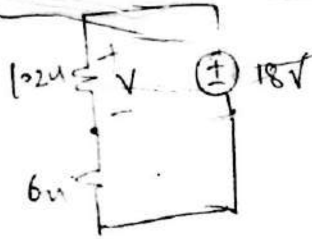
Sol:-



Soln. Step 1. 18V source alone $V \Rightarrow V_{x_1}$

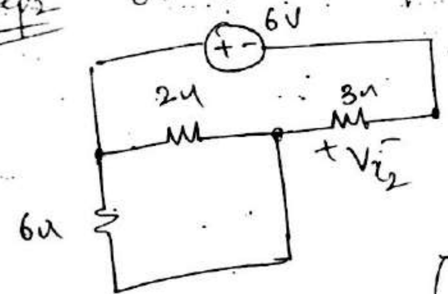






$$V_{x_1} = \frac{18}{(1.2+6)}(1.2) = 3 \text{ volts}$$

Step 2. 6V source acting alone $V \Rightarrow V_{x_2}$

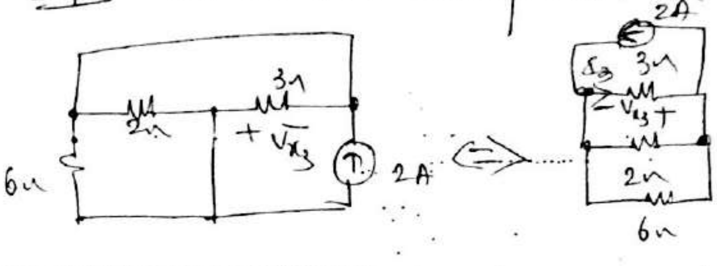


$$6 \| 2 = 1.5\,\Omega$$

$$V_{x_2} = \frac{6}{(1.5+3)} \times 3$$

$$\boxed{V_{x_2} = 4 \text{ volts}}$$

Step 3. 2A current source acting alone. $V \Rightarrow V_{x_3}$



$$\therefore I_3 = \frac{2.(2\|6)}{3+2\|6} = \frac{2\times 1.5}{3+1.5}$$

$$\boxed{I_3 = 0.666\text{ A}} = \frac{2}{3}\text{A}$$

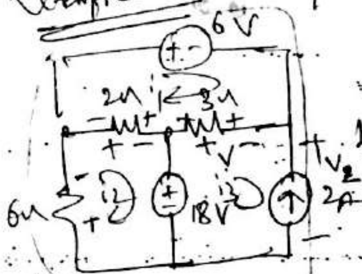
$$V_{x_3} = -I_3 \times 3 = -2 \text{ volts}$$

using SPT

$$V = V_{x_1} + V_{x_2} + V_{x_3}$$

$$= 3 + 4 - 2 = 5 \text{ volts}$$

Verification by using Mesh analysis



$V = 3(i_3 - i_2)$ volts

$= 3[-2+2.5] = 1.5$ volts

Loop 1: $-6i_2 - 2(i_2 - i_1) - 18 = 0$

$-6i_2 - 2i_2 + 2i_1 = 18$

$2i_1 - 8i_2 = 18 \leftarrow (1)$

Verified: $\boxed{V_1 = 5 \text{ volts}}$

loop 2: $18 - 3(i_3 - i_1) - V_2 = 0 \leftarrow$

$18 - 3i_3 + 3i_1 - V_2 = 0$

$+3i_1 - 3i_3 = V_2 + 18 \leftarrow (2)$

$6 + 2(i_1 - i_2) + 3(i_1 - i_3) = 0$

$2i_1 - 2i_2 + 3i_1 - 3i_3 = -6 \Rightarrow 5i_1 - 2i_2 - 3i_3 = -6 \leftarrow (3)$

$5i_1 - 2i_2 = 0 \leftarrow (3)$

kvl to outer loop

$V_2 + 6 + 6i$

$V_2 = -6 -$

eq + (3) becomes

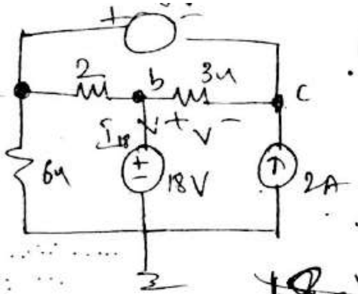
$3i_1 - 3i_3 + 6i$

$3i_1 + 6i_2 - 3i$

$i_3 = -2$

$i_1 = -1A$

$i_2 = -2.5$



(01)

$V_a - V_c = 6V \leftarrow (1)$

$\boxed{V = V_b - V_c}$ volts

$V_b = 18$ volts

$\cancel{\frac{18-V_a}{2} + I_{18} + \frac{18-V_c}{3} = 0} \leftarrow (2)$

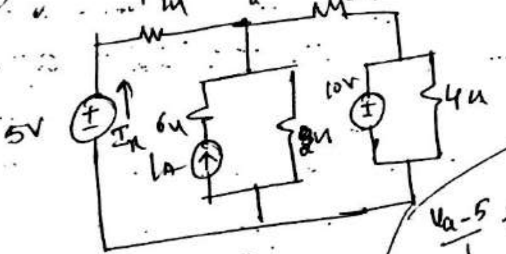
$$\frac{V_a}{6} + \frac{V_a-18}{2} + \frac{V_c-18}{3} - 2 = 0$$

$$\left[6^{-1}+2^{-1}\right]V_a + 3^{-1}V_c = \left(\frac{18}{2}+\frac{18}{3}+2\right)$$

$V_a = 19$ volts, $V_c = 13$ volts.

$V = V_b - V_c = 18 - 13 = 5$ volts Verified

(3) Use Superposition theorem to find I_x of the values shown in fig. (8m) Jan 2015



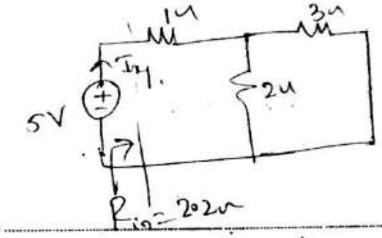
Verification using nodal

$$\frac{V_a-5}{1} + \frac{V_a-10}{3} + \frac{V_a}{2} - 1 = 0$$

$V_a = 5.0901$ A

$I_x = 5\cdot$

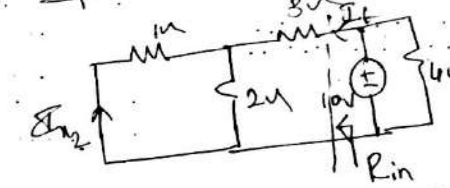
Soln: Step 1. 5V source acting alone.



$R_{in} = 2.2\Omega$

$I_{x_1} = \frac{5}{2.2} = 2.2727$ Amperes

Step 2. 10V acting alone.

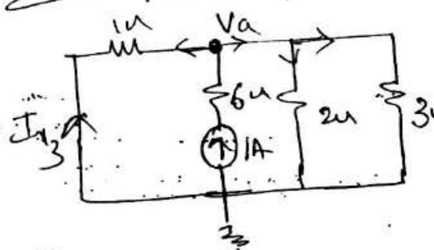


$R_{in} = 3 + 1\|2 = 3.666\Omega$

$I_t = \frac{10}{R_{in}} = \frac{10}{3.666} = 2.7227A$

$$I_{x_2} = -\left[\frac{2.7227 \times 2}{1+2}\right] = -1.01818A$$

Step 3. 1A source acting alone.



$\frac{V_a}{1} + \frac{V_a}{2} + \frac{V_a}{3} - 1 = 0$

$V_a = 0.5454$ volts

$I_{x_3} = \frac{-V_a}{1} = -0.5454$ Amperes

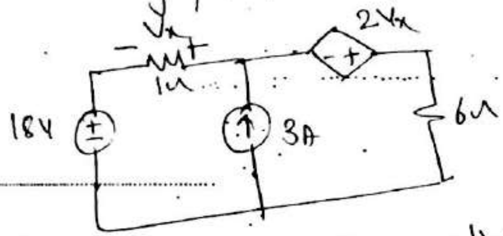
Using SPT

$I_x = I_{x_1} + I_{x_2} + I_{x_3}$

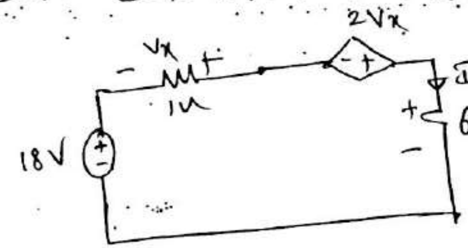
$= 2.2752 - 1.1818 - 0.5454$

$\boxed{I_x = 0.5479}$ Amperes.

④ Calculate the Current in the 6Ω resistor of the ckt shown in fig using principle of superposition. (6m) J/J 2014



Sol: Step1: 18V Source acting alone



using KVL

$$18 + V_x + 2V_x - 6I_{x_1} = 0$$

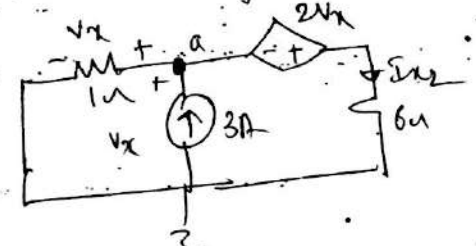
but $V_x = -I_{x_1}(1)$

$$V_x = -I_{x_1}$$

$$18 - I_{x_1} - 2I_{x_1} - 6I_{x_1} = 0$$

$$18 = +9I_{x_1} \Rightarrow \boxed{I_{x_1} = 2} \text{ Amperes}$$

Step2: 3A Source acting alone



KCL @ a

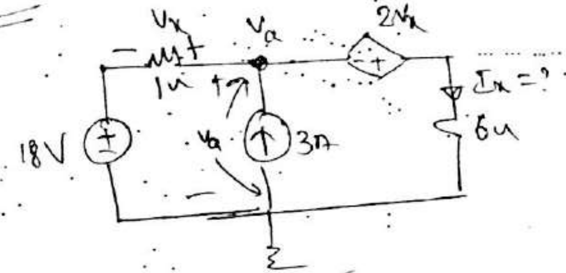
$$\frac{V_x}{1} - 3 + \frac{V_x + 2V_x}{6} = 0 \Rightarrow \boxed{V_x = 2} \text{ volts}$$

$$I_{x_2} = \frac{V_x + 2V_x}{6} = \frac{2+4}{6} = 1A$$

using SPT the current through 6Ω resistor in

$$I_x = I_{x_1} + I_{x_2} = 2 + 1 = 3A$$

Verification



$$\frac{V_a - 18}{1} - 3 + \frac{V_a + 2V_x}{6} = 0$$

$$[1 + 6^{-1}]V_a + \left(\frac{1}{3}\right)V_x = (3+18) \leftarrow ①$$

$$18 + V_x - V_a = 0$$

$$\Rightarrow -V_a + V_x = -18 \leftarrow ②$$

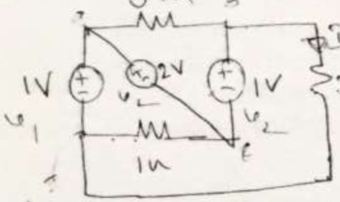
$V_a = 18$ volts $\quad V_x = 0$ volts.

$$\Rightarrow I_x = \frac{V_a + 2V_x}{6} = \frac{18+0}{6} = 3A.$$

Verified

(5) Find the current through 2Ω resistor in the n/w shown in fig. using Superposition theorem.

Jan 2014 (8m)



$V_a = 1V \leftarrow (1)$ $\quad V_b - V_c = 1V \leftarrow (2)$

$$\frac{V_b - 1}{1} + \frac{V_c}{2} + \frac{V_b}{1} = 0$$

$$2V_b + 0.5V_c = 1 \leftarrow (3)$$

$$V_a - V_c = 2V \leftarrow (4)$$

$V_a = 1$ Volt.

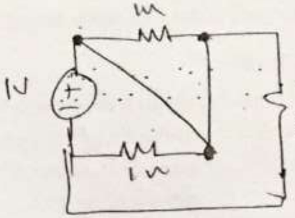
$\boxed{V_c = 1 - 2 = -1 \text{ Volt}}$

$V_b = 1 + V_c = 1 - 1 = 0$ Volt.

$$I_x = \frac{V_b - V_c}{2} = 0A$$

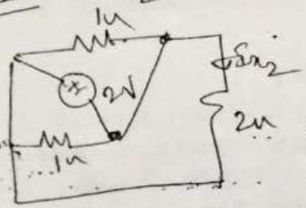
Verified

Soln - Step1. $V_1 = 1V$ Source alone



$\boxed{I_{x_1} = \frac{1}{2} A}$

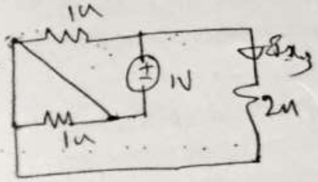
Step2. $V_2 = 2V$ Source alone.



$I_{x_2} = \frac{-2}{2} = A$

$\boxed{I_{x_2} = -1 A}$

Step3 $V_3 = 1V$ acting alone.



$I_{x_3} = \frac{1}{2} A$

Using SPT

$I = I_{x_1} + I_{x_2} + I_{x_3} = 0.5 - 1 + 0.5 = 0A$

PROGRAMME EDUCATIONAL OBJECTIVES (PEQs)

PEO1 : To educate to be a Electronics and Communication Engineering graduate with an ability to pursue higher studies in blobal scenario.

PEO2 : To educate the learners to be highly competent Electronics and Communication Engineers with in-depth knowledge in the engineering fundamentals and chosen domain.

PEO3 : To impart the knowledge to the students to be able to function in a team with varied professional background or fields of Engineering and Technology to be able to meet the challenges of competitive field.

PEO4 : To enable the Electronics and communication engineering graduates in a truly professional way with ethical approach in solving and serving the needs of the society with humane approach.

PEO5 : To motivate the Electronic and communication engineering graduates to keep abreast with modern ever changing engineering and techonologies and applications to evolve with innovative solutions and to build a carrier of their own with leader ship qualities.

PROGRAMME OUTCOMES

a) An ability to apply knowledge of mathematics, science, and engineering,

b) An ability to design and conduct experiments, as well as to analyze and interpret data,

c) An ability to design a system, components, or process to meet desired needs within realistic constraints such as economic, environmental, social, political, ethical, health and safety, manufacturability, and sustainability,

d) An ability to function on multidisciplinary teams,

e) An ability to identify, formulate, and solve engineering problems,

f) An understanding of professional and ethical responsibility,

g) An ability to communicate effectively,

h) The board education necessary to understand the impact of engineering solutions in a global, economic, environmental, and societal context,

i) A recognition of the need for and an ability to engage in life-long learning,

j) A knowledge of contemporary issues, and

k) An ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.

l) Knowledge of advanced mathematics, including differential equations, linear algebra, complex variables, an probability and statistics, including applications to electronics and communication engineering.

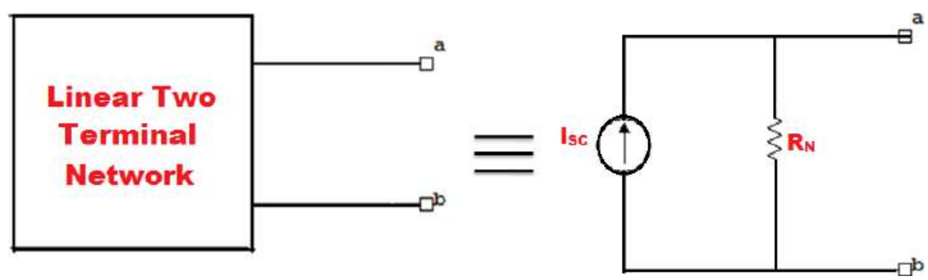
Theorem 1: Norton's Theorem

Statement :

Norton's Theorem states that a linear two terminal network can be replaced by an equivalent circuit consisting of a current I_N in parallel with a resistor R_N , where

- R_N is the equivalent resistance at the terminals when the independent sources are turned off
- I_N is short circuit current through the terminals.

If the circuit consists of the dependent sources the Norton's resistance has to be found out as R_N = Voc / Isc

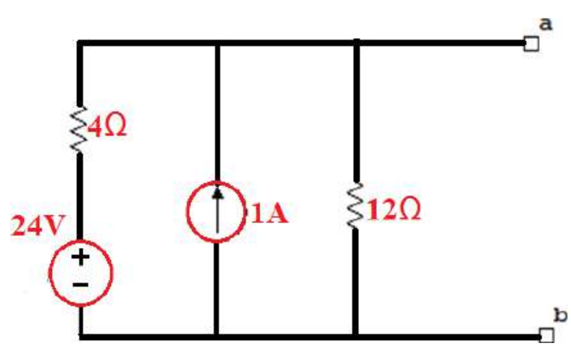


There can be two types of problems,

1. To find the Norton's equivalent circuit across the open circuit terminals
2. To find a voltage or a current in the circuit by Norton's Theorem.

Problems:

P1. Find the Norton's equivalent circuit across the terminals a-b

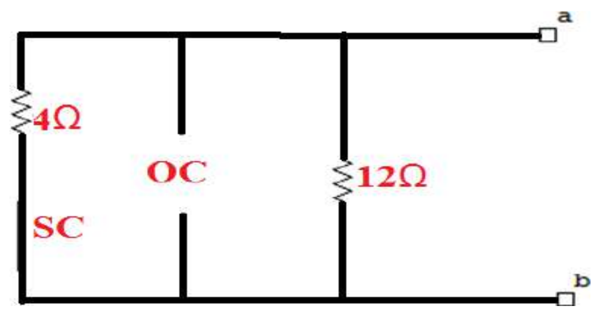


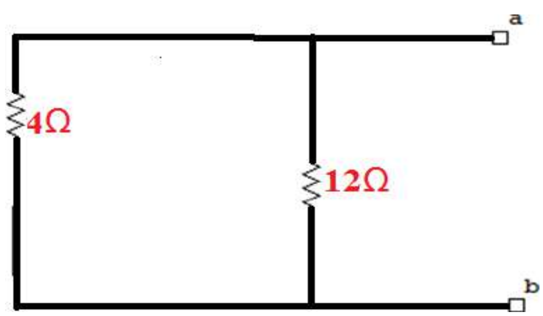
Solution:

Steps to find out the Norton's Resistance R_N :

Step 1: Turn off the independent sources

(open-circuit the current source and short-circuit the voltage source)





Step 2: Find the equivalent resistance looking into the open circuit terminals

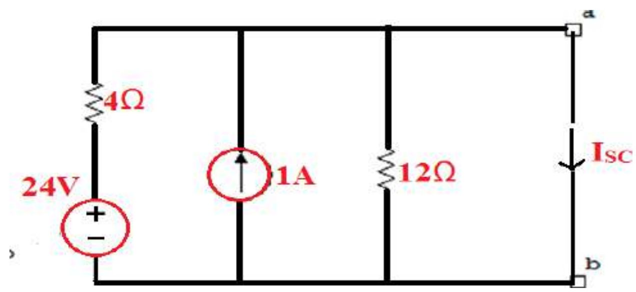
R_N= 12 x 4 / 12 + 4

R_N = 3 Ω

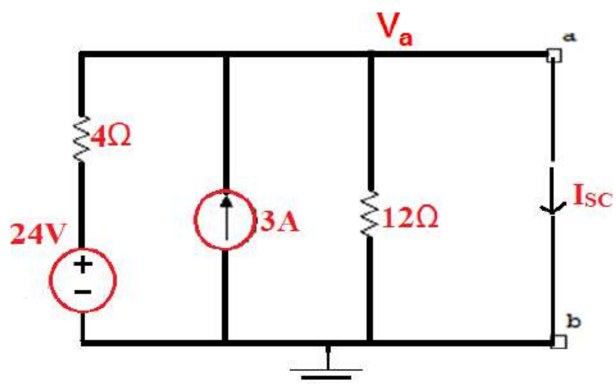
<u>Steps to find out the Norton's Current I_N (Short circuit current):</u>

Step 1: Short circuit the open circuit terminals and mark the I_{SC} as shown.

Step 2: Find the short circuit current by a suitable technique



By Node Analysis:



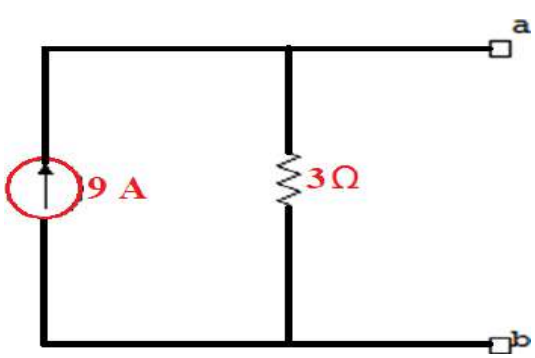
Applying KCL at node a :

$$\frac{Va - 24}{4} + \frac{Va}{12} + Isc = 3$$

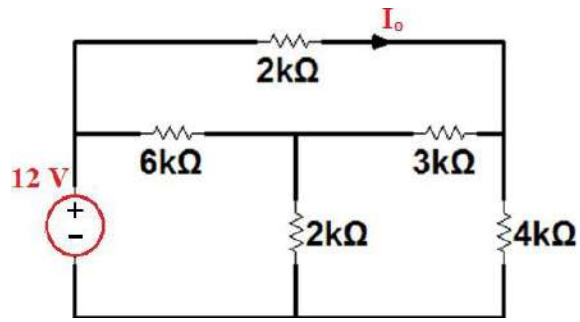
Substituting Va = 0 V in the above equation implies

Isc= 9 A

Therefore the Norton's equivalent circuit across terminals a-b is



P2. Find I_0 in the network shown, using Norton's Theorem



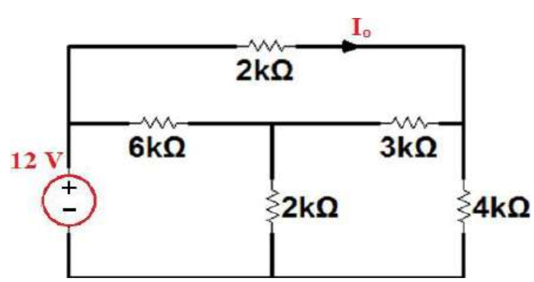
<u>Solution:</u>

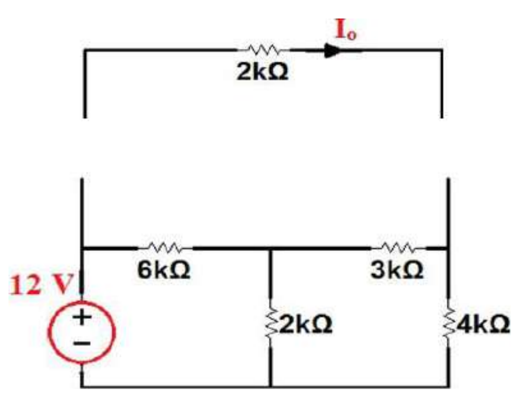
Step 1: Separate the branch through which I_0 is flowing

Step 2: Find the Norton's equivalent network across the open circuit terminals

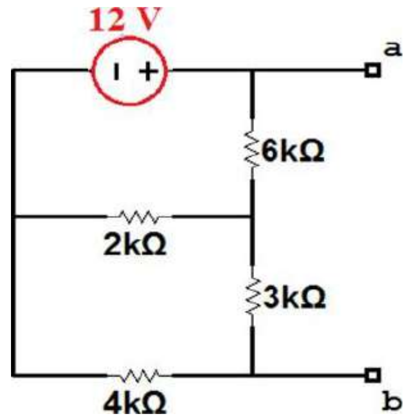
Step 3: Connect the branch separated, back to the Norton's equivalent circuit to find I_0

<u>Step 1: Separate the branch through which I_0 is flowing</u>

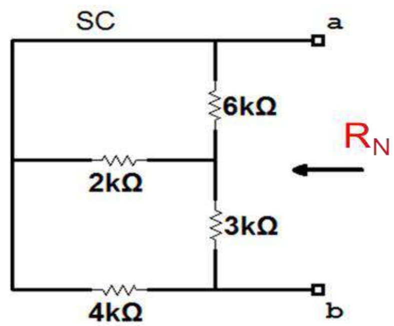




Step 2: Find the Norton's equivalent network across the open circuit terminals a-b



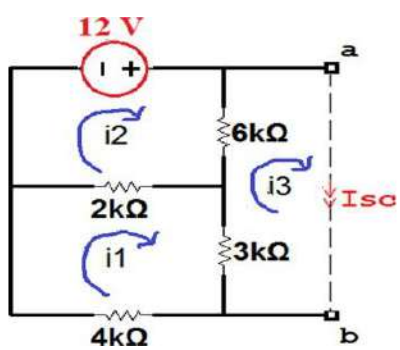
Find the R_N across the open circuit terminals a-b by short-circuiting12 V source



R_N = [(6 K || 2 K) + 3 K] || 4 K

R_N = 2.12 K Ω

Find the I_{SC} or I_N through terminals a-b by short-circuiting a-b as shown



By Mesh Analysis:

Mark i1, i2, i3 as shown

KVL to Mesh 1:

4Ki1+ 2K(i1 - i2) + 3K(i1 - i3) = 0

9K i1 – 2K i2 – 3K i3 = 0 .... Eq1

KVL to mesh 2:

-12 + 6K(i2 – i3) + 2K(i2 – i1)=0

-2K i1 + 8K i2 – 6K i3 = 12 ....Eq2

KVL to mesh 3:

3K (i3 – i1) + 6K (i3 – i2)=0

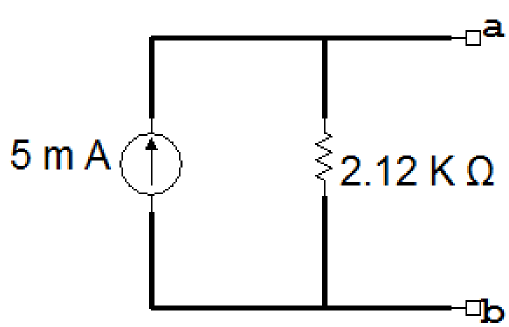
-3K i1 – 6K i2 + 9K i3 = 0 .... Eq3

Solving Eq1, Eq2 and Eq3 we have,

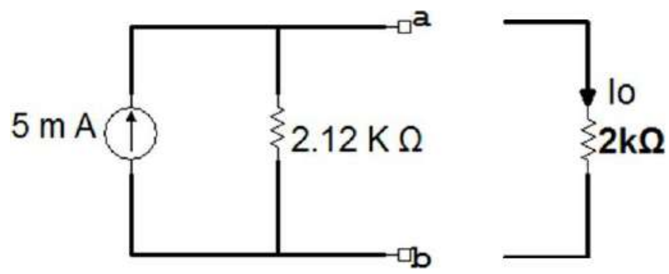
i1= 3mA, i2=6mA, i3=5mA

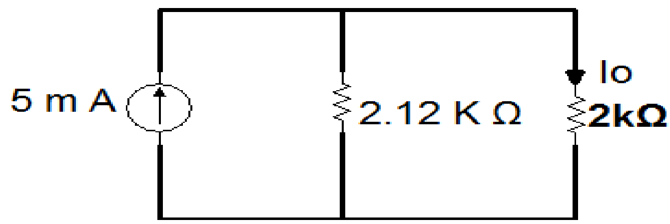
Isc = i3 = 5mA

Therefore the Norton's equivalent circuit across terminals a-b is



Step 3: Connect the branch separated, back to the Norton's equivalent circuit to find I_0

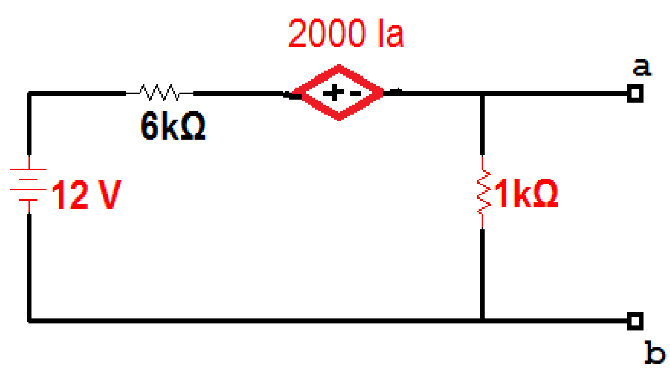




By Current Division Method

$$Io = \frac{5m \times 2.12\ K}{2\ K + 2.12\ K} = 2.57\ mA$$

P3. Find the Norton's Equivalent network across the terminals a-b



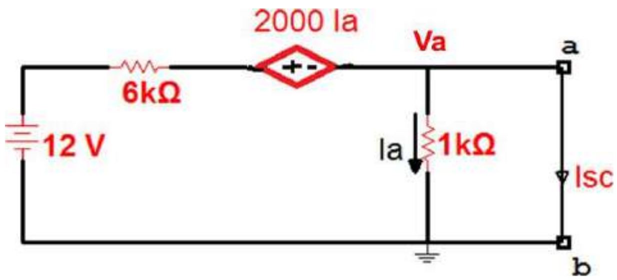
Solution:

Since the network consists of the dependent source (Dependant sources cannot be turned off) the Norton's resistance has to be found out as

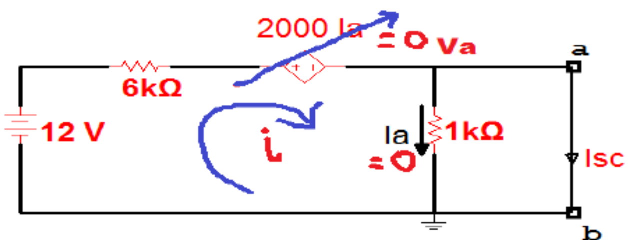
$$R_N = Voc / Isc$$

<u>Step 1: To find out I_{SC} (I_N)</u>

Short Circuit the terminals a-b and mark I_{SC} as shown

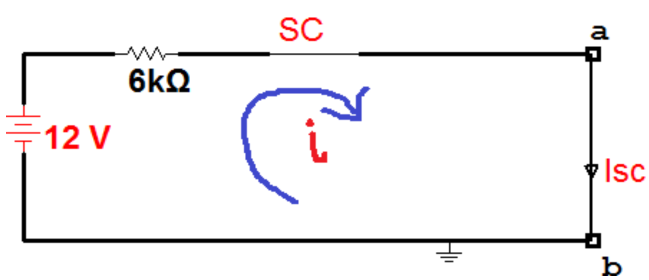


Va = Ia = 0



Since Va is connected to ground through short circuit terminals a-b Va=0.

Hence the circuit gets reduced to…

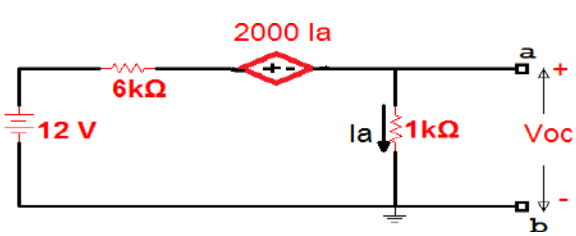


KVL: -12 + 6K i =0

i = 12/6K = 2 m A

I_{SC} = i = 2 m A

Step 2: To find out V_{OC}



KCL at node a:

$$\frac{Voc + 2000Ia - 12}{6K} + \frac{Voc}{1K} = 0$$

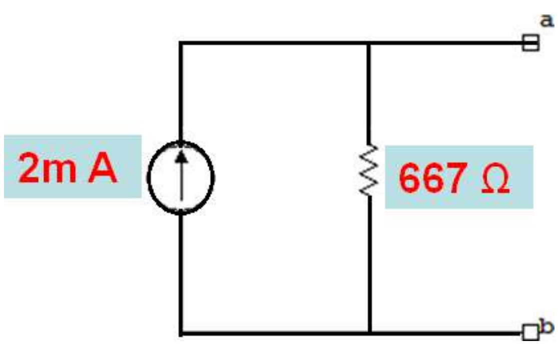
2000 Ia + 7 Va =12

Substituting $Ia = \frac{Voc}{1K}$

$\underline{V_{OC} = 4/3\ V}$

Therefore $R_N = V_{OC} / I_{SC} = 667\ \Omega$

Therefore Norton's equivalent circuit across the terminals a-b is given by



Theorem 2: Thevenin's Theorem

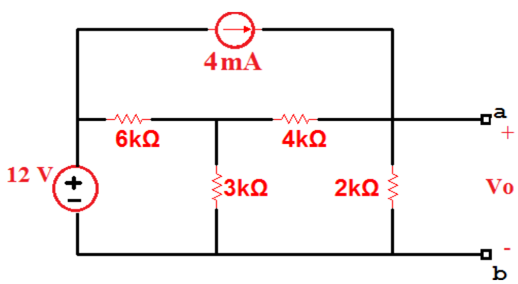
Definition :

Thevenin's Theorem states that a linear two terminal network can be replaced by an equivalent network consisting of an Voltage V_T in series with a resistor R_T , where

- R_T is the equivalent resistance at the terminals when the independent sources are turned off
- V_T is open circuit voltage across the terminals.

If the circuit consists of the dependent sources the Norton's resistance has to be found out as R_T = Voc / Isc

P1. Find V_o by Thevenin's Theorem



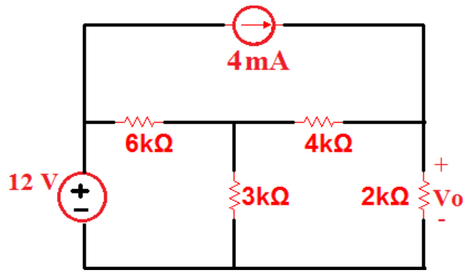
Solution:

Step 1: Remove resistor 2K Ω from the circuit across which V_O is dropping

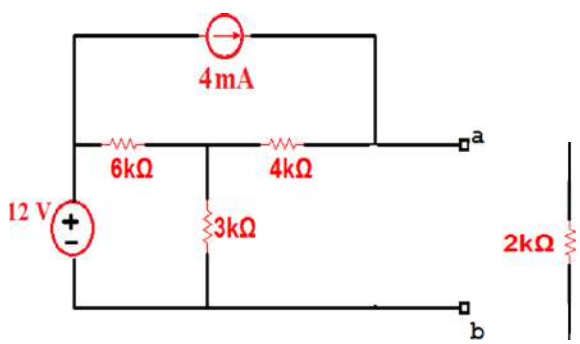
Step 2: Find the Thevenin's network across the open circuit terminals a-b

Step 3: Connect 2K Ω (Disconnected in Step 1) across the open circuit terminals a-b and find V_O.

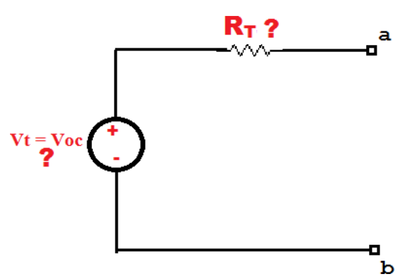
Circuit can be visualized as,



Step 1: Remove resistor 2K Ω from the circuit across which V_O is dropping and mark terminals a-b

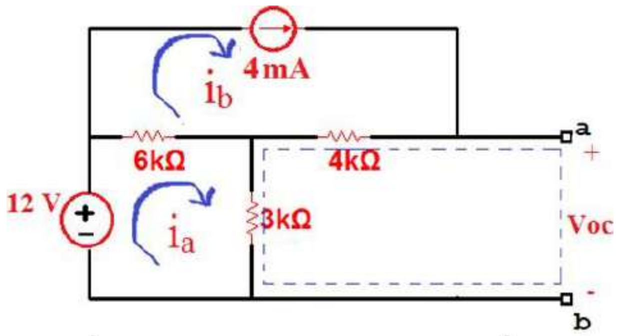


Step 2: Find the Thevenin's network across the open circuit terminals a-b



<u>To find V_{OC}:</u>

Mark V_{OC} across the open circuit terminals as shown:



Mark Mesh currents i $_a$ and i $_b$:

By Observation:

I_a = 4 m A

Applying KVL to Mesh 1:

$- 12 + 6K (i_a - i_b) + 3K i_a = 0$

$9K i_a - 6K i_b = 12$

Sub. I_a = 4 mA,

I_b = 4 m A

To find V_{oc} apply KVL along the dotted path:

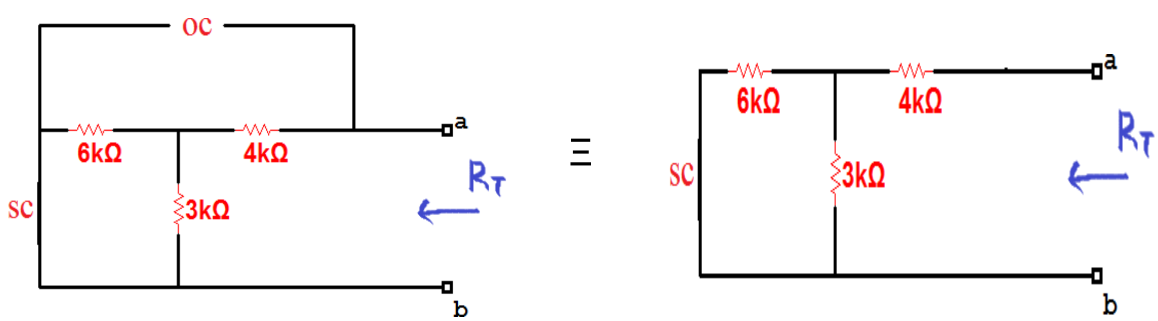
$- 3K I_a - 4K I_b + Voc = 0$

Sub. I_a and I_b,

$\boxed{Voc = 28 V}$

To find R_T :

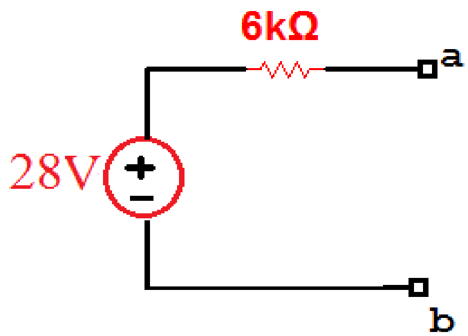
Deactivate the independent sources



RT = (6K || 3K) + 4K

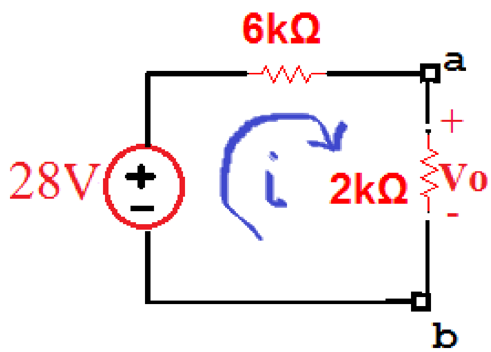
$\boxed{R_T = 6\ K}$

Therefore the Thevenini's network is



Step 3: To find V_O

Now connect 2 K Ω across a-b to find V_O



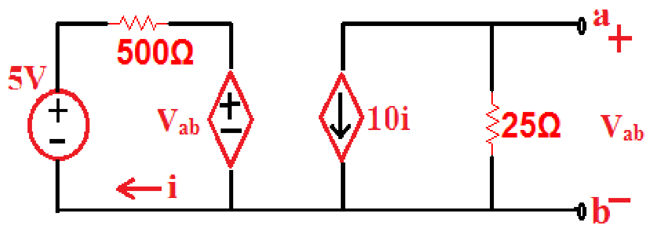
KVL gives,

-28 + 6K i + 2K i = 0

i = 28/8K = 3.5 mA

$\boxed{Vo= 2K\ i1 = 7\ V}$

P2. Find the Thevenin's Equivalent circuit across terminals a-b

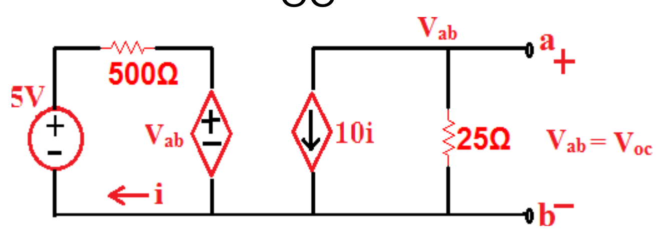


Solution:

Since the dependant sources are involved R_T is given by

$$\mathbf{R_T = V_{oc} / I_{SC}}$$

Step 1: To find V_{OC}



Applying KVL to LHS part:

$-5 + 500\ i + V_{ab} = 0$

$500\ i + V_{ab} = 5$

Applying KCL to RHS part:

$10\ i + V_{ab}/25 = 0$

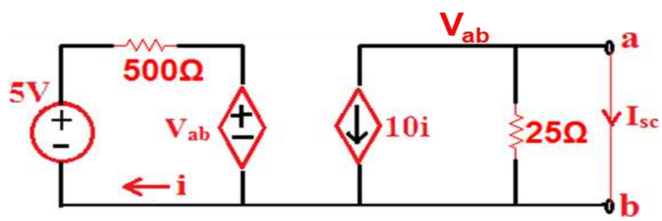
$250\ i + V_{ab} = 0$

Solving equations we have

$i = 0.02$ A $\quad V_{ab} = -5$ V

$V_{oc} = V_{ab} = -5$ V

Step 2:To find I_{SC}



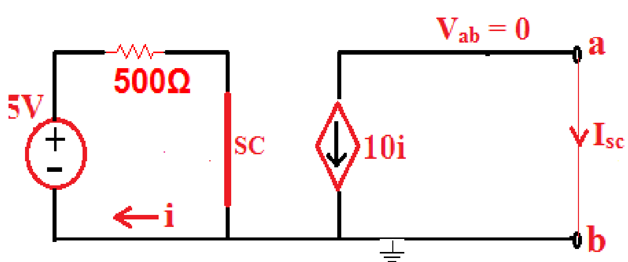
Short circuit terminals a-b and mark I_{SC} as shown

Mark V_{ab}

Since V_{ab} is connected to ground through a-b, $\mathbf{V_{ab} = 0}$

Since 25 Ω is in parallel with a short, 25 Ω is redundant

Therefore the circuit reduces to,



From LHS part, KVL gives

$-5 + 500\, i = 0$

From RHS part,

$I_{SC} = -10i$

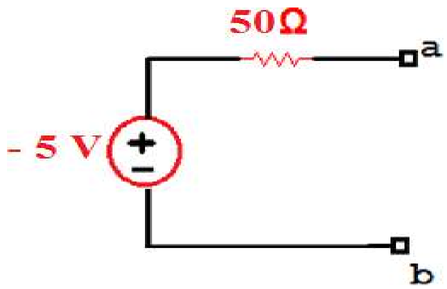
and sub. i = 0.01 A

$\boxed{I_{SC} = -0.1\text{ A}}$

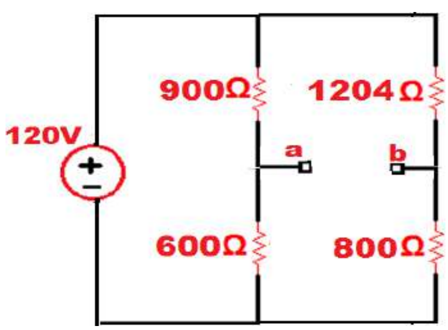
Therefore $R_T = V_{OC} / I_{SC} = -5 / -0.1$

$R_T = 50\ \Omega$

Therefore the Thevenin's network is,



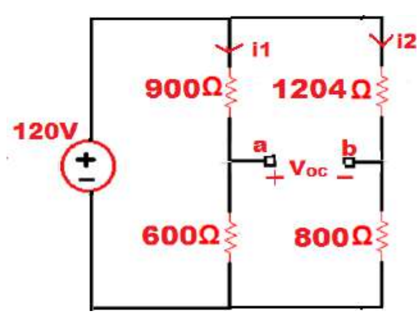
P3. Find the Thevenin's Equivalent network across terminals a-b



Solution:

Step1: To find Mark V_{OC} (V_T) across terminals a-b

Mark the branch currents i1 and i2 as shown



Applying KVL to mesh1

$-120 + 900\ i1 + 600\ i1 = 0$

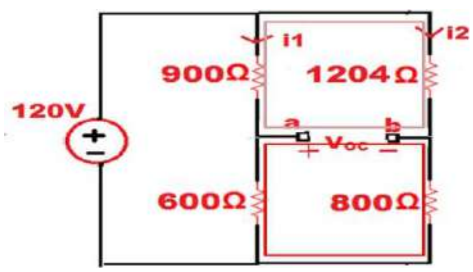
$i1 = 0.08$ A

Applying KVL to mesh2

$-120 + 1204\ i2 + 800\ i2 = 0$

$i2 = 0.05988$ A

To find V_{OC}:



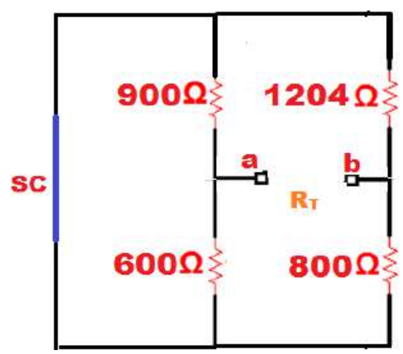
Applying KVL along the pink path

$-900\ i1 + 1204\ i2 - V_{OC} = 0$

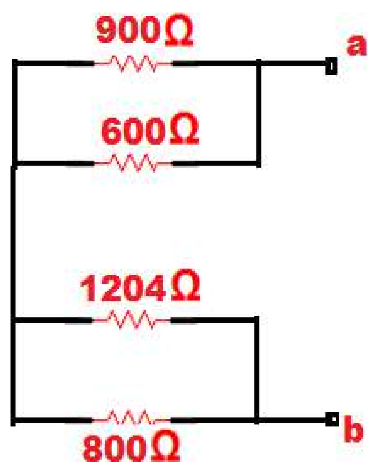
$V_{OC} = 0.095$ V

Step 2: To find R_T

Turning off 120 V source



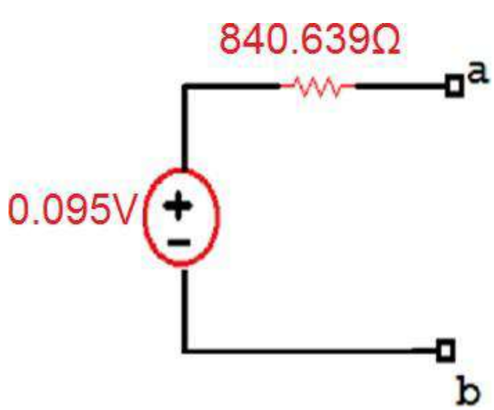
which can be visualized as



R_T = (900 || 600) + (1204 || 800)

R_T = 840.638 Ω

Therefore Thevenin's network is



Summary:

1. Thevenin's network is a Voltage in series with a resistor

2. Thevenin's voltage is V_{OC} across the terminals

3. Thevenin's resitance and Norton's resistance are the same.

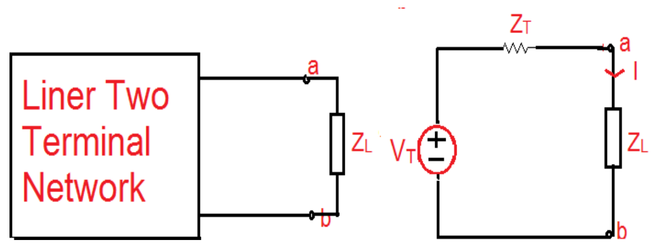
4. Thevenin's and Norton's equivalent networks can be obtained by source trensformatiom.

Theorem 3: Maximum Power Transfer Theorem

There are three cases to be considered in this

1. AC circuits with Impedance (Z_L) as load
2. AC circuits with purely resistive load (R_L)
3. DC circuits with resistive load (R_L)

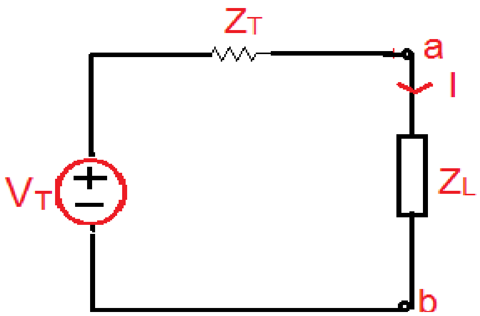
Conditions for Maximum Power Transfer :



where,

$Z_T = R_T + j\,X_T$

$Z_L = R_L + j\,X_L$



KVL to closed path:

$-V_T + Z_T\, I + Z_L I = 0$

$$I = \frac{V_T}{Z_T + Z_L} = \frac{V_T}{(R_T + j\, X_L) + (R_L + jX_L)}$$

The average power delivered to the load is

$$P = \frac{1}{2} |I^2| R \qquad \ldots\ldots\ldots\ldots\ldots \qquad 1$$

$$I^2 = \frac{V_T^2}{[(R_T + j\, X_T) + (R_L + jX_L)]^2}$$

$$I^2 = \frac{V_T^2}{[(R_T + R_L) + j(X_T + X_L)]^2}$$

$$|I|^2 = \frac{|V_T|^2}{\left[\sqrt{(R_T + R_L)^2 + (X_T + X_L)^2}\right]^2}$$

Subtituting in equation in 1

$$P = \frac{R_L}{2} \frac{|V_T|^2}{(R_T + R_L)^2 + (X_T + X_L)^2}$$

For this P to be P_{Max} we can vary two parameters

– R_L and X_L in the load impedance.

Mathematically it can be done by differentiating P with respect to R_L and X_L partially and equating it to zero respectively.

i.e,

$$\frac{\partial P}{\partial R_L} = 0 \quad \text{and} \quad \frac{\partial P}{\partial X_L} = 0$$

Performing $\frac{\partial P}{\partial R_L} = 0$ results in

$$(R_T + R_L)^2 + (X_T + X_L)^2 - 2R_L(R_T + R_L) = 0$$

This implies

$$R_L = \sqrt{R_T^2 + (X_T + X_L)^2} \quad \ldots\ldots\ldots\ldots 2$$

Performing $\frac{\partial P}{\partial X_L} = 0$ results in

$$X_L = -X_T \quad \ldots\ldots\ldots\ldots 3$$

Substituting 3 in 2

$$R_L = R_T \quad \ldots\ldots\ldots\ldots\ldots 4$$

From equations 3 and 4

$$Z_L = R_L + j\,X_L = R_T - j\,X_T$$

$$Z_L = Z_T^*$$

If the Load Z_L is purely resistive then

$X_L = 0$ and $Z_L = R_L$

Substituting $X_L = 0$ in 2

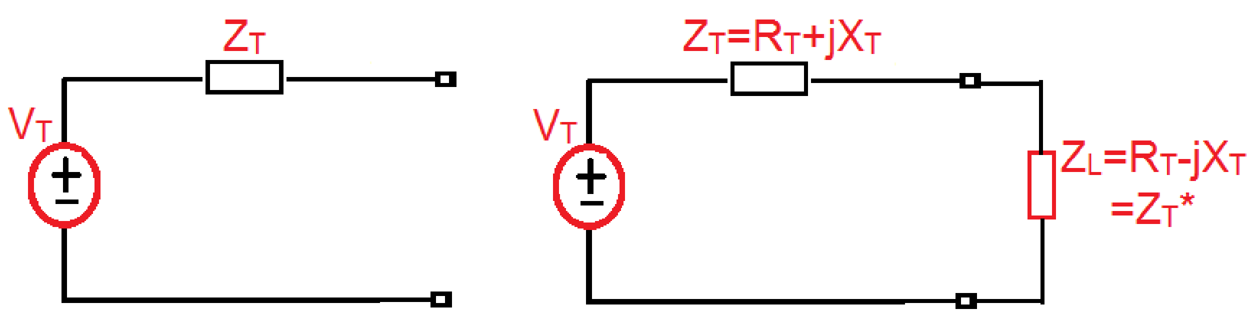
$$R_L = \sqrt{R_T^2 + X_T^2} \quad \text{..................} 5$$

$$R_L = |Z_T| \quad \text{...............} 6$$

Equations 4 , 5 and 6 are the conditions for which the maximum power would be transferred to the load.

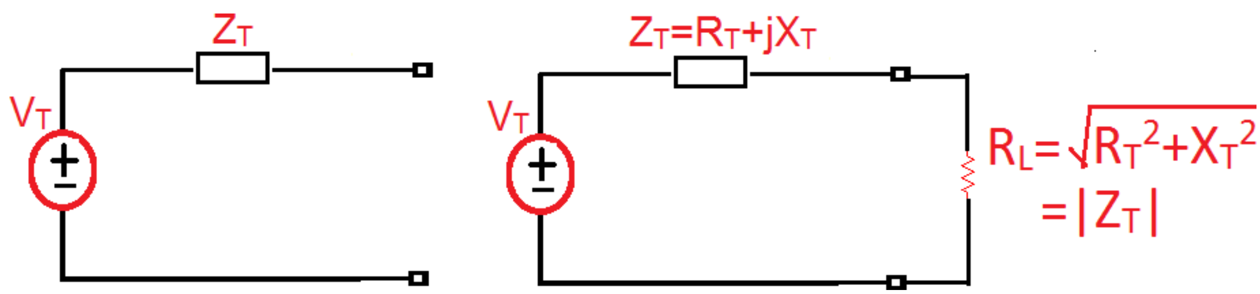
Highlights:

1. AC circuits with Impedance (Z_L) as load



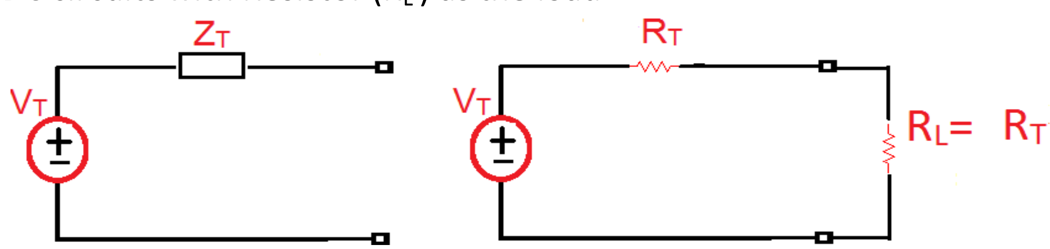
$P_{max} = |\ i\ |^2 R_L$

2. AC circuits with Pure Resistive (R_L) load



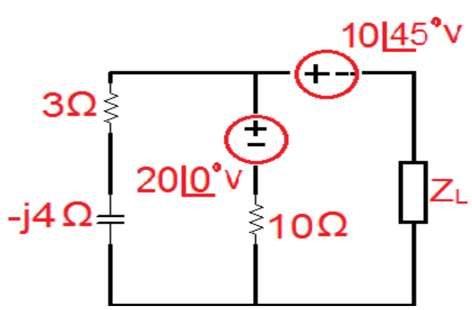
$P_{max} = |\ i\ |^2\ R_L$

3. DC circuits with Resistor (R_L) as the load



$P_{max} = i^2\ R_L$

P1. Calculate the value of Z_L for maximum power transfer and also calculate the maximum power.



<u>Solution:</u>

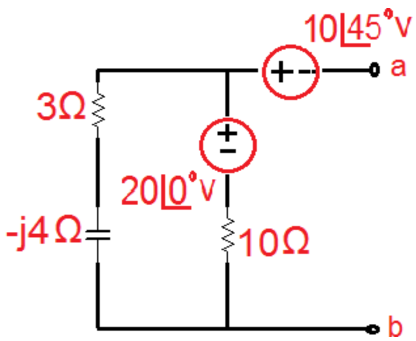
<u>Step1.</u> Remove the Impedance Z_L

<u>Step2</u>. Find the Thevenin's equivalent network across the terminals a-b

<u>Step3</u>. Connect Z_L=Z_T* across the terminals a-b for the maximum power transfer.

<u>Step4.</u> Find $P_{max} = |\ I\ |^2\ R_L$

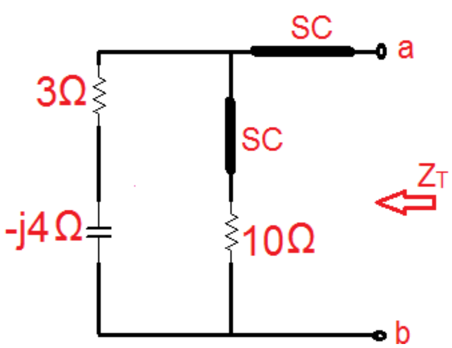
Step1. Remove the Impedance Z_L and mark terminals a-b



Step2. Find the Thevenin's equivalent network across the terminals a-b.

To find Thevenin's Impedance Z_L:

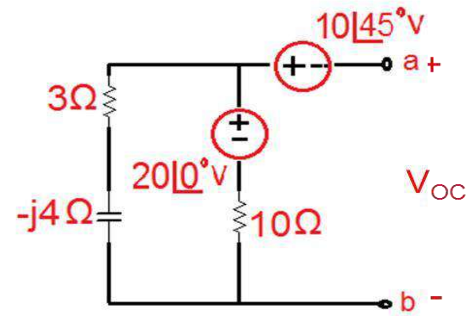
Deactivating the independent sources we have,

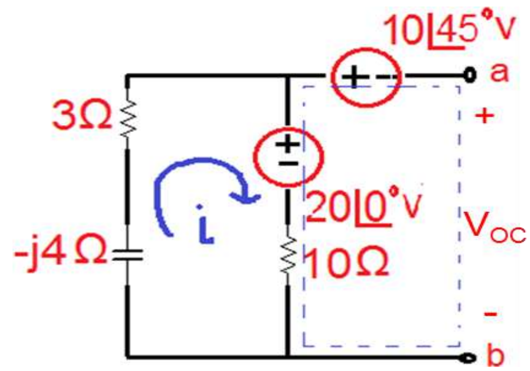


$Z_T = 10 \,||\, (3 - j\,4)$

$Z_T = 2.97 - j\,2.16\ \Omega$

To find Thevenin's Voltage V_T or V_{OC}:





<u>KVL implies:</u>

(3-j4) i + 20 +10 i = 0

i = -1.405 - j 0.432

KVL along the dotted path to find V_{OC}:

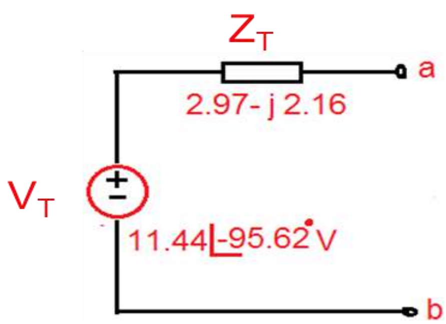
- 10 i – 20 + 10∟45 + V_{OC} = 0

Substituting i

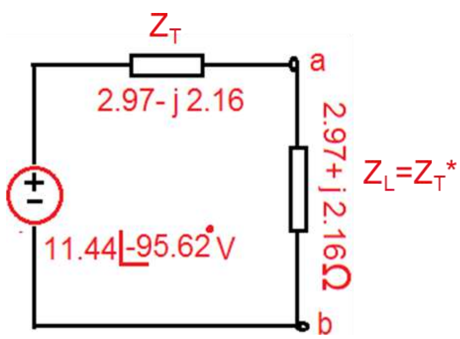
V_T = -1.121- j 1.391

= 11.44 ∟-95.62 V

Therefore Thevenin's equivalent network is



<u>Step3. Connect Z_L=Z_T* across the terminals a-b to find the maximum power transfer.</u>



<u>KVL implies:</u>

$-11.44 \angle -95.62 + (2.9729)i + (2.9729)i = 0$

$i = -0.185 - j\,1.916$ A

$i = 1.925 \angle -95.62$ A

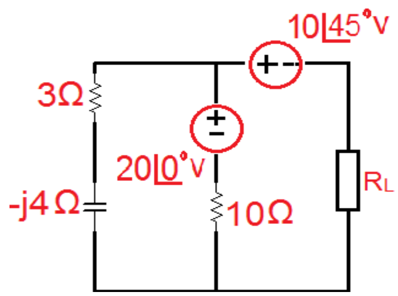
<u>Step 4. To find P_{max}</u>

$P_{max} = |\, i \,|^{2} R_L$

$= (1.925)^2 x\ 2.9729$

P_{max} = 11 Watts

P2. Calculate the value of R_L for maximum power transfer and also calculate the maximum power.



Solution:

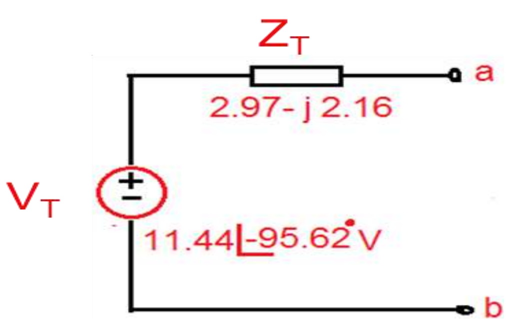
Step1. Remove the Impedance Z_L

Step2. Find the Thevenin's equivalent network across the terminals a-b

Step3. Connect Z_L=| Z | across the terminals a-b for the maximum power transfer.

Step4. Find $P_{max} = |I|^2 R_L$

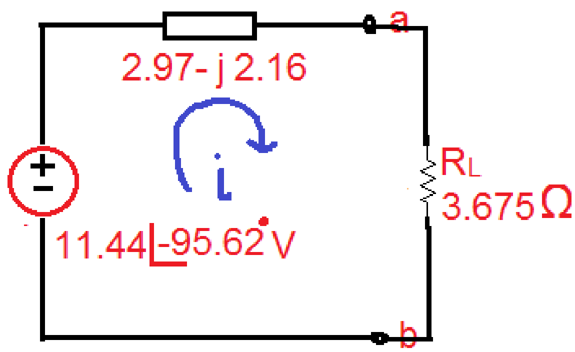
From Step1 and Step2 (Refer P1), the Thevenin's equivalent is



Step3. Connect R_L=| Z| across the terminals a-b to find the maximum power transfer.

$$R_L = |Z_T| = \sqrt{(2.97)^2 + (2.16)^2}$$

R_L = 3.675 Ω



KVL implies

$-11.44 \angle -95.62 + (2.97 - j\,2.16)\,i + 3.675\,i = 0$

$i = 1.6377 \angle -77.62$ A

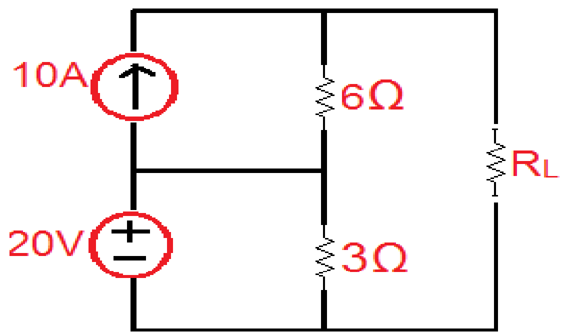
<u>Step 4. To find P_{max}</u>

$P_{max} = |\,i\,|^{2} R_L$

$= (1.6377)^2 \times 3.675$

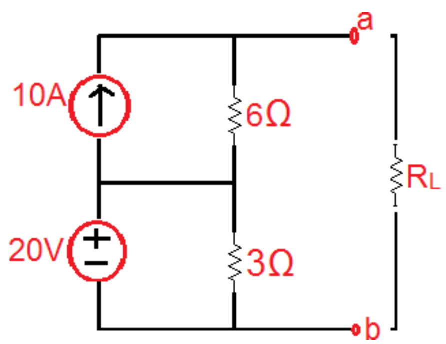
$P_{max} = 9.85$ W

P3. Find the R_L across the load for which maximum power will be transferred to the load and hence find the maximum power



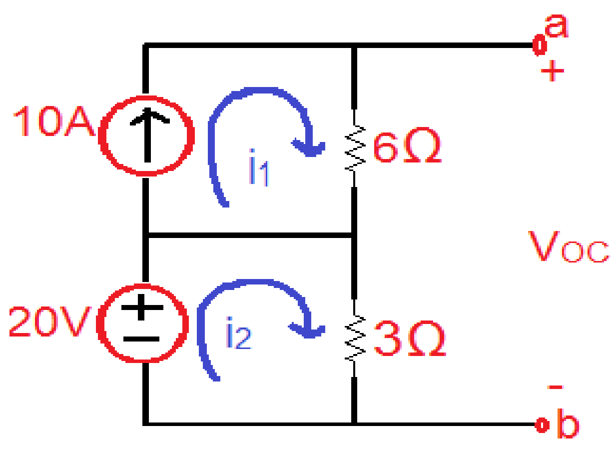
Solution:

Step 1: Remove the resistor R_L and mark terminals a-b as shown



Step 2: Find the Thevenin's network across the terminals a-b

To find V_{OC}:



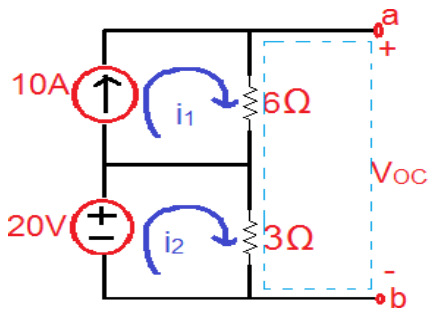
By observation:

$i_1 = 10$ A

KVL to mesh 2:

$-20 + 3\ i_2 = 0$

$i_2 = 20/3$ A



$-\ 3i_2 - 6i_1 + V_{OC} = 0$

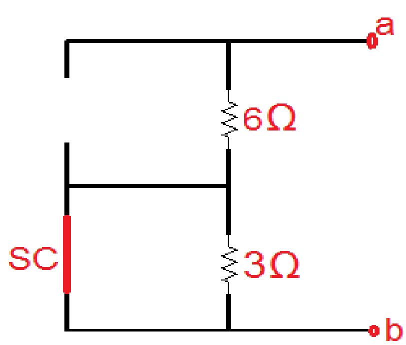
KVL along the dotted path

$V_{OC} = 6\ i_1 + 3\ i_2$

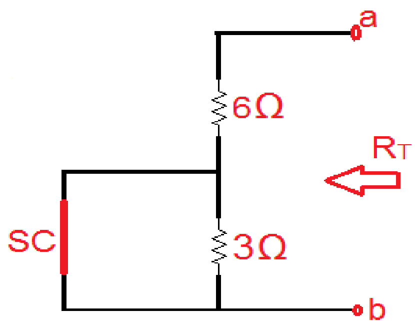
Substituting i_1 and i_2

$V_T = V_{OC} = 80$ V

To find R_T:



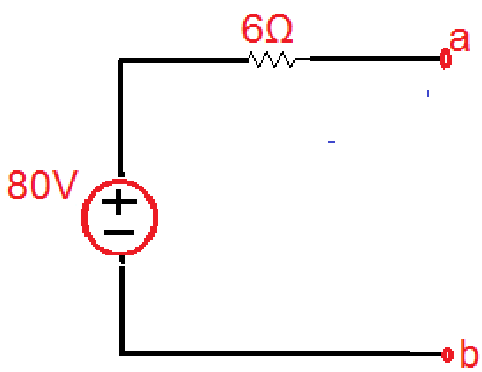
which can be visualized as



Since 3 Ω is in parallel with the short, it is redundant.

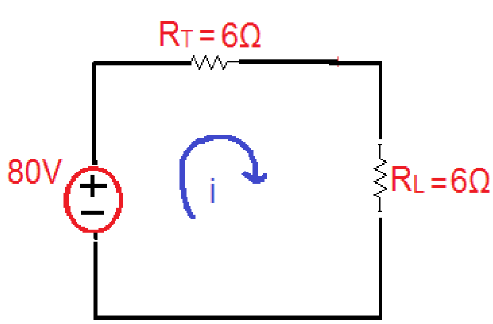
Therefore R_T= 6 Ω

Therefore Thevenin's network is



<u>Step 3: To find P_{max}</u>

Connect R_L = R_T across the terminals a-b



KVL implies:

$- 80 + 6 i + 6 i = 0$

$i = 20/3$ A

$$P_{max} = i^2 R_L = (20/3)^2 \times 6 = 266.66 \text{ W}$$

<u>Summary:</u>

1. Maximum power transfer theorem is the extention of Thevenin's theorem.
2. The coditions for Maximum power to be transferred to the load are

    i) For AC circuits if load is impedance then $Z_L = Z_T^*$

    ii)For AC circuits if load is purely resistive then $R_L = | Z_T |$

    iii)For DC circuits $R_L = R_T$

3. Power is always a real entity and therefore for power calculations always real part of Z_L (i.e.,R_L) is used.

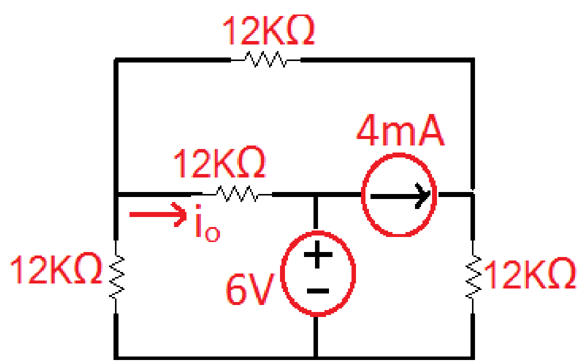
Theorem 4: Superposition Theorem

Statement:

In any Linear circuit containing multiple independent sources, a current or a voltage at any point in the circuit can be calculated as algebraic sum of Individual contributions of each source when acting alone.

Problems:

P1. Find i_o by Super position theorem.



Solution:

Let $i_0 = i_{01} + i_{02}$

where,

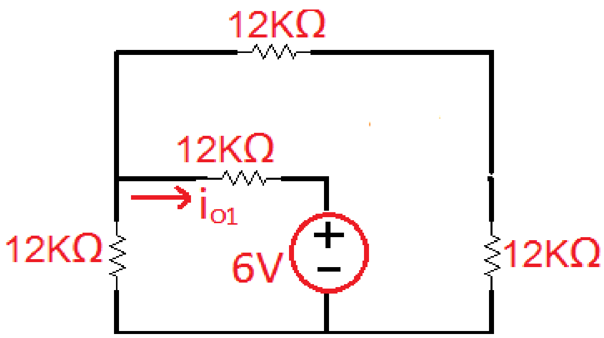
i_{01} is the contribution of 6 V source when acting alone and

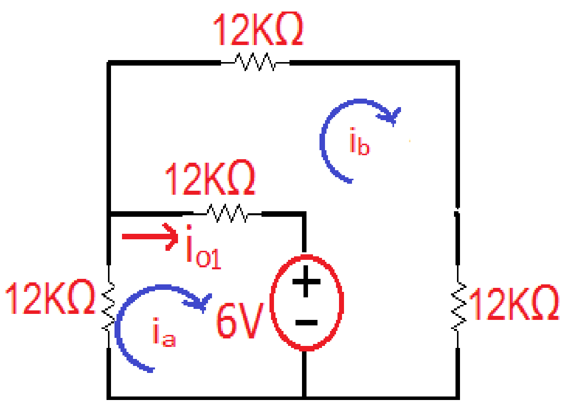
i_{02} is the contribution of 4mA source when acting alone

Steps:

Step 1 : To find i_{01} which is the contribution of 6 V acting alone

Deactivating the 4mA source the circuit becomes





<u>Applying KVL to mesh 1:</u>

$12K\ i_a + 12K\ (i_a - i_b) + 6 = 0$

$24K\ i_a - 12K\ i_b = -6$ Eq1

<u>Applying KVL to mesh 2:</u>

$12K\ (i_b - i_a) + 12K\ i_b + 12K\ i_b - 6 = 0$

$-12K\ i_a + 36K\ i_b = 6$ Eq2

Solving equations Eq1 and Eq2,

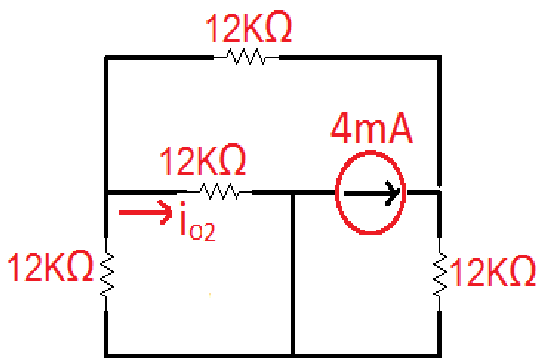
$i_a = -0.2$ mA

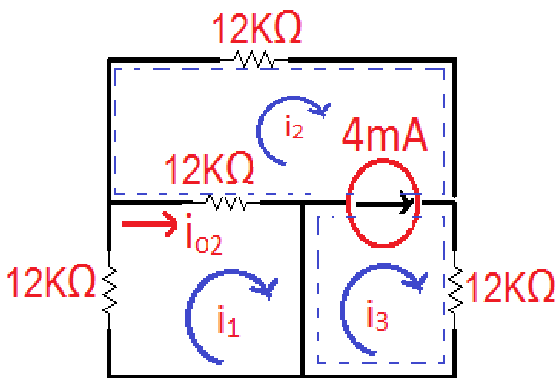
$i_b = 0.1$ mA

$\mathbf{i_{o1} = i_a - i_b = -0.3\ mA}$

Step 2 : To find i_{o2} which is the contribution of 4mA source acting alone

Deactivating the 6 V source the circuit becomes





Constraint equation:

$i_3 - i_2 = 4mA$

Applying KVL to mesh 1:

$12K\, i_1 + 12K\,(i_1 - i_2) = 0$

$24K\, i_1 - 12K\, i_2 = 0$

Applying KVL to Supermesh:

$12K(\, i_2 - i_1) + 12K\, i_2 + 12K\, i_3 = 0$

$-12K\, i_1 + 24K\, i_2 + 12K\, i_3 = 0$

Applying KVL to mesh 1:

$12K\ i_1 + 12K\ (i_1 - i_2) = 0$

$24K\ i_1 - 12K\ i_2 = 0$

Solving equations 1, 2 and 3

$i_1 = -0.8$ mA; $i_2 = -1.6$ mA; $i_3 = 2.4$mA

$\mathbf{i_{o2} = i_1 - i_2 = 0.8}$ **mA**

Step 3 : To find i_o

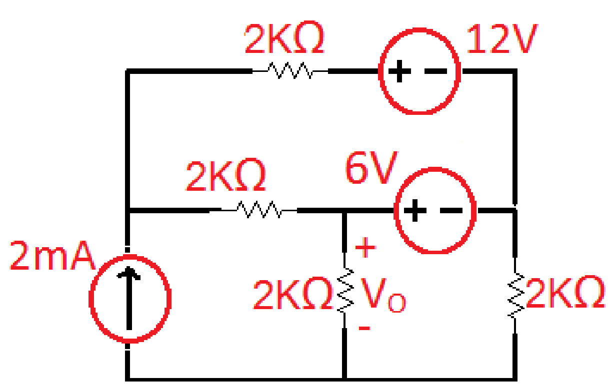
By Super Position Theorem,

$i_0 = i_{01} + i_{02}$

$i_o = -0.3m + 0.8m$

$i_o = 0.5$ m A

P2. Find V_o by Super position theorem.



Solution:

Let $V_0 = V_{01} + V_{02} + V_{03}$

where,

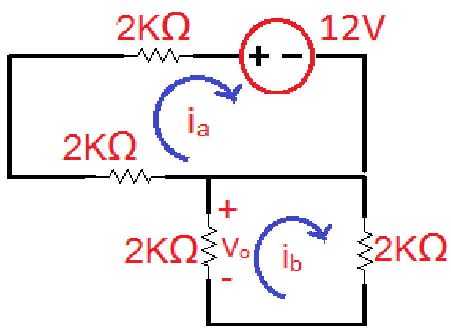
V_{01} is the contribution of 12V source when acting alone

V_{02} is the contribution of 6V source when acting alone

V_{03} is the contribution of 2mA source when acting alone

Step 1: To find V_{o1}

Deactivate 6V and 2mA sources



KVL to mesh2:

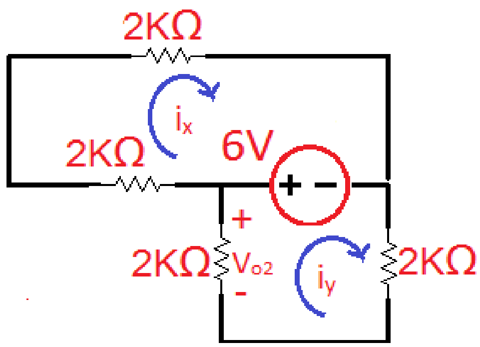
$2K\, i_b + 2K\, i_b = 0$

$i_b = 0$

$V_{o1} = -\,2K\, i_b = 0V$

Step 2: To find V_{o2}

Deactivate 12V and 2mA sources



<u>KVL to mesh2:</u>

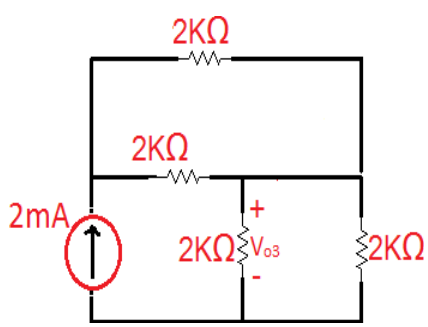
$2K\ i_y + 6 + 2K\ i_y = 0$

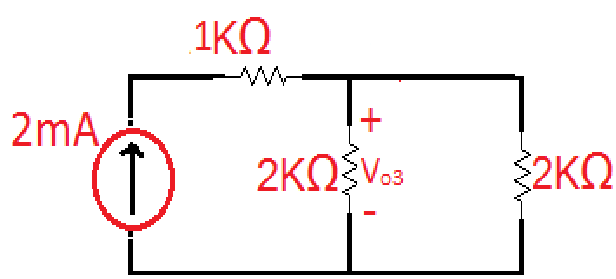
$i_Y = -1.5mA$

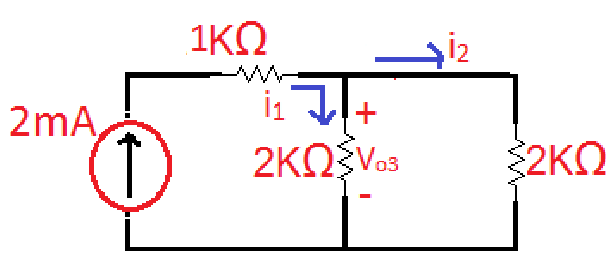
$V_{O2} = -\ 2K\ i_Y = 3\ V$

<u>Step 3: To find V_{o3}</u>

Deactivate 12V and 6V sources







$i_1 = i_2 = 1mA$

$V_{O3} = 2K\ i_1 = 2V$

Step 4:

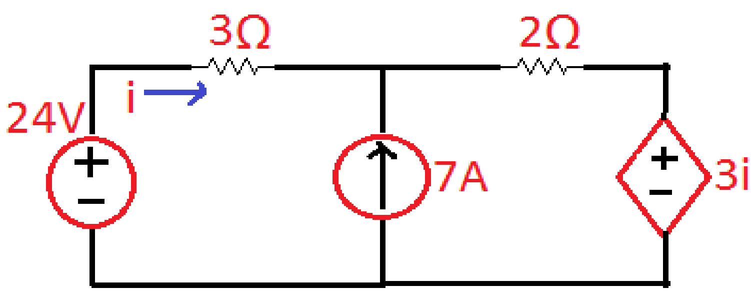
By Super position Theorem

$V_0 = V_{01} + V_{02} + V_{03}$

$V_0 = 0 + 3 + 2$

$V_0 = 5\ V$

P3. Find i by Super position theorem.



Solution:

Let $i = i_1 + i_2$

where,

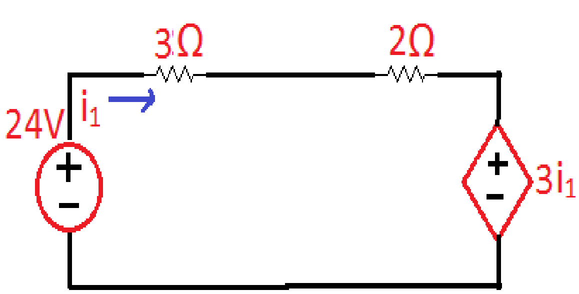
i_1 is the contribution of 24V source when acting alone

i_2 is the contribution of 7A source when acting alone

The dependant voltage source cannot be deactivated - keep it as it is.

Step 1: To find i_1

Deactivate 7A source



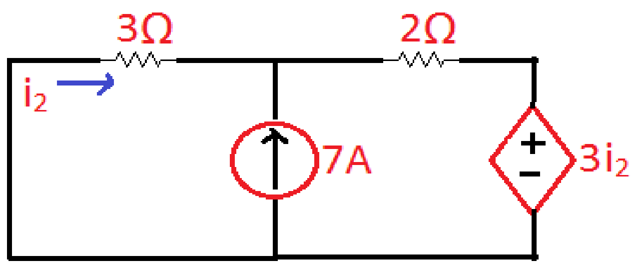
Applying KVL:

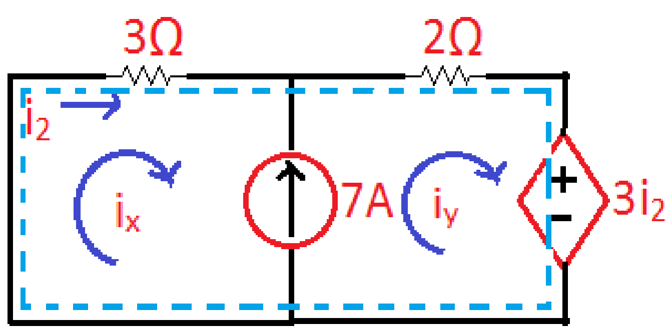
$-24 + 3\,i_1 + 2\,i_1 + 3\,i_1 = 0$

$i_1 = 3$ A

<u>Step 2: To find i_2</u>

Deactivate 24V source





<u>Constraint equation:</u>

$-i_X + i_Y = 7A$

<u>KVL to Supermesh:</u>

$3\,i_X + 2\,i_Y + 3\,i_2 = 0$

Sub. $i_2 = i_X$

$3\,i_X + 2\,i_Y + 3\,i_X = 0$

$6\,i_X + 2\,i_Y = 0$

Solving the equations

$-i_X + i_Y = 7A$

$6\ i_X + 2\ i_Y = 0$

Implies,

$i_X = -1.75$ A and $i_Y = 5.25$A

$i_2 = i_x = -1.75$A

<u>Step 3:</u>

By Super position Theorem

$i = i_1 + i_2$

$i = 3 - 1.75$

$i = 1.25$ A

<u>Summary:</u>

1. Superposition theorem is applicable to circuits with multiple independent sources only.
2. Dependant sources can be present.
3. At a time only one independent source should be acting, which gives its individual contribution.
4. Algebraic summation of the individual contributions gives the actual current/voltage in a circuit.
5. It is as good as cutting down complex problems into simpler ones.

NETWORK ANALYSIS (18EC32)

Syllabus:-

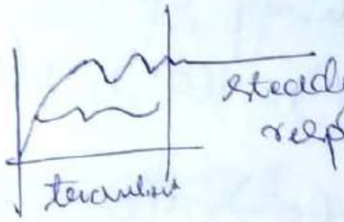
Module -3

Transient Behavior and initial conditions

Initial Conditions

Any electrical n/w consists of Vg sources, current sources, L & C.
when such n/ws are to be analysed, the integro-differential eqns are written & solved.
general soln to such eqn consists of 2 parts
i) complementary fun → general soln
ii) particular intergral → particular soln

Any response consists of ~~initial state~~ transient response & steady state response



Complimentary fun is soln of homogeneous eqn which also represent the response of the system.
Transient response depends on type, value & arrangement of elements in the n/w.
Complimentary fun is general soln of homogenous eqn & particular integral is the particular soln of non homogenous eqn.

while solving eqn ~~of~~ of n^{th} order, we come across n no of constants in the complementary fn, which are to be evaluated to get ~~partial~~ exact soln. To evaluate these constants, n no of initial conditions are required.

The initial conditions of n/w are the conditions prevailing in the elements of the n/w at the [instant] of closing the switch at $t=0$.

In a switching opn, $\underline{t=0}$ is taken as ref.

The initial conditions in n/w may be the vgs [across] the various elements, currents through them or charges existing on them at time of switching opn i.e at $t=0$.

Immediately before the switching opn, these quantities are referred to as

$v(0^-)$, $i(0^-)$, $q(0^-)$ at $t=0^-$.

Immediately after switching opn, these quantities are referred to as $v(0^+)$, $i(0^+)$, $q(0^+)$ at $t=0^+$.

~~Knowing these values of $v(0^-)$, $i(0^-)$, $q(0^-)$~~,

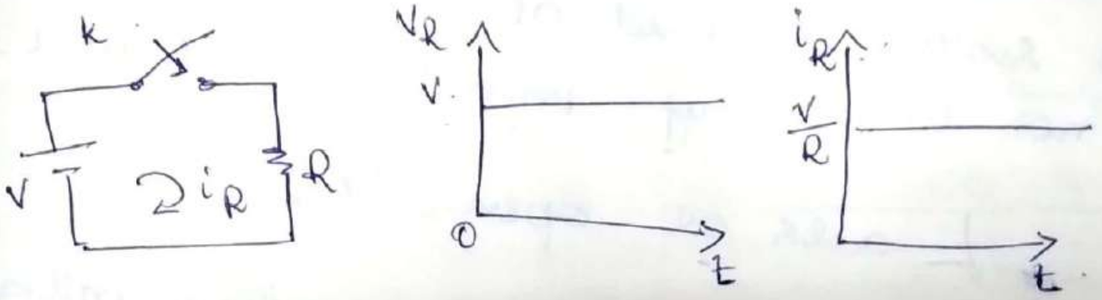
The initial conditions in a n/w depends on the past history of the n/w prior to ~~to~~ $t=0^-$ and n/w structure at $t=0^+$ just after switching. They also depend on the nature of elements in the n/w.

The knowledge of initial values of one or more derivative of response, are helpful in anticipating the form of response, thus we can check the soln.

knowing the values of Vg & currents of elements at $t = 0^-$, finding these values at $t = 0^+$, constitutes the evaluation of initial conditions

Initial conditions of element.

Resistor :



In resistor, current & Vgs are related by

$V = iR$

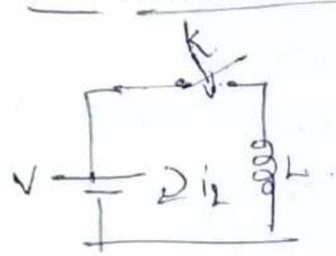
when a step response of V vg is applied to resistor by closing switch at $t=0$, the current ~~is~~ is also a step function & is given by $I = \frac{V}{R}$.

The waveform of current is same as w/t of Vg i.e

current through R changes instantaneously, if Vg changes instantaneously.

$|||^{ly}$ the Vg across resistor also changes instantaneously when the current through it changes instantaneously

The inductor:

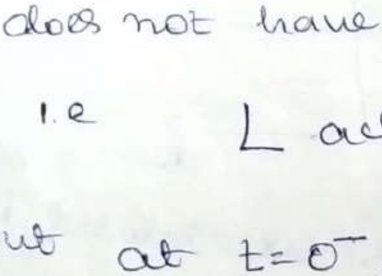


Current through inductor does not change instantaniously.
when switch is closed at $t=0$, if inductor does not have any initial current at $t=0^+$,

i.e L acts as open ckt.

But at $t=0^-$, if the conductor has initial current I_0. then at $t=0^+$, current in inductor continues to be I_0, inductor acts as a current source of I_0 ampers

The capacitor: $c=\frac{Q}{V} \Rightarrow V=\frac{Q}{C}$

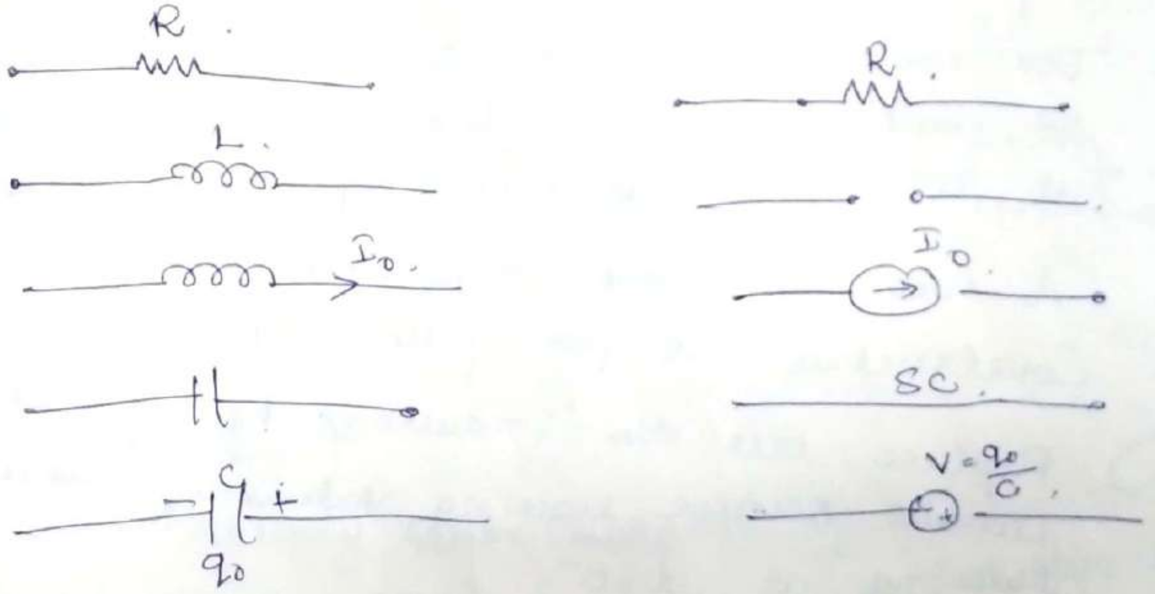


The vg across the C cannot change instantanously.
when uncharged capacitor is connected to a DC vg source V by closing switch K at $t=0$,
when there is no charge on C, vg across it is zero & hence acts as short ckt.
If capacitor has initial charge of q_0 coloumbs at $t=0^-$, then at $t=0^+$, the capacitor is equivalent to vg source of $V=\frac{Q_0}{C}$

ummary.

Initial Conditions

Condition of element at $t = 0^-$ Condition of element at $t = 0^+$.



There are some exceptions to the initial Conditions of the elements. They are.

1. when the impulse Vg is applied to an inductance, its Current Changes instantaneously.
2. when an impulse current is applied to a Capacitor, its Vg changes instantaneously.

procedure for finding initial Conditions:

There is no unique procedure ~~to be followed~~ for to find the initial Conditions. It is like a game of chess, strategy is Choosen depending on ~~other~~ opposing party move. Here the procedure depends on the particular n/w being Considered.

general procedure is as follows:

1. Initial values of V or currents before switch at $t=0^-$ can be found directly from the schematic diagram of given n/w

2. For each ~~value~~ element of the n/w, we must find out, what happens to element at $t=0^+$ i.e after closing the switch.

3. A new equivalent n/w at $t=0^+$ is constructed as per following rules.

a) Replace all the inductors by open ckt or current source having values of current flowing at $t=0^-$.

b) Replace all the capacitors by short ckt or Vg source of $V=\frac{q_0}{C}$, if there is any initial charge

c) Resistors are left in the n/w without any change

4. From n/w at $t=0^+$, first initial values of Vg & currents are solved. Then their derivatives are found.

VI relations of n/w elements

$R \longrightarrow v(t) = R\,i(t)$ and $i(t) = \frac{v(t)}{R}$

$L \longrightarrow v(t) = L\frac{di(t)}{dt}$ & $i(t) = \frac{1}{L}\int_{0^-}^{t} v(t)dt$

$= \frac{1}{L}\int_{0}^{t} v(t)dt + i_L(0^-)$

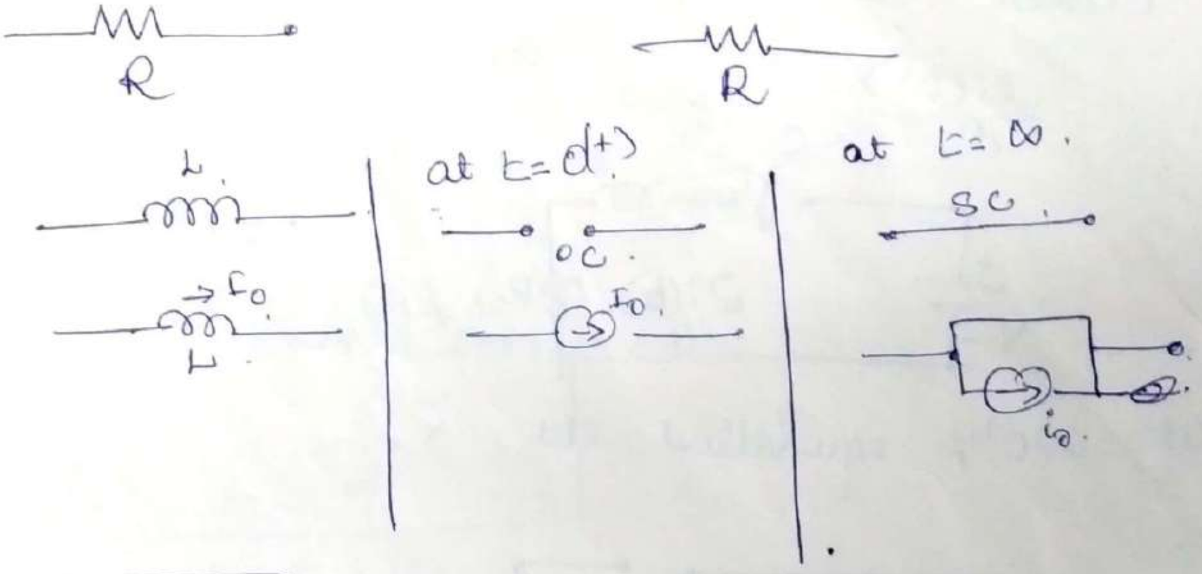
$C=\frac{Q}{V}$

$V=\frac{Q}{V}=\frac{I(t)}{C}$

$C \rightarrow v(t) = \frac{1}{C}\int_{0^-}^{t} i(t)dt$ & $i(t) = C\frac{dv(t)}{dt}$

$v(t) = \frac{1}{C}\int_{0}^{t} i(t)dt + V_C(0^-)$

Final Conditions in a n/w. i.e at $t=-\infty$.

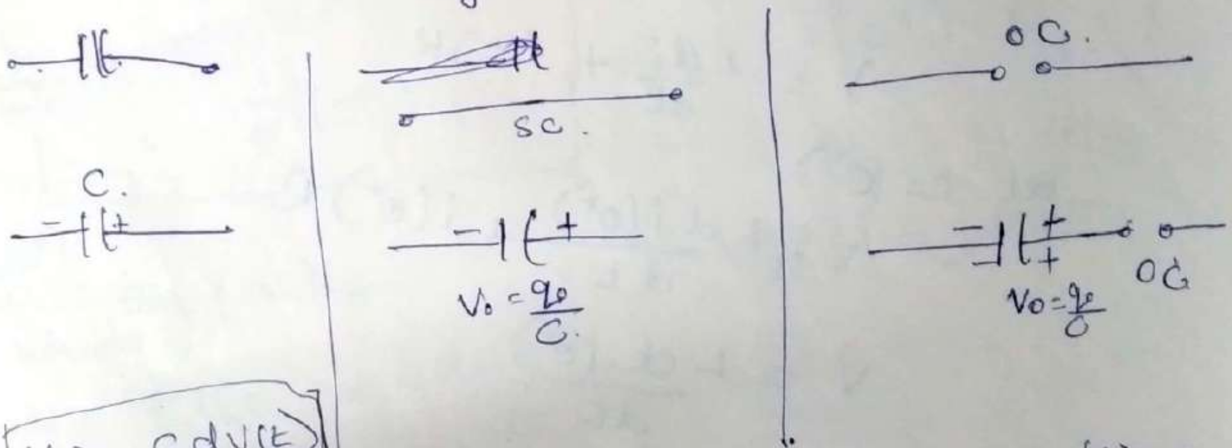


$$\boxed{V = L\frac{di}{dt}}$$

under steady state condition $\frac{di}{dt}=0$.

This means $v=0$ & hence L acts as short ckt at $t=(\infty)$

uncharged & $v=0 \Rightarrow$ SC

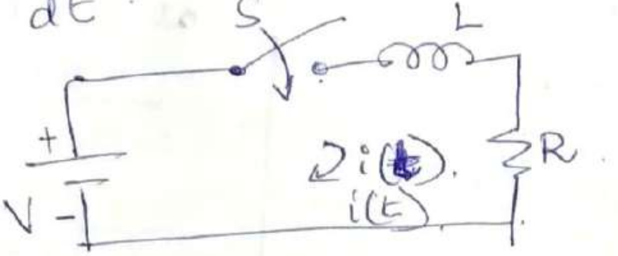


$$\boxed{i(t) = C\frac{dv(t)}{dt}}$$

under steady state. $\frac{dv(t)}{dt}=0$ i.e at $t=\infty$, $i(t)=0$.

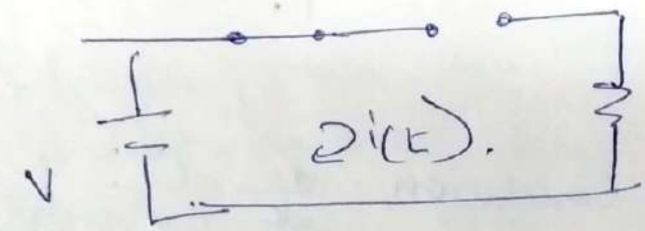
i.e capacitors acts an open ckt.

1. For the n/w shown below, switch S closed at t=0, Find the conditions $i(0^+)$ & $\frac{di(0^+)}{dt}$.



$$V = L\frac{di}{dt} + iR$$

at $t=0^+$, equivalent ckt is



$$i(0^+) = 0.$$

KVL to given loop.

$$V = L\frac{di}{dt} + i(t)R.$$

at $t=(0^+)$

$$V = L\frac{di(0^+)}{dt} + i(0^+)R.$$

$$V = L\frac{di(0^+)}{dt}.$$

$$\frac{di(0^+)}{dt} = \frac{V}{L}.$$

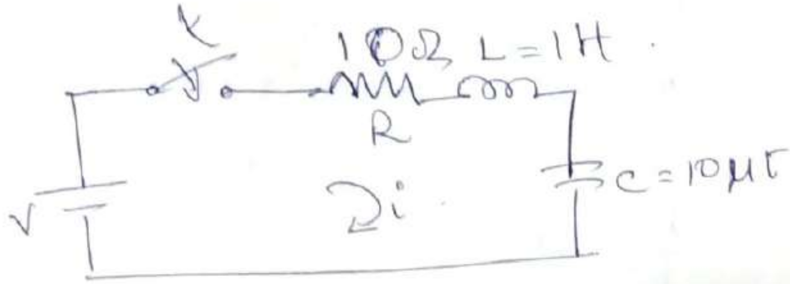
5

For the ckt shown.

$V = 10\,V$, $R = 10\,\Omega$ $L = 1H$ $C = 10\,\mu F$

& $V_c(0) = 0$ find

$i(0^+)$, $\frac{di}{dt}(0^+)$ and $\frac{d^2 i}{dt^2}(0^+)$.

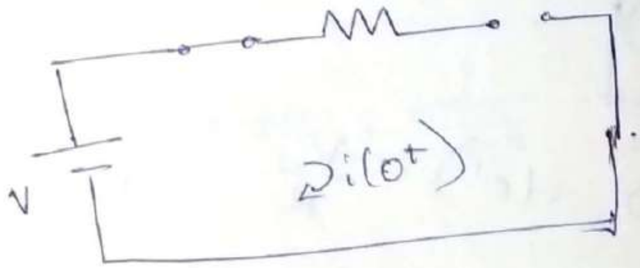


$i = C\frac{dV_c}{dt}$

switch is closed at $t=0$

$$V = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \quad \text{———} (1).$$

At $t = 0^{(+)}$.



$\therefore\ i(0^+) = 0$.

at $t = 0^{(+)}$. ~~V = R i(0+) +~~ $V = i(0^+)R + L\frac{di(0^+)}{dt} + \frac{1}{C}\int i(0^+)\,dt$

KVL to this loop

$$V = Ri(0^+) + L\frac{di(0^+)}{dt}$$

$$V = 0 + L\frac{di(0^+)}{dt}$$

$$\frac{di(0^+)}{dt} = \frac{V}{L} = \frac{10}{1} = 10\,A/sec.$$

differentiating (1).

$\because V$ is const.

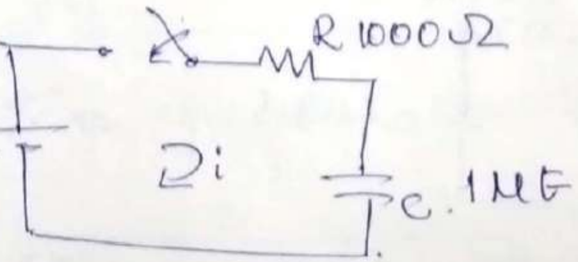
$$R\frac{di(0^+)}{dt} + L\frac{d^2 i(0^+)}{dt} + \frac{1}{C}\,i(0^+) = 0$$

$$10 \times 10 + L\frac{d^2 i(0^+)}{dt} + \frac{1}{C}(0) = 0.$$

$$\frac{d^2 i(0^+)}{dt} = \frac{-100}{L} = \frac{-100}{1} = -100\,A/sec^2.$$

3) in the n/w shown, the switch K is cl... at $t=0$, with capacitor uncharged. Find the values for i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t=0^+$, for the element values as follows.

$V = 100V$, $R = 1000\Omega$ and $C = 1\mu F$

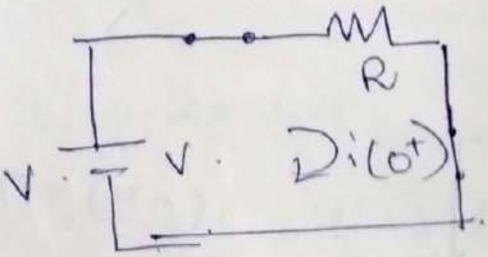


when closed at $t=0$,

$$V = iR + \frac{1}{C}\int i\,dt \quad \text{——} (1)$$

n/w at $t = 0^{(+)}$.

C → uncharged → V_g is 0 → SC.



$V = iR$.

$$V = i(0^+)R \Rightarrow i(0^+) = \frac{V}{R} = \frac{100}{1000} = 0.1A$$

diff (1).

$$R\frac{di(0^+)}{dt} + \frac{1}{C}i(0^+) = 0. \quad \text{——} (2)$$

$$R\frac{di(0^+)}{dt} + \frac{1}{1\mu F}(0.1) = 0.$$

$$\frac{di(0^+)}{dt} = -\frac{0.1\times10^6}{R}$$

$$\frac{di(0^+)}{dt} = -\frac{0.1\times10^{6\,3}}{1000} = 0.1\times10^3.$$

$$= -1\times10^{+2}.$$

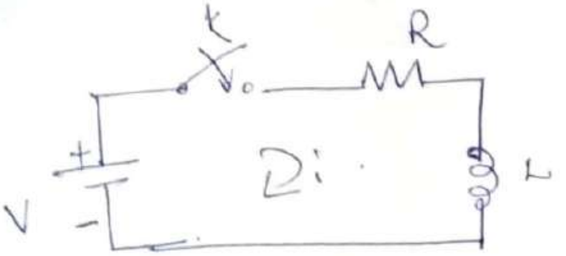
$$= -100A/sec.$$

diff (2) $R\frac{d^2i(0^+)}{dt} + \frac{1}{C}\frac{di(0^+)}{dt} = 0.$

$$\frac{d^2i(0^+)}{dt} = -\frac{1}{C}\frac{di(0^+)}{dt}/R = \frac{-(-100)}{\frac{10^{-6}}{10^3}} = \frac{100}{10} = 10^5 A/s$$

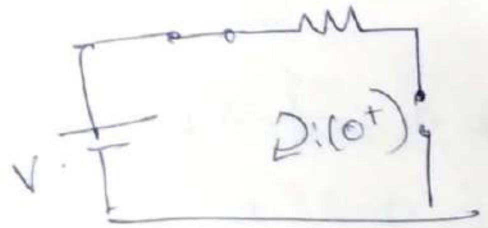
(6)

In the n/w shown, K is closed at $t=0$, with zero current in the inductor. Find i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t=0^+$. if $R = 10\Omega$, $L = 1H$ and $V = 100V$.



$$V = iR + L\frac{di}{dt} \quad \text{—} (1).$$

at $t=(0^+)$



$$i(0^+) = 0.$$

~~dif~~ (1) $\cancel{R\frac{di}{dt} + L\frac{d^2i}{dt^2}}$ $\frac{di}{dt} = \frac{V - i(0^+)R}{L}$

$$= \frac{100}{1}$$

$$= 100A/sec.$$

diff (1)

$$R\frac{di(0^+)}{dt} + L\frac{d^2i(0^+)}{dt} = 0.$$

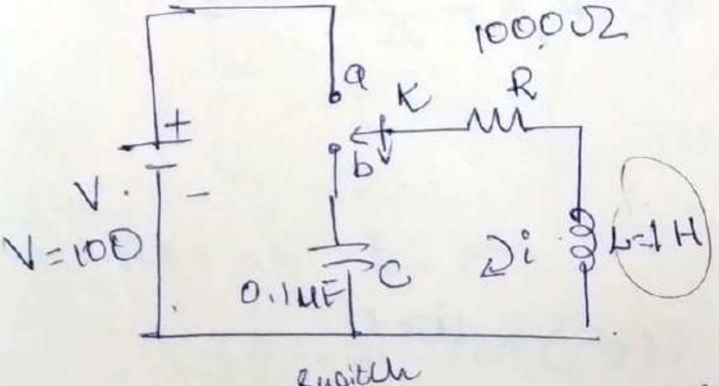
$$10 \times 100 + 1\frac{d^2i(0^+)}{dt} = 0.$$

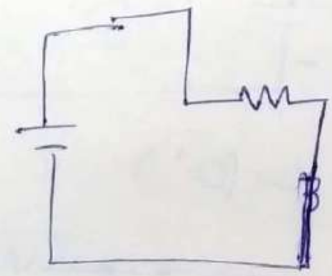
$$\frac{d^2i(0^+)}{dt} = -1000 \, A/sec^2.$$

5) For the n/w shown, k is changed from position a to b at $t=0$. Solve for i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t=0^+$.

if $R = 1000\,\Omega$, $L = 1H$, $C = 0.1\mu F$ and $V = 100V$.

Assume that capacitor is initially uncharged.





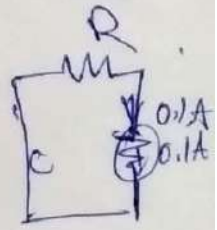
switch

when k is at position a. $\longrightarrow L \to SC$.

$$? - i(0^-) = \frac{V}{R} = \frac{100}{1000} = 0.1A.$$

$$\therefore \quad i(0^+) = 0.1A.$$

when k is changed from a to b,



$$i(0^+) = 0.1A$$

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0 \text{ —— } (1)$$

$$Ri(0^+) + L\frac{di(0^+)}{dt} + \underbrace{\frac{1}{C}\int i(0^+)\,dt}_{} = 0.$$

initially unch

$C \to SC$.

$$R\,i(0^+) + L\frac{di(0^+)}{dt} + V_C(0^+) = 0.$$

$$R(i(0^+)) + L\frac{di(0^+)}{dt} = 0.$$

$$\frac{di(0^+)}{dt} = \frac{-R \times 0.1}{L} = \frac{-1000 \times 0.1}{1} = -100\,A/sec.$$

diff ①

$$R\frac{di(0^+)}{dt} + L\frac{d^2 i}{dt^2} + \frac{1}{C}\, i\cancel{dt} = 0.$$

$$1000 \times (-100) + \cancel{1} \times \frac{d^2 i(0^+)}{dt^2} + \frac{i(0^+)}{C} = 0.$$

$$-10^5 + L\frac{d^2 i(0^+)}{dt} + \frac{0.1}{0.1 \times 10^{-6}} = 0.$$

$$\frac{d^2 i(0^+)}{dt^2} = 10^5 - 1 \times 10^{-1} \times 10^7.$$

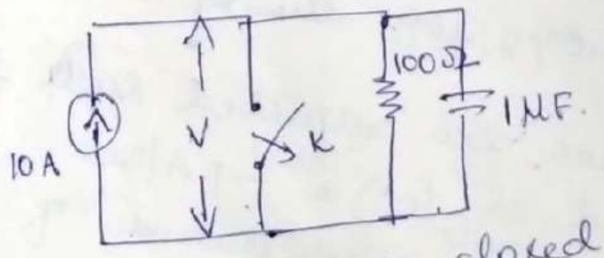
$$= 10^5 - 10^6.$$

$$= -9 \times 10^5\ A/sec^2.$$

1000000
100000
900000

68). In the circuit shown in fig. Switch k is opened at $t=0$. find the values of V, $\frac{dV}{dt}$ and $\frac{d^2V}{dt^2}$ at $t = 0^+$.

$i = C\frac{dv}{dt}$.



kcl.

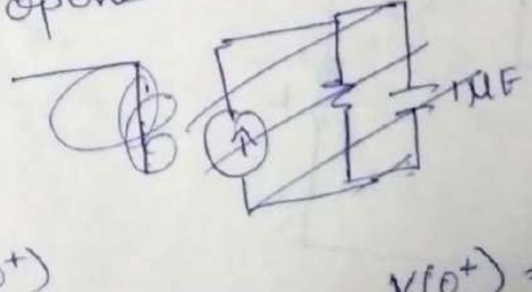
$$I = 10 + \frac{V}{100} + C\frac{dv}{dt}.$$

$$\frac{V}{100} + C\frac{dv}{dt} = 10 \quad \text{—} ①$$

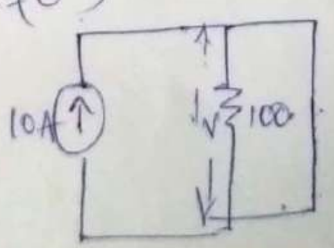
when switch is closed, all the current flows through ~~so~~ switch, & capacitor is not charged

$$\therefore V_c(0^-) = 0 = V_c(0^+).$$

when opened.



at $t = (0^+)$ $\qquad V(0^+) = 0 \qquad (\because SC)$



$$\frac{v(0^+)}{100} + C\frac{dv(0^+)}{dt} = 10.$$

$$0 + C\frac{dv(0^+)}{dt} = 10.$$

$$\frac{dv(0^+)}{dt} = \frac{10}{C} = \frac{10}{10^{-6}} = 10^{7}\ \text{V/sec}.$$

diff (1).

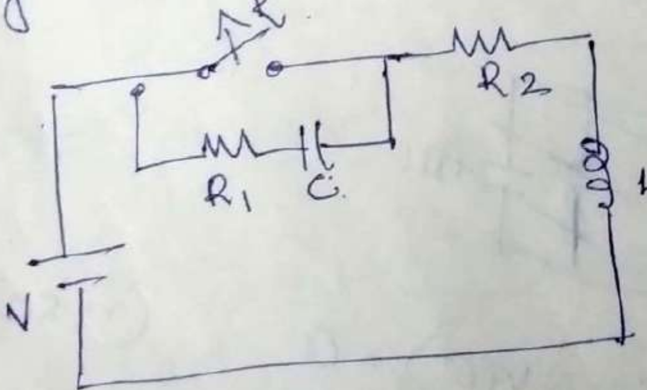
$$\frac{\frac{dv}{dt}}{100} + C\frac{d^2v}{dt^2} = 0.$$

$$\frac{10^{\cancel{7}5}}{10^{2}} + C\frac{d^2v}{dt^2} = 0$$

$$C\frac{d^2v}{dt^2} = -10^{5}$$

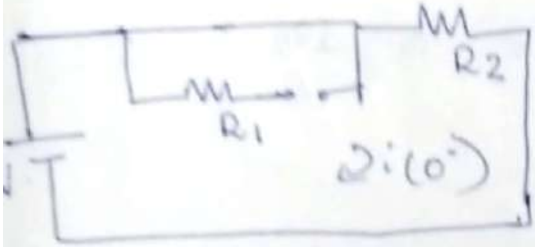
$$\frac{d^2v}{dt^2} = \frac{-10^{5}}{C} = \frac{-10^{5}}{10^{-6}} = -10^{11}\ \text{V/sec}^2.$$

(7) In the n/w shown, the switch is opened at $t=0$, after the n/w has attained the steady state with the switch closed.
a) Find an expression for the Vg across the switch at $t=0^+$.
b) If the parameters are adjusted such that $i(0^+) = 1\,A$ and $\frac{di}{dt}(0^+) = -1\ A/sec$, what is the value of the derivative of the Vg across the switch?



when k is closed, steady state attained

L acts as SC & C act as OC

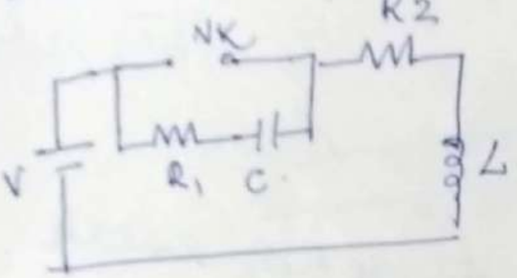


$i(0^-) = \frac{V}{R_2} = i(0^+)$

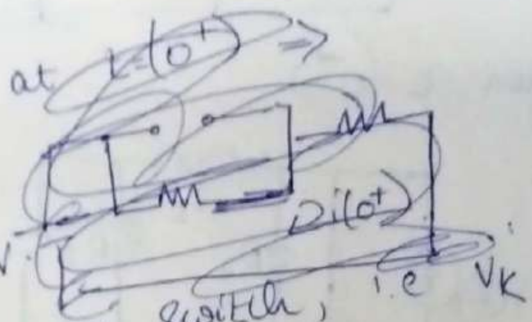
& capacitor is uncharged,

$V_c(0^-) = 0 = V_c(0^+)$

when k is opened



at



i.e. V_K across switch,

They ask to find V_g across switch, — (1)

$V_K = i(t)R_1 + \frac{1}{C}\int i\,dt$ — (1).

at $t = (0^+)$ $V_K = i(0^+)R_1 + \cancel{V_c(0^+)}$.

$V_K = i(0^+)R_1$

$V_K = \frac{V}{R_2} \cdot R_1 =$

$V_K = V\frac{R_1}{R_2}$.

$\frac{dV_K}{dt} = \frac{di}{dt}R_1 + \frac{i}{C}$.

$\frac{dV_K(0^+)}{dt} = R_1(-1) + \frac{1}{C}$.

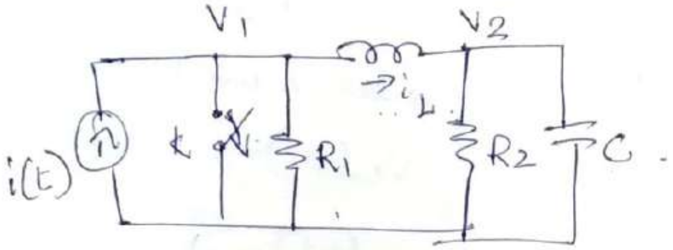
$\frac{dV_K}{dt} = -R_1 + \frac{1}{C}$

$\frac{dV_K}{dt} = \frac{1}{C} - R_1$.

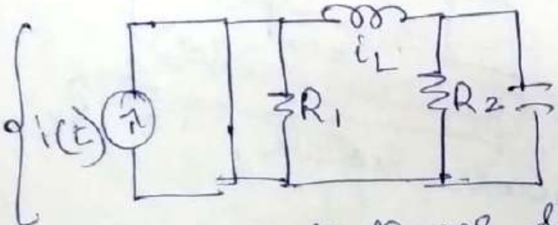
(8) The n/w shown in fig has 2 independent pairs. If the switch k is opened at t. At n find the following quantities at $t = 0^+$,

i) V_1 ii) V_2 iii) $\frac{dV_1}{dt}$ & iv) $\frac{dV_2}{dt}$.

at



when $t = (0^-)$



all current flows through closed switch.

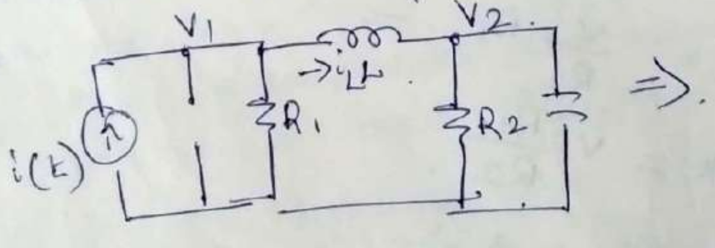
$\therefore$ hence $i_L(0^-) = 0 = \cancel{i_L(0^+)}$. $V_c(0^-) = 0$.

$\therefore i_L(0^+) = 0$ & $V_c(0^+) = 0$.

$\therefore V_2(0^+) = 0$

when switch k is opened at $t=0$,

at node



at node V_1 $\quad -i(t) + \frac{V_1}{R_1} + \cancel{V_1 - V_2}\; i_L = 0$ ——— (1)

$$i(t) = \frac{V_1}{R_1} + \cancel{i_L(0^+)}$$

$$i(0^+) = \frac{V_1(0^+)}{R_1} + 0$$

$\therefore$ $V_1(0^+) = R_1[i(0^+)]$. ——— (2)

At node V_2

$$-i_L + \frac{V_2}{R_2} + C\frac{dV_2}{dt} = 0 \quad \text{--- (3)}$$

at $t=0^+$ $\quad -i_L(0^+) + \frac{V_2(0^+)}{R_2} + C\frac{dV_2(0^+)}{dt} = 0.$

$$0 + 0 + C\frac{dV_2(0^+)}{dt} = 0$$

$$\Rightarrow \frac{dV_2}{dt}(0^+) = 0.$$

diff (1)

eqn (1) $i(t) = \frac{V_1}{R_1} + i_L(0^+)$

$$\cancel{\frac{di(t)}{dt} + \frac{1}{R_1}\frac{dV_1(0^+)}{dt} + i_L(0^+) = 0}.$$

$$\cancel{\frac{di}{dt} + 0}$$

$$\frac{di(t)}{dt} = \frac{1}{R}\frac{dV_1}{dt} + \frac{di_L}{dt}.$$

$$\frac{di}{dt}(0^+) = \frac{1}{R}\frac{dV_1}{dt}(0^+) + \frac{R_1\, i(0^+)}{L}.$$

$$\frac{1}{R}\frac{dV_1}{dt}(0^+) = \left[\frac{di(0^+)}{dt} - \frac{R_1}{L}i(0^+)\right]$$

$$\frac{dV_1(0^+)}{dt} = R_1\left[\frac{di(0^+)}{dt} - \frac{R_1}{L}i(0^+)\right]$$

$V = L\frac{di_L}{dt}$

$\therefore i_L = \frac{1}{L}\int(V_1 - V_2)\,d$

$\frac{di_L}{dt} = \frac{V_1 - V_2}{L}$

$\frac{di_L(0^+)}{dt} = \frac{1}{L}[V_1 - V_2]$

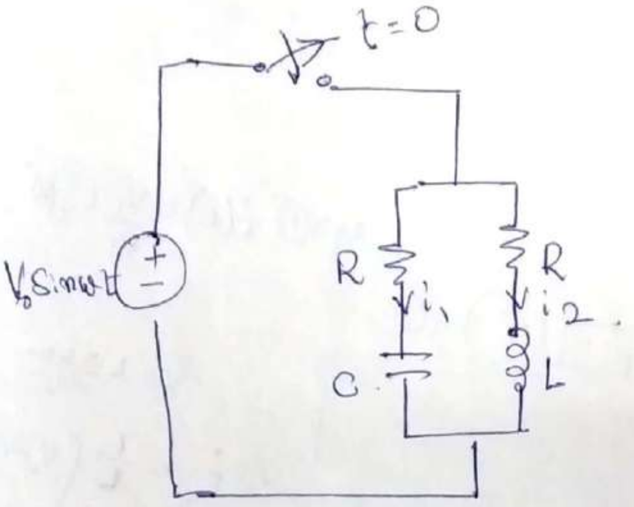
$= \frac{1}{L}[R_1 i(0^+) - 0]$

$= \frac{R_1 i(0^+)}{L}.$

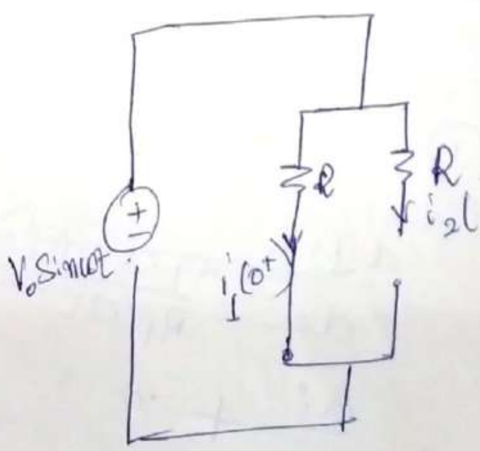
9). In the ckt shown. the switch K is at $t=0$. S.T. at $t=0^+$;

$$\frac{di_1}{dt} = \frac{V_0}{R}\left[\omega\cos\omega t - \frac{\sin\omega t}{RC}\right] \;\&$$

$$\frac{di_2}{dt} = \frac{V_0 \sin\omega t}{L}.$$



At $t=0^+$



$$i_1(0^+) = \frac{V_0 \sin\omega t}{R} \qquad i_2(0^+) = 0.$$

~~writing loop eqn for i_1~~

$$V_0 \sin\omega t = i_1 R + \frac{1}{C}\int i_1\, dt \quad \text{—— (1)}$$

~~loop eqn for i_2~~,

$$V_0 \sin\omega t = i_2 R + L\frac{di_2}{dt} \quad \text{—— (2)}$$

diff (1)

$$R\frac{di_1}{dt} + \frac{i}{C} = V_0(\cos\omega t)\,\omega.$$

$$R\frac{di_1(0^+)}{dt} + \frac{i(0^+)}{C} = V_0\omega\cos\omega t.$$

$$R\frac{di_1(0^+)}{dt} = V_0\omega\cos\omega t - \frac{i(0^+)}{C}$$

$$= V_0\omega\cos\omega t - \frac{V_0\sin\omega t}{RC}$$

$$\frac{di_1(0^+)}{dt} = \frac{V_0}{R}\left[\omega\cos\omega t - \frac{\sin\omega t}{RC}\right]$$

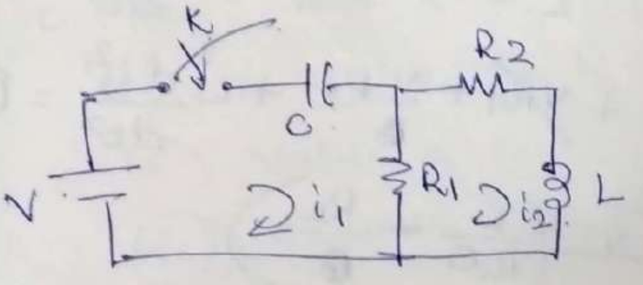
diff (2)

$$\cancel{V_0\omega\cos\omega t = \frac{di_2}{dt}.R + L\,d}$$

$$\frac{di_2}{dt} = \frac{1}{L}\left[V_0\sin\omega t - i_2(0^+)R\right] = \frac{V_0\sin\omega t}{L}$$

In the n/w shown. Switch k is closed at $t=0$. The n/w being initially unenergized.

find $i_1(0^+)$, $i_2(0^+)$, $\frac{di_1(0^+)}{dt}$, $\frac{di_2(0^+)}{dt}$, $\frac{d^2i_1(0^+)}{dt^2}$, $\frac{d^2 i_2(0^+)}{dt^2}$

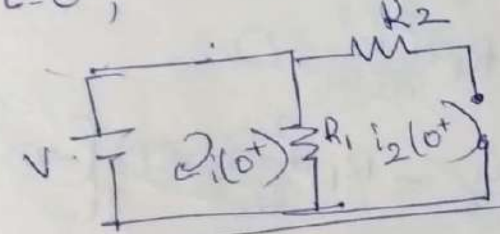


$$V = \frac{1}{C}\int i_1 dt + R_1(i_1 - i_2) \quad \text{—— (1)}$$

$$R_1(i_2 - i_1) + R_2 i_2 + L\frac{di_2}{dt} = 0$$

$$-R_1 i_1 + i_2(R_1+R_2) + L\frac{di_2}{dt} = 0 \quad \text{—— (2)}$$

at $t=0^+$,



$$\boxed{i_1(0^+) = \frac{V}{R_1} \quad \& \quad i_2(0^+) = 0}$$

from (2)
$$-R_1 i_1(0^+) + i_2(0^+)(R_1+R_2) + L\frac{di_2(0^+)}{dt} = 0$$

$$-\cancel{R_1}\frac{V}{\cancel{R_1}} + 0 + L\frac{di_2(0^+)}{dt} = 0$$

$$L\frac{di_2(0^+)}{dt} = V \Rightarrow \boxed{\frac{di_2(0^+)}{dt} = \frac{V}{L}}$$

diff (1) $$0 = \frac{i_1(0^+)}{C} + \frac{di_1(0^+)}{dt}R_1 - R_1\frac{di_2(0^+)}{dt} = 0 \quad \text{—— (3)}$$

$$= \frac{V}{R_1 C} + \frac{di_1(0^+)}{dt}R_1 - R_1\frac{V}{L} = 0$$

$$\frac{di_1(0^+)}{dt}R_1 = +V\left(\frac{R_1}{L} - \frac{1}{R_1 C}\right) \Rightarrow \frac{di_1(0^+)}{dt} = \frac{V}{R_1}\left(\frac{R_1}{L} - \frac{1}{R_1 C}\right)$$

diff eqn (2).

$$-R_1\frac{di_1}{dt} + R_1\frac{di_2}{dt} + R_2\frac{di_2}{dt} + L\frac{di_2^2}{dt^2} = 0.$$

$$-R_1\left[\frac{V}{R_1}\left(\frac{R_1}{L} - \frac{1}{R_1C}\right)\right] + R_1\left(\frac{V}{L}\right) + R_2\frac{V}{L} + L\frac{di_2^2}{dt^2} = 0$$

$$-V\left(\frac{R_1}{L} - \frac{1}{R_1C}\right) + \frac{V}{L}(R_1+R_2) + L\frac{di_2^2}{dt^2} = 0.$$

$$-\frac{V}{L}\left(R_1 - \frac{L}{R_1C}\right) + \frac{V}{L}(R_1+R_2) + L\frac{di_2^2}{dt^2} = 0.$$

$$-\cancel{VR_1} + \frac{VL}{R_1C} + \cancel{VR_1} + VR_2 + L^2\frac{di_2^2}{dt^2} = 0.$$

$$L^2\frac{di_2^2}{dt^2} = -V\left(\frac{L}{R_1C} + R_2\right).$$

$$\frac{di_2^2}{dt^2} = -\frac{V}{L^2}\left(\frac{L}{R_1C} + R_2\right)$$

$$= -V\left[\frac{1}{R_1LC} + \frac{R_2}{L^2}\right)$$

In th
une
The

diff eqn (3).

$$\frac{1}{C}\frac{di_1}{dt} + R_1\frac{d^2i_1}{dt^2} - R_1\frac{d^2i_2}{dt^2} = 0$$

$$\frac{1}{C}\left(\frac{V}{L} - \frac{V}{R_1^2C}\right) + R_1\frac{d^2i_1}{dt^2} - R_1\left[(-V)\left(\frac{1}{R_1LC} + \frac{R_2}{L^2}\right)\right].$$

$$\frac{1}{C}\left(\frac{V}{L} - \frac{V}{R_1^2C}\right) + R_1\frac{d^2i_1}{dt^2} + R_1V\left(\frac{1}{R_1LC} + \frac{R_2}{L^2}\right) = 0.$$

$$R_1\frac{d^2i_1}{dt^2} = -\left(\frac{V}{LC} - \frac{VR_1R_2}{L^2}\right) - \frac{V}{CL} + \frac{V}{R_1^2C^2}.$$

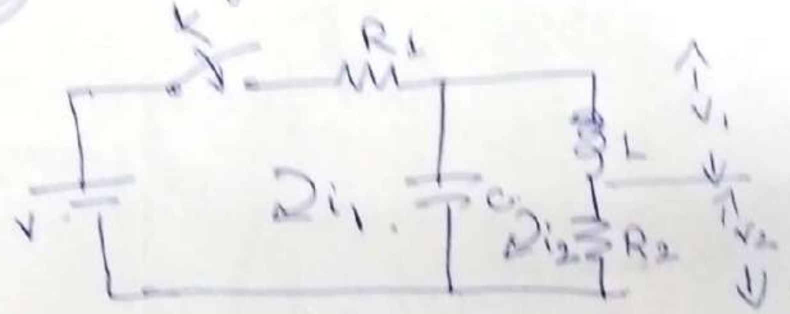
$$= -\frac{2V}{LC} - \frac{VR_1R_2}{L^2} + \frac{V}{R_1^2C^2}.$$

$$\frac{d^2i_1}{dt^2} = \frac{1}{R}\left[\frac{1}{C}\left(\frac{V}{R_1^2C} - \frac{V}{L}\right) - VR_1\left(\frac{1}{R_1LC} + \frac{R_2}{L^2}\right)\right].$$

For the ckt shown. Switch K is closed at $t=0$, connecting the battery to an unenergized n/w. Determine

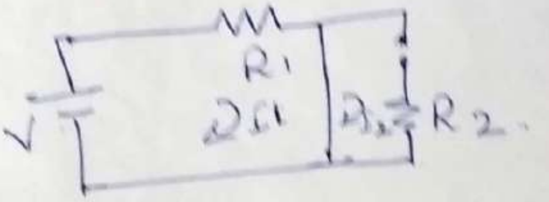
i) V_1 and V_2 at $t=0^+$ & $t=\infty$.

2) 1st & second derivative of V_1 & V_2 at $t=0^+$.



at $t=0^{(+)}$

$i_2(0^+)=0$ $L \rightarrow OC$, $C \rightarrow SC$.



$i_1(0^+) = \frac{V}{R_1}$.

$\therefore I_2(0^+)$

$V_1+V_2=0$. but $V_2(0^+)=0$

$\therefore V_1=0 \Rightarrow V_1(0^+)=0$.

At $t=\infty$, $L \rightarrow SC$ C acts as OC.

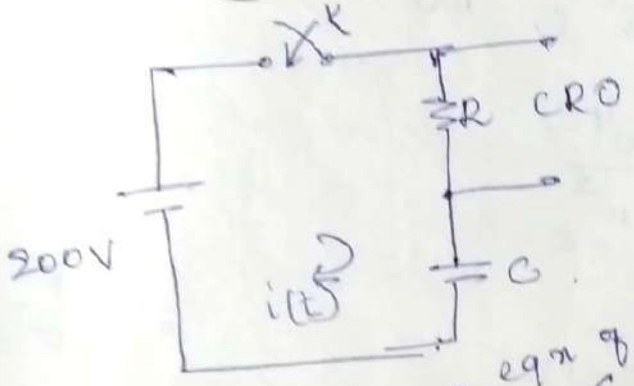
$V_1(\infty)=0$. $V_2(\infty) = \frac{V}{R_1+R_2}R_2$.

In the ckt shown the capacitor is initially uncharged, switch k is closed at time t=0. The initial value of current is found to be 25mA through CRO. The transient disappears (reduces 2% of its initial value) after a time 0.1 sec. By classical method By Determine the

i) the value of R

ii) value of C

iii) expression for the current i(t) for t>0.



when k is closed at t=0.

$$200 = iR + \frac{1}{C}\int i\,dt \quad \text{---} (1)$$

diff (1).

$$\frac{di}{dt}R + \frac{i}{C} = 0$$

eqn of type

solving soln of $\left(RS + \frac{1}{C}\right) i = 0 \Rightarrow RS+1 = 0$

$(RSC+1) = 0$ & $S = -\frac{1}{RC}$

$i = ke^{+St} \Rightarrow i = k\,e^{-t/RC}$ —— (1) where k is constant

$RC\left(\frac{-1}{RC}\right)$

At $t(0^+)$. $i(0^+) = 25mA$ (given)

$e^0 = 1$

$$25m = k\,e^{-(0/RC)} = k$$

$k = 25m$

$\therefore i = 25\times10^{-3}\,e^{-t/RC}$.

$i(0^+) =$

At $t = 0^+$, $C \to SC$. (given)

$i(0^+) = \frac{V}{R} = \frac{200}{R} = 25mA$

$\frac{200}{R} = 25m \Rightarrow R = \frac{200}{25m} = 8000\,\Omega$

$R = 8k\Omega$.

After 0.1 sec

$i = 2\%$ of initial V.

$= \frac{2}{100} \times 25 \times 10^{-3}$.

$\frac{2}{100} \times 25 \times 10^{-3} = k\, e^{-t/RC}$.

$5 \times 10^{-4} = 25 \times 10^{-3}\, e^{-0.1/RC}$.

$e^{-0.1/RC} = \frac{5 \times 10^{-4}}{25 \times 10^{-3}} =$

$e^{-0.1/RC} = \frac{1}{50}$. $\qquad e^{-t} = \frac{1}{e^t}$.

$\frac{1}{e^{0.1/RC}} = \frac{1}{50}$.

$\therefore e^{0.1/RC} = 50$.

$\frac{0.1}{RC} = \ln(50) = 3.91$.

$\frac{0.1}{RC} = 3.91$

$RC = \frac{0.1}{3.91} = 0.0256$.

$C = \frac{0.0256}{R} = \frac{0.0256}{8k} = 3.195\,\mu F$.

NETWORK ANALYSIS (18EC32)

Syllabus:-

Module -4

Laplace Transform and its Applications

Laplace transformation

$$\mathcal{L}[f(t)] = F(s) = \int_{0^-}^{\infty} f(t)\, e^{-st}\, dt$$

where $s = \sigma + j\omega$ is a Complex number.

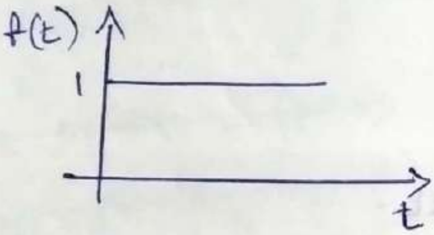
provided $\int_{0^-}^{\infty} |f(t)|\, e^{-\sigma t}\, dt < \infty$ for real +ve σ.

$s = \sigma + j\omega$ is a Complex number.

Laplace transform of standard functions

1. unit step function

$f(t) = u(t)$



$u(t) = 1$ for $t \geq 0$
$= 0$ for $t < 0^-$

$$\mathcal{L}[u(t)] = \int_{0}^{\infty} 1 \cdot e^{-st}\, dt = \left[\frac{e^{-st}}{-s}\right]_{0^-}^{\infty} = -\frac{1}{s}\left[e^{-st}\right]_{0}^{\infty}$$

$$= -\frac{1}{s}\left[e^{-\infty} - e^{-0}\right] = -\frac{1}{s}[0-1]$$

$$= \frac{1}{s}$$

$$\boxed{\mathcal{L}[u(t)] = \frac{1}{s}}$$

2. $f(t) = e^{at}$ where a is constant.

$$\mathcal{L}[e^{at}] = \int_{0^-}^{\infty} e^{at}\, e^{-st}\, dt = \int_{0^-}^{\infty} e^{-(s-a)t}\, dt = \left[\frac{-1}{(s-a)} e^{-(s-a)t}\right]_{0^-}^{\infty}$$

$$= \frac{-1}{s-a}\left[e^{-\infty} - e^{0}\right] = \frac{-1}{s-a}[0-1] = \frac{1}{s-a}$$

$$\boxed{\mathcal{L}[e^{at}] = \frac{1}{s-a}}$$

3&4. $f(t) = \sin\omega t$ and $f(t) = \cos\omega t$.

$$\mathcal{L}\left[e^{j\omega t}\right] = \mathcal{L}\left[\cos\omega t + j\sin\omega t\right]$$

$$= \frac{1}{s - j\omega} \times \frac{s + j\omega}{s + j\omega}$$

$$= \frac{s}{s^2+\omega^2} + j\,\frac{\omega}{s^2+\omega^2}$$

$e^{at} = \frac{1}{s-a}$

$e^{-j\omega t} = \frac{1}{s-j\omega}$

$$\boxed{\begin{aligned}\mathcal{L}\left[\sin\omega t\right] &= \frac{\omega}{s^2+\omega^2} \\ \mathcal{L}\left[\cos\omega t\right] &= \frac{s}{s^2+\omega^2}\end{aligned}}$$

5. $f(t) = t^n$

$$\mathcal{L}\left[t^n\right] = \int_0^\infty t^n e^{-st}\,dt.$$

$$= t^n \left.\frac{e^{-st}}{-s}\right]_0^\infty - \int_0^\infty \left(\frac{e^{-st}}{-s}\right) n\, t^{n-1}\,dt$$

$$= \cancel{t^n}(\cancel{0} - 0) + \frac{n}{s}\int_0^\infty t^{n-1} e^{-st}\,dt$$

$$= \frac{n}{s}\,\mathcal{L}\left[t^{n-1}\right]$$

$$= \frac{n}{s}\,\frac{(n-1)}{s}\,\mathcal{L}\left[t^{n-2}\right]$$

$$= \frac{n}{s}\left(\frac{n-1}{s}\right)\cdot\left(\frac{n-2}{s}\right)\cdots\frac{2}{s}\cdot\frac{1}{s}\,\mathcal{L}\left[t^{n-n}\right]$$

$$= \frac{n}{s}\,\frac{(n-1)}{s}\,\frac{(n-2)}{s}\cdots\frac{2}{s}\,\frac{1}{s}\cdot\frac{1}{s}$$

$$\boxed{\mathcal{L}\left[t^n\right] = \frac{n!}{s^{n+1}}}$$

(2).

$$\mathcal{L}[t^{(n-1)}] = \frac{(n-1)!}{s^{(n-1)+1}} = \frac{(n-1)!}{s^n}$$

$$\mathcal{L}[t^3] = \frac{3!}{s^4}$$

$$\mathcal{L}[t^2] = \frac{2!}{s^3}$$

$t^n f(t)$

$$\mathcal{L}[f(t)] = F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$

$$\mathcal{L}[f'(t)] = F'(s) = \int_{0^-}^\infty (-t) f(t)\, e^{-st}\, dt = \mathcal{L}[-t\, f(t)]$$

$$\mathcal{L}\left[f''(t)\right] = F''(s) = \int_{0^-}^{\infty} t^2 f(t)\, e^{-st}\, dt = \mathcal{L}\left[t^2 f(t)\right]$$

$\vdots$

$$\mathcal{L}\left[f^n(t)\right] = F^n(s)(-1)^n = \int_{0^-}^{\infty} t^n f(t)\, e^{-st}\, dt$$

$$\boxed{\therefore \mathcal{L}\left[t^n f(t)\right] = (-1)^n F^n(s).}$$

7) $f(t) = \sinh \omega t$

$$\mathcal{L}[\sinh \omega t] = \mathcal{L}\left[\frac{e^{\omega t} - e^{-\omega t}}{2}\right]$$

$$= \frac{1}{2}\left[\frac{1}{s-\omega} - \frac{1}{s+\omega}\right] = \frac{1}{2}\left[\frac{s+\omega-s+\omega}{s^2-\omega^2}\right] = \frac{1}{\cancel{2}}\left[\frac{\cancel{2}\omega}{s^2-\omega^2}\right]$$

$$\boxed{\mathcal{L}[\sinh \omega t] = \frac{\omega}{s^2-\omega^2}}$$

8) $f(t) = \cosh \omega t$

$$\mathcal{L}[\cosh \omega t] = \mathcal{L}\left[\frac{e^{\omega t} + e^{-\omega t}}{2}\right] =$$

$$= \frac{1}{2}\left[\frac{1}{s-\omega} + \frac{1}{s+\omega}\right] = \frac{1}{\cancel{2}}\left[\frac{\cancel{2}s}{s^2+\omega^2}\right]$$

$$\boxed{\mathcal{L}[\cosh \omega t] = \frac{s}{s^2+\omega^2}}$$

9] Laplace transform of derivatives

$$\mathcal{L}[f'(t)] = \int_{0^-}^{\infty} f'(t)e^{-st}\,dt = \left[e^{-st} f(t)\right]_{0^-}^{\infty} - \int_{0}^{\infty} f(t)\,e^{-st}(-s)\,dt$$

$$= -f(0^-) + s\int_{0}^{\infty} f(t)e^{-st}\,dt$$

$$= sF(s) - f(0^-)$$

In general

$$\mathcal{L}[f^n(t)] = s^n F(s) - s^{n-1} f(0^-) - s^{n-2} f'(0) - \dots - f^{n-1}(0^-)$$

③

Laplace transform of integrals

$$\mathcal{L}\left[\int_{0^-}^{t} f(t)dt\right] = \int_{0^-}^{\infty}\left[\int_{0^-}^{t} f(t)\,dt\right] e^{-st}.dt$$

$$= \left[-\frac{e^{-st}}{S}\int_{0^-}^{t} f(t)dt\right]_{0^-}^{\infty} + \int_{0^-}^{\infty} f(t)\frac{e^{-st}}{S}dt \quad \left\{\begin{array}{l}\text{first term} \times \\ \text{derivation of} \\ \text{of } 2^{nd}\text{ fun}\end{array}\right.$$

$$= 0 + \frac{F(S)}{S}$$

If the integral has the limits $-\infty$ to t instead of (0^-) to t, then

$$\int_{-\infty}^{t} f(t)dt = \underbrace{\int_{-\infty}^{0^-} f(t)dt}_{\text{const}} + \int_{0^-}^{t} f(t)dt$$

and can be represented ~~The first term on the right hand~~ as $f(-\infty)$ or $f(0^-)$

$$\boxed{\therefore \mathcal{L}\left[\int_{-\infty}^{t} f(t)dt\right] = \mathcal{L}\left[f(0^-) + \int_{0^-}^{t} f(t)dt\right] = \frac{f(0^-)}{S} + \frac{F(S)}{S}}$$

if $f(t)$ is a current, then $f(0^-)$ represents the initial charge $q(0^-)$ .. if $f(t)$ is a voltage, then $f(0^-)$ represents flux linkages $\phi(0^-) = Li(0^-)$.

$$f(0^-) \Rightarrow \begin{array}{l} i(t) \longrightarrow \text{initial charge } q(0^-) \\ v(t) \longrightarrow \text{flux linkage } \phi(0^-) = Li(0^-). \end{array}$$

(1)

11. <u>property</u> of <u>linearity</u>:

$F(s) \longrightarrow f(t)$ then

$\mathcal{L}[k f(t)] = k \mathcal{L}[f(t)] = k F(s)$ where

12. <u>property</u> of <u>Superposition</u>

If $F_1(s), F_2(s) \ldots F_n(s)$ are the Laplace transform
$f_1(t), f_2(t), \ldots f_n(t)$ then

$$\mathcal{L}[f_1(t) + f_2(t) + \ldots f_n(t)] = F_1(s) + F_2(s) + \ldots F_n(s)$$

<u>Inverse Laplace transform</u>

$$f(t) = \mathcal{L}^{-1}[F(s)]$$

$$f(t) = \frac{1}{2\pi j} \underbrace{\int_{\sigma_1 - j\infty}^{\sigma_1 + j\infty}}_{\text{Complex inversion integral}} F(s) e^{st} dt \quad \Bigg\} \rightarrow \text{Not used}$$

If $F(s)$ is not in standard form for which $f(t)$ can be readily found, it must be Converted into the std form and then its inverse is found.

The uniqueness property of Laplace transformation i.e no two different functions have the same Laplace transformation, helps to find $f(t)$ for given $F(s)$

procedure for to use Laplace transformation

1. The integro differential equations are written for given N/w.
2. On applying LT, the transformed eqns are written, inserting the initial conditions to them
3. The transformed eqns are manipulated algebraically, such that they are in std form for which inverse LT can be found.
4. The inverse LT of these eqns give required solns

First shifting theorem:

If $F(S)$ is LT of $f(t)$ then

$$\mathcal{L}\left[e^{-at} f(t)\right] = F(S+a)$$

ex: $\mathcal{L}\left[\sin \omega t\right] = \frac{\omega}{s^2+\omega^2}$, $\mathcal{L}\left[e^{-at}\sin \omega t\right] = \frac{\omega}{(S+a)^2+\omega^2}$

Second shifting theorem:

shifted.

If $\mathcal{L}[f(t)] = F(S)$ then $\mathcal{L}\left[f(t-a)\, u(t-a)\right] = e^{-as} F(S)$

ex: $\mathcal{L}\left[t\, u(t)\right] = \frac{1}{s^2}$ $\therefore$ $\mathcal{L}\left[(t-a)\, u(t-a)\right] = e^{-as}\frac{1}{s^2}$

Convolution theorem:

If $F_1(s)$ and $F_2(S)$ are Laplace transforms of $f_1(t)$ and $f_2(t)$ respectively, then

$$\mathcal{L}\int_0^t f_1(\tau)\, f_2(t-\tau)\, d\tau = \mathcal{L}\int_0^t f_1(t-\tau)\, f_2(\tau)\, d\tau$$

$$= \mathcal{L}\left[f_1(t) * f_2(t)\right] = F_1(S)\, F_2(S)$$

i.e LT of convolution of 2 fun = product of 2 transforms

Initial value theorem:

This theorem helps us to find the initial value of function $f(t)$ directly from the transformed functions $F(S)$

unit S $f(t)$

Statement: If $f(t)$ and $f'(t)$ are Laplace transform then the behaviour of $f(t)$ in the neighbourhood $t=0^-$ corresponds to the behaviour of $SF(S)$ in the neighbourhood of $S=\infty$.

i.e $$f(0^-) = \lim_{t \to 0^-} f(t) = \lim_{S \to \infty} SF(S)$$

Final value theorem:

This theorem helps us to find the final value function $f(t)$ directly from transformed function

i.e $$f(\infty) = \lim_{t \to \infty} f(t) = \lim_{S \to 0} SF(S).$$

Laplace transform of periodic functions :-

$f(t)$ be a periodic function with T as period.

Let $f_1(t)$, $f_2(t)$, $f_3(t)$ ... represent the first, second, third, .. etc cycles of the periodic wave. Then

$$f(t) = f_1(t) + f_2(t) + f_3(t) + \cdots$$

$$= f_1(t) + f_1(t-T)u(t-T) + f_1(t-2T)u(t-2T)$$
$$+ f_3(t-3T)u(t-3T) + \cdots$$

Then $F(s) = F_1(s) + e^{-Ts} F_1(s) + e^{-2Ts} F_1(s) + e^{-3Ts} F_1(s) + \cdots$

$$= F_1(s)\left[1 + e^{-Ts} + e^{-2Ts} + e^{-3Ts} + \cdots\right].$$

$$= F_1(s)\left[1 - e^{-Ts}\right]^{-1}$$

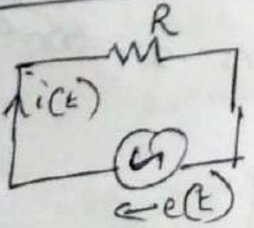
$$\boxed{F(s) = \frac{F_1(s)}{1 - e^{-Ts}}}$$

Transformed N/ws :

For solving electrical N/ws using LT, it is necessary for us to know the transformed equivalents of all the elements present in the N/w, considering initial values on them.

The elements $\longrightarrow$ R, L, C.

1) Resistance :

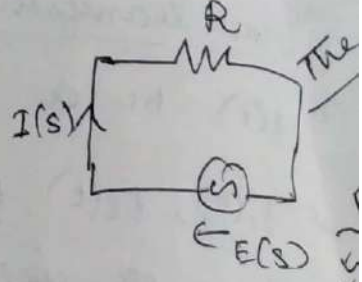


$$i(t) = \frac{e(t)}{R}$$

or $$I(s) = \frac{E(s)}{R}.$$

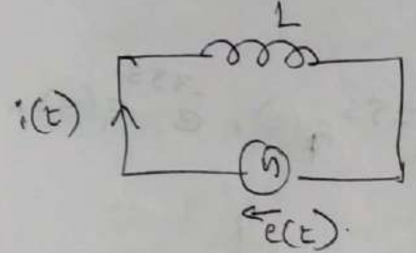
then transformed n/w is

we can observed that, resistance remains unchanged in the transformed n/w.



The

2) The inductance:



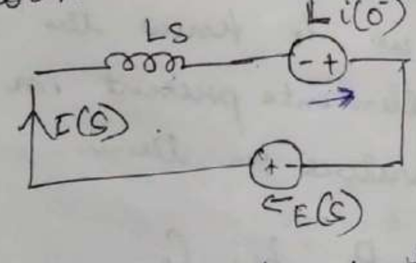
$$e(t) = L\frac{di}{dt}$$

$$\boxed{E(s) = L[sI(s) - i(0^-)]}$$

$$E(s) - LsI(s) + Li(0^-) = 0$$

$$I(s) = \frac{E(s) + Li(0^-)}{Ls} \quad \text{—— (1)}$$

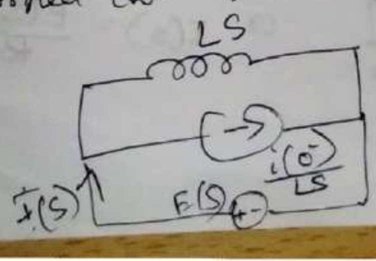
The transformed ckt satisfying eqn is ~~two~~ shown



eqn (1) can be written as

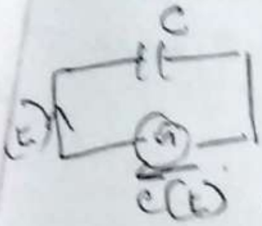
$$I(s) = \frac{E(s)}{Ls} + \frac{i(0^-)}{s} \quad \text{—— (2)}$$

The transformed ckt for eqn 2 may be



$C = \frac{q}{V} \Rightarrow q = CV$ (7)

The capacitance:



$$i(t) = C\frac{de(t)}{dt}$$

$$I(s) = C\left[sE(s) - V_c(0^-)\right]$$

$$= CsE(s) - CV_c(0^-)$$

$$I(s) = \frac{E(s)}{1/Cs} - CV_c(0^-) \quad \text{— (1)}$$

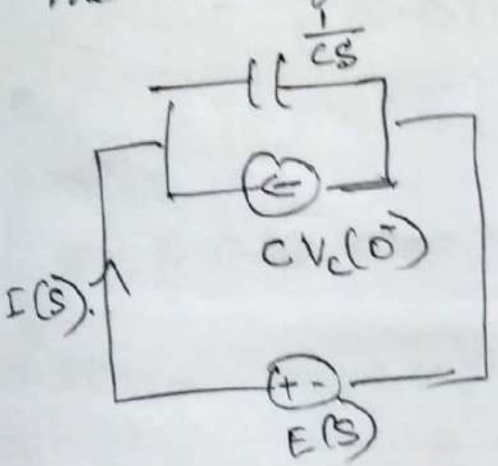
$$I(s) = E(s)Cs - CV_c(0^-)$$

$E(s)Cs = I(s) + CV_c(0^-)$

$E(s) = \frac{I(s)}{Cs} + \frac{V_c(0^-)}{s}$

$V_c(0^-) \rightarrow$ initial Vg on the capacitor.

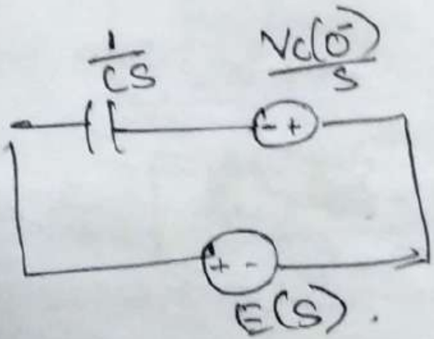
The transformed ckt satisfying eqn (1) is



eqn (1) can also be written

$$\boxed{E(s) = \frac{I(s)}{Cs} + \frac{V_c(0^-)}{s}}$$

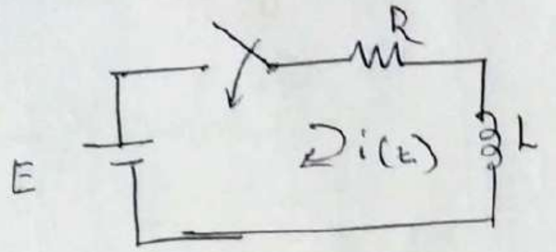
$$\left\{\begin{array}{l} I(s) + CV_c(0^-) = E(s)Cs \\ \frac{I(s)}{Cs} + \frac{V_c(0^-)}{s} \end{array}\right.$$



Laplace transformation.

For the ckt shown, find an expression for $i(t)$, when the switch k is closed at $t=0$.

9



when k is closed at $t=0$.

$$L\frac{di}{dt} + Ri = E.$$

$$L\left[sI(s) - \cancel{i(0^-)}\right] + RI(s) = \frac{E}{s}$$

$$i(0^-) = i(0^+) = 0.$$

$$I(s)\left[Ls+R\right] = \frac{E}{s}$$

$$I(s) = \frac{E}{s(Ls+R)} = \frac{E}{sL\left(s+\frac{R}{L}\right)}$$

$$= \frac{E}{L}\cdot\frac{1}{s\left(s+\frac{R}{L}\right)}$$

$$= \cancel{\frac{E}{R}\cdot\frac{1}{S.P}}$$

$$= \frac{E}{L}\left[\frac{A}{s} + \frac{B}{s+\frac{R}{L}}\right]$$

$$= \frac{E}{L}\left[\frac{L/R}{s} - \frac{L/R}{s+R/L}\right]$$

$$= \frac{E}{L}\left[\frac{L}{R} - \frac{L}{R}e^{-R/L\,t}\right].$$

$$\boxed{i(t) = \frac{E}{R}\left[1 - e^{-\frac{R}{L}t}\right]}$$

$$\frac{A}{s} + \frac{B}{s+R/L}$$

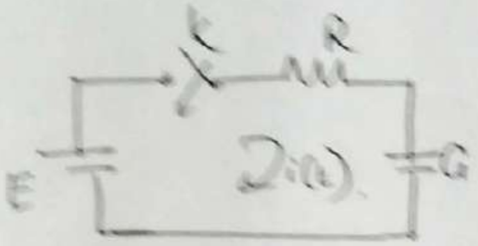
$$A = \left(\frac{1}{s+R/L}\right)_{s=0} \quad A \Rightarrow L/R$$

$$B = \frac{1}{s}\bigg|_{s=-R/L} = \quad B \Rightarrow -$$

4

2

2) For the ckt shown, find an expression for $i(t)$, when the switch K is closed at $t=0$. Assume that there is no initial charge on the capacitor. 10



$C = \frac{q}{V}$

$q = CV$

when switch K is closed at $t=0$.

$$Ri + \frac{1}{C}\int i\,dt = E$$

on taking LT

$$R\,I(s) + \left[\frac{I(s)}{Cs} - \frac{\cancel{q(0^-)}}{Cs}\right] = \frac{E}{s}$$

$$I(s)\left[R + \frac{1}{Cs}\right] = \frac{E}{s}$$

$$I(s) = \frac{E}{s\left(R + \frac{1}{Cs}\right)} = \left(\frac{E}{sR + \frac{1}{C}}\right)$$

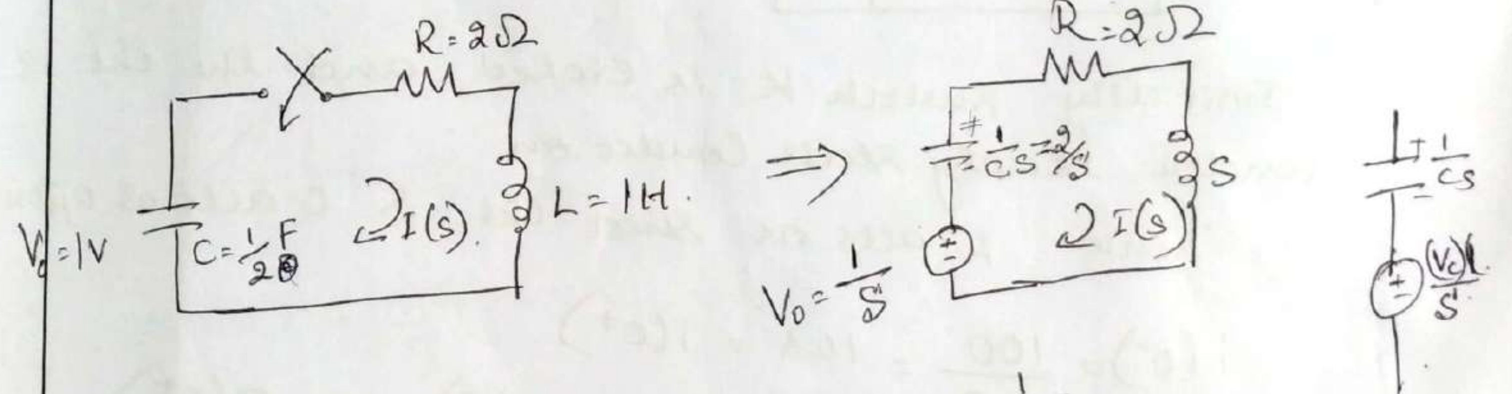
$$= \frac{E}{R\left(s + \frac{1}{CR}\right)} = \cancel{\frac{E}{R}\left(\frac{E}{\ }\right)}$$

$$= \frac{E}{R}\left(\frac{1}{s + \frac{1}{CR}}\right)$$

$$\boxed{i(t) = \frac{E}{R}\, e^{-t/RC}}$$

3) In the ckt shown in fig, if the capacitor is initially charged to 1V, find an expression for i(t) when the switch K is closed at t=0.

11



$$R\,I(s) + S\,I(s) + \frac{2}{S}I(s) = \frac{1}{S}$$

$$I(s)\left(R + S + \frac{2}{S}\right) = \frac{1}{S}$$

$$I(s)\,\frac{SR + S^2 + 2}{\cancel{S}} = \frac{1}{\cancel{S}}$$

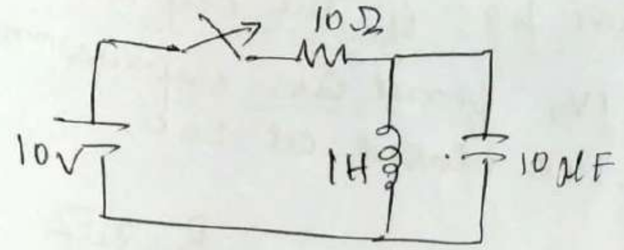
$$I(s) = \frac{1}{S^2 + SR + 2}$$

$R = 2\Omega$

$$I(s) = \frac{1}{S^2 + 2S + 2} = \frac{1}{(S+1)^2 + 1}$$

$$\boxed{i(t) = e^{-t}\sin t}$$

4) In the ckt shown, the switch K is closed and the steady state is reached. At t=0, the switch is opened. Find the expression for the current in the inductor using Laplace transform.



Initially switch k is closed and the ckt is under steady state condition.

∴ hence L acts as short ckt & C acts as open ckt

$$i(0^-) = \frac{100}{10} = 10A = i(0^+)$$

$$V_c(0^-) = 0 = V_c(0^+) \quad \because q(0^-) = 0 = q(0^+)$$

when k is opened.

$$L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0.$$

$$L\left[sI(s) - i(0^-)\right] + \left[\frac{I(s)}{CS} + \frac{q_c(0^-)}{S}\right] = 0$$

$$LSI(s) - L\,i(0^-) + \frac{I(s)}{CS} = 0.$$

$$\cancel{I(s)}\ LSI(s) - 10L + \frac{I(s)}{10\mu F\, S} = 0.$$

$$\cancel{I(s)\left[S - 10 + \frac{10}{S}\right.}$$

$$SI(s) + \frac{I(s)}{10^{-5}\, s} = 10$$

$$I(s)\left[s + \frac{10^5}{s}\right] = 10.$$

$$I(s)\left[\frac{s^2 + 10^5}{s}\right] = 10.$$

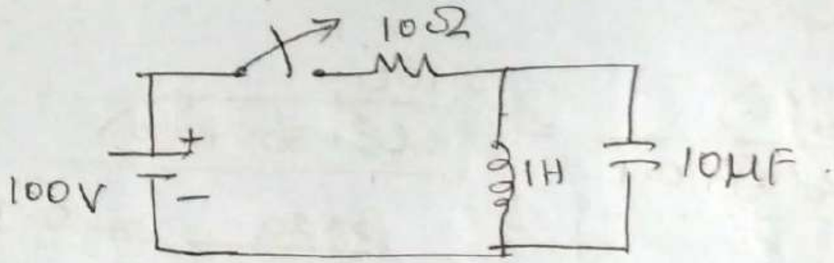
$$I(s) = \frac{10s}{s^2 + 10^5} = \frac{10s}{s^2 + (10^{5/2})^2}$$

ut pages

$$\boxed{I(t) = 10\cos 10^{5/2}t.}$$

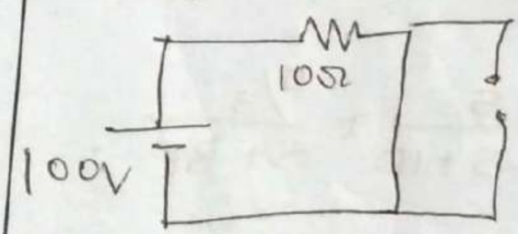
(16)

1) In the n/w shown, switch k is closed and steady state is reached. At $t=0$, the switch is opened. Find the expression for the current in the inductor using Laplace transform. 24



when k is closed, the ckt is under steady state condition. L acts as SC & C → O.C

$t=0^-$

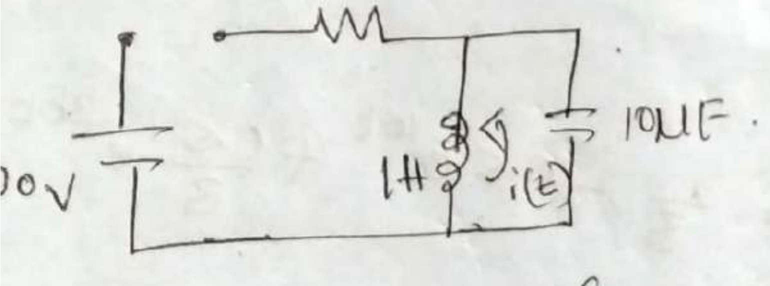


$$i(0^-) = i(0^+) = \frac{100}{10} = 10A.$$

$$V_c(0^-) = 0 = V_c(0^+)$$

$$q(0^-) = 0 = q(0^+)$$

when k is opened



$$L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0.$$

Taking L.T

$$L\left[sI(s) - i(0^-)\right] + \frac{1}{C}\left[\frac{I(s)}{Cs} - \frac{q(0^-)}{s}\right] = 0.$$

$$1\left[sI(s) - 10\right] + \frac{1}{10\times10^{-6}}\left(\frac{I(s)}{s}\right) = 0$$

$$I(s)\left[s + \frac{10^5}{s}\right] = 10.$$

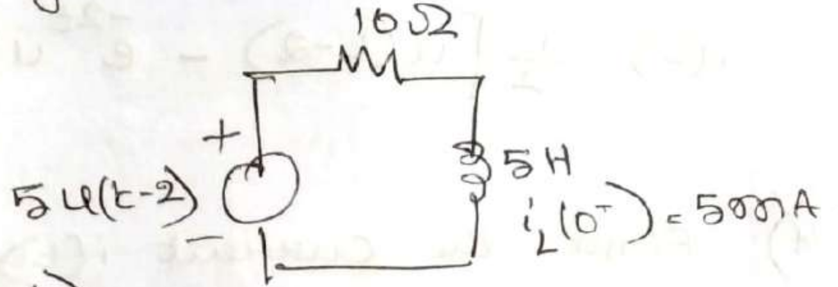
$$I(s) = \frac{10}{s + \frac{10^5}{s}} = \frac{10s}{s^2 + 10^5}$$ (17)

25

$$= \frac{10s}{s^2 + (10^{5/2})^2}.$$

$$\boxed{i(t) = 10 \cos 10^{5/2} t.}$$

Determine the response current $i(t)$ in the ckt shown using Laplace transform 27



$i_L(0^-) = 5mA = i_L(0^+)$

$$10\,I(s) + L[s\,I(s) - i(0^-)] = \frac{5e^{-2s}}{s}$$

$$10\,I(s) + 5(s\,I(s)) - 5 \times 5 = \frac{5e^{-2s}}{s}$$

$$I(s)[10 + 5s] - 25m = \frac{5e^{-2s}}{s}$$

$$I(s) = \frac{\frac{5e^{-2s}}{s} + 25 \times 10^{-3}}{10 + 5s}$$

$$= \frac{5e^{-2s} + (25 \times 10^{-3})s}{5s(2+s)}$$

$$= \frac{e^{-2s} + 5 \times 10^{-3} s}{s(s+2)}$$

$$= \frac{e^{-2s}}{s(s+2)} + \frac{5 \times 10^{-3}}{(s+2)}$$

$$= e^{-2s}\left[\frac{A}{s} + \frac{B}{s+2}\right] + \frac{5 \times 10^{-3}}{s+2}$$

$$= e^{-2s}\left[\frac{1/2}{s} - \frac{1/2}{s+2}\right] + \frac{5 \times 10^{-3}}{s+2}$$

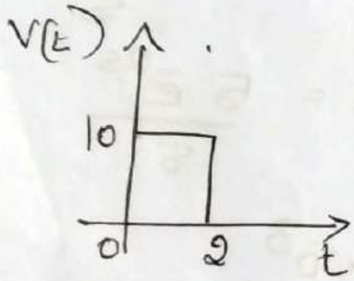
$A = \left.\frac{1}{s+2}\right|_{s=0} = \frac{1}{2}$

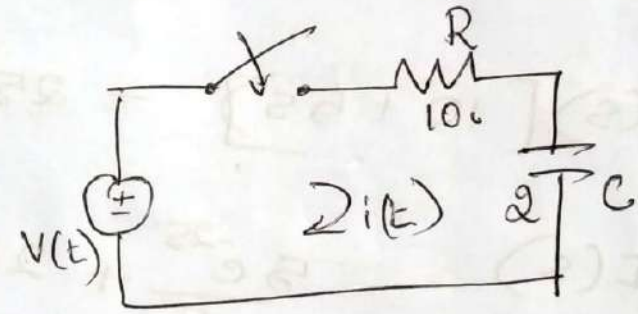
$B = \left.\frac{1}{s}\right|_{s=-2} = \frac{-1}{2}$

$$= \frac{1}{2} e^{-2s}\left[\frac{1}{s} - \frac{1}{s+2}\right] + \frac{5\times10^{-3}}{s+2}$$

$$i(t) = \frac{1}{2}\left[u(t-2) - e^{-2t}u(t-2)\right] + 5\times10^{-3} e^{-2t} u(t)$$

4) Find the current $i(t)$ assuming zero initial conditions, when switch K is closed at $t=0$. The excitation $V(t)$ is pulse magnitude 10V. and duration of 2sec. consider $R = 10\Omega$, $C = 2F$.





$$V(t) = 10u(t) - 10u(t-2) = 10\left[u(t) - u(t-2)\right]$$

$$\left[R\,I(s) + \frac{1}{C}\left[\frac{I(s)}{s}\right] + \cancel{\frac{V_c(0^-)}{s}}\right] = V(s).$$

$$\left[10\,I(s) + \frac{1}{2}\frac{I(s)}{s}\right] = 10\left[\frac{1}{s} - e^{-2s}\frac{1}{s}\right].$$

$$I(s)\left[10 + \frac{1}{2s}\right] = 10\left[\frac{1}{s} - \frac{e^{-2s}}{s}\right]$$

$$I(s)\left[\frac{20s+1}{2\cancel{s}}\right] = 10\left(\frac{1-e^{-2s}}{\cancel{s}}\right)$$

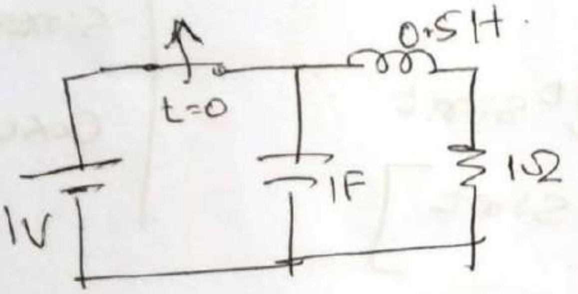
$$I(s) = \frac{20\left(1 - e^{-2s}\right)}{1+20s}$$

$$= \frac{20(1-e^{-2s})}{20\left(s+\frac{1}{20}\right)} = \frac{1}{s+\frac{1}{20}} - \frac{e^{-2s}}{s+\frac{1}{20}}.$$

(21) 29

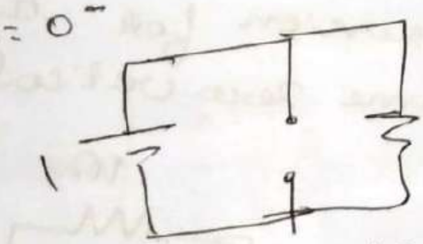
$$\boxed{i(t) = e^{-\frac{1}{20}t}\,u(t) - e^{-\frac{1}{20}(t-2)}\,u(t-2)}$$

5) The n/w shown in fig was in steady state before $t=0^+$. The switch is opend at $t=0$. Find $i(t)$ for $t>0$ using L.T.



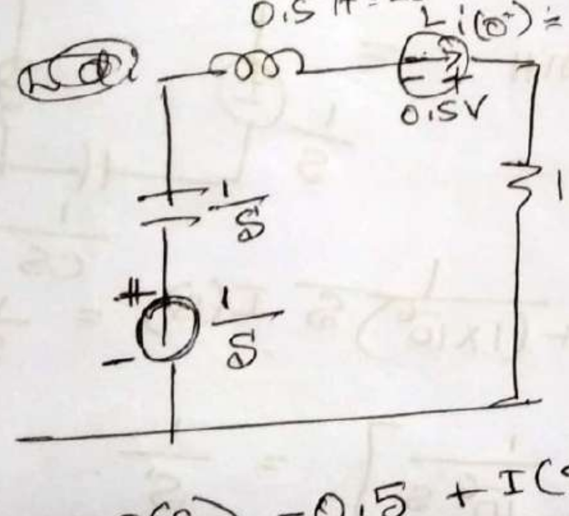
steady C → O.C
L → S.C

$t = 0^-$



$i(0^-) = \frac{1}{1} = 1\,A = i(0^+)$

$V_c(0^-) = 1V = V_c(0^+)$



$0.5\,I(s) - 0.5 + I(s) - \frac{1}{s} + \frac{1}{s}I(s) = 0.$

$I(s)\left[0.5 + 1 + \frac{1}{s}\right] = \frac{1}{s} + 0.5$

$$I(S) = \frac{\frac{1}{S}+\frac{1}{2}}{1+\frac{1}{S}+\frac{1}{2}S} = \frac{\frac{S+2}{2S}}{\frac{2S+2+S^2}{2S}}$$

$$= \frac{S+2}{S^2+2S+2}$$

$$= \frac{(S+1)+1}{(S+1)^2+1}$$

$$= \frac{S+1}{(S+1)^2+1} + \frac{1}{(S+1)^2+1}$$

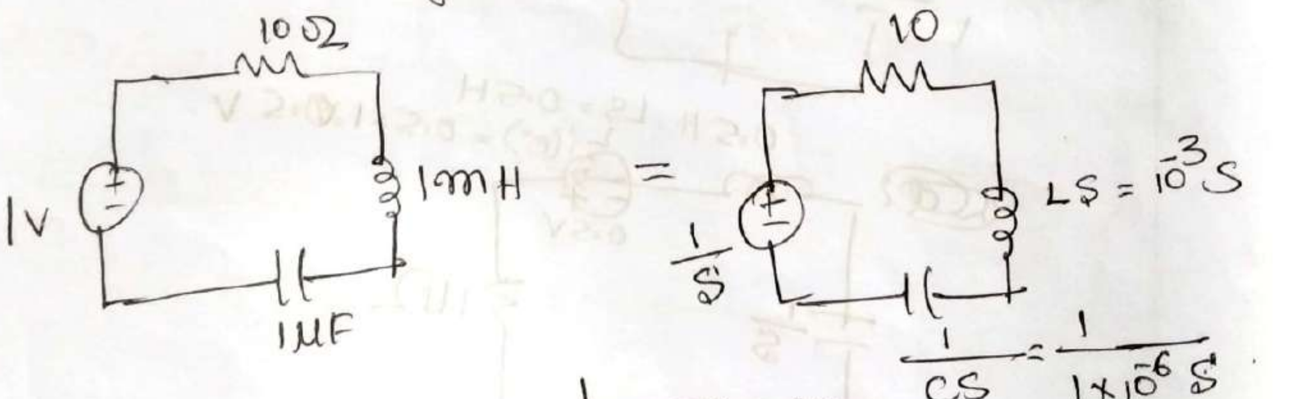
$$i(t) = e^{-t}\cos t + e^{-t}\sin t$$

$$i(t) = e^{-t}[\cos t + \sin t]$$

$\sin\omega t = \frac{\omega}{S^2+\omega^2}$

$\cos\omega t = \frac{S}{S^2+\omega^2}$

6) using LT obtain an expression for the Current $i(t)$ in the n/w of fig. Assume zero critical Conditions



$$10\,I(S) + 10^{-3}S\,I(S) + \frac{1}{(1\times10^{-6})S}\,I(S) = \frac{1}{S}$$

$$I(S)\left[10 + 10^{-3}S + \frac{1}{10^{-6}S}\right] = \frac{1}{S}$$

$$I(S)\left[\frac{10^{-5}S + 10^{-9}S^2 + 1}{10^{-6}S}\right] = \frac{1}{S}$$

23

31

$$I(s)\left[\frac{10^6(10^{-3}s+10^{-9}s^2+1)}{\cancel{s}} = \frac{1}{\cancel{s}}\right]$$

$$= I(s)\left[10s + 10^{-3}s^2 + 10^6\right] = 1$$

$$I(s) = \frac{1}{10s + 10^{-3}s^2 + 10^6} \times \frac{10^3}{10^3}$$

$$= \frac{10^3}{s^2 + 10^4 s + 10^9}$$

$$= \frac{1000}{s^2 + 10{,}000s + 10^9}$$

$$= \frac{1000}{(s+5000)^2 + 31225^2} \times \frac{31225}{31225}$$

$$= 0.032 \times \frac{31225}{(s+5000)^2 + 31225^2}$$

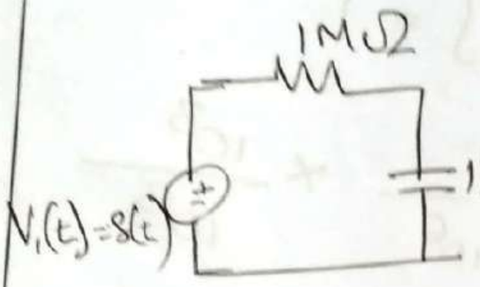
$10^9 s^2$

$5000s \quad 5000s = 10{,}000$

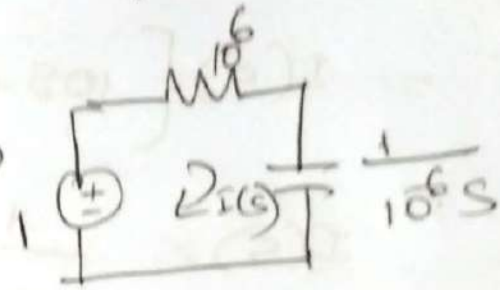
$10^9 - (5000)^2 = 31225^2$

$$\boxed{i(t) = 0.032\, e^{-5000t} \sin 31225t}$$

7) For the critically relaxed n/w of shown obtain expression for the current $i(t)$ 32 use Laplace transform. Given $V_i(t) = \delta(t)$



$\Rightarrow$



$$\left[10^6 + \frac{1}{10^{-6} s}\right] I(s) = 1.$$

$$\left[\frac{s+1}{10^{-6} s}\right] I(s) = 1.$$

$$I(s) = 10^{-6}\left(\frac{s}{s+1}\right)$$

$$= 10^{-6}\left[\frac{s+1-1}{s+1}\right]$$

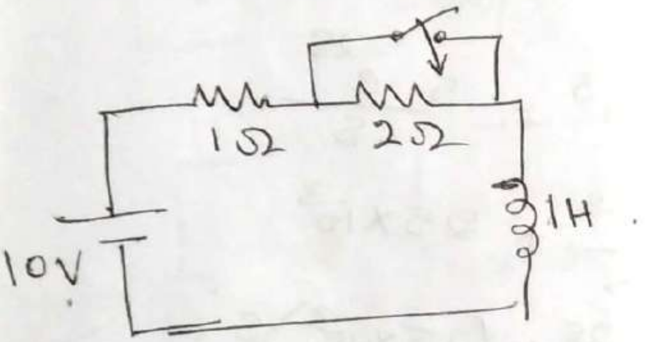
$$= 10^{-6}\left[1 - \frac{1}{s+1}\right]$$

impulse $= \delta(t)$

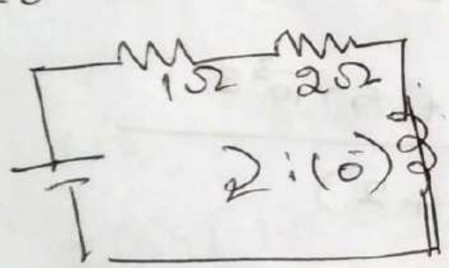
$$\boxed{i(t) = 10^{-6}\left[\delta(t) - e^{-t}\right]}.$$

(1)

1. The battery V_g 10V is applied for a steady state period with switch K open. Obtain the complete expression for the current after closing the switch K. Use L.T.



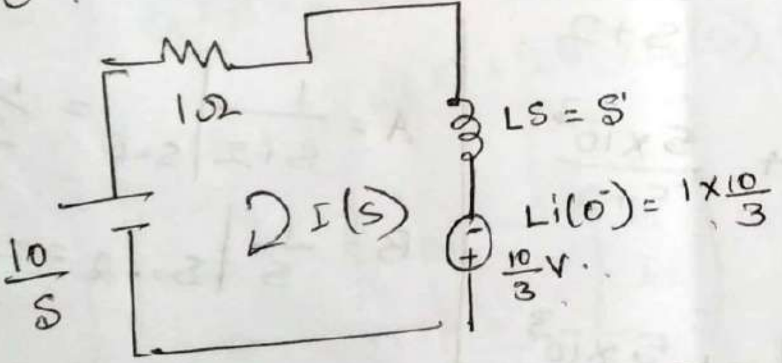
$t = 0^-$



$L \to SC$

$$i(0^-) = \frac{10}{3}A = i(0^+)$$

$t = 0^+$:



$$\frac{10}{S(S+1)} = \frac{A}{S} + \frac{B}{S+1}$$

$$A = \left.\frac{10}{S+1}\right|_{S=0} = 10$$

$$B = \left.\frac{10}{S}\right|_{S=-1} = -10$$

$$I(S) + S\,I(S) - \frac{10}{3} = \frac{10}{S}$$

$$I(S)\left[1+S\right] = \frac{10}{S} + \frac{10}{3} = \frac{30+10S}{3S}$$

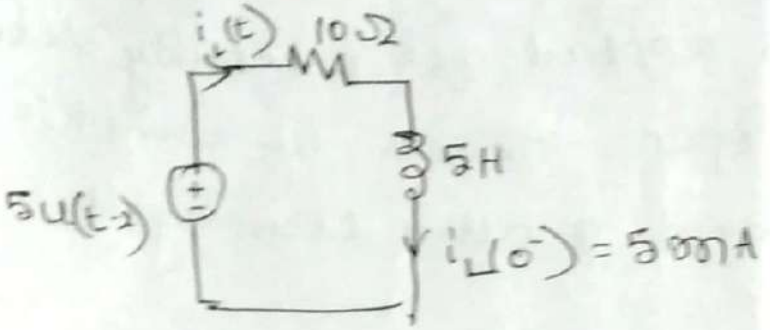
$$I(S) = \frac{30+10S}{3S(S+1)} = \frac{\cancel{30}^{10}}{\cancel{3}S(S+1)} + \frac{10\cancel{S}}{3\cancel{S}(S+1)}$$

$$= \frac{10}{S(S+1)} + \frac{10/3}{S+1}$$

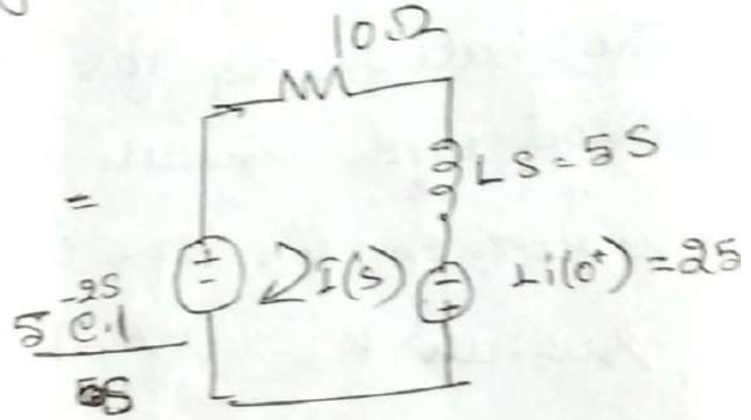
$$\frac{10}{S} - \frac{10}{S+1} + \frac{10/3}{S+1} \Rightarrow \boxed{10u(t) - 10e^{-t}u(t)}$$

2)

Solve for $i_L(t)$ using L.T



=



$$10\,I(S) + 5S\,I(S) - 25\times10^{-3} = \frac{5\,e^{-2S}}{S}$$

$$I(S)[10+5S] = 5\frac{e^{-2S}}{S} + 25\times10^{-3}$$

$$= \frac{5e^{-2S} + (25\times10^{-3})S}{S(10+5S)}$$

$$= \frac{\cancel{5}\;e^{-2S} + 5\times10^{-3}S}{\cancel{5}.\,S(S+2)}$$

$$= \frac{e^{-2S}}{S(S+2)} + \frac{5\times10^{-3}}{S+2}$$

$$= e^{-2S}\left[\frac{A}{S} + \frac{B}{S+2}\right] + \frac{5\times10^{-3}}{S+2}$$

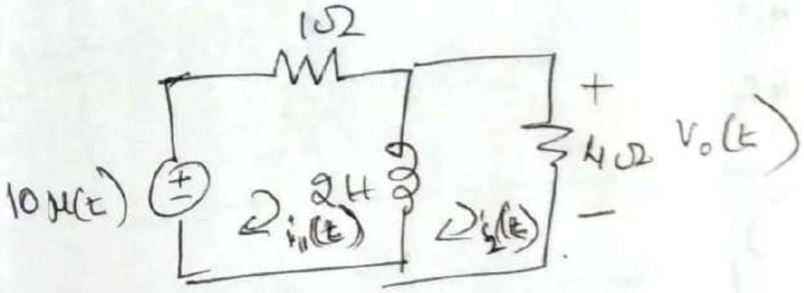
$$A = \left.\frac{1}{S+2}\right|_{S=0} = \frac{1}{2}$$

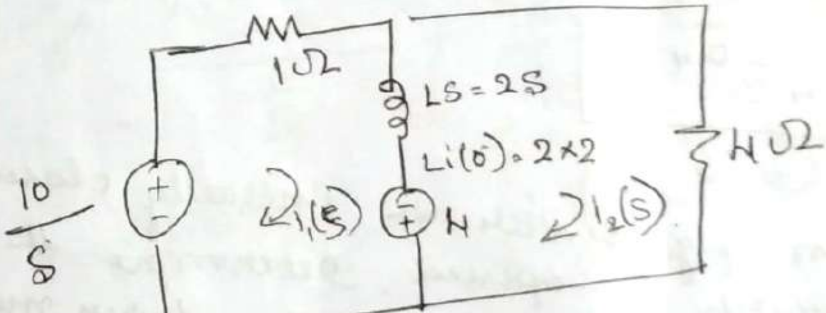
$$B = \left.\frac{1}{S}\right|_{S=-2} = -\frac{1}{2}.$$

$$I(S) = e^{-2/s}\left[\frac{1/2}{S} - \frac{1/2}{S+2}\right] + \frac{5\times10^{-3}}{S+2}$$

$$i(t) = \frac{1}{2}\left[u(t-2) - e^{-2(t-2)}.u(t-2)\right] + 5\times10^{-3}e^{-2t}.u(t).$$

For the ckt shown in fig, the switch is closed at $t=0$, The initial current through the inductance is $2A$. Obtain the expression for $V_0(t)$ for $t>0$.





$$I_1(S) + 2S(I_1(S) - I_2(S)) - 4 = \frac{10}{S}$$

$$I_1(S)[1+2S] - 2S\,I_2(S) = \frac{10}{S} + 4 = \frac{10+4S}{S} \quad (1)$$

$$4I_2(S) + 4 + 2S(I_2(S) - I_1(S)) = 0.$$

$$-2S\,I_1(S) + (4+2S)I_2(S) = -4 \quad (2)$$

$$\begin{bmatrix} 1+2S & -2S \\ -2S & 4+2S \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{10+4S}{S} \\ -4 \end{bmatrix}$$

$$\Delta = (1+2S)(4+2S) - 4S^2$$
$$= 4 + 8S + 2S + \cancel{4S^2} - \cancel{4S^2} = 4 + 10S$$

$$I_2 = \frac{\begin{vmatrix} 1+2S & \frac{10+4S}{S} \\ -2S & -4 \end{vmatrix}}{\Delta} = \frac{(1+2S)(-4) + \frac{2\cancel{S}(10+4S)}{\cancel{S}}}{\Delta}$$

$$= \frac{-4 - \cancel{8S} + 20 + \cancel{8S}}{\Delta} = \frac{16}{\Delta}.$$

$$I_2 = \frac{16}{\Delta} = \frac{16}{4+10S} = \frac{16}{10\left(S+\frac{4}{10}\right)}$$

$$= \frac{1.6}{S+0.4}$$

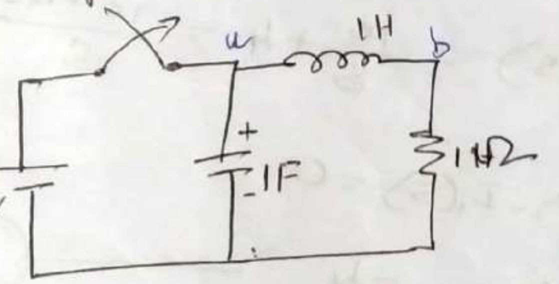
$$= 1.6\, e^{-0.4t}$$

$$V_0(t) = 4 \times i_2(t)$$

$$= 4 \times 1.6\, e^{-(0.4)t}$$

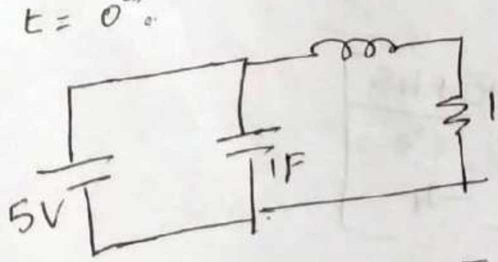
$$\boxed{V_0(t) = 6.4\, e^{-0.4t}}$$

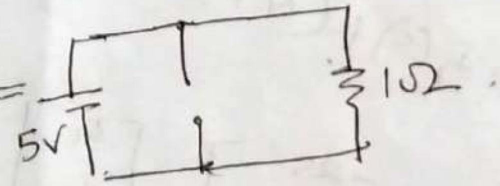
4] In the ckt shown in fig switch is initially closed. After steady state, the switch is opened. Determine the nodal Vgs $V_a(t)$ and $V_b(t)$ using Laplace transform method



=

At $t = 0^-$.

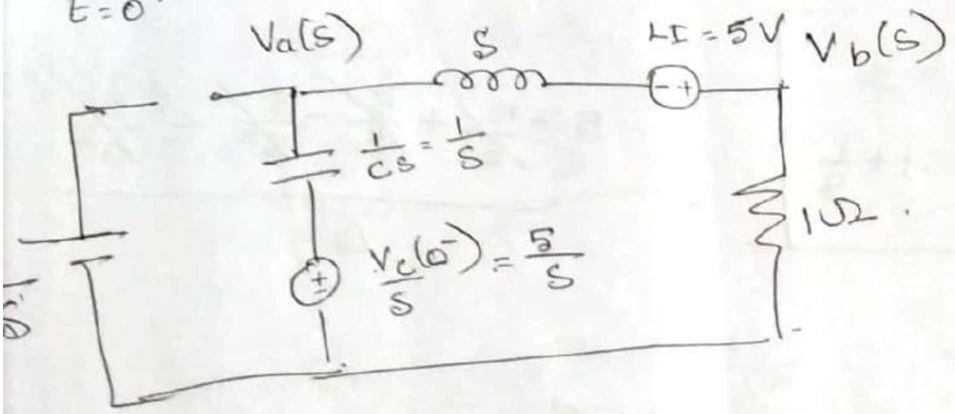




$$i(0^-) = \frac{5}{1} = 5\,A = i(0^+)$$

$$V_c(0^-) = 5V = V_c(0^+)$$

$t=0^+$



$$\frac{V_a - \frac{5}{s}}{\frac{1}{s}} + \frac{V_a - V_b + 5}{s} = 0$$

$$s\left(V_a - \frac{5}{s}\right) + \frac{1}{s}(V_a - V_b) + \frac{5}{s} = 0.$$

$$\left(s+\frac{1}{s}\right)V_a - \frac{1}{s}V_b = 5 - \frac{5}{s} \quad \text{——— (1).}$$

$$\frac{V_b}{1} + \frac{V_b - V_a - 5}{s} = 0.$$

$$-\frac{1}{s}V_a + \left(1+\frac{1}{s}\right)V_b = \frac{5}{s} \quad \text{——— (2).}$$

$$\begin{bmatrix} s+\frac{1}{s} & -\frac{1}{s} \\ -\frac{1}{s} & 1+\frac{1}{s} \end{bmatrix} \begin{matrix} V_a(s) \\ V_b(s) \end{matrix} = \begin{bmatrix} 5-\frac{5}{s} \\ \frac{5}{s} \end{bmatrix}$$

$$\Delta = \left(s+\frac{1}{s}\right)\left(1+\frac{1}{s}\right) - \frac{1}{s^2}$$

$$= \frac{(s^2+1)}{s}\left(\frac{s+1}{s}\right) - \frac{1}{s^2}$$

$$= \frac{s^3+s+s^2+1}{s^2} - \frac{1}{s^2}$$

$$= s + \frac{1}{s} + 1 + \cancel{\frac{1}{s^2}} - \cancel{\frac{1}{s^2}} = \boxed{\frac{s^2+s+1}{s}}$$

$$V_a(s) = \frac{\begin{vmatrix} 5-\frac{5}{s} & -\frac{1}{s} \\ \frac{5}{s} & 1+\frac{1}{s} \end{vmatrix}}{\Delta} = \frac{5 - \cancel{\frac{5}{s}} + \cancel{\frac{5}{s}} - \cancel{\frac{5}{s^2}} + \cancel{\frac{5}{s^2}}}{\Delta}$$

$$= \frac{5}{\frac{s^2+s+1}{s}} = \frac{5s}{s^2+s+1}$$

$$= \frac{5s}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$s^2 + \frac{1}{4} + s + 1 - \frac{1}{4}$$

$$= (s+1)^2 + \left(\frac{3}{4}\right)$$

$$= 5\, \frac{s+\frac{1}{2}-\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\frac{s}{s^2+a^2} = \cos at$$

$$\frac{a}{s^2+a^2} = \sin$$

$$= \cancel{5e^{\frac{1}{2}}\cos}$$

$$= 5\, \frac{\left[\left(s+\frac{1}{2}\right) - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}}\right]}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= 5\, e^{-\frac{1}{2}t} \cos \frac{\sqrt{3}}{2}t - e^{\frac{1}{2}t} 5 \cdot \frac{1}{\sqrt{3}} \sin \frac{\sqrt{3}}{2} t$$

$$= \cancel{5\left[e^{-\frac{1}{2}} \cos \frac{\sqrt{3}}{2}t - \frac{1}{\sqrt{3}} \sin\right.}$$

$$\boxed{V_a(t) = 5e^{-\frac{1}{2}t}\left[\cos \frac{\sqrt{3}}{2}t - \frac{1}{\sqrt{3}} \sin \frac{\sqrt{3}}{2}t\right]}$$

$$V_b(S) = \frac{\begin{vmatrix} S+\frac{1}{S} & 5-\frac{5}{S} \\ -\frac{1}{S} & \frac{5}{S} \end{vmatrix}}{\Delta} = \frac{5+\frac{5}{S^2}+\frac{1}{S}\left(5-\frac{5}{S}\right)}{\Delta}$$

$$= \frac{5+\cancel{\frac{5}{S^2}}+\frac{5}{S}-\cancel{\frac{5}{S^2}}}{\Delta} = \frac{\frac{5S+5}{S}}{\Delta}$$

$$= \frac{\frac{5S+5}{\cancel{S}}}{\frac{S^2+S+1}{\cancel{S}}} = \frac{5(S+1)}{S^2+S+1}.$$

$S^2+S+\frac{1}{4}$

$= (S+\frac{1}{2})^2$

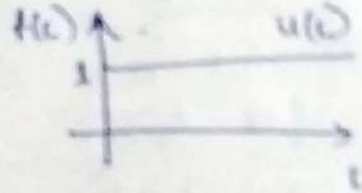
$$= \frac{5(S+1)}{(S+\frac{1}{2})^2}$$

$$= 5\left\{\frac{(S+\frac{1}{2})+\frac{1}{2}\sqrt{3}\times\frac{1}{\sqrt{3}}}{(S+\frac{1}{2})^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right\}$$

$$= 5\left[e^{-\frac{1}{2}t}\cos\frac{\sqrt{3}}{2}t + \frac{1}{\sqrt{3}}e^{-\frac{1}{2}t}\sin\frac{\sqrt{3}}{2}t\right]$$

$$\boxed{V_b(S) = 5e^{-\frac{1}{2}t}\left[\cos\frac{\sqrt{3}}{2}t + \frac{1}{\sqrt{3}}\sin\frac{\sqrt{3}}{2}t\right]}$$

unit step function

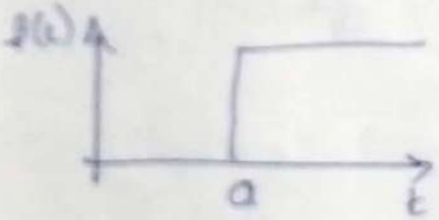


$u(t) = 1$ for $t \geq 0$
$\quad\; 0$ for $t < 0$.

$$\mathcal{L}[u(t)] = \int_{0^-}^{\infty} 1\, e^{-st}\, dt = \left[-\frac{1}{s} e^{-st}\right]_0^{\infty} = \frac{1}{s}$$

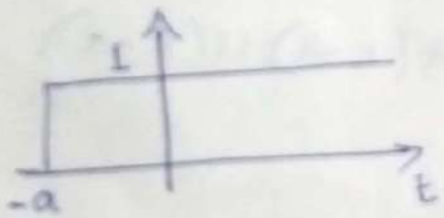
$$\boxed{\mathcal{L}[u(t)] = \frac{1}{s}}$$

)



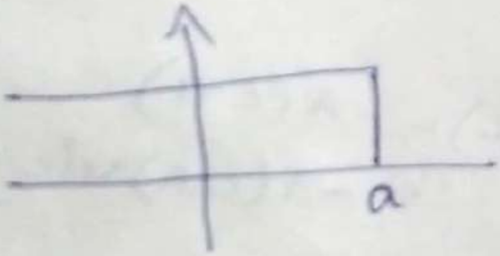
$u(t-a) = 1$ for $t \geq a$
$\qquad\quad 0$ for $t < a$.

b)



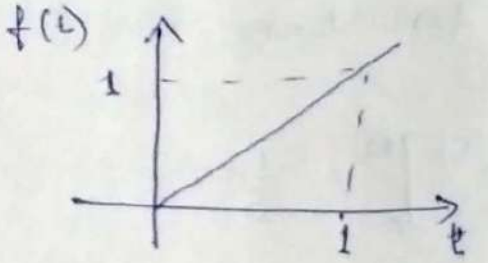
$u(t+a) = 1$ for $t \geq -a$
$\qquad\quad 0$ for $t < -a$

c)



$u(-(t-a)) = u(a-t) = 1$ for $t \leq a$
$\qquad\qquad\qquad\qquad\quad 0$ for $t > a$

Unit ramp functions:



$r(t) = t \quad$ for $t \geq 0$

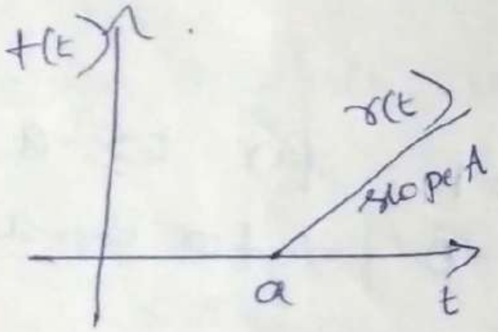
$\quad\;\; 0 \quad$ for $t < 0$.

$$\boxed{\mathcal{L}[r(t)] = \mathcal{L}[t] = \frac{1}{s^2}}$$

slope = y/x

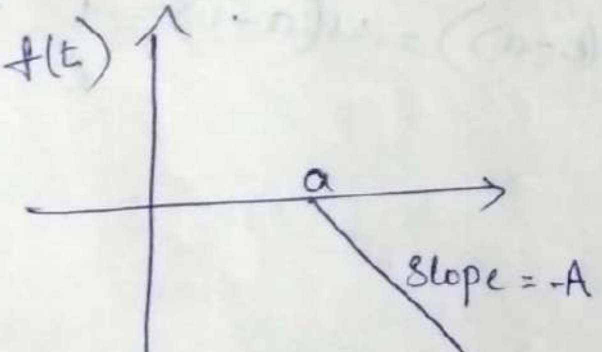
if slope is $At = \quad \mathcal{L}[At] = \dfrac{A}{s^2}$.

Shifted versions of ramp



$r(t) = A(t-a)$

$\quad\;\; = A(t-a)\,u(t-a)$



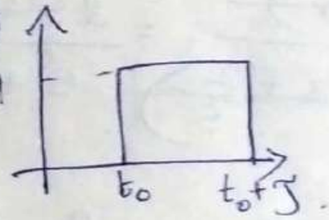
$r(t) = -A(t-a)$

$\quad\;\; = -A(t-a)\,u(t-a)$

Gate function:

The gate fun helps to determine the Laplace transform of discrete periodic functions.

Gate fun has height 1 and a period of τ.



it starts at $t=t_0$ & ends at $t=t_0+\tau$

$\tau \rightarrow$ period of gate fun.

$$g_{t_0}(t) = u(t-t_0) - u\left[t-(t_0+\tau)\right]$$

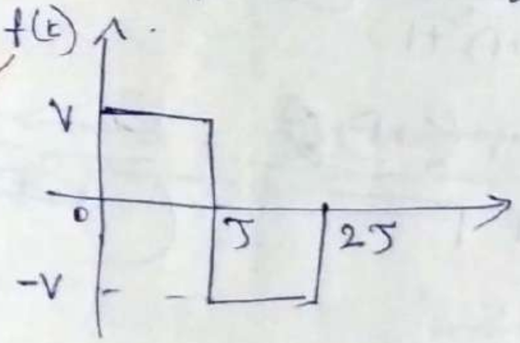
$$G_{t_0}(s) = e^{-t_0 s}\frac{1}{s} - e^{-(t_0+\tau)s}\cdot\frac{1}{s}$$

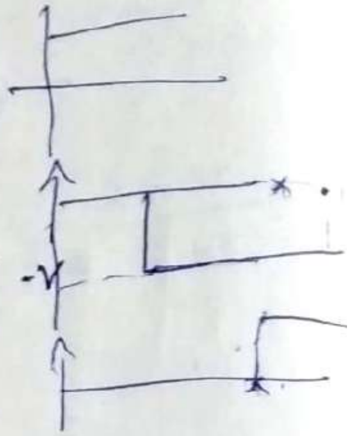
$$= \frac{e^{-t_0 s}}{s}\left[1-e^{-\tau s}\right].$$

$t_0=0$, $G_0(s) = \frac{1}{s}\left(1-e^{-\tau s}\right).$

Find Laplace transform of following signal

1)





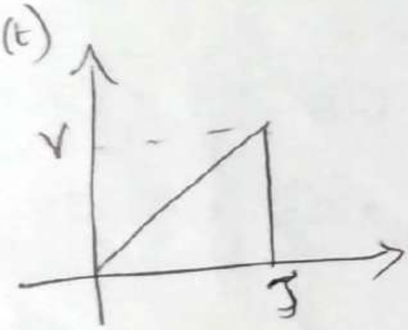
$$f(t) = V\,u(t) - 2V\,u(t-T) + V\,u(t-2T)$$

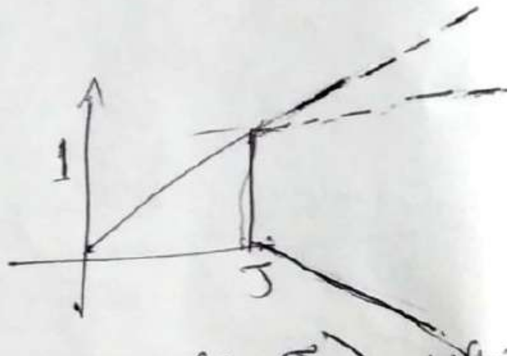
$$= \frac{V}{s} - 2V e^{-Ts}\frac{1}{s} + V e^{-2Ts}\frac{1}{s}$$

$$= \frac{V}{s}\left(1 - 2e^{-Ts} + e^{-2Ts}\right)$$

$$F(s) = \frac{V}{s}\left[\left(1 - e^{-Ts}\right)^2\right]$$

2)





$r(t) - r(t-T) - u(t-$

$t - (t-T) - u(t-$

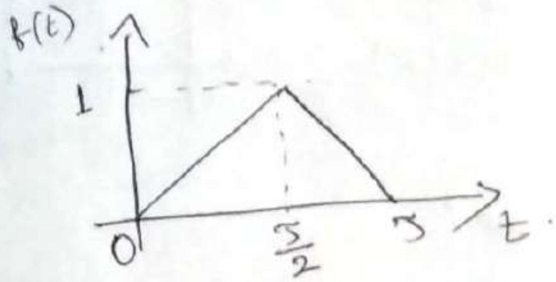
~~$f(t) = t$~~ unit slope $\frac{V}{T}$

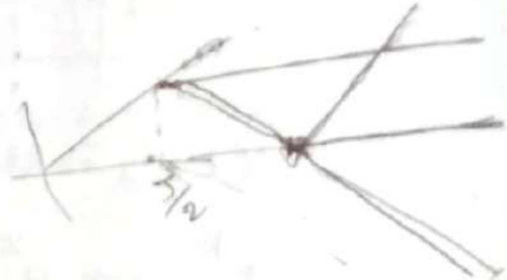
$$f(t) = \frac{V}{T}t - \frac{V}{T}(t-T) - V\,u(t-T)$$

$$= \frac{V}{T}t\,u(t) - \frac{V}{T}(t-T)\,u(t-T) - V\,u(t-T)$$

$$F(s) = \frac{V}{T}\frac{1}{s^2} - \frac{V}{T}e^{-Ts}\frac{1}{s^2} - V e^{-Ts}\frac{1}{s}$$

For the w/f shown in fig write Laplace transform.





$$r(t) - r\left(t-\frac{T}{2}\right) - r\left(t-\frac{T}{2}\right) + r(t-T)$$

$$= r(t) - 2r\left(t-T/2\right) + r(t-T)$$

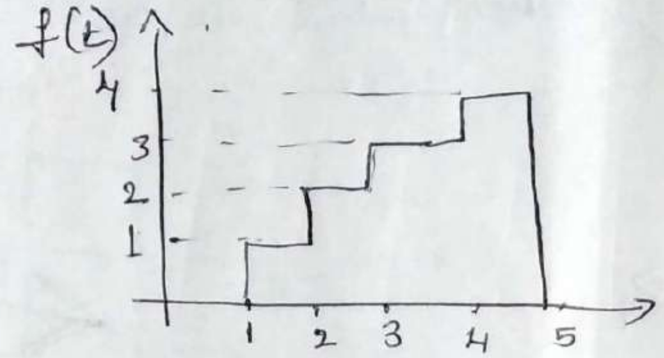
$$= \text{slope } t - \text{slope}\cdot 2\left(t-\frac{T}{2}\right) + \text{Slope}(t-T)$$

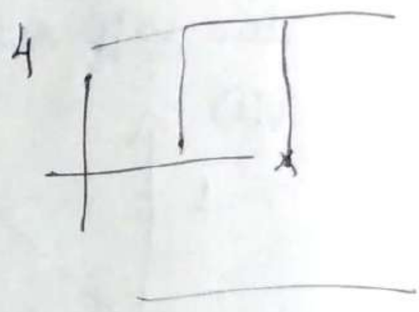
$$= \frac{1}{\frac{T}{2}}t - 2\frac{1}{\frac{T}{2}}\left(t-\frac{T}{2}\right) + \frac{1}{T/2}(t-T)$$

$$= \frac{2}{T}t - \frac{4}{T}\left(t-\frac{T}{2}\right) + \frac{2}{T}(t-T)$$

$$= \frac{2}{T}t\,u(t) - \frac{4}{T}\left(t-\frac{T}{2}\right)u\left(t-\frac{T}{2}\right) + \frac{2}{T}(t-T)u(t-T)$$

3)



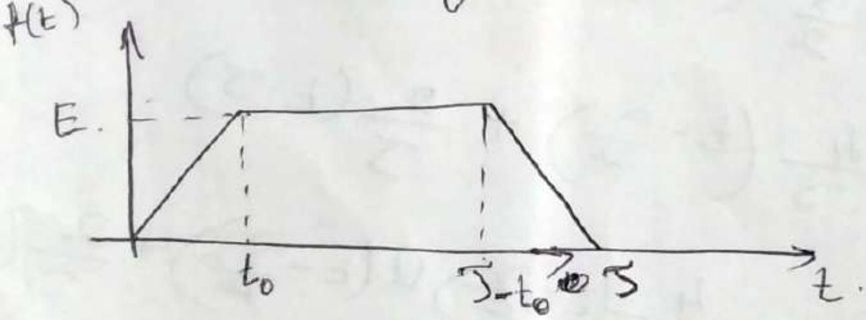


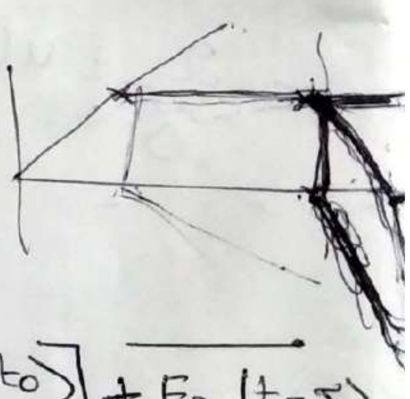
$$f(t) = u(t-1) + u(t-2) + u(t-3) + u(t-4) - 4u(t-5)$$

$$F(s) = e^{-s}\frac{1}{s} + e^{-2s}\frac{1}{s} + e^{-3s}\frac{1}{s} + e^{-4s}\frac{1}{s} - 4e^{-5s}\frac{1}{s}$$

$$= \frac{1}{s}\left[e^{-s} + e^{-2s} + e^{-3s} + e^{-4s} - 4e^{-5s}\right].$$

4) write the eqn for the waveform and find its Laplace transform.





$$f(t) = r(t) - r(t-t_0) - r(t-(5-t_0)) + r(t-5)$$

$$f(t) = t - t(t-t_0) - (t-(5-t_0)) + (t-5)$$

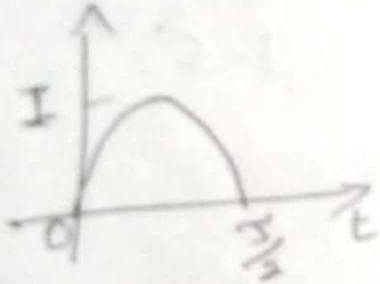
$$f(t) = \frac{E}{t_0}t - \frac{E_0}{t_0}(t-t_0) - \frac{E_0}{t_0}\left[t-(5-t_0)\right] + \frac{E_0}{t_0}(t-5)$$

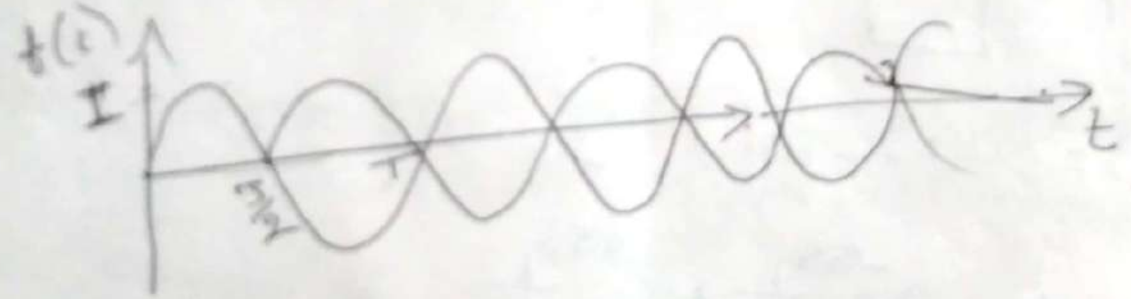
$$= \frac{E}{t_0}t\,u(t) - \frac{E_0}{t_0}(t-t_0)\,u(t-t_0) - \frac{E_0}{t_0}\left[t-(5-t_0)\right]u(t-(5-t_0))$$
$$+ \frac{E_0}{t_0}(t-5)\,u(t-5)$$

$$= \frac{E}{t_0}\cdot\frac{1}{s^2} - \frac{E_0}{t_0}e^{-t_0 s}\frac{1}{s^2} - \frac{E_0}{t_0}e^{-(T-t_0)s}\frac{1}{s^2} + \frac{E_0}{t_0}e^{-Ts}\frac{1}{s^2}$$

$$= \frac{E_0}{t_0 s^2}\left[1 - e^{-t_0 s} - e^{-(T-t_0)s} + e^{-Ts}\right]$$

write the eqn for sinusoidal waveform shown & find its Laplace transform





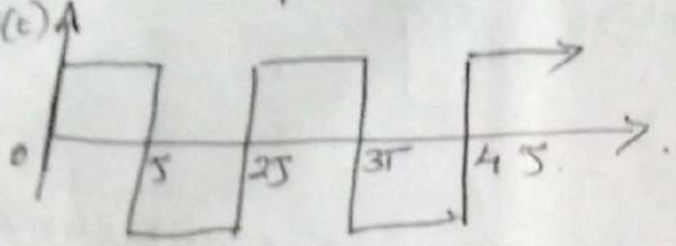
$$w = 2\pi f = \frac{2\pi}{T}$$

$$f(t) = I\sin\left(\frac{2\pi}{T}t\right)u(t) + I\sin\frac{2\pi}{T}\left(t - \frac{T}{2}\right)u\left(t - \frac{T}{2}\right)$$

$$= I\,\frac{\frac{2\pi}{T}}{s^2 + \left(\frac{2\pi}{T}\right)^2} + I e^{-\frac{T}{2}s}\left(\frac{\frac{2\pi}{T}}{s^2 + \left(\frac{2\pi}{T}\right)^2}\right)$$

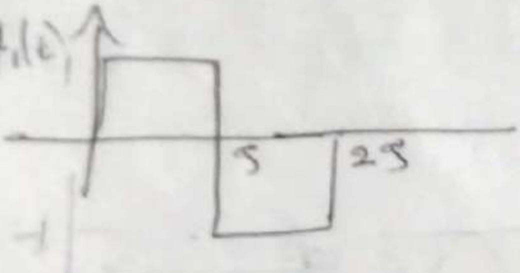
$$\boxed{F(s) = I\,\frac{\frac{2\pi}{T}}{s^2 + \left(\frac{2\pi}{T}\right)^2}\left[1 + e^{-\frac{T}{2}s}\right]}$$

6) For the rectangular waveform shown write down the Laplace transform equation

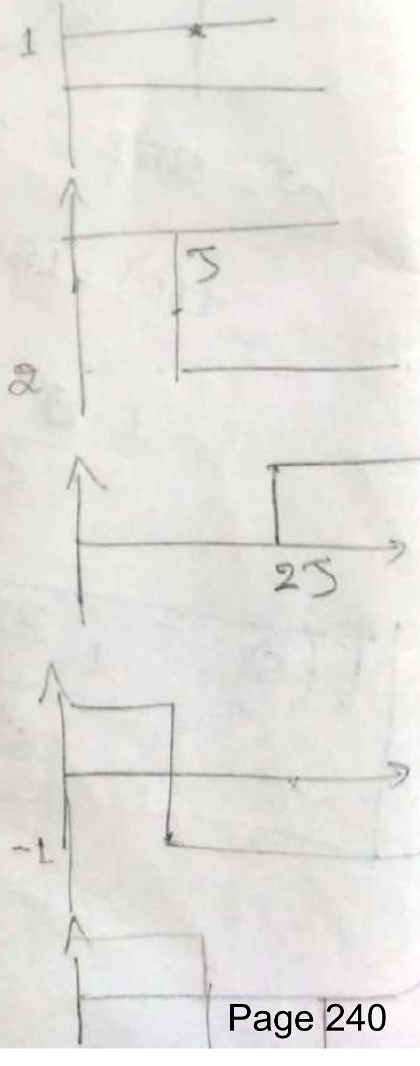


whenever there is a periodic fun.

Laplace transform $F(S) = \dfrac{F_1(S)}{1-e^{-2TS}}$



$$f_1(t) = u(t) - 2u(t-T) + u(t-2T)$$



$$F_1(S) = \frac{1}{S} - 2e^{-TS}\frac{1}{S} + e^{-2TS}\frac{1}{S}$$

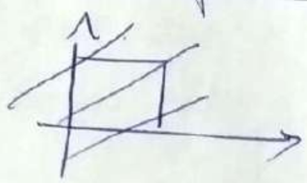
$$= \frac{1}{S}\left[1 - 2e^{-TS} + e^{-2TS}\right]$$

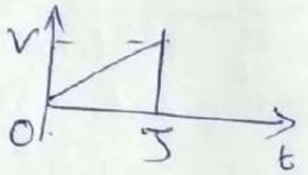
$$= \frac{1}{S}\left(1 - e^{-TS}\right)^2$$

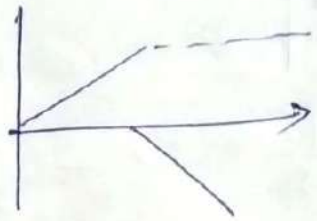
$$F(S) = \frac{F_1(S)}{1-e^{-2TS}}$$

$$= \frac{1\left(1-e^{-TS}\right)^2}{S\left(1-e^{-2TS}\right)}$$

1) Waveform Synthesis:



1) 



$$f(t) = r(t) - r(t-\tau) - u(t-\tau)$$

$$f(t) = \frac{V}{\tau} t - \frac{V}{\tau}(t-\tau) - V\,u(t-\tau)$$

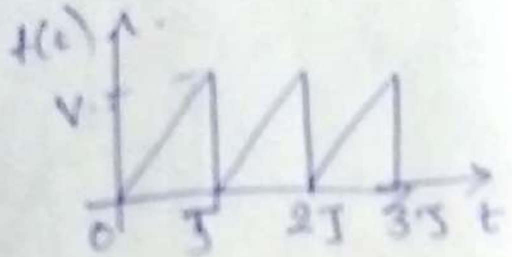
$$= \frac{V}{\tau} t\, u(t) - \frac{V}{\tau}(t-\tau)\,u(t-\tau) - V\,u(t-\tau)$$

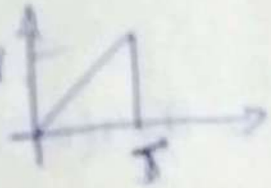
$$F(s) = \frac{V}{\tau}\frac{1}{s^2} - \frac{V}{\tau} e^{-\tau s}\frac{1}{s^2} - V e^{-\tau s}\frac{1}{s}$$

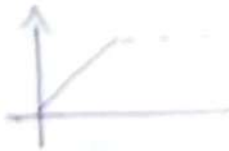
$$= \frac{V}{\tau s^2}\left[1 - e^{-\tau s}\right] - V\frac{e^{-\tau s}}{s}$$

$$= \frac{V}{\tau s^2}\left[1 - e^{-\tau s} - s\tau\, e^{-\tau s}\right]$$

$$\boxed{F(s) = \frac{V}{\tau s^2}\left[1 - (1+\tau s)e^{-\tau s}\right]}$$





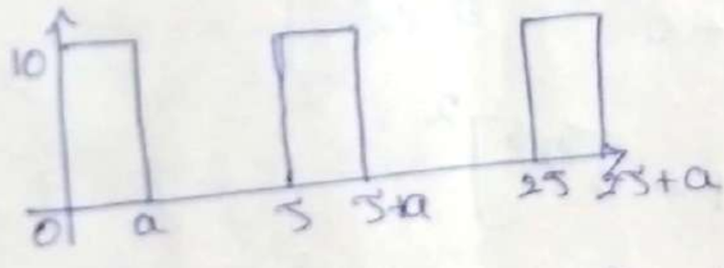


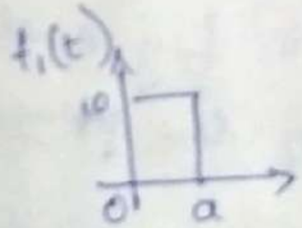
$$\cancel{f_1(t) = \frac{V}{T} r(t) - \frac{V}{T} r(t-T)}$$

$$F_1(s) = \frac{V}{Ts^2}\left[1-(1+Ts)e^{-Ts}\right]$$

$$F(s) = \frac{F_1(s)}{1-e^{-Ts}}$$

For the rectangular pulse shown in fig write the eqn and find its Laplace transform.





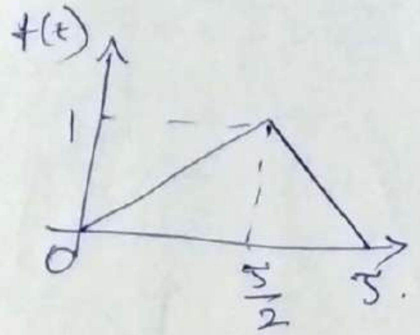
$$f_1(t) = 10u(t) - 10u(t-a)$$

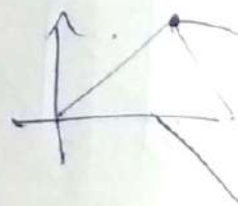
$$F_1(s) = 10\frac{1}{s} - 10e^{-as}\frac{1}{s}$$

$$= \frac{10}{s}\left[1-e^{-as}\right]$$

$$F(s) = \frac{F_1(s)}{1-e^{-Ts}} = \frac{10}{s}\left[\frac{1-e^{-as}}{1-e^{-Ts}}\right]$$

4)





$$f(t) = \frac{1}{T/2} r(t) - \frac{2}{T} r(t - T/2) - \frac{2}{T} r(t - T/2) + \frac{2}{T} r(t - T)$$

$$= \frac{2}{T} t - \frac{2}{T}(t - T/2) - \frac{2}{T}(t - T/2) + \frac{2}{T}(t - T)$$

$$f(t) = \frac{2}{T} t - \frac{4}{T}(t - T/2) + \frac{2}{T}(t - T)$$

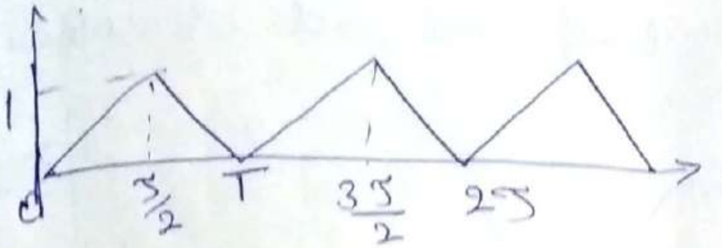
$$= \frac{2}{T} t\, u(t) - \frac{4}{T}(t - T/2)\, u(t - T/2) + \frac{2}{T}(t - T) u(t$$

$$F(s) = \frac{2}{T}\frac{1}{s^2} - \frac{4}{T} e^{-T/2 s} \frac{1}{s^2} + \frac{2}{T} e^{-Ts} \frac{1}{s^2}.$$

$$= \frac{2}{Ts^2}\left[1 + e^{-Ts} - 2 e^{-T/2 s}\right]$$

$$F(s) = \frac{2}{Ts^2}\left[1 - e^{-T/2 s}\right]^2.$$

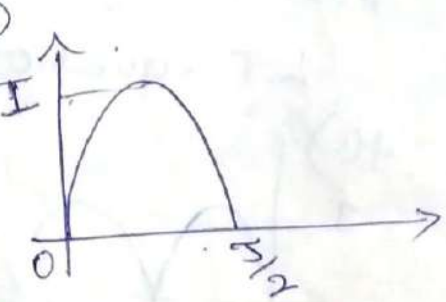
5] Synthesis the w/f shown in fig and find the laplace transform of the periodic waveform.

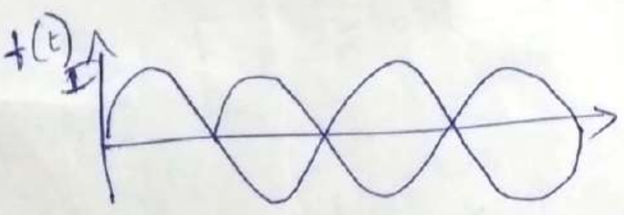


$$F(s) = \frac{F_1(s)}{1-e^{-Ts}}$$

$$= \frac{\frac{2}{Ts^2}\left[1-e^{-\frac{T}{2}s}\right]^2}{1-e^{-Ts}}.$$

write the equation for the sinusoidal waveform and find its Laplace transform.





$\cancel{f(t) =}$ $\omega = 2\pi f = \frac{2\pi}{T}$

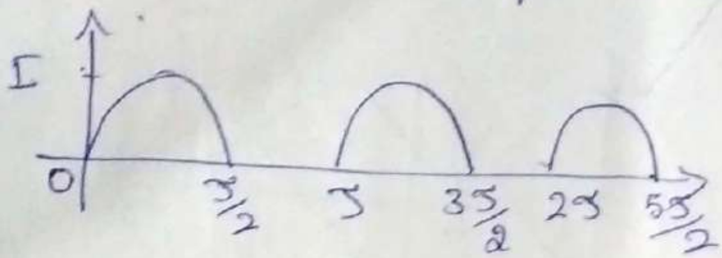
$$f(t) = I \sin \omega t + I \sin \omega(t - T/2)$$

$$f(t) = I \sin \frac{2\pi}{T} t\, u(t) + I \sin \frac{2\pi}{T}(t - T/2)\, u(t - T/2)$$

$$F(s) = I \frac{\frac{2\pi}{T}}{s^2+\left(\frac{2\pi}{T}\right)^2} + I e^{-\frac{T}{2}s} \cdot \frac{\frac{2\pi}{T}}{s^2+\left(\frac{2\pi}{T}\right)^2}.$$

$$= I \frac{\frac{2\pi}{T}}{s^2+\left(\frac{2\pi}{T}\right)^2}\left[1+e^{-\frac{T}{2}s}\right]$$

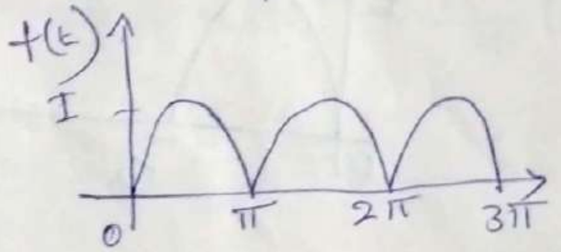
7). Find the LT for half rectified sine wave



$$F(s) = \frac{F_1(s)}{1-e^{-Ts}} = I$$

$$= I \, \frac{\frac{2\pi}{T}}{s^2 + \left(\frac{2\pi}{T}\right)^2} \left[\frac{1+e^{-\frac{T}{2}s}}{1-e^{-Ts}} \right]$$

8) For the full rectified waveform, find the LT equation



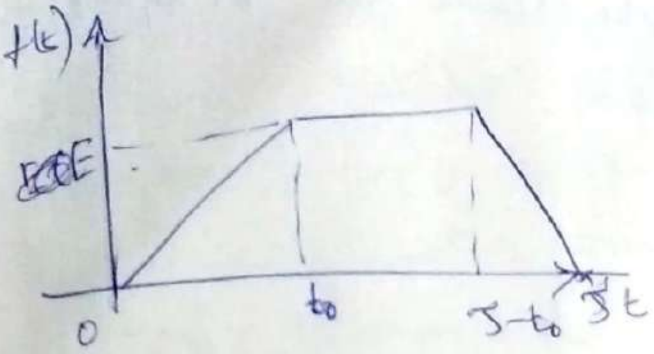
$$F(s) = \frac{F_1(s)}{1-e^{-Ts}}$$

$T = 2\pi$

$$\therefore F_1(s) = \frac{I}{s^2+1}\left(1+e^{-\pi s}\right)$$

$$F(s) = I \, \frac{1}{(s^2+1)} \, \frac{(1+e^{-\pi s})}{(1-e^{-\pi s})}$$

$$= \frac{I}{s^2+1} \coth \frac{\pi s}{2}$$



$$f(t) = \frac{E}{t_0}\, r(t) - \frac{E}{t_0}\, r(t-t_0) - \frac{E}{t_0}\, r(t-(\tau-t_0)) + \frac{E}{t_0}\, r(t-\tau)$$

$$= \frac{E}{t_0} t - \frac{E}{t_0}(t-t_0) - \frac{E}{t_0}(t-(\tau-t_0)) + \frac{E}{t_0}(t-\tau)$$

$$= \frac{E}{t_0} t\, u(t) - \frac{E}{t_0}(t-t_0)\, u(t-t_0) - \frac{E}{t_0}\left[t-(\tau-t_0)\, u(t-\tau-t_0)\right]$$
$$+ \frac{E}{t_0}(t-\tau)\, u(t-\tau)$$

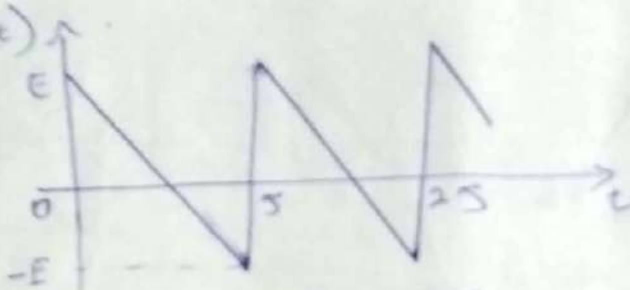
$$= \frac{E}{t_0}\cdot\frac{1}{s^2} - \frac{E}{t_0} e^{-t_0 s}\frac{1}{s^2} - \frac{E}{t_0} e^{-(\tau-t_0)s}\frac{1}{s^2} + \frac{E}{t_0} e^{-\tau s}\frac{1}{s^2}.$$

$$= \frac{E}{t_0 s^2}\left[1 - e^{-t_0 s} - e^{-(\tau-t_0)s} + e^{-\tau s}\right]$$

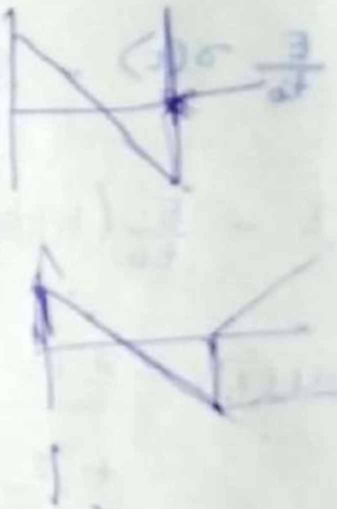
$$= \frac{E_0}{t_0 s^2}\left\{1 - e^{-t_0 s} + e^{-\tau s}\left(1 - e^{t_0 s}\right)\right\}$$

$$= \cancel{\frac{E_0}{t_0 s^2}\left[1 - e^{t_0 s}\right.}$$

10) For the w/f shown. write down the LT of



$\cancel{f(t) = -\frac{E}{T/2}t + \frac{E}{\frac{T}{2}}(t-T)}$



$$f_1(t) = E\,u(t) - \frac{E}{T/2}t + \frac{E}{T/2}(t-T) + E\,u(t-T)$$

$$= E\,u(t) - \frac{2E}{T}t + \frac{2E}{T}(t-T) + E\,u(t-T)$$

$$F_1(s) = E\frac{1}{s} - \frac{2E}{T}\frac{1}{s^2} + \frac{2E}{T}e^{-Ts}\frac{1}{s^2} + E\,e^{-Ts}\frac{1}{s}$$

$$= \frac{E}{s}\left(1+e^{-Ts}\right) - \frac{2E}{Ts^2}\left(1-e^{-Ts}\right)$$

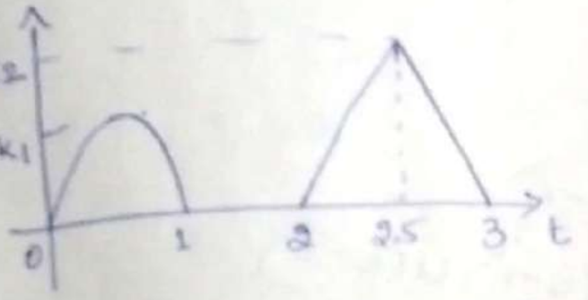
$\Longrightarrow$

$$F(s) = \frac{F_1(s)}{1-e^{-Ts}}$$

$$= \frac{E(1+e^{-Ts})}{s(1-e^{-Ts})} - \frac{2E}{Ts^2}$$

$$F(s) = -\frac{2E}{Ts^2} + \frac{E}{s}\coth\left(\frac{Ts}{2}\right).$$

11) The w/f shown in is sinusoidal in the interval t=0 to t=1 and is an isosceles triangle from t=2 to t=3. For all other t, V=0. write the expression for V(t), using step, ramp and sine functions and find its laplace transform.



$f_1(t) = k_1 \sin\omega t + k_1 \sin\omega(t-1)$

$$\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \text{ rad/sec}$$

$$= k_1 \sin\pi t\, u(t) + k_1 \sin\pi(t-1)u(t-1)$$

$$F_1(s) = k_1 \frac{\pi}{s^2+\pi^2} + k_1 e^{-s}\frac{\pi}{s^2+\pi^2}$$

$$f_2(t) = \frac{k_2}{0.5}r(t-2) - \frac{k_2}{0.5}r(t-2.5) - \frac{k_2}{0.5}r(t-2.5)$$

$$+ \frac{k_2}{0.5}r(t-3)$$

$$= \frac{k_2}{0.5}(t-2)u(t-2) - \frac{2}{0.5}k_2(t-2.5)u(t-2.5)$$

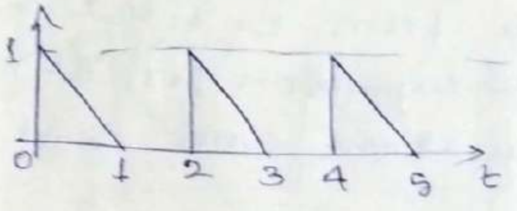
$$+ \frac{k_2}{0.5}(t-3)u(t-3).$$

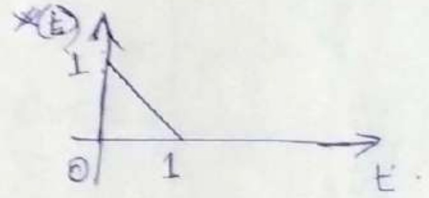
$$= 2k_2(t-2)u(t-2) - 4k_2(t-2.5)u(t-2.5) + 2k_2(t-3)u(t-3)$$

$$F_2(s) = 2k_2\frac{e^{-2s}}{s^2} - 4k_2 e^{-2.5s}\frac{1}{s^2} + 2k_2 e^{-3s}\frac{1}{s^2}$$

$$F(s) = F_1(s) + F_2(s)$$

12) Find LT of periodic signal $x(t)$





$$f_1(t) = u(t) - r(t) + r(t-1)$$

$$f_1(t) = u(t) - t\,u(t) + (t-1)\,u(t-1)$$

$$F_1(S) = \frac{1}{s} - \frac{1}{s^2} + e^{-s}\frac{1}{s^2}$$

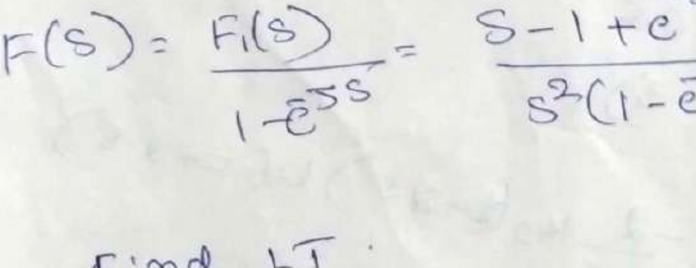
$$= \cancel{\frac{1}{s} - \frac{1}{s^2}\left[1+e^{-s}\right]} = \frac{s-1+e^{-s}}{s^2}$$

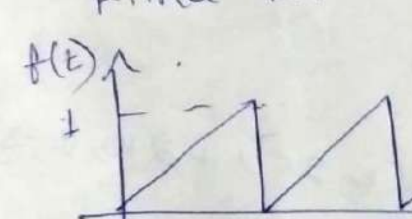
$$= \frac{s-1+e^{-s}}{s^2}$$

here $S = 2$

$$F(S) = \frac{F_1(s)}{1-e^{-Ts}} = \frac{s-1+e^{-s}}{s^2(1-e^{-2s})}$$

13) Find LT.





$$f_1(t) = r(t) - r(t-a) - u(t-a$$

$$= t - (t-a) - u(t-a)$$

$$= \frac{1}{a}t\,u(t) - \frac{1}{a}(t-a)\,u(t-a) - u$$

$$F_1(s) = \frac{1}{a}\frac{1}{s^2} - \frac{1}{a} e^{-as} \frac{1}{s^2} - e^{-as}\frac{1}{s}$$

$$F_1(s) = \frac{1}{as^2}\left(1 - e^{-as}\right) - e^{-as}\frac{1}{s}$$

$$F(s) = \frac{F_1(s)}{1 - e^{-Ts}}$$

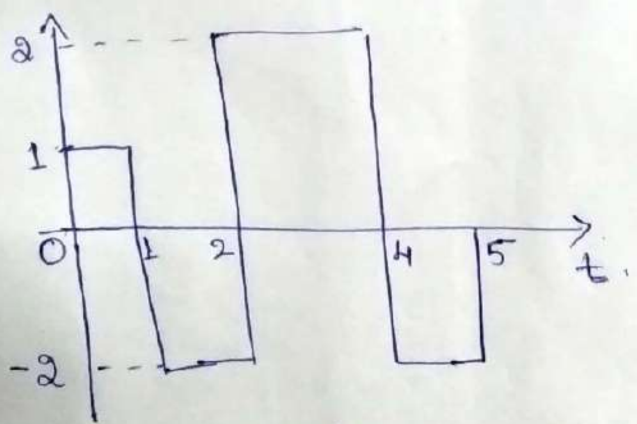
T

$$= T = a$$

$$= \frac{F_1(s)}{1 - e^{-as}}$$

$$\boxed{F(s) = \frac{1}{as^2} - e^{-as}\frac{1}{s(1 - e^{-as})}}$$

14]. Find LT.



$$f(t) = u(t) - 3u(t-1) + 4u(t-2) - 4u(t-4) - 2u(t-5)$$

$$F(s) = \frac{1}{s}\left[1 - 3e^{-s}\frac{1}{s} + 4e^{-2s}\frac{1}{s} - 4e^{-4s}\frac{1}{s} - 2e^{-5s}\frac{1}{s}\right]$$

$$\boxed{F(s) = \frac{1}{s}\left[1 - 3e^{-s} + 4e^{-2s} - 4e^{-4s} - 2e^{-5s}\right]}$$

NETWORK ANALYSIS (18EC32)

Syllabus:-

Module -5

- ❖ **Two Port Network Parameters**
- ❖ **Resonance**

Introduction.

Electrical n/w consists of passive and active elements.

To energise a passive n/w, the n/w needs to be connected to an energy source.

Two terminals are provided for the passive n/w which may be represented as a box, and to these terminals the energy source is connected.

If only one pair of terminals available for internal connections, the n/w is termed as one port n/w.

If 2 pairs of terminals are available, → 2 port n/w.

one of them is called → i/p port.
o/p port.

The vg & I at the 2 ports are interrelated and these relationships are Expressed in terms of n/w parameters.

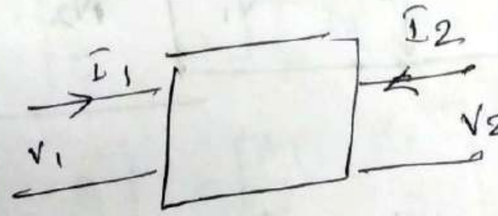


Fig. 7.1(a)

7.3 Open-circuit Impedance Parameters (z parameters):

The defining equations for these parameters are:

$$z_{11} I_1 + z_{12} I_2 = V_1 \quad \text{.... (7.7)}$$

$$z_{21} I_1 + z_{22} I_2 = V_2 \quad \text{.... (7.8)}$$

By putting $I_1 = 0$ or $I_2 = 0$ in the above equations, we get

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad \text{.... (7.9)} \qquad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \quad \text{.... (7.11)}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \quad \text{.... (7.10)} \qquad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad \text{.... (7.12)}$$

z_{11}, z_{12}, z_{21} and z_{22} are called *open-circuit impedance parameters*, as they are obtained by putting $I_1 = 0$ or $I_2 = 0$ i.e. by open-circuiting the two ports alternately.

For reciprocal or bilateral networks, $z_{12} = z_{21}$.

The equivalent network of a two-port network in terms of z parameters is as shown in Fig.7.1(b).

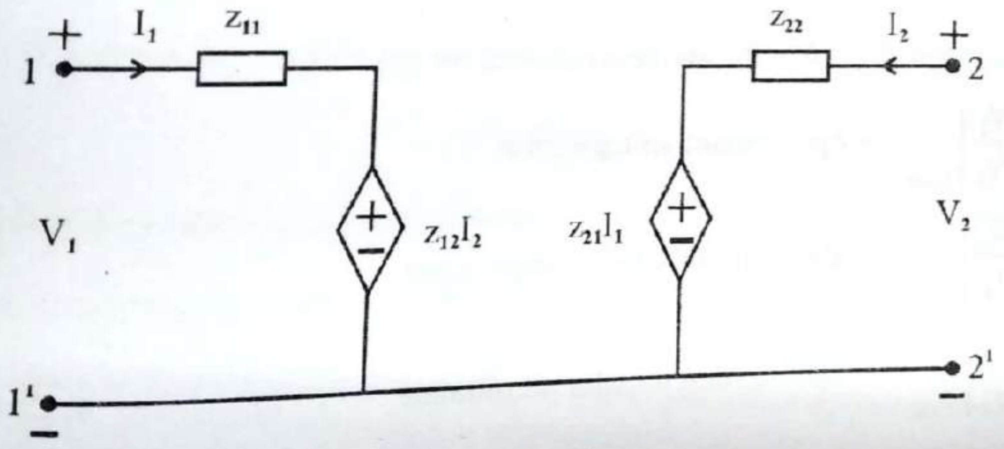


Fig. 7.1(b)

$I = EY$

2). Short circuited admittance parameters.

I_1, I_2 dependent, $V_1\ V_2 \rightarrow$ independent

$I_1, I_2 \rightarrow$ dependent.

$I_1 = f_1(\underbrace{V_1, V_2}_{in})$

$I_2 = f_2(V_1, V_2)$

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} \Rightarrow$ i/p admittance with o/p port shorted

$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} \Rightarrow$ Transfer admittance with i/p port short circuited.

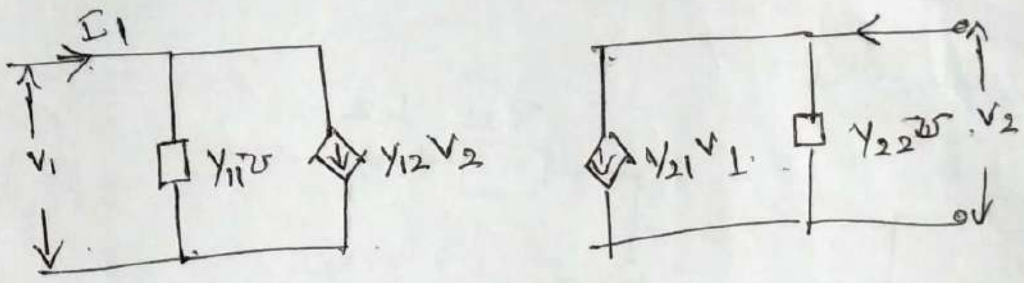
$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} \Rightarrow$ Transfer admittance with o/p port short circuited.

$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} \Rightarrow$ output admittance with i/p port short circuited.

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$I_1 = Y_{11}V_1 + Y_{12}V_2$ · Tru
$I_2 = Y_{21}V_1 + Y_{22}V_2$ —
· Gi
tr

where $\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = Y_{11}Y_{22} - Y_{12}Y_{21} = \Delta y$.



Hybrid parameters (h parameters) i/p current o/p vg

dependent V_1, I_2 independent - I_1, V_2

i/p vg, o/p current. p

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$ → i/p impedence with o/p port short ckted

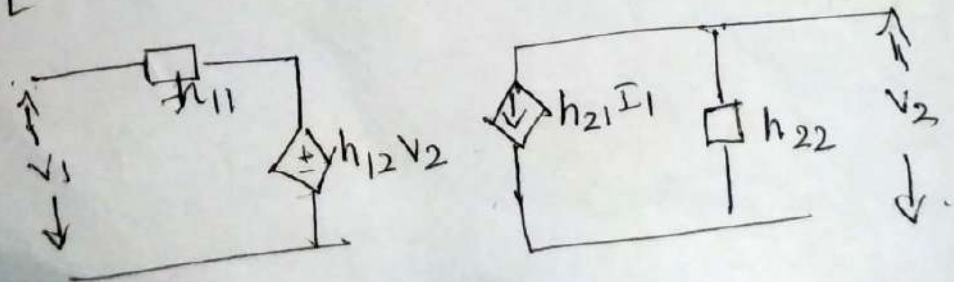
$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$ → Reverse Vg gain with i/p port open ckted.

$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$ ⇒ Forward current gain with o/p port short ckted.

$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$ ⇒ o/p admittance with i/p port open ckted.

$\Delta = h_{11}h_{22} - h_{21}h_{12}$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$



Transmission parameters (T).

Gives relation b/w Vg & current at one port to the Vg & current at the other port.

V_1, I_1 , $V_2\ I_2$
dependent independ.

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$$A = \frac{V_1}{V_2}\Big|_{I_2=0}$$

$$\frac{1}{A} = \frac{V_2}{V_1}\Big|_{I_2=0}$$ ⟹ Forward vg gain with o/p port open circuited.

$$-B = \frac{V_1}{I_2}\Big|_{V_2=0}$$

$$-\frac{1}{B} = \frac{I_2}{V_1}\Big|_{V_2=0}$$ ⟶ Transfer Conductance with o/p port short ckted.

$$C = \frac{I_1}{V_2}\Big|_{I_2=0}$$

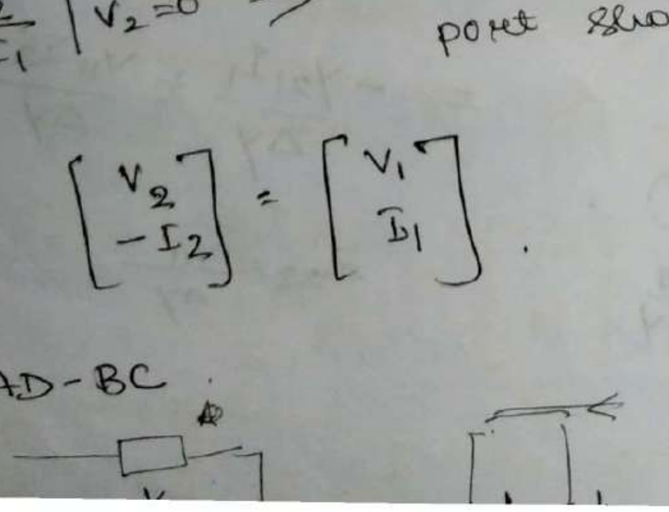
$$\frac{1}{C} = \frac{V_2}{I_1}\Big|_{I_2=0}$$ ⟹ Transfer impedence with o/p port open circuited.

$$-D = \frac{I_1}{I_2}\Big|_{V_2=0}$$

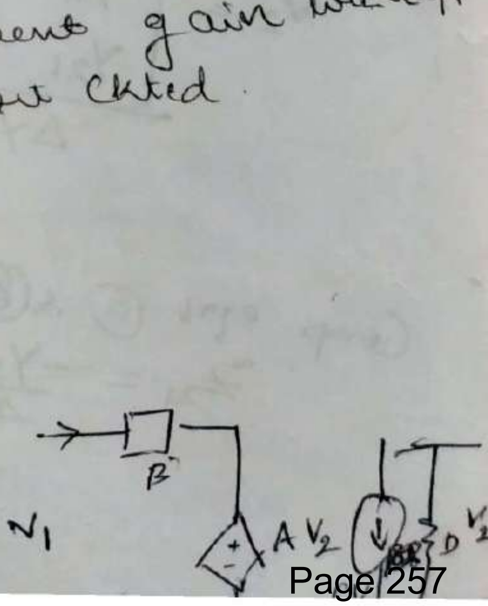
$$-\frac{1}{D} = \frac{I_2}{I_1}\Big|_{V_2=0}$$ ⟹ Forward current gain with o/p port short ckted.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}.$$

$$\Delta T = AD - BC$$



ckt ?



Relationship b/w Z and Y parameters:

Z parameters
$$\begin{cases} V_1 = Z_{11} I_1 + Z_{12} I_2 & \text{--- (1)} \\ V_2 = Z_{21} I_1 + Z_{22} I_2 & \text{--- (2)} \end{cases}$$

Y parameters
$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{--- (3)}$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{--- (4)}$$

from (3) & (4), express V_1 & compare with (1) & (2)

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}.$$

$$V_1 = \frac{\begin{bmatrix} I_1 & Y_{12} \\ I_2 & Y_{22} \end{bmatrix}}{\Delta y} = \frac{Y_{22} I_1 - Y_{12} I_2}{\Delta y}.$$

$$V_1 = \frac{Y_{22} I_1}{\Delta y} - \frac{Y_{12} I_2}{\Delta Y} \quad \text{--- (5)}$$

Compare (1) & (5)

$$Z_{11} = \frac{Y_{22}}{\Delta Y} \qquad Z_{12} = -\frac{Y_{12}}{\Delta Y}.$$

$$V_2 = \frac{\begin{bmatrix} Y_{11} & I_1 \\ Y_{21} & I_2 \end{bmatrix}}{\Delta Y} = \frac{Y_{11} I_2}{\Delta Y} - \frac{Y_{21} I_1}{\Delta Y}.$$

$$= -\frac{Y_{21} I_1}{\Delta Y} + \frac{Y_{11} I_2}{\Delta Y} \quad \text{--- (6)}.$$

Comp eqn (2) & (6).

$$Z_{21} = \frac{-Y_{21}}{\Delta Y} \qquad Z_{22} = \frac{Y_{11}}{\Delta Y}$$

Relation b/w Z and h parameter.

V_1, I_2, I_1, V_2.

h parameter
$$\begin{cases} V_1 = h_{11} I_1 + h_{12} V_2 & \text{--- } \cancel{(1)} \text{ --- } (3) \\ I_2 = h_{21} I_1 + h_{22} V_2 & \text{--- } \cancel{(2)} \text{ --- } (4) \end{cases}$$

&

Z parameters
$$\begin{cases} V_1 = Z_{11} I_1 + Z_{12} I_2 & \text{--- } \cancel{(3)} \text{ --- } (1) \\ V_2 = Z_{21} I_1 + Z_{22} I_2 & \text{--- } \cancel{(4)} \text{ --- } (2) \end{cases}$$

from (4).

$$h_{22} V_2 = I_2 - h_{21} I_1$$

$$V_2 = \frac{I_2}{h_{22}} - \frac{h_{21}}{h_{22}} I_1$$

$$V_2 = -\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2 \text{ --- (5)}$$

Compare (2) & (5).

$$Z_{21} = -\frac{h_{21}}{h_{22}}, \quad Z_{22} = \frac{1}{h_{22}}$$

Subst. eqn (5) in (3).

$$V_1 = h_{11} I_1 + h_{12}\left(-\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2\right)$$

$$= I_1\left(h_{11} - \frac{h_{12} h_{21}}{h_{22}}\right) + \frac{h_{12}}{h_{22}} I_2 \text{ --- (6)}$$

Compare eqn (6) & (1)

$$Z_{11} = h_{11} - \frac{h_{12} h_{21}}{h_{22}} = \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}} = \frac{\Delta h}{h_{22}}$$

$$Z_{12} = \frac{h_{12}}{h_{22}}$$

Relation b/w Z and T parameters :-

Z $\quad V_1 = Z_{11} I_1 + Z_{12} I_2$ —— (1)

$V_2 = Z_{21} I_1 + Z_{22} I_2$ —— (2).

T parameter.

$V_1 = A V_2 - B I_2$ —— (3)

$I_1 = C V_2 - D I_2$ —— (4).

from eqn (4).

$C V_2 = I_1 + D I_2$.

$V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2$ —— (5).

Compare (2) & (5)

$Z_{21} = \frac{1}{C} \qquad Z_{22} = \frac{D}{C}$.

Subst (5) in eqn (3).

$V_1 = A\left(\frac{1}{C} I_1 + \frac{D}{C} I_2\right) - B I_2$.

$V_1 = \frac{A}{C} I_1 + I_2\left(\frac{AD}{C} - B\right)$ —— (6).

Compare (1) & (6)

$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$

$Z_{11} = \frac{A}{C} \qquad Z_{12} = \frac{AD}{C} - B$

$= \frac{AD - BC}{C}$

$= \frac{\Delta T}{C}$

Summary:

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{Y_{22}}{\Delta Y} & \frac{-Y_{12}}{\Delta Y} \\ \frac{-Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{bmatrix} = \begin{bmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix} = \begin{bmatrix} \frac{A}{C} & \frac{\Delta T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$$

(5)

Relation b/w Y parameters & other type of parameters.

Y parameters $\rightarrow$ Z
h
T.

Z par $V_1 = Z_{11} I_1 + Z_{12} I_2$ —— (1) $V_1, V_2 \rightarrow$ depend
$V_2 = Z_{21} I_1 + Z_{22} I_2$ —— (2) $I_1, I_2 \rightarrow$ independent

Y $I_1 = Y_{11} V_1 + Y_{12} V_2$ —— (3) $I_1, I_2 \rightarrow$ depend
$I_2 = Y_{21} V_1 + Y_{22} V_2$ —— (4) $V_1, V_2 \rightarrow$ independ

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad \Delta Z = Z_{11} Z_{22} - Z_{21} Z_{12}$$

$$I_1 = \frac{\begin{bmatrix} V_1 & Z_{12} \\ V_2 & Z_{22} \end{bmatrix}}{\Delta} = \frac{V_1 Z_{22} - V_2 Z_{12}}{\Delta Z}$$

$$I_1 = \frac{Z_{22}}{\Delta Z} V_1 - \frac{Z_{12} V_2}{\Delta Z} \quad \text{—— (5)}$$

Comparing (3) & (5).

$$Y_{11} = \frac{Z_{22}}{\Delta Z}, \quad Y_{12} = -\frac{Z_{12}}{\Delta Z}.$$

$$I_2 = \frac{\begin{bmatrix} Z_{11} & V_1 \\ Z_{21} & V_2 \end{bmatrix}}{\Delta Z} = \frac{Z_{11} V_2 - Z_{21} V_1}{\Delta Z}.$$

$$I_2 = \frac{Z_{11}}{\Delta Z} V_2 - \frac{Z_{21}}{\Delta Z} V_1.$$

$$= -\frac{Z_{21}}{\Delta Z} V_1 + \frac{Z_{11}}{\Delta Z} V_2$$

$\therefore$

$$Y_{21} = -\frac{Z_{21}}{\Delta Z}, \quad Y_{22} = \frac{Z_{11}}{\Delta Z}$$

Y and h parameter.

Y P.
$$\begin{cases} I_1 = Y_{11} V_1 + Y_{12} V_2 & \text{--- (1)} \\ I_2 = Y_{21} V_1 + Y_{22} V_2 & \text{--- (2)} \end{cases}$$

h par
$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{--- (3)}$$
$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{--- (4)}$$

from (3)
$$h_{11} I_1 = V_1 - h_{12} V_2$$
$$I_1 = \frac{1}{h_{11}} V_1 - \frac{h_{12}}{h_{11}} V_2 \quad \text{--- (5)}$$

Compare (1) & (5)
$$Y_{11} = \frac{1}{h_{11}} \qquad Y_{12} = -\frac{h_{12}}{h_{11}}$$

subs (5) in (4)
$$I_2 = h_{21}\left(\frac{1}{h_{11}} V_1 - \frac{h_{12}}{h_{11}} V_2\right) + h_{22} V_2$$
$$= \frac{h_{21}}{h_{11}} V_1 + \left(-\frac{h_{21} h_{12}}{h_{11}} + h_{22}\right) V_2$$
$$I_2 = \frac{h_{21}}{h_{11}} V_1 + \left(h_{22} - \frac{h_{21} h_{12}}{h_{11}}\right) V_2 \quad \text{--- (6)}$$

Compare (2) & (6).
$$Y_{21} = \frac{h_{21}}{h_{11}} \qquad Y_{22} = \left(\frac{h_{11} h_{22} - h_{21} h_{12}}{h_{11}}\right) V_2$$
$$Y_{21} = \frac{h_{21}}{h_{11}} \qquad Y_{22} = \frac{\Delta h}{h_{11}}$$

Y par

12

6

Y in terms of T parameters

Y par
$$\begin{cases} I_1 = Y_{11}V_1 + Y_{12}V_2 & \text{--- (1)} \\ I_2 = Y_{21}V_1 + Y_{22}V_2 & \text{--- (2)} \end{cases}$$

T parameters
$$\begin{cases} V_1 = AV_2 - BI_2 & \text{--- (3)} \\ I_1 = CV_2 - DI_2 & \text{--- (4)} \end{cases}$$

Rewrite eqn (3).

$$BI_2 = AV_2 - V_1$$

$$I_2 = \frac{A}{B}V_2 - \frac{1}{B}V_1$$

$$I_2 = -\frac{1}{B}V_1 + \frac{A}{B}V_2 \quad \text{--- (5)}$$

$$\therefore \boxed{Y_{21} = -\frac{1}{B}} \quad \boxed{Y_{22} = \frac{A}{B}}$$

subs (5) in (4).

$$I_1 = CV_2 - D\left(-\frac{1}{B}V_1 + \frac{A}{B}V_2\right)$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = AD-BC$$

$$= \left(C - \frac{AD}{B}\right)V_2 + \frac{D}{B}V_1$$

$$= \left(\frac{BC-AD}{B}\right)V_2 + \frac{D}{B}V_1$$

$$= -\left(\frac{AD-BC}{B}\right)V_2 + \frac{D}{B}V_1$$

$$I_1 = \frac{D}{B}V_1 - \left(\frac{\Delta T}{B}\right)V_2$$

$$\therefore \boxed{Y_{11} = \frac{D}{B}} \quad \boxed{Y_{12} = -\frac{\Delta T}{B}}$$

Relation b/w h.parameters & other 4 parameters

$$h \rightarrow \begin{matrix} z \\ y \\ T \end{matrix}$$

$z \quad \begin{cases} v_1 = z_{11} I_1 + z_{12} I_2 & \text{--- (1)} \\ v_2 = z_{21} I_1 + z_{22} I_2 & \text{--- (2)} \end{cases}$

$h \quad \begin{cases} v_1 = h_{11} I_1 + h_{12} V_2 & \text{--- (3)} \\ I_2 = h_{21} I_1 + h_{22} V_2 & \text{--- (4)} \end{cases}$

rewriting eqn (2)

$$z_{22} I_2 = V_2 - z_{21} I_1$$

$$I_2 = \frac{1}{z_{22}} V_2 - \frac{z_{21}}{z_{22}} I_1$$

$$I_2 = -\frac{z_{21}}{z_{22}} I_1 + \frac{1}{z_{22}} V_2 \quad \text{--- (5)}$$

comp (4) & (5)

$$\boxed{h_{21} = -\frac{z_{21}}{z_{22}}} \quad \boxed{h_{22} = \frac{1}{z_{22}}}$$

sub eqn (5) in (1)

$$V_1 = z_{11} I_1 + z_{12}\left(-\frac{z_{21}}{z_{22}} I_1 + \frac{1}{z_{22}} V_2\right)$$

$$= \left(z_{11} - \frac{z_{21} z_{12}}{z_{22}}\right) I_1 + \frac{z_{12}}{z_{22}} V_2$$

$$V_1 = \left(\frac{z_{11} z_{22} - z_{21} z_{12}}{z_{22}}\right) I_1 + \frac{z_{12}}{z_{22}} V_2 \quad \text{--- (6)}$$

$$V_1 = \left(\frac{\Delta z}{z_{22}}\right) I_1 + \frac{z_{12}}{z_{22}} V_2$$

$$\boxed{h_{11} = \frac{\Delta z}{z_{22}}} \quad \boxed{h_{12} = \frac{z_{12}}{z_{22}}}$$

3

Relation b/w $T \longrightarrow \left.\begin{matrix} z \\ y \\ h \end{matrix}\right\}$

$h \rightarrow y$.

$$y \begin{cases} I_1 = Y_{11}V_1 + Y_{12}V_2 & \text{—— (1)} \\ I_2 = Y_{21}V_1 + Y_{22}V_2 & \text{—— (2)} \end{cases}$$

$$h \begin{cases} V_1 = h_{11}I_1 + h_{12}V_2 & \text{—— (3)} \\ I_2 = h_{21}I_1 + h_{22}V_2 & \text{—— (4)} \end{cases}$$

Rewriting eqn (1)

$$Y_{11}V_1 = I_1 - Y_{12}V_2,$$

$$V_1 = \frac{1}{Y_{11}} I_1 - \frac{Y_{12}}{Y_{11}} V_2 \quad \text{—— (5)}$$

compare (3) & (5)

$$\boxed{h_{11} = \frac{1}{Y_{11}}} \qquad \boxed{h_{12} = \frac{-Y_{12}}{Y_{11}}}$$

Subst. (5) in (2).

$$I_2 = Y_{21}\left(\frac{1}{Y_{11}} I_1 - \frac{Y_{12}}{Y_{11}} V_2\right) + Y_{22}V_2.$$

$$I_2 = \frac{Y_{21}}{Y_{11}} I_1 + \left(Y_{22} - \frac{Y_{21}Y_{12}}{Y_{11}}\right) V_2 \quad \text{—— (6)}$$

compare (4) & (6).

$$h_{21} = \frac{Y_{21}}{Y_{11}} \qquad h_{22} = \frac{\Delta Y}{Y_{11}}$$

h and T

$$h \begin{cases} V_1 = h_{11} I_1 + h_{12} V_2 & \text{(1)} \\ I_2 = h_{21} I_1 + h_{22} V_2 & \text{(2)} \end{cases}$$

$$T \begin{cases} V_1 = A V_2 - B I_2 & \text{(3)} \\ I_1 = C V_2 - D I_2 & \text{(4)} \end{cases}$$

~~Rewriting eqn (3)~~

$$\cancel{B I_2 = V_1 - A V_2}$$

$$\cancel{I_2 = \frac{1}{B} V_1 - \frac{A}{B} V_2} \quad \text{(5)}$$

Rewrit eqn (4)

$$D I_2 = C V_2 - I_1$$

$$I_2 = \frac{C}{D} V_2 - \frac{1}{D} I_1$$

$$I_2 = -\frac{1}{D} I_1 + \frac{C}{D} V_2 \quad \text{(5)}$$

Compare (2) & (5)

$$h_{21} = -\frac{1}{D} \qquad h_{22} = \frac{C}{D}$$

Sub eqn (5) in (3)

$$V_1 = A V_2 - B \left(-\frac{1}{D} I_1 + \frac{C}{D} V_2 \right)$$

$$V_1 = \left(A - \frac{BC}{D} \right) V_2 + \frac{B}{D} I_1$$

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = AD - BC$$

$$V_1 = \frac{B}{D} I_1 + \left(\frac{AD - BC}{D} \right) V_2$$

$$V_1 = \frac{B}{D} I_1 + \frac{\Delta T}{D} V_2 \quad \text{(6)}$$

(1) & (6):

$$h_{11} = \frac{B}{D} \qquad h_{12} = \frac{\Delta T}{D}$$

~~Series~~ Connection of two ports!

cascade connection of two port networks.

(show that resultant ABCD matrix of cascade connection is the product of individual ABCD matrix).

The transmission parameters A, B, C and D are useful in describing two-port n/ws which are connected in cascade.

in cascade

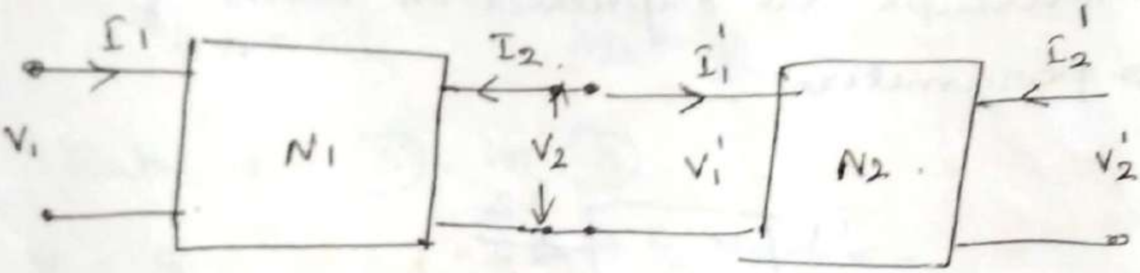


Fig shows 2 port n/ws connected in cascade. In the cascade connection the o/p port of the first network becomes the i/p port of the second n/w.

Here $I_1' = -I_2$.

Let A_1, B_1, C_1, D_1 be the transmission parameters of n/w N_1

III^ly A_2, B_2, C_2 and D_2 be the transmission parameters of the n/w N_2

W.K.T ABCD parameters are given as

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

Expressing this in matrix form for N_1

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \quad ? \quad \text{—(1)}$$

||ly

$$\begin{bmatrix} V_1' \\ I_1' \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix} \quad \text{—(2)}$$

Since $V_1' = V_2$ & $I_1' = -I_2$.

$$\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix} \quad \text{—(3)}$$

put (3) in (1).

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix}.$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2' \\ I_2' \end{bmatrix}$$

hence $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$.

This eqn shows that the resultant ABCD matrix of cascade connection is the product of individual ABCD matrices.

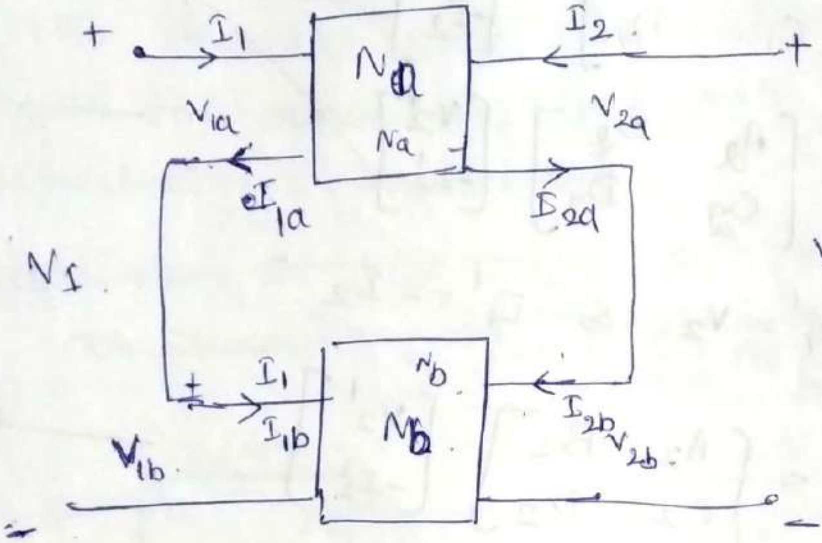
Two ports are said to be connected in cascade, if o/p port of one is i/p port for the second.

This connection is also called as <u>Tandem-Connection</u>

This can be conveniently studied by ABCD parameters.

Series Connection of 2 ports:

Two two port networks N_a and N_b are said to be connected in series if corresponding ports are connected in series



$V = I2$

In this connection, the i/p & o/p currents at the corresponding ports are forced to be the same. The overall port Vgs are equal to the sum of the corresponding port Vgs of the individual 2 ports

$V_1 = V_{1a} + V_{1b}$

$V_2 = V_{2a} + V_{2b}$

$= \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} + \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} + \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$$

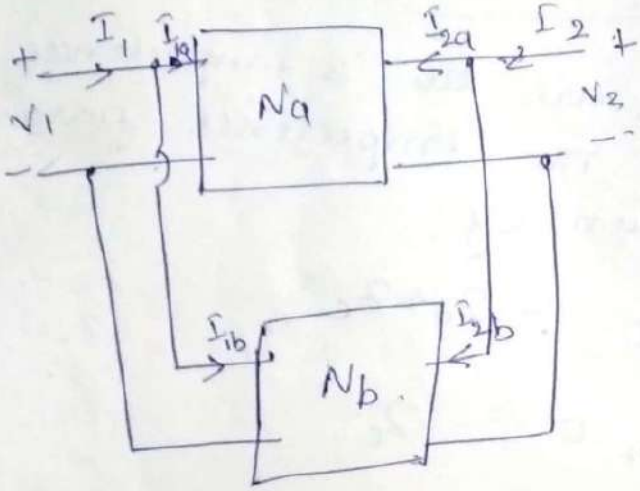
$$= \begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{21a} & Z_{22a} \end{bmatrix} \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} + \begin{bmatrix} Z_{11b} & Z_{12b} \\ Z_{21b} & Z_{22b} \end{bmatrix} \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

But $I_{1a} = I_{1b} = I_1$ & $I_{2a} = I_{2b} = I_2$.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11a}+Z_{11b} & Z_{12a}+Z_{12b} \\ Z_{21a}+Z_{21b} & Z_{22a}+Z_{22b} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

parallel Connection of 2 ports:-

Two 2 port n/ws are said to be connected in parallel, if the corresponding ports are connected in parallel as shown in fig.



In this connection the i/p & o/p vgs of the corresponding ports are forced to be the same. The overall port currents are equal to the sum of the corresponding port currents at the individual 2 ports.

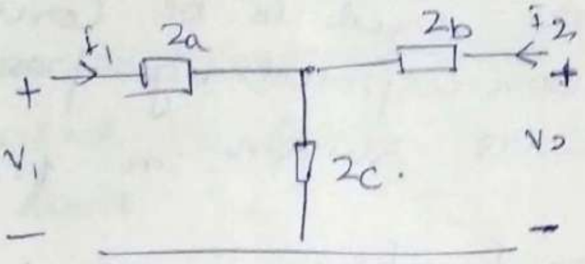
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} + \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

$$= \begin{bmatrix} Y_{11a} & Y_{12a} \\ Y_{21a} & Y_{22a} \end{bmatrix} \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} + \begin{bmatrix} Y_{11b} & Y_{12b} \\ Y_{21b} & Y_{22b} \end{bmatrix} \begin{bmatrix} V_{a1b} \\ V_{2b} \end{bmatrix}$$

But $V_{1a} = V_{1b} = V_1$ &
$V_{2a} = V_{2b} = V_2$.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11a} + Y_{11b} & Y_{12a} + Y_{12b} \\ Y_{21a} + Y_{21b} & Y_{22a} + Y_{2b} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

T section repn of 2 port n/w.



$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

Z_a, Z_b & Z_c are the 3 impedences Connected as a T n/w. The impedence parameters for the n/w are given by.

$Z_a I_1 + Z_c (I_1 + I_2) = V_1$

$Z_a I_1 + Z_c I_1 = V_1$

$Z_a I_1 + Z_{12} I_1 = V_1$

$(Z_a + Z_c) I_2 Z_c \cdot V_1$

$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} = Z_a + Z_c$$

$$Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0} = Z_c$$

$V_2 = Z_b I_2 + Z_c (I_1 + I_2)$

$V_1 = I_2 Z_c$

$$Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0} = Z_c$$

$V_2 = I_2 Z_c + I_2 Z_b$

$\frac{V_2}{I_2} =$

$$Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0} = Z_b + Z_c$$

$[T]$ T in terms of Z

$$A = \frac{Z_{11}}{Z_{21}} \qquad B = \frac{\Delta Z}{Z_{21}} \qquad \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$C = \frac{1}{Z_{21}} \qquad D = \frac{Z_{22}}{Z_{21}}$$

$$A = \frac{Z_{11}}{Z_{21}} = \frac{Z_a + Z_c}{Z_c} = 1 + \frac{Z_a}{Z_c} = 1 + Z_a Y_c.$$

$$B = \frac{\Delta Z}{Z_{21}} = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}} = \frac{(Z_a + Z_c)(Z_b + Z_c) - Z_c^2}{Z_c}$$

$$\frac{Z_a Z_b + Z_c Z_b + Z_a Z_c + Z_c^2 - Z_c^2}{Z_c}$$

$$= Z_a + Z_b + \frac{Z_a Z_b}{Z_c}$$

$$= Z_a + Z_b + Z_a Z_b Y_c.$$

$$C = \frac{1}{Z_{12}} = \frac{1}{Z_c} = Y_c.$$

$$D = \frac{Z_{22}}{Z_{21}} = \frac{Z_b + Z_c}{Z_c} = 1 + \frac{Z_b}{Z_c} = 1 + Z_b Y_c.$$

(12)

Conversely

$A = 1 + Z_a Y_c \Rightarrow A - 1 = Z_a Y_c = Z_a C$

$\therefore C = \frac{1}{Z_{12}} = Y$

$\therefore Z_a = \frac{A-1}{C}$

$D = 1 + Z_b Y_c$. $\quad D - 1 = Z_b Y_c = Z_b C$.

$\therefore Z_b = \frac{D-1}{C}$.

& $Z_c = \frac{1}{Y_c} = C$.

To show that $AD - BC = 1$

$AD - BC = (1 + Z_a Y_c)(1 + Z_b Y_c) - (Z_a + Z_b + Z_a Z_b Y_c)$

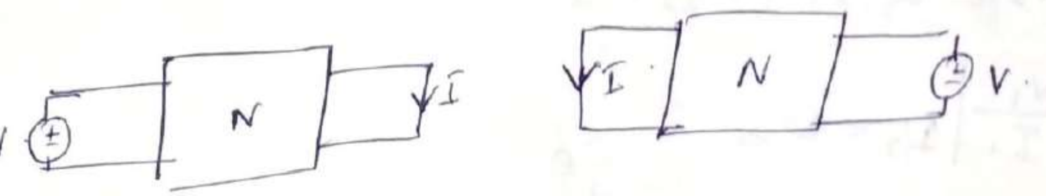
$= 1 + Z_a Y_c + Z_b Y_c - Z_a Y_c + Z_b Y_c - Z_a \cancel{Z_b Y_c^2}$
$+ \cancel{Z_a Z_b Y_c^2} Z_a Z_b Y_c^2$

$= 1$.

Reciprocal & Symmetrical n/ws:

Reciprocal n/w:-

Any 2 port n/w in which, the ratio of response to the excitation remains constant, when the positions of excitation & response are interchanged ~~is~~ such n/w is called a reciprocal n/w.



$Z_{12} = Z_{21}$
$Y_{12} = Y_{21}$.

Symmetrical n/w:

A 2 port n/w is said to be symmetrical if n/w characteristics are not changed whenever the 2 ports are interchanged.

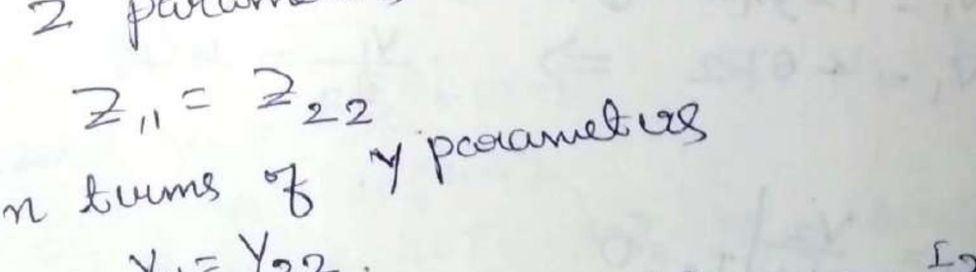
The condition for symmetry in terms of Z parameters

$Z_{11} = Z_{22}$

in terms of Y parameters

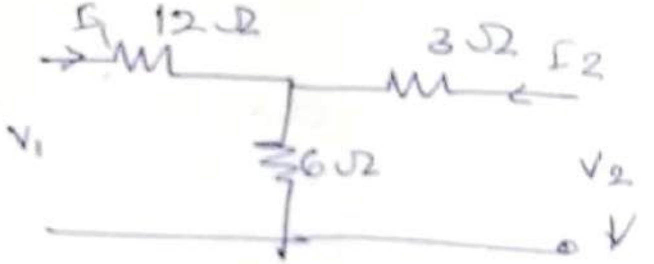
$Y_{11} = Y_{22}$.

ex:



if impedance measured at one port is equal to the impedance measured at the other port with the remaining port open ckted.

1. Find the Z parameters for the ckt & D



$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$

$V_1 = 12 I_1 + 6(I_1 + \cancel{I_2}^{\,0})$

$V_1 = 18 I_1 \Rightarrow \frac{V_1}{I_1} = 18\,\Omega$

$\therefore \boxed{Z_{11} = 18\,\Omega}$

$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$

$V_1 = 12\cancel{I_1} + 6(\cancel{I_1} + I_2)$

$V_1 = +6 I_2 \Rightarrow \frac{V_1}{I_2} = 6\,\Omega$

$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$

$V_2 = 6(I_1 + \cancel{I_2}) \Rightarrow \frac{V_2}{I_1} = 6\,\Omega$

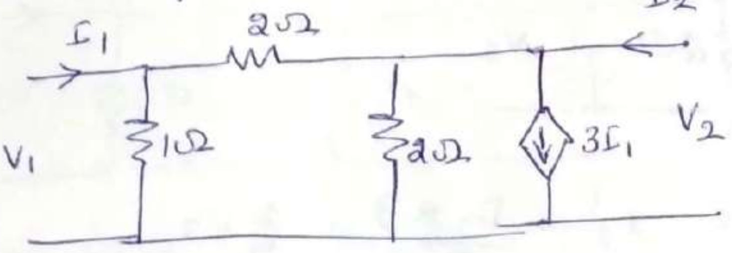
$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$

$V_2 = 3I_2 + 6(\cancel{I_1} + I_2)$

$V_2 = 9 I_2 \Rightarrow \frac{V_2}{I_2} = 9\,\Omega$

Z parameters :

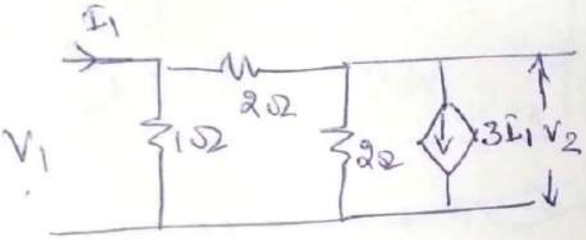
1) The n/w shown in fig contains controlled current source. Find Z parameters.



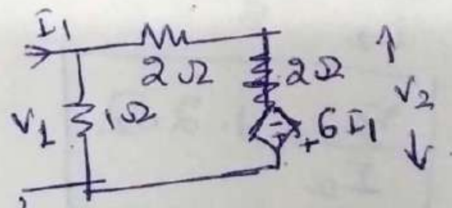
$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

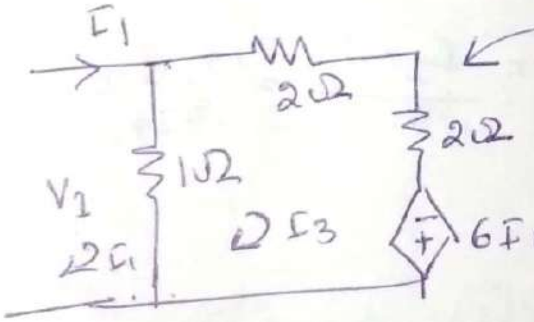
$$V_2 = Z_{21} I_1 + Z_{22} I_2 .$$

$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} \quad \& \quad Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0} .$$



$\Rightarrow$





$$V_1 = 1(I_1 - I_3) \quad \& \quad 4I_3 - 6I_1 + I_3 - I_1 = 0.$$

$$5I_3 = 7I_1$$

$$I_3 = \frac{7}{5} I_1$$

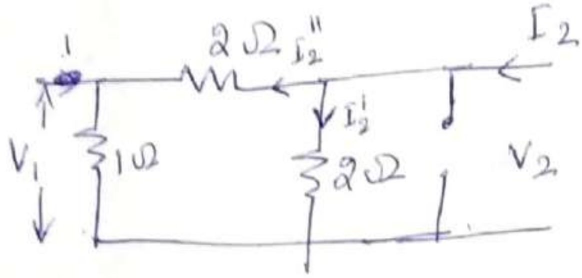
$$V_1 = I_1 - \frac{7}{5} I_1$$

$$V_1 = -\frac{2}{5} I_1 .$$

$$\frac{V_1}{I_2} = -\frac{2}{5} = -0.4\,\Omega$$

$$\boxed{\frac{V_1}{I_2} = -0.4\,\Omega}$$

6 $Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$ & $Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$



$V_2 = 2 I_2'$ & $I_2' = \frac{I_2 \times 3}{5} = \frac{3}{5} I_2$

$= 2 \frac{3}{5} I_2$

$V_2 = \frac{6}{5} I_2$

$\frac{V_2}{I_2} = \frac{6}{5} = 1.2$

$$\boxed{\frac{V_2}{I_2} = 1.2\,\Omega}$$

$V_1 = \cancel{1} I_2''$ $\qquad$ $I_2'' = \frac{I_2 \times 2}{5} = \frac{2}{5} I_2$

$V_1 = \frac{2}{5} I_2$

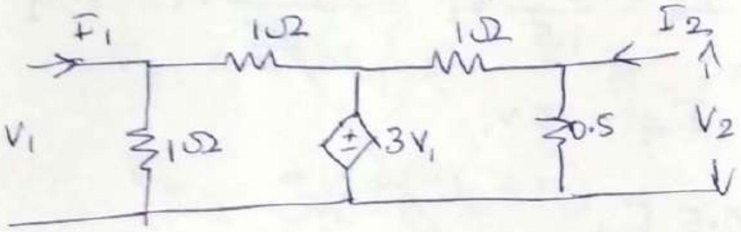
$\frac{V_1}{I_2} = \frac{2}{5}$

$$\boxed{\frac{V_1}{I_2} = 0.4\,\Omega}$$

(2)

Find Z parameters for the n/w shown which Contains a controlled Vg Source.

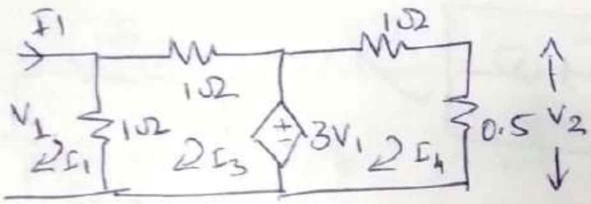
31



$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} \qquad Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0}.$$



$\cancel{V_1 = (I_1 - I_3)} + I_3 \cancel{+ 3V_1 = 0}.$

(1) loop $\quad V_1 = I_1 - I_3. \qquad$ (1)

(2) loop $\quad -I_1 + I_3 + I_3 + 3V_1 = 0 \Rightarrow -I_1 + 2I_3 + 3V_1 = 0.$

$$2I_3 = -3V_1 + I_1$$

$$I_3 = -\frac{3}{2}V_1 + \frac{1}{2}I_1 \quad (2)$$

sub (2) in (1)

$$V_1 = I_1 - \left(-\frac{3}{2}V_1 + \frac{1}{2}I_1\right)$$

$$V_1 = I_1 + \frac{3}{2}V_1 - \frac{1}{2}I_1$$

$$V_1 - \frac{3}{2}V_1 = \frac{1}{2}I_1$$

$$-\frac{1}{\cancel{2}}V_1 = \frac{1}{\cancel{2}}I_1$$

$$\boxed{\frac{V_1}{I_1} = -1\Omega}$$

3^{rd} loop.

32

$$1.5\, I_4 - 3V_1 = 0.$$

$$1.5\, I_4 = 3V_1.$$

$$I_4 = \frac{3}{1.5} V_1 = \frac{\cancel{3}}{\frac{\cancel{3}}{2}} V_1 = 2V_1$$

$$I_4 = 2V_1$$

$$V_2 = 0.5\, I_4$$

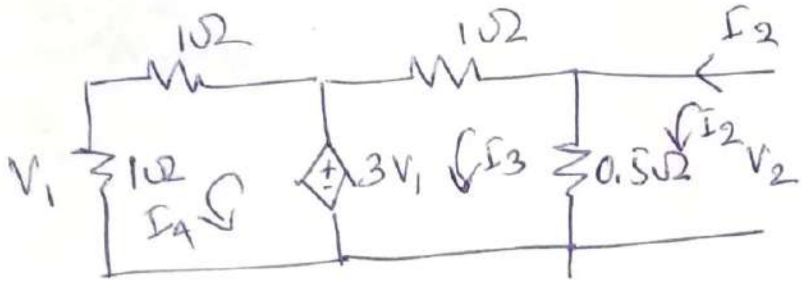
$$V_2 = 0.5\,(2V_1)$$

$$V_2 = V_1. \quad \& \quad V_1 = -I_1.$$

$$V_2 = -I_1$$

$$\boxed{\frac{V_2}{I_1} = -1\,\Omega}$$

$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$ & $Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$.



$V_1 = I_4$.

$2I_4 - 3V_1 = 0 \Rightarrow 2I_4 - 3I_4 = 0 \Rightarrow I_4 = 0$.

$\therefore I_4 = V_1 = 0$.

$I_3 + 3V_1 + 0.5(I_3 - I_2) = 0$.

$I_3 + \cancel{3V_1} + 0.5I_3 - 0.5I_2 = 0$.

$1.5I_3 = 0.5I_2 \Rightarrow I_3 = \frac{0.5}{1.50}I_2$

$I_3 = \frac{1/\cancel{2}}{3/\cancel{2}}I_2$

$I_3 = \frac{1}{3}I_2$.

$$\boxed{Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} = \frac{I_4}{I_2} = 0}$$

$$V_2 = 0.5\,(I_2 - I_3)$$

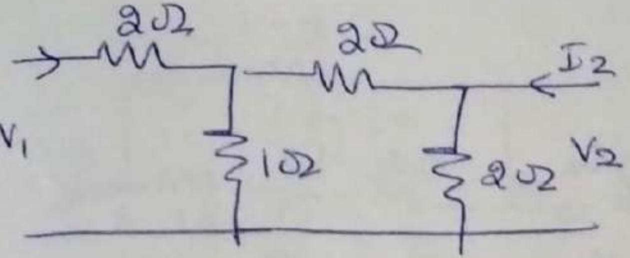
8 $$V_2 = 0.5\left[I_2 - \frac{1}{3} I_2\right]$$

$$V_2 = \cancel{0.5}\,\frac{1}{\cancel{2}}\left[\frac{\cancel{2}}{3} I_2\right]$$

$$\boxed{\frac{V_2}{I_2} = \frac{1}{3}\,\Omega}$$

k them solve:

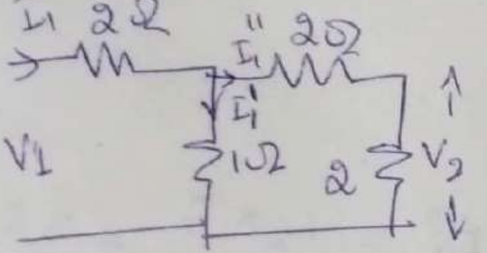
Determine the Z parameters of ckt shown. (4)



29

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$ $\qquad$ $Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$.



$V_1 = 2I_1 + I_1'$ $\quad$ & $\quad I_1' = \frac{I_1 \times 4}{5} = \frac{4I_1}{5}$

$V_1 = 2I_1 + \frac{4}{5} I_1$

$V_1 = \frac{14}{5} I_1$

$$\boxed{\begin{array}{l} \frac{V_1}{I_1} = \frac{14}{5}\,\Omega \\ \frac{V_1}{I_1} = 2.8\,\Omega \end{array}}$$

$I_1'' = \frac{I_1 \times 1}{5} = \frac{1}{5} I_1$

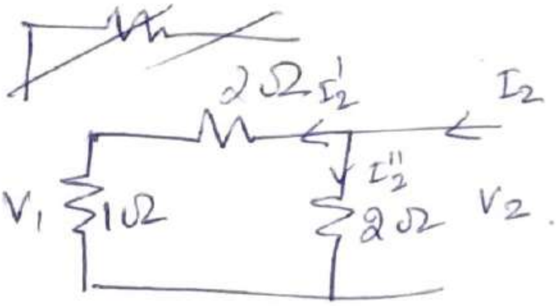
$V_2 = 2I_1''$

$V_2 = 2 \frac{1}{5} I_1 = \frac{2}{5} I_1$

$$\boxed{\frac{V_2}{I_1} = \frac{2}{5}\,\Omega}$$

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} \qquad Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}.$$

30



$$V_1 = I_2' \qquad I_2' = \frac{I_2 \times 2}{5} = \frac{2}{5} I_2$$

$$V_1 = \frac{2}{5} I_2$$

$$\boxed{\frac{V_1}{I_2} = \frac{2}{5}\,\Omega}$$

$$V_2 = 2 I_2'' \qquad \& \qquad I_2'' = \frac{I_2 \times 3}{5} = \frac{3 I_2}{5}$$

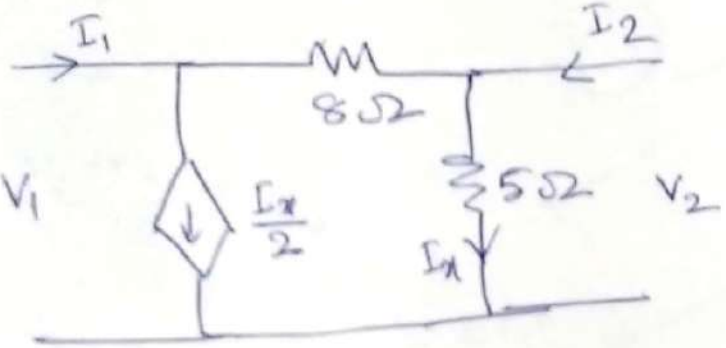
$$V_2 = 2\,\frac{3}{5} I_2$$

$$V_2 = \frac{6}{5} I_2$$

$$\boxed{\frac{V_2}{I_2} = \frac{6}{5}\,\Omega}$$

Find Z parameters for the n/w shown

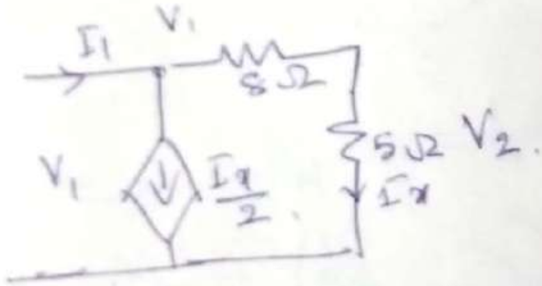
11



$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$ $\qquad Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}.$



$I_x = I_1 - \frac{I_x}{2} \Rightarrow I_x + \frac{I_x}{2} = I_1$

$\frac{3I_x}{2} = I_1$

$I_x = \frac{2}{3} I_1.$

$V_1 = (8+5)\, I_x$

$V_1 = 13\, I_x$

$V_1 = 13\left(\frac{2}{3}\right) I_1$

$\boxed{\frac{V_1}{I_1} = \frac{26}{3}\,\Omega}$ ✓

& $V_2 = 5 I_x.$

$V_2 = 5\left(\frac{2}{3}\right) I_1$

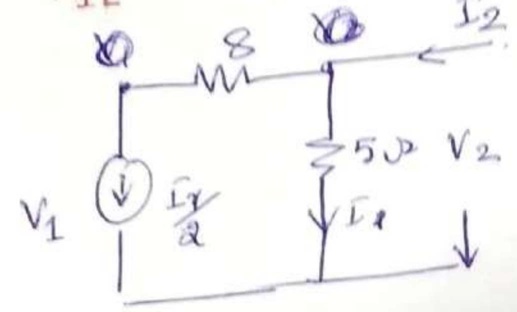
$V_2 = \frac{10}{3} I_1$

$\boxed{\frac{V_2}{I_1} = \frac{10}{3}\,\Omega}$ ✓

$$z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

$$Z_{21} = Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$$

12



$$I_x = I_2 - \frac{I_x}{2} \Rightarrow \quad I_x + \frac{I_x}{2} = I_2$$

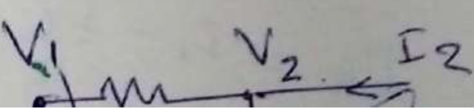
$$\frac{3I_x}{2} = I_2 \Rightarrow I_x = \frac{2}{3}I$$

$$V_2 = 5I_x$$

$$V_2 = 5\,\frac{2}{3}\,I_2$$

$$V_2 = \frac{10}{3}I_2$$

$$\boxed{\frac{V_2}{I_2} = \frac{10}{3}\,\Omega}$$



$$\frac{V_2}{I_2} = \frac{10}{3}.$$

$$V_2 = \frac{10}{3} I_2. \quad \text{——— } (1)$$

25

$$\frac{V_1 - V_2}{8} + \frac{I_x}{2} = 0.$$

$$\frac{V_1}{8} - \frac{V_2}{8} + \frac{I_x}{2} = 0.$$

$$\frac{V_1}{8} - \frac{\frac{10}{3} I_2}{8} + \frac{\frac{2}{3} I_2}{2} = 0$$

$$\frac{V_1}{8} - \frac{10}{24} I_2 + \frac{2}{6} I_2 = 0$$

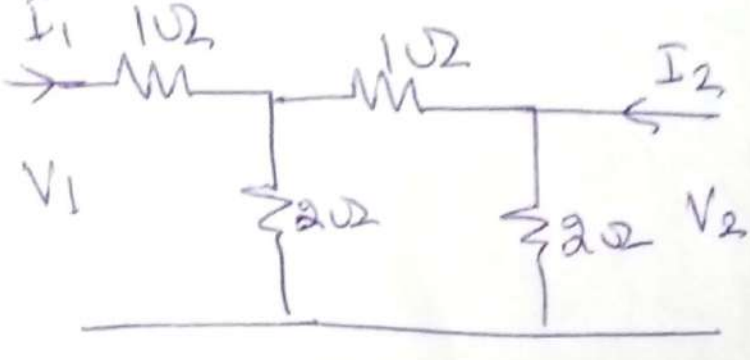
$$\frac{V_1}{8} - \frac{5}{12} I_2 + \frac{1}{3} I_2 = 0$$

$$\frac{V_1}{8} = I_2 \left[\frac{5}{12} - \frac{1}{3} \right]$$

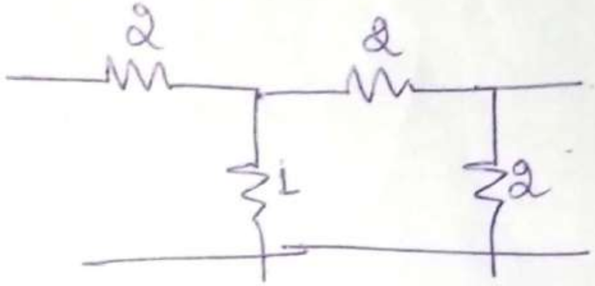
$$\frac{V_1}{8} = I_2 \left[\frac{5-4}{12} \right]$$

$$\frac{V_1}{8} = \frac{1}{12} I_2$$

$$\boxed{\frac{V_1}{I_2} = \frac{8}{12} = \frac{2}{3} \Omega}$$



ans :- $\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} \frac{11}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{6}{5} \end{bmatrix}$

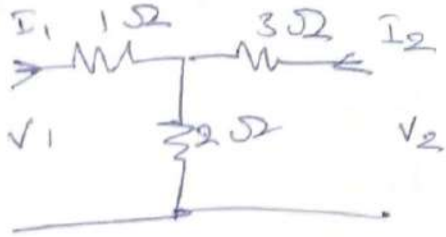


$z_{11} = \frac{14}{5}$ $z_{12} = \frac{2}{5}\Omega$

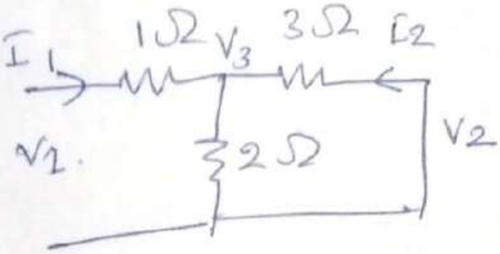
$z_{21} = \frac{2}{5}\Omega$ $z_{22} = \frac{6}{5}\Omega$

Y parameters

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$



$$Y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0}$$



$$\frac{V_3 - V_1}{1} + \frac{V_3}{2} + \frac{V_3}{3} = 0$$
$$\left(1 + \frac{1}{2} + \frac{1}{3}\right) V_3 = V_1$$
$$\left(\frac{6+3+2}{6}\right) V_3 = V_1$$
$$\frac{11}{6} V_3 = V_1$$
$$V_3 = \frac{6}{11} V_1$$

$$I_1 = \frac{V_1 - V_3}{1}$$
$$I_1 = V_1 - \frac{6}{11} V_1$$
$$I_1 = \frac{5}{11} V_1$$
$$\boxed{\frac{I_1}{V_1} = \frac{5}{11}}$$

$$Y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0}$$

$$I_2 = \frac{V_2 - V_3}{3} =$$

$$I_2 = \frac{V_2 - V_3}{3}$$
$$I_2 = \frac{0 - \frac{6}{11} V_1}{3}$$
$$\cancel{3} I_2 = -\frac{\cancel{6}\,2}{11} V_1$$
$$\boxed{\frac{I_2}{V_1} = -\frac{2}{11}\,\mho}$$

$$Y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0}$$

$$I_1 = \frac{V_1 - V_3}{1\Omega}$$

$$I_1 = -V_3$$

$$I_1 = \cancel{-\frac{2}{5}V_2}$$

$$I_1 = -\frac{2}{11}V_2$$

$$\frac{I_1}{V_2} = -\frac{2}{11}\,\mho$$

$$Y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0}$$

$$I_2 = \frac{V_2 - V_3}{3}$$

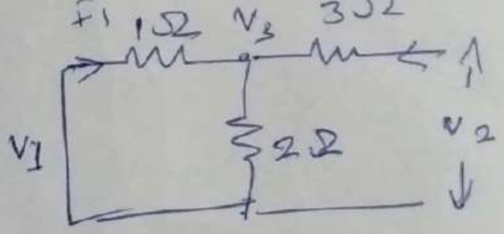
$$I_2 = \frac{V_2 - \frac{2}{11}V_2}{3} = \frac{11V_2 - 2V_2}{11\times 3} = \frac{9V_2}{33}$$

$$\cancel{3I_2 = \frac{9}{11}V_2}$$

$$\cancel{33 I_2 = 9V_2}$$

$$\cancel{\frac{I_2}{V_2} = \frac{9}{33}} = 3$$

$$Y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0}$$



$$\frac{V_3}{2} + \frac{V_3 - V_2}{3} - I_1 = 0$$

$$\left(\frac{1}{2}+\frac{1}{3}\right)V_3 = \frac{V_2}{3} + I_1$$

$$\left(\frac{3+2}{6}\right)V_3 = \frac{V_2}{3} + \frac{V_1 - V_3}{1}$$

$$\cancel{\frac{5}{2}V_3 = V_2}$$

$$\cancel{V_3 = \frac{2}{5}V_2}$$

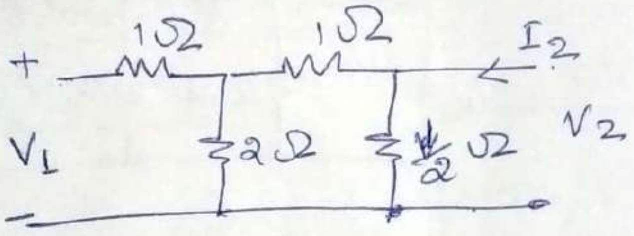
$$\left(\frac{1}{2}+\frac{1}{3}+1\right)V_3 = \frac{V_2}{3}$$

$$\left(\frac{3+2+6}{6}\right)V_3 = \frac{V_2}{3}$$

$$\frac{11}{2}V_3 = V_2$$

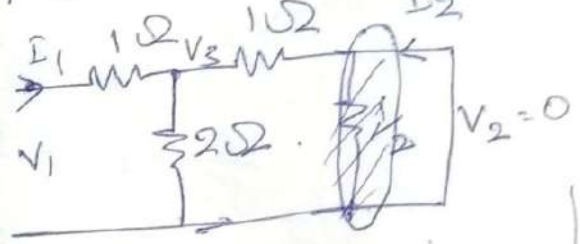
$$V_3 = \frac{2}{11}V_2$$

$$\frac{I_2}{V_2} = \frac{9}{33} = \frac{3}{11}\,\mho$$



$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$ $\quad Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$



$-I_1$
$-(V_1-V_3)$

$$I_1 = \frac{V_1 - V_3}{1}$$
$$I_1 = V_1 - \tfrac{2}{5}V_1$$
$$I_1 = \tfrac{3}{5}V_1$$
$$\frac{I_1}{V_1} = \frac{3}{5}\,\mho$$

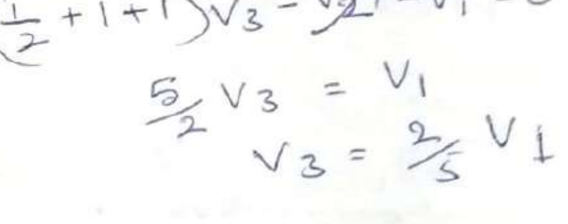
$$\frac{V_3}{2} + \frac{V_3 - V_2}{1} + \frac{V_3 - V_1}{1} = 0$$
$$\left(\tfrac{1}{2}+1+1\right)V_3 - \cancel{V_2} - V_1 = 0$$
$$\tfrac{5}{2}V_3 = V_1$$
$$V_3 = \tfrac{2}{5}V_1$$

$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$

$$I_2 = \frac{V_2 - V_3}{1} = 0 - \tfrac{2}{5}V_1$$

$$\boxed{\frac{I_2}{V_1} = -\frac{2}{5}\,\mho}$$

$\therefore Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$ $\quad$ & $\quad Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$



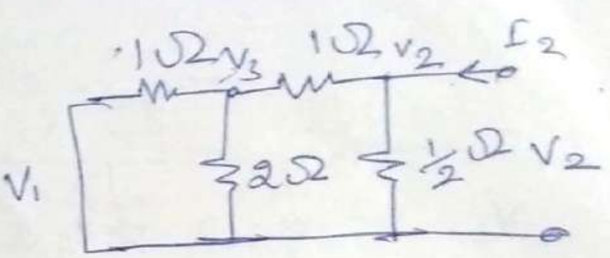
$$I_1 = \frac{\cancel{V_1} - V_3}{1}$$
$$I_1 = -\tfrac{2}{5}V_2$$
$$\frac{I_1}{V_2} = -\frac{2}{5}\,\mho$$

$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$ $\quad I_2 = V_2 - V_3$

$$\frac{V_3}{2} + \frac{V_3 - V_2}{1} + \frac{V_3 - \cancel{V_1}}{1} = 0$$
$$\left(\tfrac{1}{2}+1+1\right)V_3 = V_2$$
$$\tfrac{5}{2}V_3 = V_2$$
$$V_3 = \tfrac{2}{5}V_2$$

Find

$$Y_{22} = \frac{I_2}{V_2}\Bigg|_{V_1=0}.$$



kcl at node (2)

$$\frac{V_2 - V_3}{1\,\Omega} + \frac{V_2}{\frac{1}{2}\,\Omega} - I_2 = 0.$$

$$V_3 = \frac{2}{5} V_2$$

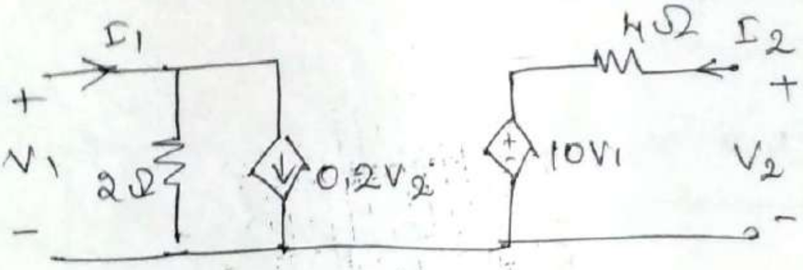
$$V_2 + 2V_2 - V_3 - I_2 = 0.$$

$$3V_2 - \frac{2}{5} V_2 - I_2 = 0$$

$$\frac{13}{5} V_2 = I_2.$$

$$\frac{I_2}{V_2} = \frac{5}{13}\,\mho.$$

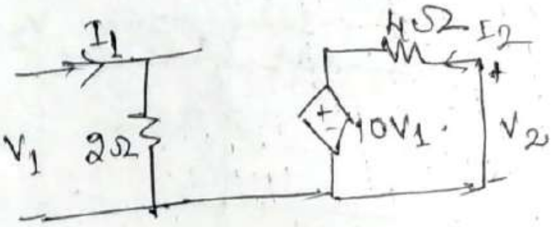
Find Y parameters



$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} \qquad Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$$



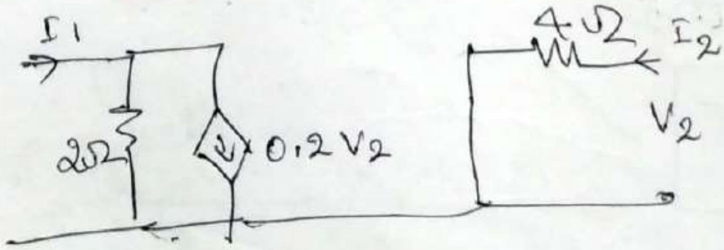
$$V_1 = 2I_1 \Rightarrow \frac{I_1}{V_1} = \frac{1}{2} = \boxed{0.5\,\mho}$$

$$4I_2 + 10V_1 = 0 \Rightarrow 4I_2 = -10V_1 \Rightarrow \frac{I_2}{V_1} = \frac{-10}{4}$$

4

$$\boxed{Y_{21} = -2.5\,\mho}$$

$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} \qquad Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$

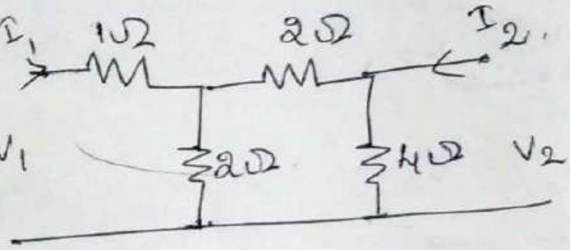


$$4I_2 = V_2 \Rightarrow \frac{I_2}{V_2} = \frac{1}{4} = \boxed{0.25\,\mho}$$

$$\cancel{2I_1 = V_1 \;\&\; V_1 = 2\times 0.2V_2}$$

$$I_1 = 0.2V_2 \Rightarrow \boxed{\frac{I_1}{V_2} = 0.2\,\Omega}$$

4) Y parameters ?

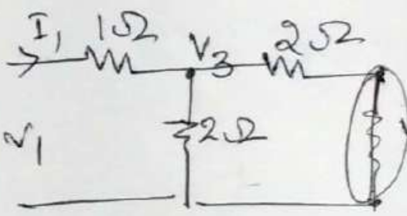


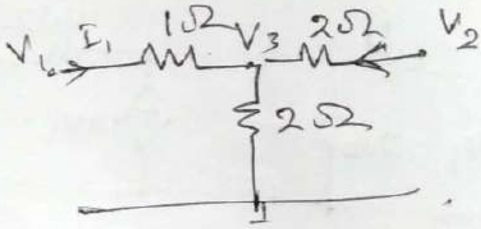
$Y_{12} =$

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2 = 0} \qquad Y_{21} = \frac{I_2}{V_1}\Big|_{V_2 = 0}$$





$\cancel{V_1 = V_3}$ $\quad I_1 = \frac{V_1 - V_3}{1}$

$$\frac{V_3 - V_1}{1} + \frac{V_3}{2} + \frac{V_3 - \cancel{V_2}}{2} = 0$$

$$\left(1 + \tfrac{1}{2} + \tfrac{1}{2}\right) V_3 = V_1$$

$$2V_3 = V_1$$

$$V_3 = \tfrac{1}{2} V_1$$

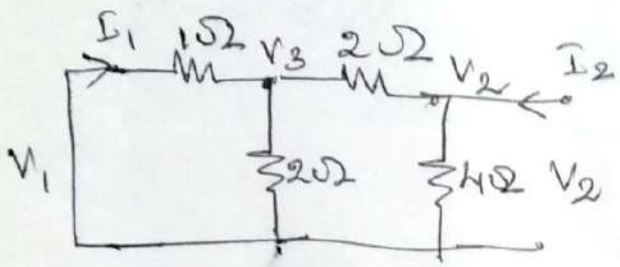
$$I_1 = V_1 - \tfrac{1}{2} V_1 = \tfrac{1}{2} V_1$$

$$\boxed{Y_{11} = \frac{I_1}{V_1} = \frac{1}{2}\ ℧}$$

$$I_2 = \frac{\cancel{V_2} - V_3}{2} = \frac{-\frac{1}{2} V_1}{2} = -\frac{1}{4} V_1$$

$$\boxed{\frac{I_2}{V_1} = -\frac{1}{4}\ ℧}$$

$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$ $Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$ (7)



$I_1 = \frac{V_1 - V_3}{1} = -V_3$

$\frac{V_3}{1} + \frac{V_3}{2} + \frac{V_3 - V_2}{2} = 0.$

$\left(1 + \frac{1}{2} + \frac{1}{2}\right) V_3 = \frac{1}{2} V_2$

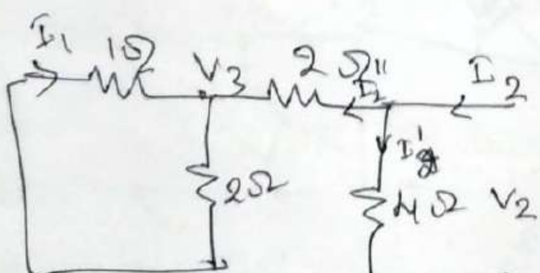
$2V_3 = \frac{1}{2} V_2$

$V_3 = \frac{1}{4} V_2.$

$I_1 = -\frac{1}{4} V_2$

$\boxed{\frac{I_1}{V_2} = -\frac{1}{4} \mho}$

$I_2 =$



Next page

$$\frac{V_2 - V_3}{2} + \frac{V_2}{4} - I_2 = 0.$$

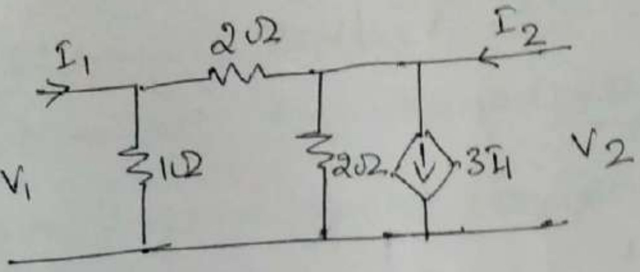
$$\frac{3}{4}V_2 - \frac{1}{2}V_3 = I_2 \qquad V_3 = \frac{1}{4}V_2$$

$$\frac{3}{4}V_2 - \frac{1}{2}\frac{1}{4}V_2 = I_2$$

$$\frac{3}{4}V_2 - \frac{1}{8}V_2 = I_2$$

$$\left(\frac{3}{4} - \frac{1}{8}\right)V_2 = I_2$$

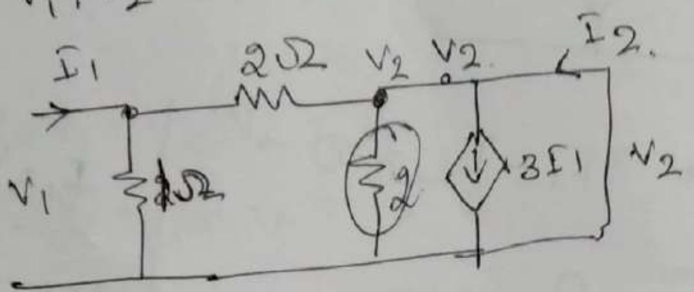
$$\left(\frac{6-1}{8}\right)V_2 = I_2 \implies \boxed{\frac{I_2}{V_2} = \frac{5}{8}\,℧}$$

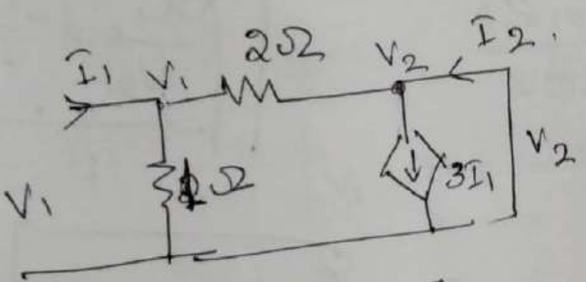


$I_1 = y_{11}V_1 + y_{12}V_2$

$I_2 = y_{21}V_1 + y_{22}V_2$

$y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$ $\qquad$ $y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$.





$-I_1 + \frac{V_1}{\cancel{2}1} + \frac{V_1 - \cancel{V_2}}{2} = 0$

$-I_1 + \frac{3}{2}V_1 = 0 \implies I_1 = \frac{3}{2}V_1 \implies \boxed{\frac{I_1}{V_1} = \frac{3}{2}\mho}$

$\frac{\cancel{V_2} - V_1}{2} + \cancel{\frac{V_2}{2}} + 3I_1 - I_2 = 0$.

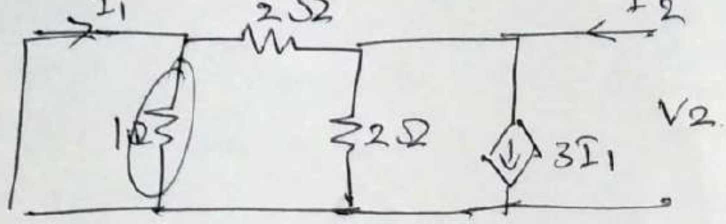
$-\frac{V_1}{2} + 3\left(\frac{3}{2}V_1\right) - I_2 = 0$.

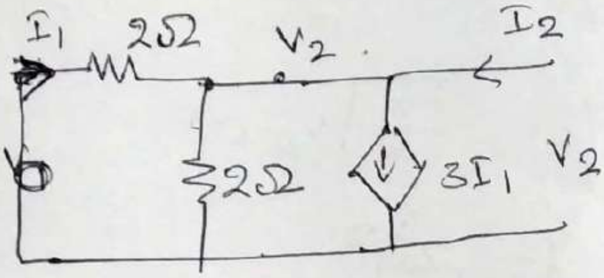
$-\frac{1}{2}V_1 + \frac{9}{2}V_1 - I_2 = 0$

$\frac{\cancel{8}\,4}{\cancel{2}}V_1 = I_2$

$\boxed{\frac{I_2}{V_1} = 4\mho}$

$$y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} \qquad y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}.$$





$$\frac{V_2}{2} + \frac{V_2 - \cancel{V_1}}{2} + 3I_1 - I_2 = 0.$$

$$V_2 + 3I_1 - I_2 = 0.$$

$$V_2 + 3\left(-\tfrac{1}{2}V_2\right) - I_2 = 0.$$

$$V_2 - \frac{3}{2}V_2 = I_2$$

$$-\frac{1}{2}V_2 = I_2 \implies \boxed{\frac{I_2}{V_2} = -\frac{1}{2}\,℧}$$

$$I_1 = \frac{\cancel{V_1} - V_2}{2} = -\frac{1}{2}\ldots$$

$$\boxed{\frac{I_1}{V_2} = -\frac{1}{2}\,℧}$$

3) Following short circuit currents and voltages are obtained experimentally for a two port n/w.

1. with output short circuited, $I_1 = 5mA$, $I_2 = -0.3mA$ & $V_1 = 25V$.

ii) with i/p short circuited, $I_1 = -5mA$, $I_2 = 10mA$, $V_2 = 30V$

Determine y parameters

$$Y_{11}V_1 + Y_{12}V_2 = I_1$$
$$Y_{21}V_1 + Y_{22}V_2 = I_2$$

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} = \frac{5\times10^{-3}}{25} = 0.2\times10^{-3}$$

$$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} = \frac{-0.3\times10^{-3}}{25} = -0.012\times10^{-3}\,℧$$

$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} = \quad = -0.1667\times10^{-3}$$

$$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} \quad = 0.333\times10^{-3}\,℧$$

4) The Z parameters of a two port n/w are $Z_{11} = 20\Omega$, $Z_{22} = 30\Omega$, $Z_{12} = Z_{21} = 10\Omega$. Find y and ABCD parameters of the n/w.

y in terms of Z.

ABCD in terms of Z.

$$Y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{Z_{22}}{Z_{11}Z_{22} - Z_{12}Z_{21}} = \frac{30}{20\times30 - 10\times10} = \frac{30}{500}\,℧$$

$$Y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{20}{500} = 0.04\,℧$$

$$Y_{12} = \frac{-Z_{12}}{\Delta_Z} = \frac{-10}{500} = -0.01 \mho = Y_{21}$$

ii) $$A = \frac{Z_{11}}{Z_{21}} = \frac{20}{10} = 2.$$

$$B = -\frac{\Delta Z}{Z_{21}} = -\frac{500}{10} = -50\Omega$$

$$C = \frac{1}{Z_{11}} = \frac{1}{10} = 0.1 \mho.$$

$$D = \frac{Z_{22}}{Z_{21}} = 3.$$

For the network shown in Fig.7.14, find y and z parameters. (Karnataka University)

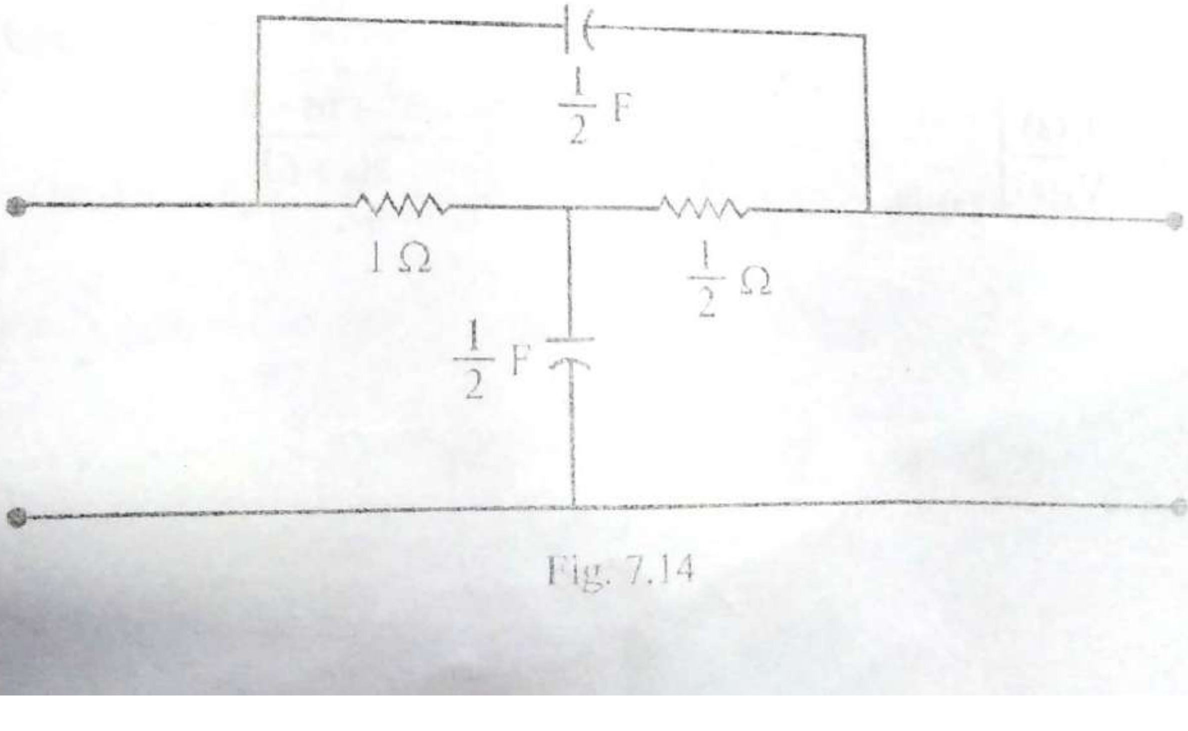


Fig. 7.14

Solution: The transformed network is as shown in Fig.1.

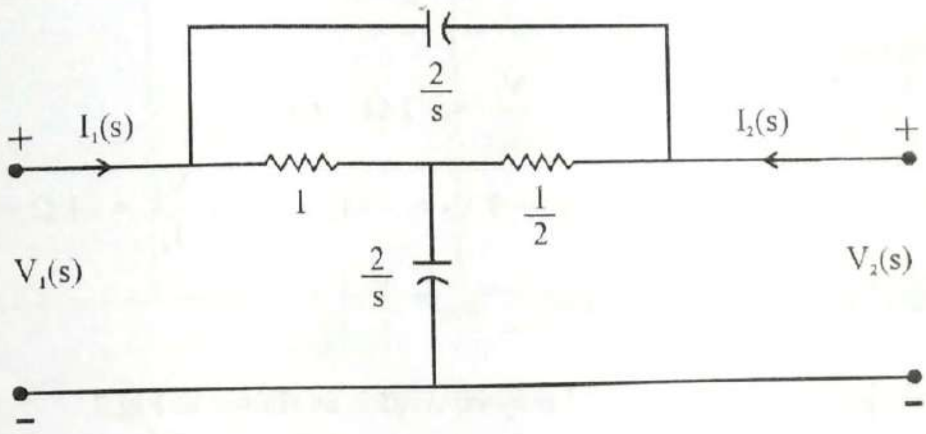


Fig. 1

Converting the star network into delta and simplifying, the network in Fig.1 can be written as in Fig.2.

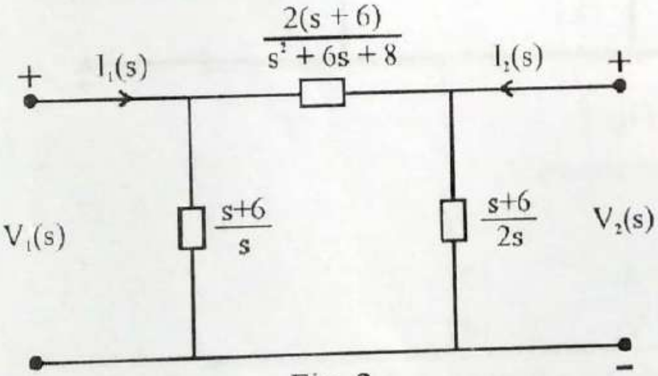


Fig. 2

$$y_{11}(s) = \left.\frac{I_1(s)}{V_1(s)}\right|_{V_2(s)=0}$$

$$= \frac{s^2+6s+8}{2(s+6)} + \frac{s}{s+6} = \frac{s^2+8s+8}{2(s+6)}$$

$$y_{21}(s) = \left.\frac{I_2(s)}{V_1(s)}\right|_{V_2(s)=0}$$

$$= \frac{-V_1(s)\frac{s^2+6s+8}{2(s+6)}}{V_1(s)} = -\frac{s^2+6s+8}{2(s+6)}$$

$$y_{22}(s) = \left.\frac{I_2(s)}{V_2(s)}\right|_{V_1(s)=0} = \frac{s^2+6s+8}{2(s+6)} + \frac{2s}{s+6} = \frac{s^2+10s+8}{2(s+6)}$$

$$y_{12}(s) = \left.\frac{I_1(s)}{V_2(s)}\right|_{V_1(s)=0} = \frac{-V_2(s)\frac{s^2+6s+8}{2(s+6)}}{V_2(s)} = -\frac{s^2+6s+8}{2(s+6)}$$

7.12 Define Z parameters. Determine Z parameters for the network shown in Fig. 7.22. (June/July 2011) (10 marks)

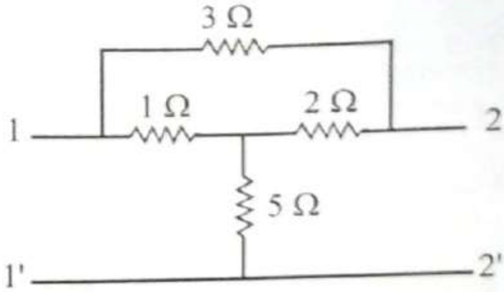


Fig. 7.22

Soln.: $Z_{11}I_1 + Z_{12}I_2 = V_1$, $Z_{21}I_1 + Z_{22}I_2 = V_2$

$$\therefore\ Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 0, \qquad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 0\ , \qquad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

Converting the star network of 1 Ω, 2 Ω and 5 Ω into delta and simplifying further, the N.W in fig. 7.22 may be written as in Fig. 1.

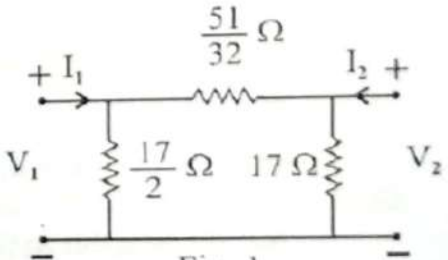


Fig. 1

When $I_2 = 0$

$$Z_{11} = \frac{V_1}{I_1} = \frac{\frac{17}{2}\times\left(\frac{51}{32}+17\right)}{\frac{17}{2}+\frac{51}{32}+17} = \frac{10{,}115}{1{,}734} = \frac{35}{6}\ \Omega$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{17\times\left(\frac{51}{32}+\frac{17}{2}\right)}{17+\frac{51}{32}+\frac{17}{2}} = \frac{19}{3}\ \Omega$$

When $I_2 = 0$, $\quad I_{17\,\Omega} = \dfrac{I_1\times\frac{17}{2}}{\frac{17}{2}+\frac{51}{32}+17} = \dfrac{\frac{17}{2}I_1}{\frac{867}{32}} = \dfrac{272}{867}\ I_1$

$\therefore\ V_2 = 17\times I_{17\,\Omega} = 17\times\dfrac{272}{867}\ I_1$, $\qquad \therefore\ Z_{21} = \dfrac{V_2}{I_1} = \dfrac{17\times 272}{867} = \dfrac{16}{3}\ \Omega = Z_{12}$

Fig. 7.32

2 **Find the y parameters for the network shown in Fig. 7.33 (Karnataka University)**

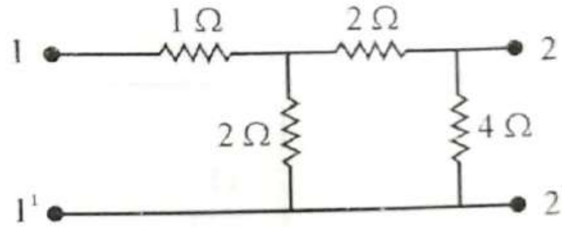


Fig. 7.33

3 **Find z parameters for the network shown in Fig. 7.34. (Mysore University)**

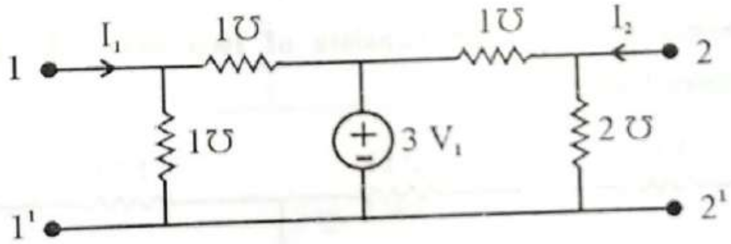


Fig. 7.34

.4 **Find y parameters for the network shown in Fig. 7.35. (Kuvempu University)**

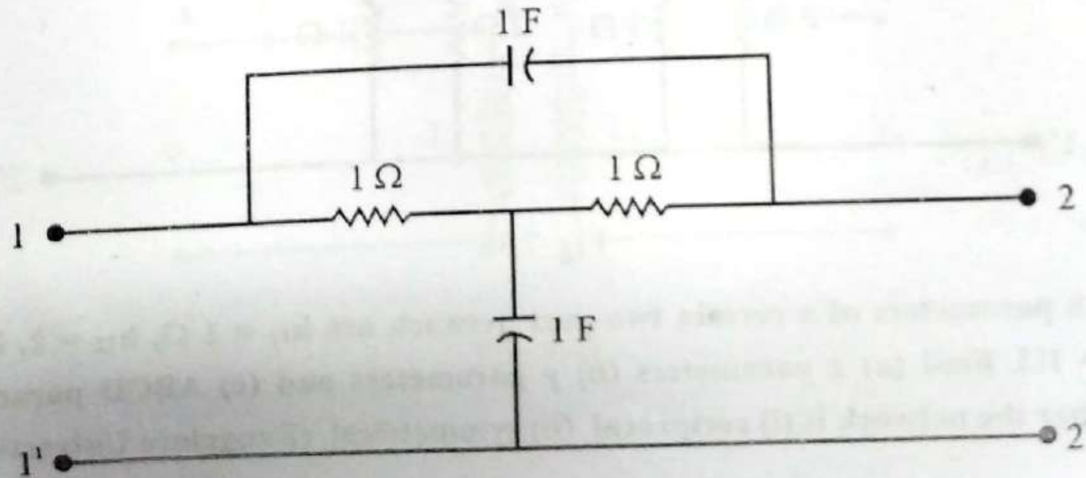


Fig. 7.35

7.2 $y_{11} = 0.5\ \mho$, $y_{12} = y_{21} = -0.25\ \mho$, $y_{22} = 0.625\ \mho$

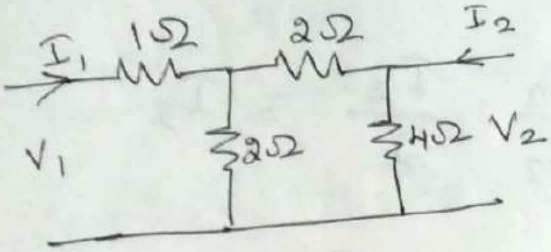
7.3 $z_{11} = z_{21} = -1\ \Omega$, $z_{12} = 0\ \Omega$, $z_{22} = \dfrac{1}{3}\ \Omega$

7.4 $y_{11}(s) = y_{22}(s) = \dfrac{s^2 + 3s + 1}{s + 2}$, $y_{12}(s) = y_{21}(s) = -\dfrac{s^2 + 2s + 1}{s + 2}$

h parameters: V_1, I_2 depen

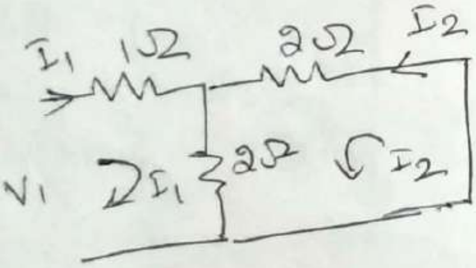
$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2.$$

1) Find h parameters of the n/w shown.



$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

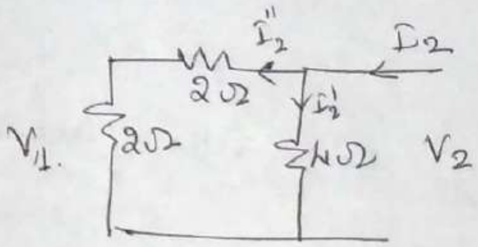
$$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0} \qquad h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}$$



$$V_1 = I_1 + 2(I_1 + I_2)$$
$$V_1 = 3I_1 + 2I_2$$
$$V_1 = 3I_1 + \cancel{2}\left(-\frac{I_1}{\cancel{2}}\right)$$
$$V_1 = 3I_1 - I_1$$
$$V_1 = 2I_1$$
$$\boxed{\frac{V_1}{I_1} = 2\Omega}$$

$$4I_2 + 2I_1 = 0.$$
$$I_1' = \frac{I_1 \times 2}{4} = \frac{I_1}{2}$$
$$I_1' = I_1'' = \frac{I_1}{2}$$
$$I_2 = -\frac{I_1}{2}.$$
$$\therefore \boxed{\frac{I_2}{I_1} = -\frac{1}{2}}$$

$$h_{12} = \frac{V_1}{V_2}\Bigg|_{I_1=0} \quad \& \quad h_{22} = \frac{I_2}{V_2}\Bigg|_{I_1=0}$$



$$I_2'' = \frac{I_2 \times 4}{4+4} = \frac{\cancel{4}I_2}{\cancel{8}_2} = \frac{I_2}{2} = I_2'$$

$$V_2 = 4I_2'$$

$$V_2 = \cancel{4}^2\frac{I_2}{\cancel{2}} = 2I_2$$

$$\boxed{\frac{I_2}{V_2} = \frac{1}{2}\,\mho}$$

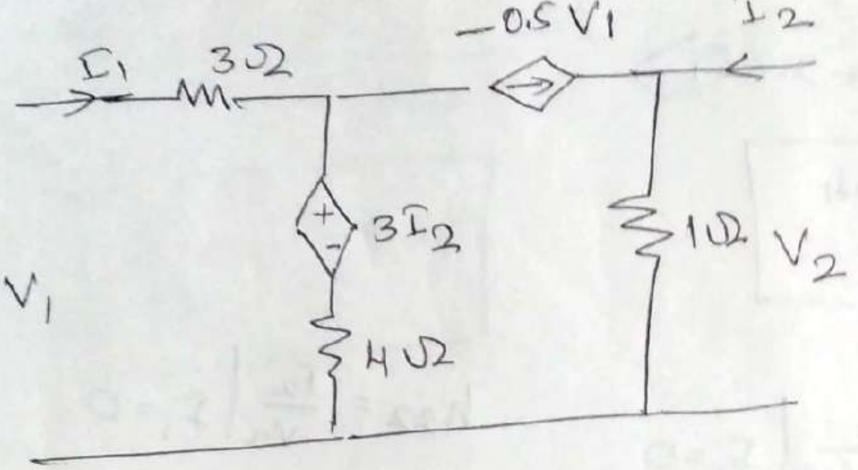
$$V_2 = 4\,I_2'$$

$$V_1 = 2\,I_2''$$

$$\frac{V_1}{V_2} = \frac{2I_2''}{4\,I_2'} = \frac{\cancel{2}\times\frac{I_2}{\cancel{2}}}{\cancel{4}^2\times\frac{I_2}{\cancel{2}}} = \frac{1}{2}$$

$$\boxed{\frac{V_1}{V_2} = \frac{1}{2}}$$

Find h

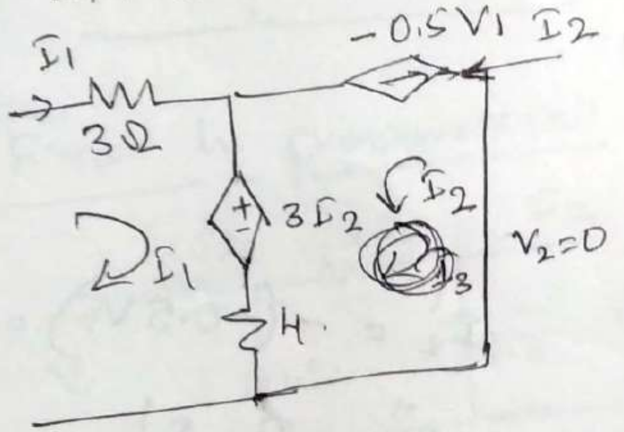


$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}$$

$$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}$$



$$I_2 = -(-0.5V_1) = 0.5V_1$$

$$3I_1 + 3I_2 + 4(I_1 + I_2) = V_1$$

$$7I_1 + 7I_2 = V_1$$

$$7I_1 + 7(+0.5V_1) = V_1$$

$$7I_1 + 3.5V_1 = V_1$$

$$7I_1 = V_1 + 3.5V_1 = 2.5V_1$$

$$\frac{V_1}{I_1} = -\frac{7}{2.5} = \frac{70}{25} = -2.8\,\Omega$$

$$\boxed{\frac{V_1}{I_1} = -2.8\,\Omega}$$

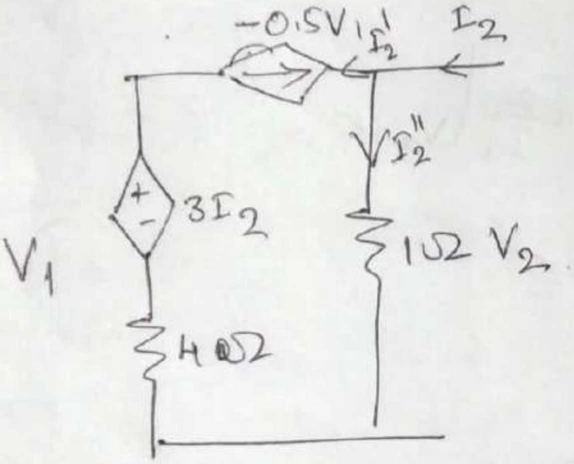
$I_2 = 0.5\, V_1$

$I_2 = 0.5(-2.8 I_1)$

$$\boxed{\frac{I_2}{I_1} = -1.4}$$

$\cancel{h_{21}} \Rightarrow h_{12} = \frac{V_1}{V_2} \Big| I_1 = 0.$

$h_{22} = \frac{I_2}{V_2} \Big| I_1 = 0.$



$V_1 = 3I_2 + 4I_2'$

$V_1 = 3I_2 + 4(0.5V_1)$

$V_1 = 3I_2 + 2V_1$

$-3I_2 = V_1.$

$I_2' = -(-0.5V_1) = 0.5V_1.$

$I_2'' = I_2 - I_2'$

$= I_2 - 0.5V_1.$

$V_2 = I_2''$

$V_2 = I_2 - 0.5V_1.$

$V_2 = I_2 - 0.5(-3I_2)$

$V_2 = I_2 + 1.5I_2$

$V_2 = 2.5I_2.$

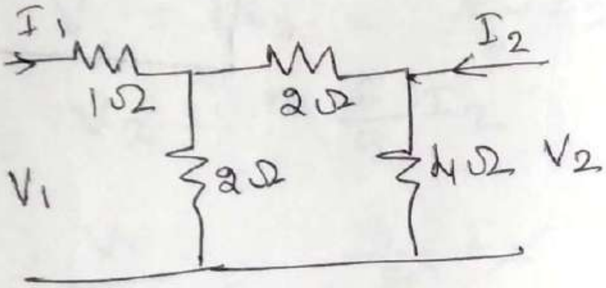
$$\boxed{\frac{I_2}{V_2} = \frac{1}{2.5} = 0.4}$$

$$\frac{V_1}{V_2} = \frac{-3I_2}{2.5I_2} = -\frac{3}{2.5} = \frac{-30^{6}}{25_{5}} = -1.2$$

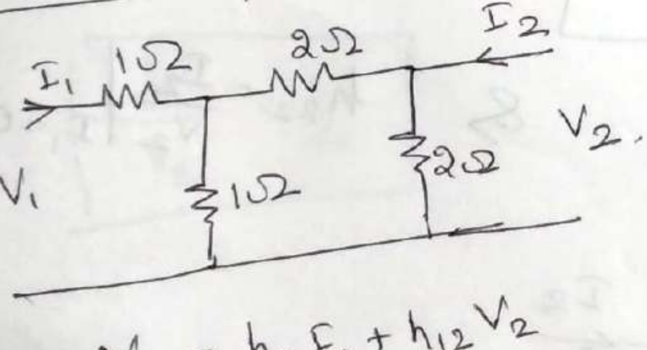
$$\boxed{\frac{V_1}{V_2} = -1.2}$$

3) Find h parameters of the n/w shown.

done



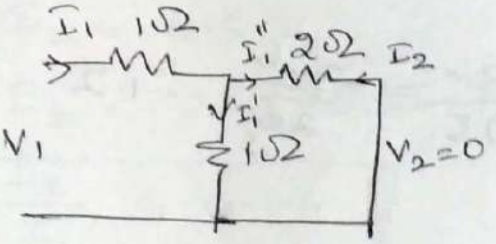
3) Find h parameters:



$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0} \qquad h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}.$$



$$I_1'' = \frac{I_1 \times 1}{3} = \frac{1}{3} I_1$$

$$\& \quad I_2 = -I_1'' = -\tfrac{1}{3} I_1$$

$$I_1' = \frac{I_1 \times 2}{3} = \frac{2}{3} I_1$$

$$\boxed{\frac{I_2}{I_1} = -\frac{1}{3}}$$

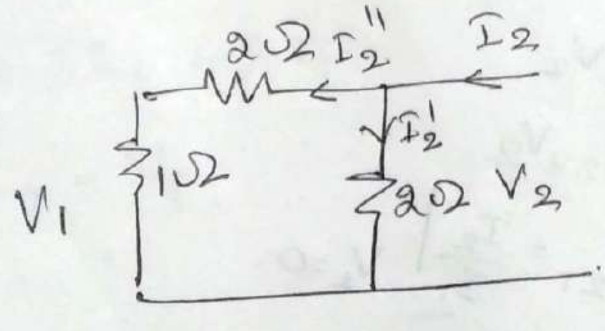
$$V_1 = I_1 + I_1'$$

$$V_1 = I_1 + \frac{2}{3} I_1$$

$$V_1 = \frac{5}{3} I_1$$

$$\boxed{\frac{V_1}{I_1} = \frac{5}{3}\,\Omega}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \quad \& \quad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$



$$I_2' = \frac{I_2 \times 3}{3+2} = \frac{3 I_2}{5}$$

$$I_2'' = \frac{I_2 \times 2}{5} = \frac{2}{5} I_2$$

$$V_2 = 2 I_2'$$

$$V_2 = 2 \times \frac{3}{5} I_2$$

$$V_2 = \frac{6 I_2}{5}$$

$$\boxed{\frac{I_2}{V_2} = \frac{5}{6} \mho}$$

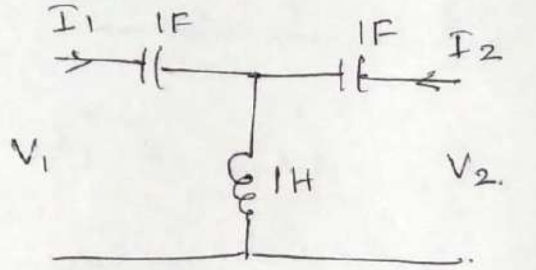
$$V_1 = 1 I_2'' = \frac{3}{5} I_2$$

$$V_2 = \frac{6}{5} I_2$$

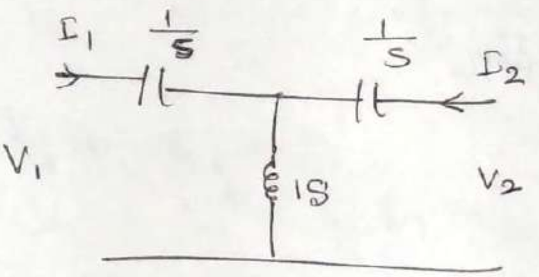
$$\frac{V_1}{V_2} = \frac{\frac{3}{\cancel{5}} \cancel{I_2}}{\frac{6}{\cancel{5}} \cancel{I_2}} = \frac{1}{2}$$

$$\boxed{\frac{V_1}{V_2} = \frac{1}{2}}$$

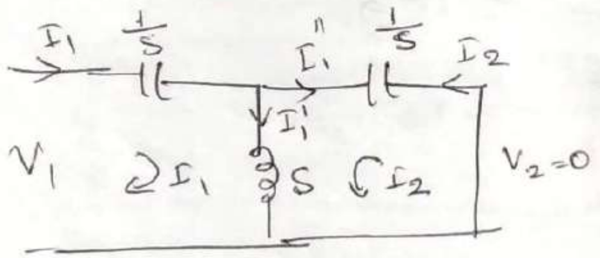
4] Determine h parameters after writing transform N/w



$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2.$$



$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$



$$I_1'' = -I_2.$$

$$I_1' = \frac{I_1 \times \frac{1}{s}}{s + \frac{1}{s}} = \frac{I_1}{s^2+1} = \frac{I_1}{1+s^2}$$

$$I_1'' = \frac{I_1 \times s}{s + \frac{1}{s}} = \frac{I_1 \cdot s \cdot s}{1+s^2} = \frac{s^2 I_1}{1+s^2}.$$

$$V_1 = \frac{1}{s} I_1 + s I_1'$$
$$= \frac{1}{s} I_1 + \frac{s\, I_1}{1+s^2} = \left[\frac{1+s^2+s^2}{(1+s^2)s}\right] I_1$$

$$\boxed{\frac{V_1}{I_1} = \frac{1+2s^2}{s(1+s^2)}\,\Omega}$$

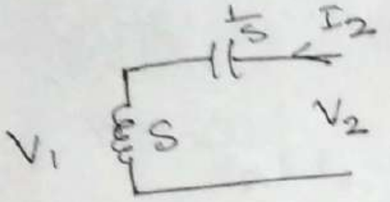
5

$$I_1'' = -I_2$$

$$\frac{s^2 I_1}{1+s^2} = -I_2$$

$$\boxed{\frac{I_2}{I_1} = -\left(\frac{s^2}{1+s^2}\right)}$$

$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0} \quad \& \quad h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0}.$$



$$V_2 = \left(s + \frac{1}{s}\right) I_2 = \left(\frac{1+s^2}{s}\right) I_2$$

$$\boxed{\frac{I_2}{V_2} = \frac{s}{1+s^2} \ ℧}$$

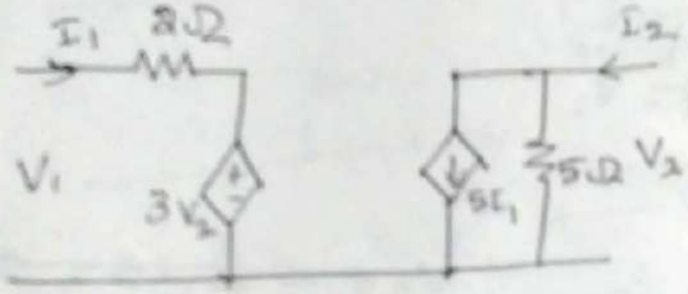
$$V_1 = s I_2.$$

$$V_2 = \left(\frac{1+s^2}{s}\right) I_2$$

$$\frac{V_1}{V_2} = \frac{s I_2}{\left(\frac{1+s^2}{s}\right) I_2} = \frac{s^2}{1+s^2}.$$

$$\boxed{\frac{V_1}{V_2} = \frac{s^2}{1+s^2}}$$

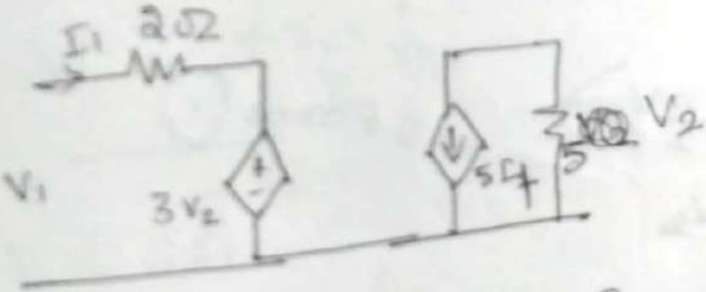
1. Determine the transmission parameters for the n/w shown in fig



$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$$A = \frac{V_1}{V_2}\Big|_{I_2=0} \qquad C = \frac{I_1}{V_2}\Big|_{I_2=0}.$$



$$V_2 = -5I_1 \times 5 \Rightarrow \boxed{\frac{I_1}{V_2} = -\frac{1}{25}}$$

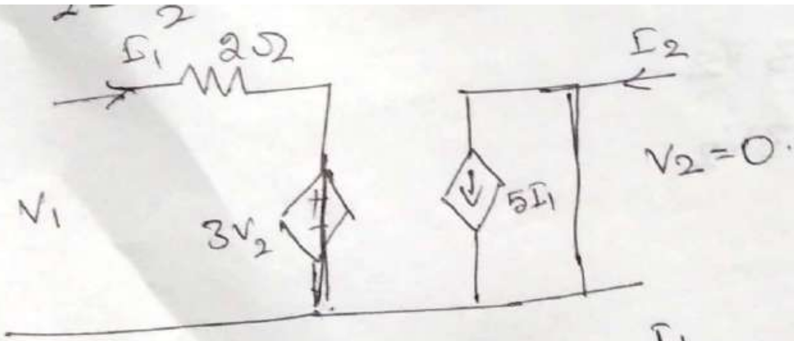
$$V_1 = 2I_1 + 3V_2$$

$$V_1 = 2\left(-\frac{1}{25}V_2\right) + 3V_2$$

$$V_1 = -\frac{2}{25}V_2 + 3V_2$$

$$V_1 = \frac{73}{25}V_2$$

$$\boxed{\frac{V_1}{V_2} = \frac{73}{25}}$$



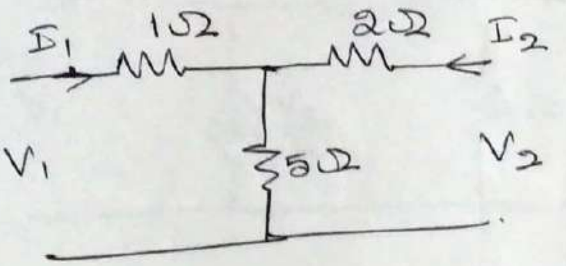
$V_2 = 0.$

$I_2 = 5I_1 \Rightarrow \frac{I_1}{I_2} = \frac{1}{5} \Rightarrow \boxed{\frac{I_1}{-I_2} = \frac{-1}{5}}$

$V_1 = 2I_1. \Rightarrow V_1 = 2\left(\frac{1}{5}\right)I_2 = \frac{2}{5} \Rightarrow \frac{V_1}{I_2} = \frac{2}{5}.$

$\boxed{\frac{V_1}{-I_2} = \frac{-2}{5}}$

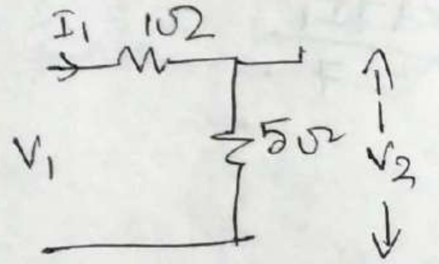
2) Find ABCD parameters: (2)



$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2.$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$



$$V_1 = 6I_1$$

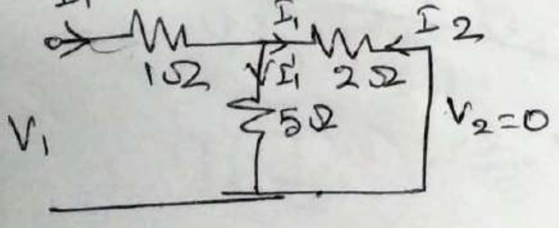
$$V_2 = 5I_1$$

$$\boxed{\frac{V_1}{V_2} = \frac{6}{5}}$$

$$V_2 = 5I_1$$

$$\boxed{\frac{I_1}{V_2} = \frac{1}{5}\,\mho}$$

$$-B = \left.\frac{V_1}{I_2}\right|_{V_2=0} \qquad -D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$$



$$I_1'' = -I_2.$$

$$I_1' = \frac{I_1 \times 2}{7} = \frac{2I_1}{7}.$$

$$I_1'' = \frac{I_1 \times 5}{7} = \frac{5I_1}{7}.$$

$$I_1'' = -I_2$$

$$\frac{5I_1}{7} = -I_2$$

$$\frac{I_1}{I_2} = -\frac{7}{5}.$$

$$-D = -\frac{7}{5}$$

$$\boxed{D = \frac{7}{5}}$$

$$V_1 = I_1 + 5I_1'$$

$$V_1 = I_1 + 5\left(\frac{2I_1}{7}\right) = \cancel{\frac{9I_1}{10}} \quad \frac{17I_1}{7}.$$

$$V_1 = \frac{17I_1}{7}$$

$$V_1 = \frac{17}{7}\left(-\frac{7}{5}\right) I_2$$

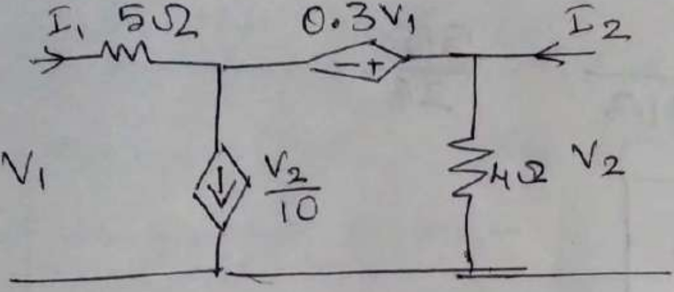
$$V_1 = \cancel{-\frac{9}{5}I_2} \quad -\frac{17}{5} I_2$$

$$\cancel{-B = \frac{V_1}{I_2} = -\frac{9}{5}\Omega} \qquad \frac{V_1}{I_2} = -\frac{17}{5}.$$

$$\cancel{B = \frac{9}{5}} \qquad -B = -\frac{17}{5}$$

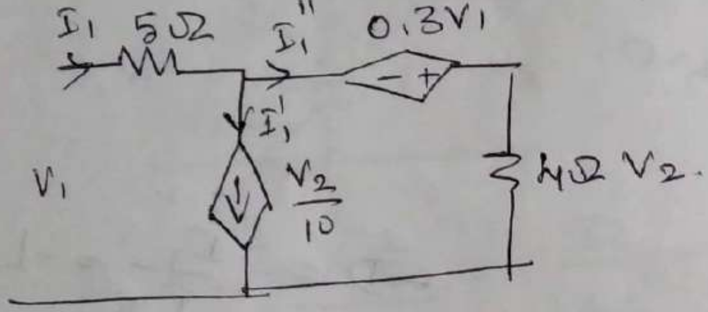
$$\boxed{B = \frac{17}{5}}$$

3). Find transmission parameters:



$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$$A = \frac{V_1}{V_2}\Big|_{I_2=0} \qquad C = \frac{I_1}{V_2}\Big|_{I_2=0}.$$



$$I_1' = \frac{V_2}{10}. \quad \& \quad I_1'' = I_1 - \frac{V_2}{10}.$$

$$5I_1 - 0.3V_1 + 4I_1'' = V_1.$$
$$5I_1 + 4\left(I_1 - \frac{V_2}{10}\right) = V_1 + 0.3V_1$$
$$5I_1 + 4I_1 - \frac{2}{5}V_2 = 1.3V_1.$$
$$9I_1 - \frac{2}{5}V_2 = 1.3V_1$$
$$9\left(\frac{7V_2}{20}\right) - \frac{2}{5}V_2 = 1.3V_1$$
$$\frac{63V_2}{20} - \frac{2}{5}V_2 = 1.3V_1$$
$$\left(\frac{63-8}{20}\right)V_2 = 1.3V_1$$
$$\frac{55}{20}V_2 = 1.3V_1$$

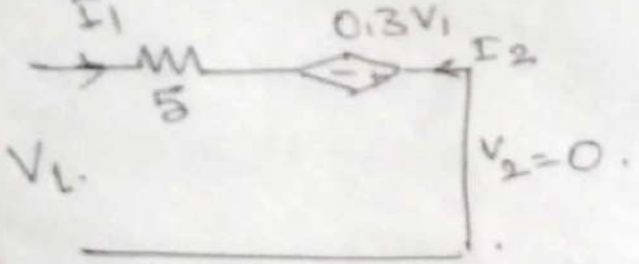
$$V_2 = 4I_1''$$
$$V_2 = 4\left(I_1 - \frac{V_2}{10}\right)$$
$$V_2 = 4I_1 - \frac{2}{5}V_2$$
$$V_2 + \frac{2}{5}V_2 = 4I_1$$
$$\frac{7V_2}{5} = 4I_1$$
$$\frac{V_2}{I_1} = \frac{20}{7}$$
$$\boxed{\frac{I_1}{V_2} = \frac{7}{20}}$$

$$\frac{55}{20}V_2 = 1.3V_1$$

$$\frac{V_1}{V_2} = \frac{55}{20\times1.3} = \frac{55}{26}$$

$$\boxed{\frac{V_1}{V_2} = \frac{55}{26}}$$

$$-B = \left.\frac{V_1}{I_2}\right|_{V_2=0} \qquad -D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$$



$$I_1 = -I_2.$$

$$V_1 = 5I_1 - 0.3V_1$$

$$V_1 = -5I_2 - 0.3V_1$$

$$V_1 + 0.3V_1 = -5I_2$$

$$1.3V_1 = -5I_2$$

$$\frac{V_1}{I_2} = \frac{-5}{1.3}$$

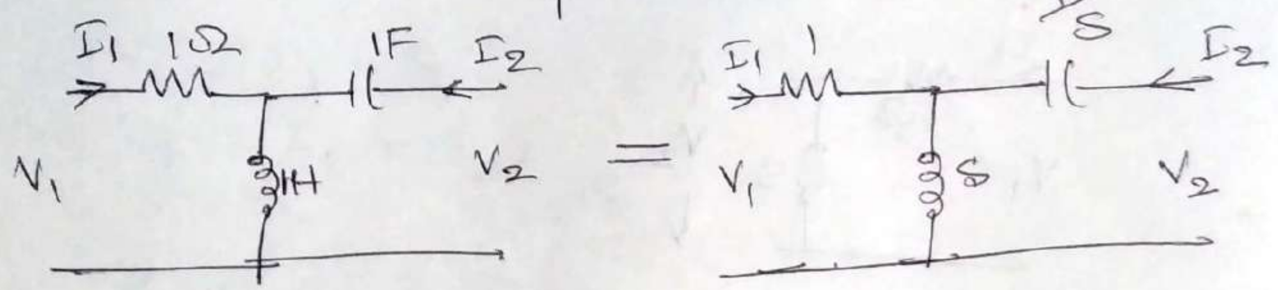
$$\boxed{B = \frac{5}{1.3} = \frac{50}{13}\,\Omega}$$

$$-D = \frac{I_1}{I_2} = -1$$

$$\boxed{D = 1}$$

(7)

(4) Find transmission parameters.

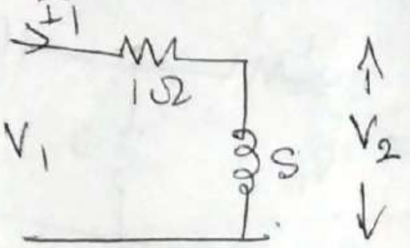


$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$$A = \frac{V_1}{V_2}\bigg|_{I_2=0} \qquad C = \frac{I_1}{V_2}\bigg|_{I_2=0}$$

With $I_2 = 0$



$$V_1 = (1+S)I_1$$

$$V_2 = S I_1.$$

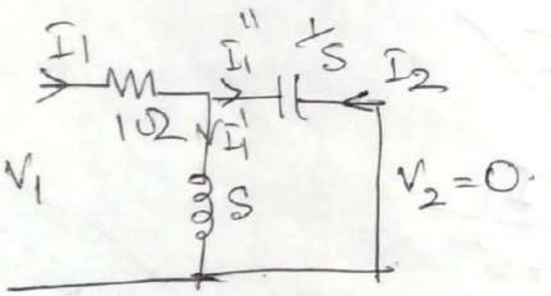
$$\boxed{\frac{V_1}{V_2} = \left(\frac{1+S}{S}\right)}.$$

$$V_2 = S I_1.$$

$$\boxed{\frac{I_1}{V_2} = \frac{1}{S}}$$

$$-B = \left.\frac{V_1}{I_2}\right| V_2 = 0$$

$$-D = \left.\frac{I_1}{I_2}\right| V_2 = 0.$$



$$I_1'' = -I_2$$

$$I_1'' = \frac{I_1 \times S}{S + \frac{1}{S}} = \frac{S I_1 \cdot S}{(S^2+1)} = \frac{S^2 I_1}{(1+S^2)} = -I_2$$

$$I_1' = \frac{I_1 \times \frac{1}{S}}{S + \frac{1}{S}} = \left(\frac{I_1}{1+S^2}\right).$$

$$\frac{I_1}{I_2} = -\left(\frac{1+S^2}{S^2}\right).$$

$$\boxed{D = \frac{1+S^2}{S^2}}$$

$$V_1 = I_1 + sI_1'$$

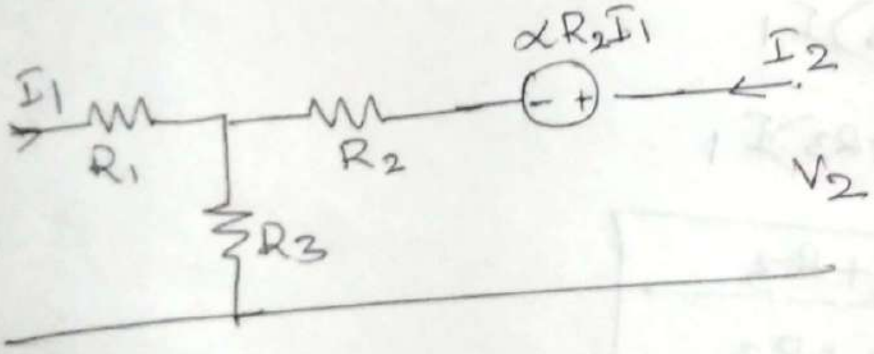
$$V_1 = I_1 + s\frac{I_1}{(1+s^2)}$$

$$V_1 = \frac{(1+s^2+s)I_1}{1+s^2}$$

$$V_1 = \left(\frac{1+s+s^2}{\cancel{1+s^2}}\right)(-)\left(\frac{\cancel{1+s^2}}{s^2}\right)I_2$$

$$\boxed{\frac{V_1}{I_2} = \frac{1+s+s^2}{s^2}}$$

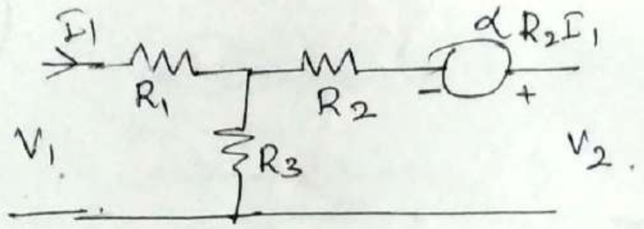
5] Determine ABCD parameters:



$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$$A = \frac{V_1}{V_2}\bigg|_{I_2=0} \qquad C = \frac{I_1}{V_2}\bigg|_{I_2=0}$$



$$\cancel{-\alpha R_2 I_1 + V_2}$$

$$-\alpha R_2 I_1 + V_2 - I_1 R_3 = 0$$

$$I_1(-\alpha R_2 - R_3) = -V_2$$

$$-I_1(\alpha R_2 + R_3) = -V_2$$

$$\boxed{\frac{I_1}{V_2} = \frac{1}{\alpha R_2 + R_3}}$$

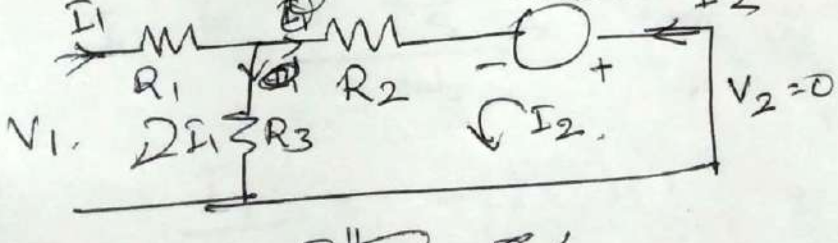
$$V_1 = I_1 R_1 + I_1 R_3 = I_1(R_1 + R_3)$$

$$V_1 = (R_1 + R_3) I_1$$

$$V_2 = (\alpha R_2 + R_3) I_1$$

$$\boxed{\frac{V_1}{V_2} = \frac{R_1 + R_3}{\alpha R_2 + R_3}}$$

$$-B = \left.\frac{V_1}{I_2}\right|_{V_2 = 0} \quad \& \quad -D = \left.\frac{I_1}{I_2}\right|_{V_2 = 0}$$



$$I_1'' = -I_2$$

Network

Resonant circuits
Series & parallel resonance
frequency response of series & parallel circuits
Q factor
Bandwidth

Resonance is a phenomenon which takes place in ac circuits
- very imp especially in field of Comm
ex: Radio Rx has the ability to select certain desired freq, transmitted by station. & rejects all other unwanted frequencies transmitted by the other station
Such a selection of required freq & rejection of unwanted freq is based on the principle of resonance.

Resonance is a phenomenon in which applied vg and resulting current are in phase
OR
Ac ckt is said to be under resonance if it exhibits unity power factor condition. [cosθ = 1]

The resonance Condition can be achieved either by keeping the n/w elements Constant & varying freq
OR
keeping freq Constant & varying freq dependent Elements

A resonant ckt must have an inductance & Capacitance
The resistance will be always present either due to lack of ideal elements or due to the resistance itself.

when resonance occurs, energy absorbed by one reac element is exactly equal to the energy released by another reactive element; within the system.

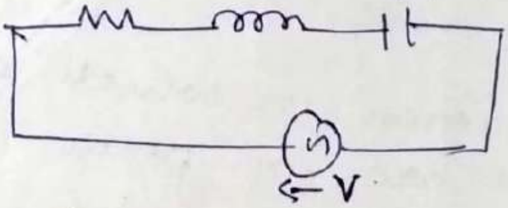
The total apparent power is simply the avg power dissipated by resistive element.

The avg power absorbed by the system will also be max at resonance.

Two types:

1. Series resonant circuit
2. parallel resonant ckt

Series resonant circuit: R, L, C connected in series across alternating vg of varying freq



$$Z = (R + jX_L - jX_C)$$
$$= R + j\omega L - j\frac{1}{\omega C}$$
$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

at resonance $Z = R$, $\omega \to \omega_0$.

$$\omega_0 L - \frac{1}{\omega_0 C} = 0 \implies \omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$2\pi f_0 = \frac{1}{\sqrt{LC}}$$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

$f_r \rightarrow$ resonant freq.

At resonance, current through ckt is maximum. & is given by

$$I_r = \frac{E}{R} = I_m$$

$$I = \frac{V}{R + j(X_C - X_L)}$$

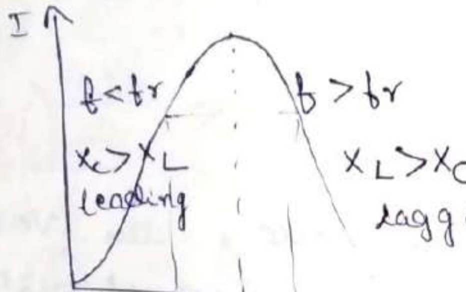
$$X_L = j\omega L$$

$$X_C = \frac{-j}{\omega C}$$

for low f_r

For freq $< f_r$,
$X_C > X_L$ &

For freq $> f_r$, $X_L > X_C$.

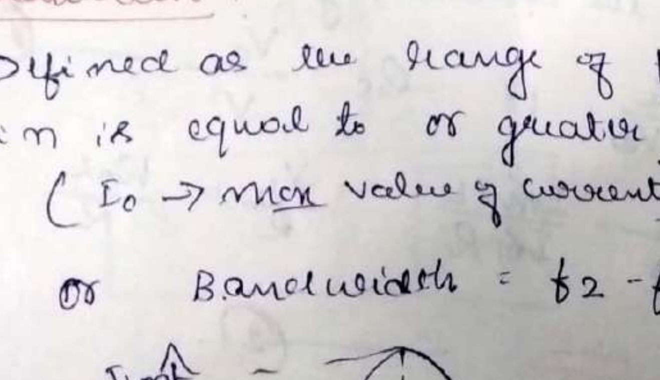


Xc- Current leading V_g Current lagging V_g.

Bandwidth:

Defined as the range of frequencies over which the gain is equal to or greater than $\frac{1}{\sqrt{2}} I_0$.
($I_0 \rightarrow$ max value of current)

or   Bandwidth $= f_2 - f_1$.



$f_1 f_2 \rightarrow$ cut off freq
or $\rightarrow$ band freq
or $\rightarrow$ half power freq.

keeping
(E Constant)

at resonce, $P_m = I_m^2 R$

or f_1 or f_2 $P = \left(\frac{I_m}{\sqrt{2}}\right)^2 R = \frac{I_m^2}{2} R = \frac{P_m}{2}$

$\therefore f_1 f_2 \rightarrow$ half power frequencies

A ~~series~~ resonant ckt is always adjusted to select the band of frequencies lying b/w f_1 & f_2

Selectivity:

It is defined as the ratio of resonant freq to B.W

i.e $\boxed{\text{Selectivity} = \frac{f_0}{f_2 - f_1}}$

Q_s factor (Quality factor) or (Vg magnification

During series resonance, the voltages across reactive ~~resistive~~ elements i.e inductance & capacitance is many times more than the applied Vg.

i.e $Q \rightarrow$ ratio of Vg across inductor or capacitor to the applied Vg.

$$Q = Q_s = \frac{V_L}{V} \quad \text{or} \quad Q_s = \frac{V_C}{V} \quad \text{—— (1)}$$

$$Q_s = \frac{V_L}{V} = \frac{I_0 X_L}{I_0 R} = \frac{I_m X_L}{I_m R} = \frac{X_L}{R} = \frac{\omega_0 L}{R}$$

i.e. $$Q_s = \frac{\omega_0 L}{R} \quad \text{—— (2)}$$

also ~~$V_L = I_0 X_L$~~

$$Q_s = \frac{V_C}{V} = \frac{I_0 X_C}{I_0 R} = \frac{I_m X_C}{I_0 R} = \frac{X_C}{R} = \frac{1}{\omega C R}$$

$$Q_s = \frac{1}{\omega_0 C R} \quad \text{—— (3)}$$

Parallel resonance!

From (2) & (3)

$$\frac{W_0 L}{R} = \frac{1}{WCR}.$$

Quality factor can also be defined as the ratio of inductive reactance or capacitive reactance to the resistance.

~~(2) & (3)~~ $Q_s = \frac{W_0 L}{R} = \frac{1}{W_0 C R}$.

$$\cancel{Q_s = \frac{1}{W_0^2 C L}}.$$

$$Q = \frac{X_L}{R} \quad \& \quad Q = \frac{X_C}{R} \qquad X_C = \frac{1}{WC}.$$

$$Q = \frac{W_0 L}{R} \quad \text{--- (5)} \qquad Q = \frac{1}{W_0 C R}.$$

$$W_0 = \frac{1}{QCR} \quad \text{--- (4)}.$$

Subst: (4) in (5)

$$Q = \frac{\frac{1}{QCR} \cdot L}{R}$$

$$Q = \frac{L}{QCR^2}$$

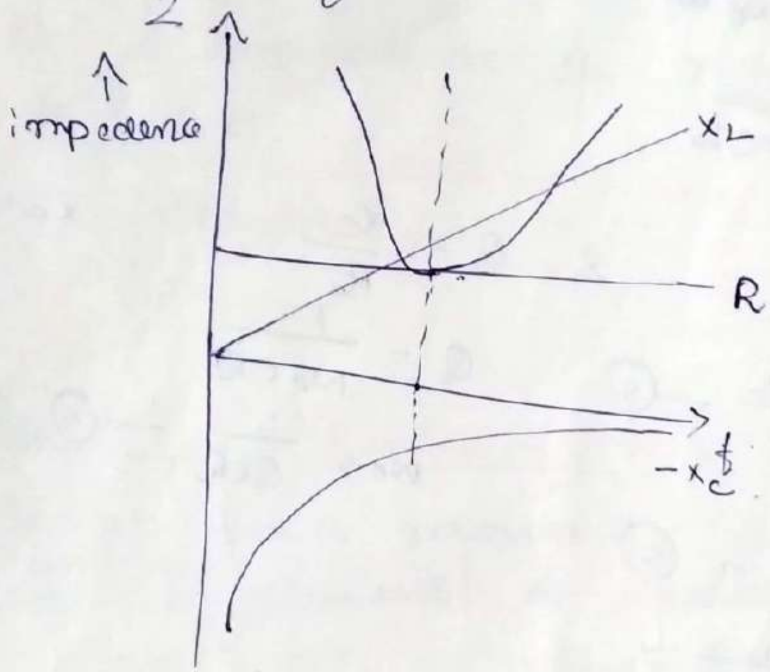
$$Q^2 = \frac{L}{R^2 C}.$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}.}$$

Basically a resonant ckt is freq selective. The behaviour of reactive components changes with variation in freq.

So it is necessary to study variation of diff parameters with variation in freq

1. Variation of reactance with frequency



$X_L = j\omega L$

$f \uparrow \Rightarrow X_L = \uparrow$

$X_C = \frac{j}{\omega C}$

$f \uparrow \Rightarrow X_C \downarrow$

R remains constant for all freq.

The inductive reactance X_L follows st. line.

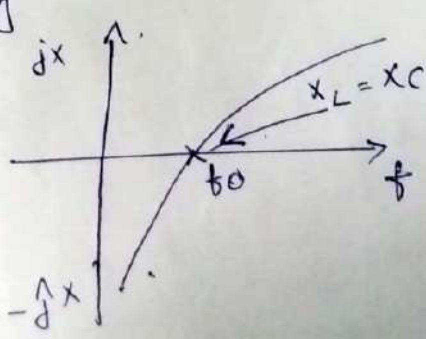
X_C follows hyperbolic curve.

At f_r, $X_L = X_C$. The variation of Z is shown

at f_r, I max $\Rightarrow$ Z min.

i.e. $f < f_r$ for lower value f, $Z \uparrow$ due to increased value of X_C

for higher value f $(f > f_r)$ $Z \uparrow$ due to increased value of X_L



variation of impedence with freq.

$$Z = R + j(X_L - X_C)$$

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right).$$

~~under~~

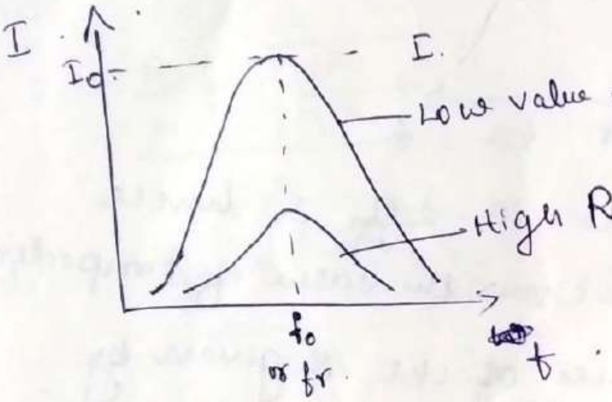
under resonce $Z = R$.

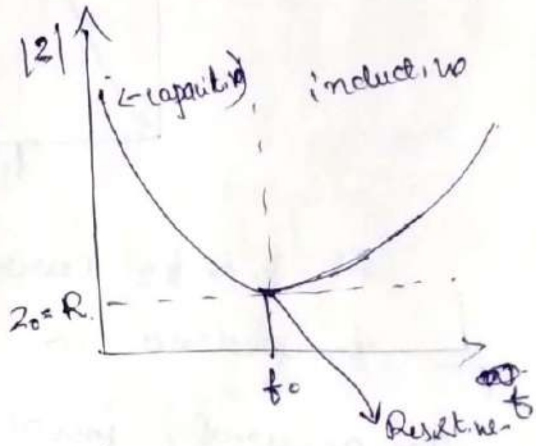
Current in series resonant ckt is given by

$$I = \frac{V}{|Z|} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}.$$

at resonance $I = I_0$

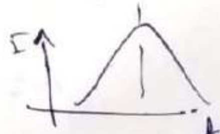
$$I_0 = \frac{V}{R}.$$

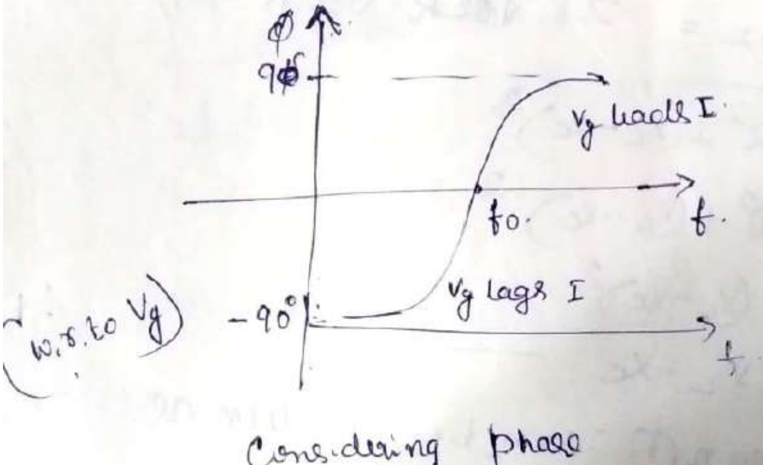


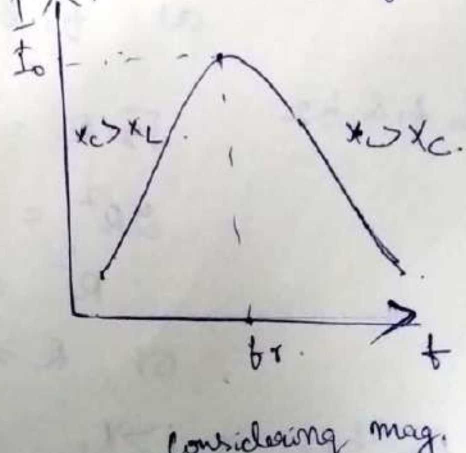


$X_C = \frac{1}{\omega C}$ $X_L = \omega L$.

variation of current with freq







Considering phase

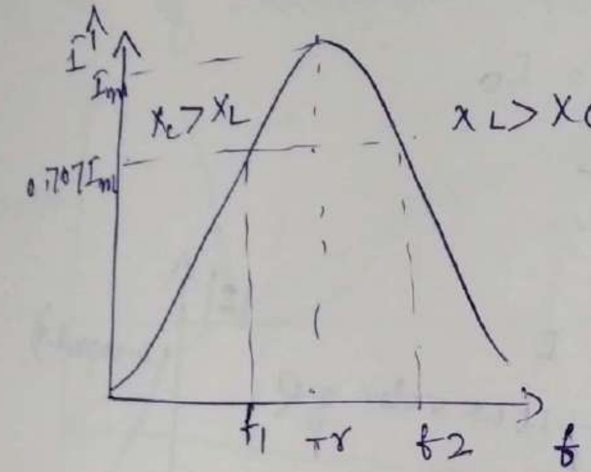
Considering mag.

$f > f_0(f_r) \Rightarrow$ inductive effect.

w.r.to current

Expression for f_1 & f_2 :-
Relation b/w, f_1, f_2, f_r.
expression for $f_{C max}$, $f_{L max}$.
expression for B.W

Expression for f_1 & f_2 & B.W



At f_1 & f_2 current is $\frac{I_m}{\sqrt{2}}$ & hence impedence is $\sqrt{2}$ time the value of impedence

In general, impedence ~~of~~ ckt is given by.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

At f_r $Z_r = R$. $Z = \sqrt{2} Z_r$

at f_1 & f_2 = $Z = \sqrt{2} R$

& f_1 & f_2

$$\sqrt{2} R = \sqrt{R^2 + (X_L - X_C)^2}$$

$$2R^2 = R^2 + (X_L - X_C)^2$$

$$R^2 = (X_L - X_C)^2$$

or $R = X_L - X_C$ ——— (1)

At f_1, $X_C > X_L$, eqn (1) can be written as

$$R = X_C - X_L$$

$$R = X_C - X_L$$

$$= \frac{1}{\omega_1 C} - \omega_1 L$$

$$R = \frac{1 - \omega_1^2 LC}{\omega_1 C}$$

$$R\omega_1 C = 1 - \omega_1^2 LC$$

$$\omega_1^2 LC + R\omega_1 C - 1 = 0 \qquad \div LC$$

$$\omega_1^2 + \frac{R}{L}\omega_1 - \frac{1}{LC} = 0$$

$$ax^2 + bx + C$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\omega_1 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4\left(\frac{-1}{LC}\right)}}{2}$$

$$= \frac{-\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} + \frac{4}{LC}}}{2}$$

$$\omega_1 = \frac{-\frac{R}{L} + \sqrt{\frac{R^2}{L^2} + \frac{4}{LC}}}{2}$$

discard

$$\omega_1 = \frac{-\frac{R}{L} - \sqrt{\frac{R^2}{L^2} + \frac{4}{LC}}}{2}$$

$-ve$ sign $\rightarrow -ve\ \omega$

$$\omega_1 = -\frac{R}{2L} + \frac{1}{2}\sqrt{\frac{R^2}{L^2} + \frac{4}{LC}}$$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

$$\boxed{f_1 = \frac{1}{2\pi}\left[-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]}$$

At f_2, $X_L > X_C$

at f_2,
$$R = X_L - X_C$$

$$= \omega_2 L - \frac{1}{\omega_2 C}$$

$$R = \frac{\omega_2^2 LC - 1}{\omega_2 C}$$

$$\omega_2 RC = \omega^2 LC - 1$$

$$\omega_2^2 LC - \omega_2 RC - 1 = 0.$$

$$\omega_2^2 - \omega_2 \frac{R}{L} - \frac{1}{LC} = 0. \qquad \div LC.$$

$$\omega_2 = \frac{\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 + 4\left(\frac{1}{LC}\right)}}{2}.$$

$$\omega_2 = \frac{\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} + \frac{4}{LC}}}{2}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}.$$

$$\boxed{f_2 = \frac{1}{2\pi}\left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]}$$

Bandwidth $: f_2 - f_1$

$$= \frac{1}{2\pi}\left\{\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right\} - \left[\frac{1}{2\pi}\left(\frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]\right.$$

$$= \frac{1}{2\pi}\left[\frac{R}{2L} + \frac{R}{2L}\right]$$

$$= \frac{1}{2\pi} \cdot \frac{\cancel{2}R}{\cancel{2}L}$$

$$\boxed{f_2 - f_1 = \frac{R}{2\pi L}.}$$

$\frac{X_C}{R} = \frac{1}{\omega CR}$

$= \frac{1}{2\pi f_r CR}$

f_r:

at resonance $Q = Q_s = \frac{X_L}{R} = \frac{\omega L}{R}$

$$= \frac{2\pi f_r L}{R}$$

$$= \frac{2\pi f_r}{R}\frac{L}{\cancel{L}}. \qquad \frac{R}{2\pi L} = f_2 - f_1$$

$$Q = \frac{f_r}{f_2 - f_1}.$$

$$Q_s = \frac{f_r}{f_2 - f_1}$$

Characterises resonance B.W relative to its Central freq.

$$f_2 - f_1 = \frac{f_r}{Q}$$

$$B.W = f_2 - f_1 = \frac{f_r}{Q} = \frac{f_r}{Q_s}$$

$$\boxed{\begin{aligned} f_2 - f_1 &= \frac{R}{2\pi L} \\ f_2 - f_1 &= \frac{f_r}{Q_s} \end{aligned}}$$

$$\frac{f_2 - f_1}{f_r} = \frac{1}{Q_s}$$

Sometimes $\frac{f_2 - f_1}{f_r} \rightarrow$ is referred to as fractional

Relation b/w f_r, f_1 & f_2:

The impedences of an RLC resonant ckt at f_1 & f_2 are given by

$X_C > X_L$ $\quad Z_1 = \sqrt{R^2 + (X_{C_1} - X_{L_1})^2}$ magnitude equal.

$$Z_2 = \sqrt{R^2 + (X_{L_2} - X_{C_2})^2}$$

But $Z_1 = Z_2$

$$\cancel{R^2} + (X_{C_1} - X_{L_1})^2 = \cancel{R^2} + (X_{L_2} - X_{C_2})^2$$

$$X_{C_1} - X_{L_1} = X_{L_2} - X_{C_2}$$

$$X_{C_1} + X_{C_2} = X_{L_1} + X_{L_2}$$

$$\frac{1}{\omega_1 C} + \frac{1}{\omega_2 C} = \omega_1 L + \omega_2 L$$

$$\frac{1}{C}\left[\frac{\cancel{\omega_1 + \omega_2}}{\omega_1 \omega_2}\right] = L\left[\cancel{\omega_1 + \omega_2}\right]$$

$$\omega_1 \omega_2 = \frac{1}{LC} \quad \text{—— (1)}$$

but $f_0 = \frac{1}{2\pi\sqrt{LC}}$.

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \omega_0^2 = \frac{1}{LC} \quad \text{--- (2)}$$

$\therefore$ from (1) & (2)

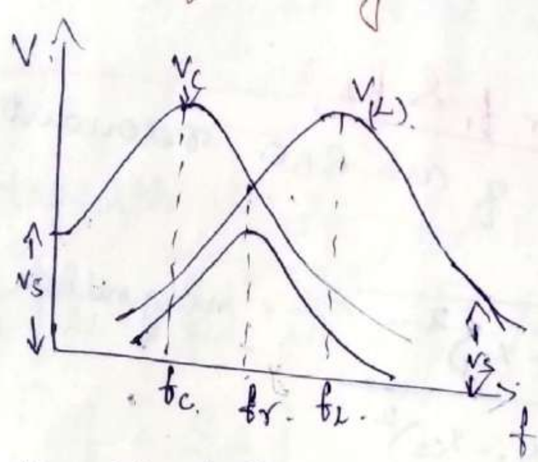
$$\omega_1 \omega_2 = \omega_0^2.$$

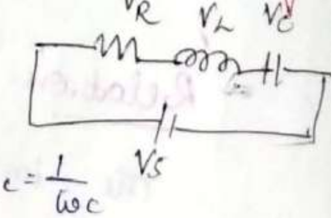
$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

or $\boxed{f_r = \sqrt{f_1 f_2}}$

i.e resonant freq is geometric mean of half power frequencies.

Variation of Voltages across L & C with frequency





Initially at $f=0 \Rightarrow C \rightarrow$ acts as open ckt & blocks current.

$\therefore$ Then across capacitor we have total i/p V_g V_s

As freq $\uparrow$, reactance of C $\downarrow$ & that.

but freq $\uparrow$, reactance of L $\uparrow$ &

so $X_C - X_L = \downarrow$. & current $\uparrow$.

As current $\uparrow$, V_g across R V_R $\uparrow$ & also both V_L & V_C $\uparrow$.

hen frequency $= f_r$, Impedence $Z = R$,

$\therefore$ current is max $\Rightarrow$ so V_R reaches max value.

As freq is still $\uparrow$ above f_r,

reactance of L $\uparrow$ & reactance of C $\downarrow$.

& hence $(X_L - X_C)$, $\uparrow$ $\Rightarrow$ current $\downarrow$,

$\Rightarrow$ so V_R $\downarrow$ & also both V_C & V_L $\downarrow$.

As frequency becomes very high, both V_R & V_C value approaches zero while V_L value approaches V_s.

From the graph, it is clear that, V_g across C & V_g across L is not max at f_r.

At resonance V_L & V_C are equal in mag. but opposite in phase

V_g V_C is max at freq f_C $(f_C < f_r)$

& V_g V_C is max at freq V_L. $(f_L > f_r)$.

Frequencies for maximum V_g across L & C

$f_{C\max}$ is freq at which $V_{C\max}$ occurs &

is $f_{C\max} < f_r$ for which $X_C > X_L$

$V \rightarrow$ supply V

$V_C = I X_C$.

$$V_C = \frac{V}{Z} \cdot \frac{1}{\omega C}$$

$$= \frac{V}{\sqrt{R^2 + (X_C - X_L)^2}} \cdot \frac{1}{\omega C}.$$

$$V_c^2 = \frac{V^2}{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2} \times \frac{1}{\omega^2 C^2}$$

$$= \frac{V^2}{\omega^2 C^2 R^2 + \omega^2 C^2 \left(\frac{1}{\omega^2 C^2} + \omega^2 L^2 - 2\frac{L}{C}\right)}$$

$$= \frac{V^2}{\omega^2 C^2 R^2 + \left(1 + \omega^4 C^2 L^2 - 2\omega^2 LC\right)}$$

$$= \frac{V^2}{\omega^2 C^2 R^2 + \left(\omega^2 LC - 1\right)^2}$$

V_c is max when $\frac{dV_c^2}{d\omega} = 0$

$$\frac{dV_c^2}{d\omega} = \frac{\cancel{\omega^2 C^2 R^2 (\omega^2}\, d r \left[\frac{d}{dr}(nr)\right] - nr\left[\frac{d}{dr}(dr)\right]}{(dr)^2}$$

$$= -V^2 \left\{2\omega C^2 R^2 + 4\omega^3 C^2 L^2 - 4\omega LC\right] = 0.$$

$$2\omega C^2 R^2 + 4\omega^3 L^2 C^2 - 4\omega LC = 0.$$

$$2\omega C\left(CR^2 + 2\omega^2 L^2 C - 2L\right) = 0.$$

$$CR^2 + 2\omega^2 L^2 C - 2L = 0.$$

$$2\omega^2 L^2 C - 2L + CR^2 = 0. \qquad \div 2L^2C$$

$$\omega^2 - \frac{1}{LC} + \frac{R^2}{2L^2} = 0.$$

$$\omega^2 = \frac{1}{LC} - \frac{R^2}{2L^2}.$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

$$\therefore \boxed{f_{c\,max} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}}$$

$\frac{1}{x} \cdot \frac{1}{x^2}$ $\frac{1 \cdot 3x}{(x+3x)^2} = 0$. $n \cdot x^{n-1}$

$\frac{1}{1+3x}$ $+ \frac{V^2}{\omega^4 C^2 R^2 (\omega^2 LC - 1)^2} = 0$

$\frac{V^2 \cdot 2\omega C^2 R^2 + 4\omega^3 C^2 L^2}{(\quad)^2}$

$V_L = IX_L$

calle

$f_L max = ?$

$f_L max = \frac{1}{2\pi}\sqrt{}$

$V_L = IX_L = \frac{V}{\sqrt{R^2+(X_L-X_C)^2}} \omega L.$

$= \frac{V \omega L}{\sqrt{R^2+\left(\omega L - \frac{1}{\omega C}\right)^2}}$

$V. = V$

Frequency deviation (δ) :

The frequency deviation of an R.L.C series ckt is defined as the ratio of the difference b/w operating frequency & resonant frequency to the resonant frequency

$$\delta = \frac{\omega - \omega_r}{\omega_r} = \frac{f - f_r}{f_r}.$$

$\omega \rightarrow$ operating freq in rad/sec

$\omega_r \rightarrow$ resonant freq in rad/sec.

$f \rightarrow$ operating frequency in Hz

$f_r \rightarrow$ resonant frequency in Hz.

The impedence of an RLC series ckt is given by

$$Q = \frac{\omega L}{R} \qquad Q = \frac{1}{\omega C R}$$

$$Z = R + jX_L - jX_C.$$

$$= R + j\omega L - j\frac{1}{\omega C}.$$

$$= R\left[1 + j\left(\frac{\omega L}{R} - \frac{1}{\omega C R}\right)\right]$$

$$= R\left[1 + j\left[\frac{\omega_r L}{R} \cdot \frac{\omega}{\omega_r} - \frac{1}{\omega_r C R} \times \frac{\omega_r}{\omega}\right)\right]$$

$$= R\left[1 + j\left(Q_s \frac{\omega}{\omega_r} - Q_s \cdot \frac{\omega_r}{\omega}\right)\right]$$

$$= R\left[1 + jQ_s\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)\right]$$

$$= R\left[1 + jQ_s\left(1 + \delta - \frac{1}{1+\delta}\right)\right]$$

$$= R\left[1 + jQ_s\left(\frac{\cancel{1} + \delta^2 + 2\delta - \cancel{1}}{1+\delta}\right)\right]$$

$$= R\left[1 + jQ\,\delta\left(\frac{2+\delta}{1+\delta}\right)\right]$$

$$\boxed{Z = R\left[1 + jQ\delta\left(\frac{2+\delta}{1+\delta}\right)\right]}.$$

$$\delta = \frac{\omega - \omega_r}{\omega_r}$$

$$\delta = \frac{\omega}{\omega_r} - 1$$

$$\frac{\omega}{\omega_r} = 1 + \delta.$$

1) A series resonant ckt ~~Containing~~ resonating 1000KHz with effective value of Q as 100 having total value of resistance in ckt is ... The applied Vg is 10V. Calculate the magn & phase angle of current & impedence of the at frequency 10KHz below resonant freq.

$f_r = 1000KHz$

$Q = 100$

$Z = R = 50\Omega$

$V = 10\Omega$

$f = 1000k - 10k = 990$

$I = \frac{V}{Z}$

$Z = R[1 + jQ\delta \frac{2+\delta}{1+\delta}]$

$\delta = \frac{f - f_r}{f_r}$

$= \frac{990k - 1000k}{1000k}$

$= \cancel{-0.99}$

$= -0.01$

$= 50[1 + j(100)(-0.01) \cdot \frac{2-0.01}{1-0.01}]$

$= 50[1 - j\,2.010]$

$= 50[$

$= 50 - 100.5j$

$= 112.25\angle{-63.55}\ \Omega$

$I = \frac{V}{Z}$

$= \frac{10}{112.25\angle{-63.55}}$

$= 0.0891\angle{63.55}$

Magnitude $= 0.0891$ $\quad \phi = \angle 63.55$

i.e Current leads Vg.

i.e below resonant freq is capacitive

E.g. It is required that a series RLC circuit should resonate at 1MHz. Determine values of R, L & C if B.W of ckt is 5KHz and its impedance is 50Ω at resonance

$B.W = 5\,KHz$

$Z_0 = R = 50\Omega$

$f_0 = 1MHz = 1\times10^6\,Hz.$

The impedence of series RLC ckt at resonance is given by

$Z_0 = R$

$R = 50\Omega.$

$$B.W = f_2 - f_1 = \frac{R}{2\pi L} = 5KHz$$

$$L = \frac{R}{2\pi(B.W)} = \frac{50}{2\pi(5000)}$$

$L = 1.5915\,mH$

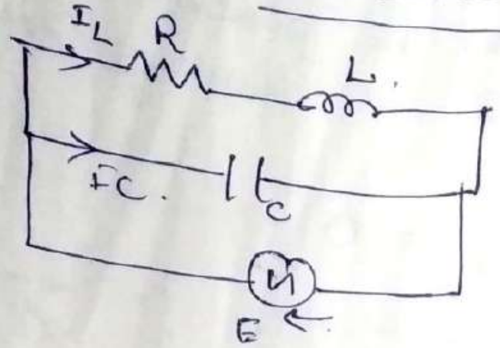
$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$C = \frac{1}{(2\pi f_0)^2 L}$$

$$= \frac{1}{(2\pi\times1\times10^6)^2(1.5915\times10^{-3})}$$

$= 15.9159\,mH$

Parallel resonance:



Consists of an inductive coil of resistance R & inductance L placed in parallel with C & Connected to an alternating supply of Vg E.

The impedence of coil is

$$Z_L = R + jwL.$$

Admittance of coil is

$$Y_L = \frac{1}{Z_L} = \frac{1}{R+jwL} \times \frac{R-jwL}{R-jwL} = \frac{R-jwL}{R^2+w^2L^2}.$$

$$Z_C = \frac{-j}{wC}$$

&

$$Y_C = \frac{1}{Z_C} = \frac{1}{\frac{-j}{wC}} = \frac{wC}{-j} \times \frac{j}{j} = jwC.$$

Total admittance of ckt is

$$Y = Y_L + Y_C$$

$$= \frac{R-jwL}{R^2+w^2L^2} + jwC.$$

$$= \frac{R}{R^2+w^2L^2} + j\left(wC - \frac{wL}{R^2+w^2L^2}\right)$$

ckt at resc...
impedence of ckt be purely resist...
or admittance must be purelly conduct...

$$\therefore \omega_r C - \frac{\omega_r L}{R^2+\omega_r^2 L^2} = 0.$$

$$\cancel{\omega_r} C = \frac{\cancel{\omega_r} L}{R^2+\omega_r^2 L^2}$$

$$R^2+\omega_r^2 L^2 = \frac{L}{C} \quad \text{———} \quad (1)$$

$$\omega_r^2 = \frac{\frac{L}{C}-R^2}{L^2} = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$\boxed{f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

At resonance, admittance of ckt is purelly condu...

$$Y_r = \frac{R}{R^2+\omega^2 L^2} \quad \text{———} \quad (2)$$

From (1) $\quad R^2+\omega^2 L^2 = \frac{L}{C}$

sub (1) in (2) $\quad Y_r = \frac{R}{\frac{L}{C}} = \frac{RC}{L}$

$$Y_r = \frac{RC}{L}$$

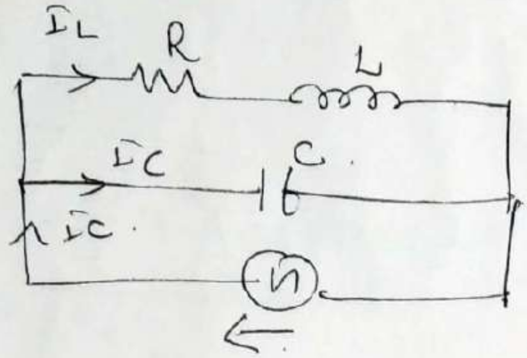
or $\boxed{Z_r = \frac{L}{RC}}$ — dynamic resistance

(1)

Parallel Resonance

Resonance → V_g & I in phase.

Here elements are Connected in parallel.



Impedence → admittace
R → Conductance.
Reactance → susceptance.

practical practical resonant ckt

Consises of an inductive coil of resistance R & an inductance L placed in parallel with Capacitance C Connected to an alternating V_g .V of variable freq f.

Impedence of coil is

$$Z_L = R + jwL.$$

Admittance of coil is

$$Y_L = \frac{1}{Z_L} = \frac{1}{R+jwL} \times \frac{R-jwL}{R-jwL} = \frac{R-jwL}{R^2+w^2L^2}.$$

$$Z_C = -\frac{j}{wC}.$$

$$Y_C = \frac{1}{Z_C} = \frac{wC}{-j} \times \frac{j}{j} = jwC.$$

Total admittance of the coil is

$$Y = Y_L + Y_C.$$

$$= \frac{R - jwL}{R^2+w^2L^2} + jwC.$$

$$= \frac{R}{R^2+w^2L^2} + j\left(wC - \frac{wL}{R^2+w^2L^2}\right)$$

At resonance, impedence of ckt is purely resistive admittance must be purely conductive.

$\therefore$ imaginary part is zero.

At resonance,

$$\omega_r C - \frac{\omega_r L}{R^2 + \omega_r^2 L^2} = 0$$

$$\cancel{\omega_r} C = \frac{\omega_r L}{R^2 + \omega_r^2 L^2} \Rightarrow \boxed{R^2 + \omega_r^2 L^2 = \frac{L}{C}} \quad \text{---} (1)$$

$$\omega_r^2 L^2 = \frac{L}{C} - R^2$$

$$\omega_r^2 = \frac{\frac{L}{C} - R^3}{L^2} = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$\& \therefore \boxed{f_r = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

At resonance, admittance of ckt is purely conductive.

$$Y_r = \frac{R}{R^2 + \omega^2 L^2}$$

$$Y_r = \frac{R}{\frac{L}{C}} \qquad \because \text{from } (1) \; R^2 + \omega^2 L^2 = \frac{L}{C}$$

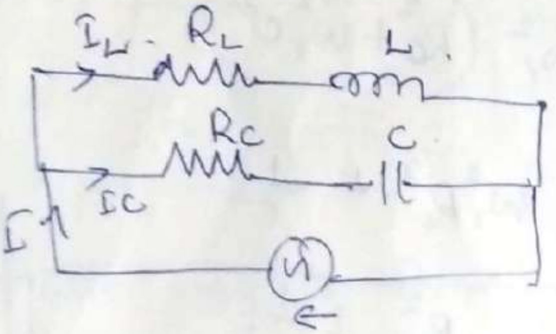
$$Y_r = \frac{RC}{L}$$

or $\boxed{Z_r = \frac{L}{RC}}$ $\Rightarrow$ dynamic resistance.

Current at resonance is given by

$$I_r = \frac{E}{Z} = \frac{E \times 1}{\frac{L}{RC}} = \frac{E \, RC}{L}$$

Parallel Resonant cht Considering capacitance to have Resistance



Consider a parallel cht as shown.

$Z_L = R_L + j\omega L$.

i.e $Y_L = \frac{1}{R_L + j\omega L} \times \frac{R_L - j\omega L}{R - j\omega L} = \frac{R_L - j\omega L}{R^2 + \omega^2 L^2}$.

$Z_C = R_C - j\frac{1}{\omega C}$.

$Y_C = \frac{1}{R_C - j\frac{1}{\omega C}} \times \frac{R_C + j\frac{1}{\omega C}}{R_C + j\,\omega C} = \frac{R_C + j\frac{1}{\omega C}}{R_C^2 + \frac{1}{\omega^2 C^2}}$.

Total admittance Y

$$Y = Y_L + Y_C.$$

$$= \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2} + \frac{R_C + j\frac{1}{\omega C}}{R_C^2 + \frac{1}{\omega^2 C^2}}$$

$$= \left(\frac{R_L}{R_L^2 + \omega^2 L^2} + \frac{R_C}{R_C^2 + \frac{1}{\omega^2 C^2}}\right) + j\left[\frac{1/\omega C}{R_C^2 + \frac{1}{\omega^2 C^2}} - \frac{\omega L}{R_L^2 + \omega^2 L^2}\right]$$

At resonance,

Admittance of ckt is purely Conductive.

$\therefore$ imaginary part is 0.

i.e. $$\frac{\frac{1}{\omega_r C}}{R_C^2 + \frac{1}{\omega^2 C^2}} = \frac{\omega_r L}{R_L^2 + \omega_r^2 L^2}.$$

$$\frac{R_L^2 + \omega_r^2 L^2}{\omega_r C} = \omega_r L \left(R_C^2 + \frac{1}{\omega_r^2 C^2} \right).$$

$$\frac{1}{LC}\left(R_L^2 + \omega_r^2 L^2\right) = \omega_r^2 \left(R_C^2 + \frac{1}{\omega_r^2 C^2} \right)$$

$$\frac{R_L^2}{LC} + \frac{\omega_r^2 L}{C} = \omega_r^2 R_C^2 + \frac{1}{C^2}$$

$$\boxed{\omega_r^2 \left(R_C^2 - \frac{L}{C} \right) = \frac{R_L^2}{LC} - \frac{1}{C^2} = \frac{1}{LC}\left(R_L^2 - \frac{L}{C} \right).}$$

Condition should be 0

$\omega_r^2\left(R_C^2 - \frac{L}{C}\right) = \frac{1}{LC}$ [illegible]

$R_C^2 - \frac{L}{C} = 0$

or $R_L^2 - \frac{L}{C} = 0$

$$\therefore \omega_r^2 = \frac{\frac{1}{LC}\left(R_L^2 - \frac{L}{C} \right)}{R_C^2 - \frac{L}{C}}.$$

$$\omega_r = \frac{1}{\sqrt{LC}} \sqrt{\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}}.$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}}.}$$

The admittance at resonance is purely conductive

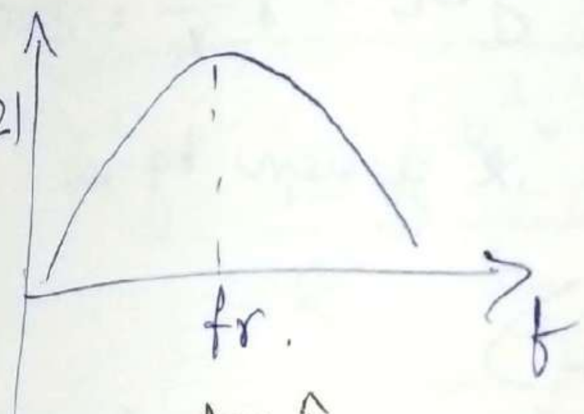
$$Y_r = \frac{R_L}{R_L^2 + \omega_r^2 L^2} + \frac{R_C}{R_C^2 + \frac{1}{\omega^2 C^2}}.$$

Current at resonance is given by

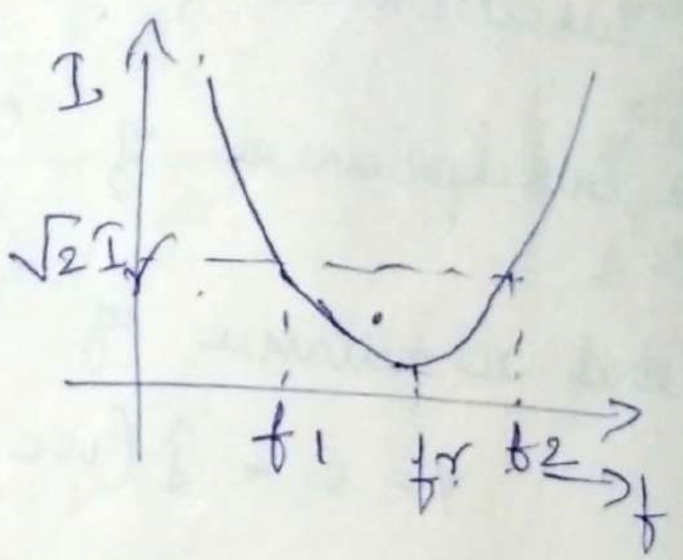
$$I_r = E\,Y_r = \frac{E}{Z}.$$

$$= E \left[\frac{R_L}{R^2 + \omega_r^2 L^2} + \frac{R_C}{R_C^2 + \frac{1}{\omega^2 C^2}} \right]$$

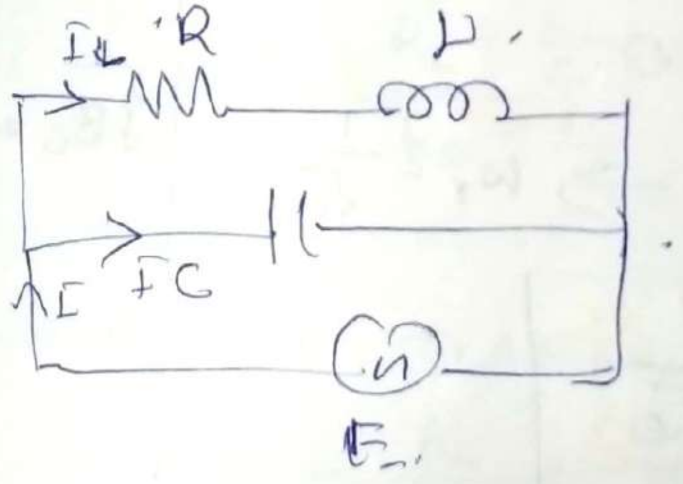
2



$2\pi f L \rightarrow f\uparrow \Rightarrow X_L \uparrow$



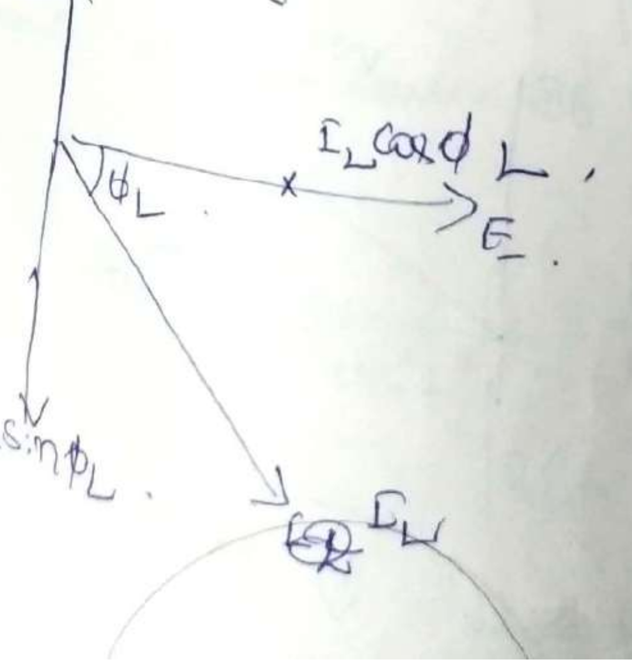
Q factor of a parallel resonant ckt.



VI

L →

Vector diagram is shown



I_L lags V

I_C leads V

$Z = \sqrt{R^2 + (\omega L)^2}$

$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$

Q factor of a parallel resonance

At resonance only reactive currents flow through 2 branches. ($I_L \sin\phi_L$) through R-L branch & I_C through capacitance. These currents will be many times more than the total current at resonance.

The quality factor of parallel resonant ckt is defined as current magnification.

$$Q_P = \frac{\text{current through capacitor at resonance}}{\text{Total current at resonance.}}$$

$$X_C = \frac{-j}{wC} \Rightarrow Y_C = \frac{wC}{-j} \times \frac{j}{j}$$

$$= \frac{I_{Cr}}{I_r}$$

Acceptance/susceptance

$$= \frac{E Y_C}{E Y_2} = \frac{E Y_C}{\cancel{E} Y_2}$$

$$= \frac{E W_r C}{\cancel{E}\ \frac{RC}{L}} = \frac{W_r C}{\frac{RC}{L}}$$

$$= \frac{W_r C}{\frac{RC}{L}}$$

$$= \frac{W_r L}{R}$$

$$= \frac{X_L}{R}$$

$$= Q_S.$$

$$Y_r = \frac{R}{R^2 + w^2 L^2}$$

$$Y_r = \frac{R}{L/C}$$

$$Y_r = \frac{RC}{L}$$

$$\text{or } Z_r = \frac{L}{RC}.$$

$$X_L = jwL$$

$$\frac{1}{X_L} = \frac{1}{jwL} \times \frac{j}{j} = \frac{j}{-wL}$$

$$\frac{EY_L}{EY_2} = \frac{\frac{1}{wL}}{\frac{RC}{L}}$$

$$=$$

∴ quality factor of series & parallel resonance ckt is same.

$\sqrt{2}$ ... at Res

Imp
To derive f_1, f_2, => impedance of parallel reso Freq is 0.707 times value of max impedance at reso

i.e $Z = \frac{R_L Q_0^2}{1 + j 2 Q_0 \delta}$ } deriv not required

freq deviation
$$\delta = \pm \frac{1}{2 Q_0}$$

~~$f_2 - f_r = f$~~

frequency deviation → ratio of diff b/w operating freq & resonant freq to reson freq

$$\delta = \frac{f - f_r}{f_r} = \pm \frac{1}{2 Q_0}.$$

B.W of parallel ckt is given by

$$B.W = f_2 - f_1 = \frac{\text{resonant freq}}{\text{Quality factor}} = \frac{f_r}{Q_P}.$$

$$\boxed{f_2 - f_1 = \frac{f_r}{Q_P}} \Rightarrow Q_P = \frac{f_r}{f_2 - f_1}$$

$$\text{Selectivity} = \frac{\text{Resonant freq}}{B.W} = \frac{f_r}{f_2 - f_1}$$

$$\boxed{\text{Selectivity} = \frac{f_r}{f_2 - f_1} = Q_P}$$

in series res

$f_2 - f_1 = \frac{R}{2\pi L}$

$Q = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{2\pi f_r L}{R}$

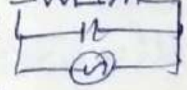
$Q = \frac{f_r}{f_2 - f_1}$

$\boxed{Q = \frac{f_r}{B.W}}$

& also,

$$I_L = Q_0 I$$
$$I_C = Q_0 I$$

Current through inductor & capacitor are Q times supplied current at antiresonance.

Frequency of parallel resonance can be  written in another form as follows

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}}$$

$$= \frac{1}{2\pi}\frac{1}{\sqrt{LC}}\sqrt{1 - \frac{CR_L^3}{L}}$$

$$=$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{1}{Q^2}}$$

$$Q = \frac{\omega L}{R} = \frac{1}{\omega R_L C}$$

$$Q = \frac{1}{\omega R_L C} \Rightarrow \omega = \frac{1}{QR_L C}$$

$$\cancel{Q = \frac{\frac{L}{QRC}}{R}}$$

$$\cancel{Q = \frac{RL}{QRC}}$$

$$Q^2 = \frac{L}{C}$$

$$Q = \frac{\left(\frac{1}{QRC}\right)L}{R}$$

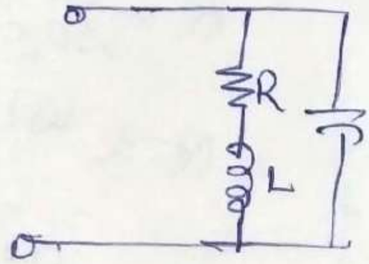
$$Q = \frac{L}{QR^2C}$$

$$Q^2 = \frac{L}{CR^2}$$

This eqn indicates that freq differs from that of series ckt with the same elements by a factor $\sqrt{1 - \frac{1}{Q_0^2}}$.

For high quality factor circuits, frequency of series & parallel ckts are almost same.

In the ckt given below, an inductance of ... having a Q of 5 is in parallel with capa... Determine the value of capacitance & coil ... at resonant freq of 500 rad/sec.



$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

or $$f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{1}{Q_0^2}}$$

$$(2\pi f_r =) \frac{1}{\sqrt{LC}}\sqrt{1 - \frac{1}{Q_0^2}}$$

$$(2\pi f_r)^2 = \frac{1}{LC}\left(1 - \frac{1}{Q_0^2}\right)$$

$$C = \frac{1}{(2\pi f_r)^2 \times L}\left(1 - \frac{1}{Q_0^2}\right)$$

$$= \frac{1}{(500)^2 \times 0.1}\left(1 - \frac{1}{5^2}\right)$$

$$= 38.4 \times 10^{-6}\ F$$

$$\boxed{C = 38.4\,\mu F}$$

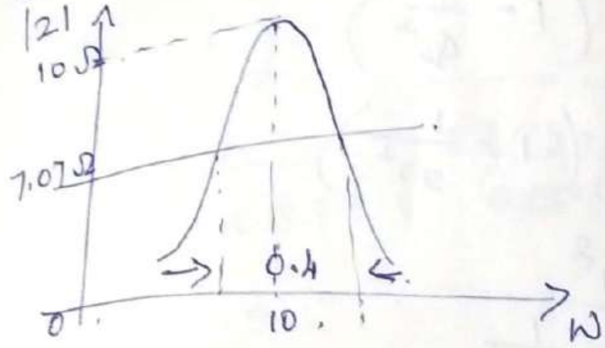
$$Q = \frac{WL}{R} \Rightarrow R = \frac{WL}{Q}$$

$$R = \frac{\overset{100}{\cancel{500}} \times 0.1}{\cancel{5}}$$

$$\boxed{R = 10\,\Omega}$$

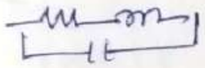
6

Determine the R-L-C parallel ckt parameters whose response curve is as shown in fig. what are the new values of ω_r and bandwidth if C is increased 4 times?



From fig. $Z_r = 10\Omega$. $\omega_r = 10$ rad/sec.

$B.W = 0.4$ rad/S.

Consider Std parallel ckt 

$\therefore Z_r = \frac{L}{RC} = 10\Omega$.

$Q_p = \frac{f_r}{BW} = \frac{\omega_r}{BW}$.

$Q_p = \frac{10}{0.4} = 25$.

1) $\boxed{Z_r = R(1+Q_0^2)}$.

$10 = R(1+25^2)$.

$R = \frac{10}{1+25^2}$

$= 0.01597\Omega$.

$Z_r = \frac{L}{RC}$.

$10 = \frac{L}{RC} \Rightarrow \frac{L}{C} = R(10)$

$\Rightarrow \frac{L}{C} = (0.01597)(10)$

$\frac{L}{C} = 0.1597$.

$B.W = \frac{f_r}{Q_p}$.

In derv:

$R^2 + \omega^2 L^2 = \frac{L}{C}$.

$R^2\left(1 + \frac{\omega^2 L^2}{R^2}\right) = \frac{L}{C}$

$R^2(1+Q_0^2) = \frac{L}{C}$.

$R(1+Q_0^2) = \frac{L}{CR}$

$R(1+Q_0^2) = Z$

$$f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{1-\frac{1}{Q_0^2}}.$$

$$\omega_r = \frac{1}{\sqrt{LC}}\sqrt{1-\frac{1}{Q_0^2}}.$$

$$\omega_r^2 = \frac{1}{LC}\left(1-\frac{1}{Q_0^2}\right).$$

$$(10)^2 = \frac{1}{LC}\left(1-\frac{1}{25^2}\right)$$

$$\frac{1}{LC} = \frac{10^2}{1-\frac{1}{25^2}}$$

$$\frac{1}{LC} = 100.16$$

$$LC = 9.984\times10^{-3}. \quad \text{—— (2)}$$

we have $\frac{L}{C} = 0.1597 \Rightarrow C = \frac{L}{0.1597}$ —— (3)

Sⁱ (3) in (2).

$$L\,\frac{L}{0.1597} = 9.984\times10^{-3}$$

$$L = 0.03993H.$$

$$L = 39.93mH$$

from (3):

$$C = \frac{L}{0.1597} = 0.25F.$$

∴

$$\boxed{\begin{array}{l} L = 39.93mH \\ C = 0.25F \\ R = 0.01597. \end{array}}$$

(7)

Now if $c' = 4C = 4(0.25) = 1F$ used in the ckt.

$$f_{r_{new}} = \frac{1}{\sqrt{LC}} \sqrt{1 - \frac{1}{Q_0^2}}$$

$$Q_0' = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$= \frac{1}{0.01597}\sqrt{\frac{39.93\,mH}{1F}}$$

$$= 12.51$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}\left(\sqrt{1 - \frac{1}{Q_0^2}}\right)$$

$$\omega_r = \frac{1}{2\pi\sqrt{LC}}$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$\therefore$ 5 rad/s.

$$\omega_{r_{new}} = \frac{1}{\sqrt{LC}}\left(\sqrt{1 - \frac{1}{Q_0^2}}\right)$$

$$= \frac{1}{\sqrt{39.93m \times 1}}\left(\sqrt{1 - \frac{1}{(12.51)^2}}\right)$$

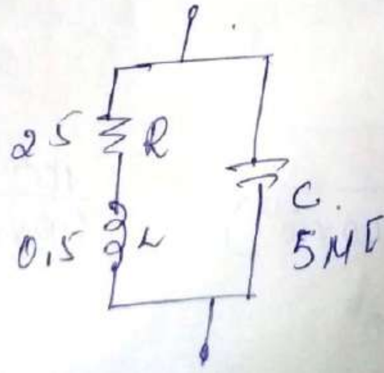
0.9967

$$= 4.988 \text{ rad/s}$$

$$B.W = \frac{\omega_r}{Q_0} = \frac{4.988}{12.5} = 0.399 \text{ rad/sc}$$

$\simeq$ 0.4 rad/sc.

3) If $R = 25\Omega$, $L = 0.5H$ $C = 5\mu F$. find ω_r, Q and bandwidth for the ckt shown.



$$\omega_r = \frac{1}{\sqrt{LC}}($$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$\omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}.$$

$$= \sqrt{\frac{1}{0.5 \times 5 \times 10^{-6}} - \frac{(25)^2}{(0.5)^2}}$$

$$= 630.476 \text{ rad/sec.}$$

$$Q = \frac{\omega L}{R}$$

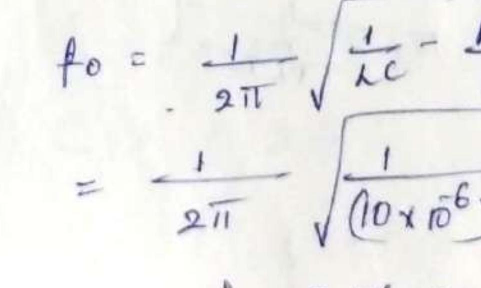
$$= \frac{630.476 \times 0.5H}{25}$$

$$= 12.6095.$$

$\omega_2 - \omega_1$ $\quad B.W = \dfrac{\omega_r}{Q_0} = \dfrac{630.476}{12.6095} =$

$$= 50 \text{ rad/Sec.}$$

For the parallel resonant ckt shown in fig find I_0, I_L, I_C, f_0 & dynamic resistance.



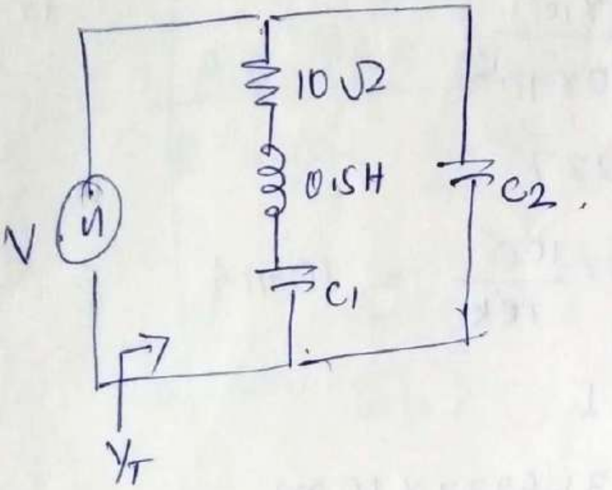
$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}.$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{(10 \times 10^{-6})(100 \times 10^{-12})} - \frac{10^2}{(10 \times 10^{-6})^2}}$$

$$= \frac{1}{2\pi} 31.6067 \times 10^{6}.$$

$$f_0 = 5.0298 \text{ MHz}.$$

A coil of $R = 10\Omega$ and $L = 0.5H$ is Connected in Series with a capacitor. The current is maximum when $f = 50Hz$. A second capacitor is Connected in parallel with this ckt. what capacitance must it have so that the Combn acts like a non inductive resistor at 100Hz. calculate the total current supplied in each case if the applied vg is 220V.



$f_0 = 50H.$

Let us calculate C_1 first,

$$f_0 = \frac{1}{2\pi\sqrt{LC_1}}$$

$$\sqrt{LC_1} = \frac{1}{2\pi f_0}$$

$$LC_1 = \frac{1}{(2\pi f_0)^2}$$

$$C_1 = \frac{1}{(2\pi f_0)^2 \times L}$$

$$= 20.258 \times 10^{-6} F$$

$C_1 = 20.258\, \mu F.$

$$Z = \frac{L}{CR}$$

$$= \frac{10\mu}{100P \times 10}$$

$$= 10k\,\Omega$$

$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$= \frac{1}{10}\sqrt{\frac{10 \times 10^{-6}}{100 \times 10^{-12}}}$$

$$= 31.6227$$

$$I_0 = \frac{V}{Z} = \frac{100}{10k} = 10mA$$

$$I_C = I_L = Q\,I$$

$$= 31.6227 \times 10m$$

$$= 316.227mA.$$

$$\omega_r^2 = \frac{1}{LC}\frac{(R_L^2 - \frac{L}{C})}{R_C^2 - \frac{L}{C}}$$

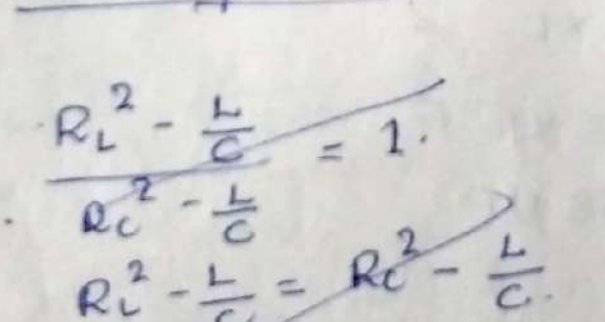
$$\omega_r = \frac{1}{\sqrt{LC}}\frac{(R_L^2 - \frac{L}{C})}{R_C^2 - \frac{L}{C}}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}$$

$$R_L^2 - \frac{L}{C} = 0 \quad \text{or} \quad R_C^2 -$$

$$R_L^2 = \frac{L}{C}$$

Determine R_L and R_C for which ckt shown in fig resonate at all frequencies.



$$f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}}$$

$$\cancel{\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}} = 1}$$

$$\cancel{R_L^2 - \frac{L}{C} = R_C^2 - \frac{L}{C}}$$

$$R_L^2 - R_C^2 =$$

$$R_L^2 - \frac{L}{C} = 0$$

$$R_L^2 = \frac{L}{C}$$

$$R_L = \sqrt{\frac{L}{C}}$$

$$= \sqrt{\frac{4 \times 10^{-3}}{40 \times 10^{-6}}}$$

$$= 10\Omega$$

$$R_C = \sqrt{\frac{L}{C}}$$

$$R_C = 10\,\Omega$$

C_2 is connected in parallel with the above ckt. 8

Admittance

$$Y_T = \frac{1}{R + j\left(\omega L - \frac{1}{\omega C_1}\right)} + \frac{1}{\frac{-j}{\omega C_2}}$$

$$\frac{\omega C_2}{-j} \times \frac{j}{j} = \frac{\omega C_2 j}{1}$$

$$= \frac{1}{R + j\left(\omega L - \frac{1}{\omega C_1}\right)} + j\omega C_2$$

$$= \frac{R - j\left(\omega L - \frac{1}{\omega C_1}\right)}{R^2 + \left(\omega L - \frac{1}{\omega C_1}\right)^2} + j\omega C_2$$

$$= \frac{R}{R^2 + \left(\omega L - \frac{1}{\omega C_1}\right)^2} + j\left[\omega C_2 - \frac{\left(\omega L - \frac{1}{\omega C_1}\right)}{R^2 + \left(\omega L - \frac{1}{\omega C_1}\right)^2}\right]$$

Above resonance, it must be purely resistive.

hence susceptance should be 0.

At $f = 100\,Hz$

$$\omega C_2 - \frac{\left(\omega L - \frac{1}{\omega C_1}\right)}{R^2 + \left(\omega L - \frac{1}{\omega C_1}\right)^2} = 0.$$

$$\omega C_2 = \frac{\left(\omega L - \frac{1}{\omega C_1}\right)}{R^2 + \left(\omega L - \frac{1}{\omega C_1}\right)^2}.$$

$$C_2 = \frac{\cancel{\omega}\left(L - \frac{1}{\omega^2 C_1}\right)}{\cancel{\omega}\left[R^2 + \left(\omega L - \frac{1}{\omega C_1}\right)^2\right]}$$

$$C_2 = \frac{0.5 - \frac{1}{(2\pi \times 100)^2 \times (20.2642 \times 10^{-6})}}{10^2 + \left(2\pi \times 100 \times 0.5 - \frac{1}{(2\pi \times 100 \times 20.2642 \times 10^{-6})}\right)^2}$$

$$\frac{0.5 - 0.125}{(10)^2 + 55540.45}$$

$= \cancel{6.7426}$

$= 6.7397\,\mu F$

with C_1 only in series RLC ckt, max current is given by

$$I_0 = \frac{V}{R} = \frac{220}{10} = 22\,A.$$

with C_2 in ckt, impedence at ckt resonance

$$Y_r = \frac{R}{R^2 + \left(\omega L - \frac{1}{\omega C_1}\right)^2}$$

$$Z = \frac{R^2 + \left(\omega L - \frac{1}{\omega C_1}\right)^2}{R}$$

$$= \frac{(10)^2 + \left[2\pi \times 100 \times 0.5 - \left(\frac{1}{(2\pi \times 100 \times 20.2642 \times 10^{-6})}\right)\right]^2}{10}$$

$$= \frac{100 + \left(314.2 - \cancel{7852.47}\ 78.529\right)^2}{10}$$

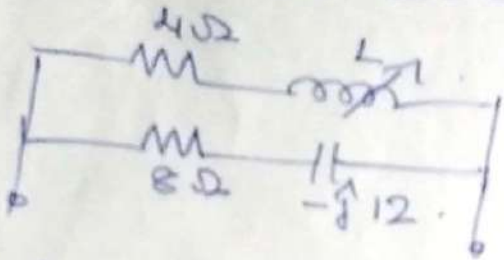
$$= \cancel{\frac{56.833 \times 10^6}{10}} \quad \frac{100 + 55540.82}{10}$$

$$= \cancel{5.6833 \times 10^6\,\Omega} \quad 5.564 \times 10^3\,\Omega.$$

Max Current

$$I_0 = \frac{220}{Z_{ar}} = \frac{220}{5.5616 \times 10^3} = 39.55\,mA.$$

Find the values of L for which given ckt. resonates at $\omega = 5000$ rad/sec



Total admittance of ckt is given by.

$$Y = \left[\frac{1}{4+jX_L} + \frac{1}{8-j12}\right] ℧$$

$$= \frac{4-jX_L}{16+X_L^2} + \frac{8+j12}{8^2+12^2}$$

$$= \left(\frac{4}{4^2+X_L^2} + \frac{8}{8^2+12^2}\right) + j\left(\frac{12}{8^2+12^2} - \frac{X_L}{16+X_L^2}\right).$$

At resonance, imaginary part is zero.

$$\frac{12}{8^2+12^2} - \frac{X_L}{16+X_L^2} = 0.$$

$$\frac{12}{8^2+12^2} = \frac{X_L}{16+X_L^2}.$$

$$\frac{3}{52} = \frac{X_L}{16+X_L^2}.$$

$$3X_L^2 + 48 = 52X_L.$$

$$3X_L^2 - 52X_L + 48 = 0.$$

$$X_L^2 - \frac{52}{3}X_L + 16 = 0.$$

$$X_L = 16.36 \text{ or } 0.978.$$

$$X_L = \frac{\frac{52}{3} \pm \sqrt{\left(\frac{52}{3}\right)^2 - 4}}{2}$$

$$= \frac{\frac{52}{3} \pm 15.377}{2}$$

$$= 16.355 \text{ or } 0.9$$

or $\omega L = 16.36$ or $\omega L = 0.978$

$L = 3.27$ mH $\quad L = 0.196$ mH.

problems on series resonance $Z = R\left[1+jQ_s\delta\left(\frac{2+\delta}{1+\delta}\right)\right]$

$f_0 = \frac{1}{2\pi\sqrt{LC}}$, $\quad Q = \frac{V_L}{V}$ or $V_L = QV$

$f_0 = \sqrt{f_1 f_2}$. $\quad Q = \frac{V_C}{V}$ or $V_C = QV$

$B.W = f_2 - f_1$

$B.W = \frac{f_0}{Q}$ $\quad Q = \frac{X_L}{R} = \frac{\omega_0 L}{R}$

$B.W = \frac{R}{2\pi L}$ $\quad Q = \frac{1}{\omega_0 C R}$

$f_C = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$ $\quad Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

$Q = \frac{f_0}{f_2 - f_1}$

$f_L = \frac{1}{2\pi\sqrt{LC - \frac{R^2C^2}{2}}}$

1. In a series RLC ckt driven with a sinusoidal a.c voltage source determine value of C required to achieve resonance in a ckt at 5kHz if value of resistance & inductance are 2Ω and 1mH respectively.

$R = 2\Omega \quad L = 1mH, \quad f_0 = 5kHz$.

at resonance,

$V_L = I_0(jX_L)$
$= j\frac{V}{R}(\omega_0 L)$
$= j(Q_0)V$

$V_C = I_0(-jX_C)$
$= \frac{V}{R}(-j/\omega C)$
$= -jVQ_0$

$f_0 = \frac{1}{2\pi\sqrt{LC}}$.

$\sqrt{LC} = \frac{1}{2\pi f_0}$.

$LC = \frac{1}{(2\pi f_0)^2}$.

$C = \frac{1}{(2\pi f_0)^2 \times L}$

$= 1.013 \times 10^{-6}$ F

$= 1.013 \mu F$.

A series RLC ckt consists of a resistance of
1 kΩ & an inductance of 100 mH in series with
capacitance of 10 PF. If 100 V is applied as
i/p across the combn determine
i) resonant freq ii) max current in the ckt
iii) Q factor of ckt
iv) half-power frequencies

$R = 1 k\Omega$
$L = 100 m$
$C = 10 PF$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{100 \times 10^{-3} \times 10 \times 10^{-12}}}$$

$$= 159.13 \cdot kHz.$$

ii) $I_0 = \frac{V}{R} = \frac{100}{1000} = 0.1 A.$

iii) $Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{100\Omega}\sqrt{\frac{100 m}{10 \times 10^{-12}}} = 100.$

or $Q = \frac{\omega L}{R} = \frac{2\pi f_0 \times L}{R} = 100.$

iv)

iv) f_1 & f_2

$$f_1 = \frac{1}{2\pi}\left[-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$$

$$f_2 = \frac{1}{2\pi}\left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right.$$

$$\sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} = \sqrt{\left(\frac{1000}{2\times 100\times 10^{-3}}\right)^2 + \frac{1}{100\times 10^{-3}\times 10\times 10^{-12}}}$$

$$= 1000002.5$$

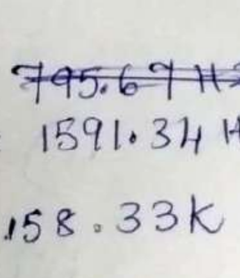
$$\cancel{f_1} = \cancel{\frac{1}{2\pi}}\ \frac{R}{2L} = \frac{1000}{2\times 100\times 10^{-3}} = 5000$$

$$f_1 = \frac{1}{2\pi}\left[-5000 + 1000002.5\right]$$

$$= 158.339\,\text{KHz}$$

$$f_2 = \frac{1}{2\pi}\left[5000 + 1000002.5\right] \qquad \pi = 3.142$$

$$= 159.93\,\text{KHz}$$



or B.W $\Delta f = f_2 - f_1 = \frac{R}{2\pi L}$

$$\Delta f = \frac{R}{2\pi L}$$

$$\Delta f = \frac{R}{2\pi L} = \frac{1000}{2\pi\times 100\times 10^{-3}} = \cancel{795.67\,\text{Hz}} = 1591.34\,\text{Hz}$$

$$f_1 = f_0 - \frac{\Delta f}{2} = 159.13\text{K} - 795.67 = 158.33\text{K}$$

$$f_2 = f_0 + \frac{\Delta f}{2} = 159.13\text{K} + 795.67 = 159.93\text{K}$$

(4)

do it later

In series RLC ckt with variable capacitance, the current is at max value with capacitance of 20 μF and the current reduces 0.707 times max value with capacitance of 30 μF. Find the values of R and L. What is the B.W of ckt if supply v_g is $20 \sin(6.28 \times 10^3)t$ Volts.

$V = V_m \sin \omega t$

$\omega = 6.28 \times 10^3$.

This is tuning ckt $\Rightarrow \therefore f_i = f_r$.

$f_r = \frac{\omega}{2\pi} = \omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi}$

$f_0 = f_r = \frac{6.28 \times 10^3}{2 \times \pi} = 1000 Hz$.

$f_0 = \frac{1}{2\pi\sqrt{LC}}$ at resonance $C = 20 \mu F$.

$\sqrt{LC} = \frac{1}{2\pi f_0}$.

$L = \frac{1}{(2\pi f_0)^2 C}$.

$= \frac{1}{(2\pi \times 1000)^2 \times 20 \times 10^{-6}}$

$\boxed{L = 1.2662 mH}$

At half power freq current becomes 0.707 ti[mes]
of its max value with $C = 30\mu F$. & ~~the~~ Resista[nce]
of ckt equals resistance of the ckt.
at this stage reactance is given by

$$|X| = X_L - X_C = \left(\omega L - \frac{1}{\omega C}\right)$$

$$\cancel{X =} \omega L = 2\pi f_0 L = 2\pi \times 1000 \times 1.2662 \times 10^{-3} = 7.957\,\Omega$$

$$\omega C = 2\pi f_0 C = 2\pi f_0 \times 30 \times 10^{-6} = \cancel{0.1885} \; 5.307\,\Omega$$

$$|X| = \left(7.957 - \frac{1}{0.1885}\right)$$

$$X = 2.652\,\Omega$$

$$\cancel{Z = R + jX =}$$

$\therefore$ resistance $R = X = 2.652\,\Omega$.

At
$Z = R + j(X_L - X_C)$
At, f_0 $Z = R$
at $f_1 = Z = \sqrt{2}R$.
f_1 $R = X_L - X_C$.
f_2 $R = X_C - X_L$.

$$B.W = \frac{f_0}{Q_0} = \frac{f_0}{\frac{\omega_0 L}{R}} = \cancel{100}$$

$$Q = \frac{\omega_0 L}{R} = \frac{7.957}{2.652} = 3.$$

$$\therefore B.W = \frac{f_0}{Q} = \frac{1000\,Hz}{3} = 333.33\,Hz.$$

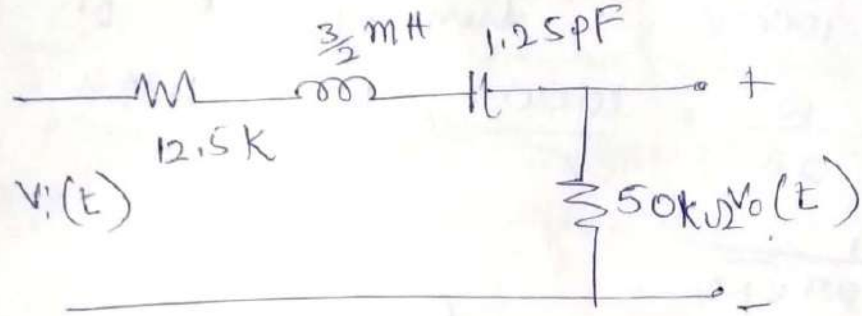
For the ckt shown. Determine the following

i) f_0

ii) Q

iii) Half power freq.

iv) Bandwidth



total $R = 12.5k + 50k = 62.5k$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{\frac{3}{2}\times10^{-3}\times1.25\times10^{-12}}}$$

$$= 3.675\,MHz = 3675049.454 = 36750.495\times10^{3}$$

$$Q = \frac{\omega L}{R} = \frac{2\pi f_0 L}{R} = \frac{2\pi f_0 L}{R} = 5.543$$

or $Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} =$

$$B.W = \Delta f = f_2 - f_1 = \frac{f_0}{Q_0} = \frac{36750.495\times10^{3}}{5.543}$$

$$\Delta f = 6630.073\,KHz$$

$$\cancel{\frac{\Delta f}{2} = 3315.037\,KHz}$$

$$f_1 = f_0 - \frac{\Delta f}{2} = 33435.458\times10^{3} = 33.435\times10^{6}.$$

$$f_2 = f_0 + \frac{\Delta f}{2} = 40065.5\,KHz = 40.066\,MHz$$

6) A coil is connected in series with a Variab Capacitor across $V(t) = 10\cos 1000t$.
The capacitor is ~~varying~~ & current is max when $C = 10\mu F$. when $C = 12.5\mu F$, the current is 0.707 times the max value. Find L, R & Q of the coil.

$\omega = 1000$. tuning ckt $f_1 = f_r$

$$f_0 = \frac{\omega}{2\pi} = \frac{1000}{2\pi} \rightarrow 159.134\,Hz.$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

at f_0 $C = 10\times10^{-6}$.

$$\sqrt{LC} = \frac{1}{2\pi f}$$

$$L = \frac{1}{(2\pi f)^2 \times C} = \frac{1}{(2\pi\times159.134)^2 \times 10\times10^{-6}}$$

$$L = 0.1\,H.$$

f_1 & f_2

$$R = |X_L - X_C|$$

$$= \omega L - \frac{1}{\omega C}$$

$$\begin{cases} \omega L = 1000\times0.1 = 100. \\ \omega C = 1000\times12.5\times10^{-6} = 0.0125 \end{cases}$$

$$R = \left[100 - \frac{1}{0.0125}\right]$$

$$R = 20\,\Omega.$$

$$Q = \frac{\omega L}{R} = \frac{1000\times0.1}{20} = \frac{100}{20} = 5.$$