BMS
INSTITUTE OF TECHNOLOGY AND MANAGEMENT
Avalahalli, Doddaballapur Main Road, Bengaluru – 560064

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

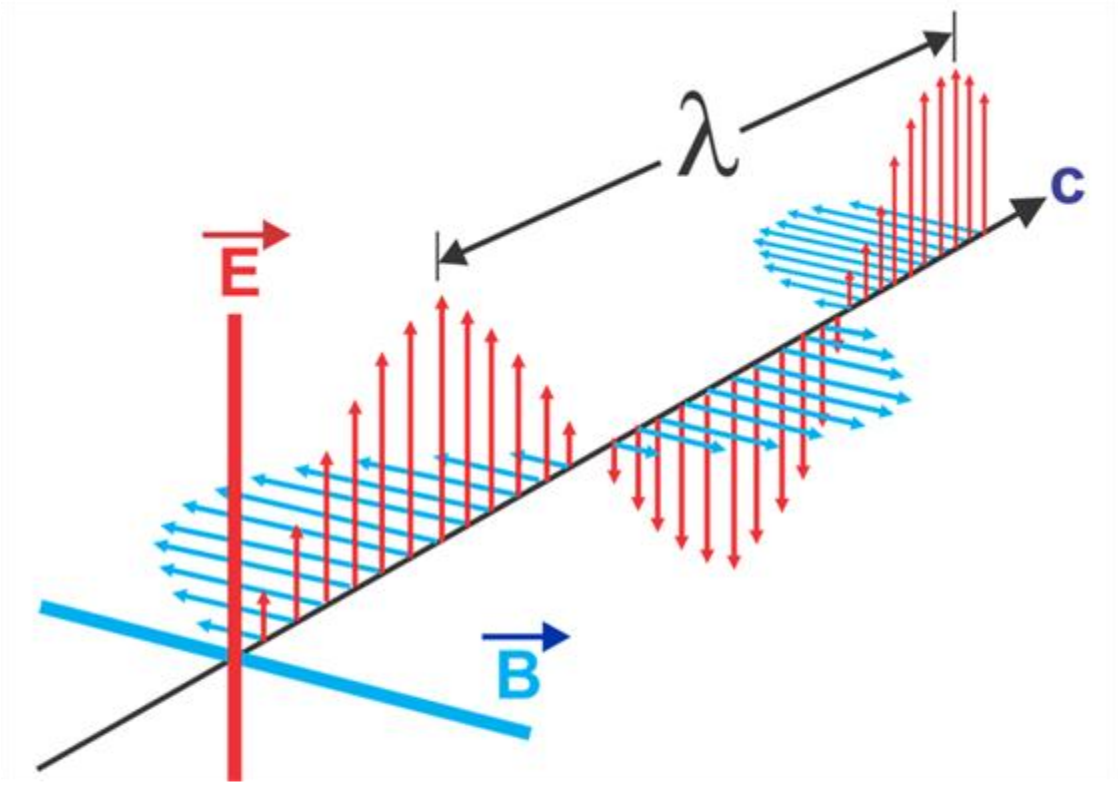
ELECTROMAGNETIC WAVES
18EC55

STUDY MATERIAL

V SEMESTER

B.M.S INSTITUTE OF TECHNOLOGY & MANAGEMENT

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING



5TH SEM   NOTES

SUBJECT: ELECTROMAGNETIC WAVES (18EC55)

Subject: Engineering Electromagnetics

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- Important Applications of Engineering Electromagnetics
- Comparision between Network analysis and Electromagnetic field theory
- Symbols of scalar Parameters
- Symbols of Vector Parameters
- Small value representation
- Large Value representation
- List of Physical constants

Fundamentals of Vector Algebra

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2. Definition of vector
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   b. Position and Distance vector
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5. Orthogonal Co-ordinate System
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      - Point transformation
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6. ∇, Gradient, Divergence, Curl ,Laplace's and poison's Equation
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* INTRODUCTION

Engineering Electromagnetics deals with electric field, magnetic field and also electromagnetic fields and phenomena.

"Electromagnetics is a branch of physics (or) electrical engineering in which electric and magnetic phenomena are studied."

Electromagnetic theory is essential to design and analyze all communication and radar systems.

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* Important Applications of Engineering Electromagnetics

Electromagnetic principles are used in various disciplines such as microwaves, antennas, electric machines, satellite communications, bioelectromagnetics, plasmas, nuclear research, fiberoptics, radar meteorology and remote sensing.

The important application areas of Electromagnetic fields are

> Remote Sensing radars
> Radio astronomy radars
> Electromagnetic interference and Compatibility.
> Electric motors
> Wireless and mobile communications.
> Radio broadcast.
> all type of antenna analysis and design
> all types of transmission Lines and waveguides.
> Fiber optic communications.

> Electric relays. etc.

* Comparision between Network theory and Electromagnetic Field theory [•]

The design and analysis of a System, device (or) Circuit requires the use of Some theory (or) the other. The analysis of a System is universally defined as one by which the output is obtained from the given input and System details. On the otherhand, the design of a System is one by which the System details are obtained from the given input and output. These two important tasks are executed by two most popular theories, namely Network and electromagnetic theories.

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Network theory	Electromagnetic field theory
* Deals with voltage (V) and Current (I).	* Deals with Electric field ($\vec{E}$) and magnetic field ($\vec{H}$).
* V and I are scalar quantities	* E and H are Vector quantities.
* V and I are function of time (t).	* E and H are function of time (t) and Spatial Variables (x, y, z) (or) (ρ, ϕ, z) (or) (r, θ, ϕ).
* Basic Laws are Ohm's Law, Kirchoff's Laws	* Basic Laws are Coulomb's Law, Gauss's law, Ampere's Circuit Law etc.
* Basic theorem's are thevenin's, Norton's, reciprocity, Superposition, and Maximum power transfer theorems	* Basic theorem's are Stokes, Divergence and poynting theorem's.

NT	EMF
* Basic equations are Mesh/Loop equations.	* Basic Equations are Maxwells, poisson's, Laplace's and Wave eqns.
* at Low frequencies the Length of connecting wires is very much Smaller than Wavelength. i.e $l << \lambda$.	* at high frequencies the Length of connecting Components are of the order of λ.
* usefull at Low frequencies. (KHz range)	* usefull at all frequencies (MHz, GHz) range.
* Cannot be applied in freespace.	* Is applicable in freespace.
* Network theory cannot be used to analyse (or) design a Complete Communication System	* Electromagnetic Field theory Can be used where Network theory fails to hold good for the analysis and design of a Communication System.
* Network theory is Simplified approximation of field theory	* More accurate theory.

* Symbols of Scalar parameters.

parameter	Symbol	Definition	Unit.
• Resistivity	ρ	reciprocal of conductivity	Ω-m.
• Conductivity	σ	reciprocal of resistivity	$\mho$-m^{-1} ($\mho$/m).
• Electric Flux	ψ	displaced charge	Coulomb (C).
• Magnetic flux	ϕ	$\phi = -\int_0^t v\,dt$	Weber (wb) 1 web = 1 Volt-Sec
• Electromotive force	Emf, V	it is the ratio of power to current	Volt 1 Volt = 1 J/C (or) watt/amp.
• Magnetomotive force	V_m (mmf)	$V_m = \int_A^B \bar{H} \cdot \bar{dl}$	Amp (A).
• Capacitance	C	$C = \frac{Q}{V}$	Farads (F) 1 F = 1 C/1 volt
• Inductance	L	$L = \frac{N\phi}{I}$	Henry (H). 1 H = 1 web/amp

• Mutual Inductance	M	$M = \frac{N_2 \phi_{12}}{I_1}$	Henry (H)

Parameter	Symbol	Definition	Unit
• frequency	f	reciprocal of Timeperiod of a periodic wave $(1/T)$	Hz. $1\,Hz = 1\,Cycle/sec$
• Energy	W	it is the work done when force is Exerted through a distance of one metre.	Joule (J) $1\,Joule = 10^7\,ergs$.
• power	P	it is the time rate of energy $\left(\frac{dw}{dt}\right)$.	Watt (W) $1\,W = 1\,J/sec$ $1\,W = 1\,Volt \times 1\,Amp$
• Charge	Q	it is the product of Current & time	$1\,C = 1\,A\text{-}Sec$.
• Resistance	R	it is the ratio of voltage and Current	ohm (Ω). $1\,\Omega = 1\,Volt/1\,amp$
• Conductance	G	it is the reciprocal of R	$\mho$ (mho). $1\,\mho = 1\,Amp/1\,volt$.
• Permeability	μ	$\mu = \frac{B}{H}$	Henry/meter (H/m)

• Permittivity	ϵ	$\epsilon = \frac{D}{E}$	Farad/metre (F/m)

Parameter	Symbol	Def^n	Unit
• Permittivity of free space	ϵ_0	8.854×10^{-12} F/m $\simeq \frac{1}{36\pi \times 10^{9}}$	F/m.
• Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H/m	H/m.
• Relative permittivity of a medium	ϵ_r	$\epsilon_r = \frac{\epsilon}{\epsilon_0}$ $\epsilon_r = 1$ for free space	[Dimensionless] No unit
• Permeability of a Medium	μ_r	$\mu_r = \frac{\mu}{\mu_0}$	No-unit.
• Electric Susceptibility	χ_e	$\chi_e = \epsilon_r - 1$ (χ - is pronounced as chi)	No unit.
• Magnetic Susceptibility	χ_m	$\chi_m = \mu_r - 1$	No unit.

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• Intrinsic impedance of free space	η_0	$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$ $= 120\pi\,\Omega$ (or) $377\,\Omega$	ohm (Ω)

(8)

Parameter	Symbol	Definition	Unit
Differential Length	dl	Small-Length	meter (m)
Area	S (or) A	product of two Lengths	m^2.
Differential area	dS	product of two differential Lengths	m^2.
• Differential Volume	dv	product of differential Length, Width and breadth	m^3
• Volume	v	product of Length, width, breadth	m^3.
• angular frequency	ω	$2\pi f = \frac{2\pi}{T}$	rad/sec.
• Wave Length	λ	c/f (or) v/f	meters
• Electric potential	V	$-\int \bar{E}\cdot\bar{dl}$	Volts

• Magnetic Scalar potential	V_m	$-\int \bar{H}\cdot\bar{dl}$	Ampere

Parameter	Symbol	Definition	Unit
Surface charge density	ρ_s	Q/S	C/m^2
Line charge density	ρ_l	Q/L	C/m
Volume charge density	ρ_v	Q/V	C/m^3
propagation Constant	γ	$(\alpha + j\beta)$	dB/m
• attenuation Constant	α	it is a measure of reduction of EM wave as it propagates	dB/m
• phase Constant	β	it is a measure of phase shift of EM wave	rad/m
• Depth of penetration (or) Skin depth	δ	it is the depth at which an EM wave is attenuated to 37% of original value	$\delta = 1/\alpha$ meter

Parameter	Symbol	Definition	Unit
• Group Velocity	V_g	it is the velocity with which energy propogates in a guided structure	m/sec
• Phase velocity	V_p	it defines a point of constant phase	m/sec.

* Symbols of Vector Parameters

Parameter	Symbol	Definition	Unit
• Force	$\bar{F}$	it is the product of mass and acceleration	Newton (N) $1\,\text{Newton} = \frac{kg\text{-}m}{Sec^2}$
• Electric Field Intensity	$\bar{E}$	Force per unit charge	Volt/m (or) Newton/C
• Conduction Current density	$\bar{J}_c$	ratio of Current to area	A/m^2
• Displacement electric flux density	$\bar{D}$	$\bar{D} = \epsilon \bar{E}$	C/m^2
• Displacement Current density	$\bar{J}_d$	$\bar{J}_d = \frac{\partial \bar{D}}{\partial t}$	A/m^2
• Magnetic Field Intensity	$\bar{H}$	Current per meter width	A/m
• Magnetic Flux density	$\bar{B}$	$\bar{B} = \mu \bar{H}$	Wb/m^2 (or) Tesla (T)

Parameter	Symbol	Definition	unit
Velocity vector	$\bar{V}$	rate of displacement	m/sec
unit velocity vector	$\bar{a}_v$	$\bar{a}_v = \frac{\bar{V}}{\lvert\bar{V}\rvert}$	
Electric dipole moment	p	Qd	Coulomb-m
Magnetic dipole moment	m	IA	$A\text{-}m^2$
Magnetisation	$\bar{M}$	$\chi_m \bar{H}$	A/m
Torque	$\bar{T}$	$\bar{R} \times \bar{F}$	N-m
Surface Current density	$\bar{J}_s$	Current per meter.	A/m
Tangential Component of $\bar{E}$	E_t	Tangential component of $\bar{E}$	V/m
Tangential component of $\bar{H}$	H_t	Tangential component of $\bar{H}$	A/m

Parameter	Symbol	Definition	Unit
• Normal Component of $\bar{D}$	D_n	Normal component of $\bar{D}$	C/m^2
• Normal component of $\bar{B}$	B_n	Normal component of $\bar{B}$	Wb/m^2
• Poynting Vector	$\bar{P}$	$\bar{E} \times \bar{H}$	$Watt/m^2$

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* Small Value Representation

Value	prefix	Symbol	Eg:- Length
• 10^{-1}	Deci	d	dm
• 10^{-2}	Centi	c	cm
• 10^{-3}	milli	m	mm
• 10^{-6}	micro	μ	μm
• 10^{-9}	nano	n	nm
• 10^{-12}	pico	p	pm
• 10^{-15}	femto	f	fm
• 10^{-18}	atto	a	am

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* LARGE Value Representation

Value	prefix	Symbol	Length	frequency Ex:
10	deka	da	dam	daHz
10^2	hecto	h	hm	hHz
10^3	kilo	K	Km	KHz
10^6	Mega	M	Mm	MHz
10^9	Giga	G	Gm	GHz
10^{12}	Tera	T	Tm	THz

* List of Physical Constants.

Quantity	Symbol	Experimental value	Approximated Value [* used for solving problems]
Electron-volt (J)	eV	1.6030×10^{-19}	1.6×10^{-19}
• permittivity of freespace (F/m)	ε_0	8.854×10^{-12}	$\frac{10^{-9}}{36\pi}$
• permeability of free space (H/m)	μ_0	$4\pi \times 10^{-7}$	12.6×10^{-7}
• Intrinsic impedance of free space (Ω)	η_0	376.6	120π
• Speed of Light in Vaccum (m/sec)	c	2.998×10^{8}	3×10^{8}
• Electron charge (Coulombs)	e	-1.6030×10^{-19}	-1.6×10^{-19}
• Electron mass (kg)	m_e	9.1066×10^{-31}	9.1×10^{-31}
• proton mass (kg)	m_p	1.67248×10^{-27}	1.67×10^{-27}
• Neutron mass (kg)	m_n	1.6749×10^{-27}	1.67×10^{-27}

Quantity	Symbol	Experimental value	Approximate value
Boltzmann Constant (J/k)	k	1.38047×10^{-23}	1.38×10^{-23}
plank's Constant (J·s)	h	6.624×10^{-34}	6.62×10^{-34}
Acceleration due to gravity (m/s^2)	g	9.81	9.8

* Fundamentals of Vector Algebra

Electromagnetic engineering is the study of electric and magnetic fields. All field quantities are vector quantities i.e direction dependent. Therefore it is necessary to study the concepts of vectors before actually we start studying the fields.

I Definition of Scalar with example.

A physical quantity having only magnitude but no direction is called Scalar.

Eg: Length, mass, charge, flux, voltage, Current, time and distance etc.

let 'ϕ' - Scalar quantity

$$\boxed{\text{Scalar}(\phi) = \text{Magnitude}}$$

1. Definition of Vector with Example

A physical quantity having both magnitude and direction.

Eg:- Velocity, Displacement, acceleration, force, torque, electric field Intensity, electric flux density, Magnetic field Intensity, Weight etc.

2a. Representation of vector ($\overline{A}$ (or) $\hat{A}$ (or) $\vec{A}$)

Vector ($\overline{A}$) = Magnitude × direction

direction is represented by using unit vector

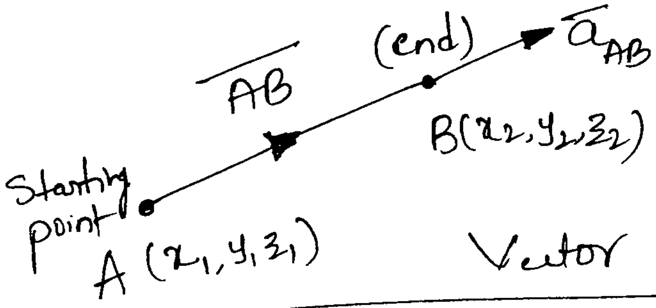
$$\overline{A} = |\overline{A}|\,\overline{a}_A = A\,\overline{a}_A$$

where $\overline{a}_A$ - unit vector

$$\boxed{\overline{a}_A = \frac{\overline{A}}{|\overline{A}|}}$$

Vector is a directed Line i.e a Line having both magnitude and direction. The vector has a Starting and Ending point.

The direction is Shown by arrow.



Vector Starts at A and ends at B.

Vector = Magnitude × direction

$$\boxed{\overline{AB} = |\overline{AB}| \times \overline{a}_{AB}}$$

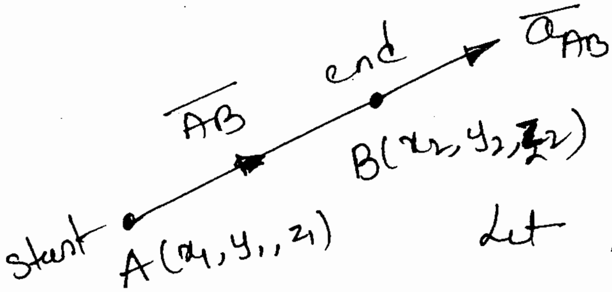
The first Letter in $\overline{AB}$ indicates the Start and the Second Letter is the end of the Vector. the arrow or bar over the head of the Letter is used to indicate Vector.

where $|\overline{AB}|$ = magnitude of vector $\overline{AB}$.

$\overline{a}_{AB}$ = unit vector in direction A to B.

$\overline{a}_{AB}$ is used to indicate unit vector i.e a vector whose magnitude is one, it gives only direction.

(21) $$\boxed{\overline{a}_{AB} = \frac{\overline{AB}}{|\overline{AB}|}}$$ i.e vector by its magnitude.



Let $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ be two points given. Then

$$\overline{AB} = (x_2 - x_1)\,\overline{a}_x + (y_2 - y_1)\,\overline{a}_y + (z_2 - z_1)\,\overline{a}_z \quad \leftarrow (1a)$$

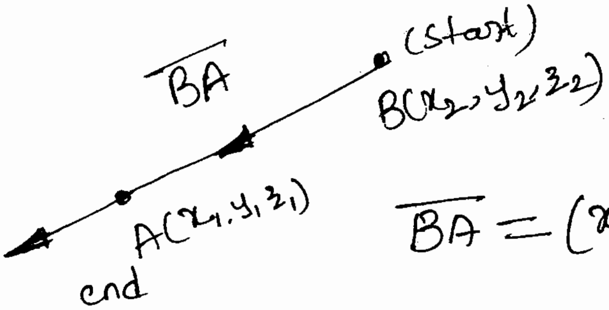
where $\overline{a}_x$, $\overline{a}_y$ and $\overline{a}_z$ are the unit vectors in Cartesian Co-ordinate System.

$$|\overline{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \quad \leftarrow (1b)$$

$$\overline{AB} = |\overline{AB}|\,\overline{a}_{AB}$$

$$\Rightarrow \overline{a}_{AB} = \frac{\overline{AB}}{|\overline{AB}|} \quad \leftarrow (1c)$$

unit vector $$\overline{a}_{AB} = \frac{(x_2 - x_1)\,\overline{a}_x + (y_2 - y_1)\,\overline{a}_y + (z_2 - z_1)\,\overline{a}_z}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$



$$\overline{BA} = (x_1 - x_2)\,\overline{a_x} + (y_1 - y_2)\,\overline{a_y} + (z_1 - z_2)\,\overline{a_z} \leftarrow (2a)$$

$$|\overline{BA}| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \leftarrow (2b)$$

$$\boxed{\overline{BA} = |\overline{BA}|\,\overline{a_{BA}}}$$

$$\overline{a_{BA}} = \frac{\overline{BA}}{|\overline{BA}|} \leftarrow (2c)$$

$$\overline{a_{BA}} = \frac{(x_1 - x_2)\,\overline{a_x} + (y_1 - y_2)\,\overline{a_y} + (z_1 - z_2)\,\overline{a_z}}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}}$$

Keypoints:-

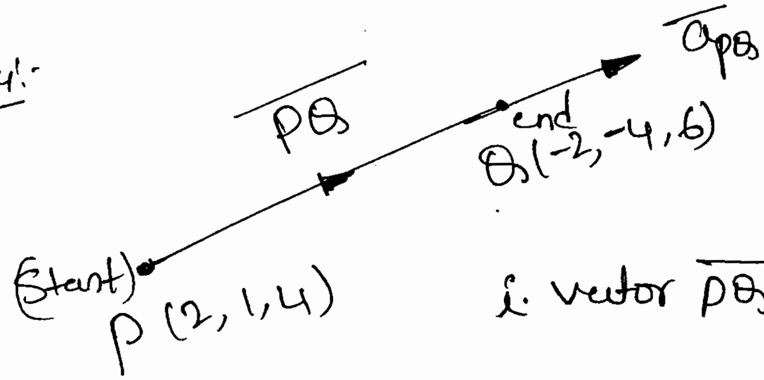
i. $\overline{AB} = -\overline{BA}$; from eq^n (1a) and (2a).

ii. $|\overline{AB}| = |\overline{BA}|$; from eq^n (1b) and (2b)

iii. $\overline{a_{AB}} = -\overline{a_{BA}}$; from eq^n (1c) and (2c).

Example: Find the vector $\overline{PQ}$ if $P(2,1,4)$m and $Q(-2,-4,6)$m also find unit vector $\overline{PQ}$.

Sol^n:-



i. vector $\overline{PQ}$ is

$$\overline{PQ} = (x_2-x_1)\overline{a_x} + (y_2-y_1)\overline{a_y} + (z_2-z_1)\overline{a_z}$$

$$\overline{PQ} = (-2-2)\overline{a_x} + (-4-1)\overline{a_y} + (6-4)\overline{a_z}$$

$$\boxed{\overline{PQ} = -4\overline{a_x} - 5\overline{a_y} + 2\overline{a_z}}$$

ii. Unit vector $\overline{a_{PQ}}$:-

$$\overline{PQ} = |\overline{PQ}|\,\overline{a_{PQ}}$$

$$\overline{a_{PQ}} = \frac{\overline{PQ}}{|\overline{PQ}|}$$

$$|\overline{PQ}| = \sqrt{(-4)^2+(-5)^2+2^2} = \sqrt{16+25+4} = \sqrt{45}\text{ m}$$

$$\boxed{\overline{a_{PQ}} = \frac{-4\overline{a_x} - 5\overline{a_y} + 2\overline{a_z}}{\sqrt{45}}}$$

Example-2

Given three point A(2, -3, 1)m, B(-4, -2, 6)m and C(0, 5, -3)m Find

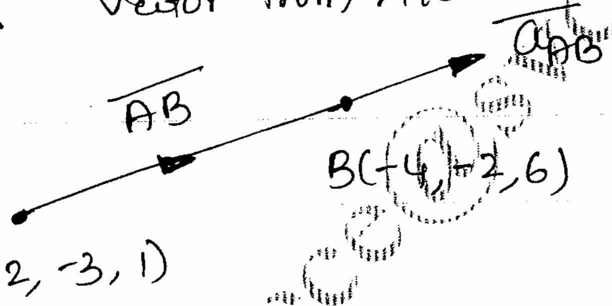
i. the vector from A to B.

ii. the vector from B to C.

iii. the unit vector from B to A.

iv. the vector from A to the mid point of the straight line joining B to C. v. Find the unit vector from A to C.

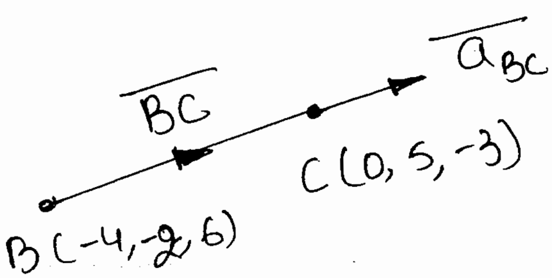
Soln: i. Vector from A to B is $\overline{AB}$



$$\overline{AB} = (-4-2)\overline{a_x} + (-2+3)\overline{a_y} + (6-1)\overline{a_z}$$

$$\boxed{\overline{AB} = -6\overline{a_x} + \overline{a_y} + 5\overline{a_z}}$$

ii. Vector from B to C $\overline{BC}$



$$\overline{BC} = (0+4)\overline{a_x} + (5+2)\overline{a_y} + (-3-6)\overline{a_z}$$

$$\boxed{\overline{BC} = 4\overline{a_x} + 7\overline{a_y} - 9\overline{a_z}}$$

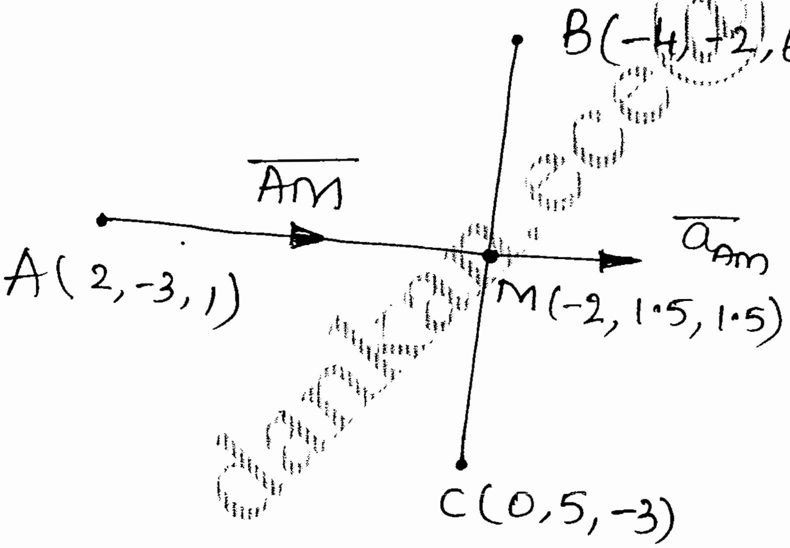
iii. distance from B to C is $|\overline{BC}|$

$$|\overline{BC}| = \sqrt{4^2 + 7^2 + (-9)^2}$$

$$|\overline{BC}| = \sqrt{16+49+81} = \sqrt{146}$$

$$\boxed{|\overline{BC}| = \sqrt{146} \text{ metres}}$$

iv.



let 'M' be the Mid point of line joining B and C

$$M \Rightarrow \left[\frac{-4+0}{2}, \frac{-2+5}{2}, \frac{6-3}{2}\right]$$

$$M(-2, 1.5, 1.5)$$

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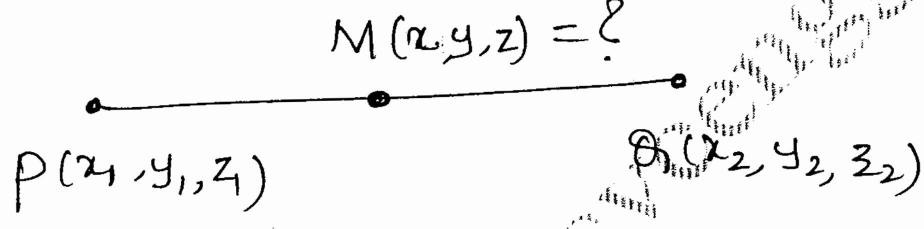
$$\overline{AM} = (-2-2)\overline{a_x} + (1.5+3)\overline{a_y} + (1.5-1)\overline{a_z}$$

$$\boxed{\overline{AM} = -4\overline{a_x} + 4.5\overline{a_y} + 0.5\overline{a_z}}$$

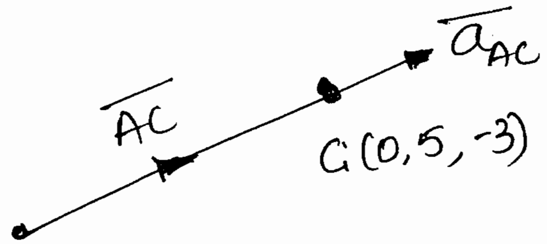
Key Note: Mid point formula



$$x = \frac{x_1+x_2}{2} \; ; \; y = \frac{y_1+y_2}{2} \; ; \; z = \frac{z_1+z_2}{2}$$

$$M(x,y,z) = M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right).$$

V. Unit vector from A to C is $\overline{a}_{AC}$



A(2,-3,1) $\qquad \overline{a}_{AC} = \dfrac{\overline{AC}}{|\overline{AC}|}$

$$\overline{AC} = (0-2)\,\overline{a}_x + (5+3)\,\overline{a}_y + (-3-1)\,\overline{a}_z$$

$$\overline{AC} = -2\overline{a}_x + 8\overline{a}_y - 4\overline{a}_z$$

$$|\overline{AC}| = \sqrt{(-2)^2 + (8)^2 + (-4)^2}$$

$$= \sqrt{4+64+16} = \sqrt{84}\ \text{m}$$

$$\overline{a}_{AC} = \frac{\overline{AC}}{|\overline{AC}|} = \frac{-2\overline{a}_x + 8\overline{a}_y - 4\overline{a}_z}{\sqrt{84}}$$

$$\boxed{\overline{a}_{AC} = -0.218\,\overline{a}_x + 0.872\,\overline{a}_y - 0.436\,\overline{a}_z}$$

2b. Position and distance Vector

position Vector (or) radius vector ($\bar{r}_p$):

The position vector (or) radius vector $\bar{r}_p$ of a point p is defined as the directed distance from origin O to point P.

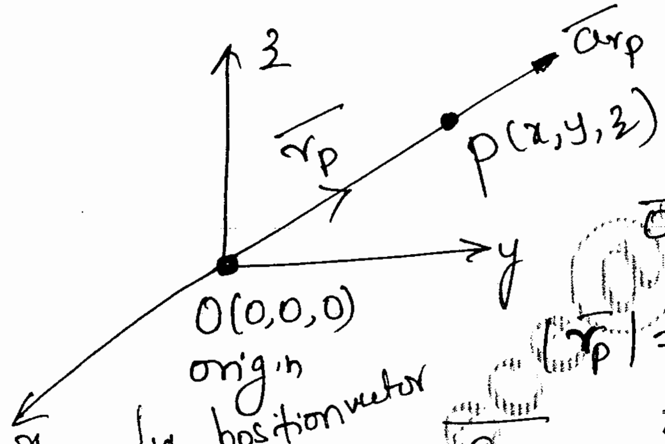


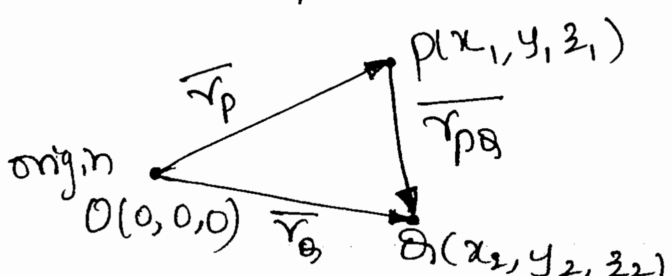
fig. position vector

$$\overline{OP} = \bar{r}_p = x\bar{a}_x + y\bar{a}_y + z\bar{a}_z$$

$$|\bar{r}_p| = \sqrt{x^2+y^2+z^2} \; ; m$$

$$\bar{a}_{r_p} = \frac{\bar{r}_p}{|\bar{r}_p|} = \frac{x\bar{a}_x + y\bar{a}_y + z\bar{a}_z}{\sqrt{x^2+y^2+z^2}}$$

distance Vector:- distance Vector is the displacement from one point to another.



the distance vector

$$\bar{r}_{pQ} = \bar{r}_Q - \bar{r}_p$$

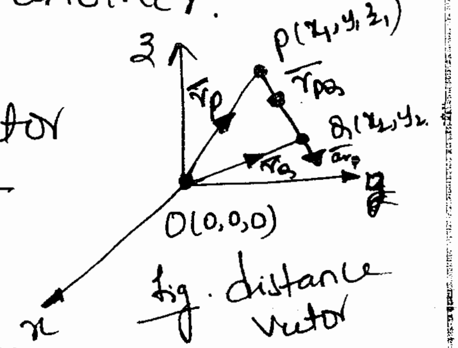


fig. distance vector

(29)

$$\boxed{\bar{r}_{pQ} = \overline{PQ} = (x_2-x_1)\bar{a}_x + (y_2-y_1)\bar{a}_y + (z_2-z_1)\bar{a}_z}$$

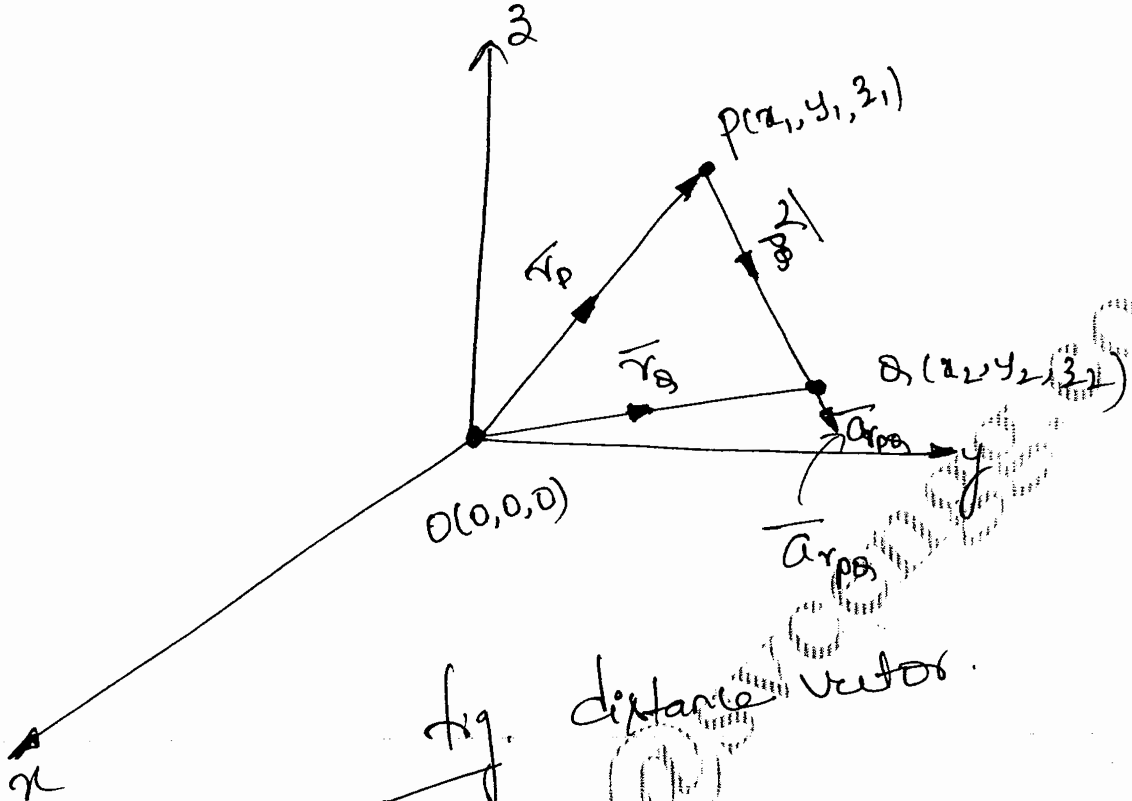


fig. distance vector.

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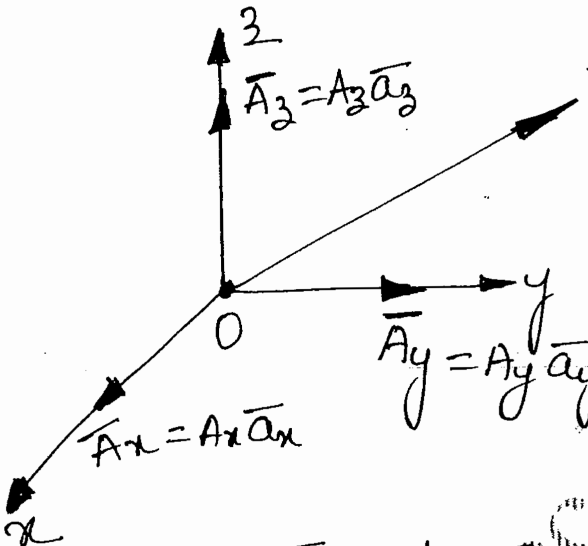
3. Operation on Vectors

3a. Addition / Subtraction of Vectors :-

While adding the vectors, add the components in the same direction.



$\bar{A} = A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_z$ ---- General vector in Cartesian Co-ordinate System

A_x, A_y, A_z - Components along x, y, and z direction respectively.

$\bar{a}_x, \bar{a}_y, \bar{a}_z$ - unit vectors along x, y, & z direction respectively.

Let $\bar{A} = A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_z$

and $\bar{B} = B_x \bar{a}_x + B_y \bar{a}_y + B_z \bar{a}_z$

$$\bar{A} + \bar{B} = A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_z + B_x \bar{a}_x + B_y \bar{a}_y + B_z \bar{a}_z$$

$$\boxed{\bar{A} + \bar{B} = (A_x + B_x)\bar{a}_x + (A_y + B_y)\bar{a}_y + (A_z + B_z)\bar{a}_z}$$

Ily

$$\bar{A}-\bar{B} = A_x\bar{a}_x + A_y\bar{a}_y + A_z\bar{a}_z - B_x\bar{a}_x - B_y\bar{a}_y - B_z\bar{a}_z$$

$$\boxed{\bar{A}-\bar{B} = (A_x-B_x)\bar{a}_x + (A_y-B_y)\bar{a}_y + (A_z-B_z)\bar{a}_z.}$$

In general

$$\boxed{\bar{A}\pm\bar{B} = (A_x\pm B_x)\bar{a}_x + (A_y\pm B_y)\bar{a}_y + (A_z\pm B_z)\bar{a}_z}$$

Example problem -3.

Four points are $A(2,3,-1)$, $B(1,5,2)$, $C(3,1,-5)$ and $D(1,2,3)$. Find

a) $\overline{AB} + \overline{CD}$

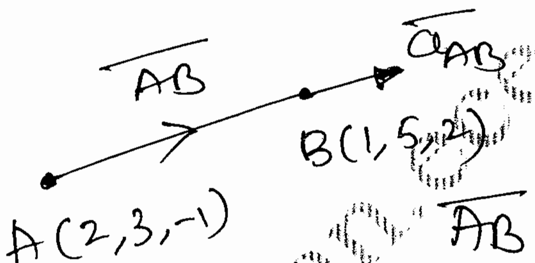
b) $\overline{AB} - \overline{CD}$ c) $\overline{AB} - \overline{DC}$

d) $|\overline{AB} + \overline{CD}|$.

Soln:

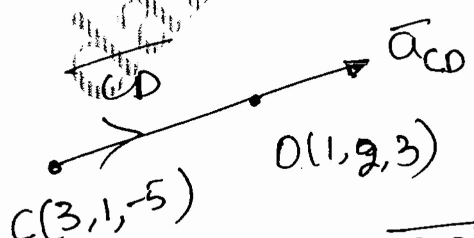
a) $\overline{AB} + \overline{CD} = ?$

$\overline{AB} = ?$



$$\overline{AB} = (1-2)\overline{a}_x + (5-3)\overline{a}_y + (2+1)\overline{a}_z$$

$$\overline{AB} = -\overline{a}_x + 2\overline{a}_y + 3\overline{a}_z$$



$$\overline{CD} = (1-3)\overline{a}_x + (2-1)\overline{a}_y + (3+5)\overline{a}_z$$

$$\overline{CD} = -2\overline{a}_x + \overline{a}_y + 8\overline{a}_z$$

$$\overline{AB} + \overline{CD} = -\overline{a}_x + 2\overline{a}_y + 3\overline{a}_z - 2\overline{a}_x + \overline{a}_y + 8\overline{a}_z$$

$$\boxed{\overline{AB} + \overline{CD} = -3\overline{a}_x + 3\overline{a}_y + 11\overline{a}_z}$$

b) $\overline{AB} - \overline{CD} = -\bar{a}_x + 2\bar{a}_y + 3\bar{a}_z + 2\bar{a}_x - \bar{a}_y - 8\bar{a}_z$

$$\boxed{\overline{AB} - \overline{CD} = \bar{a}_x + \bar{a}_y - 5\bar{a}_z}$$

c) $\overline{AB} - \overline{DC} = ?$

$\overline{DC} = -\overline{CD} = -[-2\bar{a}_x + \bar{a}_y + 8\bar{a}_z]$

$\overline{DC} = 2\bar{a}_x - \bar{a}_y - 8\bar{a}_z$

$\overline{AB} - \overline{DC} = -\bar{a}_x + 2\bar{a}_y + 3\bar{a}_z - 2\bar{a}_x + \bar{a}_y + 8\bar{a}_z$

$$\boxed{\overline{AB} - \overline{DC} = -3\bar{a}_x + 3\bar{a}_y + 11\bar{a}_z} = \overline{AC} + \overline{CD}.$$

d) $|\overline{AB} + \overline{CD}| = ?$

from bit (a).

$\overline{AB} + \overline{CD} = -3\bar{a}_x + 3\bar{a}_y + 11\bar{a}_z$

$|\overline{AB} + \overline{CD}| = \sqrt{9+9+121} = \sqrt{139}$

$$\boxed{|\overline{AB} + \overline{CD}| = \sqrt{139} \text{ meter}}$$

Example problem :- 4.

Given vectors $\bar{A} = \bar{a}_x + 3\bar{a}_z$ and $\bar{B} = 5\bar{a}_x + 2\bar{a}_y - 6\bar{a}_z$.

determine

i. $|\bar{A} + 3\bar{B}|$. $\rightarrow |\bar{A} + 3\bar{B}|$.

ii. $5\bar{A} - \bar{B}$.

iii. the component of $\bar{A}$ along $\bar{a}_y$.

iv. the direction of vector $\bar{A}$ and $\bar{B}$.

v. A unit vector parallel to $3\bar{A} + \bar{B}$.

Soln. i. $|\bar{A} + 3\bar{B}| = ?$ $\rightarrow |\bar{A} + 3\bar{B}| = ?$

$\bar{A} = \bar{a}_x + 3\bar{a}_z$ and $\bar{B} = 5\bar{a}_x + 2\bar{a}_y - 6\bar{a}_z$.

$\bar{A} + 3\bar{B} = \bar{a}_x + 3\bar{a}_z + 15\bar{a}_x + 6\bar{a}_y - 18\bar{a}_z$

$\bar{A} + 3\bar{B} = 16\bar{a}_x + 6\bar{a}_y - 15\bar{a}_z$

$|\bar{A} + 3\bar{B}| = \sqrt{16^2 + 6^2 + 15^2} = \sqrt{517}$ m

$\boxed{|\bar{A} + 3\bar{B}| = \sqrt{517}}$ m

ii. $5\bar{A} - \bar{B} = \cancel{5\bar{a}_x} + 15\bar{a}_z - \cancel{5\bar{a}_x} - 2\bar{a}_y + 6\bar{a}_z$

$\boxed{5\bar{A} - \bar{B} = -2\bar{a}_y + 21\bar{a}_z}$

iii. $\overline{A} = \overline{a_x} + 0\overline{a_y} + 3\overline{a_z}$ ← ①

$\overline{A} = A_x\overline{a_x} + A_y\overline{a_y} + 3\overline{a_z}$ ← ②

the component of $\overline{A}$ along $\overline{a_y}$ i.e A_y is

by comparing eqⁿ ① and eqⁿ ②

$$\boxed{A_y = 0}$$

iv. the direction of vector A is $\overline{a_A} = ?$

$\overline{A} = \overline{a_x} + 3\overline{a_z}$; $|\overline{A}| = \sqrt{1+9} = \sqrt{10}$ m.

$$\overline{a_A} = \frac{\overline{A}}{|\overline{A}|} = \frac{\overline{a_x} + 3\overline{a_z}}{\sqrt{10}}.$$

the direction of vector B is $\overline{a_B} = ?$

$\overline{a_B} = \frac{\overline{B}}{|\overline{B}|}$; $B = 5\overline{a_x} + 2\overline{a_y} - 6\overline{a_z}$

$|\overline{B}| = \sqrt{25+4+36} = \sqrt{65}$ m.

$$\boxed{\overline{a_B} = \frac{5\overline{a_x} + 2\overline{a_y} - 6\overline{a_z}}{\sqrt{65}}}$$

v. A unit vector parallel to $3\overline{A} + \overline{B}$ is

$\pm \overline{a_{3\overline{A}+\overline{B}}}$

$$\pm \bar{a}_{3\bar{A}+\bar{B}} = \pm\left[\frac{3\bar{A}+\bar{B}}{|3\bar{A}+\bar{B}|}\right]$$

$$3\bar{A}+\bar{B} = 3\bar{a}_x + 9\bar{a}_z + 5\bar{a}_x + 2\bar{a}_y - 6\bar{a}_z$$

$$= 8\bar{a}_x + 2\bar{a}_y + 3\bar{a}_z.$$

$$|3\bar{A}+\bar{B}| = \sqrt{64+4+9} = \sqrt{77}\ m$$

$$\pm \bar{a}_{3\bar{A}+\bar{B}} = \pm \frac{[8\bar{a}_x + 2\bar{a}_y + 3\bar{a}_z]}{\sqrt{77}}$$

$$= \pm\left[0.9117\bar{a}_x + 0.2279\bar{a}_y + 0.3418\bar{a}_z\right]$$

the unit vector parallel to $3\bar{A}+\bar{B}$ is

$$\pm\left[0.9117\bar{a}_x + 0.2279\bar{a}_y + 0.3418\bar{a}_z\right]$$

Example problem -5

if $\bar{A} = 10\bar{a}_x - 4\bar{a}_y + 6\bar{a}_z$ and $\bar{B} = 2\bar{a}_x + \bar{a}_y$; Find

a> the component of $\bar{A}$ along $\bar{a}_y$.

b> the magnitude of $3\bar{A} - \bar{B}$.

c> a unit vector along $\bar{A} + 2\bar{B}$.

Solu:-

Given

a> $\bar{A} = 10\bar{a}_x - 4\bar{a}_y + 6\bar{a}_z$

the component of $\bar{A}$ along $\bar{a}_y$ is A_y

i.e $\boxed{A_y = -4}$

b> the magnitude of $3\bar{A} - \bar{B}$.

$$3\bar{A} - \bar{B} = 30\bar{a}_x - 12\bar{a}_y + 18\bar{a}_z - 2\bar{a}_x - \bar{a}_y$$

$$3\bar{A} - \bar{B} = 28\bar{a}_x - 13\bar{a}_y + 18\bar{a}_z$$

$$|3\bar{A} - \bar{B}| = \sqrt{28^2 + 13^2 + 18^2} = \sqrt{1277} \text{ m.}$$

$$\boxed{|3\bar{A} - \bar{B}| = \sqrt{1277}} \text{ meter.}$$

c) a unit vector along $\bar{A}+2\bar{B}$ is

$$\bar{a}_{\bar{A}+2\bar{B}} = \frac{\bar{A}+2\bar{B}}{|\bar{A}+2\bar{B}|}.$$

$$\bar{A}+2\bar{B} = 10\bar{a}_x - 4\bar{a}_y + 6\bar{a}_z + 4\bar{a}_x + 2\bar{a}_y$$

$$\boxed{\bar{A}+2\bar{B} = 14\bar{a}_x - 2\bar{a}_y + 6\bar{a}_z}$$

$$|\bar{A}+2\bar{B}| = \sqrt{14^2+2^2+6^2} = \sqrt{236}\ m.$$

$$\bar{a}_{\bar{A}+2\bar{B}} = \frac{\bar{A}+2\bar{B}}{|\bar{A}+2\bar{B}|}$$

$$= \frac{14\bar{a}_x - 2\bar{a}_y + 6\bar{a}_z}{\sqrt{236}}$$

$$\boxed{\bar{a}_{\bar{A}+2\bar{B}} = 0.9113\bar{a}_x - 0.1302\bar{a}_y + 0.3906\bar{a}_z}$$

Example problem, -6.

point P and Q are Located at (0, 2, 4) and (−3, 1, 5). Calculate

a. the position of vector $\overline{r_p}$.

b. the distance vector from P to Q.

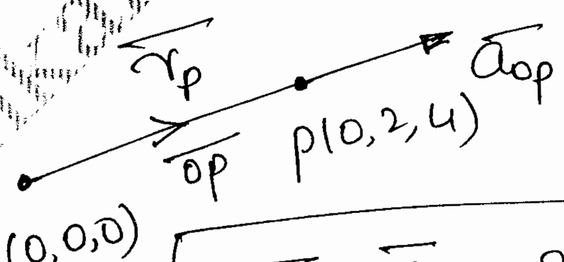
c. the distance between P and Q.

d. A vector parallel to PQ with magnitude of 10.

Sol:-

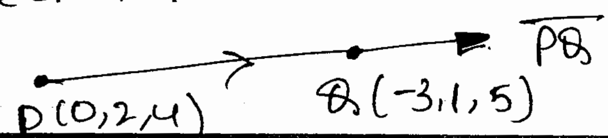
a) the position of vector $\overline{r_p}$

it is directed line from origin to point P (0, 2, 4).



$$\boxed{\overline{OP} = \overline{r_p} = 2\overline{a_y} + 4\overline{a_z}}$$

b) the distance vector from P to Q.



$$\overline{PQ} = -3\overline{a_x} + (1-2)\overline{a_y} + (5-4)\overline{a_z}$$

$$\boxed{\overline{PQ} = -3\overline{a_x} - \overline{a_y} + \overline{a_z}}$$

c). the distance between P and Q.

$$|\overline{PQ}| = ?$$

$$\overline{PQ} = -3\overline{a_x} - \overline{a_y} + \overline{a_z}$$

$$|\overline{PQ}| = \sqrt{9+1+1} = \sqrt{11}\ m.$$

$$\boxed{|\overline{PQ}| = \sqrt{11}}\ \text{meter}$$

d). A vector parallel to PQ with magnitude of 10 is $\pm 10\overline{a}_{PQ}$

$$\pm 10\,\overline{a}_{\overline{PQ}} = \pm 10\,\frac{\overline{PQ}}{|\overline{PQ}|}$$

$$= \pm 10\,\frac{[-3\overline{a_x} - \overline{a_y} + \overline{a_z}]}{\sqrt{11}}$$

$$= \pm\ [-9.04\overline{a_x} - 3.015\overline{a_y} + 3.015\overline{a_z}]$$

Example problem -7

Given points $P(1,-3,5)$, $Q(2,4,6)$ and $R(0,3,8)$

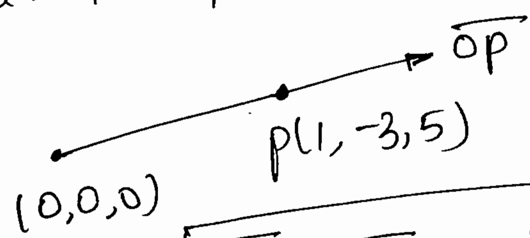
Find.

a. the position vectors of P and R.

b. the distance vector $\overline{r}_{QR}$.

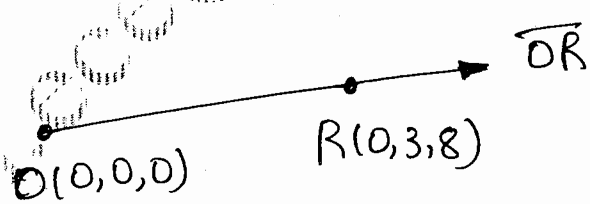
c. the distance between Q and R.

Soln a. the position vectors of P and R are



$$\boxed{\overline{r}_P = \overline{OP} = \overline{a}_x - 3\overline{a}_y + 5\overline{a}_z}$$

lly



$$\boxed{\overline{OR} = \overline{r}_R = 3\overline{a}_y + 8\overline{a}_z}$$

b) the distance vector $\overline{r}_{QR}$

$$\overline{r}_{QR} = \overline{r}_R - \overline{r}_Q = 3\overline{a}_y + 8\overline{a}_z - 2\overline{a}_x - 4\overline{a}_y - 6\overline{a}_z$$

$$\boxed{\overline{r}_{QR} = -2\overline{a}_x - \overline{a}_y + 2\overline{a}_z}$$

c) the distance between Q and R is

$$|\overline{QR}| = ?$$

$$\overline{QR} = \bar{r}_{QR} = -2\bar{a}_x - \bar{a}_y + 2\bar{a}_z$$

$$|\overline{QR}| = \sqrt{4+1+4} = \sqrt{9}\ \text{m}.$$

$$\boxed{|\overline{QR}| = \underline{3\ \text{m}}}$$

3b. Product of Vectors (or) Vector Multiplications

Like addition and subtraction one more operation can be performed on vectors, it is multiplication.

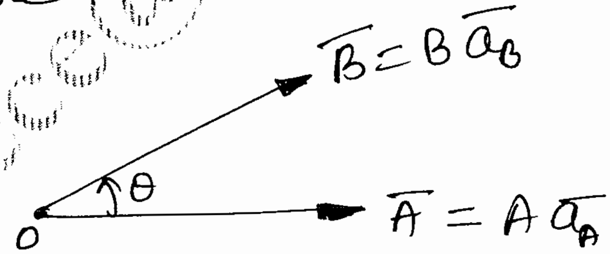
Vectors can be multiplied by two ways:

i. Scalar (or) dot product.

ii. Vector (or) Cross product.

i. Scalar (or) dot product

Let $\overline{A} = A\overline{a}_A$ and $\overline{B} = B\overline{a}_B$, be two vectors shown in figure



then

$$\boxed{\overline{A} \cdot \overline{B} = AB\cos\theta} \leftarrow (1)$$

is called the dot product of vectors $\overline{A}$ and $\overline{B}$, where θ is the angle between them.

In equation (i).

A = magnitude of $\overline{A}$.

B = magnitude of $\overline{B}$.

θ = angle between $\overline{A}$ and $\overline{B}$.

Key points:-

i. dot product between any two vectors results in Scalar.

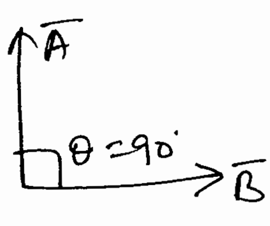
ii. $\overline{A}\cdot\overline{B} = \overline{B}\cdot\overline{A}$.

iii. $\overline{A}$ and $\overline{B}$ are parallel

$\longrightarrow \overline{A}$ then $\theta = 0$

$\longrightarrow \overline{B}$ $\therefore$ $\boxed{\overline{A}\cdot\overline{B} = AB}$

iv. $\overline{A}$ and $\overline{B}$ are perpendicular, i.e. $\theta = 90^\circ$

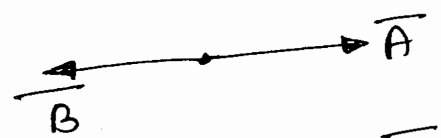


$\overline{A}\cdot\overline{B} = AB\cos(90^\circ)$ ↗ 0

$\boxed{\overline{A}\cdot\overline{B} = 0}$

if two vectors are $\perp^r$ (or) right angles to each other then their dot product is zero.

v. $\bar{A}$ and $\bar{B}$ are opposite.

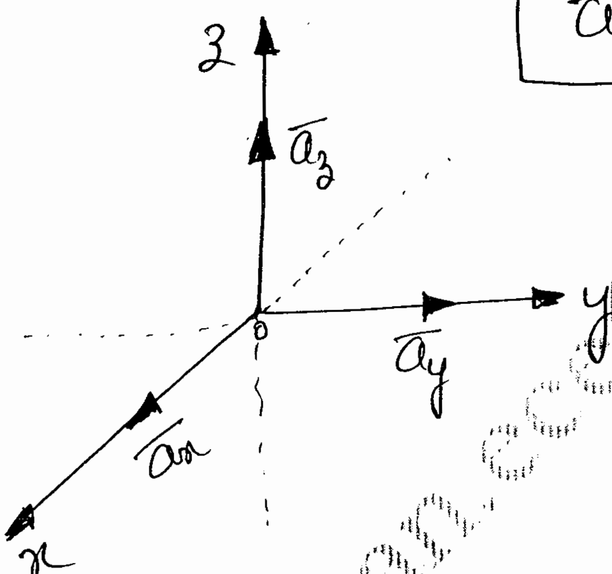


$\theta = \pi$ (or) 180°

$$\bar{A} \cdot \bar{B} = AB \cos(180^\circ) \quad (\cos(180^\circ) = -1)$$

$$\boxed{\bar{A} \cdot \bar{B} = -AB}$$

vi. Dot product of unit vectors



$$\boxed{\bar{a}_x \cdot \bar{a}_x = \bar{a}_y \cdot \bar{a}_y = \bar{a}_z \cdot \bar{a}_z = 1}$$

$$\boxed{\bar{a}_x \cdot \bar{a}_y = \bar{a}_y \cdot \bar{a}_z = \bar{a}_z \cdot \bar{a}_x = 0}$$

vii. Consider a vector $\bar{A} = A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_z$

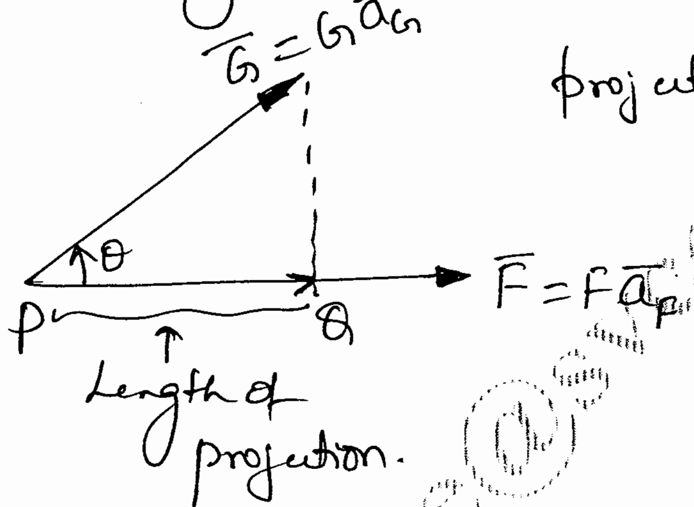
and $\bar{B} = B_x \bar{a}_x + B_y \bar{a}_y + B_z \bar{a}_z$

then

$$\boxed{\bar{A} \cdot \bar{B} = A_x B_x + A_y B_y + A_z B_z}$$

viii. Application of dot product.

The definition of dot product can be used to find length of projection of one vector on other. Let $\overline{F}$ and $\overline{G}$ are the vectors making angle θ as shown in figure.



projection of $\overline{G}$ on $\overline{F}$ is

$G\cos\theta = |\overline{PQ}|$.

the length $PQ = G\cos\theta = \overline{G}\cdot\overline{a}_F$.

ix. Vector projection of $\overline{G}$ on $\overline{F}$ is $\overline{PQ}$ = mag × direction.

$$\overline{PQ} = |\overline{PQ}|\,\overline{a}_{PQ} = (G\cos\theta)\cdot\overline{a}_F$$

i.e $\overline{PQ}$ = Length of projection × unit vector $\overline{F}$.

$$\boxed{\overline{PQ} = (\overline{G}\cdot\overline{a}_F)\,\overline{a}_F = (G\cos\theta)\,\overline{a}_F}$$

Hly. the Length $\overline{F}$ on $\overline{G}$ $= F\cos\theta = \overline{F}\cdot\overline{a}_G$.
and vector projection of $\overline{F}$ on $\overline{G}$ is

$$= (F\cos\theta)\,\overline{a}_G = (\overline{F}\cdot\overline{a}_G)\,\overline{a}_G.$$

x. $\overline{A}\cdot(\overline{B}+\overline{C}) = \overline{A}\cdot\overline{B} + \overline{A}\cdot\overline{C}$.

xi. $\overline{A}\cdot\overline{A} = |\overline{A}|^2 = A^2$.

Example problem.-8

Given vector $\overline{A} = 5\overline{a_x} + 4\overline{a_y} + 3\overline{a_z}$ and

$\overline{B} = 2\overline{a_x} + 3\overline{a_y} + 4\overline{a_z}$. Find

i) Dot product $\overline{A}\cdot\overline{B}$.

ii) angle between vectors $\overline{A}$ and $\overline{B}$.

Soln:

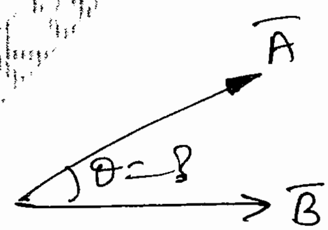
i) $\overline{A}\cdot\overline{B} = (5\overline{a_x} + 4\overline{a_y} + 3\overline{a_z})\cdot(2\overline{a_x} + 3\overline{a_y} + 4\overline{a_z})$

$\overline{A}\cdot\overline{B} = 5(2) + 4(3) + 3(4)$

$= 10 + 12 + 12 = 34$

$\boxed{\overline{A}\cdot\overline{B} = 34}$

ii. angle between vectors $\overline{A}$ and $\overline{B}$



$\overline{A}\cdot\overline{B} = AB\cos\theta$

$A = |\overline{A}| = \sqrt{25+16+9} = \sqrt{50}$ m.

$B = |\overline{B}| = \sqrt{4+9+16} = \sqrt{29}$ m.

$\cos\theta = \frac{\overline{A}\cdot\overline{B}}{AB} = \frac{34}{\sqrt{50}\times\sqrt{29}} = 0.8928$

$\theta = \cos^{-1}(0.8928) = 26.7621°$

$\boxed{\theta = 26.7621°}$

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Example problem - 9

Given Vector $\bar{A} = 5\bar{a}_x + 4\bar{a}_y + 3\bar{a}_z$ and $\bar{B} = K\bar{a}_x + 3\bar{a}_y + 4\bar{a}_z$. Find the value of 'K' such that both the vectors are right angles to each other.

Soln:

$$\bar{A} \perp \bar{B} \Rightarrow \boxed{\bar{A} \cdot \bar{B} = 0}$$

$$(5\bar{a}_x + 4\bar{a}_y + 3\bar{a}_z) \cdot (K\bar{a}_x + 3\bar{a}_y + 4\bar{a}_z) = 0$$

$$5K + 12 + 12 = 0$$

$$5K = -24$$

$$K = \frac{-24}{5} = -4.8$$

$$\boxed{K = -4.8}$$

Example problem-10

Given vectors $\bar{A} = 3\bar{a}_x + 4\bar{a}_y + \bar{a}_z$ and $\bar{B} = 2\bar{a}_y - 5\bar{a}_z$. Find the angle between $\bar{A}$ and $\bar{B}$.

Soln:-

$$\bar{A}\cdot\bar{B} = AB\cos\theta$$

$$\theta = \cos^{-1}\left[\frac{\bar{A}\cdot\bar{B}}{AB}\right]$$

$$\bar{A}\cdot\bar{B} = \left[3\bar{a}_x + 4\bar{a}_y + \bar{a}_z\right]\cdot\left[2\bar{a}_y - 5\bar{a}_z\right]$$

$$= 0 + 8 - 5 = 3$$

$$\boxed{\bar{A}\cdot\bar{B} = 3}$$

$$|\bar{A}| = A = \sqrt{9+16+1} = \sqrt{26}\ m.$$

$$|\bar{B}| = B = \sqrt{4+25} = \sqrt{29}\ m.$$

$$\theta = \cos^{-1}\left[\frac{3}{\sqrt{26}\sqrt{29}}\right] = 83.727^\circ$$

$$\boxed{\theta = 83.73^\circ}$$

Example problem -- 11

Given points A (2, 5, -1), B(3, -2, 4) and C(-2, 3, 1) find.

i. $\overline{R_{AB}} \cdot \overline{R_{AC}}$

ii. angle between $\overline{R_{AB}}$ & $\overline{R_{AC}}$.

iii. Length of projection of $\overline{R_{AB}}$ on $\overline{R_{AC}}$

iv. Vector projection of $\overline{R_{AB}}$ on $\overline{R_{AC}}$.

Soln:- i) dot product $\overline{R_{AB}} \cdot \overline{R_{AC}} = ?$

$$\overline{R_{AB}} = \overline{a_x} - 7\overline{a_y} + 5\overline{a_z}$$

$$\overline{R_{AC}} = -4\overline{a_x} - 2\overline{a_y} + 2\overline{a_z}.$$

$$\overline{R_{AB}} \cdot \overline{R_{AC}} = -4 + 14 + 10 = 20$$

$$\boxed{\overline{R_{AB}} \cdot \overline{R_{AC}} = 20}$$

ii. angle between $\overline{R_{AB}}$ and $\overline{R_{AC}}$ is

$$\theta = \cos^{-1}\left[\frac{\overline{R_{AB}} \cdot \overline{R_{AC}}}{|\overline{R_{AB}}|\,|\overline{R_{AC}}|}\right]$$

$$|\overline{R_{AB}}| = \sqrt{1+49+25} = \sqrt{75}\ m$$

$$|\overline{R_{AC}}| = \sqrt{16+4+4} = \sqrt{24}\ m.$$

$$\theta = \cos^{-1}\left[\frac{20}{\sqrt{75}\sqrt{24}}\right] = 61.874^\circ$$

$$\boxed{\theta = 61.874^\circ}$$

iii. Length of projection of $\overline{R_{AB}}$ on $\overline{R_{AC}}$ is

$$= |\overline{R_{AB}}| \cos\theta$$

$$= \sqrt{75}\cos(61.874^\circ)$$

$$= 4.0824 \text{ meter}$$

iv. Vector projection of $\overline{R_{AB}}$ on $\overline{R_{AC}}$ is

$$= |\overline{R_{AB}}| \cos\theta \, \overline{a_{AC}}.$$

$$= (4.0824)\left(\frac{-4\overline{a_x} - 2\overline{a_y} + 2\overline{a_z}}{\sqrt{24}}\right)$$

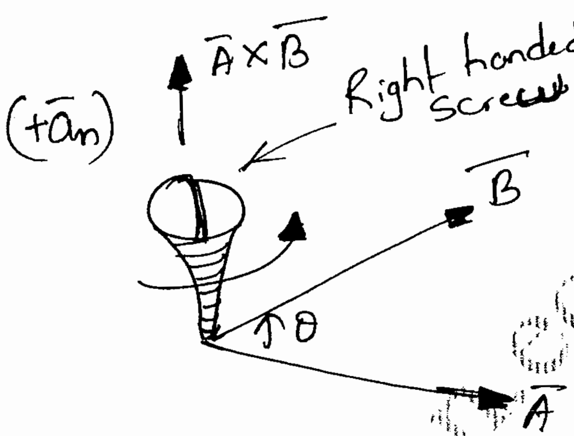
$$= -3.33\overline{a_x} - 1.667\overline{a_y} + 1.667\overline{a_z}$$

ii. Vector product (or) Cross product.

The Cross product between vectors $\bar{A}$ and $\bar{B}$ is given by

$$\boxed{\bar{A} \text{ cross } \bar{B} = \bar{A} \times \bar{B} = AB \sin\theta \, \bar{a}_n} \longleftarrow (1)$$

The $\times$ sign does not mean simple multiplication, it is a Cross product of two vectors. The term $\bar{a}_n$ indicates a unit vector, thus A cross B results in a vector.



The magnitude of $\bar{A} \times \bar{B}$ is $AB\sin\theta$ and the direction is perpendicular to the plane containing $\bar{A}$ and $\bar{B}$ and is in the sense of advance of right handed screw rotated from the first vector (i.e $\bar{A}$) to the second vector (i.e $\bar{B}$) through the smaller angle between their positive direction.

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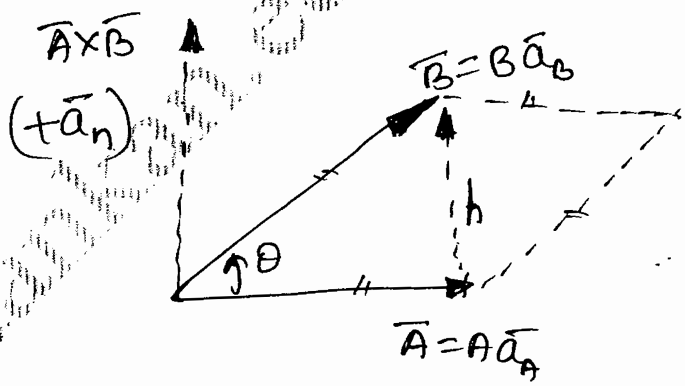
When the order of Cross product is changed i.e instead of $\bar{A} \times \bar{B}$ if it is $\bar{B} \times \bar{A}$, then right handed Screw will advance in downward direction that is $-\bar{a}_n$.

$$\therefore \boxed{\bar{A} \times \bar{B} = -\bar{B} \times \bar{A}}$$

Application of Cross product:-

The Cross product of two vectors $\bar{A}$ and $\bar{B}$, written as $\bar{A} \times \bar{B}$ is a vector quantity whose magnitude is the area of the parallelogram formed by $\bar{A}$ and $\bar{B}$ and is in the direction of advance of a right-handed Screw as A is turned into B.



$\because$ h-height (m)

Area of parallelgram is $|\bar{A} \times \bar{B}| = AB \sin\theta$

and area of triangle is given by

Area of triangle $= \frac{1}{2}|\bar{A}\times\bar{B}|$. ← (2)

$= \frac{1}{2}\times$ breadth $\times$ height ← (3)

equating eq^n (2) and eq^n (3)

$\frac{1}{2}|\bar{A}\times\bar{B}| = \frac{1}{2}\cdot|\bar{A}|\times h.$ ← (4)

but in fig. $\sin\theta = \frac{h}{|\bar{B}|}$

$\Rightarrow h = |\bar{B}|\sin\theta$

eq^n (4) becomes

$\cancel{\frac{1}{2}}|\bar{A}\times\bar{B}| = \cancel{\frac{1}{2}}|\bar{A}|\times|\bar{B}|\sin\theta$

$\Rightarrow \boxed{|\bar{A}\times\bar{B}| = |\bar{A}|\cdot|\bar{B}|\sin\theta}$

and $\boxed{\bar{A}\times\bar{B} = |\bar{A}|\cdot|\bar{B}|\sin\theta\ \bar{a}_n}$

Key note points:-

i. $\bar{A} \times \bar{B} = AB \sin\theta \, \bar{a}_n$.

ii. $\bar{A} \times \bar{B} = -\bar{B} \times \bar{A}$.

iii. Cross product of any two vectors result in Scalar.

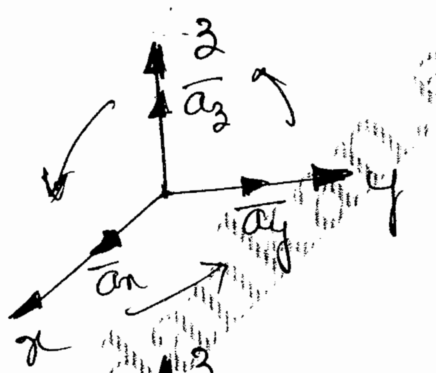
iv. If $\bar{A}$ and $\bar{B}$ are parallel.

i.e $\theta = 0$. $\sin(0) = 0$.

$\therefore \boxed{\bar{A} \times \bar{B} = 0.}$

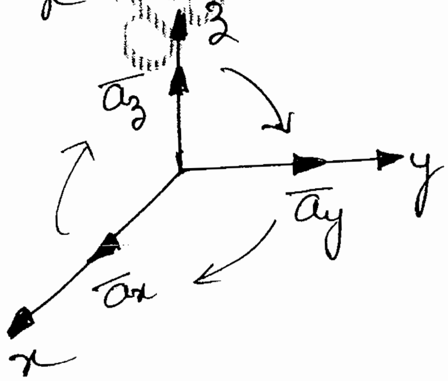
$\text{lly} \quad \bar{A} \times \bar{A} = 0.$

v. Cross product of unit vectors.



$\bar{a}_x \times \bar{a}_y = +\bar{a}_z$
$\bar{a}_y \times \bar{a}_z = +\bar{a}_x$
$\bar{a}_z \times \bar{a}_x = +\bar{a}_y$

rotating Anticlock-wise direction.



$\bar{a}_x \times \bar{a}_z = -\bar{a}_y$
$\bar{a}_z \times \bar{a}_y = -\bar{a}_x$
$\bar{a}_x \times \bar{a}_z = -\bar{a}_y$

rotating clockwise direction.

and

$$\bar{a}_x \times \bar{a}_x = \bar{a}_y \times \bar{a}_y = \bar{a}_z \times \bar{a}_z = 0.$$

because $\theta = 0$. no rotational field Exists.

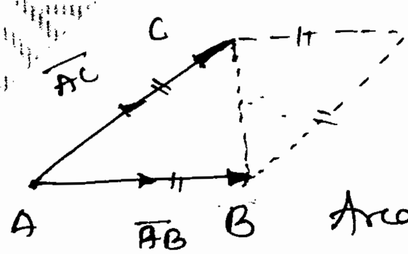
vi. if given vectors $\bar{A} = A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_z$

and $\bar{B} = B_x \bar{a}_x + B_y \bar{a}_y + B_z \bar{a}_z$.

then

$$\bar{A} \times \bar{B} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \text{ and } ; \quad \bar{B} \times \bar{A} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ B_x & B_y & B_z \\ A_x & A_y & A_z \end{vmatrix}$$

vii. when two vectors $\overline{AB}$ and $\overline{AC}$ are given then area of parallelogram formed by two vectors is



Area of parallelogram $= |\overline{AB} \times \overline{AC}|$.

Area of the triangle ABC

$$= \frac{1}{2} |\overline{AB} \times \overline{AC}|.$$

viii. Basic properties.

$$\overline{A} \times \overline{B} \neq \overline{B} \times \overline{A}.$$

$$\overline{A} \times \overline{B} = -\overline{B} \times \overline{A}.$$

$$\overline{A} \times (\overline{B} + \overline{C}) = \overline{A} \times \overline{B} + \overline{A} \times \overline{C}.$$

$$\overline{A} \times \overline{A} = 0.$$

Example Problem -12

Two Vectors are represented by

$\overline{A} = 2\overline{a}_x + 2\overline{a}_y + 0\overline{a}_z$ and

$\overline{B} = 3\overline{a}_x + 4\overline{a}_y - 2\overline{a}_z$ Find $\overline{A} \times \overline{B}$.

Show that $\overline{A} \times \overline{B}$ is at right angle to $\overline{A}$.

Soln:

Given $\overline{A} = 2\overline{a}_x + 2\overline{a}_y + 0\overline{a}_z$

$\overline{B} = 3\overline{a}_x + 4\overline{a}_y - 2\overline{a}_z$.

$$\overline{A} \times \overline{B} = \begin{vmatrix} \overline{a}_x & \overline{a}_y & \overline{a}_z \\ 2 & 2 & 0 \\ 3 & 4 & -2 \end{vmatrix}$$

$$= (-4-0)\overline{a}_x - (-4-0)\overline{a}_y + (8-6)\overline{a}_z$$

$$\boxed{\overline{A} \times \overline{B} = -4\overline{a}_x + 4\overline{a}_y + 2\overline{a}_z}$$

To show $(\bar{A} \times \bar{B}) \perp^{r} \bar{A}$

when

$$(\bar{A} \times \bar{B}) \cdot \bar{A} = 0.$$

$$= [-4\bar{a}_x + 4\bar{a}_y + 2\bar{a}_z] \cdot [2\bar{a}_x + 2\bar{a}_y + 0\bar{a}_z]$$

$$= -8 + 8 + 0 = \underline{0}.$$

Example problem -13

Given vectors $\bar{A} = 3\bar{a}_x + 4\bar{a}_y + \bar{a}_z$ and $\bar{B} = 2\bar{a}_y - 5\bar{a}_z$. Find the angle between $\bar{A}$ and $\bar{B}$.

Solu:
$$\theta = \sin^{-1}\left[\frac{|\bar{A}\times\bar{B}|}{|\bar{A}|\,|\bar{B}|}\right] \quad : \bar{A} = 3\bar{a}_x + 4\bar{a}_y + \bar{a}_z, \quad \bar{B} = 2\bar{a}_y - 5\bar{a}_z$$

$$|\bar{A}| = \sqrt{9+16+1} = \sqrt{26}\ m.$$

$$|\bar{B}| = \sqrt{4+25} = \sqrt{29}\ m.$$

$$\bar{A}\times\bar{B} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 3 & 4 & 1 \\ 0 & 2 & -5 \end{vmatrix} = -22\bar{a}_x + 15\bar{a}_y + 6\bar{a}_z$$

$$|\bar{A}\times\bar{B}| = \sqrt{22^2+15^2+6^2} = \sqrt{745}$$

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$$\theta = \sin^{-1}\left[\frac{|\bar{A}\times\bar{B}|}{|\bar{A}|\,|\bar{B}|}\right] = \sin^{-1}\left[\frac{\sqrt{745}}{\sqrt{26}\sqrt{29}}\right] = \underline{83.727^\circ}$$

$$\boxed{\theta = 83.72^\circ}$$

Example problem - 14

If $\overline{A} = \overline{a_x} + 3\overline{a_z}$ and $\overline{B} = 5\overline{a_x} + 2\overline{a_y} - 6\overline{a_z}$.
Find θ_{AB} using i. dot product ii. Cross product.

Solu: i. using dot product.

$\overline{A} = \overline{a_x} + 3\overline{a_z}$; $\overline{B} = 5\overline{a_x} + 2\overline{a_y} - 6\overline{a_z}$.

$|\overline{A}| = \sqrt{1+9} = \sqrt{10}$ m. $|\overline{B}| = \sqrt{25+4+36}$

$|\overline{B}| = \sqrt{65}$ m.

$\overline{A} \cdot \overline{B} = 5 - 18 = -13$

$\overline{A} \cdot \overline{B} = AB \cos\theta_{AB}$

$$\theta_{AB} = \cos^{-1}\left[\frac{\overline{A} \cdot \overline{B}}{|\overline{A}|\,|\overline{B}|}\right]$$

$$= \cos^{-1}\left[\frac{-13}{\sqrt{10}\,\sqrt{65}}\right] = 120.657^\circ$$

$$\boxed{\theta_{AB} = 120.657^\circ}$$

ii. using Cross product

$$\theta_{AB} = \sin^{-1}\left[\frac{|\bar{A}\times\bar{B}|}{|\bar{A}|\cdot|\bar{B}|}\right]$$

$$|\bar{A}\times\bar{B}| = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 1 & 0 & 3 \\ 5 & 2 & -6 \end{vmatrix}$$

$$\bar{A}\times\bar{B} = -6\,\bar{a}_x - (-6-5)\bar{a}_y + 2\bar{a}_z$$

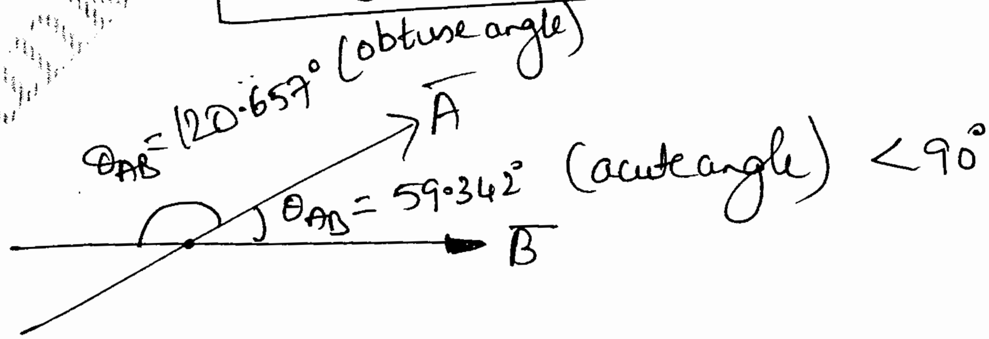
$$\bar{A}\times\bar{B} = -6\bar{a}_x + 21\bar{a}_y + 2\bar{a}_z$$

$$|\bar{A}\times\bar{B}| = \sqrt{6^2+21^2+2^2} = \sqrt{481}$$

$|\bar{A}| = \sqrt{10}\ m$ and $|\bar{B}| = \sqrt{65}\ m.$

$$\theta_{AB} = \sin^{-1}\left[\frac{|\bar{A}\times\bar{B}|}{|\bar{A}|\,|\bar{B}|}\right] = \sin^{-1}\left[\frac{\sqrt{481}}{\sqrt{10}\,\sqrt{65}}\right]$$

$$\boxed{\theta_{AB} = 59.342^\circ}$$



Example problem -15

three field quantities are given by

$\overline{P} = 2\overline{a_x} - \overline{a_z}$, $\overline{Q} = 2\overline{a_x} - \overline{a_y} + 2\overline{a_z}$ and

$\overline{R} = 2\overline{a_x} - 3\overline{a_y} + \overline{a_z}$. Determine

i. $(\overline{P} + \overline{Q}) \times (\overline{P} - \overline{Q})$.

ii. $\overline{Q} \cdot \overline{R} + \overline{P}$

iii. $\overline{P} \cdot \overline{Q} \times \overline{R}$

iv. $\sin \theta_{QR}$

v. $\overline{P} \times (\overline{Q} \times \overline{R})$.

vi. a unit vector $\perp^r$ to both $\overline{Q}$ and $\overline{R}$.

vii. ~~the~~ component of $\overline{P}$ along $\overline{Q}$.

Soln: i. $(\overline{P} + \overline{Q}) \times (\overline{P} - \overline{Q})$

$$= \cancel{\overline{P} \times \overline{P}}^{\,0} - \overline{P} \times \overline{Q} + \overline{Q} \times \overline{P} - \cancel{\overline{Q} \times \overline{Q}}^{\,0}$$

$$= \overline{Q} \times \overline{P} + \overline{Q} \times \overline{P}$$

$$= 2\overline{Q} \times \overline{P}.$$

$$2\overline{Q} \times \overline{P} = \begin{vmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ 4 & -2 & 4 \\ 2 & 0 & -1 \end{vmatrix}$$

$$= [2\overline{a_x} - (-4-8)\overline{a_y} + 4\overline{a_z}]$$

$$2\bar{Q}\times\bar{P} = 2\bar{a}_x + 12\bar{a}_y + 4\bar{a}_z$$

$$\therefore \boxed{(\bar{P}+\bar{Q})\times(\bar{P}-\bar{Q}) = 2\bar{a}_x + 12\bar{a}_y + 4\bar{a}_z}$$

ii. $\bar{Q}\cdot\bar{R}\times\bar{P} = ?$

$$\bar{R}\times\bar{P} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 2 & -3 & 1 \\ 2 & 0 & -1 \end{vmatrix}$$

$$= [+3-0]\,\bar{a}_x - [-2-2]\,\bar{a}_y + [0+6]\,\bar{a}_z$$

$$\boxed{\bar{R}\times\bar{P} = 3\bar{a}_x + 4\bar{a}_y + 6\bar{a}_z}$$

$$\bar{Q}\cdot(\bar{R}\times\bar{P}) = (2\bar{a}_x - \bar{a}_y + 2\bar{a}_z)\cdot(3\bar{a}_x + 4\bar{a}_y + 6\bar{a}_z)$$

$$= 6-4+12 = 14$$

$$\boxed{\bar{Q}\cdot(\bar{R}\times\bar{P}) = 14}$$

iii. $\bar{P}\cdot\bar{Q}\times\bar{B}$

$$\bar{Q}\times\bar{B} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 2 & -1 & 2 \\ 2 & -3 & 1 \end{vmatrix}$$

$$= [-1+6]\,\bar{a}_x - [2-4]\,\bar{a}_y + [-6+2]\,\bar{a}_z$$

$$\bar{Q}\times\bar{B} = 5\bar{a}_x + 2\bar{a}_y - 4\bar{a}_z$$

$$\bar{P}\cdot(\bar{Q}\times\bar{B}) = (2\bar{a}_x - \bar{a}_z)\cdot(5\bar{a}_x + 2\bar{a}_y - 4\bar{a}_z)$$

$$= 10 + 0 + 4 = 14.$$

$$\boxed{\bar{P}\cdot\bar{Q}\times\bar{B} = 14}$$

iv. $\sin\theta_{QB} = ?$

$$\bar{Q}\times\bar{B} = |\bar{Q}|\,|\bar{R}|\,\sin\theta_{QB}\,\bar{a}_n$$

$$\Rightarrow \quad |\bar{Q}\times\bar{B}| = |\bar{Q}|\,|\bar{R}|\,\sin\theta_{QB}$$

$$\sin\theta_{QR} = \frac{|\bar{Q} \times \bar{R}|}{|\bar{Q}|\,|\bar{R}|}.$$

$$\bar{Q} \times \bar{R} = 5\bar{a}_x + 2\bar{a}_y - 4\bar{a}_z$$

$$|\bar{Q} \times \bar{R}| = \sqrt{25+4+16} = \sqrt{45}.$$

$$|\bar{Q}| = \sqrt{4+1+4} = \sqrt{9} = 3m.$$

$$|\bar{R}| = \sqrt{4+9+1} = \sqrt{14}\,m$$

$$\sin\theta_{QR} = \frac{\sqrt{45}}{3 \times \sqrt{14}} = 0.59761$$

$$\boxed{\sin\theta_{QR} = 0.59761}$$

V. $\bar{P} \times (\bar{Q} \times \bar{R}) = ?$

$$\bar{P} = 2\bar{a}_x - \bar{a}_z;$$

$$\bar{Q} \times \bar{R} = 5\bar{a}_x + 2\bar{a}_y - 4\bar{a}_z$$

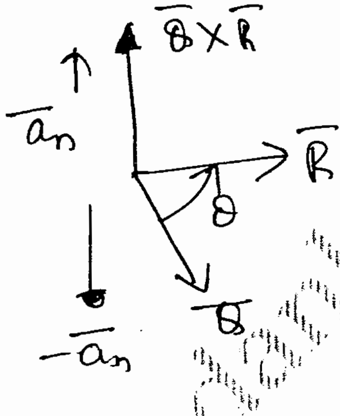
$$\bar{P} \times (\bar{Q} \times \bar{R}) = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 2 & 0 & -1 \\ 5 & 2 & -4 \end{vmatrix}$$

$$= [+2]\,\bar{a}_x - [-8+5]\,\bar{a}_y + [4-0]\,\bar{a}_z$$

$$\boxed{\bar{P} \times (\bar{Q} \times \bar{R}) = 2\bar{a}_x + 3\bar{a}_y + 4\bar{a}_z}$$

vi. a unit vector perpendicular to both $\bar{Q}$ and $\bar{R}$ is

$$\bar{Q} \times \bar{R} = QR\sin\theta\,\bar{a}_n = |\bar{Q} \times \bar{R}|\,\bar{a}_n$$



$$\bar{a}_n = \pm \frac{\bar{Q} \times \bar{R}}{|\bar{Q} \times \bar{R}|}$$

$$\bar{a}_n = \pm \frac{[5\bar{a}_x + 2\bar{a}_y - 4\bar{a}_z]}{\sqrt{25+4+16}}$$

$$\boxed{\bar{a}_n = \pm\,[0.745\,\bar{a}_x + 0.298\,\bar{a}_y - 0.596\,\bar{a}_z]}$$

vii. the component of $\overline{P}$ along $\overline{Q}$ is

$$= P\cos\theta_{PQ}\,\overline{a_Q}$$

$$= (\overline{P}\cdot\overline{a_Q})\,\overline{a_Q} = \overline{P}\cdot\frac{\overline{Q}}{|\overline{Q}|}\left(\frac{\overline{Q}}{|\overline{Q}|}\right)$$

$$= \frac{(\overline{P}\cdot\overline{Q})\,\overline{Q}}{|\overline{Q}|^2} = \frac{[4-2]\,[2\overline{a_x}-\overline{a_y}+2\overline{a_z}]}{[4+1+4]}$$

$$= \frac{2}{9}\,[2\overline{a_x}-\overline{a_y}+2\overline{a_z}]$$

$$\boxed{P\cos\theta_{PQ}\,\overline{a_Q} = 0.44\,\overline{a_x} - 0.22\,\overline{a_y} + 0.44\,\overline{a_z}}$$

Example problem -16

Let $\bar{E} = 3\bar{a}_y + 4\bar{a}_z$ and $\bar{F} = 4\bar{a}_x - 10\bar{a}_y + 5\bar{a}_z$.

a. Find the Component of $\bar{E}$ along $\bar{F}$.

b. Determine a unit vector perpendicular to both $\bar{E}$ and $\bar{F}$.

Soln:-

a) Component of $\bar{E}$ along $\bar{F}$ = $E\cos\theta_{EF}\,\bar{a}_F$.

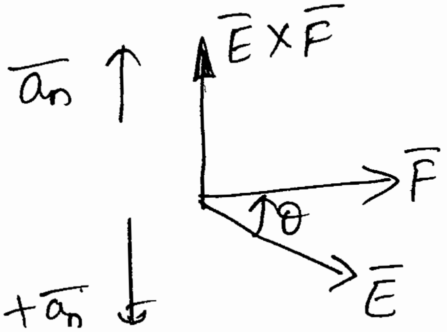
$$E\cos\theta_{EF}\,\bar{a}_F = \frac{(\bar{E}\cdot\bar{F})\,\bar{F}}{|\bar{F}|^2}$$

$$= \frac{[-30+20]\,[4\bar{a}_x - 10\bar{a}_y + 5\bar{a}_z]}{[16+100+25]}$$

$$= \frac{-10}{141}[4\bar{a}_x - 10\bar{a}_y + 5\bar{a}_z]$$

$$\boxed{E\cos\theta_{EF}\,\bar{a}_F = -0.283\,\bar{a}_x + 0.70\,\bar{a}_y - 0.354\,\bar{a}_z}$$

b. a unit vector perpendicular to both $\bar{E}$ and $\bar{F}$ is $\pm\bar{a}_n$



$$\bar{E}\times\bar{F} = EF\sin\theta\,\bar{a}_n$$

$$\bar{E}\times\bar{F} = |\bar{E}\times\bar{F}|\,\bar{a}_n$$

$$\Rightarrow \bar{a}_n = \pm\frac{[\bar{E}\times\bar{F}]}{|\bar{E}\times\bar{F}|}.$$

$$\bar{E}\times\bar{F} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 0 & 3 & 4 \\ 4 & -10 & 5 \end{vmatrix}$$

$$= [15+40]\bar{a}_x - [0-16]\bar{a}_y + [0-12]\bar{a}_z$$

$$\bar{E}\times\bar{F} = 55\bar{a}_x + 16\bar{a}_y - 12\bar{a}_z$$

$$|\bar{E}\times\bar{F}| = \sqrt{3425}.$$

$$\bar{a}_n = \pm\frac{55\bar{a}_x + 16\bar{a}_y - 12\bar{a}_z}{\sqrt{3425}}$$

$$= \pm[0.9398\bar{a}_x + 0.2734\bar{a}_y - 0.205\bar{a}_z]$$

Example problem - 17

if $\bar{A} = 4\bar{a}_x - 2\bar{a}_y + 6\bar{a}_z$ and $\bar{B} = 12\bar{a}_x + 18\bar{a}_y - 8\bar{a}_z$.

determine.

a. $\bar{A} - 3\bar{B}$.

b. $(2\bar{A} + 5\bar{B}) / |\bar{B}|$.

c. $\bar{a}_x \times \bar{A}$.

d. $(\bar{B} \times \bar{a}_x) \cdot \bar{a}_y$

Solu: a. $\bar{A} = 4\bar{a}_x - 2\bar{a}_y + 6\bar{a}_z$

$\bar{B} = 12\bar{a}_x + 18\bar{a}_y - 8\bar{a}_z$.

$\bar{A} - 3\bar{B} = 4\bar{a}_x - 2\bar{a}_y + 6\bar{a}_z - 36\bar{a}_x - 54\bar{a}_y + 24\bar{a}_z$

$$\boxed{\bar{A} - 3\bar{B} = -32\bar{a}_x - 56\bar{a}_y + 30\bar{a}_y}$$

b. $2\bar{A} + 5\bar{B} = 8\bar{a}_x - 4\bar{a}_y + 12\bar{a}_z + 60\bar{a}_x + 90\bar{a}_y - 40\bar{a}_z$

$2\bar{A} + 5\bar{B} = 68\bar{a}_x + 86\bar{a}_y - 28\bar{a}_z$

$|\bar{B}| = \sqrt{12^2 + 18^2 + 8^2} = \sqrt{532}$ m

$$\frac{2\bar{A}+5\bar{B}}{|\bar{B}|} = \frac{68\,\bar{a}_x + 86\,\bar{a}_y - 28\,\bar{a}_z}{\sqrt{532}}$$

$$= 2.948\,\bar{a}_x + 3.728\,\bar{a}_y - 1.214\,\bar{a}_z$$

c) $\bar{a}_x \times \bar{A} = \bar{a}_x \times (4\bar{a}_x - 2\bar{a}_y + 6\bar{a}_z)$

$$= 4\cancel{(\bar{a}_x \times \bar{a}_x)}^{0} - 2(\bar{a}_x \times \bar{a}_y) + 6(\bar{a}_x \times \bar{a}_z)$$

$$= -2(\bar{a}_z) + 6(-\bar{a}_y)$$

$$= -6\,\bar{a}_y - 2\bar{a}_z$$

d) $(\bar{B} \times \bar{a}_x) \cdot \bar{a}_y$

$$\bar{B} \times \bar{a}_x = (12\bar{a}_x + 18\bar{a}_y - 8\bar{a}_z) \times \bar{a}_x$$

$$= 12\cancel{(\bar{a}_x \times \bar{a}_x)}^{0} + 18(\bar{a}_y \times \bar{a}_x) - 8(\bar{a}_z \times \bar{a}_x)$$

$$= 18(-\bar{a}_z) - 8(+\bar{a}_y)$$

$$= -8\,\bar{a}_y - 18\,\bar{a}_z$$

$$(\bar{B} \times \bar{a}_x) \cdot \bar{a}_y = (-8\bar{a}_y - 18\bar{a}_z) \cdot \bar{a}_y = -8$$

$$\boxed{(\bar{B} \times \bar{a}_x) \cdot \bar{a}_y = -8}$$

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Example problem -18

Determine the dot product, Cross product and angle between $\bar{P} = 2\bar{a}_x - 6\bar{a}_y + 5\bar{a}_z$ and $\bar{Q} = 3\bar{a}_y + \bar{a}_z$.

Soln: i. $\bar{P}\cdot\bar{Q} = (2\bar{a}_x - 6\bar{a}_y + 5\bar{a}_z)\cdot(3\bar{a}_y + \bar{a}_z)$

$$= 0 - 18 + 5 = -13$$

dot product $\boxed{\bar{P}\cdot\bar{Q} = -13}$; $|\bar{P}| = \sqrt{4+36+25} = \sqrt{65}$ m.

$|\bar{Q}| = \sqrt{9+1} = \sqrt{10}$ m.

ii. Cross product

$$\bar{P}\times\bar{Q} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 2 & -6 & 5 \\ 0 & 3 & 1 \end{vmatrix}$$

$$= [-6-15]\bar{a}_x - [2-0]\bar{a}_y + [6-0]\bar{a}_z$$

$$\boxed{\bar{P}\times\bar{Q} = -21\bar{a}_x - 2\bar{a}_y + 6\bar{a}_z}$$

$$\theta_{PQ} = \cos^{-1}\left[\frac{\bar{P}\cdot\bar{Q}}{|\bar{P}|\,|\bar{Q}|}\right] = \cos^{-1}\left[\frac{-13}{\sqrt{65}\sqrt{10}}\right] = \underline{120.65^\circ}$$

$$\boxed{\theta_{PQ} = 120.65^\circ}$$

Example problem - 19

Find the area of the parallelogram formed by the vectors $\bar{D} = 4\bar{a}_x - \bar{a}_y + 5\bar{a}_z$ and

$\bar{E} = -\bar{a}_x + 2\bar{a}_y + 3\bar{a}_z$.

Solu: Area of parallelogram is given by

$|\bar{D} \times \bar{E}|$

$$\bar{D} \times \bar{E} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 4 & -1 & 5 \\ -1 & 2 & 3 \end{vmatrix}$$

$$= [-3-10]\,\bar{a}_x - [12+5]\,\bar{a}_y + [8-1]\,\bar{a}_z$$

$$\bar{D} \times \bar{E} = -13\bar{a}_x - 17\bar{a}_y + 7\bar{a}_z.$$

$$|\bar{D} \times \bar{E}| = \sqrt{13^2 + 17^2 + 7^2}$$

$$\boxed{|\bar{D} \times \bar{E}| = \sqrt{507}}\ m^2$$

Example problem - 20

if $\bar{A} = 4\bar{a}_x - 6\bar{a}_y + \bar{a}_z$ and $\bar{B} = 2\bar{a}_x + 5\bar{a}_z$ Find

a) $\bar{A} \cdot \bar{B} + 2|\bar{B}|^2$

b) A unit vector $\perp^r$ to both $\bar{A}$ and $\bar{B}$.

Soln:

a) $\bar{A} \cdot \bar{B} + 2|\bar{B}|^2 = ?$

$\bar{A} \cdot \bar{B} = [4\bar{a}_x - 6\bar{a}_y + \bar{a}_z] \cdot [2\bar{a}_x + 5\bar{a}_z]$

$= 8 + 0 + 5 = 13$

$\boxed{\bar{A} \cdot \bar{B} = 13}$

$\bar{B} = 2\bar{a}_x + 5\bar{a}_z$

$|\bar{B}|^2 = 4 + 25 = 29.$

$\bar{A} \cdot \bar{B} + 2|\bar{B}|^2 = 13 + 2(29) = 71.$

$\boxed{\bar{A} \cdot \bar{B} + 2|\bar{B}|^2 = 71}$

b) A unit vector $\perp^{r}$ to both $\bar{A}$ and $\bar{B}$.

$$\bar{a}_n = \pm \frac{\bar{A} \times \bar{B}}{|\bar{A} \times \bar{B}|}$$

$$\bar{A} \times \bar{B} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 4 & -6 & 1 \\ 2 & 0 & 5 \end{vmatrix}$$

$$= [-30-0]\bar{a}_x - [20-2]\bar{a}_y + [0+12]\bar{a}_z$$

$$\boxed{\bar{A} \times \bar{B} = -30\bar{a}_x - 18\bar{a}_y + 12\bar{a}_z}$$

$$|\bar{A} \times \bar{B}| = \sqrt{30^2 + 18^2 + 12^2} = \sqrt{1368}$$

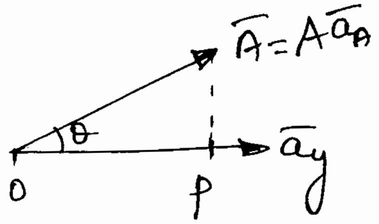
$$\bar{a}_n = \pm \frac{[-30\bar{a}_x - 18\bar{a}_y + 12\bar{a}_z]}{\sqrt{1368}}$$

$$\boxed{\bar{a}_n = \pm [-0.8111\bar{a}_x - 0.4867\bar{a}_y + 0.3244\bar{a}_z]}$$

Example problem - 21

Given $\bar{A} = -6\bar{a}_x + 3\bar{a}_y + 2\bar{a}_z$. The projection of $\bar{A}$ along $\bar{a}_y$ is ?

Solu:
$$\boxed{\bar{A}\cdot\bar{a}_y = A_y = 3}$$



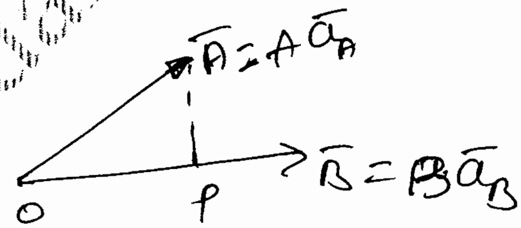
$$\boxed{OP = A\cos\theta = \bar{A}\cdot\bar{a}_y = A_y = 3}$$

Example problem - 22

The component of $6\bar{a}_x + 2\bar{a}_y - 3\bar{a}_z$ along $3\bar{a}_x - 4\bar{a}_y$ is ?

Solu: let $\bar{A} = 6\bar{a}_x + 2\bar{a}_y - 3\bar{a}_z$

and $\bar{B} = 3\bar{a}_x - 4\bar{a}_y$.



the component of $\bar{A}$ along $\bar{B}$ is $= A\cos\theta$

$$= \bar{A}\cdot\bar{a}_B$$

$$\bar{A}\cdot\bar{a}_B = \frac{\bar{A}\cdot\bar{B}}{|\bar{B}|} = \frac{[6\bar{a}_x + 2\bar{a}_y - 3\bar{a}_z]\cdot[3\bar{a}_x - 4\bar{a}_y]}{\sqrt{9+16}}$$

$$= \frac{18-8}{\sqrt{25}} = \frac{10}{5} = 2$$

$$\boxed{A\cos\theta = \bar{A}\cdot\bar{a}_B = 2}$$

Example problem -23

Given that $\bar{A} = \bar{a}_x + \alpha\bar{a}_y + \bar{a}_z$ and $\bar{B} = \alpha\bar{a}_x + \bar{a}_y + \bar{a}_z$. if $\bar{A}$ and $\bar{B}$ are normal to each other, α is ?

Solu: $\bar{A}\cdot\bar{B} = 0$ when $\bar{A} \perp^r \bar{B}$.

$$\bar{A}\cdot\bar{B} = (\bar{a}_x + \alpha\bar{a}_y + \bar{a}_z)\cdot(\alpha\bar{a}_x + \bar{a}_y + \bar{a}_z)$$

$$\bar{A}\cdot\bar{B} = [\alpha + \alpha + 1] = 0$$

$$2\alpha + 1 = 0$$

$$\boxed{\alpha = -1/2}$$

Example problem -24

Let $\bar{F} = 2\bar{a}_x - 6\bar{a}_y + 10\bar{a}_z$ and $\bar{G} = \bar{a}_x + G_y\bar{a}_y + 5\bar{a}_z$. If $\bar{F}$ and $\bar{G}$ have same unit vector then G_y is ?

Soln: given $\bar{a}_F = \bar{a}_G$

$$\frac{2\bar{a}_x - 6\bar{a}_y + 10\bar{a}_z}{\sqrt{4+36+100}} = \frac{\bar{a}_x + G_y\bar{a}_y + 5\bar{a}_z}{\sqrt{1 + G_y^2 + 25}}$$

Equating $\bar{a}_y$ components on bothside.

$$\frac{-6}{\sqrt{140}} = \frac{G_y}{\sqrt{G_y^2 + 26}}$$

Square on bothside

$$\frac{36}{140} = \frac{G_y^2}{G_y^2 + 26}$$

$$36G_y^2 + 936 = 140G_y^2$$

$$104G_y^2 = 936$$

$$G_y^2 = \frac{936}{104}$$

$$G_y^2 = 9$$

$$G_y = \pm\sqrt{9} = \pm 3$$

$$\boxed{G_y = \pm 3}$$

but

$\Rightarrow$ the value G_y, for $\vec{F}$ and $\vec{G}$ have same vector is -3

i.e $\boxed{G_y = -3}$

Example problem - 25

A triangle is defined by the three points $A(2,-5,1)$, $B(-3,2,4)$ and $C(0,3,1)$ Find

i. $\overline{R_{BC}} \times \overline{R_{BA}}$

ii. the Area of the triangle.

iii. a unit vector $\perp^r$ to the plane in which the triangle is located.

soln:- i. $\overline{R_{BC}} \times \overline{R_{BA}} = ?$

$$\overline{R_{BC}} = 3\overline{a_x} + \overline{a_y} - 3\overline{a_z}$$

$$\overline{R_{BA}} = 5\overline{a_x} - 7\overline{a_y} - 3\overline{a_z}$$

$$\overline{R_{BC}} \times \overline{R_{BA}} = \begin{vmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ 3 & 1 & -3 \\ 5 & -7 & -3 \end{vmatrix}$$

$$= [-3-21]\overline{a_x} - [-9+15]\overline{a_y} + [-21-5]\overline{a_z}$$

$$\boxed{\overline{R_{BC}} \times \overline{R_{BA}} = -24\overline{a_x} - 6\overline{a_y} - 26\overline{a_z}}$$

ii. Area of the triangle $= \frac{1}{2} \left| \overline{R_{BC}} \times \overline{R_{BA}} \right|$

$$\overline{R_{BC}} \times \overline{R_{BA}} = -24\bar{a}_x - 6\bar{a}_y - 26\bar{a}_z$$

$$\left| \overline{R_{BC}} \times \overline{R_{BA}} \right| = \sqrt{24^2 + 6^2 + 26^2} = \sqrt{1288}$$

Area of $\Delta^{le} = \frac{1}{2}\sqrt{1288} = \underline{17.944 \text{ m}^2}$

iii. A unit vector $\perp^r$ to the plane in which the triangle is located is nothing but a unit vector in the direction of Cross product.

$$\bar{a}_n = \pm \frac{\overline{R_{BC}} \times \overline{R_{AC}}}{\left| \overline{R_{BC}} \times \overline{R_{AC}} \right|} = \pm \frac{[-24\bar{a}_x - 6\bar{a}_y - 26\bar{a}_z]}{\sqrt{24^2 + 6^2 + 26^2}}$$

$$= \pm \frac{[-24\bar{a}_x - 6\bar{a}_y - 26\bar{a}_z]}{\sqrt{24^2 + 6^2 + 26^2}}$$

$$\boxed{\bar{a}_n = \pm [0.669\bar{a}_x + 0.1672\bar{a}_y + 0.7245\bar{a}_z]}$$

Example problem -26

Show that $\bar{A} = 4\bar{a}_x - 2\bar{a}_y - \bar{a}_z$ and $\bar{B} = \bar{a}_x + 4\bar{a}_y - 4\bar{a}_z$ are perpendicular by considering their dot product.

(4m). Jan 2014 (EEE).

Soln: If $\bar{A}$ and $\bar{B}$ are perpendicular to each other then $\bar{A} \cdot \bar{B} = 0$.

$$\bar{A} \cdot \bar{B} = [4\bar{a}_x - 2\bar{a}_y - \bar{a}_z] \cdot [\bar{a}_x + 4\bar{a}_y - 4\bar{a}_z]$$

$$= \cancel{4} - \cancel{8} + \cancel{4} = 0$$

$$\boxed{\bar{A} \cdot \bar{B} = 0}$$

$\therefore \bar{A} \perp^r \bar{B}$.

Keynote points

1. Scalar Tripple product :-

$$\bar{A} \cdot (\bar{B} \times \bar{C}) = \bar{B} \cdot (\bar{C} \times \bar{A}) = \bar{C} \cdot (\bar{A} \times \bar{B})$$

$$= \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

ii. Vector tripple product :

$$\bar{A} \times (\bar{B} \times \bar{C}) = \bar{B}(\bar{A} \cdot \bar{C}) - \bar{C}(\bar{A} \cdot \bar{B})$$

...... [BAC − CAB rule]

4. The DEL OPERATOR (∇) / Spatial operator.

The ∇ (del) operator in Cartesian Co-ordinate System is defined as

$$\nabla = \frac{\partial}{\partial x}\overline{a_x} + \frac{\partial}{\partial y}\overline{a_y} + \frac{\partial}{\partial z}\overline{a_z} \quad ; \; m^{-1} \text{ (or) } 1/m.$$

- it is a vector differential operator.
- del can operate on a Scalar as well as vector.
- when del operates on a Scalar, the operation is called gradient.
- when it operates on a vector, it can operate two ways, either dot product (or) cross product. the operations are called as divergence and Curl respectively.

Del operator (∇)

Scalar (ϕ) — Vector ($\overline{A}$)

dot product — Cross product

i. $\nabla\phi$ (Gradient) [it is a vector]

ii. $\nabla\cdot\overline{A}$ (Divergence) [it is a Scalar]

iii. $\nabla\times\overline{A}$ (curl) [it is a vector]

* Divergence of a gradient $\nabla\cdot(\nabla\phi) = \nabla^2\phi$ called Laplacian of a Scalar ϕ.

4ai. Concept of Gradient

* The gradient of a scalar field ϕ is a vector that represents both the magnitude and the direction of the maximum space rate of increase of ϕ.
* when ∇ operates on a scalar the operation called as Gradient.
* Gradient result's in vector.
* if ϕ is a scalar quantity then

$$\nabla\phi = \text{gradient of } \phi = \text{grad } \phi$$

thus

$$\boxed{\nabla\phi = \frac{\partial\phi}{\partial x}\bar{a}_x + \frac{\partial\phi}{\partial y}\bar{a}_y + \frac{\partial\phi}{\partial z}\bar{a}_z} = \text{Vector}$$

grad operation result in vector.

Example problem - 27

Find the gradient of function ϕ

i) $\phi = \cosh xyz$.

ii) $\phi = x^2+y^2+z^2$.

Solu:- i. $\nabla\phi = \frac{\partial\phi}{\partial x}\bar{a}_x + \frac{\partial\phi}{\partial y}\bar{a}_y + \frac{\partial\phi}{\partial z}\bar{a}_z$

$$\nabla\phi = \frac{\partial}{\partial x}[\cosh xyz]\bar{a}_x + \frac{\partial}{\partial y}[\cosh xyz]\bar{a}_y + \frac{\partial}{\partial z}[\cosh xyz]\bar{a}_z$$

$$\nabla\phi = yz\sinh xyz\,\bar{a}_x + xz\sinh xyz\,\bar{a}_y + xy\sinh xyz\,\bar{a}_z$$

$$\boxed{\nabla\phi = \sinh xyz[yz\bar{a}_x + xz\bar{a}_y + xy\bar{a}_z]}$$

ii. $\nabla\phi = \frac{\partial}{\partial x}[x^2+y^2+z^2]\bar{a}_x + \frac{\partial}{\partial y}[x^2+y^2+z^2]\bar{a}_y + \frac{\partial}{\partial z}[x^2+y^2+z^2]\bar{a}_z$

$$\nabla\phi = 2x\bar{a}_x + 2y\bar{a}_y + 2z\bar{a}_z$$

$$\boxed{\nabla\phi = 2[x\bar{a}_x + y\bar{a}_y + z\bar{a}_z]}$$

Solved ~~Pr~~ example - 28

If $\phi(x,y,z) = 3x^2y - y^3z^2$. Find $\nabla\phi$ at the point $(1,-2,-1)$.

$$\nabla\phi = \frac{\partial\phi}{\partial x}\bar{a}_x + \frac{\partial\phi}{\partial y}\bar{a}_y + \frac{\partial\phi}{\partial z}\bar{a}_z$$

$$\nabla\phi = \frac{\partial}{\partial x}\left[3x^2y - y^3z^2\right]\bar{a}_x + \frac{\partial}{\partial y}\left[3x^2y - y^3z^2\right]\bar{a}_y + \frac{\partial}{\partial z}\left[3x^2y - y^3z^2\right]\bar{a}_z$$

$$\nabla\phi = 6xy\,\bar{a}_x + (3x^2 - 3y^2z^2)\bar{a}_y - 2y^3z\,\bar{a}_z$$

$\nabla\phi$ at point $P(1,-2,-1)$

$$\nabla\phi_P = 6(1)(-2)\bar{a}_x + \left[3(1)^2 - 3(-2)^2(-1)^2\right]\bar{a}_y - 2(-2)^3(-1)\bar{a}_z$$

$$\boxed{\nabla\phi_P = -12\bar{a}_x - 12\bar{a}_y - 16\bar{a}_z}$$

Solved example - 29

Find the gradient of following scalar fields.

i. $V = e^{-z} \sin(2x) \cosh y$.

ii. $U = x^2 y + xyz$.

iii. $w = x^2 y^2 + xyz$.

Soln:- i. $\nabla V = \frac{\partial V}{\partial x} \bar{a}_x + \frac{\partial V}{\partial y} \bar{a}_y + \frac{\partial V}{\partial z} \bar{a}_z$

$$\nabla V = \frac{\partial}{\partial x}\left[e^{-z} \sin(2x) \cosh y\right] \bar{a}_x + \frac{\partial}{\partial y}\left[e^{-z} \sin(2x) \cosh y\right] \bar{a}_y + \frac{\partial}{\partial z}\left[e^{-z} \sin(2x) \cosh y\right] \bar{a}_z$$

$$\boxed{\nabla V = 2e^{-z} \cos(2x) \cosh y \, \bar{a}_x + e^{-z} \sin(2x) \sinh y \, \bar{a}_y - e^{-z} \sin(2x) \cosh y \, \bar{a}_z}$$

ii. $U = x^2 y + xyz$

$$\nabla U = \frac{\partial U}{\partial x} \bar{a}_x + \frac{\partial U}{\partial y} \bar{a}_y + \frac{\partial U}{\partial z} \bar{a}_z$$

$$= \frac{\partial}{\partial x}\left[x^2 y + xyz\right] \bar{a}_x + \frac{\partial}{\partial y}\left[x^2 y + xyz\right] \bar{a}_y + \frac{\partial}{\partial z}\left[x^2 y + xyz\right] \bar{a}_z$$

$$= [2xy + yz]\,\overline{a_x} + [x^2 + xz]\,\overline{a_y} + (xy)\,\overline{a_z}$$

$$\boxed{\nabla U = y(2x+z)\,\overline{a_x} + x(x+z)\,\overline{a_y} + xy\,\overline{a_z}}$$

iii. $W = x^2y^2 + xyz$.

$$\nabla W = \frac{\partial W}{\partial x}\,\overline{a_x} + \frac{\partial W}{\partial y}\,\overline{a_y} + \frac{\partial W}{\partial z}\,\overline{a_z}$$

$$\nabla W = \frac{\partial}{\partial x}[x^2y^2 + xyz]\,\overline{a_x} + \frac{\partial}{\partial y}[x^2y^2 + xyz]\,\overline{a_y} + \frac{\partial}{\partial z}[x^2y^2 + xyz]\,\overline{a_z}$$

$$\boxed{\nabla W = 2xy^2 + yz\,\overline{a_x} + (2x^2y + xz)\,\overline{a_y} + xy\,\overline{a_z}}$$

Fundamental properties of Gradient

Let U and V are the Scalar fields.

i. $\nabla(V+U) = \nabla V + \nabla U$.

ii. $\nabla(UV) = U\nabla V + V\nabla U$.

iii. $\nabla\left(\frac{V}{U}\right) = \frac{U\nabla V - V\nabla U}{U^2}$

iv. $\nabla V^n = nV^{n-1}\nabla V$

v. The magnitude of ∇V equals the maximum rate of change in V per unit distance.

vi. ∇V points in the direction of the maximum rate of change in V.

4b. Divergence ($\nabla \cdot \bar{A}$)

when ∇ operates on a vector $\bar{A}$ then $\nabla \cdot \bar{A}$ is called as divergence of $\bar{A}$.

if $\bar{A} = A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_z$, then

divergence of $\bar{A} = \operatorname{div} \bar{A} = \nabla \cdot \bar{A}$

$$= \left[\frac{\partial}{\partial x} \bar{a}_x + \frac{\partial}{\partial y} \bar{a}_y + \frac{\partial}{\partial z} \bar{a}_z\right] \cdot \left[A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_z\right]$$

thus

$$\boxed{\nabla \cdot \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}} = \text{Scalar.}$$

The divergence operation results in a Scalar quantity.

Fundamental properties of divergence

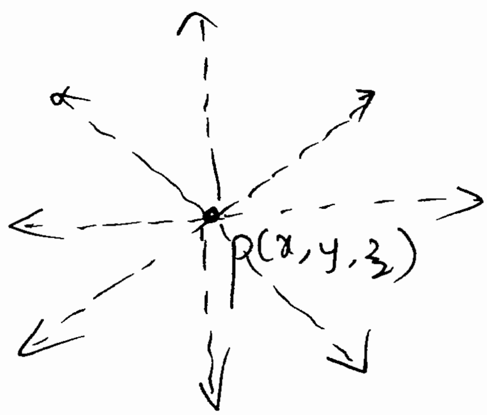
i. it produces a Scalar field. (because Scalar product is involved).

ii. $\nabla\cdot(\bar{A}+\bar{B}) = \nabla\cdot\bar{A} + \nabla\cdot\bar{B}$.

iii. $\nabla\cdot(V\bar{A}) = V\,\nabla\cdot\bar{A} + \bar{A}\cdot\nabla V$

where V – Scalar.

iv. if $\nabla\cdot\bar{A} > 0$ @ point $p(x,y,z)$ i.e $\nabla\cdot\bar{A} = +ve$.



$\nabla\cdot\bar{A} > 0$.

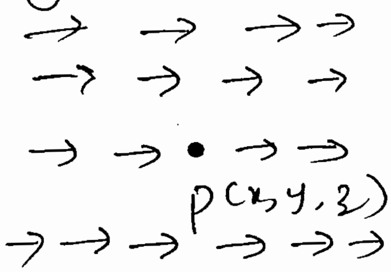
if $\nabla\cdot\bar{A}$ is +ve at a particular point 'p' is nothing but there is an Source at point 'p' which generates the field.

v. if $\nabla\cdot\bar{A} = -ve$ i.e $\nabla\cdot\bar{A} < 0$ at point $p(x,y,z)$

$\nabla\cdot\bar{A} < 0$

if the $\nabla\cdot\bar{A}$ is –ve at a particular point $p(x,y,z)$ is nothing but there is an Sink at that point which absorbs the field.

vi. if $\nabla \cdot \bar{A} = 0$ at point $p(x,y,z)$ is nothing but whatever field is converging same is diverging then $\nabla \cdot \bar{A} = 0$ at point p.



$\nabla \cdot \bar{A} = 0$. at $p(x,y,z)$

Notes-

vii. ~~if $\nabla \cdot \bar{A} = 0$ which is nothing 'Solenoidal field'~~

if $\bar{A}$ is said to be 'Solenoidal' field only when $\nabla \cdot \bar{A} = 0$.

Solved Examples - 30

if $\overline{A} = x^2 z\,\overline{a_x} - 2y^2 z^2\,\overline{a_y} + xy^2 z\,\overline{a_z}$. Find $\nabla \cdot \overline{A}$ at the point $p(1,-1,1)$

Solu:

$$\nabla \cdot \overline{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\overline{A} = x^2 z\,\overline{a_x} - 2y^2 z^2\,\overline{a_y} + xy^2 z\,\overline{a_z}$$

$$A_x = x^2 z \; ; \; A_y = -2y^2 z^2 \quad A_z = xy^2 z$$

$$\nabla \cdot \overline{A} = \frac{\partial}{\partial x}(x^2 z) + \frac{\partial}{\partial y}(-2y^2 z^2) + \frac{\partial}{\partial z}(xy^2 z)$$

$$\nabla \cdot \overline{A} = 2xz - 4yz + xy^2$$

$\nabla \cdot \overline{A}$ at $p(1,-1,1)$.

$$\nabla \cdot \overline{A}_p = 2(1)(1) - 4(-1)(1) + (1)(-1)^2$$

$$\nabla \cdot \overline{A}_p = 2 + 4 + 1 = 7$$

$$\boxed{\nabla \cdot \overline{A}_p = 7}$$

Solved example - 31

Determine the Divergence of the following vectorfields and Evaluate them at the Specified points.

i. $\bar{P} = x^2yz\,\bar{a_x} + xz\,\bar{a_z}$

ii. $\bar{A} = xyz\,\bar{a_x} + 4xy\,\bar{a_y} + y\,\bar{a_z}$ at $p(1,-2,3)$.

Soln:- i. $\nabla\cdot\bar{P} = \dfrac{\partial P_x}{\partial x} + \dfrac{\partial P_y}{\partial y} + \dfrac{\partial P_z}{\partial z}$

$P_x = x^2yz$, $P_y = 0$ and $P_z = xz$

$$\nabla\cdot\bar{P} = \frac{\partial}{\partial x}(x^2yz) + \frac{\partial}{\partial z}(xz)$$

$$\nabla\cdot\bar{P} = 2xyz + x$$

$\nabla\cdot\bar{P}$ at $p(1,-2,3)$

$$\nabla\cdot\bar{P} = 2(1)(-2)(3) + 1 = -12 + 1 = -11$$

$$\boxed{\nabla\cdot\bar{P} = -11}$$

ii. $\overline{A} = yz\,\overline{a_x} + 4xy\,\overline{a_y} + y\,\overline{a_z}$

$$\nabla \cdot \overline{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$P(1, -2, 3)$

$A_x = yz \;;\quad A_y = 4xy \;;\quad A_z = y$

$$\nabla \cdot \overline{A} = \cancel{\frac{\partial}{\partial x}(yz)}^{\,0} + \frac{\partial}{\partial y}(4xy) + \cancel{\frac{\partial}{\partial z}(y)}^{\,0}$$

$$\nabla \cdot \overline{A} = 4x = 4(1) = 4$$

$$\boxed{\nabla \cdot \overline{A} \text{ at } P = 4}$$

4c. Curl :-

When ∇ operates on a vector $\bar{A}$ as a ~~cross~~ product i.e $\nabla \times \bar{A}$ is called Curl of $\bar{A}$.

if $\bar{A} = A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_z$, ~~then~~ and

$$\nabla = \frac{\partial}{\partial x}\bar{a}_x + \frac{\partial}{\partial y}\bar{a}_y + \frac{\partial}{\partial z}\bar{a}_z$$

then

$$\text{Curl of } \bar{A} = \nabla \times \bar{A} = \text{Curl } \bar{A} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla \times \bar{A} = \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\bar{a}_x - \left[\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right]\bar{a}_y + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\bar{a}_z = \text{Vector}.$$

* the Curl results in a vector which is perpendicular to ∇ as well as $\bar{A}$.

* A vector $\bar{A}$ is said to be irrotational field only when $\nabla \times \bar{A} = 0$.

* $\nabla \times \bar{A} = \text{vector} = (\text{mag})\ \bar{a}_n$.

where $\bar{a}_n$ is unit normal vector which is $\perp^{r}$ to both ∇ and $\bar{A}$.

Fundamental properties of curl

i. $\nabla \times (\bar{A} + \bar{B}) = \nabla \times \bar{A} + \nabla \times \bar{B}$.

ii. $\nabla \times (V \bar{A}) = V \nabla \times \bar{A} + \nabla V \times \bar{A}$

... where V - scalar.

iii. The Divergence of the curl of a vector field vanishes. i.e $\nabla \cdot (\nabla \times \bar{A}) = 0$.

iv. The curl of the gradient of a scalar field vanishes. ie $\nabla \times \nabla V = 0$.

Example problem -32

Determine the curl of a vector fields

i. $\overline{P} = x^2yz\,\overline{a_x} + xz\,\overline{a_z}$

ii. $\overline{A} = yz\,\overline{a_x} + 4xy\,\overline{a_y} + y\,\overline{a_z}$ at point $P(1,-2,3)$

Soln:- i. $\overline{P} = x^2yz\,\overline{a_x} + xz\,\overline{a_z}$

$P_x = x^2yz$, $P_y = 0$ and $P_z = xz$.

$$\nabla\times\overline{P} = \begin{vmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ P_x & P_y & P_z \end{vmatrix}$$

$$= \begin{vmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2yz & 0 & xz \end{vmatrix}$$

$$\nabla\times\overline{P} = -\frac{\partial(x^2yz)}{\partial y}\,\overline{a_x} - \left[\frac{\partial(xz)}{\partial x} - \frac{\partial}{\partial z}(x^2yz)\right]\overline{a_y} + \left[0 - \frac{\partial(x^2yz)}{\partial y}\right]\overline{a_z}$$

$$\nabla \times \bar{P} = -x^2 z \bar{a}_x - (z - x^2 y)\bar{a}_y - x^2 z \bar{a}_z$$

$$\boxed{\nabla \times \bar{P} = -x^2 z \bar{a}_x + (x^2 y - z)\bar{a}_y - x^2 z \bar{a}_z.}$$

$\nabla \times \bar{P}$ at $P(1,-2,3)$

$$\nabla \times \bar{P} = -(1)^2(3)\bar{a}_x + [(1)^2(-2) - 3]\bar{a}_y - (1)^2(3)\bar{a}_z$$

$$\boxed{\nabla \times \bar{P} = -3\bar{a}_x - 5\bar{a}_y - 3\bar{a}_z}$$

ii. $\bar{A} = yz\bar{a}_x + 4xy\bar{a}_y + y\bar{a}_z$

$$\nabla \times \bar{A} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & 4xy & y \end{vmatrix}$$

$$\nabla \times \bar{A} = \left[\frac{\partial y}{\partial y} - \frac{\partial}{\partial z}(4xy)\right]\bar{a}_x - \left[\frac{\partial y}{\partial x} - \frac{\partial}{\partial z}(yz)\right]\bar{a}_y + \left[\frac{\partial}{\partial x}(4xy) - \frac{\partial}{\partial y}(yz)\right]\bar{a}_z$$

$$\boxed{\nabla \times \bar{A} = \bar{a}_x + y\,\bar{a}_y + (4y-3)\,\bar{a}_z}$$

$\nabla \times \bar{A}$ at $P(1,-2,+3)$ is

$$\nabla \times \bar{A} = \bar{a}_x + (-2)\,\bar{a}_y + [4(-2)-(3)]\,\bar{a}_z$$

$$\boxed{\nabla \times \bar{A} = \bar{a}_x - 2\bar{a}_y - 11\,\bar{a}_z}$$

Solved example - 33

Find the gradient of function ϕ:

i) $\phi = \cosh xyz$

iii) $\phi = x^2 y^2 + z^2$

Soln:- $\phi = \cosh xyz$

$$\nabla \phi = \frac{\partial \phi}{\partial x} \bar{a}_x + \frac{\partial \phi}{\partial y} \bar{a}_y + \frac{\partial \phi}{\partial z} \bar{a}_z$$

$$\nabla \phi = \frac{\partial}{\partial x}[\cosh xyz] \bar{a}_x + \frac{\partial}{\partial y}[\cosh xyz] \bar{a}_y + \frac{\partial}{\partial z}[\cosh xyz] \bar{a}_z$$

$$\nabla \phi = yz \sinh(xyz) \bar{a}_x + xz \sinh(xyz) \bar{a}_y + xy \sinh(xyz) \bar{a}_z$$

$$\boxed{\nabla \phi = \sinh(xyz)\left[yz\bar{a}_x + xz\bar{a}_y + xy\bar{a}_z\right]}$$

ii)

ii) $\phi = x^2 + y^2 + z^2$

$$\nabla\phi = \frac{\partial\phi}{\partial x}\bar{a}_x + \frac{\partial\phi}{\partial y}\bar{a}_y + \frac{\partial\phi}{\partial z}\bar{a}_z$$

$$\nabla\phi = \frac{\partial}{\partial x}(x^2+y^2+z^2)\bar{a}_x + \frac{\partial}{\partial y}(x^2+y^2+z^2)\bar{a}_y + \frac{\partial}{\partial z}(x^2+y^2+z^2)\bar{a}_z$$

$$\nabla\phi = 2x\,\bar{a}_x + 2y\,\bar{a}_y + 2z\,\bar{a}_z$$

$$\boxed{\nabla\phi = 2\left[x\bar{a}_x + y\bar{a}_y + z\bar{a}_z\right]}$$

Example problem - 34

if $\phi(x, y, z) = 3x^2y - y^3z^2$, find $\nabla\phi$ at the point $P(1, -2, -1)$.

Soln:- $\nabla\phi = \frac{\partial\phi}{\partial x}\bar{a}_x + \frac{\partial\phi}{\partial y}\bar{a}_y + \frac{\partial\phi}{\partial z}\bar{a}_z$

$$\frac{\partial \phi}{\partial x} = 6xy \quad \frac{\partial \phi}{\partial y} = 3x^2 - 3y^2z^2$$

$$\frac{\partial \phi}{\partial z} = -2y^3z$$

$$\nabla \phi = 6xy\,\overline{a_x} + (3x^2 - 3y^2z^2)\,\overline{a_y} - 2y^3z\,\overline{a_z}$$

$\nabla \phi$ at $p(1,-2,-1)$

$$\nabla \phi = 6(1)(-2)\,\overline{a_x} + \left[3(1)^2 - 3(-2)^2(-1)^2\right]\overline{a_y} - 2(-2)^3(-1)\,\overline{a_z}$$

$$\boxed{\nabla \phi = -12\,\overline{a_x} - 9\,\overline{a_y} - 16\,\overline{a_z}}$$

Example problem - 35

if $\nabla \times \bar{V} = 0$, find constants a, b and c so that $\bar{V} = (x + 2y + az)\,\bar{a}_x + (bx - 3y - z)\,\bar{a}_y + (4x + cy + 2z)\,\bar{a}_z$ is irrotational.

Soln

$$\nabla \times \bar{V} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (x+2y+az) & (bx-3y-z) & (4x+cy+2z) \end{vmatrix} = 0$$

$$= [c+1]\,\bar{a}_x - [4-a]\,\bar{a}_y + [b-2]\,\bar{a}_z = 0$$

$$\Rightarrow (c+1) = 0$$
$$(4-a) = 0$$
$$(b-2) = 0$$

$$\Rightarrow \boxed{a = 4} \; ; \; \boxed{b = 2} \text{ and } \boxed{c = -1}$$

Example problem - 36

Determine the curl of these vector field.

i) $\bar{A} = (2x^2+y^2)\bar{a}_x + (xy-y^2)\bar{a}_y$.

ii) $\bar{A} = yz\,\bar{a}_x + 4xy\,\bar{a}_y + y\,\bar{a}_z$.

Soln: i)

$$\nabla \times \bar{A} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (2x^2+y^2) & (xy-y^2) & 0 \end{vmatrix}$$

$$= \left[0 - \cancel{\frac{\partial}{\partial z}(xy-y^2)}\right]\bar{a}_x - \left[0 - \cancel{\frac{\partial}{\partial z}(2x^2+y^2)}\right]\bar{a}_y + \left[\frac{\partial}{\partial x}(xy-y^2) - \frac{\partial}{\partial y}(2x^2+y^2)\right]\bar{a}_z$$

0 0

$$= [y - 2y]\,\bar{a}_z = -y\,\bar{a}_z$$

$$\boxed{\nabla \times \bar{A} = -y\,\bar{a}_z}$$

ii.

$$\nabla \times \bar{A} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & 4xy & y \end{vmatrix} = (1-0)\bar{a}_x - (0-y)\bar{a}_y + (4y-z)\bar{a}_z$$

$$\boxed{\nabla \times \bar{A} = \bar{a}_x + y\,\bar{a}_y + (4y-z)\bar{a}_z}$$

Solved example - 37

prove that $\bar{A} = yz\,\bar{a}_x + zx\,\bar{a}_y + xy\,\bar{a}_z$ is both irrotational and solenoidal.

Soln: i. $\bar{A}$ is said to be irrotational when

$$\nabla \times \bar{A} = 0.$$

$$\nabla \times \bar{A} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & zx & xy \end{vmatrix}$$

$$= \left[\frac{\partial}{\partial y}(xy) - \frac{\partial}{\partial z}(zx)\right]\bar{a}_x - \left[\frac{\partial}{\partial x}(xy) - \frac{\partial}{\partial z}(yz)\right]\bar{a}_y + \left[\frac{\partial}{\partial x}(xz) - \frac{\partial}{\partial y}(zy)\right]\bar{a}_z$$

$$= (x-x)\,\bar{a}_x - (y-y)\,\bar{a}_y + (z-z)\,\bar{a}_y$$

$$\boxed{\nabla \times \bar{A} = 0}$$

hence vector $\bar{A}$ is irrotational.

ii. Vector $\bar{A}$ is said to be Solenoidal when $\nabla\cdot\bar{A}=0$.

$$\nabla\cdot\bar{A}=\frac{\partial A_x}{\partial x}+\frac{\partial A_y}{\partial y}+\frac{\partial A_z}{\partial z}$$

$$=\frac{\partial}{\partial x}(yz)+\frac{\partial}{\partial y}(3x)+\frac{\partial}{\partial z}(xy)$$

$$=0+0+0$$

$$\boxed{\nabla\cdot\bar{A}=0}$$

hence vector $\bar{A}$ is ~~irrotational~~ solenoidal field.

5. Co-ordinate Systems

An orthogonal System is one in which the Co-ordinates are mutually perpendicular to Each other.

Types.

i. Cartesian/ Rectangular Co-ordinate System.

ii. Cylindrical Co-ordinate System.

iii. Spherical (or) polar co-ordinate System.

5.a. Cartesian | Rectangular Co-ordinate System

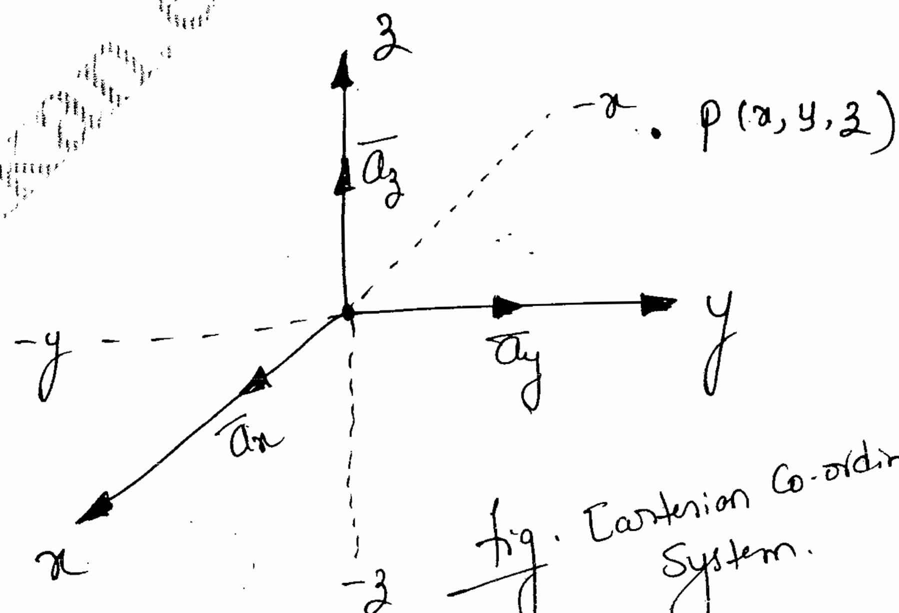
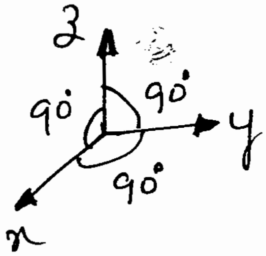


fig. Cartesian Co-ordinate System.

→ Variable used x, y, z and point $P(x, y, z)$

→ three axes x, y, z are perpendicular to each other.



→ Variables range $-\infty < x, y, z < +\infty$.

→ unit vectors $\bar{a}_x, \bar{a}_y, \bar{a}_z$

→ General vector $\bar{A} = A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_z$

where A_x, A_y, A_z are components along x, y and z direction.

$\bar{a}_x, \bar{a}_y, \bar{a}_z$ – unit vectors along x, y and z direction

→ differential Elements $P(x, y, z)$

$$P(x, y, z) \rightarrow dx \quad dy \quad dz.$$

$\Rightarrow dx, dy, dz$ --- differential element.

→ differential Length vector

$$\overline{dl} = dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z.$$

→ differential Surface (ds) and differential Surface vector

$\overline{ds} = ds\,\overline{a_n}$ .... for Closed Surface.

$\overline{ds} = ds\,(\pm\overline{a_n})$ .... for open Surface.

$ds = dx\,dy$; $z = k$ Surface.

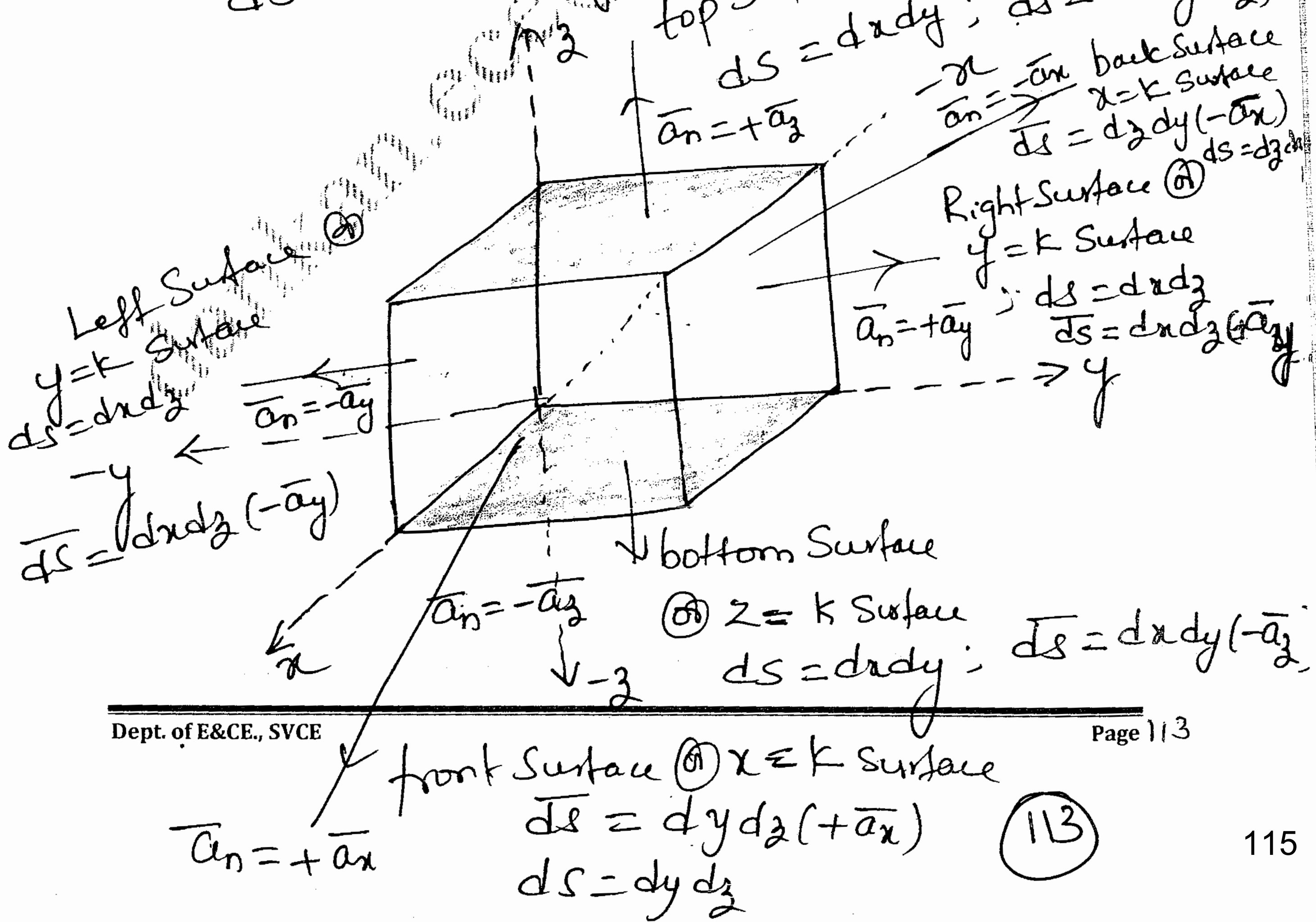
$\overline{ds} = dx\,dy\,(+\overline{a_z})$

$ds = dy\,dz$; $x = k$ Surface.

$\overline{ds} = dy\,dz\,\overline{a_x}$.

$ds = dx\,dz$; $y = k$ Surface.

$\overline{ds} = dx\,dz\,\overline{a_y}$



→ differential volume

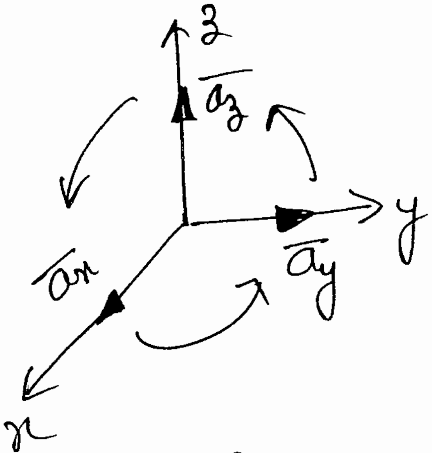
$$dv = dx\,dy\,dz.$$

→ dot product of unit vectors

$$\bar{a}_x \cdot \bar{a}_y = \bar{a}_y \cdot \bar{a}_z = \bar{a}_z \cdot \bar{a}_x = 0.$$

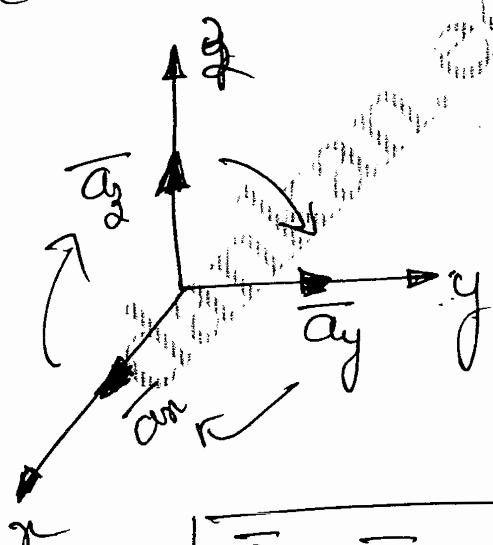
$$\bar{a}_x \cdot \bar{a}_x = \bar{a}_y \cdot \bar{a}_y = \bar{a}_z \cdot \bar{a}_z = 1.$$

→ Cross product of unit vectors.



$$\left.\boxed{\begin{aligned} \bar{a}_x \times \bar{a}_y &= \bar{a}_z \\ \bar{a}_y \times \bar{a}_z &= \bar{a}_x \\ \bar{a}_z \times \bar{a}_x &= \bar{a}_y \end{aligned}}\right\}$$

Rotating anticlockwise direction results +ve.



$$\left.\boxed{\begin{aligned} \bar{a}_x \times \bar{a}_z &= -\bar{a}_y \\ \bar{a}_z \times \bar{a}_y &= -\bar{a}_x \\ \bar{a}_y \times \bar{a}_x &= -\bar{a}_z \end{aligned}}\right\}$$

Rotating clockwise direction results -ve.

$$\boxed{\bar{a}_x \times \bar{a}_x = \bar{a}_y \times \bar{a}_y = \bar{a}_z \times \bar{a}_z = 0}$$ ... No rotational field exist.

→ The DEL (∇) operator in Cartesian Co-ordinate System.

$$\nabla = \frac{\partial}{\partial x}\overline{a_x} + \frac{\partial}{\partial y}\overline{a_y} + \frac{\partial}{\partial z}\overline{a_z}$$

Scalar function $\phi = f^n(x, y, z)$

vector $\overline{A} = A_x\overline{a_x} + A_y\overline{a_y} + A_z\overline{a_z}$

$\nabla\phi$ = Gradient

- $\nabla\phi = \frac{\partial\phi}{\partial x}\overline{a_x} + \frac{\partial\phi}{\partial y}\overline{a_y} + \frac{\partial\phi}{\partial z}\overline{a_z}$
- Gradient results in Vector.

i.e $\nabla\phi$ = vector

Dot product

$\nabla\cdot\overline{A}$ = Divergence

✓ $\nabla\cdot\overline{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

✓ $\nabla\cdot\overline{A}$ results in Scalar

i.e $\nabla\cdot\overline{A}$ = Scalar.

Cross product

$\nabla\times\overline{A}$ = Curl

✓ $\nabla\times\overline{A} = \begin{vmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$

✓ $\nabla\times\overline{A} = \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\overline{a_x} - \left[\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right]\overline{a_y} + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\overline{a_z}$

✓ Curl results in vector

i.e $\nabla\times\overline{A}$ = vector

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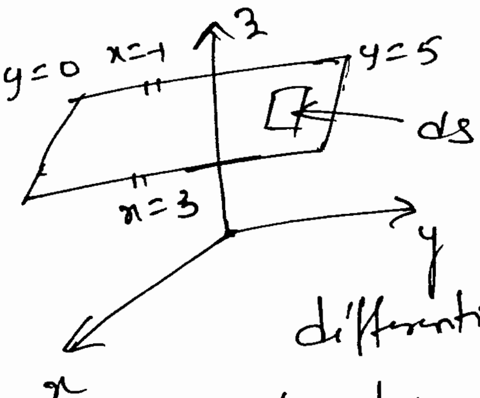
Solved problem -38

Find the area of rectangle in $z=5$ plane with $-1 \le x \le 3$ and $0 \le y \le 5$.

$\rightarrow 0 \le y \le 5$.

Solu!n



differential area ds

$$ds = dx\,dy$$

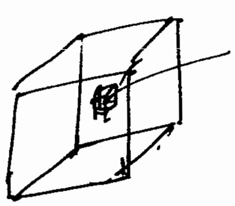
$$S = \int_{\langle S \rangle} ds = \int_{\langle S \rangle} dx\,dy = \int_{x=-1}^{3} dx \int_{y=0}^{5} dy$$

$$= [3-(-1)]\,[5-0]$$

$$= 4 \times 5 = \underline{\underline{20\ m^2}}$$

Area $\boxed{S = 20\ m^2}$

Find the volume of a closed surface bounded by $0 \le x \le 2$, $0 \le y \le 1$, $0 \le z \le 0.5$



dv. differential volume $dv = dx\,dy\,dz$

$$v = \int_{\langle v \rangle} dv = \int_0^2 dx \int_0^1 dy \int_{z=0}^{5} dz$$

$$= 2 \times 1 \times 5 = \underline{\underline{10\ m^3}}$$

volume. $\boxed{v = 10\ m^3} = \underline{\underline{10\ m^3}}$

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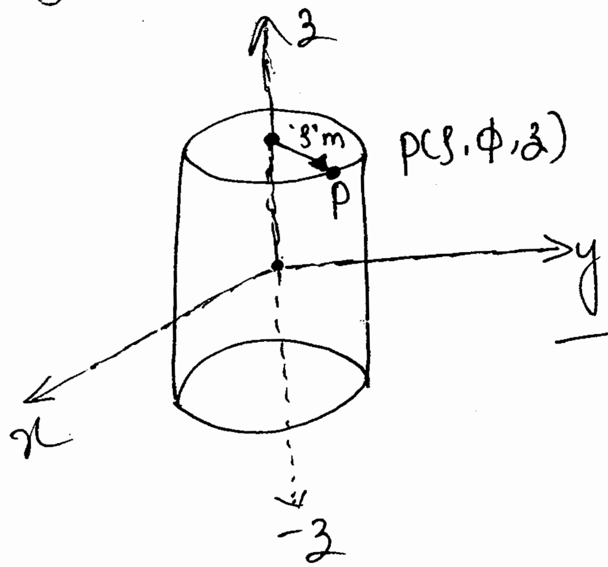
5.b. Cylindrical Co-ordinate System



→ variables used ρ, ϕ, z

→ variables range

$0 < \rho < \infty$

$0 < \phi < 2\pi$

$-\infty < z < +\infty$.

→ ρ - radius, ϕ - angle from x-axis.

z - height.

→ the axes ρ, ϕ, z are perpendicular to each other.

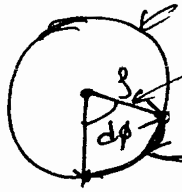
→ unit vectors $\overline{a_\rho}, \overline{a_\phi}, \overline{a_z}$.

→ General vector $\overline{A} = A_\rho \overline{a_\rho} + A_\phi \overline{a_\phi} + A_z \overline{a_z}$.

→ differential elements $P(\rho, \phi, z)$ → $d\rho$, $\rho d\phi$, dz

ρ = constant;

ϕ = constant;



arc Length $= \rho d\phi$.

Circumference $= 2\pi\rho = \phi\rho$.

→ differential Length vector

$$\overline{dl} = d\rho\,\overline{a_\rho} + \rho\, d\phi\,\overline{a_\phi} + dz\,\overline{a_z}.$$

→ differential Surface (ds) and differential Surface Vector ($\overline{ds}$).

$$\overline{ds} = ds\,\overline{a_n}.$$

$P(\rho, \phi, z)$

$d\rho \quad \rho\, d\phi \quad dz$

$$ds = d\rho\, dz \quad \ldots\ \phi = k \text{ Surface}.$$

$$\overline{ds} = d\rho\, dz\,\overline{a_\phi}\,;$$

$$ds = \rho\, d\rho\, d\phi\,; \quad \ldots\ z = k \text{ Surface}.$$

$$\overline{ds} = \rho\, d\rho\, d\phi\,\overline{a_z}.$$

$$ds = \rho\, d\phi\, dz \quad \ldots\ldots\ \rho = k \text{ Surface}.$$

$$\overline{ds} = \rho\, d\phi\, dz\,\overline{a}_\rho = \rho\, d\phi\, dz\,\overline{a_\rho}$$

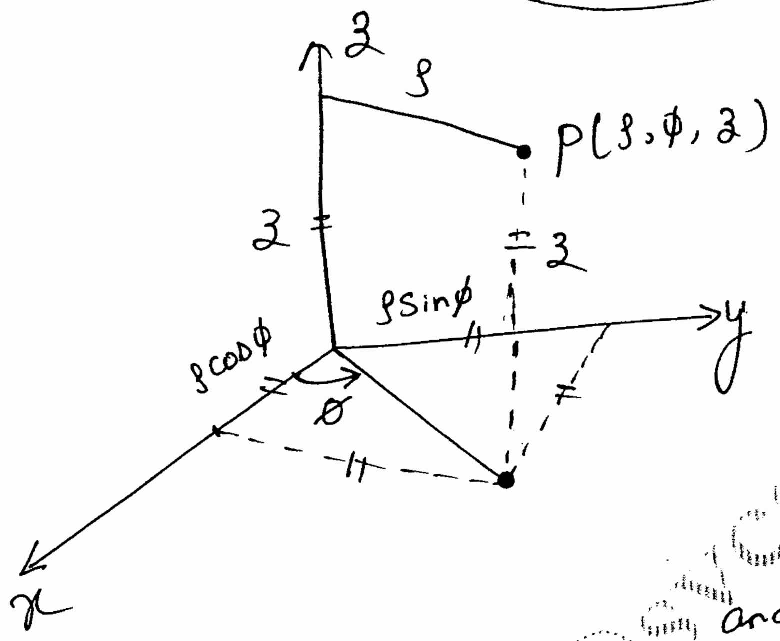
→ differential volume

$$dv = \rho\, d\rho\, d\phi\, dz.$$

→ point transformation

$$p(x,y,z) \Longleftrightarrow p(\rho,\phi,z)$$

$$\boxed{z=z}$$



In fig

$$\cos\phi = \frac{x}{\rho}$$

$$\boxed{x=\rho\cos\phi} \leftarrow (1)$$

and $\sin\phi = y/\rho$

$$\boxed{y=\rho\sin\phi} \leftarrow (2)$$

(or)

using Dot product Concept

$$\boxed{x=\rho\cos\phi}$$
$$\boxed{y=\rho\sin\phi}$$

and $\boxed{z=z}$

given $p(\rho,\phi,z) \Longrightarrow p(x,y,z) = ?$

$$\boxed{x=\rho\cos\phi,\quad y=\rho\sin\phi,\quad z=z}$$

Ily if given $p(x,y,z) \Rightarrow p(\rho,\phi,z) = ?$

Square and add eq① + eq② . $x^2+y^2=\rho^2 \Rightarrow \boxed{\rho=\sqrt{x^2+y^2}}$

$\frac{eq^n①}{eq^n②}$ $\frac{\cos\phi}{\sin\phi} = y/x$

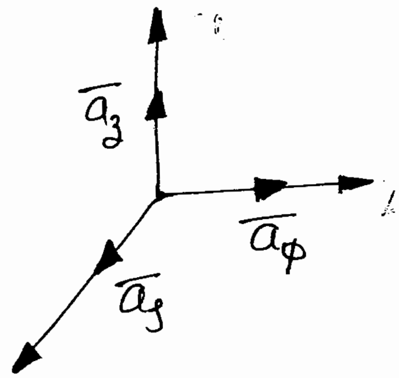
$y/x = \tan\phi \Rightarrow \boxed{\phi = \tan^{-1}(y/x)}$

and $\boxed{z=z}$

given
$p(x,y,z) \Rightarrow p(\rho,\phi,z) = ?$

$$\boxed{\rho=\sqrt{x^2+y^2} \; ; \; \phi=\tan^{-1}(y/x), \quad z=z}$$

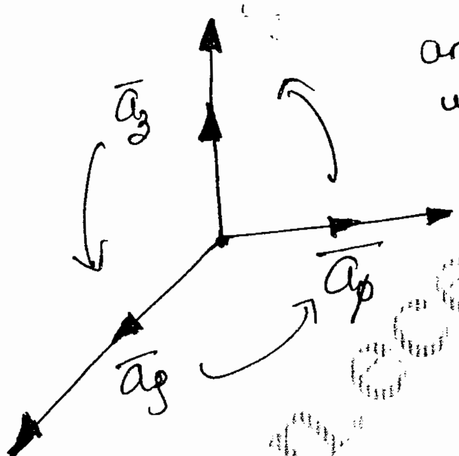
→ dot product of unit vectors.



$$\boxed{\bar{a}_\rho \cdot \bar{a}_\rho = \bar{a}_\phi \cdot \bar{a}_\phi = \bar{a}_z \cdot \bar{a}_z = 1}$$

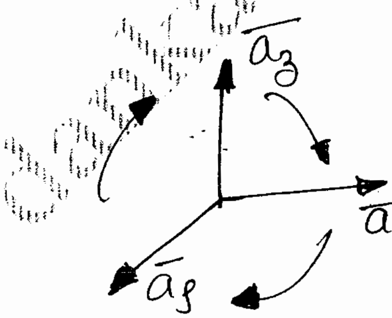
$$\boxed{\bar{a}_\rho \cdot \bar{a}_\phi = \bar{a}_\phi \cdot \bar{a}_z = \bar{a}_z \cdot \bar{a}_\rho = 0}$$

→ Cross product of unit vectors.



rotating anticlock wise direction

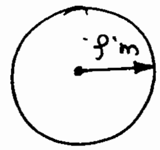
$$\bar{a}_\rho \times \bar{a}_\phi = +\bar{a}_z$$
$$\bar{a}_\phi \times \bar{a}_z = +\bar{a}_\rho$$
$$\bar{a}_z \times \bar{a}_\rho = +\bar{a}_\phi .$$



$$\bar{a}_\phi \times \bar{a}_\rho = -\bar{a}_z$$
$$\bar{a}_z \times \bar{a}_\phi = -\bar{a}_\rho$$
$$\bar{a}_\rho \times \bar{a}_z = -\bar{a}_\phi$$

rotating clockwise direction.

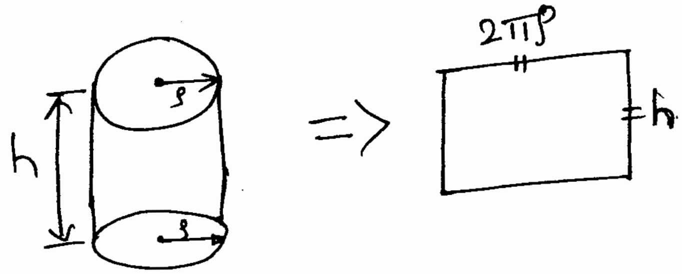
$$\boxed{\bar{a}_\rho \times \bar{a}_\rho = 0 \quad \bar{a}_\phi \times \bar{a}_\phi = 0 \quad \bar{a}_z \times \bar{a}_z = 0}$$

→ 

Area of circle $\pi\rho^2$; m^2

ρ - radius (m).

→ Surface Area of cylinder $= 2\pi\rho h$; m^2



⇒ (Circumference) height

$= 2\pi\rho \times h$ m^2

$= 2\pi\rho h$ m^2

→ Volume of cylinder $= \pi\rho^2 h$; m^3

→ Total area of cylinder $= 2\pi\rho^2 + 2\pi\rho h$.

$P(\rho, \phi, z)$

$d\rho \quad \rho d\phi \quad dz$.

* $ds = \rho\, d\phi\, dz$ ... $\rho = k$ Surface.

$$S = \rho \cdot \int_0^{2\pi} d\phi \int_0^h dz = \rho \times 2\pi \times h$$

$\boxed{S = 2\pi\rho h}$; m^2

* $ds = d\rho\, dz$; .. $\phi = k$ Surface.

$$S = \int_0^{\rho} d\rho \int_0^{h} dz = \rho h ; m^2 \qquad \boxed{S = \rho h} ; m^2$$

* $ds = \rho\, d\rho\, d\phi$ ..... $z = k$ Surface.

$$S = \int_0^{\rho} \rho \cdot d\rho \int_0^{2\pi} d\phi = \frac{\rho^2}{2}\Big|_0^{\rho} (2\pi)$$

$$= 2\pi \cdot \frac{\rho^2}{2} = \pi \rho^2$$

$$\boxed{S = \pi \rho^2} ; m^2$$

* differential volume

$$dv = \rho\, d\rho\, d\phi\, dz$$

total volume

$$v = \int_0^{\rho} \rho\, d\rho \int_0^{2\pi} d\phi \int_0^{h} dz$$

$$= \frac{\rho^2}{2}\Big|_0^{\rho} \times 2\pi \times h = \frac{\rho^2}{\cancel{2}} \cdot \cancel{2}\pi \times h$$

$$\boxed{v = \pi \rho^2 h} \; m^3$$

→ Vector transformation

Cartesian $\Longleftrightarrow$ Cylindrical

$$\bar{A} = A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_z \Longleftrightarrow \bar{A} = A_\rho \bar{a}_\rho + A_\phi \bar{a}_\phi + A_z \bar{a}_z \quad \text{— ⓐ}$$

if $\bar{A} = A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_z$ is given in Cartesian Co-ordinate System, the equivalent vector in Cylindrical Co-ordinate System $\bar{A} = A_\rho \bar{a}_\rho + A_\phi \bar{a}_\phi + A_z \bar{a}_z$.

i.e find A_ρ, A_ϕ and A_z components.

from eqⁿ ⓐ

$$A_z = A_z$$

i.e 'z' components of both the vector have equal.

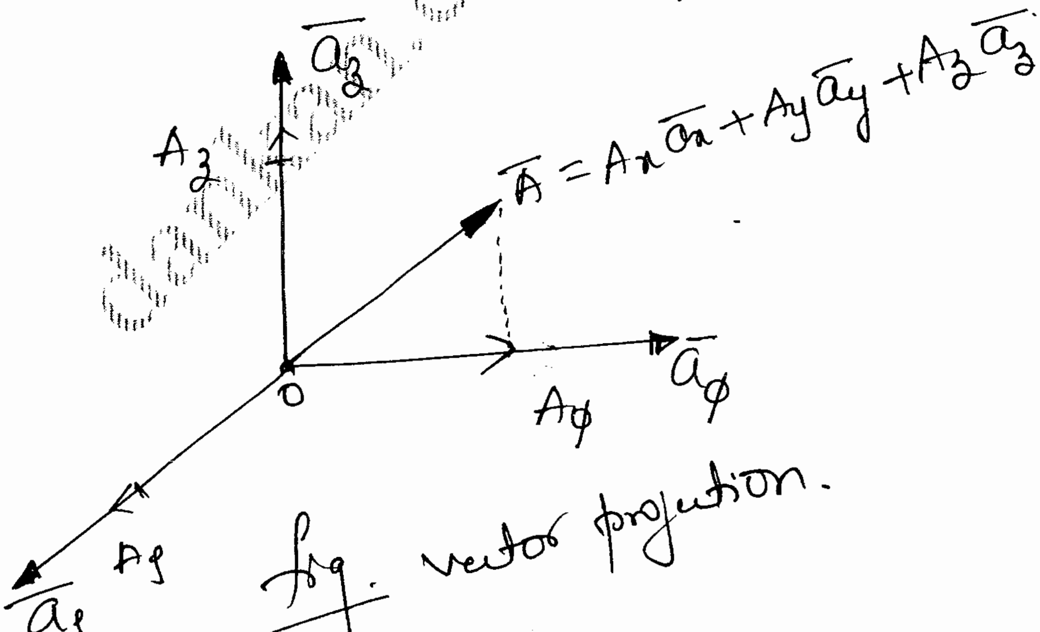


fig. vector projection.

A_ρ = projection of given vector $\bar{A}$ in $\bar{a}_\rho$ direction.

Unknown Component = [Known vector] • [unit vector of unknown Component]

i.e $A_\rho = \bar{A} \cdot \bar{a}_\rho$; $A_\phi = \bar{A} \cdot \bar{a}_\phi$; $A_z = \bar{A} \cdot \bar{a}_z$.

$$A_\rho = (A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_z) \cdot \bar{a}_\rho$$

$$A_\rho = A_x \underbrace{\bar{a}_x \cdot \bar{a}_\rho}_{\cos\phi} + A_y \underbrace{\bar{a}_y \cdot \bar{a}_\rho}_{\sin\phi} + A_z \underbrace{\bar{a}_z \cdot \bar{a}_\rho}_{0}$$

dot product table | dot product of unit vectors

$\cdot$	$\bar{a}_\rho$	$\bar{a}_\phi$	$\bar{a}_z$
$\bar{a}_x$	$\cos\phi$	$-\sin\phi$	0
$\bar{a}_y$	$\sin\phi$	$\cos\phi$	0
$\bar{a}_z$	0	0	1

$\bar{a}_x \cdot \bar{a}_\rho = \cos\phi$

$\bar{a}_y \cdot \bar{a}_\rho = \sin\phi$

$\bar{a}_z \cdot \bar{a}_\rho = 0$

$$\boxed{A_\rho = A_x \cos\phi + A_y \sin\phi}$$

lly $A_\phi = \bar{A} \cdot \bar{a_\phi}$

$$\boxed{A_\phi = -A_x \sin\phi + A_y \cos\phi}$$

and $A_z = \bar{A} \cdot \bar{a_z}$

$$\Rightarrow \boxed{A_z = A_z}$$

$$\boxed{\bar{A} = [A_x \cos\phi + A_y \sin\phi]\,\bar{a_\rho} + [-A_x \sin\phi + A_y \cos\phi]\,\bar{a_\phi} + A_z \bar{a_z}}$$

Note:- (shortcut)

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

→ If Given $\overline{A} = A_\rho \overline{a_\rho} + A_\phi \overline{a_\phi} + A_z \overline{a_z}$. Find its equivalent vector in Cartesian Coordinate system.

i.e $\overline{A} = A_x \overline{a_x} + A_y \overline{a_y} + A_z \overline{a_z} = ?$

(Shortcut)

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$$\overline{A} = A_x \overline{a_x} + A_y \overline{a_y} + A_z \overline{a_z}$$

$$\boxed{\overline{A} = [A_\rho \cos\phi - A_\phi \sin\phi]\overline{a_x} + [A_\rho \sin\phi + A_\phi \cos\phi]\overline{a_y} + A_z \overline{a_z}}$$

→ The ∇ Operator in Cylindrical Co-ordinate System.

$$P(\rho, \phi, z)$$

$d\rho \qquad \rho\, d\phi \qquad dz$

$$\boxed{\nabla = \frac{\partial}{\partial \rho}\bar{a}_\rho + \frac{1}{\rho}\frac{\partial}{\partial \phi}\bar{a}_\phi + \frac{\partial}{\partial z}\bar{a}_z} \ m^{-1}$$

→ Gradient. let $V = f^n(\rho, \phi, z)$ in a scalar function

$$\boxed{\nabla V = \text{grad}(V) = \frac{\partial V}{\partial \rho}\bar{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\bar{a}_\phi + \frac{\partial V}{\partial z}\bar{a}_z}$$

= vector.

→ Divergence $(\nabla \cdot \bar{A})$

$P(\rho, \phi, z) \Rightarrow dv = \rho\, d\rho\, d\phi\, dz$

and $\bar{A} = A_\rho \bar{a}_\rho + A_\phi \bar{a}_\phi + A_z \bar{a}_z$

$$\boxed{\nabla \cdot \bar{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}[\rho A_\rho] + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}}$$

→ Curl $(\nabla \times \bar{A}) = ?$
in Cylindrical Co-ordinate System.

$P(\rho, \phi, z)$

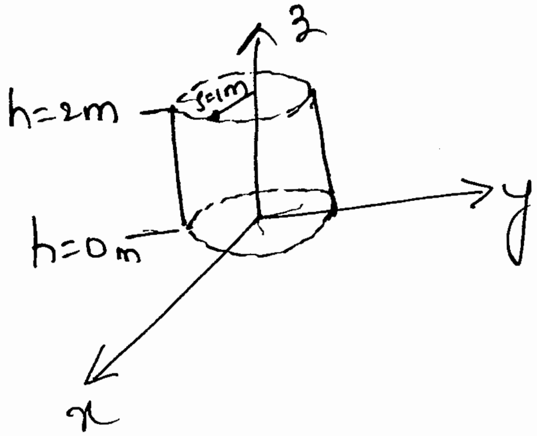
$d\rho$, $\rho\, d\phi$, $dz \Rightarrow dv = \rho\, d\rho\, d\phi\, dz$

$$\nabla \times \bar{A} = \text{Curl}(A) = \frac{1}{\rho} \begin{vmatrix} \bar{a}_\rho & \rho\,\bar{a}_\phi & \bar{a}_z \\ \partial/\partial\rho & \partial/\partial\phi & \partial/\partial z \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$\boxed{\nabla \times \bar{A} = \left[\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right]\bar{a}_\rho + \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right]\bar{a}_\phi + \frac{1}{\rho}\left[\frac{\partial (\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi}\right]\bar{a}_z}$$

Example problem - 39

Find area of the Cylindrical Surface with height $h = 2m$ and radius $\rho = 1m$.



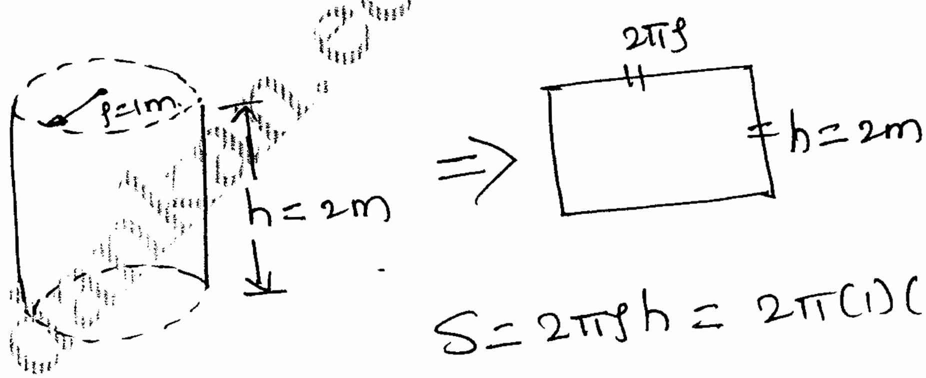
$$ds = \rho \, d\phi \, dz \quad \text{.... } \rho = 1m \text{ (constant) Surface}$$

$$S = \int_{\langle S \rangle} ds = \int_{\langle S \rangle} \rho \, d\phi \, dz \Big|_{\rho = 1m}$$

$$S = 1 \times \int_0^{2\pi} d\phi \times \int_0^2 dz = 2\pi(2)$$

$$\boxed{S = 4\pi} \, m^2$$

(or)



$$S = 2\pi\rho h = 2\pi(1)(2)$$

$$\boxed{S = 4\pi} \, m^2$$

Example problem.-40

Find the volume of the cylinder with height 3m and radius 2m.

Solu:- $dv = \rho \, d\rho \, d\phi \, dz$.

$$v = \int_{\langle vol \rangle} dv = \int_0^2 \rho \, d\rho \int_0^{2\pi} d\phi \int_0^3 dz$$

$$v = \frac{\rho^2}{2}\Big|_0^2 \times 2\pi \times 3$$

$$v = \frac{\cancel{4}^{2}}{\cancel{2}} \times 2\pi \times 3 = 12\pi \ m^3$$

$$\boxed{v = 12\pi} \ m^3$$

Example problem.-41

Convert the following points specified in Cartesian into cylindrical Co-ordinates

i. $P(0, -2, 2)$ ii. $Q(\sqrt{3}, 1, -1)$ iii. $R(-\sqrt{2}, \sqrt{2}, 3)$.

Solu:- $P(x, y, z) \Longleftrightarrow P(\rho, \phi, z)$

$$\boxed{\rho = \sqrt{x^2 + y^2} \ m \ ; \ \phi = \tan^{-1}(y/x) \ ; \ z = z}$$

i. $P(0, -2, 2)$

$x = 0, \ y = -2, \ \boxed{z = 2}$.

$\rho = \sqrt{x^2 + y^2} = \sqrt{0 + (-2)^2} = \sqrt{4} = 2. \Rightarrow \boxed{\rho = 2\,m}$

$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-2}{0}\right) = -\pi/2$

or

$\phi = -\pi/2 + 2\pi = 3\pi/2$.

$\boxed{\phi = \frac{3\pi}{2}}$

$P(0, -2, 2) \Leftrightarrow P(2, 3\pi/2, 2)$

Note:- 'ϕ' should be in ~~+ve~~.

ii. $Q(\sqrt{3}, 1, -1) \Rightarrow Q(\rho, \phi, z) = (2, \pi/6, -1)$.

iii. $R(-\sqrt{2}, \sqrt{2}, 3) \Rightarrow R(\rho, \phi, z) = (2, 3\pi/4, 3)$.

Example problem - 42

Convert the following points specified in cylindrical into Cartesian Co-ordinates

i. $P(2, 5\pi/3, -2)$ ii. $Q(4, \pi/6, 1)$ iii. $R(2, -\pi/4, 3)$.

soln:- use $\boxed{x = \rho\cos\phi}$, $\boxed{y = \rho\sin\phi}$ & $\boxed{z = z}$

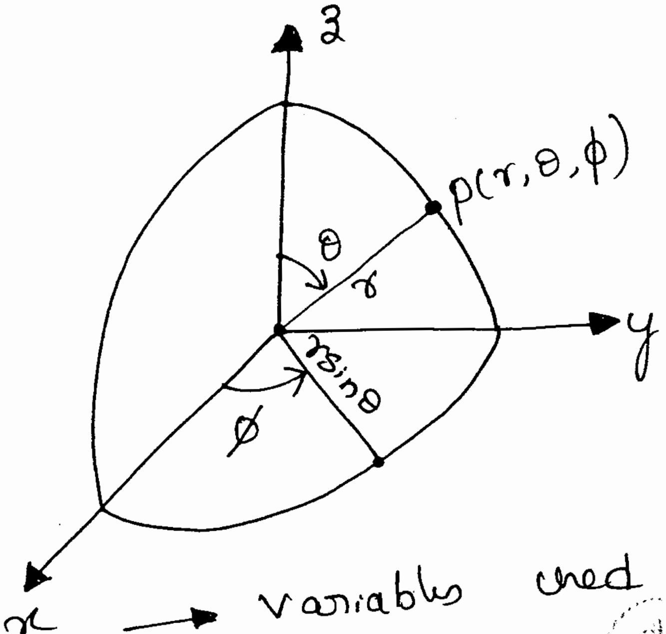
i. $P(2, 5\pi/3, -2) \Rightarrow P(1, -1.73, -2)$

ii. $Q(4, \pi/6, 1) \Rightarrow Q(3.46, 2, 1)$.

iii. $R(2, -\pi/4, 3) \Rightarrow R(-\sqrt{2}, \sqrt{2}, 3)$.

5C. Spherical / polar Co-ordinate System

The point on Spherical Co-ordinate System is represented as $p(r, \theta, \phi)$.



r - distance from origin.

θ - angle from z axis.

ϕ - angle from x axis.

→ variables used r, θ, ϕ.

→ variables range

$0 < r < \infty$

$0 < \theta < \pi$

$0 < \phi < 2\pi$.

→ unit vectors $\bar{a}_r, \bar{a}_\theta, \bar{a}_\phi$.

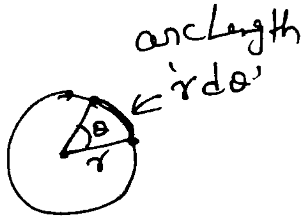
→ unit vectors $\bar{a}_r, \bar{a}_\theta, \bar{a}_\phi$ are perpendicular to each other.

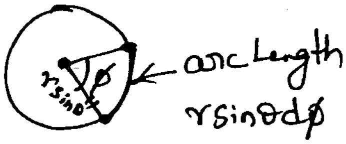
→ General Vector $\bar{A} = A_r \bar{a}_r + A_\theta \bar{a}_\theta + A_\phi \bar{a}_\phi$.

A_r, A_θ, A_ϕ are components along $\bar{a}_r, \bar{a}_\theta$ and $\bar{a}_\phi$ direction respectively.

→ differential Element of (r, θ, ϕ)

$r \rightarrow dr$, $\theta \rightarrow r\,d\theta$, $\phi \rightarrow r\sin\theta\,d\phi$.





→ point transformation.

$$p(x, y, z) \Longleftrightarrow p(r, \theta, \phi)$$

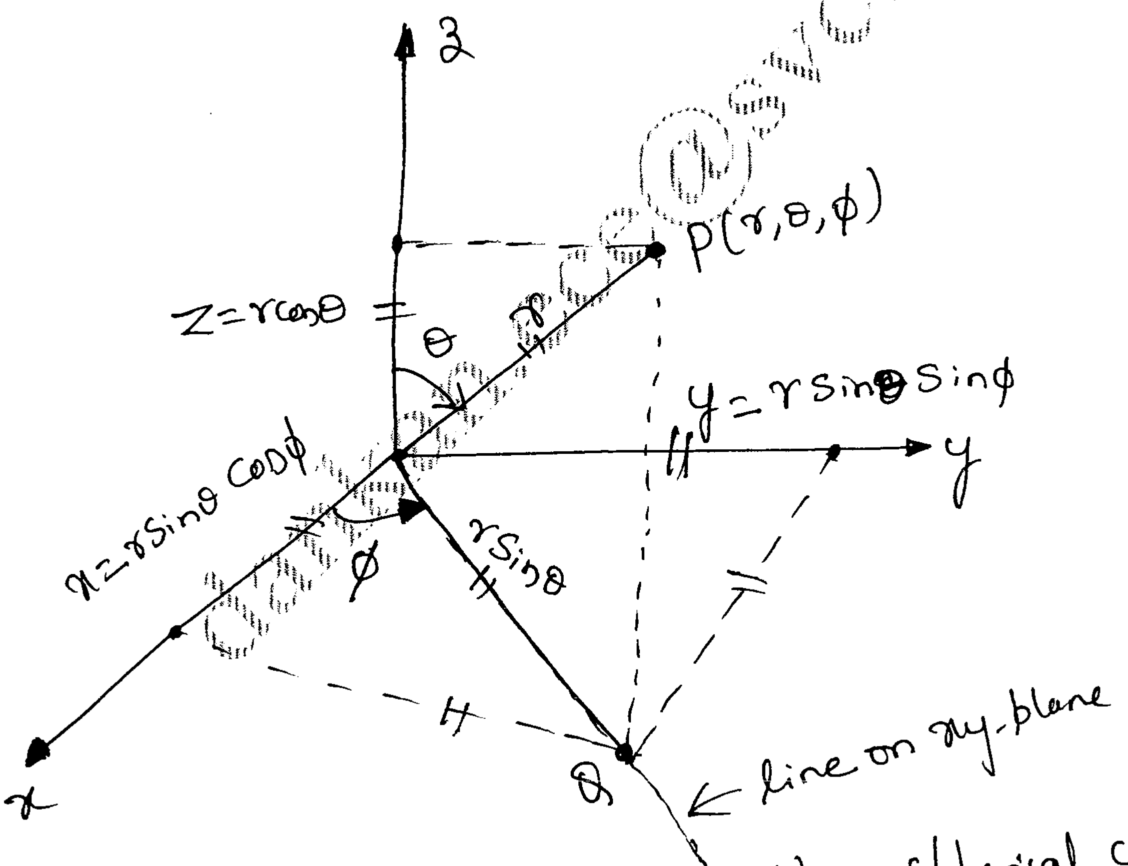


Fig. point $p(r, \theta, \phi)$ on spherical co-ordinate System.

from fig.

$$\boxed{\begin{aligned} x &= r\sin\theta\cos\phi \\ y &= r\sin\theta\sin\phi \\ z &= r\cos\theta \end{aligned}} \Longleftrightarrow \boxed{\begin{aligned} r &= \sqrt{x^2+y^2+z^2} \\ \phi &= \tan^{-1}(y/x) \\ \theta &= \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right) \end{aligned}}$$

→ differential length vector

$P(r, \theta, \phi)$

$dr \quad r\,d\theta \quad r\sin\theta\,d\phi$

$$\boxed{\overline{dl} = dr\,\overline{a_r} + r\,d\theta\,\overline{a_\theta} + r\sin\theta\,d\phi\,\overline{a_\phi}}$$

→ differential Surface (ds) and differential Surface vector $(\overline{ds})$.

$P(r, \theta, \phi)$

$dr \quad r\,d\theta \quad r\sin\theta\,d\phi$

$dS = r^2\sin\theta\,d\theta\,d\phi$; $r = k$ Surface.

$\overline{ds} = r^2\sin\theta\,d\theta\,d\phi\,\overline{a_r}$.

$ds = r\sin\theta\,dr\,d\phi$; $\theta = k$ Surface

$\overline{ds} = r\sin\theta\,dr\,d\phi\,\overline{a_\theta}$

$ds = r\,dr\,d\theta$; $\phi = k$ Surface.

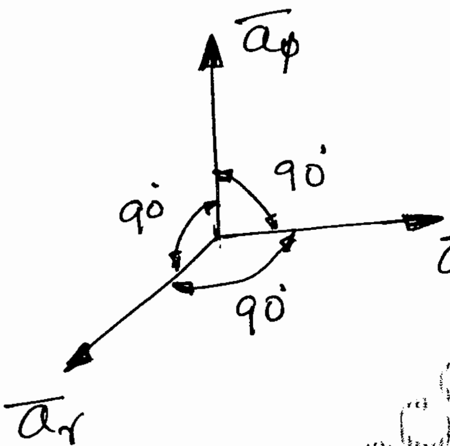
$\overline{ds} = r\,dr\,d\theta\,\overline{a_\phi}$.

→ differential volume

$P(r, \theta, \phi)$: dr, $r\,d\theta$, $r\sin\theta\,d\phi$

$$\boxed{dv = r^2 \sin\theta\, dr\, d\theta\, d\phi}$$

→ dot product of unit vectors.



$$\boxed{\overline{a_r}\cdot\overline{a_r} = \overline{a_\theta}\cdot\overline{a_\theta} = \overline{a_\phi}\cdot\overline{a_\phi} = 1}$$

$$\boxed{\overline{a_r}\cdot\overline{a_\theta} = \overline{a_\theta}\cdot\overline{a_\phi} = \overline{a_\phi}\cdot\overline{a_r} = 0}$$

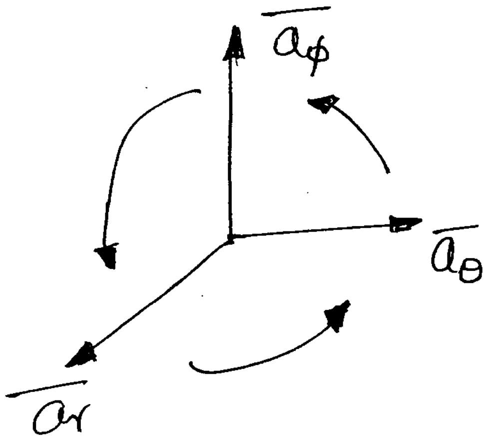
→ Surface ~~Area~~ Area of the sphere

$$\boxed{A = 4\pi r^2} \quad m^2$$

→ Volume of the Sphere

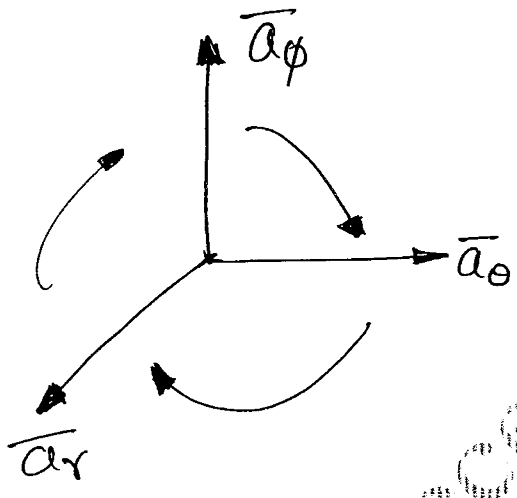
$$\boxed{V = \frac{4}{3}\pi r^3} \quad m^3.$$

→ Cross product of unit vectors :



$$\boxed{\begin{aligned} \overline{a_r} \times \overline{a_\theta} &= \overline{a_\phi} \\ \overline{a_\theta} \times \overline{a_\phi} &= \overline{a_r} \\ \overline{a_\phi} \times \overline{a_r} &= \overline{a_\theta} \end{aligned}}$$

rotating anti-clockwise direction results +ve.



$$\boxed{\begin{aligned} \overline{a_r} \times \overline{a_\phi} &= -\overline{a_\theta} \\ \overline{a_\phi} \times \overline{a_\theta} &= -\overline{a_r} \\ \overline{a_\theta} \times \overline{a_r} &= -\overline{a_\phi} \end{aligned}}$$

rotating clockwise direction results -ve.

$$\boxed{\begin{aligned} \overline{a_r} \times \overline{a_r} &= 0 \\ \overline{a_\theta} \times \overline{a_\theta} &= 0 \\ \overline{a_\phi} \times \overline{a_\phi} &= 0 \end{aligned}}$$

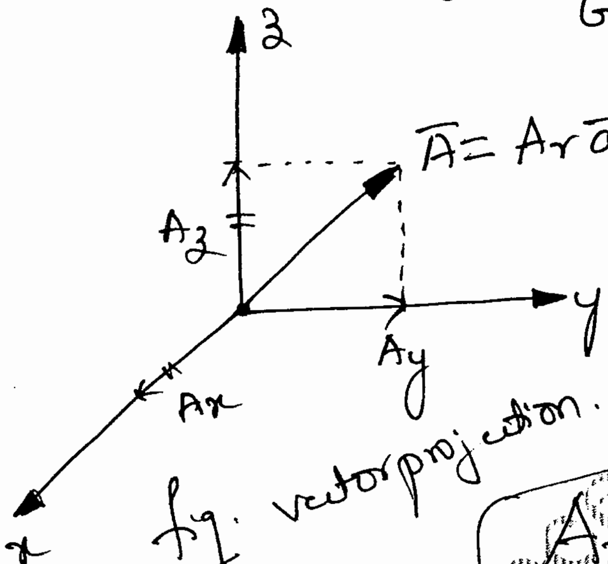
no rotational field Exist.

→ Vector transformation

$$p(x,y,z) \Longleftrightarrow p(r,\theta,\phi)$$

$$\underbrace{\bar{A} = A_x\bar{a}_x + A_y\bar{a}_y + A_z\bar{a}_z}_{\text{rectangular coordinate system}} \Longleftrightarrow \bar{A} = \underbrace{A_r\bar{a}_r + A_\theta\bar{a}_\theta + A_\phi\bar{a}_\phi}_{\text{Spherical Co-ordinate System}}.$$

Given vector $\bar{A} = A_r\bar{a}_r + A_\theta\bar{a}_\theta + A_\phi\bar{a}_\phi$ in Spherical coordinate system.



$\bar{A} = A_r\bar{a}_r + A_\theta\bar{a}_\theta + A_\phi\bar{a}_\phi$

fig. vectorprojection.

$$\boxed{\begin{aligned} A_x &= \bar{A}\cdot\bar{a}_x \\ A_y &= \bar{A}\cdot\bar{a}_y \\ A_z &= \bar{A}\cdot\bar{a}_z \end{aligned}}$$

dot product of unit vectors

$\cdot$	$\bar{a}_r$	$\bar{a}_\theta$	$\bar{a}_\phi$
$\bar{a}_x$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\bar{a}_y$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\bar{a}_z$	$\cos\theta$	$-\sin\theta$	0

$$A_x = [A_r \bar{a}_r + A_\theta \bar{a}_\theta + A_\phi \bar{a}_\phi] \cdot \bar{a}_x$$

$$A_x = [A_r \sin\theta\cos\phi + A_\theta \cos\theta\cos\phi - A_\phi \sin\phi]$$

Ily $A_y = \bar{A}\cdot\bar{a}_y$

$$A_y = \sin\theta\sin\phi\, A_r + \cos\theta\sin\phi\, A_\theta + \cos\phi\, A_\phi$$

and $A_z = \bar{A}\cdot\bar{a}_z$

$$A_z = \cos\theta\, A_r - \sin\theta\, A_\theta$$

$$\therefore \bar{A} = A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_z$$

$$\boxed{\begin{aligned}\bar{A} = {} & [\sin\theta\cos\phi\, A_r + A_\theta \cos\theta\cos\phi - A_\phi \sin\phi]\,\bar{a}_x \\ & + [\sin\theta\sin\phi\, A_r + \cos\theta\sin\phi\, A_\theta + \cos\phi\, A_\phi]\,\bar{a}_y \\ & + [\cos\theta\, A_r - \sin\theta\, A_\theta]\,\bar{a}_z\end{aligned}}$$

Note :- (Short cut)

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

My if given vector $\overline{A} = A_x\overline{a_x} + A_y\overline{a_y} + A_z\overline{a_z}$ in Cartesian Co-ordinate System, To convert it into Spherical Coordinate System use.

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\overline{A} = A_r\overline{a_r} + A_\theta\overline{a_\theta} + A_\phi\overline{a_\phi}$$

$$\boxed{\begin{aligned} \overline{A} = &\left[\sin\theta\cos\phi A_x + \sin\theta\sin\phi A_y + \cos\theta A_z\right]\overline{a_r} \\ &+ \left[\cos\theta\cos\phi A_x + \cos\theta\sin\phi A_y - \sin\theta A_z\right]\overline{a_\theta} \\ &+ \left[-\sin\phi A_x + \cos\phi A_y\right]\overline{a_\phi} \end{aligned}}$$

→ The Del (∇) Operator in Spherical Co-ordinate System.

$$P(r, \theta, \phi)$$

$$dr \qquad r\,d\theta \qquad r\sin\theta\, d\phi$$

$$\boxed{\nabla = \frac{\partial}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\bar{a}_\phi}$$

→ Gradient: ∇V or Grad(V)

let $V = \text{scalar} = f^n(r, \theta, \phi)$

$$P(r, \theta, \phi)$$

$$dr \qquad r\,d\theta \qquad r\sin\theta\, d\phi$$

$$\boxed{\nabla V = \frac{\partial V}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\bar{a}_\phi}$$

→ Divergence $(\nabla \cdot \bar{A})$:-

let $\bar{A} = A_r \bar{a}_r + A_\theta \bar{a}_\theta + A_\phi \bar{a}_\phi$

and $P(r, \theta, \phi)$

$r \rightarrow dr$, $\theta \rightarrow r d\theta$, $\phi \rightarrow r \sin\theta d\phi$

$$dv = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$\boxed{\nabla \cdot \bar{A} = \frac{1}{r^2} \frac{\partial}{\partial r}[r^2 A_r] + \frac{1}{r\sin\theta} \frac{\partial}{\partial \theta}[\sin\theta A_\theta] + \frac{1}{r\sin\theta} \frac{\partial A_\phi}{\partial \phi}}$$

→ Curl $\nabla \times \bar{A}$ (or) Curl($\bar{A}$) in Spherical Co-ordinate System

$P(r, \theta, \phi)$

$dr \quad r\,d\theta \quad r\sin\theta\,d\phi$

$dv = r^2 \sin\theta\, dr\, d\theta\, d\phi$

$$\nabla \times \bar{A} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \bar{a}_r & r\,\bar{a}_\theta & r\sin\theta\,\bar{a}_\phi \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ A_r & rA_\theta & r\sin\theta\, A_\phi \end{vmatrix}$$

$$\boxed{\nabla \times \bar{A} = \frac{1}{r\sin\theta}\left[\frac{\partial (A_\phi \sin\theta)}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right]\bar{a}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial (rA_\phi)}{\partial r}\right]\bar{a}_\theta + \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]\bar{a}_\phi}$$

Laplace's and Poisson's Equation in all the Coordinate Systems:-

→ Cartesian Coordinate System:- $P(x, y, z)$

Laplace's equation $\nabla^2 V = 0$.

$$\boxed{\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0}$$

Poisson's equation $\nabla^2 V = -\rho_v/\epsilon$ V/m²

$$\boxed{\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\rho_v/\epsilon} \text{ V/m}^2$$

→ Cylindrical Co-ordinate System:- $P(\rho, \phi, z)$

$dv = \rho \, d\rho \, d\phi \, dz$.

Laplace's equation $\nabla^2 V = 0$ V/m²

$$\boxed{\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho \frac{\partial V}{\partial \rho}\right] + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0} \text{ V/m}^2$$

Poisson's equation $\nabla^2 V = -\rho_v/\epsilon$ V/m²

$$\boxed{\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho \frac{\partial V}{\partial \rho}\right] + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = -\rho_v/\epsilon} \text{ V/m}^2$$

→ Spherical Co-ordinate System:- $P(r, \theta, \phi)$

$dv = r^2 \sin\theta \, dr \, d\theta \, d\phi$

Laplace's eqⁿ $\nabla^2 V = 0$ V/m²

$$\boxed{\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{\partial V}{\partial r}\right] + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left[\sin\theta \frac{\partial V}{\partial \theta}\right] + \frac{1}{r^2 \sin\theta}\frac{\partial^2 V}{\partial \phi^2} = 0}$$

Poisson's eqⁿ $\nabla^2 V = -\rho_v/\epsilon$; V/m²

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$$\boxed{\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{\partial V}{\partial r}\right] + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left[\sin\theta \frac{\partial V}{\partial \theta}\right] + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = -\rho_v/\epsilon} \text{ ; V/m}^2$$

6. i. ∇ - operator a. Cartesian Co-ordinate system.
$P(x, y, z)$.

vector operator

$$\boxed{\nabla = \frac{\partial}{\partial x}\bar{a}_x + \frac{\partial}{\partial y}\bar{a}_y + \frac{\partial}{\partial z}\bar{a}_z} \quad m^{-1}.$$

ii. Gradient $V = f^n(x, y, z) \Rightarrow$ Scalar f^4.
potential

; Resultn Vector quantity.

$$\boxed{\nabla V = \frac{\partial V}{\partial x}\bar{a}_x + \frac{\partial V}{\partial y}\bar{a}_y + \frac{\partial V}{\partial z}\bar{a}_z} \; ; V/m.$$

iii. Divergence $(\nabla \cdot \bar{D})$ $\bar{D}$ - electric flux density C/m^2.

let $\bar{D} = D_x\bar{a}_x + D_y\bar{a}_y + D_z\bar{a}_z$. C/m^2

$$\boxed{\nabla \cdot \bar{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}} \Rightarrow \text{Resultn Scalar.} \quad C/m^3$$

iv. Curl $(\nabla \times \bar{D})$

$$\nabla \times \bar{D} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ D_x & D_y & D_z \end{vmatrix} \Rightarrow \text{Resultn vector.} \quad C/m^3$$

V. Lapacis equation.

$$\boxed{\nabla^2 V = 0} \quad \text{i.e} \quad \boxed{\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0} \Rightarrow \text{Scalar} \quad V/m^2$$

VI. poisson's Equation.

$$\boxed{\nabla^2 V = -\rho_v/\epsilon} \; : V/m^2 \quad \boxed{\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\rho_v/\epsilon} \; : V/m^2$$

b. Cylindrical Co-ordinate System

$P(\rho, \phi, z)$ → $d\rho$, $\rho\, d\phi$, dz

$dv = \rho\, d\rho\, d\phi\, dz$

i. ∇ operator

$$\boxed{\nabla = \frac{\partial}{\partial \rho}\bar{a}_\rho + \frac{1}{\rho}\frac{\partial}{\partial \phi}\bar{a}_\phi + \frac{\partial}{\partial z}\bar{a}_z} \; ; m^{-1}$$ vector operator.

ii. Gradient $\nabla V \rightarrow V/m$

$$\boxed{\nabla V = \frac{\partial V}{\partial \rho}\bar{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\bar{a}_\phi + \frac{\partial V}{\partial z}\bar{a}_z} \; V/m$$ Result is Vector

iii. Divergence of $\bar{D}$ or $\nabla \cdot \bar{D}$ C/m^3

let electric flux density $\bar{D} = D_\rho \bar{a}_\rho + D_\phi \bar{a}_\phi + D_z \bar{a}_z$ C/m^2

$$\boxed{\nabla \cdot \bar{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho \cdot D_\rho\right] + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}} \; C/m^3$$ Result is Scalar

iv. Curl $\nabla \times \bar{H}$

let $\bar{H} = H_\rho \bar{a}_\rho + H_\phi \bar{a}_\phi + H_z \bar{a}_z$ A/m

$$\nabla \times \bar{H} = \frac{1}{\rho}\begin{vmatrix} \bar{a}_\rho & \rho\bar{a}_\phi & \bar{a}_z \\ \partial/\partial\rho & \partial/\partial\phi & \partial/\partial z \\ H_\rho & \rho H_\phi & H_z \end{vmatrix} \quad A/m^2$$

v. Poisson's and Laplace's equation

$\boxed{\nabla^2 V = -\rho_v/\epsilon}$ V/m^2 & $\boxed{\nabla^2 V = 0}$ V/m^2 result is → Scalar

Laplace's

$$\boxed{\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho\frac{\partial V}{\partial \rho}\right] + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0} \; V/m^2$$

c. Spherical Co-ordinate System $p(r, \theta, \phi)$

dr, $r\,d\theta$, $r\sin\theta\, d\phi$

$$dv = r^2 \sin\theta\, dr\, d\theta\, d\phi.$$

i. ∇ operator

$$\boxed{\nabla = \frac{\partial}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\bar{a}_\phi} \; m^{-1}$$ Vector Operator

ii. Gradient $\nabla V \rightarrow V/m$.

$$\boxed{\nabla V = \frac{\partial V}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\bar{a}_\phi} \; V/m$$ Results in vector.

iii. Divergence. $\nabla\cdot\bar{D}$ C/m^3

let $\bar{D} = D_r\bar{a}_r + D_\theta\bar{a}_\theta + D_\phi\bar{a}_\phi$ C/m^2.

$$\boxed{\nabla\cdot\bar{D} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 D_r\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\, D_\theta\right] + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi} \; C/m^3.}$$ Results in Scalar.

iv. Curl $\nabla\times\bar{H}$ A/m^2. Let $\bar{H} = H_r\bar{a}_r + H_\theta\bar{a}_\theta + H_\phi\bar{a}_\phi$

$$\boxed{\nabla\times\bar{H} = \frac{1}{r\sin\theta}\begin{vmatrix} \bar{a}_r & r\,\bar{a}_\theta & r\sin\theta\,\bar{a}_\phi \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ H_r & (r\,H_\theta) & (r\sin\theta\, H_\phi) \end{vmatrix}} \; A/m^2$$

v. Poisson's and Laplace's eq^n.

$\boxed{\nabla^2 V = -\rho_v/\epsilon}$ V/m^2 & $\boxed{\nabla^2 V = 0}$ V/m^2

Laplace eq^n

$$\boxed{\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial V}{\partial\theta}\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2} = 0} \; V/m^2$$

Algebra

Factoring Formulas

Real numbers: a, b, c
Natural number: n

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^4 - b^4 = (a^2 - b^2)(a^2 + b^2) = (a-b)(a+b)(a^2 + b^2)$$

$$a^5 - b^5 = (a-b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$$

$$a^5 + b^5 = (a+b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$$

If n is odd, then

$$a^n + b^n = (a+b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 - \ldots - ab^{n-2} + b^{n-1}).$$

If n is even, then

$$a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots + ab^{n-2} + b^{n-1}),$$

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$$a^n + b^n = (a+b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 - \ldots + ab^{n-2} - b^{n-1}).$$

Product Formulas

Real numbers: a, b, c
Whole numbers: n, k

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

Binomial Formula

$$(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n,$$

where ${}^nC_k = \dfrac{n!}{k!(n-k)!}$ are the binomial coefficients.

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$$

$$\begin{aligned}(a+b+c+\ldots+u+v)^2 &= a^2 + b^2 + c^2 + \ldots + u^2 + v^2 + \\ &+ 2(ab + ac + \ldots + au + av + bc + \ldots + bu + bv + \ldots + uv)\end{aligned}$$

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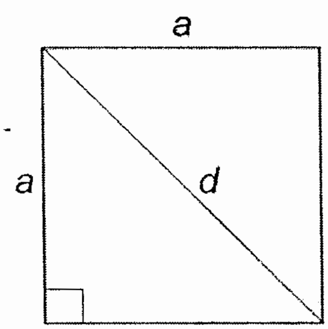
$$S = \frac{ah_a}{2} = \frac{bh_b}{2} = \frac{ch_c}{2},$$
$$S = \frac{ab\sin\gamma}{2} = \frac{ac\sin\beta}{2} = \frac{bc\sin\alpha}{2},$$
$$S = \sqrt{p(p-a)(p-b)(p-c)} \text{ (Heron's Formula)},$$
$$S = pr,$$
$$S = \frac{abc}{4R},$$
$$S = 2R^2 \sin\alpha \sin\beta \sin\gamma,$$
$$S = p^2 \tan\frac{\alpha}{2} \tan\frac{\beta}{2} \tan\frac{\gamma}{2}.$$

Square

Side of a square: a
Diagonal: d
Radius of circumscribed circle: R
Radius of inscribed circle: r
Perimeter: L
Area: S



Figure

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$\therefore\ d = a\sqrt{2}$

$\therefore\ R = \frac{d}{2} = \frac{a\sqrt{2}}{2}$

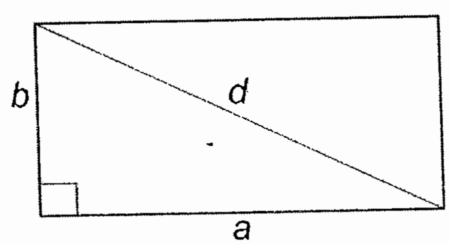
$\therefore\ r = \frac{a}{2}$

$\therefore\ L = 4a$

$\therefore\ S = a^2$

Rectangle

Sides of a rectangle: a, b
Diagonal: d
Radius of circumscribed circle: R
Perimeter: L
Area: S



$$d = \sqrt{a^2 + b^2}$$

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$$R = \frac{d}{2}$$

$$L = 2(a + b)$$

$$S = ab$$

Parallelogram

Sides of a parallelogram: a, b
Diagonals: d_1, d_2
Consecutive angles: α, β
Angle between the diagonals: φ
Altitude: h
Perimeter: L
Area: S

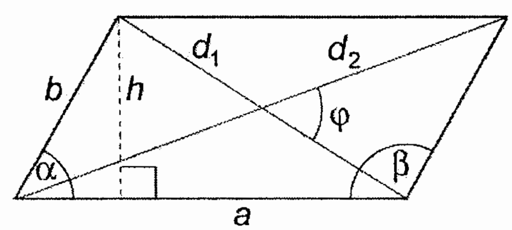

Figure 18.

$$\alpha + \beta = 180^\circ$$

$$d_1^2 + d_2^2 = 2(a^2 + b^2)$$

$$h = b \sin\alpha = b \sin\beta$$

$$L = 2(a+b)$$

$$S = ah = ab\sin(\alpha)$$

$$S = \frac{1}{2} d_1 d_2 \sin\phi.$$

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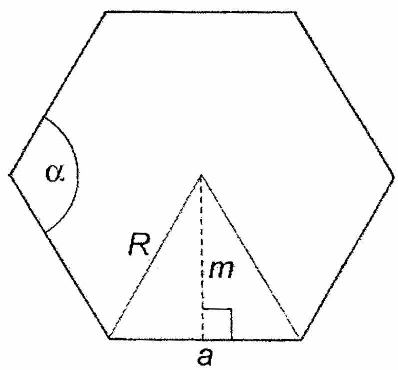
Regular Hexagon

Side: a
Internal angle: α
Slant height: m
Radius of inscribed circle: r
Radius of circumscribed circle: R
Perimeter: L
Semiperimeter: p
Area: S



Figure

$$\alpha = 120^\circ$$

$$r = m = \frac{a\sqrt{3}}{2}$$

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$$R = a$$

$$L = 6a$$

$$S = pr = \frac{a^2 3\sqrt{3}}{2},$$

where $p = \frac{L}{2}$.

Regular Polygon

Side: a
Number of sides: n
Internal angle: α
Slant height: m
Radius of inscribed circle: r
Radius of circumscribed circle: R
Perimeter: L
Semiperimeter: p
Area: S

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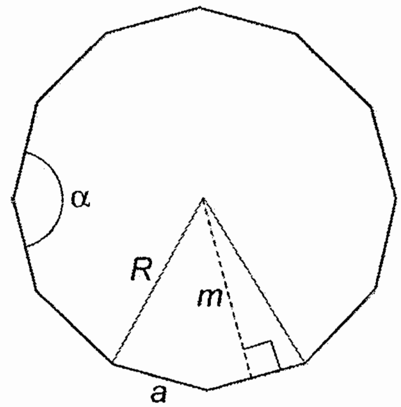
GEOMETRY



Figure

$$\alpha = \frac{n-2}{2} \cdot 180°$$

$$\alpha = \frac{n-2}{2} \cdot 180°$$

$$R = \frac{a}{2\sin\frac{\pi}{n}}$$

$$r = m = \frac{a}{2\tan\frac{\pi}{n}} = \sqrt{R^2 - \frac{a^2}{4}}$$

$$L = na$$

$$S = \frac{nR^2}{2}\sin\frac{2\pi}{n},$$

$$S = pr = p\sqrt{R^2 - \frac{a^2}{4}},$$

where $p = \frac{L}{2}$

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274. $\cancel{S = \frac{1}{2}[sR - a(R-h)] = \frac{R^2}{2}\left(\frac{\alpha\pi}{180°} - \sin\alpha\right) = \frac{R^2}{2}(x - \sin x)},$

$\cancel{S \approx \frac{2}{3}ha}.$

Cube

Edge: a
Diagonal: d
Radius of inscribed sphere: r
Radius of circumscribed sphere: r
Surface area: S
Volume: V

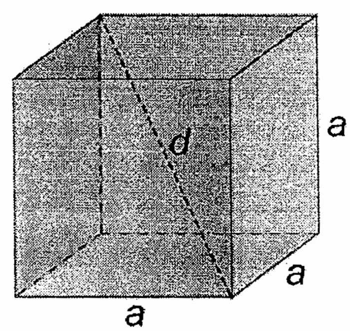

Figure 37.

275. $d = a\sqrt{3}$ $R = \frac{a\sqrt{3}}{2}$

276. $r = \frac{a}{2}$ $S = 6a^2$

$V = a^3$

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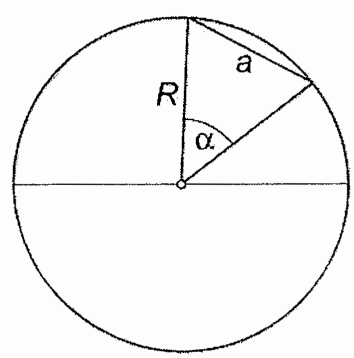
Circle

Radius: R
Diameter: d
Chord: a
Secant segments: e, f
Tangent segment: g
Central angle: α
Inscribed angle: β
Perimeter: L
Area: S

$$a = 2R\sin\frac{\alpha}{2}$$



Figure

perimeter $L = 2\pi R = \pi d$

Area $S = \pi R^2 = \frac{\pi d^2}{4}$

$= \frac{LR}{2} \ m2$

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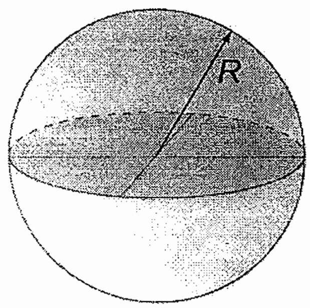
Sphere

Radius: R
Diameter: d
Surface area: S
Volume: V



Figure

$$S = 4\pi R^2$$

$$V = \frac{4}{3}\pi R^3 H = \frac{1}{6}\pi d^3 = \frac{1}{3}SR$$

Spherical Cap

Radius of sphere: R
Radius of base: r
Height: h
Area of plane face: S_B
Area of spherical cap: S_C
Total surface area: S
Volume: V

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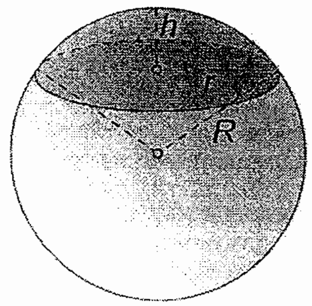

Figure .

$$R = \frac{r^2 + h^2}{2h}$$

$$S_B = \pi r^2$$

$$S_C = \pi\left(h^2 + r^2\right)$$

$$S = S_B + S_C = \pi\left(h^2 + 2r^2\right) = \pi\left(2Rh + r^2\right)$$

$$V = \frac{\pi}{6} h^2 (3R - h) = \frac{\pi}{6} h\left(3r^2 + h^2\right)$$

Spherical Sector

Radius of sphere: R
Radius of base of spherical cap: r
Height: h
Total surface area: S
Volume: V

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Right Circular Cylinder:

. GEOMETRY

Radius of bore : R

Diameter of bore : d

Height: H
Lateral surface area: S_L
Area of base: S_B
Total surface area: S
Volume: V

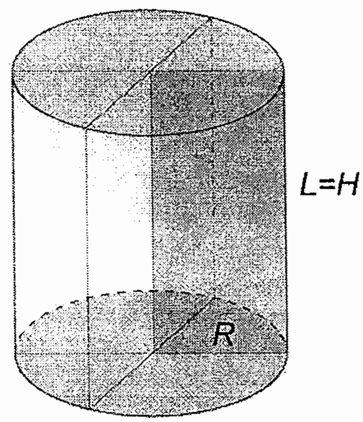

Figure .

$$S_L = 2\pi RH$$

$$S = S_L + 2S_B = 2\pi R(H + R) = \pi d\left(H + \frac{d}{2}\right)$$

$$V = S_B H = \pi R^2 H$$

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Trigonometric Functions of Common Angles

$\alpha°$	α rad	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$	$\cot\alpha$	$\sec\alpha$	$\operatorname{cosec}\alpha$
0	0	0	1	0	∞	1	∞
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2
45	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$
90	$\frac{\pi}{2}$	1	0	∞	0	∞	1
120	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$	-2	$\frac{2}{\sqrt{3}}$
180	π	0	-1	0	∞	-1	∞
270	$\frac{3\pi}{2}$	-1	0	∞	0	∞	-1
360	2π	0	1	0	∞	1	∞

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Trigonometric Formulae

$\cos^2 A + \sin^2 A = 1$ $\quad$ $\sec^2 A - \tan^2 A = 1$ $\quad$ $\operatorname{cosec}^2 A - \cot^2 A = 1$

$\sin 2A = 2 \sin A \cos A$ $\quad$ $\cos 2A = \cos^2 A - \sin^2 A$ $\quad$ $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$.

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \qquad \cos A \cos B = \frac{\cos(A+B) + \cos(A-B)}{2}$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B \qquad \sin A \sin B = \frac{\cos(A-B) - \cos(A+B)}{2}$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \sin A \cos B = \frac{\sin(A+B) + \sin(A-B)}{2}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \qquad \cos^2 A = \frac{1 + \cos 2A}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \qquad \sin^2 A = \frac{1 - \cos 2A}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \qquad \cos^3 A = \frac{3 \cos A + \cos 3A}{4}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2} \qquad \sin^3 A = \frac{3 \sin A - \sin 3A}{4}$$

Relations between sides and angles of any plane triangle

In a plane triangle with angles $A, B,$ and C and sides opposite $a, b,$ and c respectively,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = \text{diameter of circumscribed circle.}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a = b \cos C + c \cos B$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\tan \frac{A-B}{2} = \frac{a-b}{a+b} \cot \frac{C}{2}$$

$$\text{area} = \frac{1}{2} ab \sin C = \frac{1}{2} bc \sin A = \frac{1}{2} ca \sin B = \sqrt{s(s-a)(s-b)(s-c)}, \qquad \text{where } s = \frac{1}{2}(a+b+c)$$

Relations between sides and angles of any spherical triangle

In a spherical triangle with angles $A, B,$ and C and sides opposite $a, b,$ and c respectively,

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a$$

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Hyperbolic Functions

$$\cosh x = \frac{1}{2}(e^x + e^{-x}) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots \qquad \text{valid for all } x$$

$$\sinh x = \frac{1}{2}(e^x - e^{-x}) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots \qquad \text{valid for all } x$$

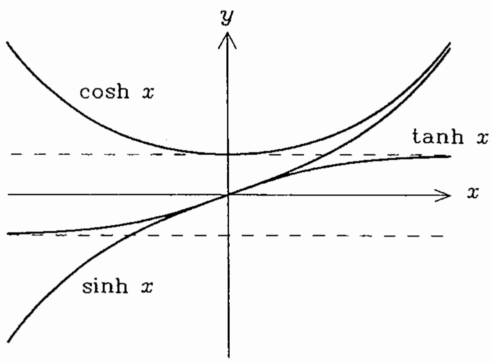
$\cosh ix = \cos x$ $\qquad$ $\cos ix = \cosh x$

$\sinh ix = i \sin x$ $\qquad$ $\sin ix = i \sinh x$

$\tanh x = \dfrac{\sinh x}{\cosh x}$ $\qquad$ $\operatorname{sech} x = \dfrac{1}{\cosh x}$

$\coth x = \dfrac{\cosh x}{\sinh x}$ $\qquad$ $\operatorname{cosech} x = \dfrac{1}{\sinh x}$

$\cosh^2 x - \sinh^2 x = 1$



For large positive x:

$\cosh x \approx \sinh x \to \dfrac{e^x}{2}$

$\tanh x \to 1$

For large negative x:

$\cosh x \approx -\sinh x \to \dfrac{e^{-x}}{2}$

$\tanh x \to -1$

Relations of the functions

$\sinh x = -\sinh(-x)$ $\qquad$ $\operatorname{sech} x = \operatorname{sech}(-x)$

$\cosh x = \cosh(-x)$ $\qquad$ $\operatorname{cosech} x = -\operatorname{cosech}(-x)$

$\tanh x = -\tanh(-x)$ $\qquad$ $\coth x = -\coth(-x)$

$$\sinh x = \frac{2\tanh(x/2)}{1 - \tanh^2(x/2)} = \frac{\tanh x}{\sqrt{1 - \tanh^2 x}} \qquad \cosh x = \frac{1 + \tanh^2(x/2)}{1 - \tanh^2(x/2)} = \frac{1}{\sqrt{1 - \tanh^2 x}}$$

$\tanh x = \sqrt{1 - \operatorname{sech}^2 x}$ $\qquad$ $\operatorname{sech} x = \sqrt{1 - \tanh^2 x}$

$\coth x = \sqrt{\operatorname{cosech}^2 x + 1}$ $\qquad$ $\operatorname{cosech} x = \sqrt{\coth^2 x - 1}$

$\sinh(x/2) = \sqrt{\dfrac{\cosh x - 1}{2}}$ $\qquad$ $\cosh(x/2) = \sqrt{\dfrac{\cosh x + 1}{2}}$

$$\tanh(x/2) = \frac{\cosh x - 1}{\sinh x} = \frac{\sinh x}{\cosh x + 1}$$

$\sinh(2x) = 2 \sinh x \cosh x$ $\qquad$ $\tanh(2x) = \dfrac{2\tanh x}{1 + \tanh^2 x}$

$$\cosh(2x) = \cosh^2 x + \sinh^2 x = 2\cosh^2 x - 1 = 1 + 2\sinh^2 x$$

$\sinh(3x) = 3\sinh x + 4\sinh^3 x$ $\qquad$ $\cosh 3x = 4\cosh^3 x - 3\cosh x$

$$\tanh(3x) = \frac{3\tanh x + \tanh^3 x}{1 + 3\tanh^2 x}$$

$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$

$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

$$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$$

$$\sinh x + \sinh y = 2 \sinh \frac{1}{2}(x+y) \cosh \frac{1}{2}(x-y) \qquad \cosh x + \cosh y = 2 \cosh \frac{1}{2}(x+y) \cosh \frac{1}{2}(x-y)$$

$$\sinh x - \sinh y = 2 \cosh \frac{1}{2}(x+y) \sinh \frac{1}{2}(x-y) \qquad \cosh x - \cosh y = 2 \sinh \frac{1}{2}(x+y) \sinh \frac{1}{2}(x-y)$$

$$\sinh x \pm \cosh x = \frac{1 \pm \tanh(x/2)}{1 \mp \tanh(x/2)} = e^{\pm x}$$

$$\tanh x \pm \tanh y = \frac{\sinh(x \pm y)}{\cosh x \cosh y}$$

$$\coth x \pm \coth y = \pm \frac{\sinh(x \pm y)}{\sinh x \sinh y}$$

Inverse functions

$$\sinh^{-1} \frac{x}{a} = \ln\left(\frac{x + \sqrt{x^2 + a^2}}{a}\right) \qquad \text{for } -\infty < x < \infty$$

$$\cosh^{-1} \frac{x}{a} = \ln\left(\frac{x + \sqrt{x^2 - a^2}}{a}\right) \qquad \text{for } x \geq a$$

$$\tanh^{-1} \frac{x}{a} = \frac{1}{2} \ln\left(\frac{a+x}{a-x}\right) \qquad \text{for } x^2 < a^2$$

$$\coth^{-1} \frac{x}{a} = \frac{1}{2} \ln\left(\frac{x+a}{x-a}\right) \qquad \text{for } x^2 > a^2$$

$$\operatorname{sech}^{-1} \frac{x}{a} = \ln\left(\frac{a}{x} + \sqrt{\frac{a^2}{x^2} - 1}\right) \qquad \text{for } 0 < x \leq a$$

$$\operatorname{cosech}^{-1} \frac{x}{a} = \ln\left(\frac{a}{x} + \sqrt{\frac{a^2}{x^2} + 1}\right) \qquad \text{for } x \neq 0$$

8. Limits

$n^c x^n \to 0$ as $n \to \infty$ if $|x| < 1$ (any fixed c)

$x^n / n! \to 0$ as $n \to \infty$ (any fixed x)

$(1 + x/n)^n \to e^x$ as $n \to \infty$, $x \ln x \to 0$ as $x \to 0$

If $f(a) = g(a) = 0$ then $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$ (l'Hôpital's rule)

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Differentiation

$$(uv)' = u'v + uv', \quad \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$(uv)^{(n)} = u^{(n)}v + nu^{(n-1)}v^{(1)} + \cdots + {}^nC_r u^{(n-r)}v^{(r)} + \cdots + uv^{(n)}$$ Leibniz Theorem

where $^nC_r \equiv \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\sinh x) = \cosh x$$
$$\frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}(\cosh x) = \sinh x$$
$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$$
$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$
$$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x \qquad \frac{d}{dx}(\coth x) = -\operatorname{cosech}^2 x$$
$$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x \qquad \frac{d}{dx}(\operatorname{cosech} x) = -\operatorname{cosech} x \coth x$$

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10. Integration

Standard forms

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c \qquad \text{for } n \neq -1$$

$$\int \frac{1}{x}\,dx = \ln x + c \qquad \int \ln x\,dx = x(\ln x - 1) + c$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c \qquad \int x e^{ax}\,dx = e^{ax}\left(\frac{x}{a} - \frac{1}{a^2}\right) + c$$

$$\int x \ln x\,dx = \frac{x^2}{2}\left(\ln x - \frac{1}{2}\right) + c$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{a^2 - x^2}\,dx = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + c = \frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right) + c \qquad \text{for } x^2 < a^2$$

$$\int \frac{1}{x^2 - a^2}\,dx = -\frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + c = \frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right) + c \qquad \text{for } x^2 > a^2$$

$$\int \frac{x}{(x^2 \pm a^2)^n}\,dx = \frac{-1}{2(n-1)}\frac{1}{(x^2 \pm a^2)^{n-1}} + c \qquad \text{for } n \neq 1$$

$$\int \frac{x}{x^2 \pm a^2}\,dx = \frac{1}{2}\ln(x^2 \pm a^2) + c$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \ln\left(x + \sqrt{x^2 \pm a^2}\right) + c$$

$$\int \frac{x}{\sqrt{x^2 \pm a^2}}\,dx = \sqrt{x^2 \pm a^2} + c$$

$$\int \sqrt{a^2 - x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\left(\frac{x}{a}\right)\right] + c$$

$$\int_0^\infty \frac{1}{(1+x)x^p}\,dx = \pi \operatorname{cosec} p\pi \qquad \text{for } p < 1$$

$$\int_0^\infty \cos(x^2)\,dx = \int_0^\infty \sin(x^2)\,dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}$$

$$\int_{-\infty}^\infty \exp(-x^2/2\sigma^2)\,dx = \sigma\sqrt{2\pi}$$

$$\int_{-\infty}^\infty x^n \exp(-x^2/2\sigma^2)\,dx = \begin{cases} 1 \times 3 \times 5 \times \cdots (n-1)\sigma^{n+1}\sqrt{2\pi} & \text{for } n \geq 2 \text{ and even} \\ 0 & \text{for } n \geq 1 \text{ and odd} \end{cases}$$

$$\int \sin x\,dx = -\cos x + c \qquad \int \sinh x\,dx = \cosh x + c$$

$$\int \cos x\,dx = \sin x + c \qquad \int \cosh x\,dx = \sinh x + c$$

$$\int \tan x\,dx = -\ln(\cos x) + c \qquad \int \tanh x\,dx = \ln(\cosh x) + c$$

$$\int \operatorname{cosec} x\,dx = \ln(\operatorname{cosec} x - \cot x) + c \qquad \int \operatorname{cosech} x\,dx = \ln[\tanh(x/2)] + c$$

$$\int \sec x\,dx = \ln(\sec x + \tan x) + c \qquad \int \operatorname{sech} x\,dx = 2\tan^{-1}(e^x) + c$$

$$\int \cot x\,dx = \ln(\sin x) + c \qquad \int \coth x\,dx = \ln(\sinh x) + c$$

$$\int \sin mx \sin nx\,dx = \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)} + c \qquad \text{if } m^2 \neq n^2$$

$$\int \cos mx \cos nx\,dx = \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)} + c \qquad \text{if } m^2 \neq n^2$$

Standard substitutions

If the integrand is a function of: substitute:

$(a^2 - x^2)$ or $\sqrt{a^2 - x^2}$	$x = a\sin\theta$ or $x = a\cos\theta$
$(x^2 + a^2)$ or $\sqrt{x^2 + a^2}$	$x = a\tan\theta$ or $x = a\sinh\theta$
$(x^2 - a^2)$ or $\sqrt{x^2 - a^2}$	$x = a\sec\theta$ or $x = a\cosh\theta$

If the integrand is a rational function of $\sin x$ or $\cos x$ or both, substitute $t = \tan(x/2)$ and use the results:

$$\sin x = \frac{2t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2} \qquad dx = \frac{2\,dt}{1+t^2}.$$

If the integrand is of the form: substitute:

$$\int \frac{dx}{(ax+b)\sqrt{px+q}} \qquad px + q = u^2$$

$$\int \frac{dx}{(ax+b)\sqrt{px^2+qx+r}} \qquad ax + b = \frac{1}{u}.$$

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Integration by parts

$$\int_a^b u\,\mathrm{d}v = uv\Big|_a^b - \int_a^b v\,\mathrm{d}u$$

Differentiation of an integral

If $f(x,\alpha)$ is a function of x containing a parameter α and the limits of integration a and b are functions of α then

$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\int_{a(\alpha)}^{b(\alpha)} f(x,\alpha)\,\mathrm{d}x = f(b,\alpha)\frac{\mathrm{d}b}{\mathrm{d}\alpha} - f(a,\alpha)\frac{\mathrm{d}a}{\mathrm{d}\alpha} + \int_{a(\alpha)}^{b(\alpha)} \frac{\partial}{\partial\alpha}f(x,\alpha)\,\mathrm{d}x.$$

Special case,

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_a^x f(y)\,\mathrm{d}y = f(x).$$

Dirac δ-'function'

$$\delta(t-\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp[\mathrm{i}\omega(t-\tau)]\,\mathrm{d}\omega.$$

If $f(t)$ is an arbitrary function of t then $\int_{-\infty}^{\infty}\delta(t-\tau)f(t)\,\mathrm{d}t = f(\tau)$.

$\delta(t) = 0$ if $t \neq 0$, also $\int_{-\infty}^{\infty}\delta(t)\,\mathrm{d}t = 1$

Reduction formulae

Factorials

$n! = n(n-1)(n-2)\ldots 1, \qquad 0! = 1.$

Stirling's formula for large n: $\ln(n!) \approx n\ln n - n.$

For any $p > -1$, $\int_0^\infty x^p\,\mathrm{e}^{-x}\,\mathrm{d}x = p\int_0^\infty x^{p-1}\,\mathrm{e}^{-x}\,\mathrm{d}x = p!. \qquad (-1/2)! = \sqrt{\pi}, \qquad (1/2)! = \sqrt{\pi}/2$, etc.

For any $p, q > -1$, $\int_0^1 x^p(1-x)^q\,\mathrm{d}x = \frac{p!q!}{(p+q+1)!}.$

Trigonometrical

If m, n are integers,

$$\int_0^{\pi/2}\sin^m\theta\,\cos^n\theta\,\mathrm{d}\theta = \frac{m-1}{m+n}\int_0^{\pi/2}\sin^{m-2}\theta\,\cos^n\theta\,\mathrm{d}\theta = \frac{n-1}{m+n}\int_0^{\pi/2}\sin^m\theta\,\cos^{n-2}\theta\,\mathrm{d}\theta$$

and can therefore be reduced eventually to one of the following integrals

$$\int_0^{\pi/2}\sin\theta\,\cos\theta\,\mathrm{d}\theta = \frac{1}{2}, \qquad \int_0^{\pi/2}\sin\theta\,\mathrm{d}\theta = 1, \qquad \int_0^{\pi/2}\cos\theta\,\mathrm{d}\theta = 1, \qquad \int_0^{\pi/2}\mathrm{d}\theta = \frac{\pi}{2}.$$

Other

If $I_n = \int_0^\infty x^n\exp(-\alpha x^2)\,\mathrm{d}x$ then $I_n = \frac{(n-1)}{2\alpha}I_{n-2}, \quad I_0 = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}, \quad I_1 = \frac{1}{2\alpha}.$

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Important Vector Identies used

Let $\overline{A}$ - be the general vector.

1. $\nabla \cdot (\nabla \times \overline{A}) = 0$ ... used in inconsistency of Ampere Law. (Module -5A)

2. $\nabla \times (\nabla \times \overline{A}) = \nabla \cdot (\nabla \overline{A}) - \nabla^2 \overline{A}$ ... used in wave eq^n derivation. (M-5B)

3. $\nabla \cdot (\overline{A} \times \overline{B}) = \overline{B} \cdot (\nabla \times \overline{A}) - \overline{A} \cdot (\nabla \times \overline{B})$ ... used in poynting theorem proof. (M-5B)

4. $\overline{A} \cdot \frac{\partial \overline{A}}{\partial t} = \frac{1}{2} \frac{\partial A^2}{\partial t}$ ... used in poynting theorem proof. (M-5B)

5. Let vector $\overline{A} = V_d \nabla V_d$

$\nabla \cdot \overline{A} = \nabla \cdot (V_d \nabla V_d) = V_d \nabla^2 V_d + \nabla V_d \cdot \nabla V_d$ .. Used in Uniqueness theorem (M3)

6. Divergence theorem

$$\oint_{\langle S \rangle} \overline{A} \cdot \overline{ds} = \int_{\langle vol \rangle} (\nabla \cdot \overline{A})\, dv. \quad (M2)$$

7. Stokes theorem $\oint_{\langle l \rangle} \overline{A} \cdot \overline{dl} = \int_{\langle S \rangle} (\nabla \times \overline{A}) \cdot \overline{ds}$; [M3B]

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8. $\nabla\cdot(V\bar{D}) = V\,\nabla\cdot\bar{D} + \bar{D}\,\nabla V$ --- used in Energy density in electrostatic field [M2]

$$\boxed{e = \frac{1}{2}\epsilon E^2}\ J/m^3$$

ly magnetic energy density

$$\boxed{e_m = \frac{1}{2}\mu H^2}\ J/m^3$$

9. Bernoulis theorem.

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

if $|x| < 1$. [M5B]

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Module 1: Coulomb's Law, Electric Field Intensity and Flux density
Experimental law of Coulomb, Electric field intensity, Field due to continuous volume charge distribution, Field of a line charge, Electric flux density.

Topics:

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1.1 Coulombs Law
   a. Statement
   b. Vector form of Coulombs Law.
      ✓ Solved Problems
   c. Force due to N-number of point charges
      ✓ Solved Problems
   d. Applications of Coulomb's Law
   e. Limitation of Coulomb's Law

1.2 Types of Charge Distribution
   a. Point charge distribution
   b. Line charge density
   c. Surface charge density
   d. Volume charge density
      ✓ Solved Problems

1.3 Electric Field Intensity
   a. Definition of Electric Field Intensity
   b. Field due to point charge
   c. Field due to N-number of point charges
      ✓ Solved Problems
   d. Field due to infinite line charge.
   e. Field due to infinite sheet charge.
      ✓ Solved Problems
   f. Field due to various charge distribution (point, line,surface,volume)
      ✓ Solved Problems

1.4 Electric Flux Density
   a. Definition of Electric Flux and its properties
   b. Definition of Electric Flux density
   c. Electric Flux density due to point charge
   d. Relationship b/w electric field intensity and electric flux density.
   e. Electric Flux density due to infinite line charge and infinite sheet charge
   f. Flux density due to various charge distributions
      ✓ Solved Problems

Summary
- List of Symbols
- List of Formulae

1.1a Coulomb's Law

Question → State and explain Coulomb's Law. clearly indicate the unit of quantities used in the force equation. [02-Dec/Jan-2009 (6M)]
[06-Dec/Jan 2009 (6M)].

(or)

→ State and explain Coulomb's Law of force between two point charges. mention the units.
[10-Dec/Jan-2014 (6M)]

(or)

→ State and Explain the Coulomb's Law of force between the two point charges.
[02-June/July 2010 (5M)]

(or)

→ State and explain Coulomb's Law of force.
[06-Dec/Jan-2012]

Statement:- The force of attraction (or) repulsion between any two point charges is directly proportional to the product of the charges and Inversely proportional to the square of the distance between them.

i.e mathematically

$$\boxed{F \propto \frac{Q_1 Q_2}{r^2}} \text{ Newton.}$$

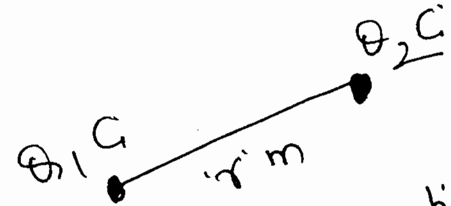

fig. coulomb's law

$$F = K \frac{Q_1 Q_2}{r^2} \quad \text{Newton.}$$

where K - constant of proportionality.

$$K = \frac{1}{4\pi\epsilon} \ m/F\text{-rad} \Rightarrow \boxed{F = \frac{Q_1 Q_2}{4\pi\epsilon r^2}} \ N.$$

where

ϵ - permittivity of the medium in which the point charges are Located (F/m).

Q_1, Q_2 - two point charges (Coulomb's).

r - distance between the two point charges (meter's).

$$\epsilon = \epsilon_0 \epsilon_r \ F/m.$$

ϵ_0 - absolute permittivity of freespace (or) Vaccum

$$\epsilon_0 = \frac{1}{36\pi} \times 10^{-9} = 8.854 \times 10^{-12} \ F/m.$$

ϵ_r - relative permittivity of the medium. (No unit)
In freespace $\boxed{\epsilon_r = 1}$

F - Force between two point charges (Newton).

Note: The value of K in freespace medium (or) Vaccum medium is

$$\boxed{K = \frac{1}{4\pi\epsilon_0} \simeq 9\times10^9} \text{ m/f-rad}$$ .... [Used in solving problems]

1.1b Vector form of Coulomb's Law

Questions.

→ State vector form of Coulomb's Law of force between two point charges and indicate the units of quantities in the force equation. [06-Dec/Jan 2014 (6M)].

(or)

→ Explain with diagram and proper units of the parameters used, Vector form of Coulomb's Law. [02-Dec 2010 (6M)]

(or)

→ State and Explain Coulomb's Law in vectorform.

[06 - June/July 2011 (6M)], [10 - Dec/Jan 2015 (6M)]

[10 - June/July 2014 (6m)], [06 - June/July 2013 (5M)]

** 15 - June/July - 2017 (4M) [CBCS-scheme].

Statement:-

The force of attraction (or) repulsion between any two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

i.e $\boxed{F \propto \frac{Q_1 Q_2}{r^2}}$ Newton. ——(1)

The force is a vector quantity and it is attractive if the charges are unlike and repulsive if the charges are alike. it acts along the straight line joining the two point charges.

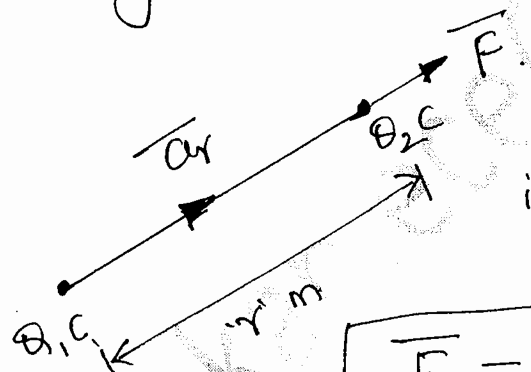

fig. vector force between two point charges

i.e mathematically

$$\boxed{\bar{F} = \frac{Q_1 Q_2}{4\pi \epsilon r^2} \bar{a_r}} \quad \text{Newton} \quad \text{——(2)}$$

where Q_1, Q_2 - point charges (Coulombs)

$\bar{F}$ - vector force between two point charges (Newton)

$\epsilon = \epsilon_0 \epsilon_r$ F/m ... permittivity of the medium.

$\bar{a_r}$ - unit vector indicates the direction of force.

$\epsilon_0 = 8.854 \times 10^{-12}$ F/m and $\epsilon_r = 1$ in freespace.

→ Force on charge Q_2 (or) Force experienced by charge Q_2 due to Q_1.
(or) Force Exerted on charge Q_2

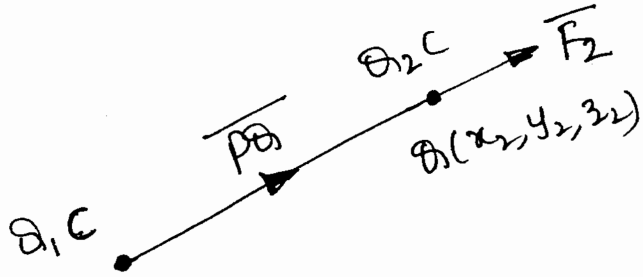

fig. vector force on charge Q_2 i.e. $\overline{F_2}$

Consider a two point charges of Q_1 and Q_2 Coulombs Located at a point $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ respectively.

the Force experience / exerted on charge Q_2 due to charge Q_1 is given by

$$\boxed{\overline{F_2} = \frac{Q_1 Q_2}{4\pi\epsilon |\overline{PQ}|^2} \overline{a_{PQ}}} \quad \text{Newton} \leftarrow (3)$$

where $\overline{PQ} = (x_2 - x_1)\overline{a_x} + (y_2 - y_1)\overline{a_y} + (z_2 - z_1)\overline{a_z}$

$$|\overline{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \text{ ; meters}$$

$$\overline{a_{PQ}} = \frac{\overline{PQ}}{|\overline{PQ}|}$$

$$\overline{a_{PQ}} = \frac{(x_2 - x_1)\overline{a_x} + (y_2 - y_1)\overline{a_y} + (z_2 - z_1)\overline{a_z}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

from eq^{n} (3)

$$\overline{F_2} = \frac{Q_1 Q_2}{4\pi\epsilon |\overline{PQ}|^2} \cdot \frac{\overline{PQ}}{|\overline{PQ}|}$$

$$\boxed{\overline{F_2} = \frac{Q_1 Q_2}{4\pi\epsilon |\overline{PQ}|^3} \cdot \overline{PQ}} \quad \text{Newton} \quad \leftarrow (4)$$

$$\Rightarrow \boxed{\overline{F_2} = \frac{Q_1 Q_2 \left[(x_2-x_1)\overline{a_x} + (y_2-y_1)\overline{a_y} + (z_2-z_1)\overline{a_z}\right]}{4\pi\epsilon \left[(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2\right]^{3/2}}} \quad \text{Newton} \quad \leftarrow (5)$$

$\underline{\text{lly}}$ Force on charge Q :

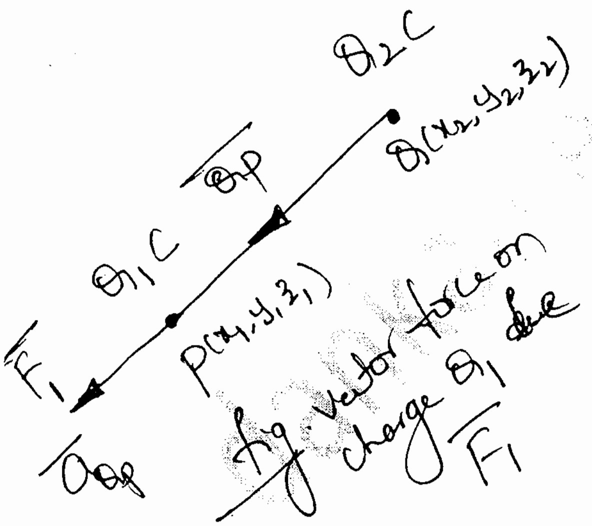

fig. vector force on charge Q_1 due $\overline{F_1}$

Force experienced by charge Q_1 due to charge Q_2 is given by

$$\boxed{\overline{F_1} = \frac{Q_1 Q_2}{4\pi\epsilon |\overline{QP}|^2} \overline{a_{QP}}} \quad \text{Newton} \quad \leftarrow (6)$$

$$\overline{QP} = (x_1 - x_2)\overline{a_x} + (y_1 - y_2)\overline{a_y} + (z_1 - z_2)\overline{a_z}$$

$$|\overline{QP}| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \text{ meter}$$

$$\overline{a}_{QP} = \frac{\overline{QP}}{|\overline{QP}|} = \frac{(x_1-x_2)\overline{a}_x + (y_1-y_2)\overline{a}_y + (z_1-z_2)\overline{a}_z}{\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}}$$

$$\overline{F}_1 = \frac{Q_1 Q_2}{4\pi\epsilon |\overline{QP}|^2} \cdot \frac{\overline{QP}}{|\overline{QP}|}$$

$$\boxed{\overline{F}_1 = \frac{Q_1 Q_2}{4\pi\epsilon |\overline{QP}|^3}\overline{QP}} \quad \text{Newton} \longrightarrow (7).$$

$$\boxed{\overline{F}_1 = \frac{Q_1 Q_2\left[(x_1-x_2)\overline{a}_x + (y_1-y_2)\overline{a}_y + (z_1-z_2)\overline{a}_z\right]}{4\pi\epsilon \left[(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2\right]^{3/2}}} \quad \text{Newton}$$

Keynote points:-

i. $|\overline{F}_1| = |\overline{F}_2|$ and $\overline{a}_{PQ} = -\overline{a}_{QP}$

ii. $\overline{F}_1 = -\overline{F}_2$ (or) $\overline{F}_2 = -\overline{F}_1$.

iii. The Resultant force F can be +ve (or) -ve that depends on nature of the charges.

Q_1	Q_2	Force (F)	Remark
+	+	+	Repulsive force. Nature of the charges are unlike.
−	−	+	
+	−	−	attractive force. Nature of the charges are alike.
−	+	−	

iv. The approximated value of $\frac{1}{4\pi\epsilon_0} \simeq 9\times10^9$.

v. The resultant force $\bar{F}$ can be expressed as

$$\bar{F} = F_x \bar{a}_x + F_y \bar{a}_y + F_z \bar{a}_z \quad \text{Newton.}$$

where F_x, F_y, F_z are force components along x, y and z direction respectively.

vi. Magnitude of Resultant force $\bar{F}$ is given by

$$\boxed{|\bar{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}} \quad \text{Newton.}$$

vii. The point at which the force experienced | exerted is considered to be the end point.

Solved problems:

problem 1. Let $Q_1 = 3\times10^{-4}$ C at $P(1,2,3)$ and a charge of $Q_2 = -10^{-4}$ C at $Q(2,0,5)$ in a Vaccum. Find

i. Force exerted on Q_2 by Q_1.

ii. Force exerted on Q_1 by Q_2.

[W.H. Hayt / [10-Dec/Jan 2015 (6M)]]

Sol^n:-

i. Force exerted on charge Q_2 by Q_1.

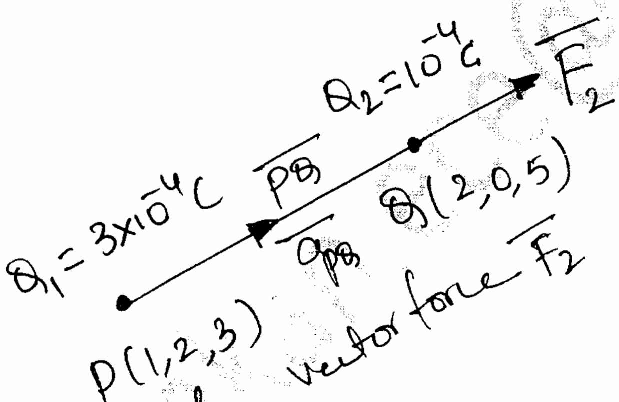

fig. vector force $\overline{F_2}$

given medium is Vaccum.

$\therefore \boxed{\epsilon = \epsilon_0}$ F/m.

; $\epsilon_r = 1$.

$$\overline{F_2} = \frac{Q_1 Q_2}{4\pi\epsilon_0 |\overline{PQ}|^2}\,\overline{a_{PQ}} \quad \text{Newton.}$$

$$\overline{PQ} = (2-1)\overline{a_x} + (0-2)\overline{a_y} + (5-3)\overline{a_z}$$

$$\overline{PQ} = \overline{a_x} - 2\overline{a_y} + 2\overline{a_z}$$

$$|\overline{PQ}| = \sqrt{1+4+4} = \sqrt{9} = 3m.$$

$$|\overline{PQ}|^2 = 9.$$

$$\overline{a}_{PQ} = \frac{\overline{PQ}}{|\overline{PQ}|} = \frac{\overline{a}_x - 2\overline{a}_y + 2\overline{a}_z}{\sqrt{9}}.$$

use $\frac{1}{4\pi\epsilon_0} = 9\times10^9$; $\boxed{\overline{F}_2 = \frac{Q_1 Q_2}{4\pi\epsilon_0 |\overline{PQ}|^3}\overline{PQ}}$ N

$$\overline{F}_2 = \frac{(3\times10^{-4})(-10^{-4})\times9\times10^9}{(3)^3}[\overline{a}_x - 2\overline{a}_y + 2\overline{a}_z]$$

$$\overline{F}_2 = -10[\overline{a}_x - 2\overline{a}_y + 2\overline{a}_z]$$

$$\boxed{\overline{F}_2 = -10\overline{a}_x + 20\overline{a}_y - 20\overline{a}_z}$$ Newton

ii. Force on Q_1 i.e force exerted on Q_1 by Q_2

$$\overline{F}_1 = -\overline{F}_2 = 10\overline{a}_x - 20\overline{a}_y + 20\overline{a}_z$$

$$\boxed{\overline{F}_1 = 10\overline{a}_x - 20\overline{a}_y + 20\overline{a}_z}$$ Newton.

$$|\overline{F}_1| = |\overline{F}_2| = \sqrt{F_x^2+F_y^2+F_z^2} = \sqrt{10^2+20^2+20^2} = 30N$$

$$\boxed{|\bar{F_1}| = |\bar{F_2}| = 30\,N}$$

the Force Components of $\bar{F_1}$ and $\bar{F_2}$ along x, y and z direction are

$F_{2x} = -10N$; $F_{2y} = 20N$; $F_{2z} = -20N$.

$F_{1x} = 10N$; $F_{1y} = -20N$; $F_{1z} = 20N$.

Problem 2.

Two point charges $Q_1 = 100\mu C$ and $Q_2 = 10\mu C$ are Located at points $(-1, 1, -3)m$ and $(3, 1, 0)m$ respectively. Find the x, y and z Components of the force on Q_1. what is the magnitude of the total force? [06-June/July 2013] soln:-

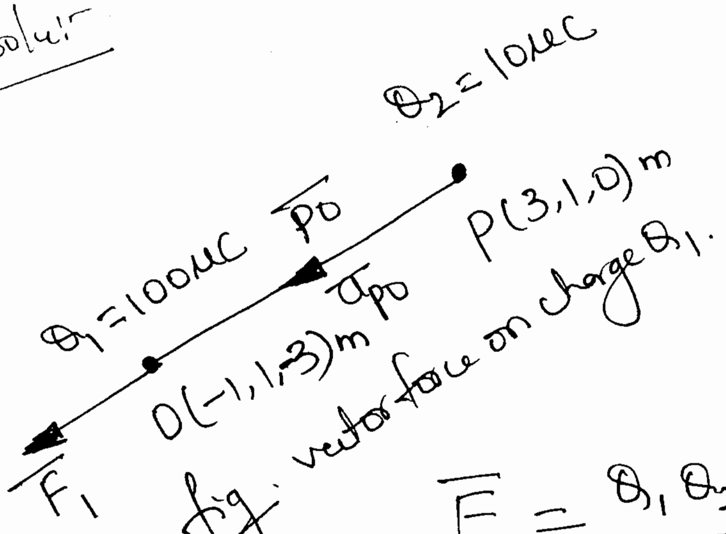

fig. vector force on charge Q_1.

$$\overline{F_1} = \frac{Q_1 Q_2}{4\pi\epsilon_0 |\overline{PO}|^2}\,\overline{a_{PO}} \text{ Newton.}$$

$$\overline{F_1} = \frac{Q_1 Q_2}{4\pi\epsilon_0 |\overline{PO}|^3}\,\overline{PO} \text{ Newton.}$$

$$\overline{PO} = -4\overline{a_x} + 0\overline{a_y} - 3\overline{a_z}$$

$$\overline{PO} = -4\overline{a_x} - 3\overline{a_z}$$

$$|\overline{PO}| = \sqrt{16+9} = \sqrt{25} = 5m$$

$$\overline{F_1} = \frac{(100\mu)(10\mu)\times 9\times 10^9}{(5)^3}\left[-4\overline{a_x} - 3\overline{a_z}\right]$$

$$\overline{F_1} = 0.072\left[-4\overline{a_x} - 3\overline{a_z}\right]$$

$$\boxed{\overline{F_1} = -0.288\overline{a_x} - 0.216\overline{a_z}} \text{ Newton}$$

$$\bar{F}_1 = F_x \bar{a}_x + F_y \bar{a}_y + F_z \bar{a}_z \text{ Newton}$$

the x, y and z components of the force on Q_1

are

$$\boxed{F_x = -0.288 \text{ N}}$$

$$\boxed{F_y = 0 \text{ N}} \text{ ; and}$$

$$\boxed{F_z = -0.216 \text{ N.}}$$

the magnitude of the total force

$$|\bar{F}_1| = \sqrt{F_x^2 + F_y^2 + F_z^2} \text{ N}$$

$$|\bar{F}_1| = \sqrt{(-0.288)^2 + 0^2 + (-0.216)^2}$$

$$\boxed{|\bar{F}_1| = 0.36} \text{ Newton}$$

Problem 3. A charge $Q_A = -20\mu C$ is located at $A(-6,4,7)$ and a charge $Q_B = 50\mu C$ is at $B(5,8,-2)$ in freespa[ce] if distances are given in meters. Find

i) $\overline{R_{AB}}$ ii) R_{AB}.

Determine the vector force exerted on Q_A by Q_B if $\epsilon_0 =$ (iii) $10^{-9}/36\pi$ F/m iv) 8.854×10^{-12} F/m.

[W.H. Hayt].

solu.

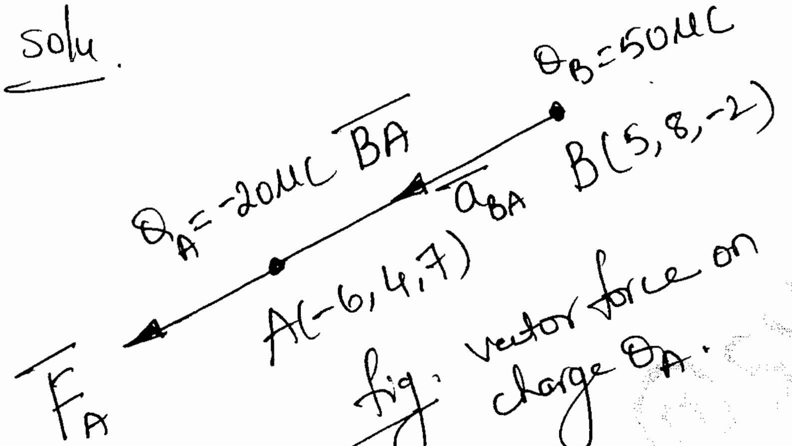

fig: vector force on charge Q_A.

i. $\overline{R_{AB}} = \overline{AB} = (5+6)\overline{a_x} + (8-4)\overline{a_y} + (-2-7)\overline{a_z}$

$$\boxed{\overline{R_{AB}} = \overline{AB} = 11\overline{a_x} + 4\overline{a_y} - 9\overline{a_z}}$$

ii. $R_{AB} = |\overline{R_{AB}}| = |\overline{AB}| = \sqrt{11^2 + 4^2 + (-9)^2}$

$$\boxed{R_{AB} = \sqrt{218} = 14.764 \text{ m}}$$

iii. Vector force exerted on Q_A due to Q_B is given by [use $\epsilon_0 = 10^{-9}/36\pi$]

$$\overline{F_A} = \frac{Q_A Q_B}{4\pi\epsilon_0 |\overline{BA}|} \overline{a_{BA}} \text{ : Newton}$$

$$\overline{F_A} = \frac{Q_A Q_B}{4\pi\epsilon_0 |\overline{BA}|^3} \overline{BA} \text{ Newton.}$$

$$\overline{BA} = -\overline{AB} = -11\overline{a_x} - 4\overline{a_y} + 9\overline{a_z}$$

and $|\overline{BA}| = |\overline{AB}| = \sqrt{218}$ m.

$$\overline{F_A} = \frac{(-20\mu)(50\mu)}{4\pi \times 10^{-9}/36\pi \, (\sqrt{218})^3} [-11\overline{a_x} - 4\overline{a_y} + 9\overline{a_z}]$$

$$\overline{F_A} = -2.7961 \times 10^{-3} [-11\overline{a_x} - 4\overline{a_y} + 9\overline{a_z}]$$

$$\boxed{\overline{F_A} = 0.03075\overline{a_x} + 0.01118\overline{a_y} - 0.02516\overline{a_z}} \text{ ; N.}$$

(or)

$$\boxed{\overline{F_A} = 30.75\overline{a_x} + 11.18\overline{a_y} - 25.16\overline{a_z}} \text{ mN}$$

iv. using $\epsilon_0 = 8.854 \times 10^{-12}$ F/m.

$$\overline{F_A} = \frac{(-20\mu)(50\mu)}{4\pi \times 8.854 \times 10^{-12} \times (\sqrt{218})^3} \left[-11\overline{a_x} - 4\overline{a_y} + 9\overline{a_z}\right]$$

$$\overline{F_A} = -2.792324 \times 10^{-3} \left[-11\overline{a_x} - 4\overline{a_y} + 9\overline{a_z}\right]$$

$$\overline{F_A} = 0.03071\overline{a_x} + 0.01116\overline{a_y} - 0.02513\overline{a_z}$$

$$\boxed{\overline{F_A} = 30.71\overline{a_x} + 11.16\overline{a_y} - 25.13\overline{a_z}} \text{ mN. (mN).}$$

obs: the value of Force Experienced in both the cases are approximately equal.

problem 4. A point charge of $Q_1 = 2\mu C$ is Located in free space at $P_1(-3,7,-4)$ while $Q_2 = 5\mu C$ is at $P_2(2,4,-1)$ m. Find $\overline{F_2}$ and $\overline{F_1}$.

Solu:

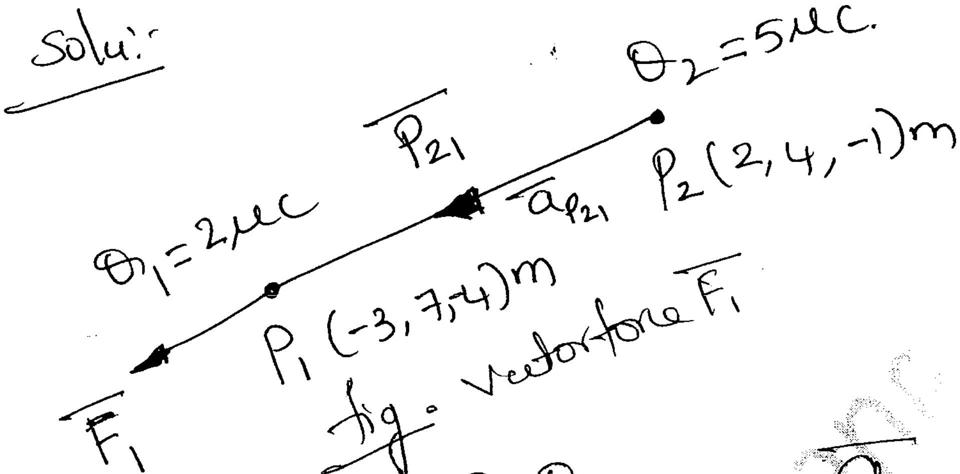

fig. vector force $\overline{F_1}$

i. $$\overline{F_1} = \frac{Q_1 Q_2}{4\pi\epsilon_0 |\overline{P_{21}}|^2}\overline{a_{P_{21}}} \text{ ; Newton}$$

$$\overline{F_1} = \frac{Q_1 Q_2}{4\pi\epsilon_0 |\overline{P_{21}}|^3}\overline{P_{21}} \text{. Newton}$$

$$\overline{P_{21}} = (-3-2)\overline{a_x} + (7-4)\overline{a_y} + (-4+1)\overline{a_z}$$

$$\overline{P_{21}} = -5\overline{a_x} + 3\overline{a_y} - 3\overline{a_z}$$

$$|\overline{P_{21}}| = \sqrt{25+9+9} = \sqrt{43}$$

$$|\overline{P_{21}}| = \sqrt{43} \text{ m}$$

$$\overline{F_1} = \frac{(2\mu)(5\mu)\times 9\times 10^{9}}{(\sqrt{43})^{3}}\left[-5\overline{a_x}+3\overline{a_y}-3\overline{a_z}\right]$$

$$\overline{F_1} = 319.18\times 10^{-6}\left[-5\overline{a_x}+3\overline{a_y}-3\overline{a_z}\right]$$

$$\boxed{\overline{F_1} = 1595.9\overline{a_x}+957.549\overline{a_y}-957.549\overline{a_z}}\ \mu N$$

$$\boxed{\overline{F_1} = 1.595\overline{a_x}+0.9575\overline{a_y}-0.9575\overline{a_z}}\ mN$$

ii. Force $\overline{F_2} = -\overline{F_1}$

$$\boxed{\overline{F_2} = -1.595\overline{a_x}-0.9575\overline{a_y}+0.9575\overline{a_z}}\ mN$$

$$|\overline{F_1}| = |\overline{F_2}| = \sqrt{1.595^2+0.9575^2+0.9575^2}\ mN$$

$$\boxed{|\overline{F_1}| = |\overline{F_2}| = 2.09228\, mN}$$

problem5. Point charge $Q_1 = 300\mu C$ Located at $(1,-1,-3)$ m experiences a force $\bar{F_1} = 8\bar{a}_x - 8\bar{a}_y + 4\bar{a}_z$ N due to point charge Q_2 at $(3,-3,-2)$ m. Determine Q_2.

solu:

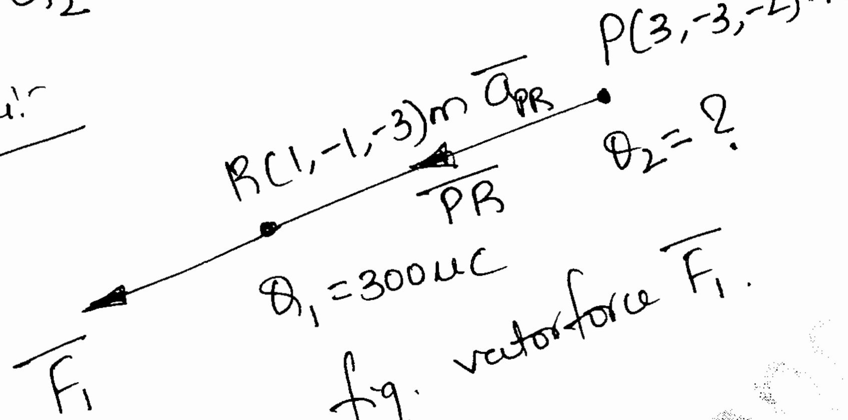

fig. vector force $\bar{F_1}$.

$$\bar{F_1} = \frac{Q_1 Q_2}{4\pi\epsilon_0 |\bar{PR}|^2}\bar{a}_{PR} \quad \text{Newton}$$

$$\bar{F_1} = \frac{Q_1 Q_2}{4\pi\epsilon_0 |\bar{PR}|^3}\bar{PR} \quad \text{Newton.}$$

$$\bar{PR} = (1-3)\bar{a}_x + (-1+3)\bar{a}_y + (-3+2)\bar{a}_z$$

$$\bar{PR} = -2\bar{a}_x + 2\bar{a}_y - \bar{a}_z$$

$$|\bar{PR}| = \sqrt{4+4+1} = \sqrt{9} = 3m$$

$$|\bar{PR}| = 3m$$

$$\bar{F}_1 = \frac{Q_1 Q_2}{4\pi\epsilon_0 (3)^3}\left[-2\bar{a}_x + 2\bar{a}_y - \bar{a}_z\right]$$

given $\bar{F}_1 = 8\bar{a}_x - 8\bar{a}_y + 4\bar{a}_z$: N

i.e

$$8\bar{a}_x - 8\bar{a}_y + 4\bar{a}_z = \frac{Q_1 Q_2}{4\pi\epsilon_0 (3)^3}\left[-2\bar{a}_x + 2\bar{a}_y - \bar{a}_z\right]$$

Equating x-component of Force [i.e F_x] on both sides

$$8 = \frac{Q_1 Q_2}{4\pi\epsilon_0 (27)}(-2)$$

$$8 = \frac{(300\mu)(Q_2)(9\times10^9)(-2)}{27}$$

$$8 = (-200\times10^3)\,Q_2$$

$$\Rightarrow Q_2 = -4\times10^{-5}\ C$$

(or) $\boxed{Q_2 = -40\,\mu C}$

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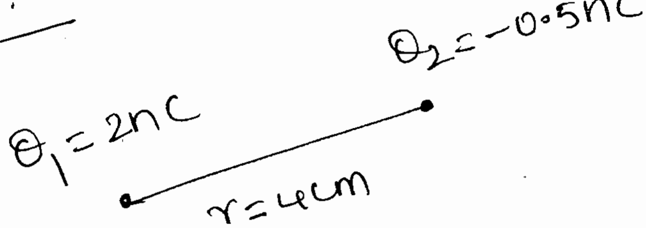
Problem 6. Two small identical conducting spheres have charges of 2×10^{-9} Coulomb and -0.5×10^{-9} C respectively. when they are placed 4cm apart, what is the force between them? if they are brough into contact and then separated by 4cm, what is the force between them.

solu:



$r = 4cm = 0.04m.$

the force between two conducting spheres is

$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}$$

$$F = \frac{(2n)(-0.5n)\times9\times10^9}{(0.04)^2}$$

$$F = -5.625\times10^{-6}\ N$$

$$\boxed{F = -5.625\ \mu N}$$

when these two conducting spheres are brought into contact and then separated, they are separated by equal amount.

the charge on each sphere is then

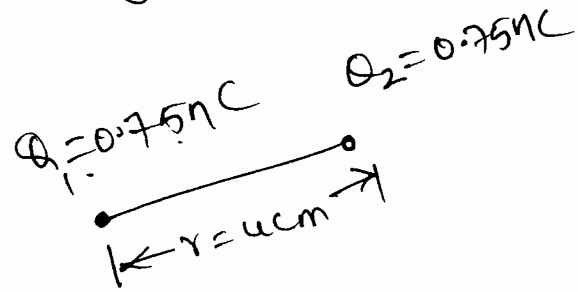
$$Q = \frac{Q_1 + Q_2}{2} = \frac{2\times10^{-9} - 0.5\times10^{-9}}{2}$$

$$Q = 0.75\times10^{-9}\ C$$

$$\boxed{Q = 0.75\,nC}$$

Now the desired force F when they are separated by 4cm apart will be

$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}$$



$$Q_1 = Q_2 = Q = 0.75nC$$

$$F = \frac{(0.75n)^2 \times 9\times10^9}{(0.04)^2}$$

$$\boxed{F = 3.1640\times10^{-6}}\ N.$$ (a)

1.1c Force due to N-number of point charges

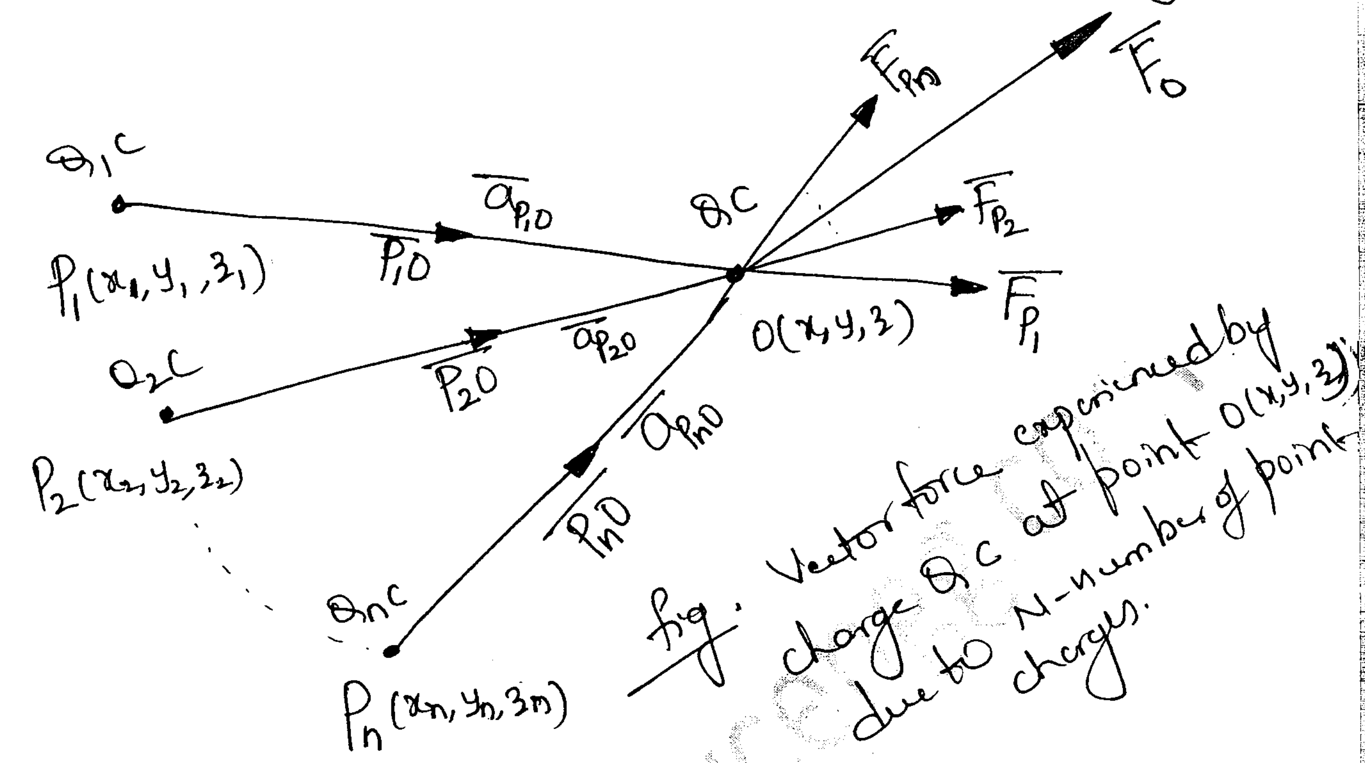


fig. Vector force experienced by charge Q C at point O(x,y,z) due to N-number of point charges.

Consider a N-number of point charges of $Q_1, Q_2, \ldots Q$ which are Located at a points $P_1, P_2, \ldots$ and P_n respectively.

the Force experienced by a point charge of Q C which is at point O(x, y, z) due to N-no of point charges Can be calculated using Superposition principle.

i.e

$$\overline{F_0} = \overline{F_{P_1}} + \overline{F_{P_2}} + \overline{F_{P_3}} + \cdots\cdots + \overline{F_{P_n}} \quad \text{Newton}$$

$$\overline{F_0} = \frac{Q_1 Q}{4\pi\epsilon |\overline{P_1 0}|^2}\overline{a_{P_1 0}} + \frac{Q_2 Q}{4\pi\epsilon |\overline{P_2 0}|^2}\overline{a_{P_2 0}} + \cdots + \frac{Q_n Q}{4\pi\epsilon |\overline{P_n 0}|^2}\overline{a_{P_n 0}}$$

$$\overline{F_0} = \frac{Q}{4\pi\epsilon}\left[Q_1 \frac{\overline{a_{P_1 0}}}{|\overline{P_1 0}|^2} + Q_2 \frac{\overline{a_{P_2 0}}}{|\overline{P_2 0}|^2} + \cdots + Q_n \frac{\overline{a_{P_n 0}}}{|\overline{P_n 0}|^2}\right]$$

$$\boxed{\overline{F_0} = \frac{Q}{4\pi\epsilon}\sum_{i=1}^{n} Q_i \frac{\overline{a_{P_i 0}}}{|\overline{P_i 0}|^2}} \text{ Newton}$$

if $Q_1 = Q_2 = \cdots = Q_n = Q$ C

$$\boxed{\overline{F_0} = \frac{Q^2}{4\pi\epsilon}\sum_{i=1}^{n} \frac{\overline{a_{P_i 0}}}{|\overline{P_i 0}|^2}} \text{ Newton.}$$

problem 7.

Two point charges Q_1 and Q_2 are located at (1, 2, 0)m and (2, 0, 0)m respectively. Find relation between the charges Q_1 and Q_2 such that the total force on a unit positive charge at (−1, 1, 0) have i) no x-component
ii) no y-component.

[10-Dec/Jan 2015 (8m)]

soln

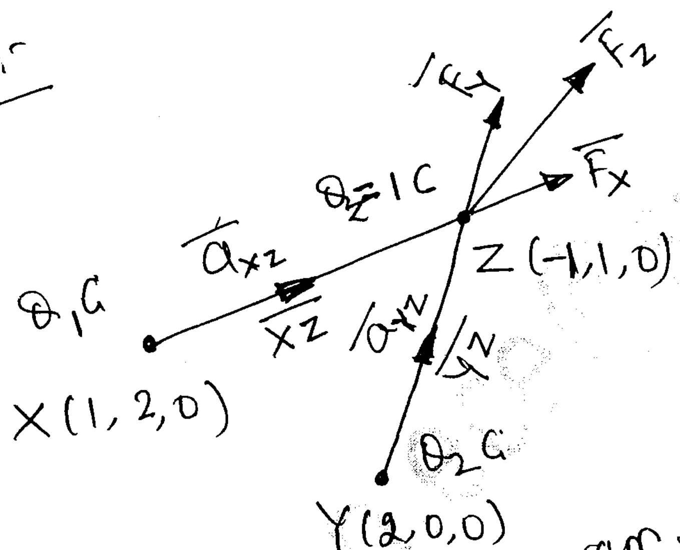


fig. vector diagram.

Assume Medium is freespace

$\boxed{\epsilon = \epsilon_0}$ F/m

using Superposition principle.

the net force experienced by charge of 1C at point Z(−1, 1, 0) due to Q_1 and Q_2 is given by

$$\boxed{\bar{F}_Z = \bar{F}_X + \bar{F}_Y} \text{ N}$$

$$\bar{F}_Z = \frac{Q_1}{4\pi\epsilon_0 |\overline{XZ}|^2}\bar{a}_{XZ} + \frac{Q_2}{4\pi\epsilon_0 |\overline{YZ}|^2}\bar{a}_{YZ} \;; N$$

$$\bar{a}_{XZ} = \frac{\overline{XZ}}{|\overline{XZ}|} \;; \quad \bar{a}_{YZ} = \frac{\overline{YZ}}{|\overline{YZ}|}$$

$$\bar{F}_Z = \frac{Q_1}{4\pi\epsilon_0 |\overline{XZ}|^3}\overline{XZ} + \frac{Q_2}{4\pi\epsilon_0 |\overline{YZ}|^3}\overline{YZ} \;; N$$

$$\overline{XZ} = -2\bar{a}_x - \bar{a}_y \;; \quad |\overline{XZ}| = \sqrt{4+1} = \sqrt{5}\, m.$$

$$\overline{YZ} = -3\bar{a}_x + \bar{a}_y \;; \quad |\overline{YZ}| = \sqrt{9+1} = \sqrt{10}\, m.$$

$$\bar{F}_Z = \frac{9\times10^9 Q_1}{(\sqrt{5})^3}\left[-2\bar{a}_x - \bar{a}_y\right] + \frac{9\times10^9 Q_2}{(\sqrt{10})^3}\left[-3\bar{a}_x + \bar{a}_y\right] ; N \quad \longleftarrow (a)$$

$$\bar{F}_Z = F_x\bar{a}_x + F_y\bar{a}_y + F_z\bar{a}_z ; N \longleftarrow (b)$$

i. Given condition.
Relationship between Q_1 and Q_2 such that the net force $\bar{F}_Z$ has NO-x component

i.e $\boxed{F_x = 0}$

by comparing eq^n (a) and eq^n (b), make F_x component is equal to zero. i.e $F_x = 0$.

$$\frac{9\times10^9\, Q_1(-2)}{(\sqrt{5})^3} + \frac{9\times10^9 (Q_2)(-3)}{(\sqrt{10})^3} = 0$$

$$\frac{\cancel{9\times10^9}\, Q_1(-2)}{(\sqrt{5})^3} = \frac{\cancel{9\times10^9}(Q_2)(3)}{(\sqrt{10})^3}$$

$$\Rightarrow \boxed{Q_1 = -0.5303\,Q_2}\ C.$$

ii. The resultant Force F_z has no y-component

i.e $F_y = 0$.

by comparing eq^n (a) and eq^n (b) make F_y component is equal to zero.

$$\frac{9\times10^9 (Q_1)(-1)}{(\sqrt{5})^3} + \frac{9\times10^9\, Q_2}{(\sqrt{10})^3} = 0$$

$$\frac{\cancel{9\times10^9}(Q_1)}{(\sqrt{5})^3} = \frac{\cancel{9\times10^9}(Q_2)}{(\sqrt{10})^2}$$

$$\Rightarrow \boxed{Q_1 = 0.3535\,Q_2}\ C.$$

The relationship between θ_1 and θ_2 such that $\overline{F}_Z$ has no 'x' component is

$$\boxed{\theta_1 = -0.5303\,\theta_2} \quad C$$

and. The $\overline{F}_Z$ has no 'y' component is

$$\boxed{\theta_1 = 0.3535\,\theta_2} \quad C$$

Solved problems:

problem 8. Point charges of 50nC each are located at A(1,0,0), B(−1,0,0), C(0,1,0) and D(0,−1,0) m. Find the total force on the charge at A and also find $\bar{E}$ at A. [10 - Jan 2013 (5M)].
CBCS: [15 - Dec|Jan - 2017 (8M)]

Solu:-

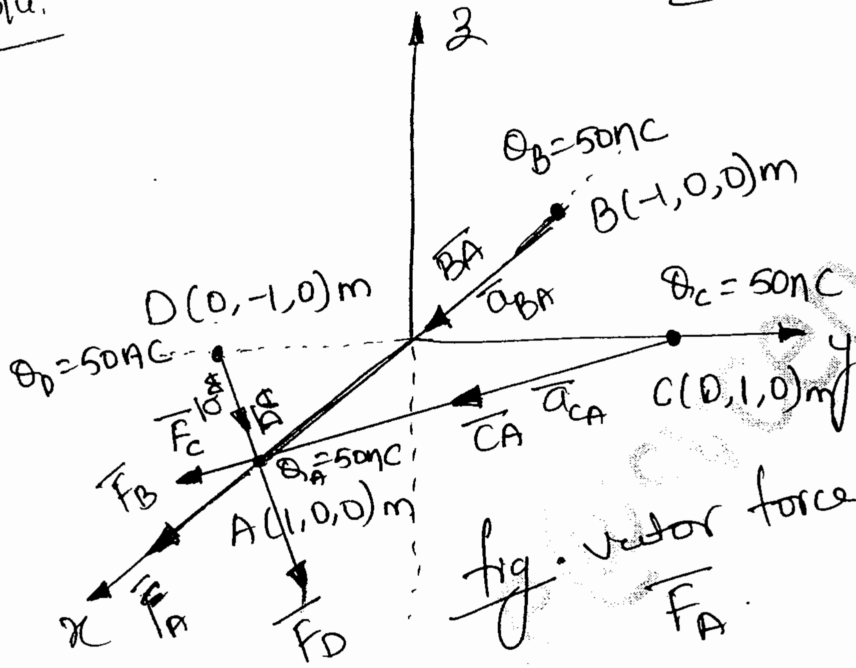


fig. vector force $\bar{F}_A$.

using Superposition principle the total force at point A due to point charges Q_B, Q_C and Q_D is given by

$$\bar{F}_A = \bar{F}_B + \bar{F}_C + \bar{F}_D \quad \text{Newton}$$

$$\bar{F}_A = \frac{Q_A Q_B}{4\pi\epsilon_0 |\overline{BA}|^2}\bar{a}_{BA} + \frac{Q_A Q_C}{4\pi\epsilon_0 |\overline{CA}|^2}\bar{a}_{CA} + \frac{Q_A Q_D}{4\pi\epsilon_0 |\overline{DA}|^2}\bar{a}_{DA} \; N$$

given $Q_A = Q_B = Q_C = Q_D = Q = 50nC$.

$$\bar{a}_{BA} = \frac{\overline{BA}}{|\overline{BA}|}; \quad \bar{a}_{CA} = \frac{\overline{CA}}{|\overline{CA}|}; \quad \bar{a}_{DA} = \frac{\overline{DA}}{|\overline{DA}|}$$

$$\therefore \overline{F_A} = \frac{Q^2}{4\pi\epsilon_0 |\overline{BA}|^3}\overline{BA} + \frac{Q^2}{4\pi\epsilon_0 |\overline{CA}|^3}\overline{CA} + \frac{Q^2}{4\pi\epsilon_0 |\overline{DA}|^3}\overline{DA} ; N$$

$$\overline{BA} = 2\overline{a_x} ; |\overline{BA}| = \sqrt{2^2} = 2m.$$

$$\overline{CA} = \overline{a_x} - \overline{a_y} ; |\overline{CA}| = \sqrt{1+1} = \sqrt{2} m.$$

$$\overline{DA} = \overline{a_x} + \overline{a_y} ; |\overline{DA}| = \sqrt{1+1} = \sqrt{2} m.$$

$$\overline{F_A} = \frac{Q^2}{4\pi\epsilon_0}\left[\frac{\overline{BA}}{|\overline{BA}|^3} + \frac{\overline{CA}}{|\overline{CA}|^3} + \frac{\overline{DA}}{|\overline{DA}|^3}\right] ; \text{Newton}$$

$$\overline{F_A} = (50\times10^{-9})^2(9\times10^9)\left[\frac{2\overline{a_x}}{2^3} + \frac{\overline{a_x}-\overline{a_y}}{(\sqrt{2})^3} + \frac{\overline{a_x}+\overline{a_y}}{(\sqrt{2})^3}\right]$$

$$\overline{F_A} = 22.5\times10^{-6}\left[0.25\overline{a_x} + \frac{2\overline{a_x}}{(\sqrt{2})^3}\right]$$

$$\overline{F_A} = 22.5\times10^{-6}\left[0.25\overline{a_x} + 0.7071\overline{a_x}\right]$$

$$\overline{F_A} = 22.5\times10^{-6}\left[0.9571\overline{a_x}\right]$$

$$\overline{F_A} = 21.5349\times10^{-6}\,\overline{a_x} \quad \text{Newton}$$

$$\boxed{\overline{F_A} = 21.5349\,\overline{a_x}\ \mu N}$$

$$F_x = 21.5349\ \mu N ; \quad F_y = 0N \quad \text{and } F_z = 0N.$$

$$|\overline{F_A}| = F_x = 21.5349\,\mu N.$$

ii. the Electric field Intensity ($\bar{E}$) at a point A

is $\bar{E}_A = \frac{\bar{F}_A}{Q_A} = \frac{21.5349 \times 10^{-6} \bar{a}_x}{50 \times 10^{-9}}$

$$\boxed{\bar{E}_A = 430.698\,\bar{a}_x} \text{ V/m}$$

$E_x = 430.698$ V/m ; $E_y = 0$ V/m ; $E_z = 0$ V/m.

$|\bar{E}_A| = E_x = 430.698$ V/m

problem 9. Find the Force on $100\mu C$ charge at $(0,0,3)m$. if Four like charges of $20\mu C$ are Located on the x-axis and y-axis at $\pm 4m$.

Sol^n:-

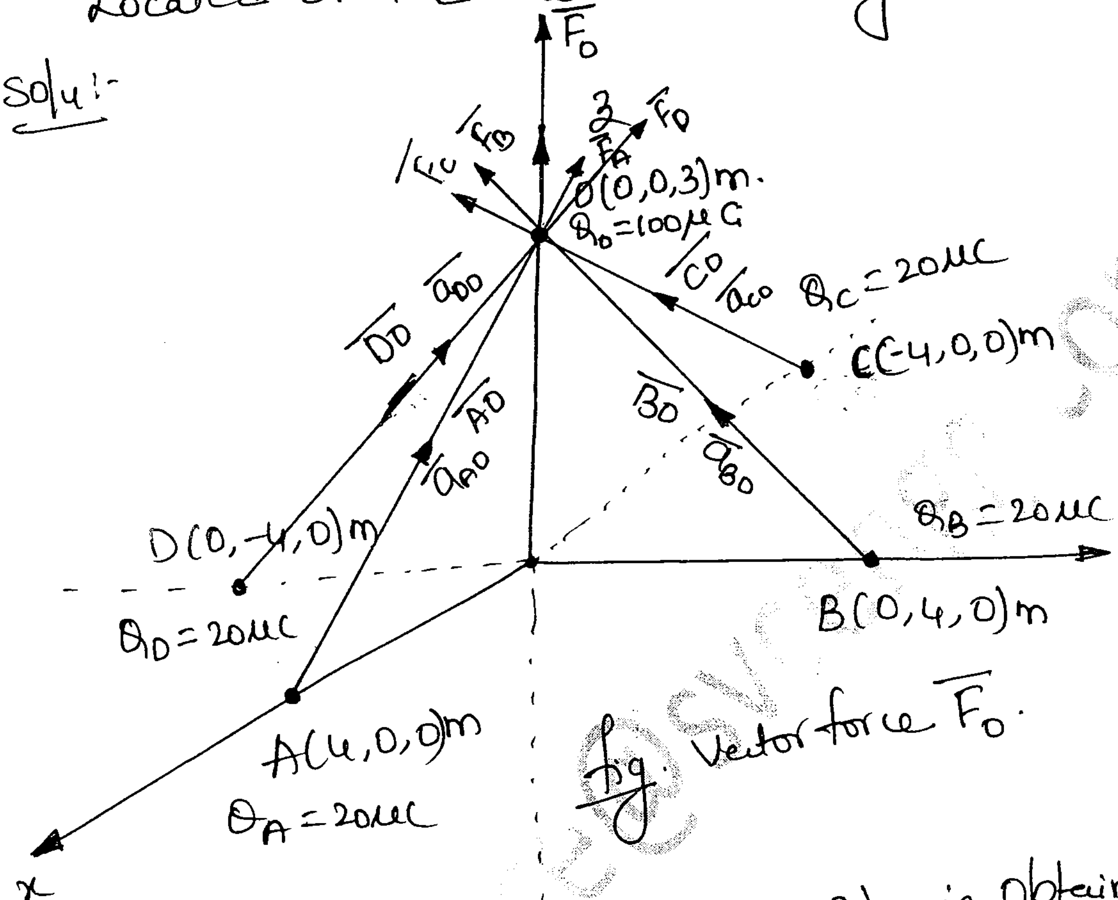


fig. Vector force $\overline{F_O}$.

the net force at point O(0,0,3)m is obtained by using Superposition principle

$$\overline{F_O}=\overline{F_A}+\overline{F_B}+\overline{F_C}+\overline{F_D} \quad \text{Newton.}$$

$$\overline{F_O}=\frac{Q_0Q_A}{4\pi\epsilon_0|\overline{AO}|^2}\overline{a_{AO}}+\frac{Q_0Q_B}{4\pi\epsilon_0|\overline{BO}|^2}\overline{a_{BO}}+\frac{Q_0Q_C}{4\pi\epsilon_0|\overline{CO}|^2}\overline{a_{CO}}+\frac{Q_0Q_D}{4\pi\epsilon_0|\overline{DO}|^2}\overline{a_{DO}}\ ;N$$

Since given $Q_A=Q_B=Q_C=Q_D=Q=20\mu C$.

$$\overline{a_{AO}}=\frac{\overline{AO}}{|\overline{AO}|}\ ;\ \overline{a_{BO}}=\frac{\overline{BO}}{|\overline{BO}|}\ ;\ \overline{a_{CO}}=\frac{\overline{CO}}{|\overline{CO}|}\ ;\ \overline{a_{DO}}=\frac{\overline{DO}}{|\overline{DO}|}$$

$$\overline{F}_0 = \frac{Q_0 Q}{4\pi\epsilon_0}\left[\frac{\overline{AO}}{|\overline{AO}|^3} + \frac{\overline{BO}}{|\overline{BO}|^3} + \frac{\overline{CO}}{|\overline{CO}|^3} + \frac{\overline{DO}}{|\overline{DO}|^3}\right]$$

$\overline{AO} = -4\overline{a}_x + 3\overline{a}_z$; $|\overline{AO}| = \sqrt{16+9} = 5m.$

$\overline{BO} = -4\overline{a}_y + 3\overline{a}_z$; $|\overline{BO}| = \sqrt{16+9} = 5m.$

$\overline{CO} = 4\overline{a}_x + 3\overline{a}_z$; $|\overline{CO}| = \sqrt{16+9} = 5m$

$\overline{DO} = 4\overline{a}_y + 3\overline{a}_z$; $|\overline{DO}| = \sqrt{16+9} = 5m.$

$|\overline{AO}| = |\overline{BO}| = |\overline{CO}| = |\overline{DO}| = 5\ m.$

$$\overline{F}_0 = \frac{Q_0 Q}{4\pi\epsilon_0|\overline{AO}|^3}\left[\overline{AO} + \overline{BO} + \overline{CO} + \overline{DO}\right]$$

$$\overline{F}_0 = \frac{100\mu \times 20\mu \times 9\times10^9}{5^3}\left[\cancel{-4\overline{a}_x} + 3\overline{a}_z \cancel{-4\overline{a}_y} + 3\overline{a}_z \cancel{+4\overline{a}_x} + 3\overline{a}_z \cancel{+4\overline{a}_y} + 3\overline{a}_z\right]$$

$$\overline{F}_0 = 0.144\,[12\overline{a}_z]$$

$$\boxed{\overline{F}_0 = 1.728\,\overline{a}_z} \quad \text{Newton}$$

$F_x = 0\ N$; $F_y = 0\ N$ and $F_z = 1.728\ N.$

$|\overline{F}_0| = F_z = 1.728\ N.$

obs: The net force along x and y direction is zero, because of charges of equal values placed over a equidistance along +ve and -ve x, y axes.

problem 10.

Eight point charges of Q C each are located at the corners of a cube of side length 'a' m with one charge at origin and with three nearest charges at (a,0,0) m, (0,a,0) m and (0,0,a) m. Find an expression for the total vector force on the charge at P(a,a,a) m assuming freespace.

[W. H. Hayt].

Sol:

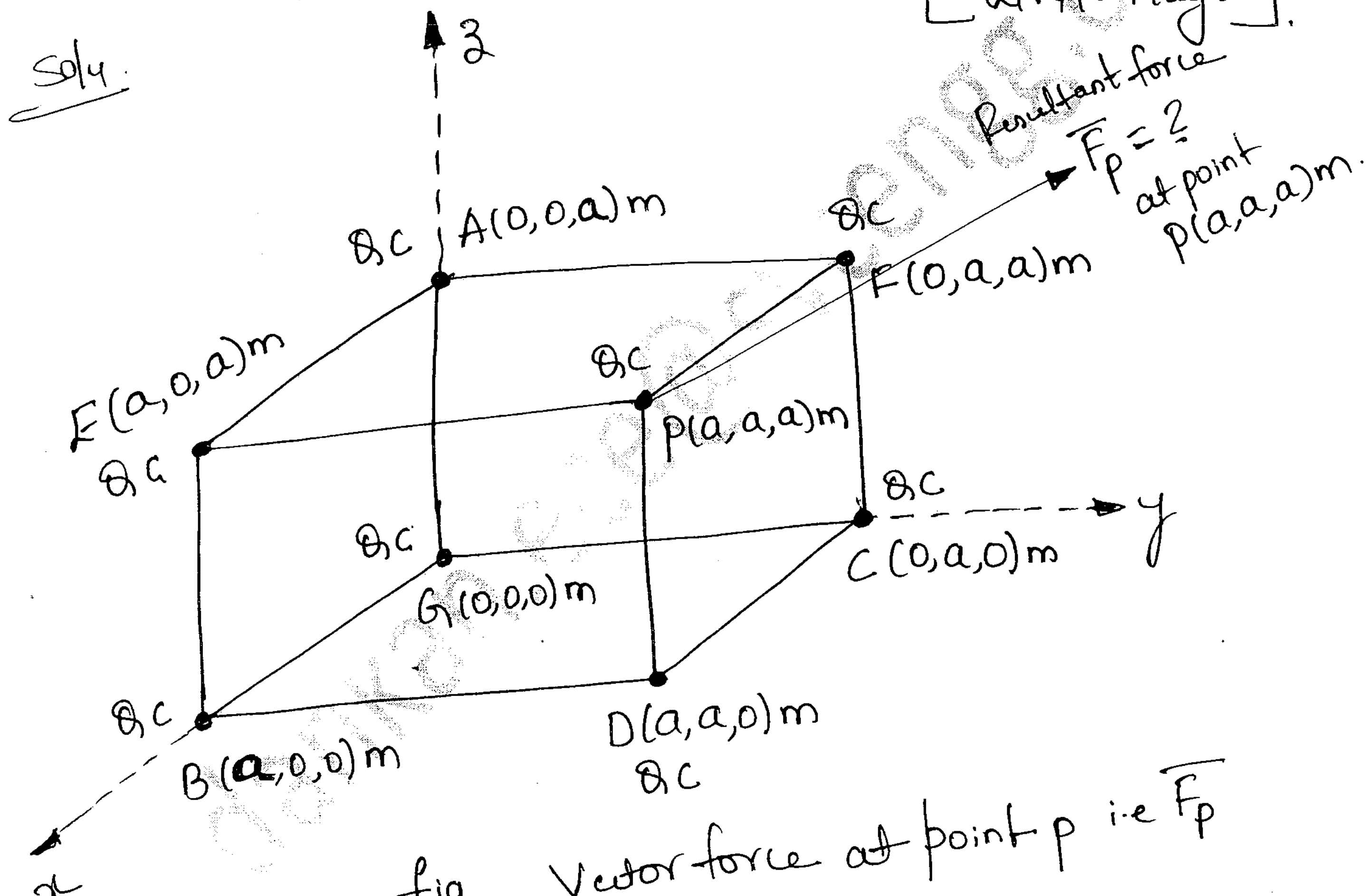


fig. Vector force at point p i.e $\overline{F_p}$

Using Superposition principle, the net force at point P is given by

$$\overline{F_p} = \overline{F_A} + \overline{F_B} + \overline{F_C} + \overline{F_D} + \overline{F_E} + \overline{F_F} + \overline{F_G} \; ; N$$

Since give all charges of equal value i.e Q

$$\overline{F_p} = \frac{Q^2}{4\pi\epsilon_0}\left[\frac{\overline{a_{Ap}}}{|\overline{Ap}|^2} + \frac{\overline{a_{Bp}}}{|\overline{Bp}|^2} + \frac{\overline{a_{Cp}}}{|\overline{Cp}|^2} + \frac{\overline{a_{Dp}}}{|\overline{Dp}|^2} + \frac{\overline{a_{Ep}}}{|\overline{Ep}|^2} + \frac{\overline{a_{Fp}}}{|\overline{Fp}|^2} + \frac{\overline{a_{Gp}}}{|\overline{Gp}|^2}\right]$$

$\overline{Ap} = a\overline{a_x} + a\overline{a_y}$; $|\overline{Ap}| = \sqrt{2}\,a\,m$; $\overline{a_{Ap}} = \frac{\overline{Ap}}{|\overline{Ap}|}$

$\overline{Bp} = a\overline{a_y} + a\overline{a_z}$; $|\overline{Bp}| = \sqrt{2}\,a\,m$; $\overline{a_{Bp}} = \frac{\overline{Bp}}{|\overline{Bp}|}$

$\overline{Cp} = a\overline{a_x} + a\overline{a_z}$; $|\overline{Cp}| = \sqrt{2}\,a\,m$; $\overline{a_{Cp}} = \frac{\overline{Cp}}{|\overline{Cp}|}$

$\overline{Dp} = a\overline{a_z}$; $|\overline{Dp}| = a\,m$; $\overline{a_{Dp}} = \frac{\overline{Dp}}{|\overline{Dp}|}$

$\overline{Ep} = a\overline{a_y}$; $|\overline{Ep}| = a\,m$; $\overline{a_{Ep}} = \frac{\overline{Ep}}{|\overline{Ep}|}$

$\overline{Fp} = a\overline{a_x}$; $|\overline{Fp}| = a\,m$; $\overline{a_{Fp}} = \frac{\overline{Fp}}{|\overline{Fp}|}$

$\overline{Gp} = a\overline{a_x} + a\overline{a_y} + a\overline{a_z}$; $|\overline{Gp}| = \sqrt{3}\,a\,m$; $\overline{a_{Gp}} = \frac{\overline{Gp}}{|\overline{Gp}|}$

Obs:- $|\overline{Ap}| = |\overline{Bp}| = |\overline{Cp}|$ and $|\overline{Dp}| = |\overline{Ep}| = |\overline{Fp}|$

$$\therefore \overline{F_p} = \frac{Q^2}{4\pi\epsilon_0}\left[\frac{\overline{Ap}}{|\overline{Ap}|^3} + \frac{\overline{Bp}}{|\overline{Bp}|^3} + \frac{\overline{Cp}}{|\overline{Cp}|^3} + \frac{\overline{Dp}}{|\overline{Dp}|^3} + \frac{\overline{Ep}}{|\overline{Ep}|^3} + \frac{\overline{Fp}}{|\overline{Fp}|^3} + \frac{\overline{Gp}}{|\overline{Gp}|^3}\right]$$

$$\overline{F_p} = \frac{Q^2}{4\pi\epsilon_0}\left[\frac{(\overline{Ap} + \overline{Bp} + \overline{Cp})}{|\overline{Ap}|^3} + \frac{(\overline{Dp} + \overline{Ep} + \overline{Fp})}{|\overline{Dp}|^3} + \frac{\overline{Gp}}{|\overline{Gp}|^3}\right]$$

$$|\overline{Ap}| = \sqrt{2}\, a\ m \ ; \ |\overline{Ap}|^3 = (\sqrt{2}a)^3 = 2^{3/2} a^3$$

$$|\overline{Dp}| = a\ m \ ; \ |\overline{Dp}|^3 = a^3.$$

$$|\overline{Gp}| = \sqrt{3}\, a\ m \ ; \ |\overline{Gp}|^3 = 3^{3/2} a^3.$$

$$\overline{F}_p = \frac{Q^2}{4\pi\epsilon_0}\left[\frac{1}{2^{3/2}a^3}\left[a\overline{a}_x + a\overline{a}_y + a\overline{a}_y + a\overline{a}_z + a\overline{a}_x + a\overline{a}_z\right] + \frac{1}{a^3}(a\overline{a}_z + a\overline{a}_y + a\overline{a}_x) + \frac{1}{3^{3/2}a^3}(a\overline{a}_x + a\overline{a}_y + a\overline{a}_z)\right]$$

$$= \frac{Q^2}{4\pi\epsilon_0}\left[\frac{1}{2^{3/2}a^3} \times a(2\overline{a}_x + 2\overline{a}_y + 2\overline{a}_z) + \frac{1}{a^3}\cdot a(\overline{a}_x + \overline{a}_y + \overline{a}_z) + \frac{1}{3^{3/2}a^3} a(\overline{a}_x + \overline{a}_y + \overline{a}_z)\right]$$

$$= \frac{Q^2}{4\pi\epsilon_0 a^2}\left[1.8995\overline{a}_x + 1.8995\overline{a}_y + 1.8995\overline{a}_z\right]$$

$$\overline{F}_p = \frac{1.8995 Q^2}{4\pi\epsilon_0 a^2}\left[\overline{a}_x + \overline{a}_y + \overline{a}_z\right]$$

$$\boxed{\overline{F}_p \simeq \frac{1.9 Q^2}{4\pi\epsilon_0 a^2}\left[\overline{a}_x + \overline{a}_y + \overline{a}_z\right]}$$ Newton.

$$\boxed{|\overline{F}_p| = \frac{3.29 Q^2}{4\pi\epsilon_0 a^2}}$$ Newton

Problem 11.

Four 10nC positive charges are Located in the z=0 plane at the corners of a square of side 8cm. A fifth 10nC positive charge is Located at a point 8cm distant from the other charges. Calculate the Magnitude of the force on the fifth charge in free space.

[W. H. Hayt | 06 - Dec/Jan 2014 (7M)]

Soln:

Notes:- Convert the distance from cm to meter(m).

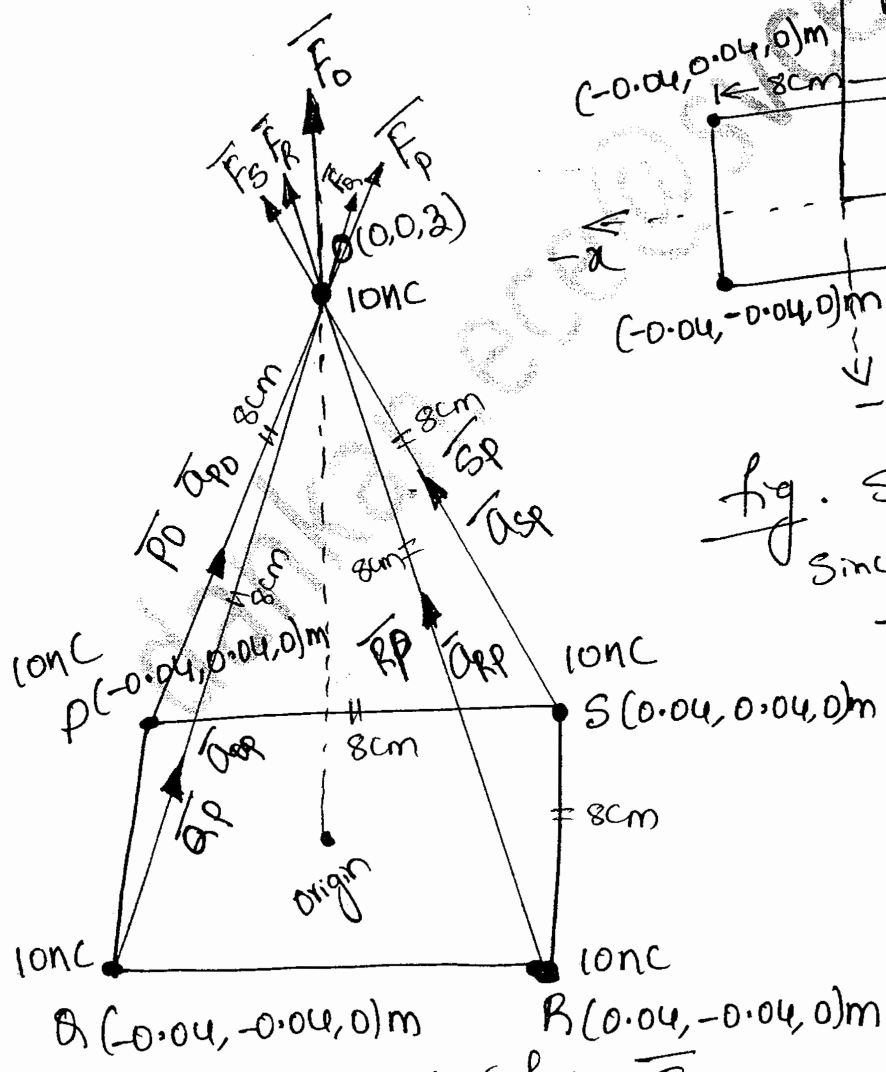


fig. Vector force $\bar{F}_0$.

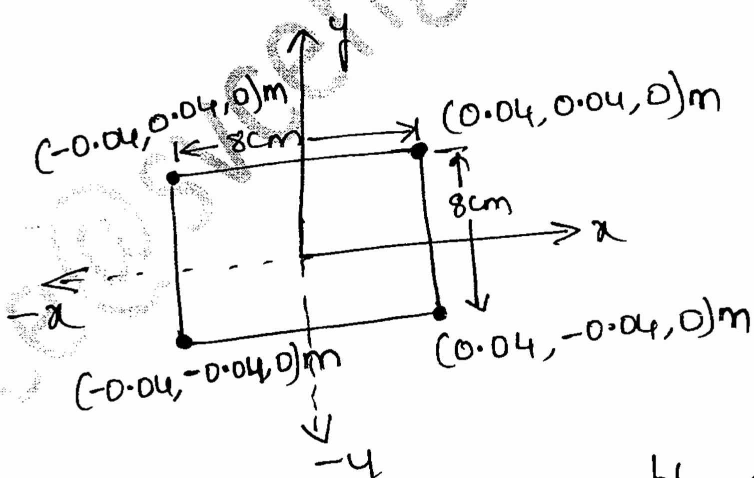


fig. Square in xy plane

Since it is in xy plane the value of z = 0.

the net force at a point $O(0,0,z)$ due to four point charges can be calculated using Superposition principle.

i.e $\boxed{\overline{F}_O = \overline{F}_P + \overline{F}_Q + \overline{F}_R + \overline{F}_S}$: Newton

To find 'z' value on z axis use distance formula

let $|\overline{PO}| = \sqrt{(0+0.04)^2 + (0-0.04)^2 + (z-0)^2} = 8cm = 0.08$

$$0.08^2 = 0.04^2 + 0.04^2 + z^2$$

$$\boxed{z = \pm 0.0565}$$

Since point $O(0,0,z)$ is on +ve z axis ∴

choose $\boxed{z = 0.0565 \text{ m}}$

∴ point $O(0,0,z) = O(0,0,0.0565)$m.

$$\overline{F}_O = \frac{Q_P Q_O}{4\pi\epsilon_0|\overline{PO}|^2}\overline{a}_{PO} + \frac{Q_R Q_O}{4\pi\epsilon_0|\overline{RO}|^2}\overline{a}_{RO} + \frac{Q_Q Q_O}{4\pi\epsilon_0|\overline{QO}|^2}\overline{a}_{QO}$$

$$+ \frac{Q_S Q_O}{4\pi\epsilon_0|\overline{SO}|^2}\overline{a}_{SO} \quad : N$$

Since $Q_P = Q_S = Q_R = Q_Q = Q = 10\times10^{-9}$ C

$$\overline{F}_O = \frac{Q^2}{4\pi\epsilon_0}\left[\frac{\overline{a}_{PO}}{|\overline{PO}|^2} + \frac{\overline{a}_{RO}}{|\overline{RO}|^2} + \frac{\overline{a}_{QO}}{|\overline{QO}|^2} + \frac{\overline{a}_{SO}}{|\overline{SO}|^2}\right] : N$$

$$\overline{F}_0 = \frac{Q^2}{4\pi\epsilon_0}\left[\frac{\overline{PO}}{|\overline{PO}|^3} + \frac{\overline{RO}}{|\overline{RO}|^3} + \frac{\overline{QO}}{|\overline{QO}|^3} + \frac{\overline{SO}}{|\overline{SO}|^3}\right].$$

$\overline{PO} = 0.04\,\overline{a}_x - 0.04\,\overline{a}_y + 0.0565\,\overline{a}_z$; $|\overline{PO}| = 0.07995\,m$

$\overline{RO} = -0.04\,\overline{a}_x + 0.04\,\overline{a}_y + 0.0565\,\overline{a}_z$; $|\overline{RO}| = 0.07995\,m$

$\overline{QO} = 0.04\,\overline{a}_x + 0.04\,\overline{a}_y + 0.0565\,\overline{a}_z$; $|\overline{QO}| = 0.07995\,m$

$\overline{SO} = -0.04\,\overline{a}_x - 0.04\,\overline{a}_y + 0.0565\,\overline{a}_z$; $|\overline{SO}| = 0.07995\,m.$

Since $|\overline{PO}| = |\overline{RO}| = |\overline{QO}| = |\overline{SO}| = 0.07995\,m.$

$$\overline{F}_0 = \frac{(10\times10^{-9})^2(9\times10^9)}{(0.07995)^3}\left[\begin{aligned}&\cancel{0.04\,\overline{a}_x} - \cancel{0.04\,\overline{a}_y} + 0.0565\,\overline{a}_z\\ &-\cancel{0.04\,\overline{a}_x} + \cancel{0.04\,\overline{a}_y} + 0.0565\,\overline{a}_z\\ &+\cancel{0.04\,\overline{a}_x} + \cancel{0.04\,\overline{a}_y} + 0.0565\,\overline{a}_z\\ &-\cancel{0.04\,\overline{a}_x} - \cancel{0.04\,\overline{a}_y} + 0.0565\,\overline{a}_z\end{aligned}\right]$$

$$\overline{F}_0 = \frac{(10\times10^{-9})^2(9\times10^9)\,4\times0.0565\,\overline{a}_z}{(0.07995)^3}$$

$$\boxed{\overline{F}_0 = 3.98014\times10^{-4}\,\overline{a}_z \simeq 4\times10^{-4}\,\overline{a}_z} \text{ Newton}$$

$$\overline{F}_0 = 4\times10^{-4}\,\overline{a}_z \text{ Newton}$$

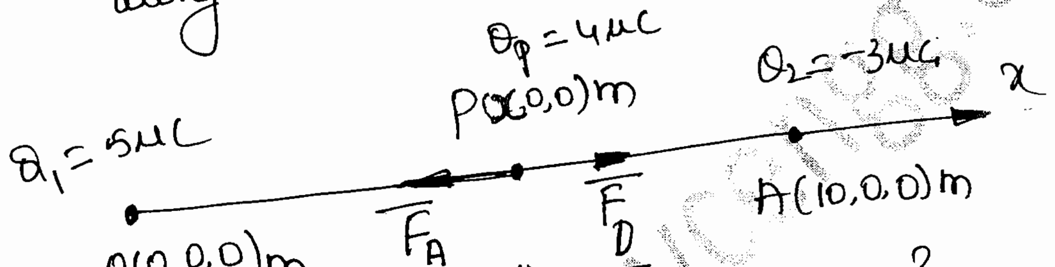
Obs:- Since all charges of same value and they Located in Symmetrical manner with equi-distance along +ve and -ve x, y axes, the net force along x and y direction is zero.

problem 12.

Two point charges of 5 and −3 μC are placed along straight line 10m apart. Determine the location of third charge of 4μC such that it is subjected to no force. [10 - June/July - 2015 (6m) EEE]

solu:

assume that all point charges are located along x-axis



given $\overline{F_p} = 0N$ @ $x = ?$

using superposition principle

$$\overline{F_p} = \overline{F_D} + \overline{F_A} \quad N$$

$$\overline{F_p} = \frac{Q_1 Q_p}{4\pi\epsilon_0 |\overline{OP}|^2}\overline{a_{op}} + \frac{Q_2 Q_p}{4\pi\epsilon_0 |\overline{AP}|^2}\overline{a_{Ap}} \; ; N$$

$$\overline{OP} = x\overline{a_x} \; ; \; |\overline{OP}| = \sqrt{x^2} = x\,m. \quad |\overline{OP}|^2 = x^2$$

$$\overline{AP} = (x-10)\;\overline{a_x} \; ; \; |\overline{AP}| = \sqrt{(x-10)^2} = (x-10)m;$$

$$|\overline{AP}|^2 = (x-10)^2$$

$$\overline{a_{op}} = \frac{\overline{OP}}{|\overline{OP}|} \; ; \; \overline{a_{Ap}} = \frac{\overline{AP}}{|\overline{AP}|}$$

$$\overline{F_p} = \frac{Q_1 Q_p}{4\pi\epsilon_0} \frac{\overline{OP}}{|\overline{OP}|^3} + \frac{Q_2 Q_p}{4\pi\epsilon_0 |\overline{AP}|^3} \overline{AP} \; : N$$

$$\overline{F_p} = \frac{Q_1 Q_p}{4\pi\epsilon_0} \frac{\cancel{x}\,\overline{a_x}}{x^{\cancel{3}\,2}} + \frac{Q_2 Q_p}{4\pi\epsilon_0} \frac{\cancel{(x-10)}\,\overline{a_x}}{(x-10)^{\cancel{3}\,2}} \; : N$$

$$\overline{F_p} = \left[\frac{Q_1 Q_p}{4\pi\epsilon_0 x^2} + \frac{Q_2 Q_p}{4\pi\epsilon_0 (x-10)^2}\right] \overline{a_x} \longleftarrow (1)$$

given that the force experienced by the third charge is zero.

ie $|\overline{F_p}| = F_x = 0$.

from eq^n (1)

$$\frac{Q_1 Q_p}{4\pi\epsilon_0 x^2} + \frac{Q_2 Q_p}{4\pi\epsilon_0 (x-10)^2} = 0$$

$$\frac{Q_1 \cancel{Q_p}}{\cancel{4\pi\epsilon_0} x^2} = \frac{-Q_2 \cancel{Q_p}}{\cancel{4\pi\epsilon_0}(x-10)^2}$$

$$\frac{5\cancel{\mu}}{x^2} = \frac{-(-3\cancel{\mu})}{(x-10)^2}$$

$$\Rightarrow \quad 5(x-10)^2 = +3x^2$$

$$5\left[x^2 + 100 - 20x\right] - 3x^2 = 0$$

$$5x^2 + 500 - 100x - 3x^2 = 0$$

$$2x^2 - 100x + 500 = 0.$$

$$\Rightarrow \boxed{x = 44.3649\,m} \text{ and } \boxed{x = 5.63508\,m}$$

$\therefore$ the points at which the force experienced by third charge is to be zero is

$P_1(44.3649, 0, 0)\,m$ and $P_2(5.63508, 0, 0)\,m$.

problem 13

Q_1 and Q_2 are the point charges Located at (0, −4, 3) and (0,1,1)m. If Q_1 is 2μC. Find Q_2 Such that the force on a test charge at (0,−3,4) has no Z-component.

Soln:-

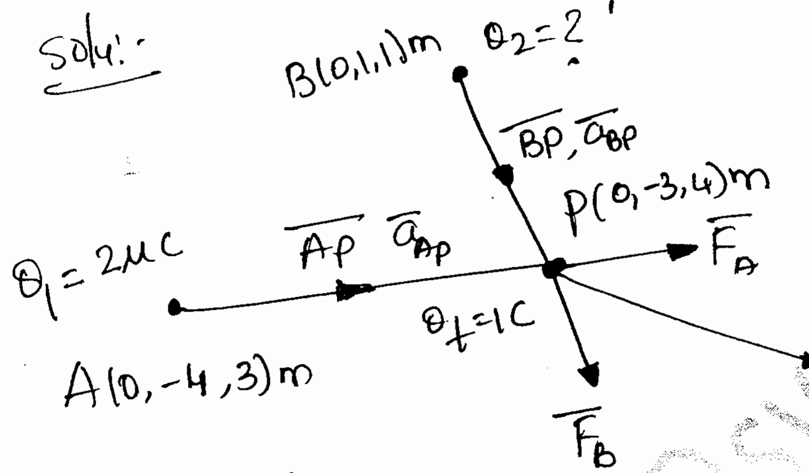


fig. Vectorforce $\overline{F}_p$

the force experience by a test charge at point P(0,−3,4)m is calculated by using Superposition principle.

i.e $\overline{F}_p = \overline{F}_A + \overline{F}_B$ Newton

$$\overline{F}_p = \frac{Q_1 Q_t}{4\pi\epsilon_0 |\overline{AP}|^2}\,\overline{a}_{AP} + \frac{Q_2 Q_t}{4\pi\epsilon_0 |\overline{BP}|^2}\,\overline{a}_{BP} \; : N$$

$\overline{AP} = \overline{a}_y + \overline{a}_z$; $|\overline{AP}| = \sqrt{2}$ m; $\overline{a}_{AP} = \frac{\overline{AP}}{|\overline{AP}|}$

$\overline{BP} = -4\overline{a}_y + 3\overline{a}_z$; $|\overline{BP}| = 5$m; $\overline{a}_{BP} = \frac{\overline{BP}}{|\overline{BP}|}$

$$\overline{F_p} = \frac{Q_1}{4\pi\epsilon_0} \frac{\overline{AP}}{|\overline{AP}|^3} + \frac{Q_2}{4\pi\epsilon_0} \frac{\overline{BP}}{|\overline{BP}|^3} \; ; N$$

$$\overline{F_p} = \frac{(2\mu)(9\times10^9)}{(\sqrt{2})^3}\left[\overline{a_y}+\overline{a_z}\right] + \frac{Q_2(9\times10^9)}{(5)^3}\left[-4\overline{a_y}+3\overline{a_z}\right] \quad \text{(1)}$$

To Find Q_2 the force on test-charge has No-'z' component.

i.e $F_z = 0N$

from eq^n (1) ; the F_z component is

$$\left\{ \frac{2\mu(9\times10^9)}{(\sqrt{2})^3} + \frac{Q_2(9\times10^9)}{(5)^3}(3) \right\} = 0$$

$$\frac{2\mu\cancel{(9\times10^9)}}{(\sqrt{2})^3} = -\frac{Q_2\cancel{(9\times10^9)}}{5^3}\times 3$$

$$Q_2 = \frac{[-2\mu\times5^3]}{3\times(\sqrt{2})^3} = -29.462\mu C$$

The value of Q_2 such that the force experienced by the test charge at point P [ie $\overline{F_p}$] has No 'z' component is $\boxed{Q_2 = -29.462}\,\mu C$

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Problem 14.

A charge $Q_1 = -20\mu C$ is Located at $A(-6,4,6)$ and a charge $Q_2 = 50\mu C$ is Located at $B(5,8,-2)$m in freespace. Find the force exerted on Q_2 by Q_1 in vector form. [10-Dec/Jan 2015 - EEE (6m)]

Soln:-

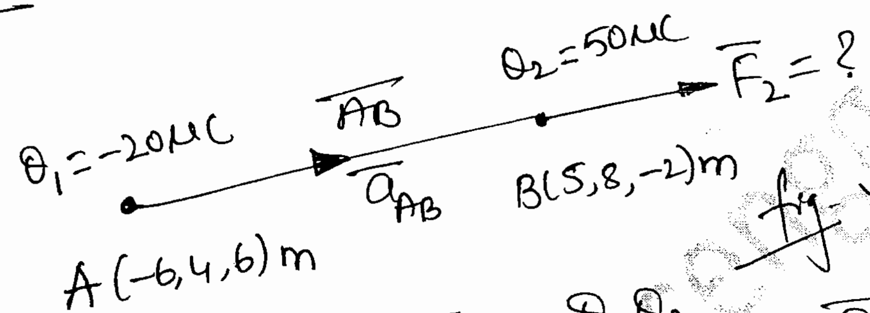


fig. vector force F_2.

the Force $\bar{F}_2 = \dfrac{Q_1 Q_2}{4\pi\epsilon_0 |\overline{AB}|^2} \bar{a}_{AB}$ N.

$\overline{AB} = (5+6)\bar{a}_x + (8-4)\bar{a}_y + (-2-6)\bar{a}_z$; $\bar{a}_{AB} = \dfrac{\overline{AB}}{|\overline{AB}|}$

$\overline{AB} = 11\bar{a}_x + 4\bar{a}_y - 8\bar{a}_z$; $|\overline{AB}| = \sqrt{201}$ m.

$$\bar{F}_2 = \frac{(-20\mu)(50\mu)(9\times10^9)}{(\sqrt{201})^3}\left[11\bar{a}_x + 4\bar{a}_y - 8\bar{a}_z\right]$$

$$\boxed{\bar{F}_2 = -0.03474\,\bar{a}_x - 0.01263\,\bar{a}_y + 0.02526\,\bar{a}_z} \text{ N.}$$

$$|\bar{F}_2| = \sqrt{(-0.03474)^2 + (-0.01263)^2 + (0.02526)^2}$$

$$\boxed{|\bar{F}_2| = 0.04477} \text{ Newton.}$$

problem 15.

Four point charges each of 10μC are placed in freespace at the points (1,0,0), (−1,0,0), (0,1,0) and (0,−1,0) m respectively. Determine the force on a point charge of 20μC located at a point (0,0,1) m.

Solu:-

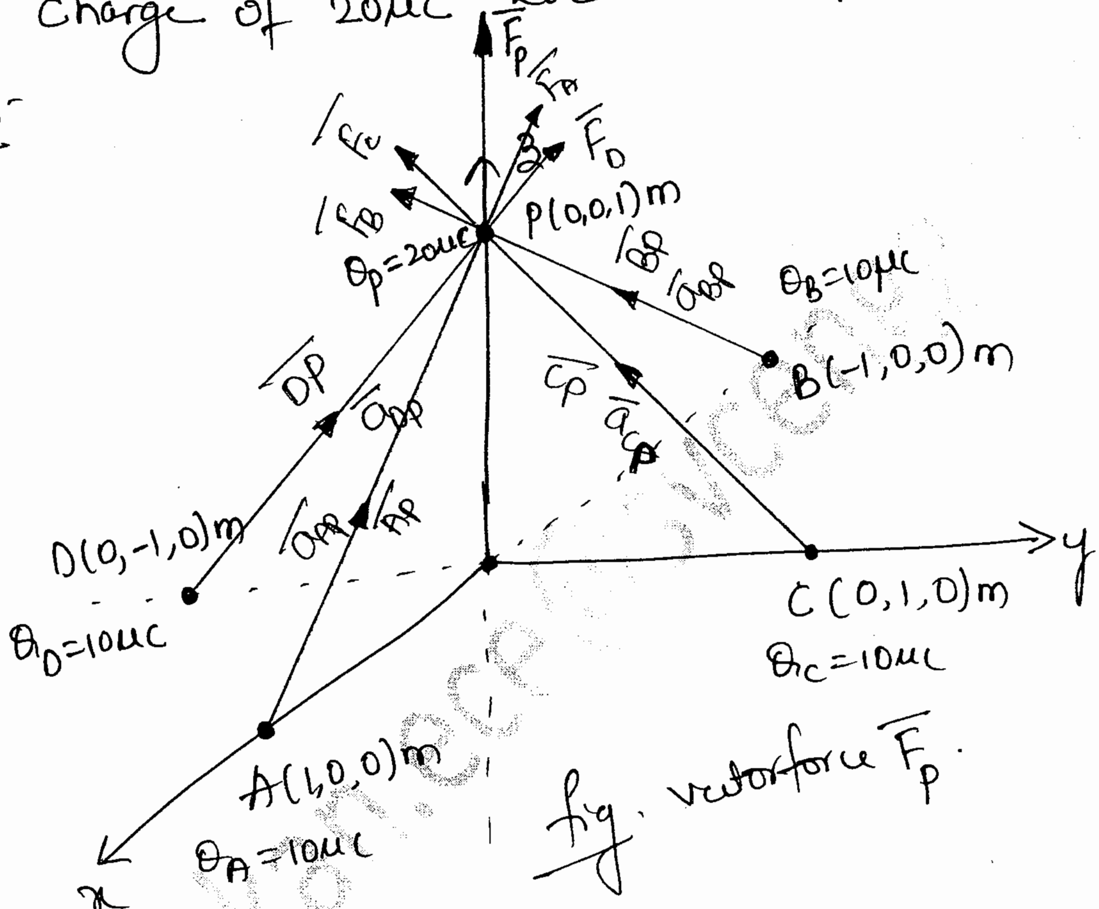


fig. vectorforce F_p.

The net force at point p is calculating using Superposition principle.

ie $\bar{F}_P = \bar{F}_A + \bar{F}_B + \bar{F}_C + \bar{F}_D$: N.

given $Q_A = Q_B = Q_C = Q_D = Q = 10\mu C$

$$\overline{F_p} = \frac{Q_1 Q_p}{4\pi\epsilon_0}\left[\frac{\overline{a_{Ap}}}{|\overline{Ap}|^2} + \frac{\overline{a_{Bp}}}{|\overline{Bp}|^2} + \frac{\overline{a_{cp}}}{|\overline{Cp}|^2} + \frac{\overline{a_{Dp}}}{|\overline{Dp}|^2}\right]$$

$$\overline{F_p} = \frac{Q_1 Q_p}{4\pi\epsilon_0}\left[\frac{\overline{Ap}}{|\overline{Ap}|^3} + \frac{\overline{Bp}}{|\overline{Bp}|^3} + \frac{\overline{Cp}}{|\overline{Cp}|^3} + \frac{\overline{Dp}}{|\overline{Dp}|^3}\right] \text{ N}$$

$\overline{Ap} = -\overline{a_x} + \overline{a_z}$; $|\overline{Ap}| = \sqrt{2}$ m.

$\overline{Bp} = \overline{a_x} + \overline{a_z}$; $|\overline{Bp}| = \sqrt{2}$ m.

$\overline{Cp} = -\overline{a_y} + \overline{a_z}$; $|\overline{Cp}| = \sqrt{2}$ m.

$\overline{Dp} = \overline{a_y} + \overline{a_z}$; $|\overline{Dp}| = \sqrt{2}$ m.

Obs:- $|\overline{Ap}| = |\overline{Bp}| = |\overline{Cp}| = |\overline{Dp}| = \sqrt{2}$ m

$$\overline{F_p} = \frac{Q Q_p}{4\pi\epsilon_0(\sqrt{2})^3}\left[\cancel{-\overline{a_x}} + \overline{a_z} + \cancel{\overline{a_x}} + \overline{a_z} - \cancel{\overline{a_y}} + \overline{a_z} + \cancel{\overline{a_y}} + \overline{a_z}\right]$$

$$\overline{F_p} = \frac{Q_1 Q_p}{4\pi\epsilon_0(\sqrt{2})^3}\, 4\overline{a_z} \text{ N.}$$

$$\overline{F_p} = \frac{10\mu\,(20\mu)\,(9\times10^9)}{(\sqrt{2})^3}\, 4\;\overline{a_z}$$

$$\boxed{\overline{F_p} = 2.54558\,\overline{a_z}}$$ Newton

$|\overline{F_p}| = F_z = 2.54558$ N.

1.1d Applications of Coulomb's Law.

Coulomb's Law is used to:

i. Find the Force between two point charges.

ii. Find the potential at a point due to a fixed charge.

iii. Find the Electric field at a point due to a fixed charge.

iv. Find the potential and Electric field due to any type of charge distribution.

1.1e Limitation of Coulomb's Law

i. Coulomb's Law is defined only for point charges.

ii. it is difficult to apply the Law when Charges are of arbitrary shape.

Topic 1.2. Types of Charge Distribution.

There are Four common types of charge distributions are

a. point charges, Q (Coulomb).
b. Line charge distribution ρ_L (C/m).
c. Surface charge distribution ρ_S (C/m^2).
d. volume charge distribution ρ_v (C/m^3).

a. point charge distribution, Q (Coulomb)

These are the charges which do not occupy any Space, that is, the volume of the point charge is Zero.

Eg. Electron is Considered to be a point charge and has a charge of 1.6×10^{-19} Coulombs.

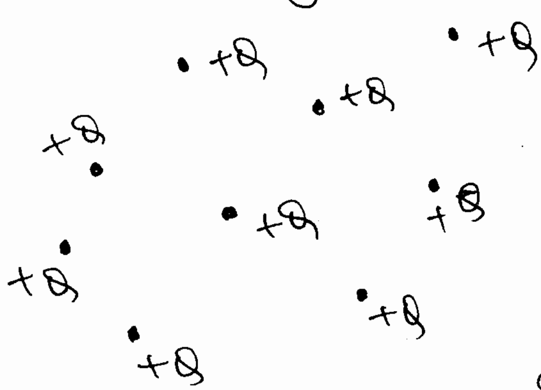


fig. point charge distribution.

b. Line charge distribution (or) Line charge density ρ_l (C/m).

Consider a Q Coulomb of charge uniformly distributed over a line of length 'L' meter.

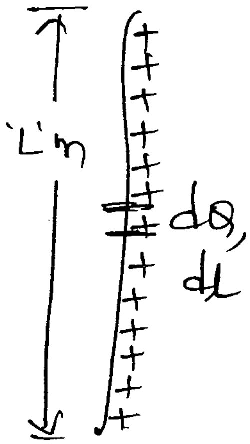


fig. line charge distribution

$$\rho_l = \frac{\text{total charge spread}}{\text{total length}} \ \text{C/m}.$$

$$\boxed{\rho_l = \frac{Q}{L}} \ \text{C/m} \quad \leftarrow (1)$$

eq (1) is valid if 'Q' is constant.

if suppose charge 'Q' is a function of Spatial variables

then $$\boxed{\rho_l = \frac{dQ}{dl}} \ \text{C/m} \quad \leftarrow (2)$$

from eq (2) the total charge 'Q' is obtained by

$$\Rightarrow \quad dQ = \rho_l \, dl \ \text{Coulomb's}$$

$$\boxed{Q = \int_{\langle l \rangle} \rho_l \, dl} \ \text{Coulomb's.}$$

c. Surface charge distribution (or) Surface charge density ρ_s (C/m²)

Consider a charge of 'Q' Coulomb's uniformly distributed over a surface of area 'S' m².

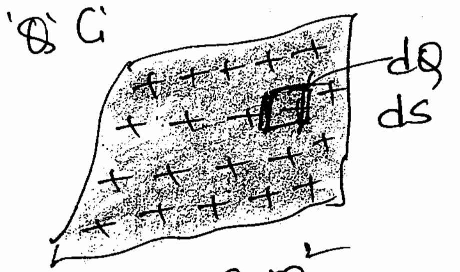


fig. Surface charge distribution.

$$\rho_s = \frac{\text{total charge spread}}{\text{total Area}} \ C/m^2$$

$$\boxed{\rho_s = \frac{Q}{S}} \ C/m^2$$

(or) $\boxed{Q = \rho_s \cdot S}$ C

if 'Q' is a function of Spatial variables then Consider ~~the~~ differential charge dQ over a surface ds.

$$\boxed{\rho_s = \frac{dQ}{ds}} \ C/m^2$$

the total charge spread Q = ?

$$\Rightarrow dQ = \rho_s \, ds$$

$$\boxed{Q = \int_{\langle S \rangle} \rho_s \cdot ds} \ \text{Coulomb's.}$$

d. Volume charge distribution (or) Volume charge density ρ_v (C/m³)

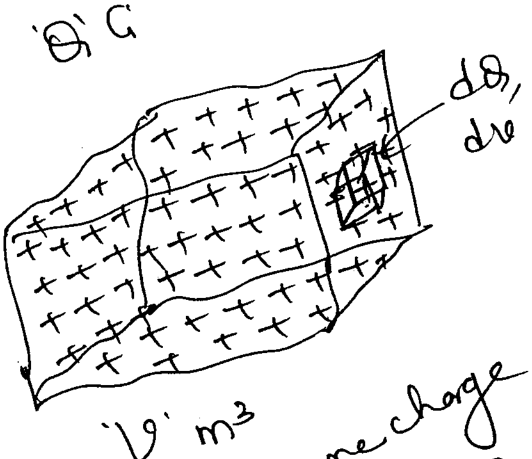


fig. volume charge distribution.

Consider a total charge of 'Q' Coulombs, uniformly distributed over a volume V

$$\rho_v = \frac{\text{total charge spread}}{\text{total volume}} \text{ C/m}^3$$

$$\boxed{\rho_v = \frac{Q}{V}} \text{ C/m}^3$$

if charge 'Q' is a function of spatial variables then consider the differential charge over a differential volume dv.

$$\boxed{\rho_v = \frac{dQ}{dv}} \text{ C/m}^3$$

the total charge spread $Q = ?$

$$dQ = \rho_v \, dv$$

$$\boxed{Q = \int_{\langle vol \rangle} \rho_v \, dv} \text{ Coulomb}$$

Keynote points:-

1. point charge Q $\longrightarrow$ Coulomb (C).

2. Line charge density $\rho_l = \frac{Q}{L}$ C/m $\Rightarrow \boxed{Q_t = \rho_l L}$ C

(or) $\rho_l = \frac{dQ}{dl}$ C/m $\Rightarrow \boxed{Q_t = \int_{\langle l \rangle} \rho_l \cdot dl}$ C.

3. Surface charge density (ρ_s).

$\rho_s = \frac{Q}{S}$ C/m^2 $\Rightarrow \boxed{Q_t = \rho_s \cdot S}$ C

(or) $\rho_s = \frac{dQ}{ds}$ C/m^2 $\Rightarrow \boxed{Q_t = \int_{\langle S \rangle} \rho_s \cdot ds}$ C.

4. volume charge density (ρ_v)

$\rho_v = \frac{Q}{v}$ C/m^3 $\Rightarrow \boxed{Q_t = \rho_v \cdot v}$ C

(or)

$\rho_v = \frac{dQ}{dv}$ C/m^3 $\Rightarrow \boxed{Q_t = \int_{\langle vol \rangle} \rho_v \, dv}$ C

5. the quantities Q, ρ_l, ρ_s, and ρ_v are scalars in nature.

6. $\int_{\langle l \rangle} = \int$ ...indicates Line integral and it is a Single integral.

$\int_{\langle S \rangle} = \iint$ ...indicates Surface integral and it is a double integral

$\int = \iiint$ .. indicates Volume integral & it is

problem 1.

Find the total charge inside a volume having charge density as $10z^2 e^{-0.1x} \sin(\pi y)$ C/m³. The volume is defined between $-2 \le x \le 2$, $0 \le y \le 2$, $3 \le z \le 4$.

soln: Given

$$\rho_v = 10z^2 e^{-0.1x} \sin(\pi y) \; C/m^3$$

$$-2 \le x \le 2, \quad 0 \le y \le 2, \quad 3 \le z \le 4.$$

$$\rho_v = \frac{dQ}{dv} \; C/m^3 \qquad dv = dx\,dy\,dz \; m^3$$

$\Rightarrow$ the total charge $Q = \int_{vol} \rho_v \, dv$ C

$$Q = \int_{vol} 10z^2 e^{-0.1x} \sin(\pi y) \, dx\,dy\,dz.$$

$$Q = 10 \int_{x=-2}^{2} e^{-0.1x} dx \int_{y=0}^{2} \sin(\pi y)\, dy \int_{z=3}^{4} z^2 dz$$

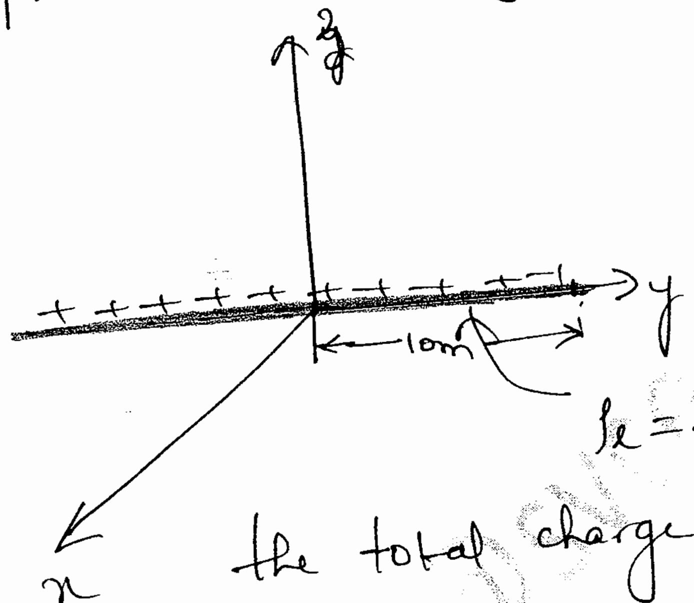
$$Q = 10 \times 4.02672 \times 0 \times 12.333$$

$$\boxed{Q = 0} \text{ Coulomb's.}$$

problem 2.

A charge is distributed on y-axis of cartesian System having a line charge density of $3y^2 \mu C/m$. Find the total charge over the Length of 10m.

Soln:



$L = 10m$

$\Rightarrow \boxed{0 \le y \le 10}$

$\rho_l = 3y^2 \mu C/m$.

the total charge distributed over a Length of 10m is given by

$$\rho_l = \frac{dQ}{dl} \; C/m^2$$

$$\Rightarrow Q = \int_{\langle l \rangle} \rho_l \cdot dl \quad C.$$

Since line charge is placed along y-axis and y-range is

$dl \rightarrow dy$ $\quad$ $0 \le y \le 10$.

$$Q = \int_{y=0}^{10} 3y^2 \cdot dy \times 1\mu = \cancel{3} \cdot \frac{y^3}{\cancel{3}}\Big|_0^{10} \times 1\mu = 10^{-3} C$$

$$= 1mC$$

$Q = 1m$ Coulomb

Problem 3. Find the charge in the volume defined by $1 \le \rho \le 2m$ and $\rho_v = \frac{5\cos^2\phi}{\rho^4}$ C/m³.

Soln:

Given $\rho_v = \frac{5\cos^2\phi}{\rho^4}$ C/m³ and $1 \le \rho \le 2$ m

ρ_v is in cylindrical Co-ordinate System.

the total charge $Q = \int_{vol} \rho_v \, dv$ C

$$P(\rho, \phi, z) \rightarrow d\rho, \ \rho d\phi, \ dz.$$

$$dv = \rho \, d\rho \, d\phi \, dz.$$

$$Q = \int_{vol} \rho_v \, \rho \, d\rho \, d\phi \, dz$$

$$Q = 5 \int_{\rho=1}^{2} 1/\rho^3 \, d\rho \int_0^{2\pi} \cos^2\phi \, d\phi \int_{z=0}^{1} dz$$

$$\left[\int_0^{2\pi} \cos^2\phi \, d\phi\right]$$

$$Q = 5 \times 0.375 \times 3.1415 \times 1 = 5.8903 \, C$$

$$\boxed{Q = 5.8903} \text{ Coulombs}$$

Problem 4. Find the total charge contained in a 2cm Length of the electron beam, cylindrical in shape with $\rho = 1cm$; height of 2cm from 2 to 4cm and $\phi = 0$ to $2\pi^c$. given charge density (8M).

$$\rho_v = -5\times10^{-6} e^{-10^5 \rho z} \ c/m^3.$$

[W.H. Hayt | 02- June/July -2010]

Solu:-

$\rho = 1cm \Rightarrow 0 \le \rho \le 0.01m$

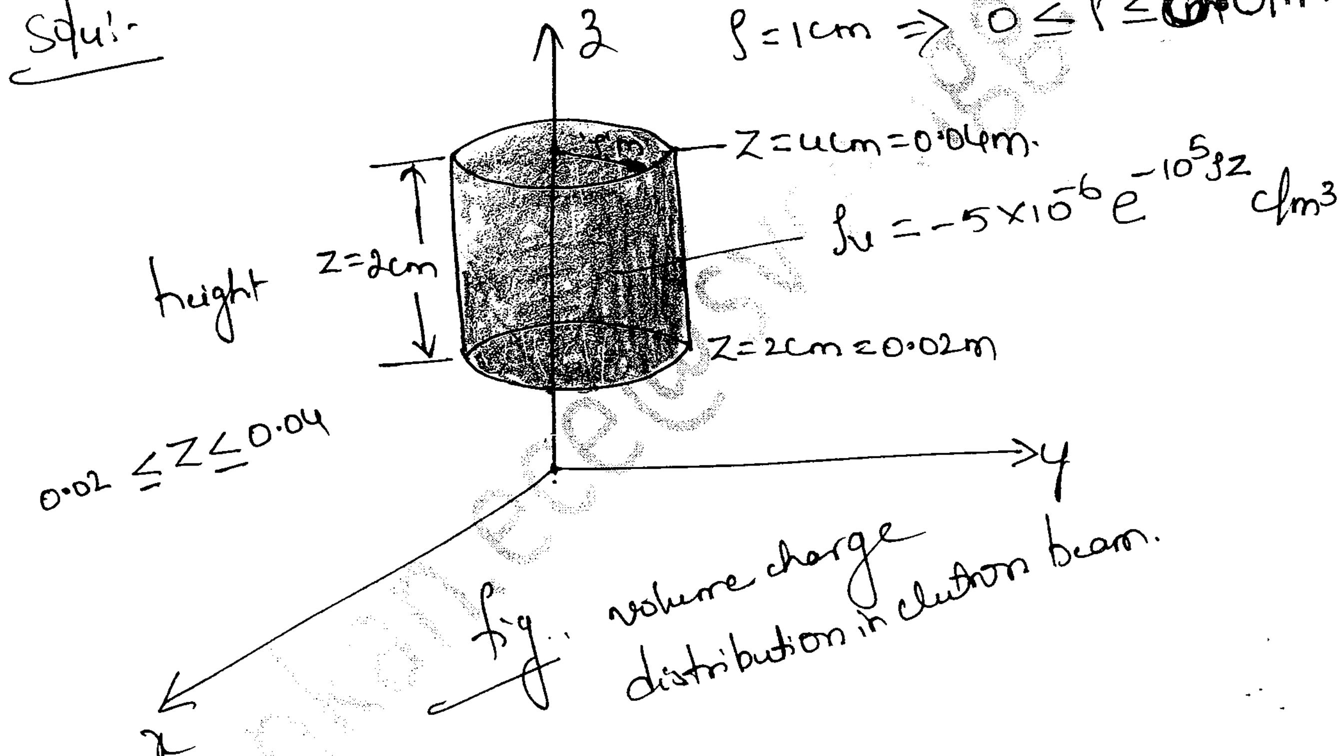


fig. volume charge distribution in electron beam.

Given. $\rho_v = -5\times10^{-6} e^{-10^5 \rho z} \ c/m^3$.

$dv = \rho \, d\rho \, d\phi \, dz$; ... Cylindrical Co-ordinate System

the total charge containing in the electron beam in

$\rho_v = \dfrac{dQ}{dv} \ c/m^3 \Longrightarrow \boxed{Q = \int_{vol} \rho_v \, dv}$ Coulomb

$$Q = \int_{\langle vol \rangle} -5\times10^{-6}\, e^{-10^5 \rho z} \,[\rho\, d\rho\, d\phi\, dz]$$

$$Q = \int_{\rho=0}^{0.01} \int_{z=0.02}^{0.04} \int_{\phi=0}^{2\pi} -5\times10^{-6}\, e^{-10^5 \rho z}\, \rho\, d\rho\, d\phi\, dz$$

$$= \int_{\rho=0}^{0.01} \int_{z=0.02}^{0.04} -5\times10^{-6}\, e^{-10^5 \rho z}\, \rho\, d\rho\, dz \int_0^{2\pi} d\phi \to 2\pi$$

$$= -5\times 2\pi \times 10^{-6} \int_{\rho=0}^{0.01} \int_{z=0.02}^{0.04} e^{-10^5 \rho z}\, \rho\, d\rho\, dz$$

first integrate w.r.t 'z' by treating 'ρ' to be constant.

$$= -10\pi\mu \int_{\rho=0}^{0.01} \cancel{\rho}\, d\rho \cdot \frac{e^{-10^5 \rho z}}{-10^5 \cancel{\rho}} \Bigg|_{0.02}^{0.04}$$

$$= +10\pi\mu\times10^{-5} \int_{\rho=0}^{0.01} \left[e^{-10^5 \rho (0.04)} - e^{-10^5 \rho (0.02)} \right] d\rho$$

$$= 10\pi\mu\times10^{-5} \left[\frac{e^{-10^5 \rho (0.04)}}{-10^5 (0.04)} \Bigg|_0^{0.01} - \frac{e^{-10^5 \rho (0.02)}}{-10^5 (0.02)} \Bigg|_0^{0.01} \right]$$

Note:- $e^{-\infty} = 0$ and $e^{-(\text{Large No})} \simeq 0$

$$= 10\pi\mu \times 10^{-5}\left[\frac{-1}{10^5(0.04)}\left[\cancelto{\simeq 0}{e^{-10^5(0.01)(0.04)}} - 1\right] + \frac{1}{10^5(0.02)}\left[\cancelto{\simeq 0}{e^{-10^5(0.01)(0.02)}} - 1\right]\right]$$

$$= 10\pi\mu \times 10^{-5}\left[+\frac{1}{10^5(0.04)} - \frac{1}{10^5(0.02)}\right]$$

$$= \frac{10\pi\mu \times 10^{-5}}{10^5}\left[(0.04)^{-1} - (0.02)^{-1}\right]$$

$$= -78.5398 \times 10^{-15}\ C$$

$$Q = -0.07853 \times 10^{-12}\ C$$

$$\boxed{Q = -0.078539\ pC}$$

Problem 5.

Find the charge in the volume defined by $1 \leq r \leq 2m$ and in Spherical Coordinate System $\rho_v = \frac{5\cos^2\phi}{r^4}$ C/m³.

Soln:-

$$Q = \int_{\langle vol \rangle} \rho_v \, dv \quad \text{Coulomb}$$

$$= \int_{\langle vol \rangle} \rho_v [r^2 \sin\theta \, dr \, d\theta \, d\phi] \; : \text{Coulomb}$$

$$Q = \int_{\langle vol \rangle} \frac{5\cos^2\phi}{r^{\cancel{4}2}} \cdot \cancel{r^2} \sin\theta \, dr \, d\theta \, d\phi$$

$$Q = 5\int_{r=1}^{2} \frac{1}{r^2} dr \int_{\theta=0}^{\pi} \sin\theta \, d\theta \int_{\phi=0}^{2\pi} \cos^2\phi \, d\phi$$

$$Q = 5 \times 0.5 \times 2 \times 3.1415$$

$$Q = \underline{15.7075} \quad \text{Coulomb's}$$

$$\boxed{Q = 15.707 \, C}$$

Problem 6.

Calculate the total charge within each of the indicated volumes

a. $0.1 \le |x|, |y|, |z| \le 0.2$; $\rho_v = \frac{1}{x^3 y^3 z^3}$ C/m^3.

b. $0 \le \rho \le 0.1$, $0 \le \phi \le \pi$, $2 \le z \le 4$ m;
$\rho_v = \rho^2 z^2 \sin(0.6\phi)$ C/m^3.

c. Universe: $\rho_v = e^{-2r}/r^2$ C/m^3.

[W. H. Hayt].

Soln:

a. Given
$\rho_v = \frac{1}{x^3 y^3 z^3}$ C/m^3 ; $dv = dx\,dy\,dz$; m^3

$0.1 \le |x|, |y|, |z| \le 0.2$

$$|x| = \begin{cases} +x ; & x \ge 0 \\ -x ; & x < 0 \end{cases}$$

$x \ge 0$ i.e. x +ve

$0.1 \le x \le 0.2 \leftarrow$ ⓐ

$x < 0$ ie x is -ve

$+0.1 \le -x \le +0.2 \Rightarrow -0.2 \le x \le -0.1 \leftarrow$ ⓑ

By comparing lower and upper limits of eqn ⓐ and eqn ⓑ

$$\boxed{-0.2 \le x \le 0.2}$$

$\therefore \quad \boxed{-0.2 \le x, y, z \le +0.2}$

$$Q = \int_{\langle vol \rangle} \rho_v \, dv = \cancel{\int_{x=-0.2}^{0.2} \frac{1}{x^3} dx}^{\,0} \; \cancel{\int_{y=-0.2}^{0.2} \frac{1}{y^3} dy}^{\,0} \; \cancel{\int_{z=-0.2}^{0.2} \frac{1}{z^3} dz}^{\,0}$$

$\boxed{Q = 0}$ Coulomb

Note:- i. if $f(x)$ is an odd function i.e $f(-x) = -f(x)$

then $\int_{-a}^{a} f(x)\,dx = 0$.

ii. if $f(x)$ is an even function ie $f(-x) = f(x)$

then $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$.

b) Given $0 \le \rho \le 0.1$, $0 \le \phi \le \pi$; $2 \le z \le 4$

and $\rho_v = \rho^2 z^2 \sin(0.6\phi)$ C/m^3 ... Cylindrical C.S

$$Q = \int_{\langle vol \rangle} \rho_v \, dv = \int_{\rho=0}^{0.1} \int_{\phi=0}^{\pi} \int_{z=2}^{4} \rho^2 z^2 \sin(0.6\phi)\, \rho \, d\rho\, d\phi\, dz$$

$$Q = \int_{\rho=0}^{0.1} \rho^3 \, d\rho \int_{\phi=0}^{\pi} \sin(0.6\phi) \, d\phi \int_{z=2}^{4} z^2 \, dz$$

$$Q = (2.5 \times 10^{-5})(2.1817)(18.666)$$

$$Q = 1.01809 \times 10^{-3} \text{ C}$$

$$\boxed{Q = 1.01809 \text{ mC}}$$

c) given $\rho_v = e^{-2r}/r^2 \text{ C/m}^3$ ... in Spherical Co-ordinate System.

$$P(r, \theta, \phi)$$

$dr \quad r\,d\theta \quad r\sin\theta\,d\phi$

$$dv = r^2 \sin\theta \, dr \, d\theta \, d\phi.$$

Universe $\Rightarrow$

$$0 \le r \le \infty$$

$$0 \le \theta \le \pi$$

$$0 \le \phi \le 2\pi.$$

$$Q = \int_{\text{vol}} \rho_v \, dv \text{ Coulomb.}$$

$$Q = \int_{\langle vol \rangle} \frac{e^{-2r}}{\cancel{r^2}} \cdot \cancel{r^2} \sin\theta \, dr \, d\theta \, d\phi$$

$$Q = \int_{r=0}^{\infty} e^{-2r} dr \int_{\theta=0}^{\pi} \sin\theta \, d\theta \int_{\phi=0}^{2\pi} d\phi$$

$$= \left. \frac{e^{-2r}}{-2} \right|_0^{\infty} \times 2 \times 2\pi$$

$$Q = -\frac{1}{2} \left[\cancelto{0}{e^{-\infty}} - \cancelto{1}{e^{0}} \right] \times 4\pi$$

$$= -\frac{1}{2} [-1] \times 4\pi$$

$$Q = +\frac{1}{2} \times 4\pi = 2\pi \text{ C}$$

$$\boxed{Q = 2\pi \text{ C} = 6.2831 \text{ Coulomb}}$$

problem 7.

A uniform volume charge density of $0.2\,\mu C/m^3$ is present throughout the spherical shell extending from $r = 3cm$ to $r = 5cm$. If $\rho_v = 0$ elsewhere, find:

a) the total charge present within the shell and

b) r_1 if half the total charge is located in the region $3cm < r < r_1$.

Soln:

a. given $\rho_v = 0.2\,\mu C/m^3$.

$r = 3cm$ to $r = 5cm \Rightarrow 0.03 < r < 0.05m$.

$0 < \theta < \pi$ and $0 < \phi < 2\pi$.

$dv = r^2 \sin\theta\, dr\, d\theta\, d\phi$.

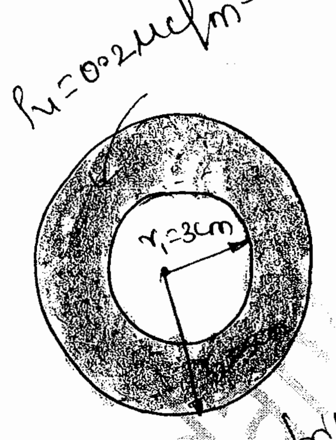


fig. concentric sphere

the total charge

$$Q = \int_{vol} \rho_v \, dv \quad \text{Coulomb}$$

$$Q = \int_{vol} (0.2\mu)\, r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$= 0.2\mu \int_{r=0.03}^{0.05} r^2 dr \int_{\theta=0}^{\pi} \sin\theta\, d\theta \int_{\phi=0}^{2\pi} d\phi$$

$$= (0.2\mu)(3.26\times10^{-5})(2)(2\pi)$$

$$= 8.1932\times10^{-11} \text{ Coulomb}$$

$$\boxed{Q = 8.1932\times10^{-11} C = 81.932\, pC}$$

b) Find $r_1 = ?$ Such that $Q_{total} = \frac{Q}{2}$ and region $3\,cm < r < r_1$

$\Rightarrow 0.03 < r < r_1$.

$$Q_{total} = \int_{vol} \rho_v\, dv$$

$$\frac{Q}{2} = \int_{vol} (0.2\mu)\, r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$\frac{Q}{2} = \int_{0.03}^{r_1} r^2 dr \int_{\theta=0}^{\pi} \sin\theta\, d\theta \int_{\phi=0}^{2\pi} d\phi \times (0.2\mu)$$

and $Q = 81.9327\times10^{-12} C$

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$$40.966\times10^{-12} = \int_{0.03}^{r_1} r^2 dr\ (2)(2\pi)(0.2\mu)$$

$$\int_{0.03}^{r_1} r^2 dr = \frac{40.966\times10^{-12}}{4\pi(0.2\mu)} = 16.3\times10^{-6}$$

$$\int_{0.03}^{r_1} r^2 dr = 16.3\times10^{-6}$$

$$\left.\frac{r^3}{3}\right|_{0.03}^{r_1} = 16.3\times10^{-6}$$

$$\frac{-1}{3}\left[(0.03)^3 - r_1^3\right] = 16.3\times10^{-6}$$

$$(0.03)^3 - r_1^3 = -48.9\times10^{-6}$$

$$r_1^3 = 0.03^3 + 48.9\times10^{-6}$$

$$r_1^3 = 75.9\times10^{-6} = 75.9\mu$$

$$r_1^3 = 75.9\mu$$

$$\Rightarrow r_1 = (75.9\times10^{-6})^{1/3} = 4.233\times10^{-2}\,m$$

$$\boxed{r_1 = 4.233\,cm = 0.04233\,m}$$

problem 8.

The charge density varies with radius in a cylindrical co-ordinate systems as $\rho_v = \dfrac{\rho_0}{(\rho^2+a^2)^2}$ C/m³.

within what distance from z-axis does half the total charge lie?

Soln: given $\rho_v = \dfrac{\rho_0}{(\rho^2+a^2)^2}$ C/m³ ... cylindrical C.S

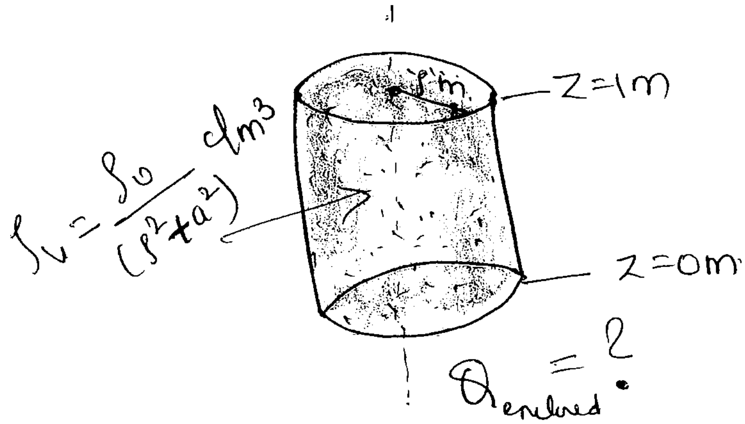
the total charge enclosed by the volume with unit length [i.e z = height = 1m].

$$Q = \int_{\langle vol \rangle} \rho_v \, dv \quad \text{Coulomb.}$$

$P(\rho, \phi, z)$

$dv = \rho \, d\rho \, d\phi \, dz$.

$$Q = \int_{\langle vol \rangle} \frac{\rho_0}{(\rho^2+a^2)^2} \rho \, d\rho \, d\phi \, dz$$



$$Q = \int_0^{\rho} \int_0^{2\pi} \int_{z=0}^{1} \frac{\rho_0}{(\rho^2+a^2)^2} \cdot \rho \, d\rho \, d\phi \, dz$$

$$Q = \rho_0 \cdot \int_{\rho=0}^{\rho} \frac{\rho}{(\rho^2+a^2)^2} d\rho \int_0^{2\pi} d\phi \int_{z=0}^{1} dz$$

$$Q = 2\pi\rho_0 \int_0^{\rho} \frac{\rho}{(\rho^2+a^2)^2} d\rho$$

$$Q = 2\pi\rho_0 \left[\frac{-1}{2(\rho^2+a^2)}\right]_0^{\rho}$$

$$Q = -\frac{\cancel{2}\pi\rho_0}{\cancel{2}} \left[\frac{1}{(\rho^2+a^2)} - \frac{1}{a^2}\right]$$

$$Q = \pi\rho_0 \left[\frac{1}{a^2} - \frac{1}{\rho^2+a^2}\right]$$

$$\boxed{Q = \frac{\pi\rho_0}{a^2}\left[1 - \frac{1}{1+\rho^2/a^2}\right] = Q(\rho)} \leftarrow (a)$$

i.e Q is a function of radius 'ρ' m.

when $\rho \to \infty$, the total charge is found to be

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$$\boxed{Q = \frac{\pi \rho_0}{a^2}}$$ Coulombs.

from eqⁿ (a)

the condition for which $Q \rightarrow Q/2$

i.e $\boxed{\rho = 'a' m}$

when $\rho = 'a'$ m in eqⁿ (1) $Q' = \frac{\pi \rho_0}{a^2}\left[1 - \frac{1}{2}\right]$

$$Q' = \frac{\pi \rho_0}{2a^2} = Q/2.$$

i.e when $\boxed{\rho = a}$ m the charge becomes half i.e

$$\boxed{\therefore Q' = \frac{Q}{2} = \frac{\pi \rho_0}{2a^2}}$$ Coulombs.

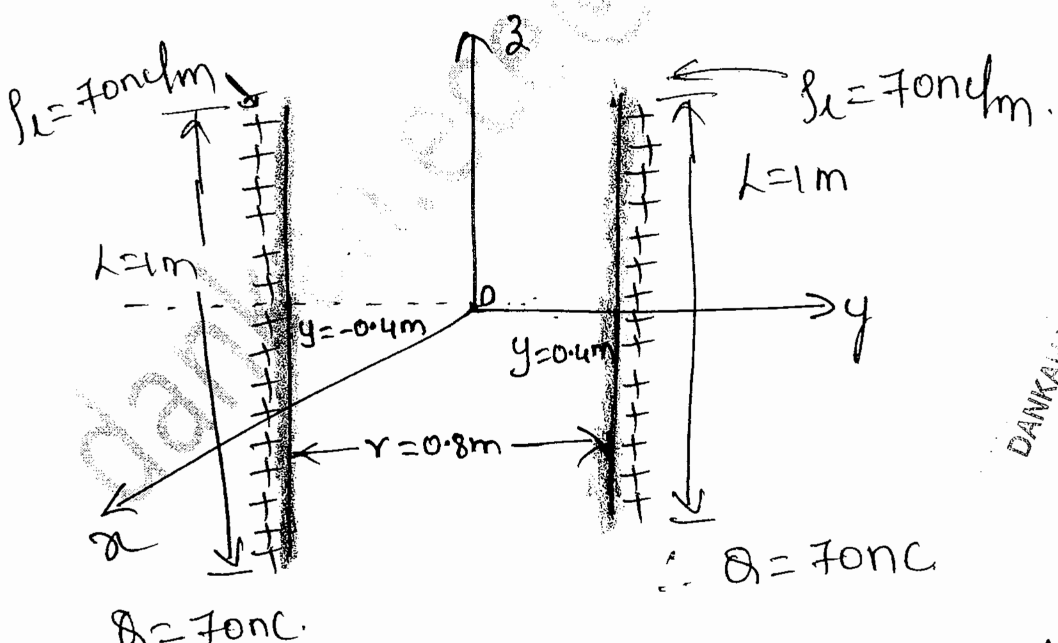
problem 9.

Two identical uniform Line charge with $\rho_L = 70\,nC/m$ are Located in free space at $x=0$; $y = \pm 0.4m$. what force per unit Length does Each Line charge Exert on other.

Solu:- given $\rho_L = 70\,nC/m$

the total charge 'Q' enclosed per Unit Length (i.e $L=1m$) is $Q = \rho_L \times L = 70\,nC/m \times 1m = 70\,nC$

$$\boxed{Q = 70\,nC}$$



the magnitude of Force exerted between both the Line charges is

$$F = \frac{QQ}{4\pi\epsilon_0 r^2} = \frac{Q^2}{4\pi\epsilon_0 r^2}$$

$$F = \frac{(70n)^2 (9\times10^9)}{(0.8)^2} = 68.906\times10^{-6}\ N.$$

$$\therefore \boxed{F = 68.906\times10^{-6} = 68.906\,\mu N}$$

Problem 10

7 Find the total charge inside a volume having volume charge density as $10z^2e^{-0.1x}\sin(\pi y)$ C/m^3 The volume is defined between $-2 \le x \le 2, 0 \le y \le 1, 3 \le z \le 4$. Ans: Q=316.16C

soln:

$$\rho_v = 10z^2 e^{-0.1x} \sin(\pi y) \quad C/m^3$$

$$-2 \le x \le 2, \quad 0 \le y \le 1, \quad 3 \le z \le 4.$$

$$Q = \int_{\langle vol \rangle} \rho_v \cdot dv = \int_{\langle vol \rangle} \rho_v \, dx\,dy\,dz \quad \text{Coulomb's}$$

$$Q = \int_{\langle vol \rangle} 10z^2 e^{-0.1x} \sin(\pi y)\, dx\,dy\,dz$$

$$Q = 10 \int_{x=-2}^{2} e^{-0.1x}\,dx \int_{y=0}^{1} \sin(\pi y)\,dy \int_{z=3}^{4} z^2\,dz.$$

using calci

$$Q = 10 \times 4.02672 \times 0.63662 \times 12.333$$

$$\boxed{Q = 316.1638} \text{ Coulomb's}$$

soln

problem 10. Find the total charge inside a volume having volume charge density as $10z^2e^{-0.1x}\sin(\pi y)$ C/m^3. The volume is defined between $-2 \le x \le 2$, $0 \le y \le 1$ and $3 \le z \le 4$.

problem 11

A charge is distributed on x-axis of Cartesian System having a line charge density of $3x^2\ \mu C/m$. Find the total charge over the Length of 10m.

soln:- $\rho_L = 3x^2\ \mu C/m$.

$0 \le x \le 10m$.

$$Q = \int_{\langle L \rangle} \rho_L \cdot dL.$$

Since the line charge density ρ_L is placed along x-axis $dL = dx$.

$$Q = \int_{x=0}^{10} (3x^2 \mu)\, dx = \frac{3x^3}{3}\Big|_0^{10} \times 1\mu$$

$$Q = 10^3 \times 1\mu = 10^{-3} C$$

$$\boxed{Q = 1mC}$$

Topic 1.3. Electric Field Intensity. [10-Jan 2014] [10-Jan-2016]

1.3a: Definition of Electric field Intensity ($\bar{E}$)
[10-June/July-2014 / 15-Dec/Jan 2017 (CBCS) / 06 may/June-2010]

Electrostatic field is produced by a charge at rest. it is defined by Coulomb's Law.

Definition: Electric field due to a charge is defined as the Coulomb's force per unit test charge. it is a Vector quantity and has the unit of Newton per Coulomb (N/C) (or) volt per meter (V/m).

i.e $$\boxed{\bar{E} = \frac{\bar{F}_t}{\cancel{Q_t}\,1C}}\ \text{V/m (or) N/C}$$

1.3b: Field due to point charge [10-Jan 2012, 10-Jan 2013, 10-June/July-2014].

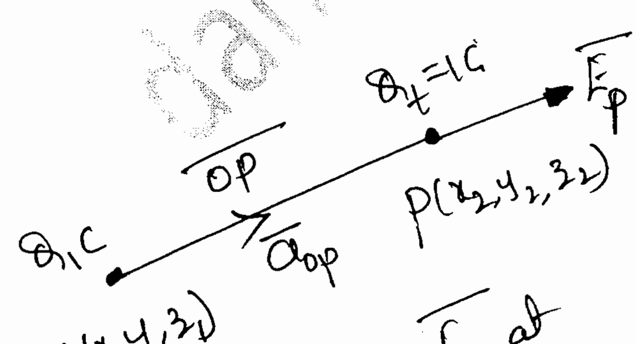


fig. field $\bar{E}_p$ at point P

In other way the Electric field Intensity at a point 'p' is nothing but the force experience by a unit positive charge at point $P(x_2, y_2, z_2)$m due to Q_1 C of charge at point $O(x_1, y_1, z_1)$m.

from Coulomb's Law

$$\overline{F}_P = \frac{Q_1 Q_t}{4\pi\epsilon_0 |\overline{OP}|^2} \overline{a}_{op} \text{ ; Newton}$$

$$\overline{E}_P = \frac{\overline{F}_P}{\cancel{Q_t} 1C} \text{ N/C (or) V/m}$$

$$\Rightarrow \boxed{\overline{E}_P = \frac{\overline{F}_P}{\cancel{Q_t} 1C} = \frac{Q_1}{4\pi\epsilon_0 |\overline{OP}|^2} \overline{a}_{op}} \text{ N/C (or) V/m}$$

Keynote points:-

i. Electric field intensity $(\overline{E})$ is a vector field and Measured in N/C (or) V/m.

ii. $\overline{E}$ at a point P is nothing but force per +ve unit charge.

ie $\boxed{\overline{E}_P = \frac{\overline{F}_P}{\cancel{Q_t} 1C}}$ N/C (or) V/m

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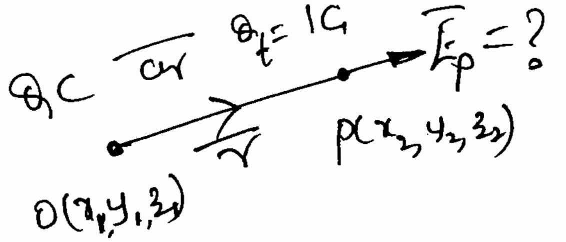
iii. In general Electric field intensity at a point 'p' due to 'Q' C of point charge is given by



$$\boxed{\bar{E_p} = \frac{Q}{4\pi\epsilon r^2}\,\bar{a_r}}\ \text{V/m (or) N/C}$$

iv. its direction is the same as that of Coulomb's force.

v. $\bar{E}$ depends on the permittivity of the medium.

vi. it depends on the distance of the charge from another charge which produces Coulomb's force.

vii. it depends on the location of the charges.

viii. When a unit charge at a distance is moved around a fixed charge, the field lines and force appear as shown in fig. below.

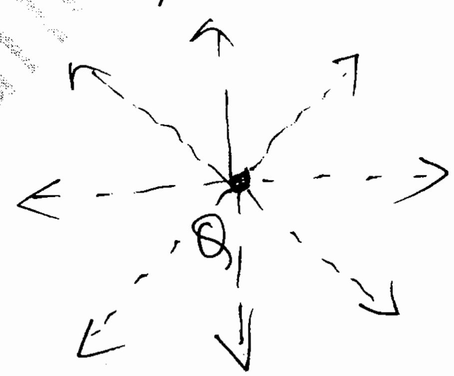


fig. Coulomb's force and Electric field.

1.3c: Electric field Intensity due to N-number of point charges.

Question. Showthat Electric field intensity at a point due to 'n' number of point charges in given by (5M).

$$\bar{E} = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \frac{Q_i}{R_i^2} \bar{a}_{R_i} \ \text{V/m}.$$

(or)

Define Electric field intensity due to point charge in a vector form. with usual notation derive Expression for field ~~due to~~ at a point due to many charges (6M).

(or)

Define and Explain Electric field intensity. State principle of Superposition and Find electric field intensity due to multiple point charge distribution. (6M).

[06-Dec 2010 | 06-Jan 2010 | 10-Jan 2013 | 10-Jan 2014 | 10-June -2014 10-Dec/Jan-2016].

[15-June/July 2017 (2M) · CBCS-Scheme]

Solu:-

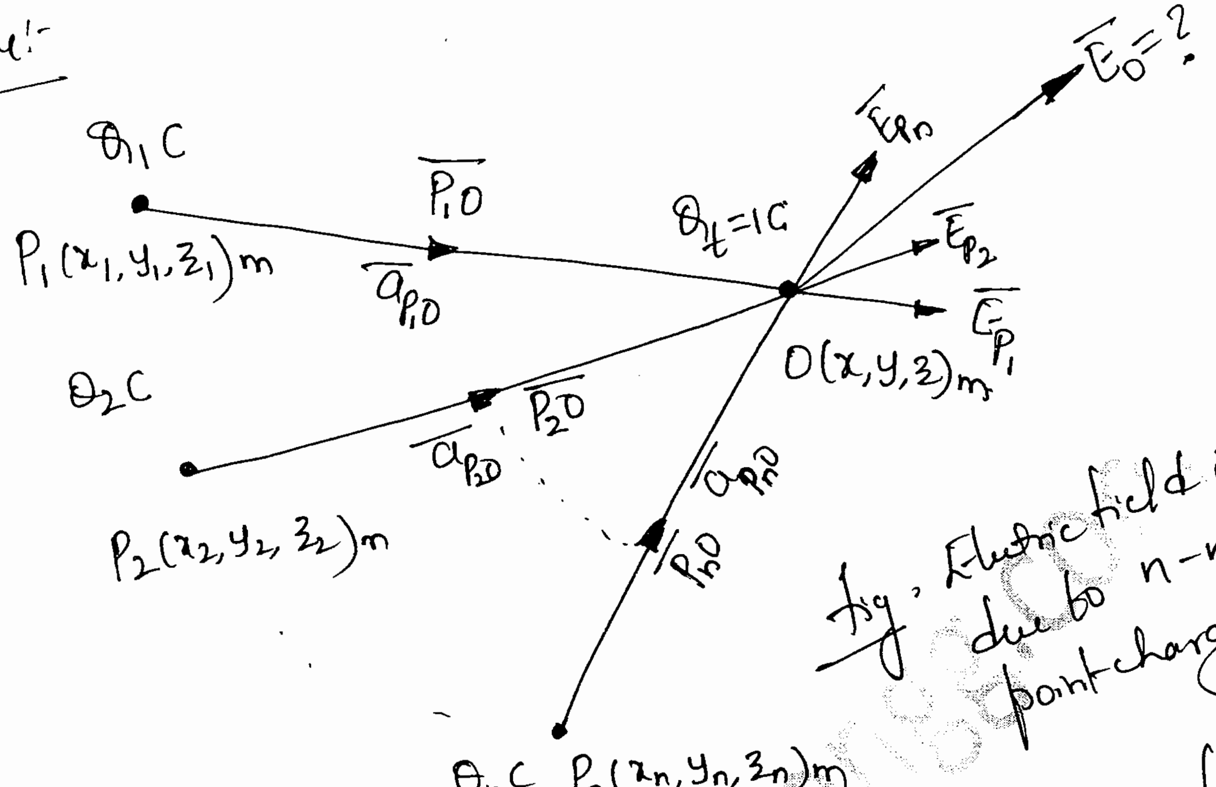


fig: Electric field intensity due to n-number of point charges.

Consider a point charges of $Q_1, Q_2, \ldots Q_n$ C placed at points $P_1, P_2, \ldots P_n$ respectively. the Electric field intensity $(\overline{E_O})$ at a point $O(x, y, z)$ is Calculated using principle of superposition.

Superposition principle:- the net field intensity at a point $O(x, y, z)$ due to n-number of point charges is equal Sum of individual field's acting one at a time.

i.e $\overline{E_O} = \overline{E_{P_1}} + \overline{E_{P_2}} + \cdots + \overline{E_{P_n}}$ V/m.

$$\overline{E}_0 = \frac{\overline{F}_0}{Q_t} \; = \frac{Q_1}{4\pi\epsilon|\overline{P_1O}|^2}\overline{a}_{P_1O} + \frac{Q_2}{4\pi\epsilon|\overline{P_2O}|^2}\overline{a}_{P_2O} + \cdots + \frac{Q_n}{4\pi\epsilon|\overline{P_nO}|^2}\overline{a}_{P_nO} \; V/m$$

$$\overline{E}_0 = \frac{1}{4\pi\epsilon}\left[Q_1 \frac{\overline{a}_{P_1O}}{|\overline{P_1O}|^2} + Q_2 \frac{\overline{a}_{P_2O}}{|\overline{P_2O}|^2} + \cdots + Q_n \frac{\overline{a}_{P_nO}}{|\overline{P_nO}|^2}\right] V/m$$

$$\boxed{\overline{E}_0 = \frac{1}{4\pi\epsilon}\sum_{i=1}^{n} Q_i \frac{\overline{a}_{P_iO}}{|\overline{P_iO}|^2}} \; V/m \quad \leftarrow (1)$$

if $Q_1 = Q_2 = \cdots = Q_n$ i.e. $= Q$

then $$\boxed{\overline{E}_0 = \frac{Q}{4\pi\epsilon}\sum_{i=1}^{n} \frac{\overline{a}_{P_iO}}{|\overline{P_iO}|^2}} \; V/m \quad \leftarrow (2)$$

∴ if the medium is considered to be free space

ie $\epsilon = \epsilon_0$ F/m & $\epsilon_r = 1$.

then eq (1) and eq (2) becomes

$$\overline{E}_0 = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n} Q_i \frac{\overline{a}_{P_iO}}{|\overline{P_iO}|^2} \; V/m.$$

and $$\overline{E}_0 = \frac{Q}{4\pi\epsilon_0}\sum_{i=1}^{n} \frac{\overline{a}_{P_iO}}{|\overline{P_iO}|^2} \; V/m.$$

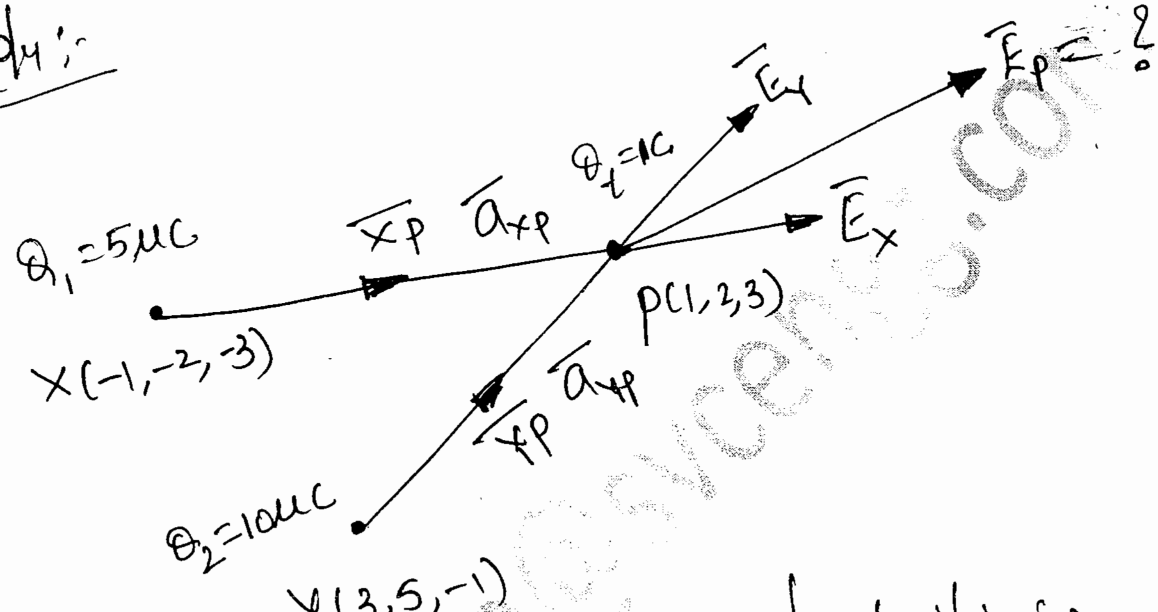
Solved problems

Problem 1.

Find $\overline{E}$ at $p(1,2,3)$ due to $Q_1 = 5\mu C$ at $(-1,-2,-3)$ and $Q_2 = 10\mu C$ at $(3,5,-1)$. (6m)

[02-Dec 2010].

Soln:-



the Electric field intensity at point 'p' is

$$\overline{E_p} = \frac{Q_1}{4\pi\epsilon|\overline{XP}|^2}\overline{a_{XP}} + \frac{Q_2}{4\pi\epsilon|\overline{YP}|^2}\overline{a_{YP}} \quad V/m.$$

$$\overline{E_p} = \frac{Q_1}{4\pi\epsilon|\overline{XP}|^3}\overline{XP} + \frac{Q_2}{4\pi\epsilon|\overline{YP}|^3}\overline{YP} \quad V/m.$$

$$\overline{XP} = (1+1)\overline{a_x} + (2+2)\overline{a_y} + (3+3)\overline{a_z}$$

$$\overline{XP} = 2\overline{a_x} + 4\overline{a_y} + 6\overline{a_z} \; ; \; |\overline{XP}| = \sqrt{4+16+36} = \sqrt{56} \text{ m.}$$

$\overline{a_{XP}} = \frac{\overline{XP}}{}$ 41 (81)

$$\overline{r_p} = (1-3)\overline{a_x} + (2-5)\overline{a_y} + (3+1)\overline{a_z} = -2\overline{a_x} - 3\overline{a_y} + 4\overline{a_z}$$

$$\overline{r_p} = \sqrt{4+9+16} = \sqrt{29}\ m \; ; \; a_{rp} = \frac{\overline{r_p}}{|\overline{r_p}|}$$

$$\overline{E_p} = \frac{Q_1}{4\pi\epsilon} \frac{\overline{x_p}}{|\overline{x_p}|^3} + \frac{Q_2}{4\pi\epsilon} \frac{\overline{r_p}}{|\overline{r_p}|^3} \ V/m.$$

$$\overline{E_p} = \frac{5\mu(9\times10^9)}{(\sqrt{56})^3}\left[2\overline{a_x} + 4\overline{a_y} + 6\overline{a_z}\right] + \frac{10\mu(9\times10^9)}{(\sqrt{29})^3}\left[-2\overline{a_x} - 3\overline{a_y} + 4\overline{a_z}\right]$$

$$\overline{E_p} = 107.381\left[2\overline{a_x} + 4\overline{a_y} + 6\overline{a_z}\right] + 576.29\left[-2\overline{a_x} - 3\overline{a_y} + 4\overline{a_z}\right]$$

$$\overline{E_p} = 214.762\overline{a_x} + 429.524\overline{a_y} + 644.286\overline{a_z} - 1152.58\overline{a_x} - 1728.87\overline{a_y} + 2305.16\overline{a_z}$$

$$\boxed{\overline{E_p} = -937.818\overline{a_x} - 1299.34\overline{a_y} + 2949.446\overline{a_z}}\ V/m.$$

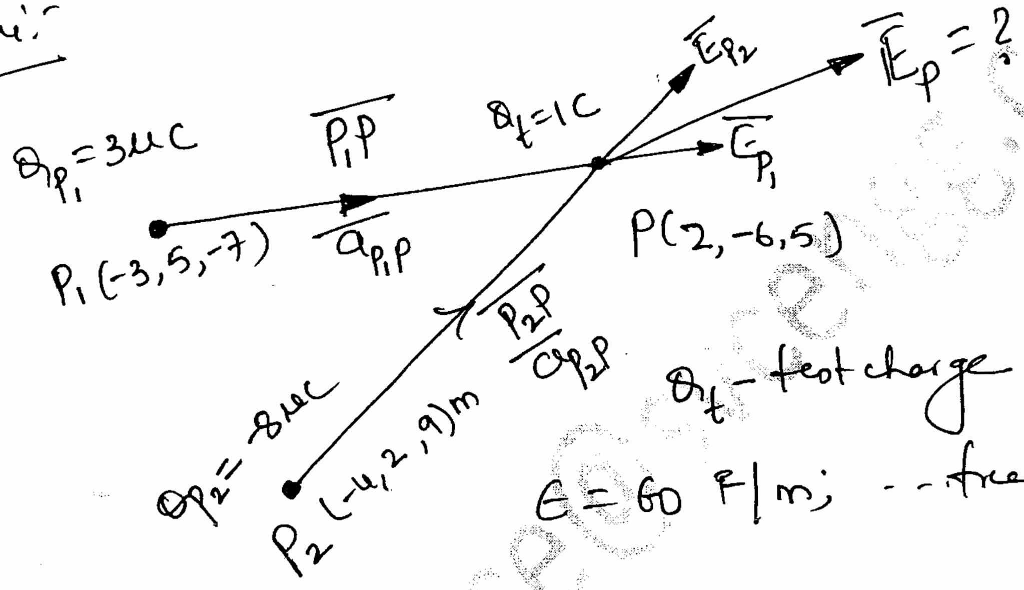
$$= -0.9378\overline{a_x} - 1.2993\overline{a_y} + 2.9494\overline{a_z}\ kV/m.$$

$$|\overline{E_p}| = 3356.638\ V/m. \text{ (or) } 3.3566\ kV/m$$

problem2. Two point charges of magnitude 3μC and -8μC are Located at place $P_1(-3, 5, -7)$ and $P_2(-4, 2, 9)$ respectively in free space. Evaluate the Electric field and also its magnitude at the point $P(2, -6, 5)$. (7M)

[Dec 2008 | Jan 2009].

Soln:



Q_t - test charge $= 1C$.

$\epsilon = \epsilon_0$ F/m; ..free space.

$$\overline{E}_p = \overline{E}_{P_1} + \overline{E}_{P_2} \text{ V/m}.$$

$$\therefore \overline{E}_p = \frac{Q_{P_1}}{4\pi\epsilon_0 |\overline{P_1P}|^2}\overline{a}_{P_1P} + \frac{Q_{P_2}}{4\pi\epsilon_0 |\overline{P_2P}|^2}\overline{a}_{P_2P} \text{ V/m}.$$

$$\overline{a}_{P_1P} = \frac{\overline{P_1P}}{|\overline{P_1P}|}; \quad \overline{a}_{P_2P} = \frac{\overline{P_2P}}{|\overline{P_2P}|}$$

$$\overline{E}_p = \frac{Q_{P_1}}{4\pi\epsilon_0 |\overline{P_1P}|^3}\overline{P_1P} + \frac{Q_{P_2}}{4\pi\epsilon_0 |\overline{P_2P}|^3}\overline{P_2P} \text{ V/m}.$$

$$\overline{P_1P} = (2+3)\bar{a}_x + (-6-5)\bar{a}_y + (5+7)\bar{a}_z$$

$$|\overline{P_1P}| = 5\bar{a}_x - 11\bar{a}_y + 12\bar{a}_z$$

$$|\overline{P_1P}| = \sqrt{25+121+144} = \sqrt{290}\ m.$$

$$\overline{P_2P} = (2+4)\bar{a}_x + (-6-2)\bar{a}_y + (5-9)\bar{a}_z$$

$$\overline{P_2P} = 6\bar{a}_x - 8\bar{a}_y - 4\bar{a}_z$$

$$|\overline{P_2P}| = \sqrt{36+64+16} = \sqrt{116}\ m$$

$$\bar{E}_p = \frac{(3\mu)(9\times10^9)}{(\sqrt{290})^3}\left[5\bar{a}_x - 11\bar{a}_y + 12\bar{a}_z\right] + \frac{(-8\mu)(9\times10^9)}{(\sqrt{116})^3}\left[6\bar{a}_x - 8\bar{a}_y - 4\bar{a}_z\right]$$

$$\bar{E}_p = 5.4672\left[5\bar{a}_x - 11\bar{a}_y + 12\bar{a}_z\right] - 57.629\left[6\bar{a}_x - 8\bar{a}_y - 4\bar{a}_z\right]$$

$$\boxed{\bar{E}_p = -318.438\bar{a}_x + 400.89\bar{a}_y + 296.1224\bar{a}_z}\ V/m$$

$E_x = -318.438\ V/m$. $E_y = 400.89\ V/m$: $E_z = 296.122\ V/m$.

← magnitude of field at point $P(2,-6,5)$ is

$$\boxed{|\bar{E}_p| = 591.44\ V/m}$$

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problem 3.

A charge of $-0.3\,\mu C$ in Located at $A(25,-30,15)$ cm and a Second charge of $0.5\,\mu C$ at $B(-10,8,12)$ cm. Find $\bar{E}$ at i> the origin ii> $P(15,20,50)$ cm. (06m)

[10 - June/July - 2013] / [02 - June/July - 2011].

Solu:- Given points are in cm Convert it into meter's.

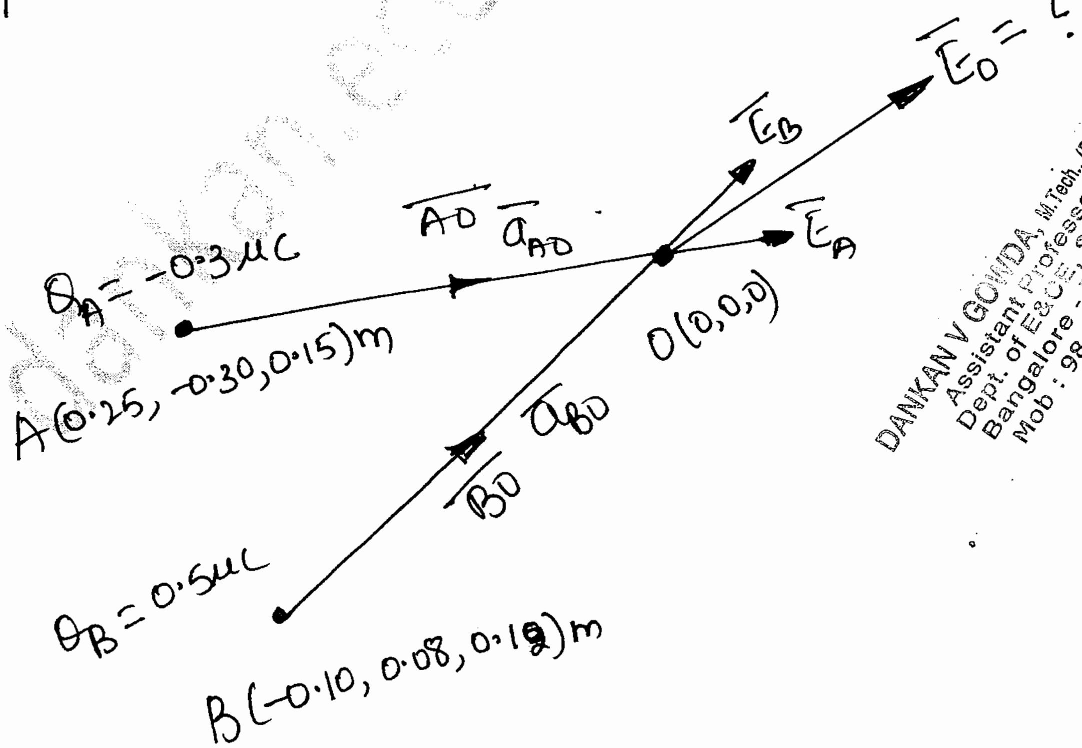
$A(25,-30,15)\,cm \longrightarrow A(0.25,-0.30,0.15)\,m.$

$B(-10,8,12)\,cm \longrightarrow B(-0.10,0.08,0.12)\,m.$

origin $O(0,0,0)\,cm \longrightarrow O(0,0,0)\,m.$

$P(15,20,50)\,cm \longrightarrow (0.15,0.20,0.50)\,m.$

i>



the Electric field intensity at point $O(0,0,0)$ is given by

$$\overline{E_0} = \overline{E_A} + \overline{E_B} \text{ V/m.}$$

$$\overline{E_0} = \frac{Q_A}{4\pi\epsilon_0 |\overline{AO}|^2}\overline{a_{AO}} + \frac{Q_B}{4\pi\epsilon_0 |\overline{BO}|^2}\overline{a_{BO}} \text{ V/m.}$$

$$\overline{E_0} = \frac{Q_A}{4\pi\epsilon_0 |\overline{AO}|^3}\overline{AO} + \frac{Q_B}{4\pi\epsilon_0 |\overline{BO}|^3}\overline{BO} \text{ V/m.}$$

$$\overline{AO} = -0.25\overline{a_x} + 0.3\overline{a_y} - 0.15\overline{a_z} \; ; \; |\overline{AO}| = \sqrt{0.175} \text{ m.}$$

$$\overline{BO} = 0.10\overline{a_x} - 0.08\overline{a_y} - 0.12\overline{a_z} \; ; \; |\overline{BO}| = \sqrt{0.0308} \text{ m.}$$

$$\overline{E_0} = \frac{(-0.3\mu)(9\times10^9)}{(\sqrt{0.175})^3}\left[-0.25\overline{a_x} + 0.3\overline{a_y} - 0.15\overline{a_z}\right]$$

$$+ \frac{(0.5\mu)(9\times10^9)}{(\sqrt{0.0308})^3}\left[0.1\overline{a_x} - 0.08\overline{a_y} - 0.12\overline{a_z}\right]$$

$$\overline{E_0} = -36881.33\left[-0.25\overline{a_x} + 0.3\overline{a_y} - 0.15\overline{a_z}\right]$$

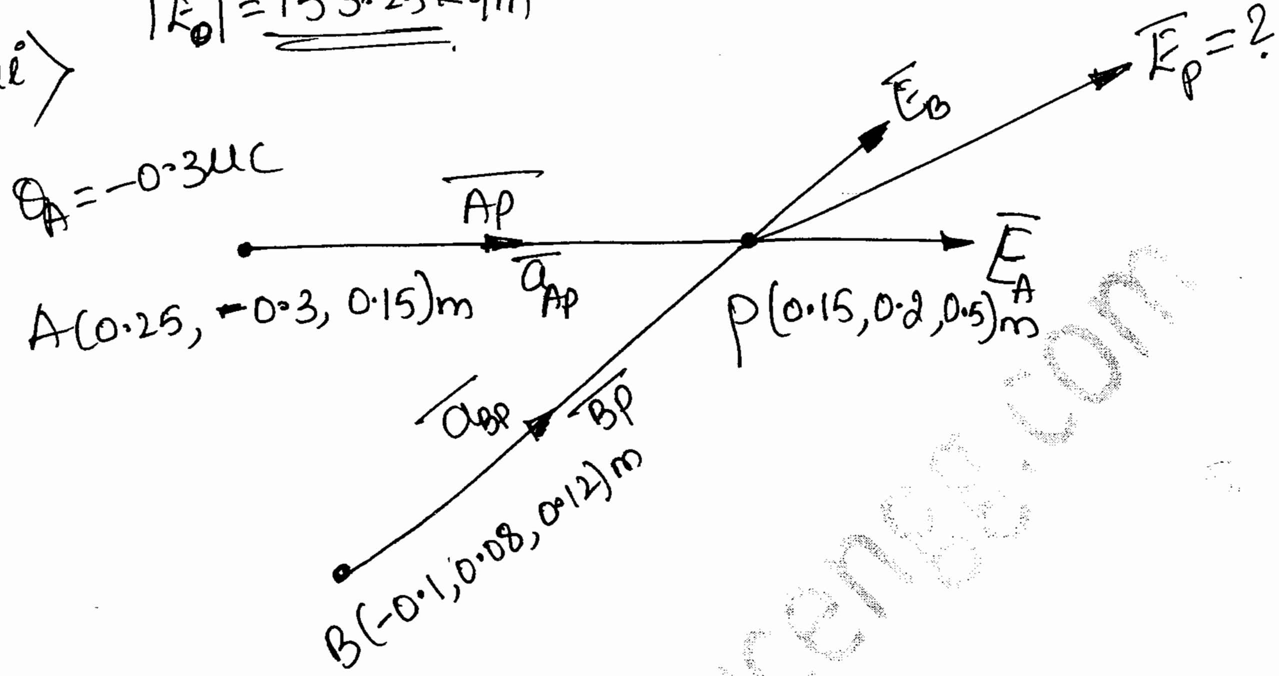
$$+ 832504.211\left[0.10\overline{a_x} - 0.08\overline{a_y} - 0.12\overline{a_z}\right]$$

$$\overline{E_0} = 92470.7536\overline{a_x} - 77664.735\overline{a_y} - 94368.30\overline{a_z} \text{ V/m}$$

$$\boxed{\bar{E}_0 = 92.47\,\bar{a}_x - 77.66\,\bar{a}_y - 94.36\,\bar{a}_z} \text{ kV/m}$$

$$|\bar{E}_0| = 153.25 \text{ kV/m}$$

ii)



$$\bar{E}_P = \frac{Q_A}{4\pi\epsilon|\overline{AP}|^2}\bar{a}_{AP} + \frac{Q_B}{4\pi\epsilon|\overline{BP}|^2}\bar{a}_{BP} \text{ V/m.}$$

$$\bar{E}_P = \frac{Q_A}{4\pi\epsilon}\frac{\overline{AP}}{|\overline{AP}|^3} + \frac{Q_B}{4\pi\epsilon}\frac{\overline{BP}}{|\overline{BP}|^3} \text{ V/m.}$$

$$\overline{AP} = -0.1\bar{a}_x + 0.5\bar{a}_y + 0.35\bar{a}_z \; ; \; |\overline{AP}| = \sqrt{0.3825} \text{ m.}$$

$$\overline{BP} = 0.25\bar{a}_x + 0.12\bar{a}_y + 0.38\bar{a}_z \; ; \; |\overline{BP}| = \sqrt{0.2213} \text{ m}$$

$$\bar{E}_P = \frac{(-0.3\mu)(9\times10^9)}{(\sqrt{0.3825})^3}\left[-0.1\bar{a}_x + 0.5\bar{a}_y + 0.35\bar{a}_z\right]$$

$$+ \frac{(0.5\mu)(9\times10^9)}{(\sqrt{0.2213})^3}\left[0.25\bar{a}_x + 0.12\bar{a}_y + 0.38\bar{a}_z\right]$$

$$\bar{E}_p = -11413.44[-0.1\bar{a}_x + 0.5\bar{a}_y + 0.35\bar{a}_z] + 43225.53[0.25\bar{a}_x + 0.12\bar{a}_y + 0.38\bar{a}_z]$$

$$\boxed{\bar{E}_p = 11947.72\bar{a}_x - 519.656\bar{a}_y + 12430.997\bar{a}_z} \text{ V/m}$$

$$\boxed{\bar{E}_p = 11.947\bar{a}_x - 0.5196\bar{a}_y + 12.4309\bar{a}_z} \text{ kV/m}.$$

the magnitude of Electric field intensity at point $P(0.15, 0.20, 0.50)$m is

$$|\bar{E}_p| = 17.2495 \text{ kV/m}.$$

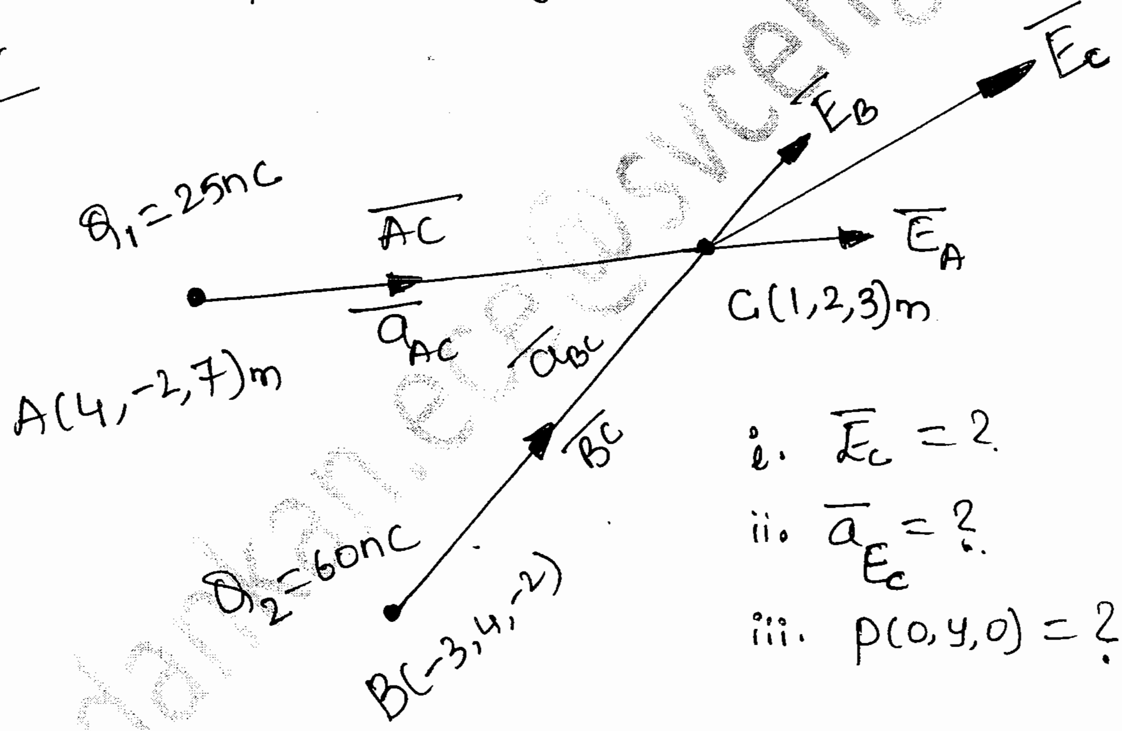
Problem 4.

Let a point charge $Q_1 = 25\,nC$ be Located at A(4, −2, 7) and a charge $Q_2 = 60\,nC$ be at B(−3, 4, −2). Find i. $\overline{E}$ at C(1, 2, 3). also ii. find the direction of the Electric field. Given $\epsilon_0 = 8.854 \times 10^{-12}$ F/m. (10M) 15 - June/July 2017 [10M - CBCS-Scheme]

[W.H. Hayt / 06 - June/July - 2011]

iii. at what point on the y-axis is $E_x = 0$ V/m

Solu:-



i. $\overline{E_c} = ?$

ii. $\overline{a_{E_c}} = ?$

iii. $P(0, y, 0) = ?$

i. $$\overline{E_C} = \overline{E_A} + \overline{E_B}$$

$$= \frac{Q_1}{4\pi\epsilon|\overline{AC}|^2}\overline{a_{AC}} + \frac{Q_2}{4\pi\epsilon|\overline{BC}|^2}\overline{a_{BC}} \text{ V/m}$$

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$$= \frac{Q_1}{4\pi\epsilon} \frac{\overline{AC}}{|\overline{AC}|^3} + \frac{Q_2}{4\pi\epsilon} \frac{\overline{BC}}{|\overline{BC}|^3} \; V/m.$$

$$\overline{AC} = (1-4)\bar{a}_x + (2+2)\bar{a}_y + (3-7)\bar{a}_z$$

$$\overline{AC} = -3\bar{a}_x + 4\bar{a}_y - 4\bar{a}_z \; ; \; |\overline{AC}| = \sqrt{41}\ m.$$

$$\overline{BC} = (1+3)\bar{a}_x + (2-4)\bar{a}_y + (3+2)\bar{a}_z$$

$$\overline{BC} = 4\bar{a}_x - 2\bar{a}_y + 5\bar{a}_z \; ; \; |\overline{BC}| = \sqrt{45}\ m.$$

$$\bar{E}_c = \frac{(25n)(9\times10^9)}{(\sqrt{41})^3}\left[-3\bar{a}_x + 4\bar{a}_y - 4\bar{a}_z\right] + \frac{(60n)(9\times10^9)}{(\sqrt{45})^3}\left[4\bar{a}_x - 2\bar{a}_y + 5\bar{a}_z\right]$$

$$\bar{E}_c = 0.85705\left[-3\bar{a}_x + 4\bar{a}_y - 4\bar{a}_z\right] + 1.788\left[4\bar{a}_x - 2\bar{a}_y + 5\bar{a}_z\right]$$

$$\boxed{\bar{E}_c = 4.58085\,\bar{a}_x - 0.1478\,\bar{a}_y + 5.5118\,\bar{a}_z} \; V/m$$

Magnitude of $\bar{E}_c$

$$|\bar{E}_c| = 7.1684 \; V/m.$$

$E_x = 4.5808$ V/m; $E_y = -0.1478$ V/m and

$E_z = 5.5118$ V/m.

ii. direction of Electric field $\bar{E}_c$ is nothing but unit vector along $\bar{E}_c$ i.e $\bar{a}_{E_c}$

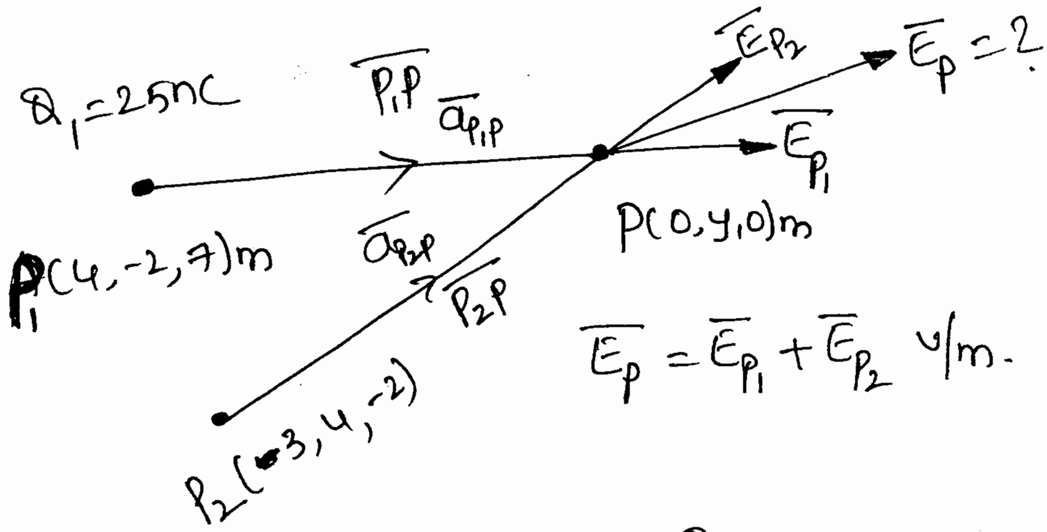
$$\bar{a}_{E_c} = \frac{\bar{E}_c}{|\bar{E}_c|} = \frac{4.5808\bar{a}_x - 0.1478\bar{a}_y + 5.5118\bar{a}_z}{7.1684}$$

$$\boxed{\bar{a}_{E_c} = 0.63906\,\bar{a}_x - 0.0206\,\bar{a}_y + 0.768\,\bar{a}_z}$$

iii. at what point on the y-axis the field component $E_x = 0$ V/m.

Consider a point on 'y' axis $p(0, y, 0)$.

Find 'y' such that $E_{P_x} = 0$ V/m.



$$\overline{E_p} = \overline{E_{P_1}} + \overline{E_{P_2}} \ \text{v/m.}$$

$$\overline{E_p} = \frac{Q_1}{4\pi\epsilon|\overline{P_1P}|^2}\overline{a_{P_1P}} + \frac{Q_2}{4\pi\epsilon|\overline{P_2P}|^2}\overline{a_{P_2P}} \quad \text{v/m}$$

$$\overline{E_p} = \frac{Q_1}{4\pi\epsilon}\frac{\overline{P_1P}}{|\overline{P_1P}|^3} + \frac{Q_2}{4\pi\epsilon}\frac{\overline{P_2P}}{|\overline{P_2P}|^3} \quad \text{v/m.}$$

$$\overline{P_1P} = -4\overline{a_x} + (y+2)\overline{a_y} + (0-7)\overline{a_z}$$

$$\overline{P_1P} = -4\overline{a_x} + (y+2)\overline{a_y} - 7\overline{a_z}.$$

$$|\overline{P_1P}| = \sqrt{4^2+(y+2)^2+7^2} \ m$$

$$\overline{P_2P} = 3\overline{a_x} + (y-4)\overline{a_y} + 2\overline{a_z};$$

$$|\overline{P_2P}| = \sqrt{9+(y-4)^2+4} \ m$$

$$\overline{E_p} = \frac{25n(9\times10^9)}{\left[4^2+(y+2)^2+7^2\right]^{3/2}}\left[-4\overline{a_x}+(y+2)\overline{a_y}-7\overline{a_z}\right]$$

$$+ \frac{6n(9\times10^9)}{\left[9+(y-4)^2+4\right]^{3/2}}\left[3\overline{a_x}+(y-4)\overline{a_y}+2\overline{a_z}\right]$$

given the Field component $E_{p_x} = 0$ V/m.

i.e

$$E_{p_x} = \frac{(25n)(9\times10^9)(-4)}{[16+(y+2)^2+49]^{3/2}} + \frac{(60n)(9\times10^9)(3)}{[9+(y-4)^2+4]^{3/2}} = 0 \text{ V/m}.$$

$$\Rightarrow \frac{(25\cancel{n})(9\cancel{\times10^9})(-4)}{[65+(y+2)^2]^{3/2}} = \frac{(-60\cancel{n})(9\cancel{\times10^9})(3)}{[13+(y-4)^2]^{3/2}}$$

$$\Rightarrow 100[13+(y-4)^2]^{3/2} = 180[65+(y+2)^2]^{3/2}$$

$$100^{2/3}[13+(y-4)^2] = (180)^{2/3}[65+(y+2)^2]$$

$$[13+y^2+16-8y] = 1.4797[65+y^2+4+4y]$$

$$\Rightarrow 0.479y^2 + 13.92y + 73.10 = 0.$$

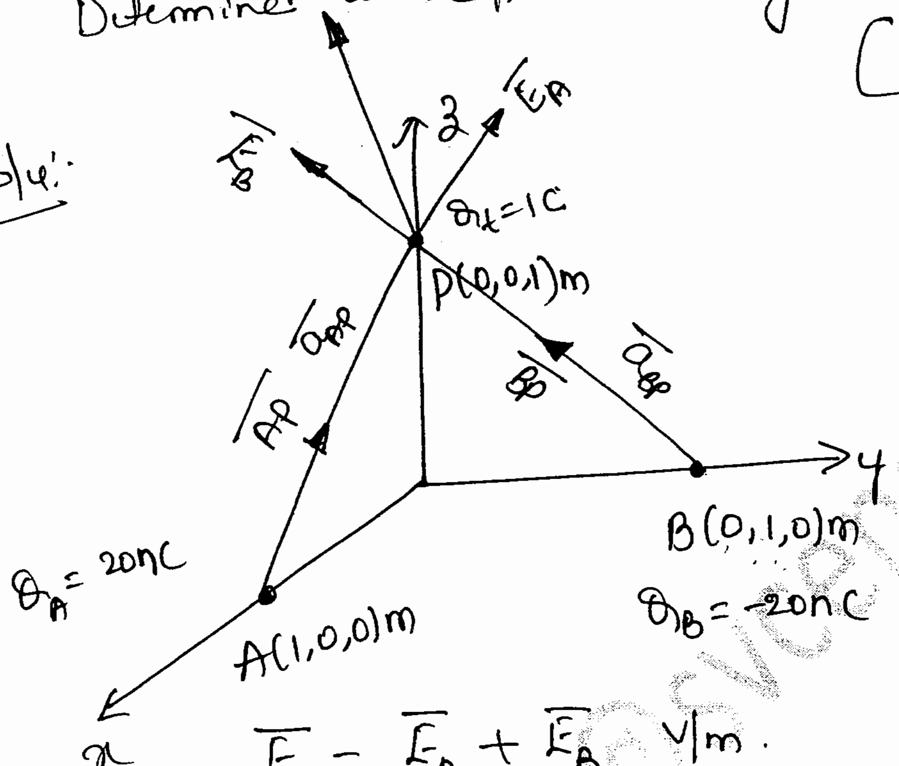
$$y_1 = -6.883 \text{ and } y_2 = -22.12.$$

$\therefore$ the points on 'y' axis such that the x component of net field $E_{p_x} = 0$ is

$$\boxed{P_1(0, -6.883, 0)} \text{ and } \boxed{P_2(0, -22.12, 0)}$$

problems. Two point charges 20nC and -20nC are situated at (1,0,0)m and (0,1,0)m in freespace. Determine the electric field intensity at (0,0,1)m. (5M) [10 - June/July 2014]

Sol^n:



$$\bar{E}_P = \bar{E}_A + \bar{E}_B \text{ V/m.}$$

$$\bar{E}_P = \frac{Q_A}{4\pi\epsilon_0 |\bar{AP}|^2}\bar{a}_{AP} + \frac{Q_B}{4\pi\epsilon_0 |\bar{BP}|^2}\bar{a}_{BP} \text{ V/m}$$

$$\bar{E}_P = \frac{Q_A}{4\pi\epsilon_0 |\bar{AP}|^3}\bar{AP} + \frac{Q_B}{4\pi\epsilon_0 |\bar{BP}|^3}\bar{BP} \text{ V/m.}$$

$$\bar{AP} = -\bar{a}_x + \bar{a}_z \; ; \quad |\bar{AP}| = \sqrt{2} \text{ m.}$$

$$\bar{BP} = -\bar{a}_y + \bar{a}_z \; ; \quad |\bar{BP}| = \sqrt{2} \text{ m.}$$

$$\bar{E}_P = \frac{(20n)(9\times10^9)}{(\sqrt{2})^3}[-\bar{a}_x + \bar{a}_z] + \frac{(-20n)(9\times10^9)}{(\sqrt{2})^3}[-\bar{a}_y + \bar{a}_z]$$

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$$\overline{E}_p = 63.639[-\overline{a}_x+\overline{a}_z] - 63.639[-\overline{a}_y+\overline{a}_z]$$

$$\boxed{\overline{E}_p = -63.639\,\overline{a}_x + 63.639\,\overline{a}_y + 127.27\,\overline{a}_z} \text{ V/m.}$$

$$|\overline{E}_p| = \underline{155.872} \text{ V/m}$$

$E_x = -63.639$ V/m, $E_y = 63.639$ V/m

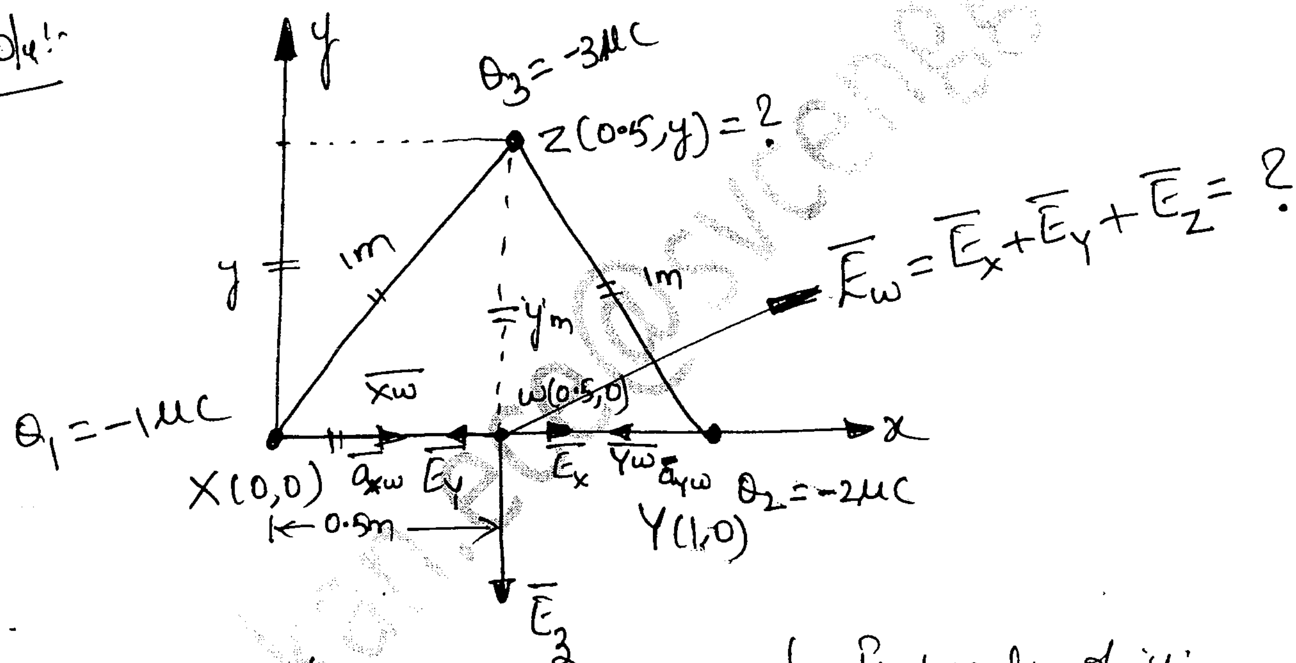
and $E_z = 127.27\,\overline{a}_z$.

Problem 6.

Three point charges $Q_1 = -1\mu C$, $Q_2 = -2\mu C$ and $Q_3 = -3\mu C$ are placed at the corners of an equilateral traingle of side 1m. Find the magnitude of the Electric field intensity at the point bisecting the joining Q_1 and Q_2. (7m)

[10- June/July -2016].

Sol^n:



using pythagorous theorem, to find value of 'y'

$$|XZ| = \sqrt{|ZW|^2 + |XW|^2}$$

$$1 = \sqrt{y^2 + 0.5^2}$$

$$y^2 + 0.5^2 = 1^2 \Rightarrow \boxed{y = 0.866\ m}$$

Choose +ve value of 'y' bcz the point $Z(0.5, y)$ is on first quadrant.

the Electric field Intensity ($\overline{E}$) at the point bisecting the joining Q_1 and Q_2 in at point W i.e

$$\overline{E}_W = ?$$

$$\overline{E}_W = \overline{E}_x + \overline{E}_y + \overline{E}_z \quad V/m.$$

$$\overline{E}_W = \frac{Q_1}{4\pi\epsilon|\overline{XW}|^2}\overline{a}_{XW} + \frac{Q_2}{4\pi\epsilon|\overline{YW}|^2}\overline{a}_{YW} + \frac{Q_3}{4\pi\epsilon|\overline{ZW}|^2}\overline{a}_{ZW} \; V/m$$

$$\overline{E}_W = \frac{Q_1}{4\pi\epsilon}\frac{\overline{XW}}{|\overline{XW}|^3} + \frac{Q_2}{4\pi\epsilon}\frac{\overline{YW}}{|\overline{YW}|^3} + \frac{Q_3}{4\pi\epsilon|\overline{ZW}|^3}\overline{ZW}$$

$$\overline{XW} = 0.5\,\overline{a}_x \; ; \; |\overline{XW}| = 0.5m$$

$$\overline{YW} = -0.5\,\overline{a}_x \; ; \; |\overline{YW}| = 0.5m.$$

$$\overline{ZW} = -0.866\,\overline{a}_y \; ; \; |\overline{ZW}| = 0.866m$$

$$\overline{E}_W = \frac{(-1\mu)(9\times10^9)}{(0.5)^3}[0.5\,\overline{a}_x] + \frac{(-2\mu)(9\times10^9)}{(0.5)^3}[-0.5\,\overline{a}_x] + \frac{(-3\mu)(9\times10^9)}{(0.866)^3}[-0.866\,\overline{a}_y]$$

$$\overline{E}_W = -36000\,\overline{a}_x + 72000\,\overline{a}_y + 36002\,\overline{a}_y \quad V/m$$

$$\overline{E}_W = 36000\,\overline{a}_x + 36002\,\overline{a}_y \quad V/m$$

$$\boxed{\overline{E}_W = 36\,\overline{a}_x + 36.002\,\overline{a}_y} \; kV/m$$

magnitude of field $\overline{E}_W$ is $|\overline{E}_W| = 50.913$ kV/m

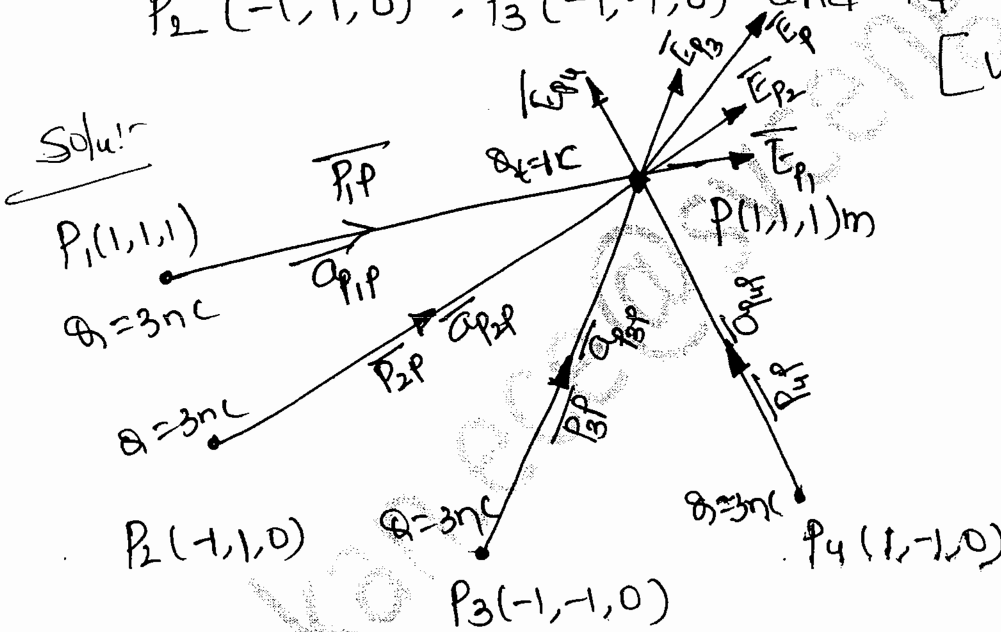
$E_x = 36\ kV/m$; $E_y = 36.002\ kV/m$.

and $|E_\omega| = \sqrt{E_x^2 + E_y^2} = 50.913\ kV/m$

problem.

Find $\bar{E}$ at $p(1,1,1)$ caused by four identical 3nC (nano-Coulomb) charges located at $P_1(1,1,0)$, $P_2(-1,1,0)$, $P_3(-1,-1,0)$ and $P_4(1,-1,0)$. [W.H. Hayt]

Solu:



using Superposition principle

$$\overline{E_P} = \bar{E}_{P_1} + \bar{E}_{P_2} + \bar{E}_{P_3} + \bar{E}_{P_4}\ V/m.$$

$$\bar{E}_P = \frac{Q}{4\pi\epsilon|\overline{P_1P}|^2}\bar{a}_{P_1P} + \frac{Q}{4\pi\epsilon|\overline{P_2P}|^2}\bar{a}_{P_2P} + \frac{Q}{4\pi\epsilon|\overline{P_3P}|^2}\bar{a}_{P_3P}$$

$$+ \frac{Q}{4\pi\epsilon|\overline{P_4P}|^2}\bar{a}_{P_4P}\ V/m.$$

$$\overline{E_p} = \frac{Q}{4\pi\epsilon}\left[\frac{\overline{P_1P}}{|\overline{P_1P}|^3} + \frac{\overline{P_2P}}{|\overline{P_2P}|^3} + \frac{\overline{P_3P}}{|\overline{P_3P}|^3} + \frac{\overline{P_4P}}{|\overline{P_4P}|^3}\right] \text{ V/m}$$

$\overline{P_1P} = \overline{a_z}$; $|\overline{P_1P}| = 1\text{m}$.

$\overline{P_2P} = 2\overline{a_x} + \overline{a_z}$; $|\overline{P_2P}| = \sqrt{4+1} = \sqrt{5}\,\text{m}$.

$\overline{P_3P} = 2\overline{a_x} + 2\overline{a_y} + \overline{a_z}$; $|\overline{P_3P}| = \sqrt{4+4+1} = \sqrt{9} = 3\,\text{m}$.

$\overline{P_4P} = 2\overline{a_y} + \overline{a_z}$; $|\overline{P_4P}| = \sqrt{4+1} = \sqrt{5}\,\text{m}$.

$$\overline{E_p} = (3n)(9\times10^9)\left[\frac{\overline{a_z}}{(1)^3} + \frac{2\overline{a_x}+\overline{a_z}}{(\sqrt{5})^3} + \frac{2\overline{a_x}+2\overline{a_y}+\overline{a_z}}{(\sqrt{9})^3} + \frac{2\overline{a_y}+\overline{a_z}}{(\sqrt{5})^3}\right]$$

$$\overline{E_p} = 27\left[0.2529\,\overline{a_x} + 0.2529\,\overline{a_y} + 1.2159\,\overline{a_z}\right] \text{ V/m}$$

$$\boxed{\overline{E_p} = 6.8283\,\overline{a_x} + 6.8283\,\overline{a_y} + 32.829\,\overline{a_z}} \text{ V/m}$$

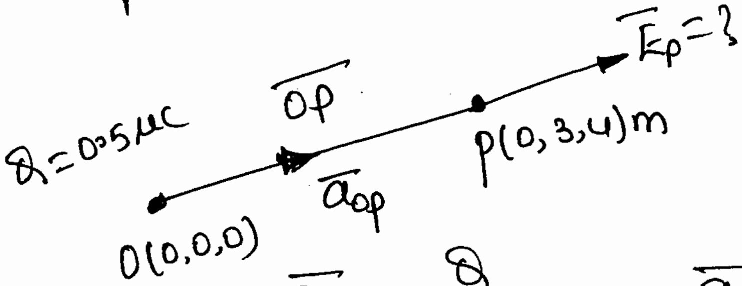
$E_x = E_y = 6.8283\ \text{V/m}$ and $E_z = 32.829\ \text{V/m}$

$$|\overline{E_p}| = \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{6.8283^2 + 6.8283^2 + 32.829^2}$$

$$\boxed{|\overline{E_p}| = 34.219 \text{ V/m}}$$

problem 8.

Find the $\overline{E}$ at $(0,3,4)$m due to a point charge of $Q = 0.5\mu C$ placed at the origin.



$$\overline{E_p} = \frac{Q}{4\pi\epsilon|\overline{OP}|^2}\,\overline{a_{op}} \quad \text{V/m}.$$

$$\overline{E_p} = \frac{Q}{4\pi\epsilon}\,\frac{\overline{OP}}{|\overline{OP}|^3} \quad \text{V/m}$$

$$\overline{OP} = 3\overline{a_y} + 4\overline{a_z}; \quad |\overline{OP}| = \sqrt{9+16} = \underline{\underline{5\,m}}.$$

$$\overline{E_p} = \frac{(0.5\mu)(9\times10^9)}{(5)^3}\left[3\overline{a_y} + 4\overline{a_z}\right]$$

$$\overline{E_p} = 36\left[3\overline{a_y} + 4\overline{a_z}\right]$$

$$\boxed{\overline{E_p} = 108\overline{a_y} + 144\overline{a_z}} \text{ V/m}$$

$E_x = 0$ V/m; $E_y = 108$ V/m and $E_z = 144$ V/m.

$$|\overline{E_p}| = \sqrt{108^2 + 144^2} = 180 \text{ V/m}.$$

$$\boxed{|\overline{E_p}| = 180} \text{ V/m}$$

Problem 9.

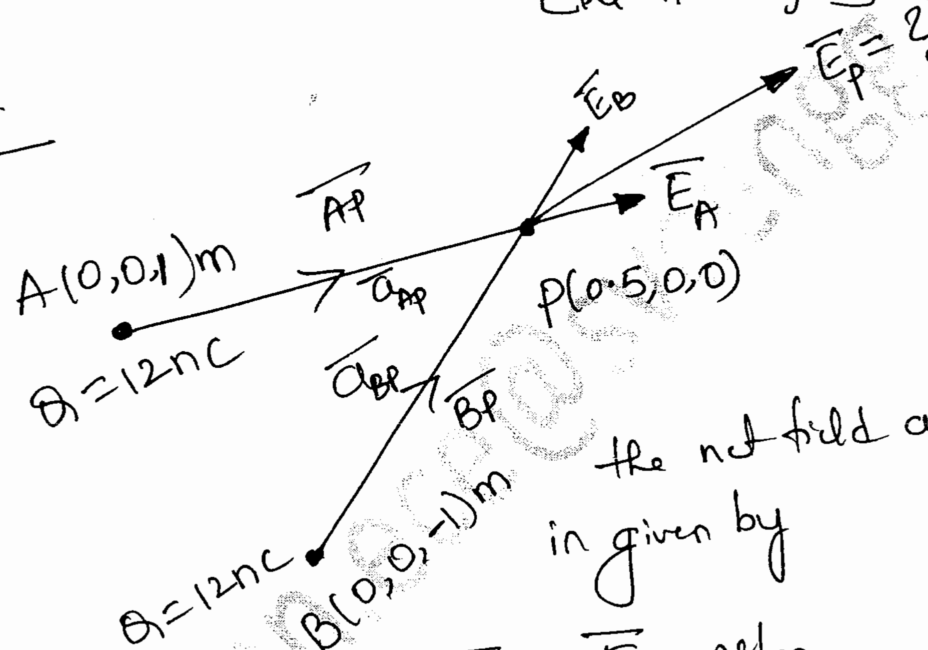
Point charges of 120nC are Located at A(0,0,1) and B(0,0,-1) in free space.

a) Find $\bar{E}$ at P(0.5,0,0)

b) what single charge at the origin would provide the identical field strength?

[W. H. Hayt].

Solu:-



the net field at point P is given by

$$\bar{E}_P = \bar{E}_A + \bar{E}_B \quad \text{v/m.}$$

$$\bar{E}_P = \frac{Q}{4\pi\epsilon|\overline{AP}|^2}\bar{a}_{AP} + \frac{Q}{4\pi\epsilon|\overline{BP}|^2}\bar{a}_{BP} \quad \text{v/m}$$

$$\bar{E}_P = \frac{Q}{4\pi\epsilon}\frac{\overline{AP}}{|\overline{AP}|^3} + \frac{Q}{4\pi\epsilon}\cdot\frac{\overline{BP}}{|\overline{BP}|^3} \quad \text{v/m.}$$

$$\overline{AP} = 0.5\,\overline{a_x} - \overline{a_z} \;:\; |\overline{AP}| = \sqrt{0.5^2+1^2} = \sqrt{1.25}\,m$$

$$\overline{BP} = 0.5\,\overline{a_x} + \overline{a_z} \;:\; |\overline{BP}| = \sqrt{0.5^2+1^2} = \sqrt{1.25}\;m.$$

$$\Rightarrow \quad |\overline{AP}| = |\overline{BP}| = \sqrt{1.25}\;m$$

$$\overline{E_p} = \frac{Q}{4\pi\epsilon(\sqrt{1.25})^3}\left[\overline{AP} + \overline{BP}\right] \;V/m.$$

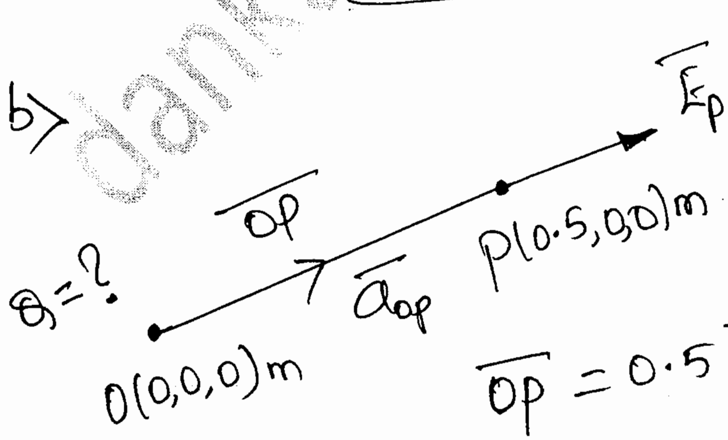
$$\overline{E_p} = \frac{(120\cancel{\pi})(9\times10^{9})}{(\sqrt{1.25})^3}\left[0.5\,\overline{a_x} - \cancel{\overline{a_z}} + 0.5\,\overline{a_x} + \cancel{\overline{a_z}}\right]$$

$$\boxed{\overline{E_p} = 772.78\,\overline{a_x}}\;V/m.$$

$$|\overline{E_p}| = 772.78 \;V/m$$

$$E_x = 772.78\;V/m \quad \text{and} \quad E_y = E_z = 0\;V/m.$$

b)



$$|\overline{E_p}| = 772.78\;V/m.$$

$$\overline{OP} = 0.5\,\overline{a_x}$$

$$|\overline{OP}| = 0.5.$$

$$\overline{E}_p = \frac{Q}{4\pi\epsilon|\overline{op}|^2}\,\overline{a}_{op} \text{ v/m.}$$

$$|\overline{op}| = 0.5\text{m.}$$

$$|\overline{E}_p| = \frac{Q}{4\pi\epsilon|\overline{op}|^2} \text{ v/m.}$$

$$Q = |\overline{E}_p| \times 4\pi\epsilon \times (|\overline{op}|)^2 \text{ Coulomb}$$

$$Q = (772.78)(9\times10^9)^{-1} \times (0.5)^2$$

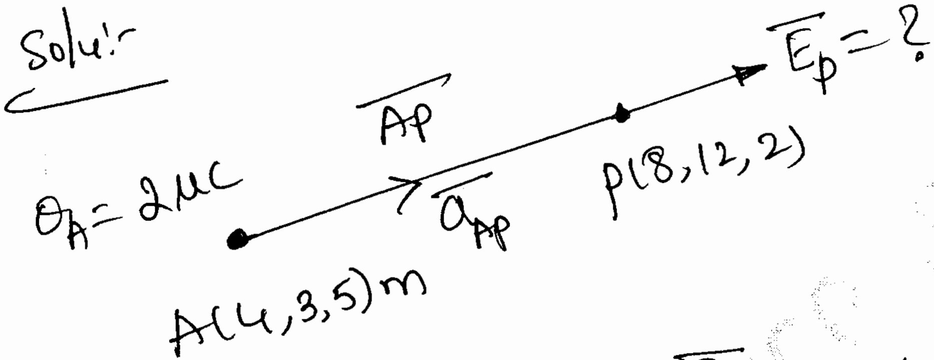
$$Q = 21.4662\times10^{-9} \text{ Coulomb}$$

$$\boxed{Q = 21.4662\,\eta C}$$

problem10

A $2\mu C$ point charge in Located at $A(4,3,5)$ in free space. Find E_ρ, E_ϕ, and E_z at $P(8,12,2)$. [W.H.Hayt]

Soln:



$$\overline{E_p} = \frac{Q_A}{4\pi\epsilon_0|\overline{AP}|^2}\overline{a_{AP}} \ \text{V/m}.$$

$$\overline{E_p} = \frac{Q_A}{4\pi\epsilon_0}\frac{\overline{AP}}{|\overline{AP}|^3} \ \text{V/m}.$$

$$\overline{AP} = 4\overline{a_x} + 9\overline{a_y} - 3\overline{a_z} \ ; \quad |\overline{AP}| = \sqrt{16+81+9} = \sqrt{106} \ m$$

$$\overline{E_p} = \frac{(2\mu)(9\times10^9)}{(\sqrt{106})^3}\left[4\overline{a_x} + 9\overline{a_y} - 3\overline{a_z}\right]$$

$$\overline{E_p} = 16.49\left[4\overline{a_x} + 9\overline{a_y} - 3\overline{a_z}\right] \ \text{V/m}.$$

$$\boxed{\overline{E}_p = 65.97\,\overline{a}_x + 148.44\,\overline{a}_y - 49.48\,\overline{a}_z} \text{ V/m.}$$

Convert the Rectangular vector into the equivalent cylindrical form.

$E_x = 65.97$ V/m ; $E_y = 148.44$ V/m & $E_z = -49.48$ V/m

$$P(8, 12, 2) \Longrightarrow P(\rho, \phi, z)$$

$$\rho = \sqrt{x^2+y^2} = \sqrt{8^2+12^2} = \sqrt{208} = 14.4 \text{ m.}$$

$$\phi = \tan^{-1}(y/x) = \tan^{-1}\left(\frac{12}{8}\right) = \underline{\underline{56.3^\circ}}$$

$$z \Rightarrow \boxed{z = 2} \text{ m}$$

$$\begin{bmatrix} E_\rho \\ E_\phi \\ E_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}_{3\times1}$$

$\overline{E}_p = E_x\overline{a_x} + E_y\overline{a_y} + E_z\overline{a_z}$ V/m. ... rectangular ~~form~~ C.S

$\Updownarrow$

$\overline{E}_p = E_\rho\overline{a_\rho} + E_\phi\overline{a_\phi} + E_z\overline{a_z}$ V/m ... cylindrical (~~form~~

$$E_\rho = E_x \cos\phi + E_y \sin\phi$$

$$E_\rho = 65.97 \cos(56.3) + 148.44 \sin(56.3^\circ)$$

$$\boxed{E_\rho = 160.09} \text{ V/m}$$

$$E_\phi = -E_x \sin\phi + E_y \cos\phi = -65.97 \sin(56.3^\circ) + 148.44 \cos(56.3^\circ)$$

$$\boxed{E_\phi = 27.477 \text{ V/m}}$$

and $\boxed{E_z = -49.48}$ V/m

$$\boxed{\bar{E}_P = 160.09\,\bar{a}_\rho + 27.477\,\bar{a}_\phi - 49.48\,\bar{a}_z} \text{ V/m}$$

→ field $\bar{E}_P$ in Cylindrical co-ordinate system.

Two particles having charges 2nC and 5nC are ~~[illegible]~~ spaced 80cm apart. Determine $\vec{E}$ at point 'A' situated at a distance of 0.5m from each of the two particles. assume dielectric constant of 5.

Problem 11.

b. ~~Two particles having charges 2nano-coulomb and 5nano-coulomb are spaced~~ 80 cm apart. Determine the electric field intensity at point "A" situated at a distance of 0.5 m from each of the two particles. Assume dielectric constant of 5. [CBCS 15-Dec/Jan-2017] (08 Marks)

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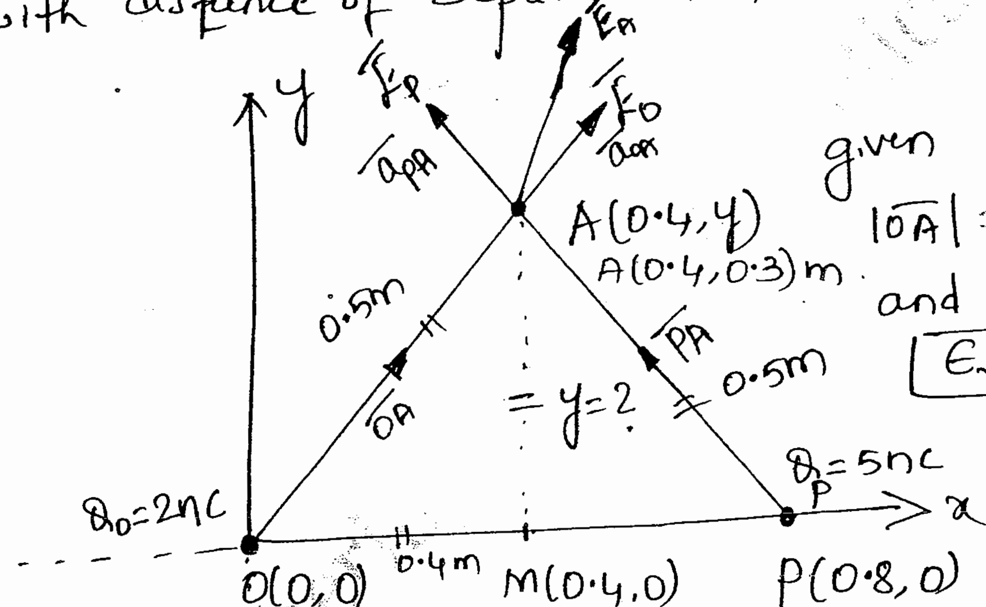
Soln:-

Consider the point charges placed along 'x' axis. with distance of Separation is 80 cm.



given $|\vec{OA}| = |\vec{PA}| = 0.5m$

and $\boxed{\epsilon_r = 5}$

M - mid point b/w O and P.

To find 'y' use pythagoras's theorem

i.e $OA^2 = OM^2 + y^2$

$\Rightarrow y^2 = OA^2 - OM^2 = 0.5^2 - 0.4^2$

$y^2 = 0.09$

$\boxed{y = 0.3 \text{ m}}$

$\therefore$ the point $A(0.4, y) = A(0.4, 0.3)$m.

07

The Electric field Intensity at a point A due to the two point charges at point's O and P is given by

$$\bar{E}_A = \bar{E}_P + \bar{E}_O \text{ V/m.}$$

$$\bar{E}_A = \frac{Q_P}{4\pi\epsilon|\overline{PA}|^2}\bar{a}_{PA} + \frac{Q_O}{4\pi\epsilon|\overline{OA}|^2}\bar{a}_{OA} \text{ V/m}$$

given $Q_P = 5nC$ and $Q_O = 2nC$; $\boxed{\epsilon = 5\epsilon_0}$ F/m

$\overline{PA} = -0.4\bar{a}_x + 0.3\bar{a}_y$ and $\overline{OA} = 0.4\bar{a}_x + 0.3\bar{a}_y$;

$|\overline{PA}| = |\overline{OA}| = 0.5m.$; $\bar{a}_{PA} = \frac{\overline{PA}}{|\overline{PA}|}$ and $\bar{a}_{OA} = \frac{\overline{OA}}{|\overline{OA}|}$

$$\bar{E}_A = \frac{Q_P}{4\pi\epsilon|\overline{PA}|^3}\overline{PA} + \frac{Q_O}{4\pi\epsilon|\overline{OA}|^3}\overline{OA} \text{ V/m.}$$

$$\bar{E}_A = \frac{5\times10^{-9}}{4\pi\epsilon}\frac{[-0.4\bar{a}_x + 0.3\bar{a}_y]}{(0.5)^3} + \frac{2\times10^{-9}}{4\pi\epsilon}\frac{[0.4\bar{a}_x + 0.3\bar{a}_y]}{(0.5)^3}$$

$$\bar{E}_A = \frac{1\times10^{-9}\times9\times10^{9}}{5(0.5)^3}\left[5(-0.4\bar{a}_x + 0.3\bar{a}_y) + 2(0.4\bar{a}_x + 0.3\bar{a}_y)\right]$$

$$\bar{E}_A = 14.4\left[-1.2\bar{a}_x + 2.1\bar{a}_y\right]$$

$$\boxed{\bar{E}_A = -17.28\bar{a}_x + 30.24\bar{a}_y} \text{ V/m}$$

$|\bar{E}_A| = 34.82$ V/m

Problem 12 - A 100nC point charge in located at A(-1,1,3) in free space.

i) Find the Locus of all points P(x,y,z) at which $E_x = 500$ V/m.

ii) Find y_1 if P(-2, y_1, 3) lies on that Locus. (8M)

[W.H.Hayt | 10-Dec-Jan 2014].

problem 13

three point charges each of 5nC are located on the x-axis at x = -1, 0 and 1m in free Space. Find

i. $\bar{E}$ at x = 5m.

ii. Determine the value and Location of the equivalent Single point charge that would produce the Same field at very large distance.

iii. Determine $\bar{E}$ at x = 5m using approximation of (ii).

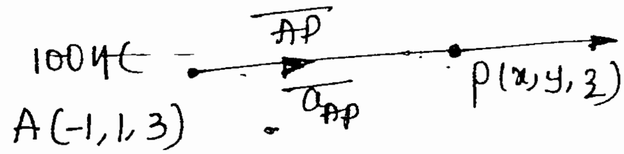
[W.H.Hayt].

A 100nc point charge is located at A(-1, 1, 3) in free space.
i) Find the locus of all points P(x, y, z) at which E_x = 500 V/m.
ii) Find y_1 if P(-2, y_1, 3) lies on that locus.

(08 Marks) 10 Dec-Jan 2014 (8M)

Problem 12

Soln:- i)



given 500 V/m

$\bar{E}_p = E_x \bar{a}_x + E_y \bar{a}_y + E_z \bar{a}_z$ V/m

$$\bar{E}_p = \frac{Q_A}{4\pi\epsilon_0 |\overline{AP}|^2} \bar{a}_{AP} = \frac{Q_A}{4\pi\epsilon_0 |\overline{AP}|^3} \overline{AP} \text{ V/m.}$$

$$\overline{AP} = (x+1)\bar{a}_x + (y-1)\bar{a}_y + (z-3)\bar{a}_z$$

$$|\overline{AP}| = \sqrt{(x+1)^2 + (y-1)^2 + (z-3)^2} \text{ m}$$

$$\bar{E}_p = \frac{100n \times 9 \times 10^9}{[(x+1)^2 + (y-1)^2 + (z-3)^2]^{3/2}} [(x+1)\bar{a}_x + (y-1)\bar{a}_y + (z-3)\bar{a}_z] \quad \leftarrow (1)$$

given

$$E_x = 500 \text{ V/m.}$$

the E_x component in eq^n (1) is

$$\frac{100\cancel{n} \times 9 \times 10^{\cancel{9}}}{[(x+1)^2 + (y-1)^2 + (z-3)^2]^{3/2}} \times (x+1) = 500 \text{ V/m.}$$

$$9(x+1) = 5[(x+1)^2 + (y-1)^2 + (z-3)^2]^{3/2}$$

$$\boxed{(x+1) = 5/9\,[(x+1)^2 + (y-1)^2 + (z-3)^2]^{3/2}}$$ in the

Locus of all points p(x, y, z) at which E_x = 500 V/m.

ii) To find y_1, given point $P(-2, y_1, 3)$ Lies on Locus

i.e $(x+1) = 5/9 \left[(x+1)^2 + (y-1)^2 + (z-3)^2\right]^{3/2}$

put $x = -2$, $y = y_1$ and $z = 3$.

$(-2+1) = 5/9 \left[(-2+1)^2 + (y_1-1)^2 + (\cancel{3-3})^2\right]^{3/2}$

$-1 = 5/9 \left[1 + (y_1-1)^2\right]^{3/2}$

$\left[1 + (y_1-1)^2\right]^{3/2} = (-9/5)$

Square on both side

$\left[1+(y_1-1)^2\right]^3 = (-9/5)^2 \Rightarrow \left[1+(y_1-1)^2\right]^3 = 3.24$

$1 + (y_1-1)^2 = (3.24)^{1/3}$

$1 + (y_1-1)^2 = 1.4797$

$1 + y_1^2 + 1 - 2y_1 = 1.4797$

$y_1^2 - 2y_1 + 0.52027 = 0$

$\boxed{y_1 = 1.6926}$ and $\boxed{y_1 = 0.3073}$

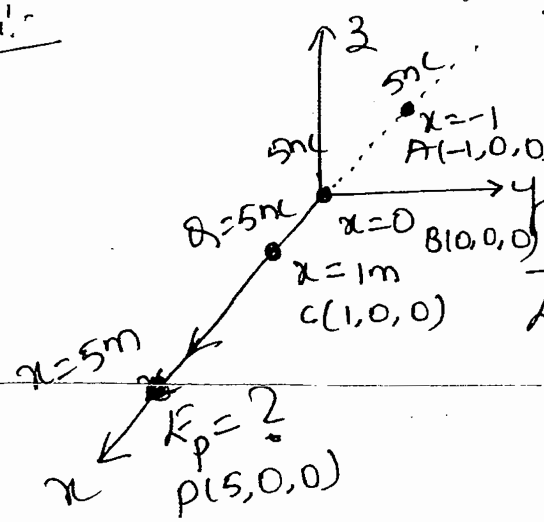
Hayt **Problem 3**

Three point charges each of 5nC are located on the x-axis at x=-1, 0 and 1 m in free space. Find

i. E at x=5m
ii. Determine the value and location of the equivalent single point charge that would produce the same field at very large distance.
iii. Determine E at x=5m using approximation of (ii)

[W.H. Hayt].

Soln:-



i) $\bar{E}$ at $x=5m$ i.e

$$\bar{E}_p = \bar{E}_A + \bar{E}_B + \bar{E}_C \text{ V/m}$$

$$\bar{E}_p = \frac{Q}{4\pi\epsilon|\bar{AP}|^2}\bar{a}_{AP} + \frac{Q}{4\pi\epsilon|\bar{BP}|^2}\bar{a}_{BP} + \frac{Q}{4\pi\epsilon|\bar{CP}|^2}\bar{a}_{CP} \text{ V/m}$$

$$\bar{E}_p = \frac{Q}{4\pi\epsilon}\left[\frac{\bar{AP}}{|\bar{AP}|^3} + \frac{\bar{BP}}{|\bar{BP}|^3} + \frac{\bar{CP}}{|\bar{CP}|^3}\right] \text{ V/m}$$

$\bar{AP} = 6\bar{a}_x$; $|\bar{AP}| = 6\,m$; $\bar{BP} = 5\bar{a}_x$; $|\bar{BP}| = 5\,m$.

$\bar{CP} = 4\bar{a}_x$; $|\bar{CP}| = 4\,m$.

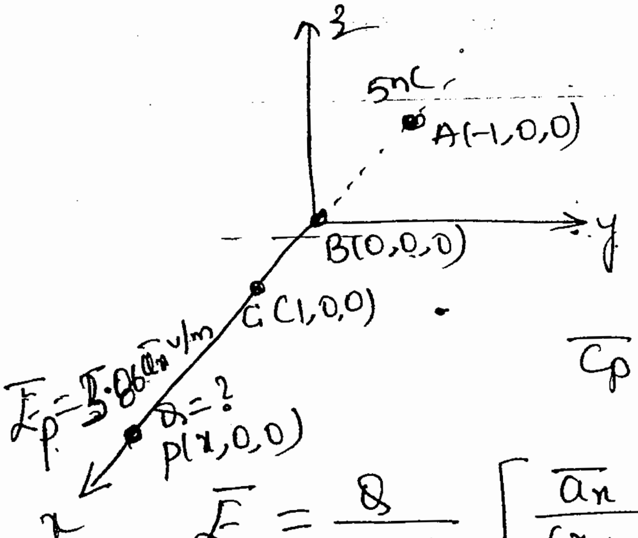
$$\bar{E}_p = 5n \times 9\times10^9\left[\frac{6\bar{a}_x}{(6)^3} + \frac{5\bar{a}_x}{(5)^3} + \frac{4\bar{a}_x}{(4)^3}\right] \text{ V/m}$$

$$\bar{E}_p = 45 \times 0.13027\,\bar{a}_x = 5.8625\,\bar{a}_x \text{ V/m}$$

$\boxed{\bar{E}_p = 5.8625\,\bar{a}_x}$ V/m and $|\bar{E}_p| = 5.8625$ V/m.

ii. To find the value and location of single point charge that would produce same $\bar{E}$ at large distance.

Let the location be general 'x' and 'x' is very large.



$$\bar{E}_p = \frac{Q}{4\pi\epsilon_0}\left[\frac{\overline{AP}}{|\overline{AP}|^3} + \frac{\overline{BP}}{|\overline{BP}|^3} + \frac{\overline{CP}}{|\overline{CP}|^3}\right] \text{ v/m}$$

$\overline{AP} = (x+1)\bar{a}_x$; $|\overline{AP}| = \sqrt{(x+1)^2}$ m $= (x+1)$ m

$\overline{BP} = x\,\bar{a}_x$; $|\overline{BP}| = x$ m.

$\overline{CP} = (x-1)\bar{a}_x$; $|\overline{CP}| = \sqrt{(x-1)^2} = (x-1)$ m

$$\bar{E}_p = \frac{Q}{4\pi\epsilon_0}\left[\frac{\bar{a}_x}{(x+1)^2} + \frac{\bar{a}_x}{x^2} + \frac{\bar{a}_x}{(x-1)^2}\right] \text{ v/m}$$

given x is very very large i.e $x >> 1$

$\therefore$ $(x+1)^2 \simeq x^2$ and $(x-1)^2 \simeq x^2$.

$\bar{E}_p = \frac{Q}{4\pi\epsilon_0} \times \frac{3\bar{a}_x}{x^2}$ v/m and given E_p to be same.

$\therefore$ $5.8625\,\cancel{\bar{a}_x} = \frac{3Q}{4\pi\epsilon_0 x^2}\,\cancel{\bar{a}_x}$ $\quad 5.8625 = \frac{3Q}{4\pi\epsilon_0 x^2}$

$\Rightarrow$ $x^2 = \frac{3Q}{4\pi\epsilon_0 \times 5.8625}$ $\quad$ let $x = \infty$ i.e $x >> 1$

Consider x to be very Large ie $\infty = 1/0$

$\therefore$ ~~This~~ is possible only when

$\Rightarrow$ $\boxed{3Q = 1.511\times10^{-8}\,C}$

$\therefore$ the point charge value $\boxed{Q = 0.537\times10^{-8} = 5.37\,nC}$

(iii) if $3Q = 1.511\times10^{-8}C$ in bit (ii)

then E_p @ $x = 5$m is $\boxed{Q = 0.483\times10^{-8}C = 4.83\,nC}$

$$\bar{E}_p = 4.83\times\cancel{10^{-9}}\times9\times\cancel{10^{9}}\left[36^{-1} + 25^{-1} + 16^{-1}\right]\bar{a}_x = 5.45\,\bar{a}_x \text{ v/m}$$

$\boxed{\bar{E}_p = 5.45\,\bar{a}_x}$ v/m using approximation of bit (ii).

problem 14.

Hayt

A charge Q_0, located at the origin in free space, produces a field for which $E_z = 1$ kV/m at point $P(-2, 1, -1)$. (*a*) Find Q_0. Find $\mathbf{E}$ at $M(1, 6, 5)$ in: (*b*) cartesian coordinates; (*c*) cylindrical coordinates; (*d*) spherical coordinates. [W.H. Hayt]

Soln:-

given 1K V/m

$Q_0 = ?$ $\overline{OP}$ $\overline{a_{op}}$ $P(-2,1,-1)$ $O(0,0,0)$

$$\overline{E_p} = E_x \overline{a_x} + E_y \overline{a_y} + E_z \overline{a_z} \text{ V/m}$$

a)

$$\overline{E_p} = \frac{Q_0}{4\pi\epsilon_0 |\overline{OP}|^2} \overline{a_{op}} \text{ V/m} = \frac{Q_0}{4\pi\epsilon_0 |\overline{OP}|^3} \overline{OP} \text{ V/m}.$$

$$\overline{OP} = -2\overline{a_x} + \overline{a_y} - \overline{a_z} \qquad |\overline{OP}| = \sqrt{4+1+1} = \sqrt{6} \text{ m}.$$

$$\overline{E_p} = \frac{Q_0 \times 9 \times 10^9}{(\sqrt{6})^3} \left[-2\overline{a_x} + \overline{a_y} - \overline{a_z}\right] \text{ V/m}$$

the E_z component in above expression is

$$E_z = \frac{-Q_0 \times 9 \times 10^9}{(\sqrt{6})^3} = 1\text{K V/m}$$

$$\Rightarrow Q_0 = \frac{-1\text{K} \times (\sqrt{6})^3}{9 \times 10^9} = -1.6329 \times 10^{-6}$$

$\therefore \boxed{Q_0 = -1.6329}\ \mu C$

b)

$Q_0 = -1.6329\,\mu C$ $O(0,0,0)$ $\overline{OM}$ $\overline{a_{om}}$ $M(1,6,5)$ $\overline{E_m} = ?$

$$\overline{E_m} = \frac{Q_0}{4\pi\epsilon_0 |\overline{OM}|^2} \overline{a_{om}} = \frac{Q_0}{4\pi\epsilon_0 |\overline{OM}|^3} \overline{OM} \text{ V/m}.$$

$\overline{OM} = \bar{a}_x + 6\bar{a}_y + 5\bar{a}_z$; $|\overline{OM}| = \sqrt{1+36+25} = \sqrt{62}$ m.

$$\bar{E}_m = \frac{-1.6399\,\mu \times 9\times10^9}{(\sqrt{62})^3}\left[\bar{a}_x + 6\bar{a}_y + 5\bar{a}_z\right]$$

$$\bar{E}_m = -30.105\left[\bar{a}_x + 6\bar{a}_y + 5\bar{a}_z\right] \text{ V/m}$$

In Cartesian Co-ordinate System.

$$\boxed{\bar{E}_m = -30.105\,\bar{a}_x - 180.63\,\bar{a}_y - 150.52\,\bar{a}_z} \text{ V/m}$$

c) $E_x = -30.105$ V/m ; $E_y = -180.63$ V/m and $E_z = -150.52$ V/m

$M(1,6,5) \Rightarrow \rho = \sqrt{1+36} = \sqrt{37}$ m ; $\phi = \tan^{-1}(6/5)$

$\boxed{\phi = 80.537^\circ}$

$E_\rho = -30.10\cos(80.537^\circ) - 180.63\sin(80.537) = \underline{-183.11 \text{ V/m}}$

$E_\phi = +30.10\sin(80.537^\circ) - 180.63\cos(80.537^\circ) = \underline{0 \text{ V/m}}$

$E_z = \underline{-150.52 \text{ V/m}} = -150.52$ V/m

In Cylindrical C.S

$$\boxed{\bar{E}_m = -183.11\,\bar{a}_\rho - 150.52\,\bar{a}_z} \text{ V/m}$$

d) E_m in Spherical C.S is $M(1,6,5) \Rightarrow r = \sqrt{1^2+6^2+5^2}$

$\boxed{r = 7.87}$ m ; $\theta = \cos^{-1}(z/r)$

$\boxed{\theta = 50.58^\circ}$

using dot product table (Fundamentals) and $\phi = \tan^{-1}(y/x) = 80.54^\circ$

$E_r = -30.11\sin\theta\cos\phi - 180.63\sin\theta\sin\phi$; $\boxed{E_\theta = 0}$ V/m

put $\theta = 50.58^\circ$ and $\phi = 80.54^\circ$

$\boxed{E_r = -273.1}$ V/m $\quad \boxed{E_{\phi(Cylin)} = E_{\phi(Sph)} = 0}$ V/m put $\phi = 80.54^\circ$

and $E_\theta = -\sin\phi\, E_x + \cos\phi\, E_y = 0$ V/m.

$\boxed{\bar{E} = -273.1\,\bar{a}_r}$ V/m ← in spherical C.S

problem 15.

A charge distributed generates a radial electric field $E = \frac{a}{r^2} e^{\frac{-r}{b}} a_r \; V/m$ where a and b are constants. Determine the total charge giving rise to this electric field. (6m) [EEE - June/July - 2016]

soln: given $\bar{E} = \frac{a}{r^2} e^{-r/b} \bar{a}_r \; V/m \leftarrow (1)$

w.k.t the $\bar{E}$ due to point charge Q is

$$\bar{E} = \frac{Q}{4\pi\epsilon r^2} \bar{a}_r \; V/m \leftarrow (2)$$

equating equation (1) and (2)

$$\frac{a}{\cancel{r^2}} e^{-r/b} \cancel{\bar{a}_r} = \frac{Q}{4\pi\epsilon \cancel{r^2}} \cancel{\bar{a}_r}$$

$$\boxed{Q = 4\pi\epsilon a e^{-r/b}} \text{ Coulombs}$$

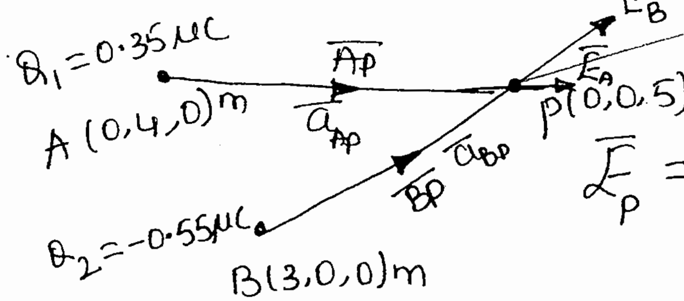
problem 15. A charge distributed generates a radial electric field $\bar{E} = \frac{a}{r^2} e^{-r/b} \bar{a}_r \; V/m$ where a and b are constants. Determine the total charge giving rise to this electric field. [EEE - June/July 2016] problem 16 $\bar{E}$ at $(0,0,5)$m at $(0,4,0)$m at $(3,0,0)$m

Find electric field intensity E at (0,0,5)m due to charge Q_1 at (0,4,0)m and charge Q_2 at (3,0,0)m. the charges are $Q_1=0.35\mu C$ and $Q_2=-0.55\mu C$ respectively. Hence find the magnitude and direction of E. (7m)

EEE June/July 2016

$Q_1 = 0.35\mu C$ $Q_2 = -0.55\mu C$. ?

soln:



$\bar{E}_p = ?$ and $\bar{a}_{E_p} = ?$

$\bar{E}_p = \bar{E}_A + \bar{E}_B$ V/m.

$$\bar{E}_p = \frac{Q_A}{4\pi\epsilon|\overline{AP}|^2}\bar{a}_{AP} + \frac{Q_B}{4\pi\epsilon|\overline{BP}|^2}\bar{a}_{BP} \text{ V/m.}$$

$$\bar{E}_p = \frac{Q_A}{4\pi\epsilon}\frac{\overline{AP}}{|\overline{AP}|^3} + \frac{Q_B}{4\pi\epsilon}\frac{\overline{BP}}{|\overline{BP}|^3} \text{ V/m}$$

$\overline{AP} = -4\bar{a}_y + 5\bar{a}_z$; $|\overline{AP}| = \sqrt{16+25} = \sqrt{41}$ m.

$\overline{BP} = -3\bar{a}_x + 5\bar{a}_z$; $\Rightarrow |\overline{BP}| = \sqrt{9+25} = \sqrt{34}$ m

$$\bar{E}_p = \frac{0.35\mu \times 9\times10^9}{(\sqrt{41})^3}\left[-4\bar{a}_y + 5\bar{a}_z\right] + \frac{(-0.55\mu)(9\times10^9)}{(\sqrt{34})^3}\left[-3\bar{a}_x + 5\bar{a}_z\right]$$

$$= 11.998\left[-4\bar{a}_y + 5\bar{a}_z\right] - 24.968\left[-3\bar{a}_x + 5\bar{a}_z\right]$$

Magnitude $\boxed{\bar{E}_p = 74.90\bar{a}_x - 47.992\bar{a}_y - 64.85\bar{a}_z}$ V/m

$$|\bar{E}_p| = \sqrt{74.9^2 + 47.99^2 + 64.85^2} = \underline{110.0852} \text{ V/m}$$

direction of $\bar{E}_p$ in $\bar{a}_{E_p} = \frac{\bar{E}_p}{|\bar{E}_p|} = \frac{74.90\bar{a}_x - 47.99\bar{a}_y - 64.85\bar{a}_z}{110.0852}$

$$\boxed{\bar{a}_{E_p} = 0.6803\bar{a}_x - 0.4359\bar{a}_y - 0.58908\bar{a}_z}$$

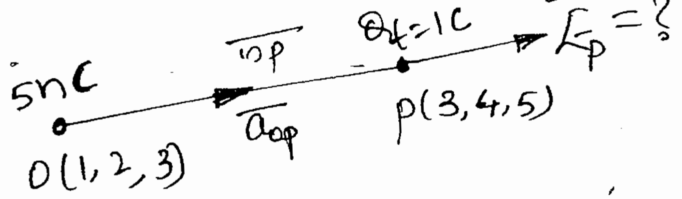
i.e unit vector along $\bar{E}_p$.

Problem 17

(3,4,5) m   5 nC   (1,2,3) m

Calculate the field intensity at a point (3,4,5) due to a charge of 5nC placed at (1,2,3).
Ans: $E=2.16a_x+2.16a_y+2.16a_z$ V/m

Sol^n:



$$\overline{E_p} = \frac{Q_o}{4\pi\epsilon|\overline{op}|^2}\overline{a_{op}} \text{ V/m}$$

$$\overline{E_p} = \frac{Q_o}{4\pi\epsilon}\frac{\overline{op}}{|\overline{op}|^3} \text{ V/m}$$

$$\overline{op} = 2\overline{a_x} + 2\overline{a_y} + 2\overline{a_z}; \quad |\overline{op}| = \sqrt{4+4+4} = \sqrt{12} \text{ m}$$

$$\overline{E_p} = \frac{5n \times 9 \times 10^9}{(\sqrt{12})^3}\left[2\overline{a_x} + 2\overline{a_y} + 2\overline{a_z}\right]$$

$$\boxed{\overline{E_p} = 2.165\overline{a_x} + 2.165\overline{a_y} + 2.165\overline{a_z}} \text{ V/m}$$

$$\boxed{|\overline{E_p}| = 3.75} \text{ V/m}$$

direction of field $\overline{a_{E_p}} = \dfrac{\overline{E_p}}{|\overline{E_p}|}$

$$\overline{a_{E_p}} = \frac{2.165\overline{a_x} + 2.165\overline{a_y} + 2.165\overline{a_z}}{3.75}$$

$$\boxed{\overline{a_{E_p}} = 0.577\overline{a_x} + 0.577\overline{a_y} + 0.577\overline{a_z}}$$

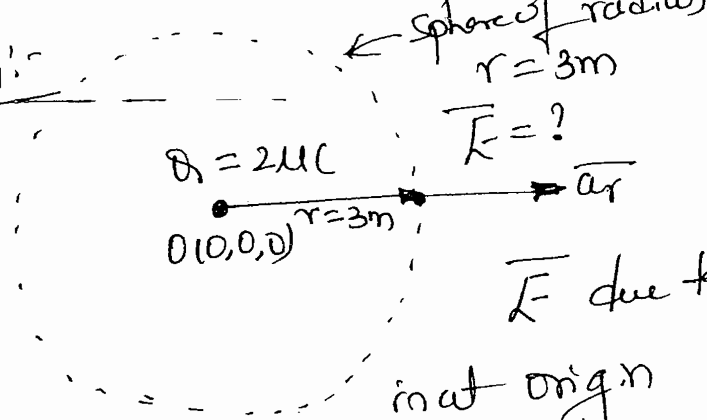
Problem 18

3m 2μC

Calculate the field intensity at a point on a sphere of radius 3m, if a positive charge of $2\mu C$ is placed at the origin of the sphere.

Ans: $E = 1.997 a_r$ kV/m $\simeq 2\bar{a}_r$ kV/m

Sol:



$\bar{E}$ due to point charge of Q C is at origin in spherical C.S is

$$\bar{E} = \frac{Q}{4\pi\epsilon r^2}\,\bar{a}_r \text{ V/m.}$$

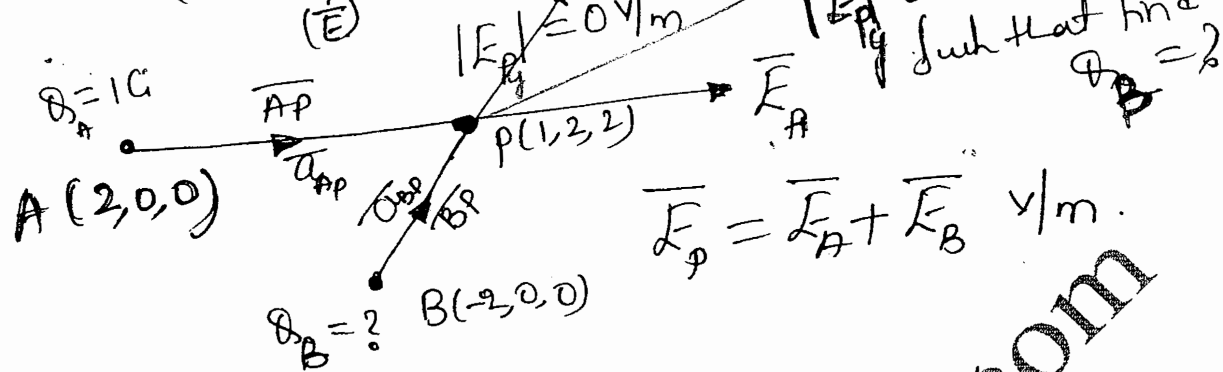
given $r = 3m$.

$$\bar{E} = \frac{2\mu \times 9\times10^9}{(3)^2}\,\bar{a}_r = 2000\,\bar{a}_r \text{ V/m}$$

$$\boxed{\bar{E} = 2\,\bar{a}_r} \text{ kV/m}$$

problem 19. 1C (2,0,0) (1,2,2)? (-2,0,0)

A charge of 1C is at (2,0,0). What charge must be placed at (-2,0,0) which will make y-component of total E zero at the point (1,2,2)?
Ans: Q=-2.59C



$$\bar{E}_P = \bar{E}_A + \bar{E}_B \ \text{V/m}.$$

$$\bar{E}_P = \frac{Q_A}{4\pi\epsilon|\overline{AP}|^2}\bar{a}_{AP} + \frac{Q_B}{4\pi\epsilon|\overline{BP}|^2}\bar{a}_{BP} \ \text{V/m}.$$

$$\overline{AP} = -\bar{a}_x + 2\bar{a}_y + 2\bar{a}_z \ ; \ |\overline{AP}| = \sqrt{1+4+4} = \sqrt{9} = 3m.$$

$$\overline{BP} = 3\bar{a}_x + 2\bar{a}_y + 2\bar{a}_z \ ; \ |\overline{BP}| = \sqrt{9+4+4} = \sqrt{17}\,m$$

$$\bar{E}_P = \frac{1\times 9\times 10^9}{(3)^3}\left[-\bar{a}_x + 2\bar{a}_y + 2\bar{a}_z\right] + \frac{Q_2\times 9\times 10^9}{(\sqrt{17})^3}\left[3\bar{a}_x + 2\bar{a}_y + 2\bar{a}_z\right]$$

$$\bar{E}_P = \frac{9}{27}\times 10^9\left[-\bar{a}_x + 2\bar{a}_y + 2\bar{a}_z\right] + \frac{9\times 10^9 Q_2}{(\sqrt{17})^3}\left[3\bar{a}_x + 2\bar{a}_y + 2\bar{a}_z\right]$$

Component along 'y' direction $E_y = 0$ (given)

ie $$\frac{9}{27}\times 10^9\times(+2) + \frac{9\times 10^9 Q_2}{(\sqrt{17})^3}\times 2 = 0$$

$$\frac{\cancel{9\times 10^9} Q_2 \times \cancel{2}}{(\sqrt{17})^3} = -\frac{\cancel{2}\times\cancel{9\times 10^9}}{27}$$

$$Q_2 = -\frac{(\sqrt{17})^3}{27} = -2.596\ C$$

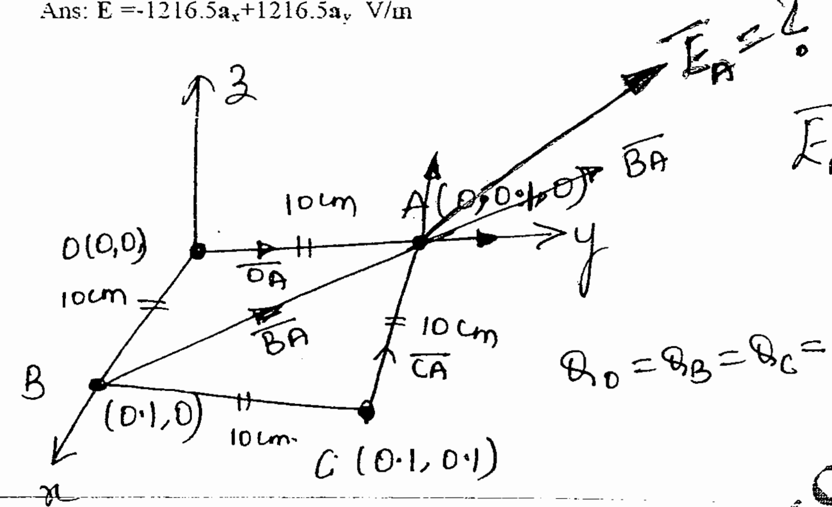
$$\therefore \boxed{Q_2 = -2.596}\ C$$

Problem 20

$1\mu C$ — 10cm side

Three equal charges of $1\mu C$ each are located at the three corners of a square of 10cm side. Find the Electric field intensity at the fourth vacant corner of the square.

Ans: E = $-1216.5\mathbf{a}_x + 1216.5\mathbf{a}_y$ V/m



$\overline{E}_A = ?$

$Q_O = Q_B = Q_C = 1\mu C$

$$\overline{E}_A = \overline{E}_O + \overline{E}_B + \overline{E}_C \text{ V/m}$$

$$\overline{E}_A = \frac{Q}{4\pi\epsilon_0}\left[\frac{\overline{OA}}{|\overline{OA}|^3} + \frac{\overline{BA}}{|\overline{BA}|^3} + \frac{\overline{CA}}{|\overline{CA}|^3}\right] \text{ V/m}$$

$\overline{OA} = 0.1\,\overline{a}_y$; $|\overline{OA}| = 0.1$ m.

$\overline{BA} = -0.1\,\overline{a}_x + 0.1\,\overline{a}_y$; $|\overline{BA}| = \sqrt{0.02}$ m.

$\overline{CA} = -0.1\,\overline{a}_x$; $|\overline{CA}| = 0.1$ m.

$$\overline{E}_A = 1\mu \times 9\times10^9\left[\frac{0.1\,\overline{a}_y}{(0.1)^3} - \frac{0.1\,\overline{a}_x}{(\sqrt{0.02})^3} + \frac{0.1\,\overline{a}_y}{(\sqrt{0.02})^3} - \frac{0.1\,\overline{a}_x}{(0.1)^3}\right]$$

$$\overline{E}_A = 9\times10^3\left[-135.35\,\overline{a}_x + 135.35\,\overline{a}_y\right]$$

$$\boxed{\overline{E}_A = -1218.19\,\overline{a}_x + 1218.19\,\overline{a}_y} \text{ kV/m}$$

$$\boxed{|\overline{E}_A| = 1722.78 \text{ kV/m}}$$

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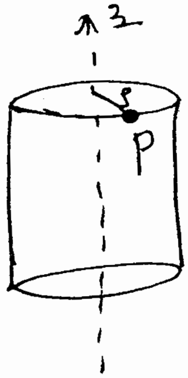
Note:- Point on 'z' axis in Cylindrical Co-ordinate System.



$P(\rho, \phi, z)$ if $\rho \to 0 \Rightarrow P(0, \phi, z)$ is the point on 'z' axis.

Ily point on xy plane [ie z=0 on xy plane] is $P(\rho, \phi, 0)$.

→ $P(0, 0, z)$ point on 'z' axis ⇒ valid in Rectangular C.S

→ $P(0, \phi, z)$ point on 'z' axis ⇒ valid in Cylindrical C.S.

1.3 d Electric Field Intensity $\bar{E}$ due to Infinite Line charge.

Questions.

Derive an expression for the electric field intensity due to infinite line charge. (8M) 10 June/July 2013.

(or)

Charge is distributed uniformly along an infinite straight line with constant density ρ_L C/m. Develop the expression for $\bar{E}$ at the general point P. (6M) [06 - June/July 2014].

[10 - June/July 2012 (8M)] [06 - May/June - 2010 (8M)]

[06 - Dec 2010 (12M)] [15 - Dec/Jan 2017 (8M) CBCS-Scheme.

[15 - June/July 2017 (6M) - CBCS - Scheme].

Electric Field Intensity ($\bar{E}$) due to Infinite Line charge ρ_ℓ

V.I.Imp

15 ~~Derive an expression for the electric field intensity due to infinite line charge.~~ (08 Marks)
10-June/July 2013

16 ~~State and explain the electric field intensity and obtain an expression for electric field intensity due to an infinitely long line charge.~~ (08 Marks)
10 - June /July 2012

17 ~~Derive the expression for $\bar{E}$ due to an infinite line of charge.~~ (08 Marks)
06 – May/June 2010

18 ~~Explain the term 'Electric field intensity' and derive the expression for field due to an infinite line of charge.~~ (12 Marks)

19 ~~Charge is distributed uniformly along an infinite straight line with constant density ρ_l. Develop the expression for $\bar{E}$ at the general point P.~~ (06 Marks)
06 –June/July 2014

20 ~~Define 'electric field intensity' and derive the expression for field due to an infinite line of charge.~~ (12 Marks)
06-DEC2010

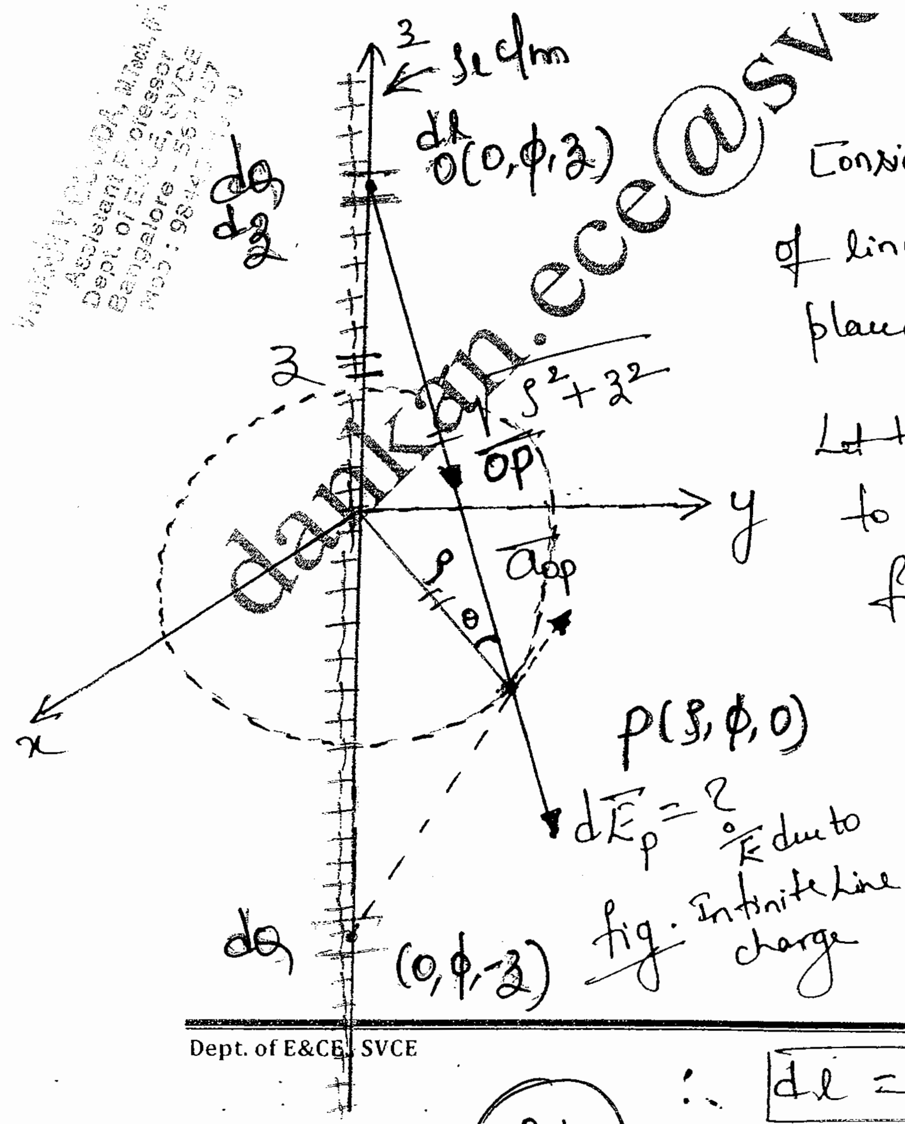


fig. Infinite Line charge

Consider the infinite Line charge of line charge density ρ_ℓ C/m. placed along 'z' axis.

Let the point where we desire to Calculate the Electric field Intensity ($\bar{E}$) to be on xy-plane ie P(ρ, φ, 0).

$\boxed{dQ = \rho_\ell \, dl}$ Coulomb's

Since line charge in placed along 'z' axis

$\therefore \boxed{dl = dz}$

(124)

$\therefore \boxed{dQ = \rho_l \cdot dz}$ Coulombs

the differential Electric field Intensity ($d\bar{E}_p$) at a point p due to differential charge dQ is

$$d\bar{E}_p = \frac{dQ}{4\pi\epsilon|\overline{op}|^2}\bar{a}_{op} \quad V/m. \qquad ; \bar{a}_{op} = \frac{\overline{op}}{|\overline{op}|}$$

$$\overline{op} = (\rho-0)\bar{a}_\rho + (\phi-\phi)\bar{a}_\phi + (0-z)\bar{a}_z$$

$$\overline{op} = \rho\bar{a}_\rho - z\bar{a}_z; \quad |\overline{op}| = \sqrt{\rho^2+z^2} \text{ m.}$$

$$d\bar{E}_p = \frac{dQ}{4\pi\epsilon|\overline{op}|^3}\overline{op} \quad V/m$$

$$d\bar{E}_p = \frac{dQ}{4\pi\epsilon(\rho^2+z^2)^{3/2}}[\rho\bar{a}_\rho - \cancel{z\bar{a}_z}^{\,0}]$$

Since for every dQ at 'z' there is another dQ at '-z', the z components of these two will gets cancel. then resulting only 'ρ' component.

$$\text{i.e } d\bar{E}_p = \frac{dQ}{4\pi\epsilon(\rho^2+z^2)^{3/2}}\rho\bar{a}_\rho$$

$$\bar{E}_p = \int_{z=-\infty}^{\infty} \frac{\rho_l \cdot dz}{4\pi\epsilon(\rho^2+z^2)^{3/2}}\rho \cdot \bar{a}_\rho \quad V/m.$$

put $\quad z = \rho\tan\theta$

$$z = \rho \tan\theta$$

L.L. $z \to -\infty$ $\theta = \tan^{-1}(z/\rho)$; $\theta = -\pi/2$.

U.L. $z \to +\infty$; $\theta = +\pi/2$.

$$dz = \rho \sec^2\theta \, d\theta$$

$1 + \tan^2\theta = \sec^2\theta$

and the term

$$(\rho^2 + z^2)^{3/2} = (\rho^2 + \rho^2 \tan^2\theta)^{3/2}$$

$$= [\rho^2 (\underbrace{1 + \tan^2\theta}_{\sec^2\theta})]^{3/2} = [\rho^2 \sec^2\theta]^{3/2}$$

$$= (\rho \sec\theta)^{2 \times 3/2} = (\rho \sec\theta)^3 = (\rho \sec\theta)^3$$

$$(\rho^2 + z^2)^{3/2} = (\rho \sec\theta)^3$$

$$\overline{E}_\rho = \int_{\theta=-\pi/2}^{\pi/2} \frac{\rho_L \times \rho \sec^2\theta \, d\theta}{4\pi\epsilon (\rho \sec\theta)^3} \rho \, \overline{a}_\rho \quad V/m$$

$$= \int_{\theta=-\pi/2}^{\pi/2} \frac{\rho_L \rho^2 \sec^2\theta \, d\theta}{4\pi\epsilon \rho^3 \sec^3\theta} \overline{a}_\rho \quad V/m$$

$$= \frac{\rho_L}{4\pi\epsilon\rho} \overline{a}_\rho \int_{\theta=-\pi/2}^{\pi/2} \frac{1}{\sec\theta} d\theta = \frac{\rho_L}{4\pi\epsilon\rho} \overline{a}_\rho \int_{\theta=-\pi/2} \cos\theta \, d\theta$$

$$= \frac{\rho_L}{4\pi\epsilon\rho} \overline{a}_\rho \times \sin\theta \Big|_{-\pi/2}^{\pi/2} = \frac{\rho_L}{4\pi\epsilon\rho} \overline{a}_\rho [\underbrace{\sin\pi/2 + \sin\pi/2}_{2}]$$

$$\left[\sin\left(\frac{\pi}{2}\right) + \sin\left(\frac{\pi}{2}\right) = 1 + 1 = 2 \right]$$

$$\overline{E}_p = \frac{\rho_L}{\cancel{4}_2 \pi \epsilon \rho} \times \cancel{2}\, \overline{a}_\rho$$

$$\boxed{\overline{E}_p = \frac{\rho_L}{2\pi\epsilon\rho}\, \overline{a}_\rho} \quad V/m \text{ (or) } N/C.$$

obs:- 1. the direction of field $\overline{E}_p$ is towards $\overline{a}_\rho$.

2. In the above expression, 'ρ' is the Length of perpendicular ~~distance from the desired point~~ to the ~~line~~ charge and $\overline{a}_\rho$ is the unit vector in the direction of perpendicular towards the desired point.

1.3e. $\overline{E}$ due to Infinite sheet charge (ρ_s) C/m²

$\overline{E}$ due to Infinite sheet charge (ρ_s) C/m² is given by

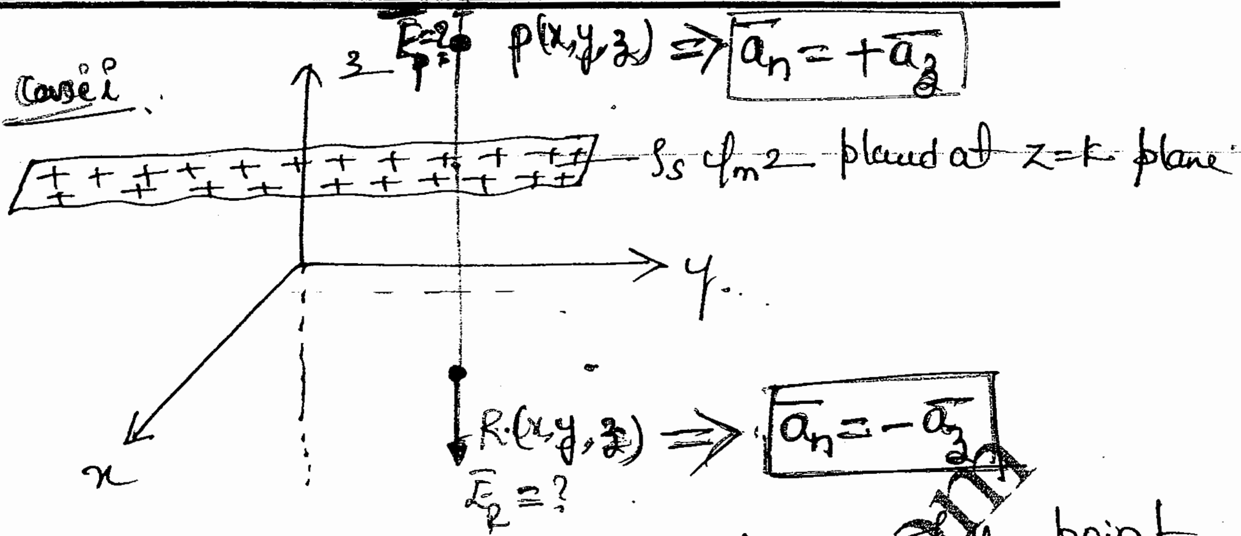
$$\boxed{\overline{E} = \frac{\rho_s}{2\epsilon}\, \overline{a}_n} \quad V/m$$

where ρ_s - sheet charge density C/m²

$\overline{a}_n$ - unit normal vector $\perp^r$ to the sheet charge.

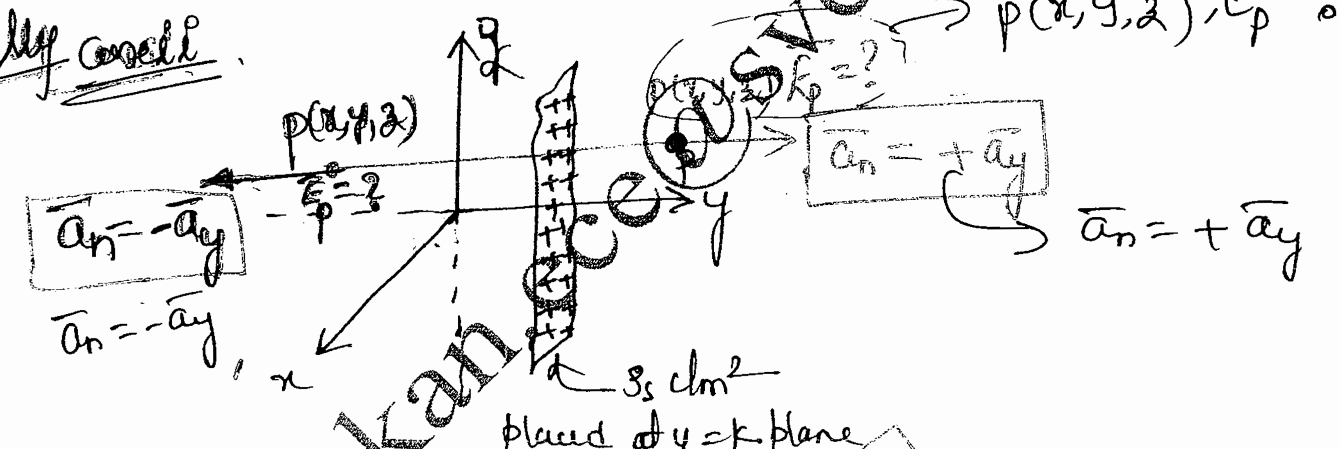
Note:- Field direction is always towards the desired point.

Cases. Case i.

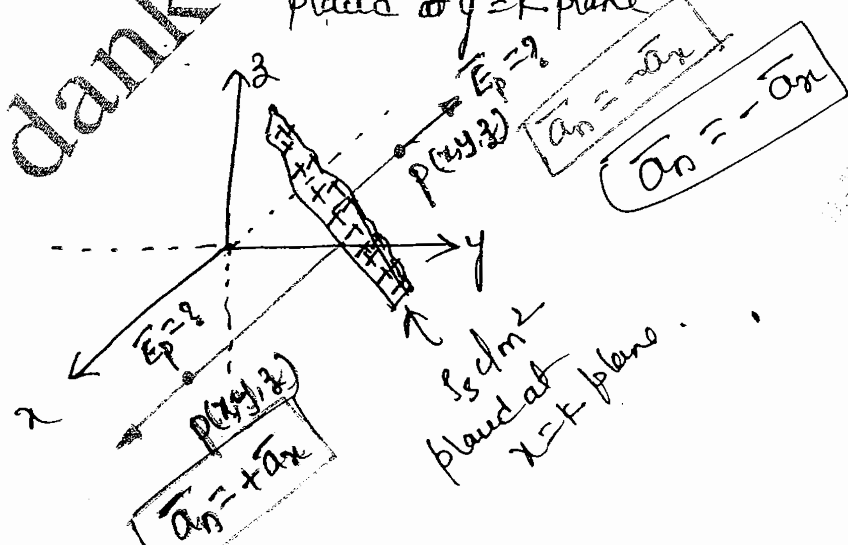


$\overline{a_n}$ is decided by the point Location. If the point Location is above the Sheet charge then $\overline{a_n} = +\overline{a_z}$ and if it Lies below the Sheet charge then $\overline{a_n} = -\overline{a_z}$.

Lly case ii.



Case iii.



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Case iv:- Field on sheet charge is zero. b/c $\overline{a_n}$ does not Exist.

Key note:- Field at a point due to infinite sheet charge is independent

Note:- the value of $\frac{1}{2\pi\epsilon_0} = 18 \times 10^9$

Notes:- $\frac{1}{2\pi\epsilon_0} = 18 \times 10^9$

Problem 1. A uniform line charge of infinite length with $\rho_L = 40$ nc/m, lies along the z-axis. Find $\bar{E}$ at (-2, 2, 8) in air. (05 Marks) 06-Dec/Jan 2016, 4m - 15-Dec/Jan-2017 CCBCS-Scheme

Ans: $E = -180\ a_x + 180 a_y$ V/m

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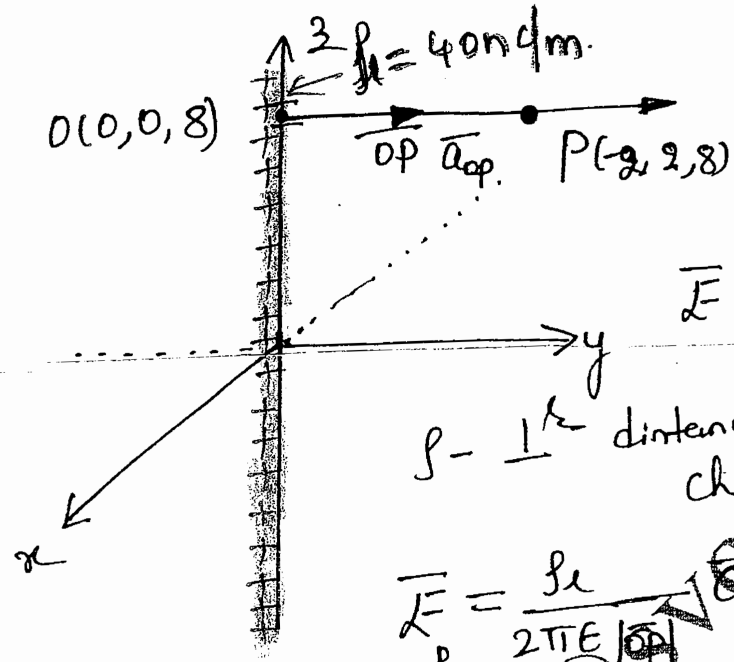


$\bar{E}_p = ?$ P(-2,2,8)

$$\bar{E} = \frac{\rho_L}{2\pi\epsilon\rho}\ \bar{a}_\rho \text{ V/m}.$$

ρ - ⊥ distance from charged point to the line charge.

$$\bar{E}_p = \frac{\rho_L}{2\pi\epsilon|\overline{OP}|}\ \bar{a}_{op} \text{ V/m} \qquad \bar{a}_{op} = \frac{\overline{OP}}{|\overline{OP}|}$$

$$\bar{E}_p = \frac{\rho_L}{2\pi\epsilon_0|\overline{OP}|^2}\ \overline{OP} \text{ V/m}.$$

$$\overline{OP} = -2\bar{a}_x + 2\bar{a}_y;$$

$$|\overline{OP}| = \sqrt{4+4} = \sqrt{8} \text{ m}.$$

$$\bar{E}_p = \frac{40 \times 10^{-9} \times 18 \times 10^9}{(\sqrt{8})^2}\left[-2\bar{a}_x + 2\bar{a}_y\right]$$

$$\bar{E}_p = 90\left[-2\bar{a}_x + 2\bar{a}_y\right]$$

$$\boxed{\bar{E}_p = -180\,\bar{a}_x + 180\,\bar{a}_y} \text{ V/m}$$

$E_x = -180$ V/m; $E_y = +180$ V/m

$\boxed{|\bar{E}_p| = 254.55}$ V/m (129)

Problem 2.

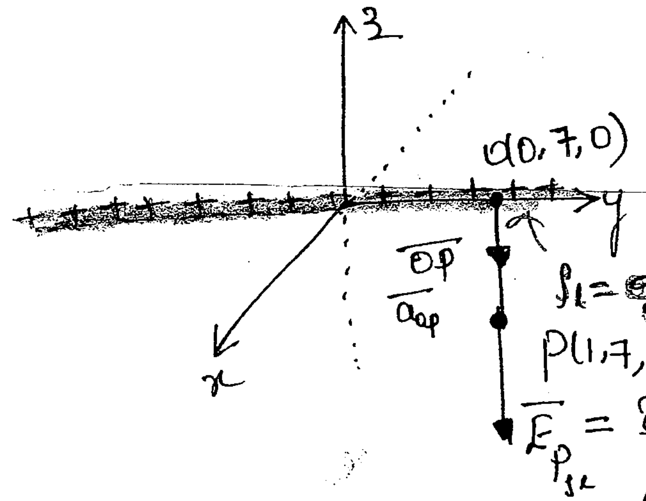
A line charge of 2 nc/m lies along y-axis while surface charge densities of 0.1 nc/m² and -0.1nc/m² exist on the plane z = 3 and z = -4m respectively. Find the $\overline{E}$ at P(1, 7, -2). (08 Marks)

2nc/m 0.1nc/m² -0.1nc/m² z=3 z=-4m $\overline{E}$ P(1,7,-2)

[07-June/July 2012]

Sol^n:- $\overline{E}_{P_{net}} = \overline{E}_{\rho_l} + \overline{E}_{\rho_{s+}} + \overline{E}_{\rho_{s-}}$ N/C @ V/m.

Case i. $\overline{E}_P$ due to line charge



$$\overline{E}_{\rho_l} = \frac{\rho_l}{2\pi\epsilon\rho}\overline{a}_\rho \text{ V/m}$$

$$\overline{E}_{\rho_l} = \frac{\rho_l}{2\pi\epsilon|\overline{OP}|}\overline{a}_{op} \text{ V/m}$$

$$\overline{E}_{\rho_l} = \frac{\rho_l}{2\pi\epsilon|\overline{OP}|^2}\overline{OP} \text{ V/m}$$

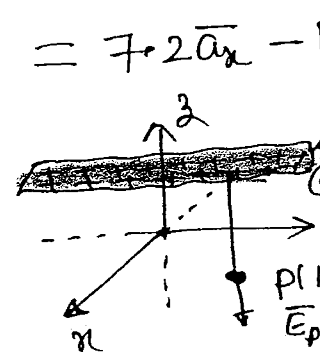
$$\overline{OP} = \overline{a}_x - 2\overline{a}_z$$

$$|\overline{OP}| = \sqrt{1+4} = \sqrt{5} \text{ m.}$$

$$\overline{E}_{\rho_l} = \frac{2n \times 18 \times 10^9}{(\sqrt{5})^2}[\overline{a}_x - 2\overline{a}_z]$$

$$\overline{E}_{\rho_l} = 7.2[\overline{a}_x - 2\overline{a}_z] = 7.2\overline{a}_x - 14.4\overline{a}_z \text{ V/m}$$

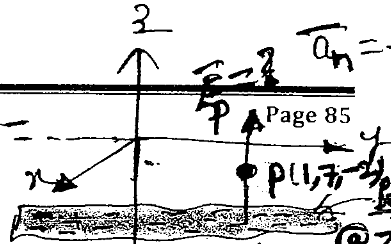
Case ii. $\overline{E}_P$ due to sheet charge ρ_{s+}.



$$\overline{E}_{\rho_{s+}} = \frac{\rho_{s+}}{2\epsilon}\overline{a}_n = \frac{0.1n}{2 \times 8.854 \times 10^{-12}}(-\overline{a}_z)$$

$$= -5.647\overline{a}_z \text{ V/m.}$$

Case iii. $\overline{E}_P$ due to sheet charge ρ_{s-}



$$\overline{E}_{\rho_{s-}} = \frac{\rho_{s-}}{2\epsilon}\overline{a}_n = \frac{-0.1n}{2 \times 8.854 \times 10^{-12}}(+\overline{a}_z)$$

$\boxed{\bar{E}_{z-} = -5.647\,\bar{a}_z \text{ V/m}}$; $\bar{E}_{net} = \bar{E}_{z_1} + \bar{E}_{z_2} + \bar{E}_{z_-}$ V/m

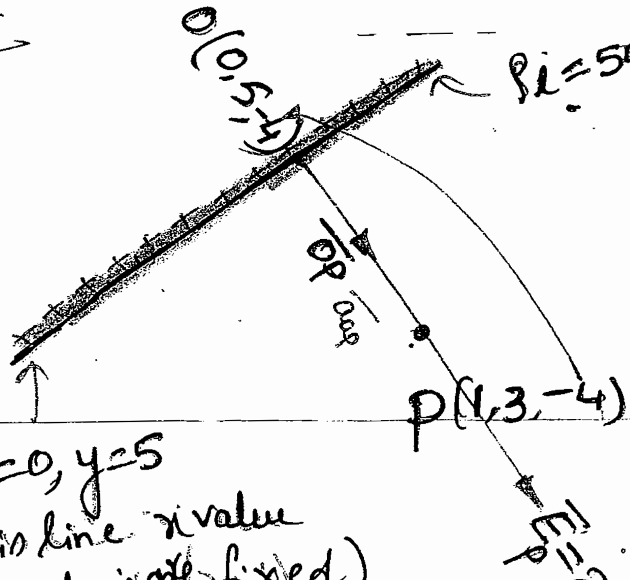
$\bar{E}_{net} = 7.2\bar{a}_x - 14.4\bar{a}_z - 5.64\bar{a}_z - 5.64\bar{a}_z$; $\boxed{\bar{E}_{net} = 7.2\bar{a}_x - 25.67\bar{a}_z}$ V/m

problem 3

A line charge density $\rho_L = 50$ nC/m is located along the line $x = 0$, $y = 5$ in free space. Find the magnitude and direction of the electric field intensity at a point P(1, 3, −4). (06 Marks)

[06 - June/July 2014]

Sol:



$|\bar{OP}|$ - is the ⊥r distance from desired point to the line charge.

$$\bar{E}_P = \frac{\rho_l}{2\pi\epsilon\rho}\,\bar{a}_\rho \text{ V/m}$$

$$\bar{a}_{op} = \frac{\bar{OP}}{|\bar{OP}|}$$

$$\bar{E}_P = \frac{\rho_l}{2\pi\epsilon|\bar{OP}|}\,\bar{a}_{op} \text{ V/m}$$

(or)

$$\bar{E}_P = \frac{\rho_l}{2\pi\epsilon|\bar{OP}|^2}\,\bar{OP} \text{ V/m}$$

$$\bar{OP} = (1-0)\bar{a}_x + (3-5)\bar{a}_y + (-4+4)\bar{a}_z$$

$$\bar{OP} = \bar{a}_x - 2\bar{a}_y \;;\; |\bar{OP}| = \sqrt{1+4} = \sqrt{5} \text{ m.}$$

$$\bar{E}_P = \frac{50n \times 18\times10^{9}}{(\sqrt{5})^2}\,[\bar{a}_x - 2\bar{a}_y]$$

$$\bar{E}_P = 180[\bar{a}_x - 2\bar{a}_y] \text{ V/m}$$

$$\boxed{\bar{E}_P = 180\bar{a}_x - 360\bar{a}_y} \text{ V/m}$$

$E_{Px} = 180$ V/m and $E_{Py} = -360$ V/m

$\boxed{|\bar{E}_P| = 402.49}$ V/m (131)

Hayt Problem 4.

Infinite uniform line charges of 5 nC/m lie along the (positive and negative) x and y axes in free space. Find E at: (a) $P_A(0, 0, 4)$; (b) $P_B(0, 3, 4)$. $P_B(0,3,4)$

y-axes $\bar{E}$ $P_A(0,0,4)$

~~Ans. $45 a_z$ V/m; $10.8 a_y + 36.9 a_z$ V/m~~ [W. H. Hayt]

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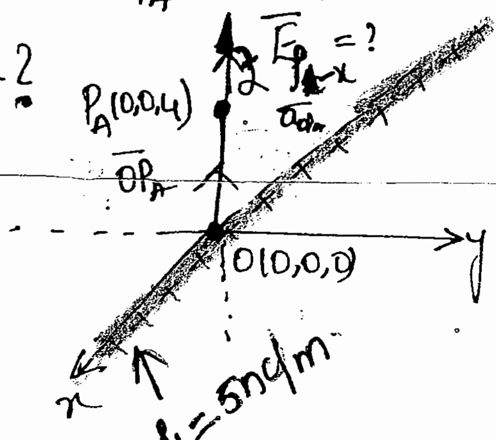
Sol^n: (a) $\bar{E}_{P_A} = ?$

$$\bar{E}_{P_A} = \bar{E}_{\rho_l - x} + \bar{E}_{\rho_l - y} \text{ V/m.}$$

$\bar{E}_{\rho_l - x} = ?$



$$\bar{E}_{\rho_l - x} = \frac{\rho_l}{2\pi\epsilon|\overline{OP_A}|} \bar{a}_{OP_A} \text{ V/m}$$

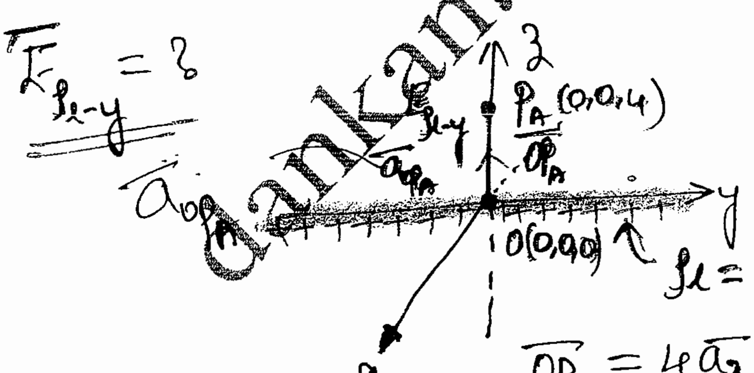
$$= \frac{\rho_l}{2\pi\epsilon|\overline{OP_A}|^2} \overline{OP_A} \text{ V/m}$$

$$\overline{OP_A} = +4\bar{a}_z$$

$$|\overline{OP_A}| = 4 \text{ m.}$$

$$\bar{E}_{\rho_l - x} = \frac{5n \times 18 \times 10^9}{(4)^2}[4\bar{a}_z] = \underline{22.5\bar{a}_z} \text{ V/m}$$

$\bar{E}_{\rho_l - y} = ?$



$$\bar{E}_{\rho_l - y} = \frac{\rho_l}{2\pi\epsilon|\overline{OP_A}|} \bar{a}_{OP_A} \text{ V/m}$$

$$\bar{E}_{\rho_l - y} = \frac{\rho_l}{2\pi\epsilon|\overline{OP_A}|^2} \overline{OP_A} \text{ V/m}$$

$$\overline{OP_A} = 4\bar{a}_z \; ; \; |\overline{OP_A}| = 4 \text{ m.}$$

$$\bar{E}_{\rho_l - y} = \frac{5n \times 18 \times 10^9}{(4)^2}[4\bar{a}_z] = \underline{22.5\bar{a}_z} \text{ V/m}$$

$$\bar{E}_{P_A} = \bar{E}_{\rho_l - x} + \bar{E}_{\rho_l - y} = 22.5\bar{a}_z + 22.5\bar{a}_z \text{ V/m}$$
$$= 45\bar{a}_z \text{ V/m}$$

$$\boxed{\bar{E}_{P_A} = 45\,\bar{a}_z} \text{ V/m}$$

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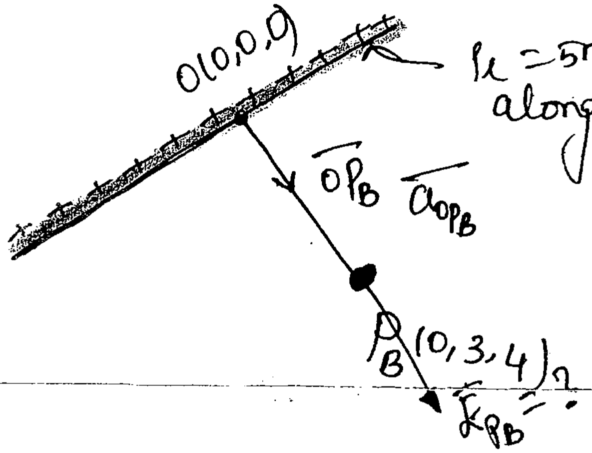
b) $\overline{E}_{P_B} = ?$ @ $P_B(0,3,4)$

<u>Case i</u> $\overline{E}_{P_{B-x}}$ due to 'x' axis line charge.



$$\overline{E}_{P_{B-x}} = \frac{\rho_L}{2\pi\epsilon|\overline{OP_B}|}\overline{a_{OP_B}} \text{ V/m}$$

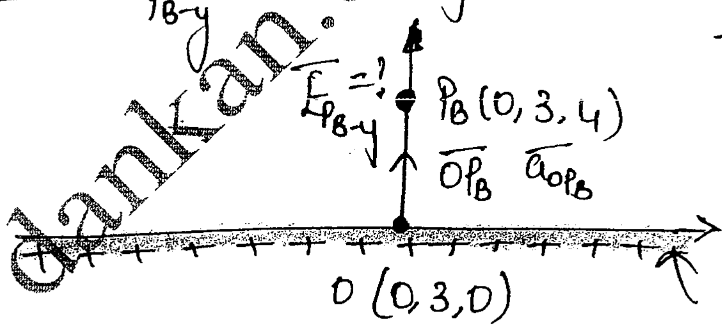
$$\overline{E}_{P_{B-x}} = \frac{\rho_L}{2\pi\epsilon|\overline{OP_B}|^2}\overline{OP_B} \text{ V/m.}$$

$$\overline{OP_B} = 3\overline{a_y} + 4\overline{a_z} \; ; \; |\overline{OP_B}| = \sqrt{9+16} = 5\text{m.}$$

$$\overline{E}_{P_{B-x}} = \frac{5\cancel{n}\times 18\times 10^{9}}{(5)^2}\left[3\overline{a_y} + 4\overline{a_z}\right] = 3.6\left[3\overline{a_y} + 4\overline{a_z}\right]$$

$$\boxed{\overline{E}_{P_{B-x}} = 10.8\,\overline{a_y} + 14.4\,\overline{a_z}} \text{ V/m.}$$

<u>Case ii</u> - $\overline{E}_{P_{B-y}}$ due to 'y' axis line charge.



$$\overline{E}_{P_{B-y}} = \frac{\rho_L}{2\pi\epsilon|\overline{OP_B}|^2}\overline{OP_B} \text{ V/m}$$

$$\overline{OP_B} = 4\overline{a_z} \; ; \; |\overline{OP_B}| = 4\text{m.}$$

$$\overline{E}_{P_{B-y}} = \frac{5\times 10^{-9}\times 18\times 10^{9}}{(4)^{\cancel{2}}}\left[\cancel{4}\overline{a_z}\right] = \underline{\underline{22.5\,\overline{a_z}}} \text{ V/m}$$

$$\overline{E}_{P} = \overline{E}_{P_{B-x}} + \overline{E}_{P_{B-y}} = 10.8\overline{a_y} + 14.4\overline{a_z} + 22.5\overline{a_z} \text{ V/m}$$

$$\boxed{\overline{E}_{P_B} = 10.8\,\overline{a_y} + 36.9\,\overline{a_z}} \text{ V/m}$$

problem 5

A uniform line charge of 16 nC/m is located along the line defined by $y = -2$, $z = 5$. If $\epsilon = \epsilon_0$: (a) find **E** at $P(1, 2, 3)$; (b) find **E** at that point in the $z = 0$ plane where the direction of **E** is given by $\frac{1}{3}\mathbf{a}_y - \frac{2}{3}\mathbf{a}_z$. [W.H. Hayt]

Ans: a. $E = 57.6a_y - 28.8a_z$ V/m

b. $E = 23a_y - 46a_z$ V/m

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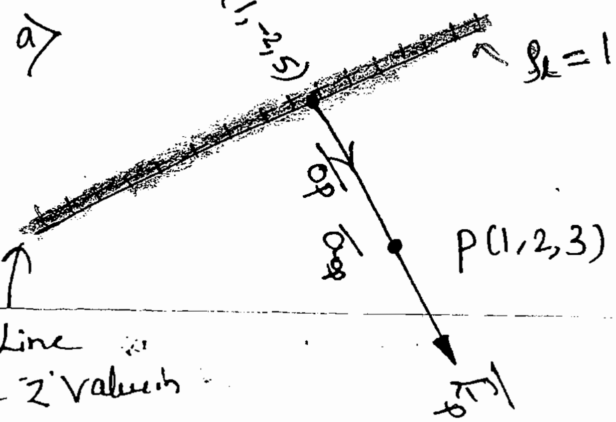
Solu:^n a)



on this Line -y and 'z' values fixed ie $y=-2$ and $z=5$.

$$\overline{E_p} = \frac{\rho_L}{2\pi\epsilon|\overline{op}|}\overline{a_{op}} \text{ V/m}$$

$$\overline{E_p} = \frac{\rho_L}{2\pi\epsilon|\overline{op}|^2}\overline{op} \text{ V/m}$$

$$\overline{op} = 4\overline{a_y} - 2\overline{a_z};$$

$$|\overline{op}| = \sqrt{16+4} = \sqrt{20} \text{ m.}$$

$$\overline{E_p} = \frac{16 \times 18 \times 10^{9}}{(\sqrt{20})^2}[4\overline{a_y} - 2\overline{a_z}]$$

$$\overline{E_p} = 14.4[4\overline{a_y} - 2\overline{a_z}]$$

$$\boxed{\overline{E_p} = 57.6\,\overline{a_y} - 28.8\,\overline{a_z}} \text{ V/m}$$

b) To Find $\overline{E}$ at that point in the $Z=0$ plane.

we need to know the point where $\overline{E} = \frac{1}{3}\overline{a_y} - \frac{2}{3}\overline{a_z}$

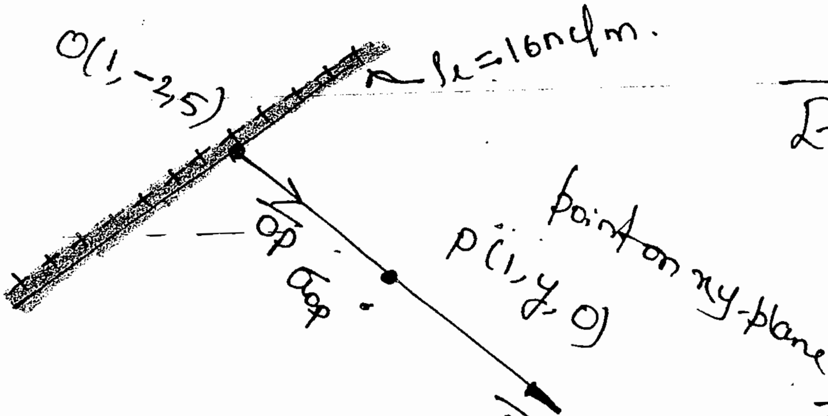
i.e $\boxed{E_z = -2E_y}$ or $E_y = -\frac{1}{2}E_z$ V/m $= -\frac{1}{2}E_z$ V/m

the point is on $z=0$ plane i.e on xy plane

that can be $(1, y, 0)$.

its a point on xy plane $\therefore z=0$.

(134) and $x=1$ bcz on point $O(1,-2,5)$. and 'y' Unknown



$$\overline{E}_p = \frac{\rho_L}{2\pi\epsilon|\overline{op}|}\overline{a}_{op} \text{ V/m}$$

$$\overline{E}_p = \frac{\rho_L}{2\pi\epsilon|\overline{op}|^2}\overline{op} \text{ V/m.}$$

$$\overline{E} = E_y\overline{a}_y + E_z\overline{a}_z$$

$$= (\tfrac{1}{3})\overline{a}_y - \tfrac{2}{3}\overline{a}_z$$

$$\boxed{E_z = -2E_y}$$

$$\overline{op} = (y+2)\overline{a}_y - 5\overline{a}_z$$

$$|\overline{op}| = \sqrt{(y+2)^2 + 25}$$

$$\overline{E}_p = \frac{16\cancel{n} \times 18 \times \cancel{10^9}}{\left[\sqrt{(y+2)^2+25}\right]^2}\left[(y+2)\overline{a}_y - 5\overline{a}_z\right] \longleftarrow (a)$$

given cond^n $\boxed{E_z = -2E_y} \Rightarrow$ to find 'y' value.

$$E_z = \frac{288(-5)}{(y+2)^2+25}$$

$$E_z = \frac{288 \times -5}{(y+2)^2+25} \text{ V/m and } E_y = \frac{16 \times 18(y+2)}{(y+2)^2+25}$$

$$E_z = -2E_y$$

$$\frac{\cancel{288} \times \cancel{-}5}{\cancel{[(y+2)^2+25]}} = \cancel{-}\frac{2 \times \cancel{16} \times \cancel{18}(y+2)}{\cancel{[(y+2)^2+25]}}$$

$$(y+2) = 5/2 \Rightarrow y = 2.5 - 2 = 0.5$$

$$\therefore \boxed{y = 1/2 \text{ (or) } 0.5} \quad \text{using eq}^n \text{ (a)}$$

$$\overline{E}_{p_{z=0\,plane}} = \frac{16 \times 18}{(0.5+2)^2+25}\left[(0.5+2)\overline{a}_y - 5\overline{a}_z\right] = 9.216\left[2.5\overline{a}_y - 5\overline{a}_z\right]$$

$$\overline{E}_{p_{z=0\,plane}} = 23.04\,\overline{a}_y - 46.08\,\overline{a}_z \text{ V/m}$$

$$\boxed{\bar{E}_{P_{z=0\,plane}} = 23.04\,\bar{a}_y - 46.08\,\bar{a}_z} \text{ V/m.}$$

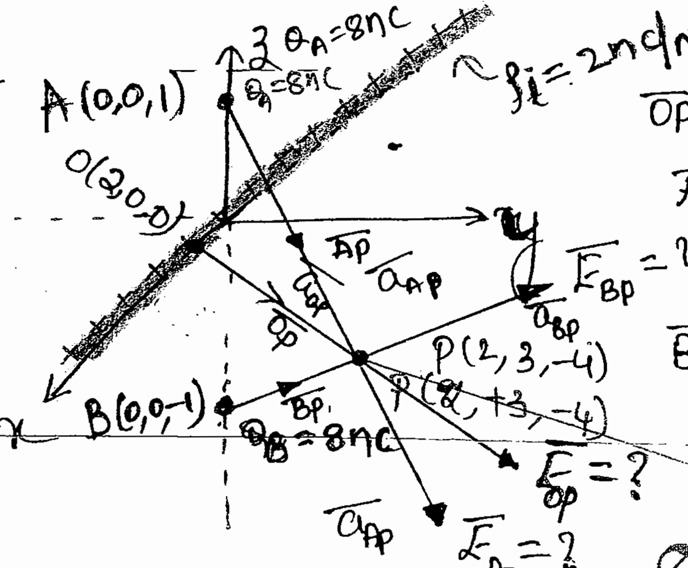
Problem 6 Hayt

An infinite Uniform Line charge $\rho_l = 2nc/m$ lies along the x-axis in free space, while point charges of 8nC each are Located at (0,0,1) and (0,0,-1). Find

i. Find **E** at (2,3,-4)

ii. To what value should ρ_l be changed to cause **E** to be zero at (0,0,3) ? [W.H. Hayt]

Sol'n:



$\overline{OP} = 3\bar{a}_y - 4\bar{a}_z$; $|\overline{OP}| = \sqrt{9+16} = 5m.$

$\overline{AP} = 2\bar{a}_x + 3\bar{a}_y - 5\bar{a}_z$;

$|\overline{AP}| = \sqrt{4+9+25} = \sqrt{38}\,m.$

$\overline{BP} = 2\bar{a}_x + 3\bar{a}_y - 3\bar{a}_z$;

$|\overline{BP}| = \sqrt{4+9+9} = \sqrt{22}\,m.$

$\bar{E}_P = \bar{E}_{OP} + \bar{E}_{AP} + \bar{E}_{BP}$ V/m

$$\bar{E}_P = \bar{E}_{OP} + \bar{E}_{AP} + \bar{E}_{BP} \text{ V/m}$$

$$\bar{E}_P = \frac{\rho_l}{2\pi\epsilon|\overline{OP}|}\bar{a}_{OP} + \frac{Q_A}{4\pi\epsilon|\overline{AP}|^2}\bar{a}_{AP} + \frac{Q_B}{4\pi\epsilon|\overline{BP}|^2}\bar{a}_{BP} \text{ V/m.}$$

$$\bar{E}_P = \frac{\rho_l}{2\pi\epsilon|\overline{OP}|^2}\overline{OP} + \frac{Q_A}{4\pi\epsilon|\overline{AP}|^3}\overline{AP} + \frac{Q_B}{4\pi\epsilon|\overline{BP}|^3}\overline{BP} \text{ V/m.}$$

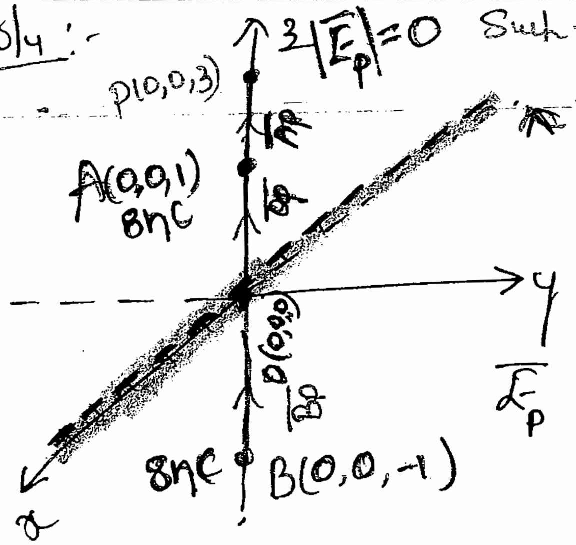
$$\bar{E}_P = \frac{2n\times 18\times 10^9}{(5)^2}[3\bar{a}_y - 4\bar{a}_z] + \frac{8n\times 9\times 10^9}{(\sqrt{38})^3}[2\bar{a}_x + 3\bar{a}_y - 5\bar{a}_z] + \frac{8n\times 9\times 10^9}{(\sqrt{22})^3}[2\bar{a}_x + 3\bar{a}_y - 3\bar{a}_z]$$

$$\bar{E}_P = 1.44[3\bar{a}_y - 4\bar{a}_z] + 0.3072[2\bar{a}_x + 3\bar{a}_y - 5\bar{a}_z] + 0.69774[2\bar{a}_x + 3\bar{a}_y - 3\bar{a}_z]$$

$$\boxed{\bar{E}_P = 2.009\,\bar{a}_x + 7.33408\,\bar{a}_y - 9.38922\,\bar{a}_z} \text{ V/m}$$

$E_x = 2.009$ V/m ; $E_y = 7.334$ V/m and $E_z = -9.389$ V/m.

b) Sol/4 :- $\exists |\overline{E}_p| = 0$ such that Find $\rho_l = ?$



$$\overline{E}_p = \frac{\rho_l}{2\pi\epsilon|\overline{OP}|}\overline{a_{OP}} + \frac{Q_A}{4\pi\epsilon|\overline{AP}|^2}\overline{a_{AP}} + \frac{Q_B}{4\pi\epsilon|\overline{BP}|^2}\overline{a_{BP}}$$

$|\overline{OP}| = 3\,m$; $|\overline{AP}| = 2\,m$; $|\overline{BP}| = 4\,m$.

$$\overline{E}_p = \frac{\rho_l \times 18\times10^9}{3}\overline{a_{OP}} + \frac{8n\times9\times10^9}{4}\overline{a_{AP}} + \frac{8n\times9\times10^9}{16}\overline{a_{BP}}$$

$$\overline{E}_{p\,@(0,0,3)} = \frac{\rho_l\times18\times10^9}{3}\overline{a_{OP}} + 18\overline{a_{AP}} + 4.5\overline{a_{BP}} \text{ V/m}$$

$|\overline{E}_p| = 0$ given. $|\overline{E}_p| = |\overline{E}_{\rho_l}| + |\overline{E}_{Q_A}| + |\overline{E}_{Q_B}|$ V/m.

$$|\overline{E}_p| = \left[\left(\frac{\rho_l\times18\times10^9}{3}\right) + 18 + 4.5\right]$$

$$\overset{\text{given}}{\underset{0}{|\overline{E}_p|}} = \left(\frac{\rho_l\times18\times10^9}{3}\right) + 18 + 4.5$$

$$\frac{\rho_l \times 18\times10^9}{3} + 22.5 = 0$$

$$\rho_l = \frac{-22.5\times3}{18\times10^{+9}} = \frac{-67.5}{18}\times10^{-9}$$

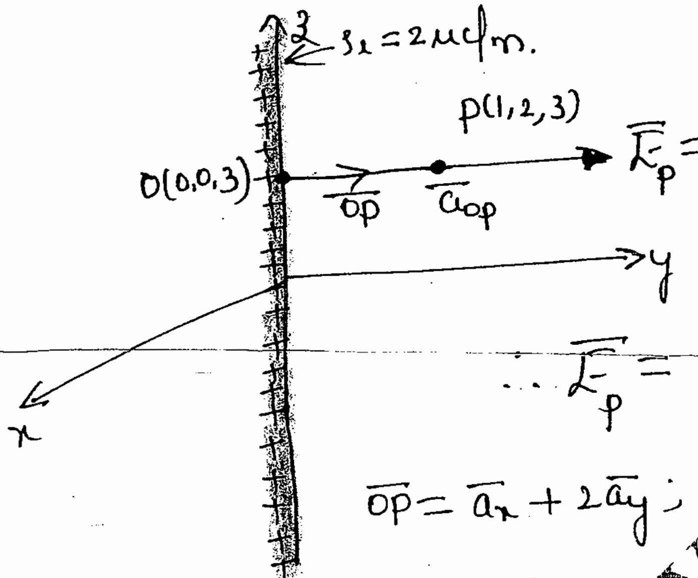
$$\boxed{\rho_l = -3.75n}\ \text{C/m}$$

problem 7.

A uniform line charge of $2\mu C/m$ is located on the z axis. Find E in cartesian coordinates at $P(1, 2, 3)$ if the charge extends from: (*a*) $z = -\infty$ to $z = \infty$; (*b*) $z = -4$ to $z = 4$.

Solu

Solu: a) $z = -\infty$ to $z = +\infty$



$$\bar{E}_p = \frac{\rho_L}{2\pi\epsilon|\overline{OP}|}\,\bar{a}_{op} \text{ V/m}$$

$$\bar{a}_{op} = \frac{\overline{OP}}{|\overline{OP}|}$$

$$\therefore \bar{E}_p = \frac{\rho_L}{2\pi\epsilon|\overline{OP}|^2}\,\overline{OP} \text{ V/m}$$

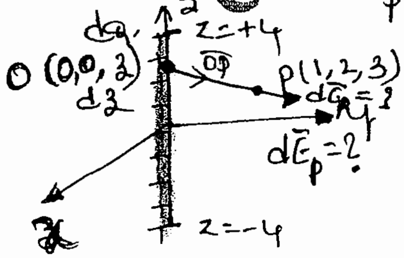
$\overline{OP} = \bar{a}_x + 2\bar{a}_y$; $|\overline{OP}| = \sqrt{1+4} = \sqrt{5}$ m

$$\bar{E}_p = \frac{2\cancel{\mu} \times 18 \times 10^{\cancel{9}\,10^3}}{(\sqrt{5})^2}\left[\bar{a}_x + 2\bar{a}_y\right]$$

$$\boxed{\bar{E}_p = 7.2\bar{a}_x + 14.4\bar{a}_y} \text{ kV/m}$$

(b) $z = -4$ to $z = +4$.

$$d\bar{E}_p = \frac{dQ}{4\pi\epsilon|\overline{OP}|^2}\,\bar{a}_{op} \text{ V/m} ; \quad dQ = \rho_L \cdot dz$$



$$\overline{OP} = \bar{a}_x + 2\bar{a}_y + (3-z)\bar{a}_z$$

$$|\overline{OP}| = \sqrt{1+4+(3-z)^2} = \sqrt{5+(3-z)^2}$$

$$\bar{E}_p = \int_{z=-4}^{4} \frac{\rho_L\,dz}{4\pi\epsilon|\overline{OP}|^2}\,\bar{a}_{op} = \int_{z=-4}^{4} \frac{\rho_L\,dz}{4\pi\epsilon|\overline{OP}|^3}\,\overline{OP} \text{ V/m}$$

$$\bar{E}_p = \frac{\rho_L}{4\pi\epsilon_0}\int_{z=-4}^{4} \frac{\bar{a}_x + 2\bar{a}_y + (3-z)\bar{a}_z}{[5+(3-z)^2]^{3/2}}\, dz$$

$$\bar{E}_p = 2\mu \times 9\times 10^9 \left[\underbrace{\int_{z=-4}^{4}\frac{dz}{[5+(3-z)^2]^{3/2}}}_{0.27217}\bar{a}_x + 2\underbrace{\int_{z=-4}^{4}\frac{dz}{[5+(3-z)^2]^{3/2}}}_{0.27217}\bar{a}_y + \int_{z=-4}^{4}\frac{(3-z)\,dz}{[5+(3-z)^2]^{3/2}}\bar{a}_z \right]$$

$$\bar{E}_p = 18000\left[0.27217\,\bar{a}_x + 0.54434\,\bar{a}_y + 0.272165\,\bar{a}_z\right]$$

$$\bar{E}_p = 4.899\,\bar{a}_x + 9.798\,\bar{a}_y + 4.8989\,\bar{a}_z \text{ kV/m}$$

$$\boxed{\bar{E}_p = 4.899\,\bar{a}_x + 9.798\,\bar{a}_y + 4.8989\,\bar{a}_z} \text{ kV/m}$$

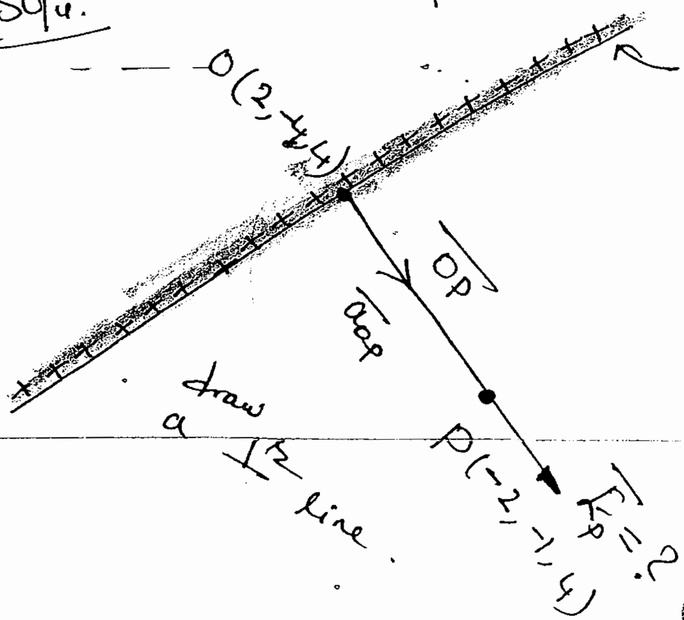
problem 8.

$\rightarrow x = 2m$ $\rightarrow y = -4m$ $\rightarrow \rho_l = 20nc/m$

On the line described by x=2m, y=-4m there is a uniform charge distribution of density $\rho_l = 20nc/m$. Determine the E at (-2,-1,4).

Sol^n:-

$\bar{E}$ $P(-2,-1,4)$



$\rho_l = 20nc/m$
$x = 2m,\ y = -4m$
on this line x & y values are fixed.

$$\overline{OP} = -4\bar{a}_x + 3\bar{a}_y$$

$$|\overline{OP}| = \sqrt{16+9} = \sqrt{25} = 5m$$

$$\bar{E}_p = \frac{\rho_l}{2\pi\epsilon|\overline{OP}|}\bar{a}_{op} \quad v/m$$

$$\bar{E}_p = \frac{\rho_l}{2\pi\epsilon|\overline{OP}|^2}\overline{OP} \quad v/m$$

$$\bar{E}_p = \frac{20\cancel{n} \times 18 \times 10^{\cancel{9}}}{(5)^2}[-4\bar{a}_x + 3\bar{a}_y]$$

$$\bar{E}_p = 14.4[-4\bar{a}_x + 3\bar{a}_y]$$

$$\boxed{\bar{E}_p = -57.6\bar{a}_x + 43.2\bar{a}_y} \ v/m$$

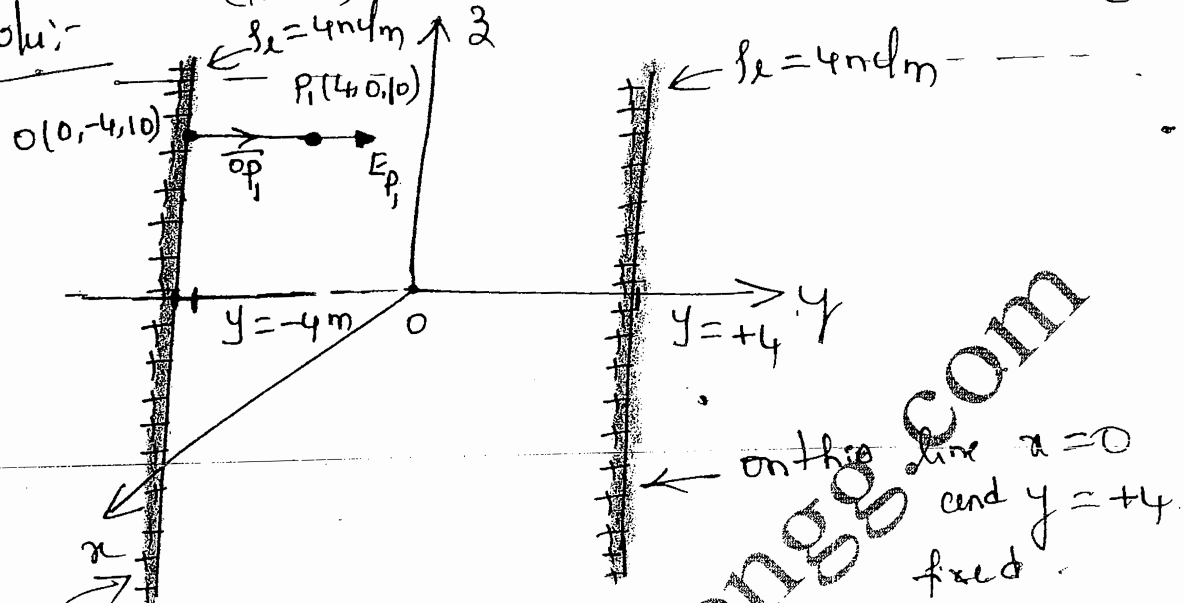
problem 9

$\rho_l = 4n\,C/m$ $\quad x=0 \quad y=\pm 4m$

Two Uniform Line charge density $\rho_l = 4n\ C/m$ lies in the x=0 plane at y=$\pm 4m$. Find E at (4,0,10)m

(4, 0, 10) m. $\quad \bar{E}$

Soln:-



on this line
x = 0 and
y = -4 fixed.

$$\bar{E}_{P_1} = \frac{\rho_l}{2\pi\epsilon|\overline{OP_1}|}\,\bar{a}_{OP_1} \ V/m$$

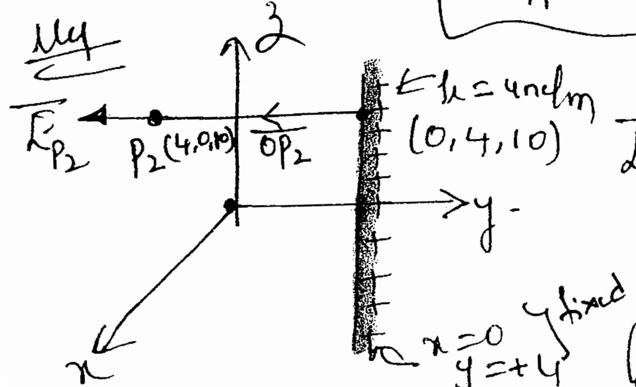
$$\overline{OP_1} = 4\bar{a}_x + 4\bar{a}_y$$

$$|\overline{OP_1}| = \sqrt{16+16} = \sqrt{32}\ m.$$

$$\bar{E}_{P_1} = \frac{4\cancel{n}\times 18\times\cancel{10^9}}{(\sqrt{32})^2}\left[4\bar{a}_x + 4\bar{a}_y\right]$$

$$\boxed{\bar{E}_{P_1} = 9\bar{a}_x + 9\bar{a}_y} \ V/m$$

||ly



$\overline{OP_2} = 4\bar{a}_x - 4\bar{a}_y$; $|\overline{OP_2}| = \sqrt{32}$ m.

$$\bar{E}_{P_2} = \frac{\rho_l}{2\pi\epsilon|\overline{OP_2}|}\,\bar{a}_{OP_2}$$

$$= \frac{4\cancel{n}\times 18\times\cancel{10^9}}{(\sqrt{32})^2}\left[4\bar{a}_x - 4\bar{a}_y\right]$$

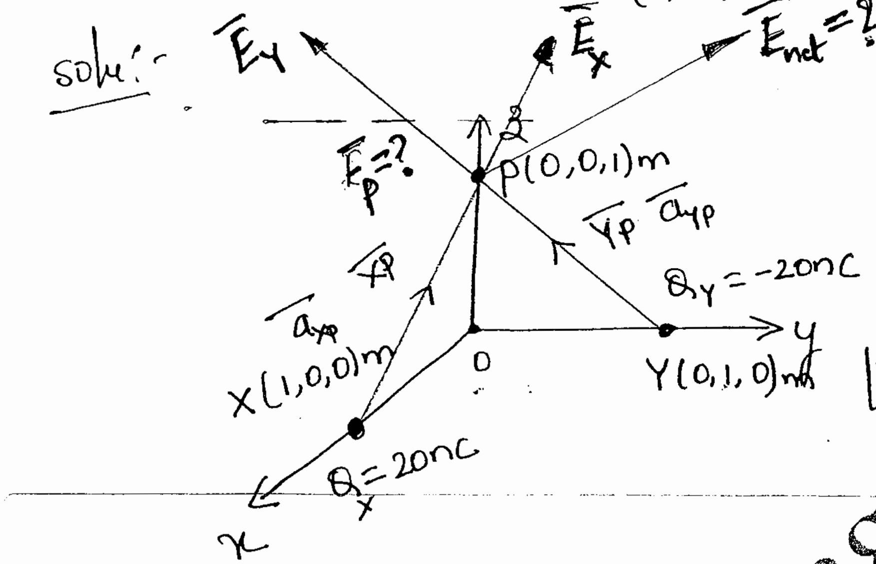
$$\boxed{\bar{E}_{P_2} = 9\bar{a}_x - 9\bar{a}_y} \ V/m$$

$$\boxed{\bar{E} = \bar{E}_{P_1} + \bar{E}_{P_2} = 18\,\bar{a}_x} \ V/m$$

problem10 $\boxed{\bar{E}_{net} = 18\,\bar{a}_x}$ V/m

Two point charges of 20nC and -20nC are situated at (1,0,0)m and (0,1,0)m in free space. Determine Electric Field Intensity at (0,0,1).

soln:



$\overline{XP} = -\bar{a}_x + \bar{a}_z$

$|\overline{XP}| = \sqrt{2}$ m.

$\overline{YP} = -\bar{a}_y + \bar{a}_z$

$|\overline{YP}| = \sqrt{2}$ m

$$\bar{E}_{net} = \bar{E}_X + \bar{E}_Y \text{ V/m.}$$

$$\bar{E}_{net} = \frac{Q_X}{4\pi\epsilon|\overline{XP}|^2}\bar{a}_{XP} + \frac{Q_Y}{4\pi\epsilon|\overline{YP}|^2}\bar{a}_{YP}$$

$$\bar{E}_{net} = \frac{Q_X}{4\pi\epsilon}\frac{\overline{XP}}{|\overline{XP}|^3} + \frac{Q_Y}{4\pi\epsilon}\frac{\overline{YP}}{|\overline{YP}|^3}$$

$$\bar{E}_{net} = \frac{20n \times 9\times10^9}{(\sqrt{2})^3}\left[-\bar{a}_x + \bar{a}_z\right] - \frac{20n\times 9\times10^9}{(\sqrt{2})^3}\left[-\bar{a}_y + \bar{a}_z\right]$$

$$\bar{E}_{net} = \frac{20n\times 9\times10^9}{(\sqrt{2})^3}\left[-\bar{a}_x + \cancel{\bar{a}_z} + \bar{a}_y - \cancel{\bar{a}_z}\right]$$

$$\boxed{\bar{E}_{net} = -63.639\,\bar{a}_x + 63.639\,\bar{a}_y} \text{ V/m}$$

$E_x = -63.639$ V/m; $E_y = 63.639$ V/m; $E_z = 0$ V/m

$$|\bar{E}_{net}| = \underline{89.99 \simeq 90} \text{ V/m}$$

problem 11.

$\rho_l = 25\,nC/m$ $x=-3$ $z=4m$ E

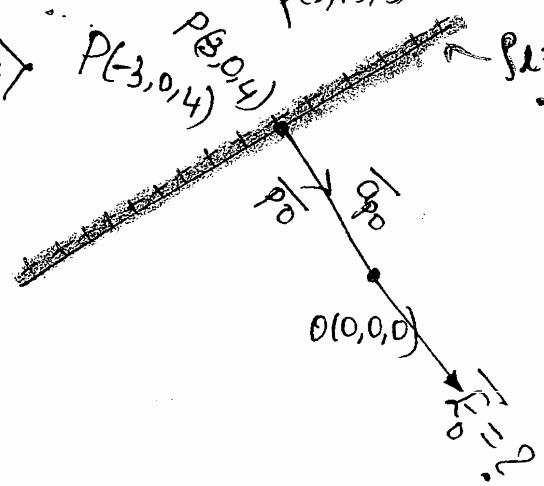
A uniform Line charge density $\rho_l = 25\ nC/m$, lies on the line x=-3, z=4m in space. Find E in Cartesian components at i. origin ii. P(2,15,3) iii. Q($\rho = 4, \varphi = 60°, z = 2$). ← $Q(\rho=4, \phi=60°, Z=2)$

Ans: i. E = $54a_x$-$72a_z$ V/m ii. E = $77.55a_x$-$31a_z$ V/m iii. E= $86.56a_x$-$17.3a_z$ V/m

P(2,15,3)

soln:

i) P(-3,0,4)



$\rho_l = 25\,nC/m$. and on this Line $x=-3, z=4m$ are fixed.

$$\overline{E_o} = \frac{\rho_l}{2\pi\epsilon|\overline{PO}|}\overline{a_{po}} = \frac{\rho_l}{2\pi\epsilon|\overline{PO}|^2}\overline{PO}\ V/m.$$

$$\overline{PO} = 3\overline{a_x} + (0-4)\overline{a_z}$$

$$\overline{PO} = 3\overline{a_x} - 4\overline{a_z}$$

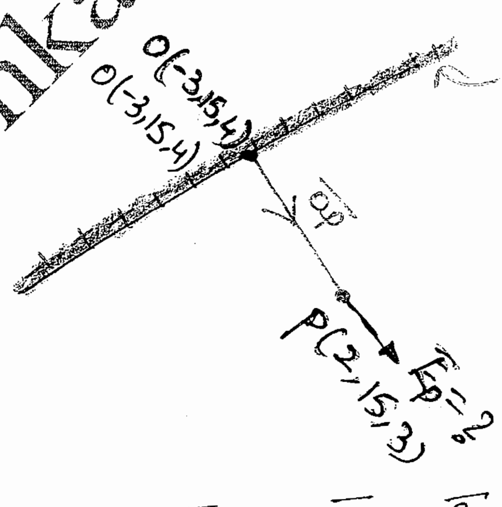
$$|\overline{PO}| = \sqrt{9+16} = \underline{5m}$$

$$\overline{E_o} = \frac{25n \times 18 \times 10^9}{(5)^2}\left[3\overline{a_x} - 4\overline{a_z}\right]$$

$$\overline{E_o} = 18\left[3\overline{a_x} - 4\overline{a_z}\right] = 54\overline{a_x} - 72\overline{a_y}\ V/m$$

$$\boxed{\overline{E_o} = 54\overline{a_x} - 72\overline{a_y}}\ V/m$$

ii)



$\rho_l = 25\,nC/m$
$x = -3m, z = 4m$.

$$\overline{E_p} = \frac{\rho_l}{2\pi\epsilon|\overline{OP}|}\overline{a_{op}}\ V/m$$

$$\overline{E_p} = \frac{\rho_l}{2\pi\epsilon|\overline{OP}|^2}\overline{OP}\ V/m.$$

$$\overline{OP} = 5\overline{a_x} - \overline{a_z}, \quad |\overline{OP}| = \sqrt{25+1} = \sqrt{26}\ m.$$

$$\overline{E}_p = \frac{25\pi \times 18\times10^{9}}{(\sqrt{26})^2}\left[5\overline{a}_x - \overline{a}_z\right]$$

$$\overline{E}_p = 17.3076\left[5\overline{a}_x - \overline{a}_z\right]$$

$$\boxed{\overline{E}_p = 86.538\,\overline{a}_x - 17.3076\,\overline{a}_z} \text{ V/m}$$

iii) $Q(\rho=4, \phi=60^\circ, z=2)$ given point is in cylindrical co-ordinate system. Convert it into equivalent point in Cartesian Co-ordinate System.

$\rho=\sqrt{x^2+y^2}$ and $\phi=\tan^{-1}(y/x)$.

$\rho^2 = x^2+y^2$ $\qquad \tan\phi = y/x$

$\qquad\qquad\qquad \tan(60^\circ) = y/x$

$16 = x^2+y^2$ — ① $\qquad \boxed{y/x = 1.732} \rightarrow$ ②

$16 = x^2+3x^2$

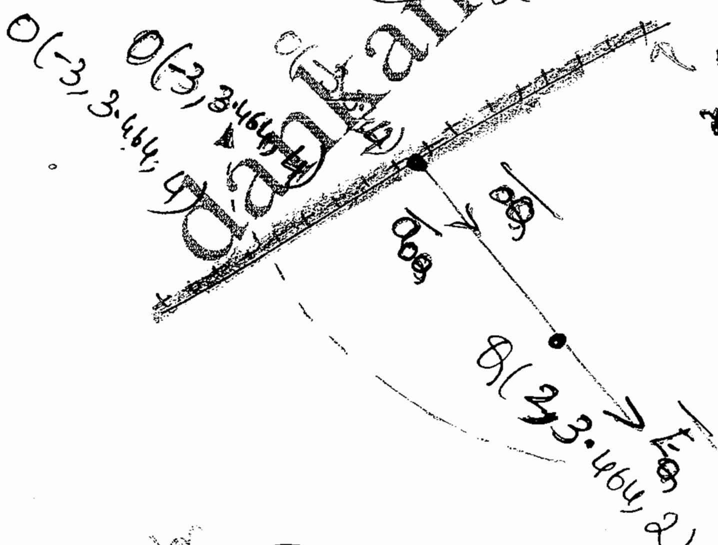
$\Rightarrow \boxed{x=\pm 2}$ choose $\boxed{x=2}$ bcz ϕ must be +ve i.e 60° $\quad y = 1.732x$

and $\boxed{y = 3.464}$ m.

$\rho_L = 25\pi$ nC/m

$x=-3, z=4$m.

$Q(2, 3.464, 2)$.



$$\overline{E}_Q = \frac{\rho_L}{2\pi\epsilon|\overline{OQ}|^2}\overline{OQ} \text{ V/m}$$

$\overline{OQ} = 5\overline{a}_x - 2\overline{a}_z$; $|\overline{OQ}| = \sqrt{25+4}$

$|\overline{OQ}| = \sqrt{29}$ m.

$$\overline{E}_Q = \frac{25\pi \times 18\times10^{9}}{(\sqrt{29})^2}\left[5\overline{a}_x - 2\overline{a}_z\right]$$

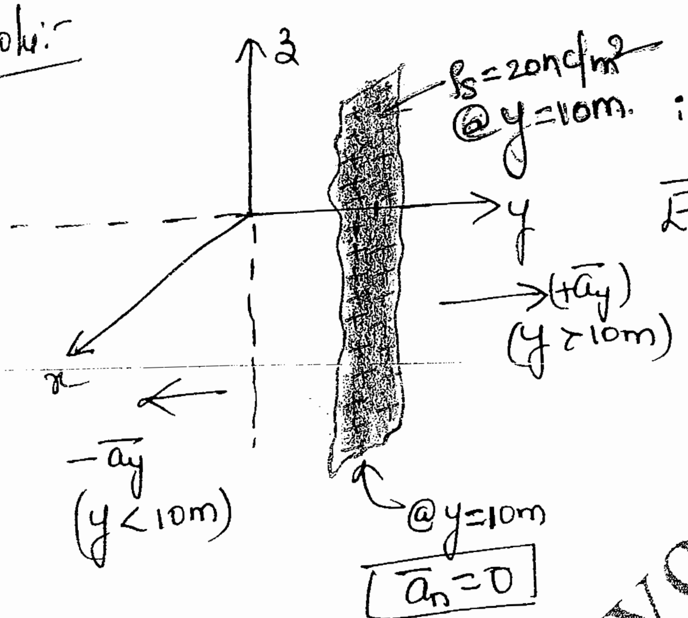
$$\boxed{\overline{E}_Q = 77.586\,\overline{a}_x - 31.034\,\overline{a}_z} \text{ V/m}.$$

problem 12 $\rho_S = 20nC/m^2$

Sheet charge lies in y=10m plane in the form of infinite square sheet with a uniform charge density of $\rho_S = 20n\ C/m^2$. Determine $\bar{E}$ at all the points.

Ans: i. if $y > 10m$; $E = 360\pi a_y$ V/m ii. if $y = 10m$: $E = 0$ V/m
iii. if $y < 10m$; $E = 360\pi(-a_y)$V/m

soln:



i) $\bar{E}$ when $y>10m$

$$\bar{E} = \frac{\rho_s}{2\epsilon}(+\bar{a}_y)$$

$$\bar{E} = 20\cancel{n} \times 18\pi \times 10^{\cancel{9}} [+\bar{a}_y]$$

$$\boxed{\bar{E} = 360\pi\, \bar{a}_y} \text{ V/m}$$

$$= 1.1309\,\bar{a}_y \text{ kV/m}$$

ii) if $y=10m$; $\bar{E} = 0$ V/m by $\bar{E} = \frac{\rho_s}{2\epsilon}\bar{a}_n^{\,\to 0}$

No unit normal vector for that surface.

$\therefore \boxed{\bar{E} = 0}$ V/m

i.e field on sheet charge is 0 V/m.

iii) if $y<10m$.

$$\bar{E} = \frac{\rho_s}{2\epsilon}\bar{a}_n = \frac{\rho_s}{2\epsilon}(-\bar{a}_y)$$

$$\bar{E} = 20\cancel{n} \times 18\pi \times 10^{\cancel{9}} [-\bar{a}_y]$$

$$\boxed{\bar{E} = -360\pi\, \bar{a}_y} \text{ V/m (or) } = -1.1309\,\bar{a}_y \text{ kV/m}$$

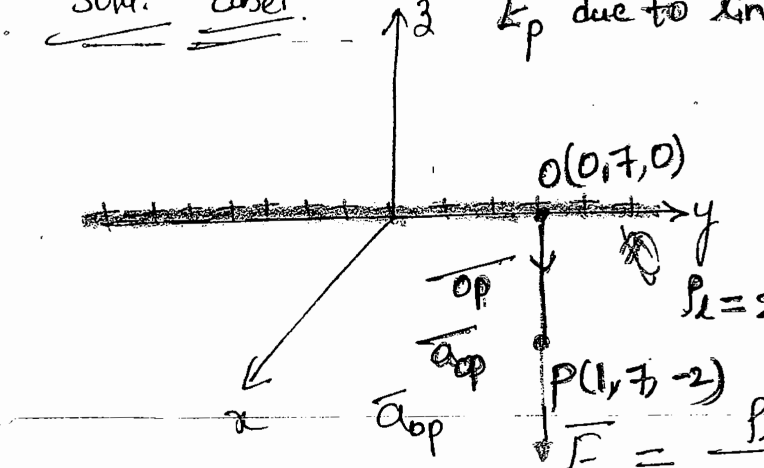
Problem 13

A line charge of $\rho_l = 2nC/m$, lies along y-axis while surface charge densities of 0.1 nC/m^2 and -0.1 nC/m^2 exist on the plane Z=3 and Z=-4m respectively. Find E at P(1,7,-2).

Ans: $E\rho_l$=7.19a_x-14.3a_z V/m ; $E\rho_{s+}$=-5.64a_z V/m ; $E\rho_{s-}$=-5.64a_z V/m ; E_{net} =7.19a_x-25.67a_z

$\rho_L = 2nC/m$, $z=3$, $z=-4m$, $0.1 nC/m^2$, $-0.1nC/m^2$, $P(1,7,-2)$

Solu: case i $\bar{E}_P$ due to line charge density



$$\bar{E}_{\rho_L} = \frac{\rho_L}{2\pi\epsilon|\overline{OP}|}\bar{a}_{op} \text{ V/m}$$

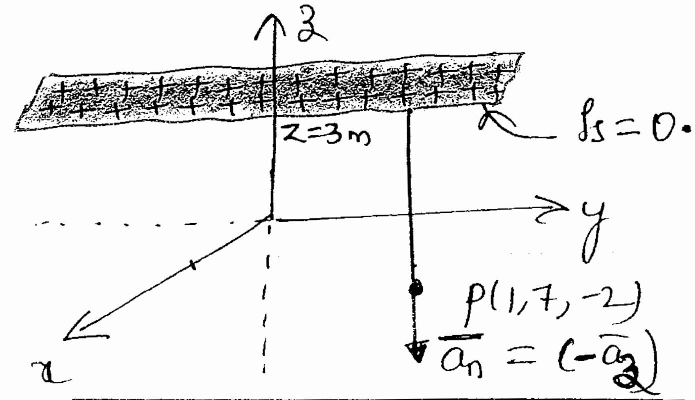
$$\bar{E}_{\rho_L} = \frac{\rho_L}{2\pi\epsilon|\overline{OP}|^2}\overline{OP} \text{ V/m} ; \quad \overline{OP} = \bar{a}_x - 2\bar{a}_z$$

$$|\overline{OP}| = \sqrt{1+4} = \sqrt{5}\text{ m.}$$

$$\bar{E}_{\rho_L} = \frac{2\cancel{\pi}\times 18\times 10^{9}}{(\sqrt{5})^2}\left[\bar{a}_x - 2\bar{a}_z\right] = 7.2\left[\bar{a}_x - 2\bar{a}_z\right]$$

$$\boxed{\bar{E}_{\rho_L} = 7.2\bar{a}_x - 14.4\bar{a}_z} \text{ V/m}$$

Case ii $\bar{E}_P$ due to sheet charge of $\rho_{s+} = 0.1nC/m^2$ located @ $z=3m$.



$\rho_s = 0.1nC/m^2$ @ $z=3m$.

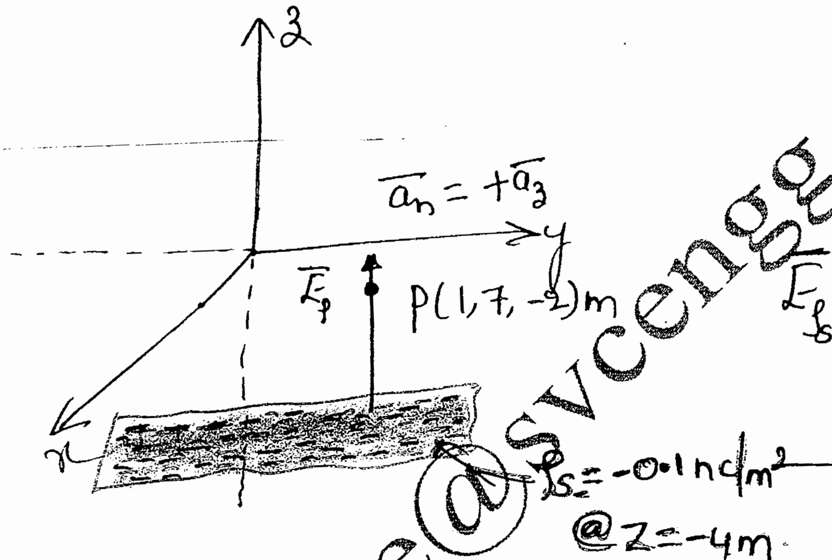
P(1,7,-2)

field is always towards the desired point when field into be measured.

$$\bar{E}_{\rho_{s+}} = \frac{\rho_s}{2\epsilon}\bar{a}_n = 0.1\cancel{n} \times 18\pi \times 10^{\cancel{9}} (-\bar{a}_z)$$

$$\boxed{\bar{E}_{\rho_{s+}} = -5.6548\,\bar{a}_z} \text{ V/m}$$

Case iii $\bar{E}_p$ due to Sheet charge of $\rho_{s-} = -0.1 nC/m^2$ placed at $z = -4m$.



$$\bar{E}_{\rho_{s-}} = \frac{\rho_{s-}}{2\epsilon}\bar{a}_n$$

$$\bar{E}_{\rho_{s-}} = -0.1\cancel{n} \times 18\pi \times 10^{\cancel{9}} \times (+\bar{a}_z)$$

$$\boxed{\bar{E}_{\rho_{s-}} = -5.6548\,\bar{a}_z} \text{ V/m}$$

net field at point 'p' is

$$\bar{E}_p = \bar{E}_{\rho_l} + \bar{E}_{\rho_{s+}} + \bar{E}_{\rho_{s-}}$$

$$= 7.2\bar{a}_x - 14.4\bar{a}_z - 5.6548\bar{a}_z - 5.6548\bar{a}_z \text{ V/m}$$

$$\boxed{\bar{E}_p = 7.2\,\bar{a}_x - 25.709\,\bar{a}_z} \text{ V/m}$$

$$|\bar{E}_p| = 26.698 \text{ V/m}$$

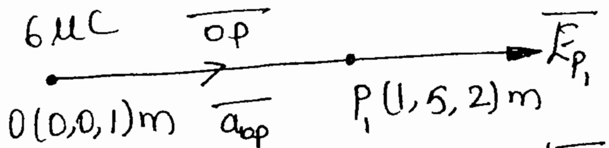
problem 14 $\leftarrow \bar{E}$ $\leftarrow P(1,5,2)m$ $\rho_l = 180nC/m$ $\leftarrow 6\mu C$ $\leftarrow (0,0,1)m$

Find E at P(1,5,2)m in free space if a point charge of $6\mu C$ is located at (0,0,1)m, the uniform line charge density $\rho_l = 180nC/m$ along x-axis and uniform sheet charge with $\rho_s = 25nC/m^2$ over the plane Z= -1m. $\leftarrow$ z = -1m.

Ans: E_Q =384.37a_x+1921.8a_y+384.3a_z V/m ; E ρ_l==557.8a_y+223.14a_z V/m ;
Eρ_{s+} =1411.79a_z V/m ; E_{net} =384.37a_x +2479.7a_y+2019.31a_z V/m

$\leftarrow \rho_s = 25nC/m^2$

Soln: Case i. $\bar{E}_{P_1}$ due to point charge



$\overline{OP} = \bar{a}_x + 5\bar{a}_y + \bar{a}_z$; $|\overline{OP}| = \sqrt{1+25+1} = \sqrt{27}$ m

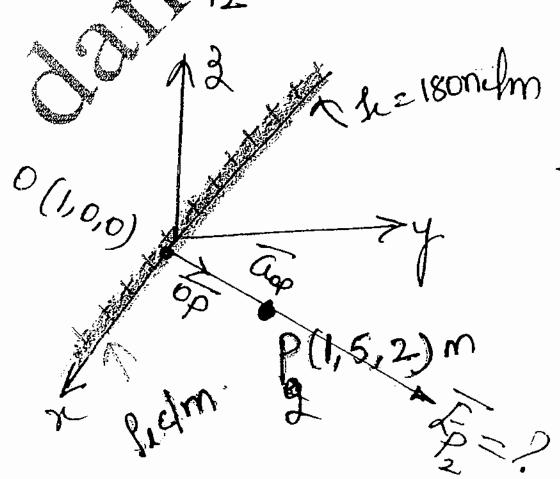
$$\bar{E}_{P_1} = \frac{Q}{4\pi\epsilon|\overline{OP}|^2}\overline{a_{op}} \text{ V/m} = \frac{Q}{4\pi\epsilon|\overline{OP}|^3}\overline{OP} \text{ V/m.}$$

$$\bar{E}_{P_1} = \frac{6\mu \times 9 \times 10^9}{(\sqrt{27})^3}\left[\bar{a}_x + 5\bar{a}_y + \bar{a}_z\right]$$

$$\bar{E}_{P_1} = 384.9\left[\bar{a}_x + 5\bar{a}_y + \bar{a}_z\right]$$

$$\boxed{\bar{E}_{P_1} = 384.9\bar{a}_x + 1924.5\bar{a}_y + 384.9\bar{a}_z} \text{ V/m}$$

Case ii $\bar{E}_{P_2}$ due to line charge.



$$\bar{E}_{P_2} = \frac{\rho_l}{2\pi\epsilon|\overline{OP}|}\overline{a_{op}} \text{ V/m}$$

$$\bar{E}_{P_2} = \frac{\rho_l}{2\pi\epsilon|\overline{OP}|^2}\overline{OP} \text{ V/m}$$

$\overline{OP} = 5\bar{a}_y + 2\bar{a}_z$

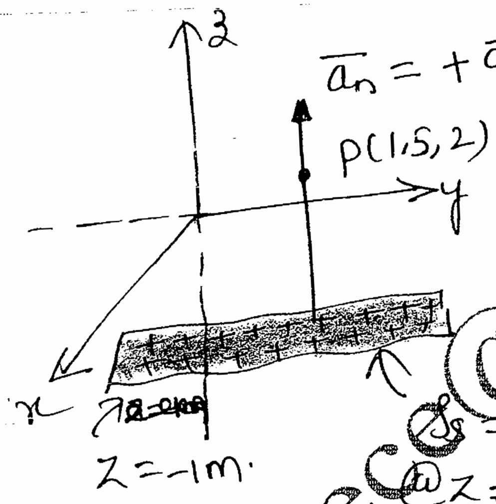
$|\overline{OP}| = \sqrt{25+4} = \sqrt{29}$ m

$$\bar{E}_{P_2} = \frac{180\cancel{n} \times 18\times 10^{\cancel{9}}}{(\sqrt{29})^2}\left[5\bar{a}_y + 2\bar{a}_z\right]$$

$$\bar{E}_{P_2} = 111.724\left[5\bar{a}_y + 2\bar{a}_z\right]$$

$$\boxed{\bar{E}_{P_2} = 558.62\,\bar{a}_y + 223.44\,\bar{a}_z} \text{ V/m.}$$

Case iii $\bar{E}_{P_3}$ due to sheet charge placed @ $z=-1$m



$$\bar{E}_{P_3} = \frac{\rho_{s+}}{2\epsilon}\bar{a}_n$$

$$\bar{E}_{P_3} = 25\cancel{n}\times 18\pi\times 10^{\cancel{9}}\,(+\bar{a}_z)$$

$$\boxed{\bar{E}_{P_3} = 1413.71\,\bar{a}_z} \text{ V/m}$$

$$\bar{E}_{net} = \bar{E}_{P_1} + \bar{E}_{P_2} + \bar{E}_{P_3} \quad \text{V/m}$$

$$= 384.9\bar{a}_x + 1924.5\bar{a}_y + 384.9\bar{a}_z + 558.62\bar{a}_y + 223.44\bar{a}_z + 1413.71\bar{a}_z$$

$$\boxed{\bar{E}_{net} = 384.9\,\bar{a}_x + 2483.12\,\bar{a}_y + 2022.05\,\bar{a}_z}$$

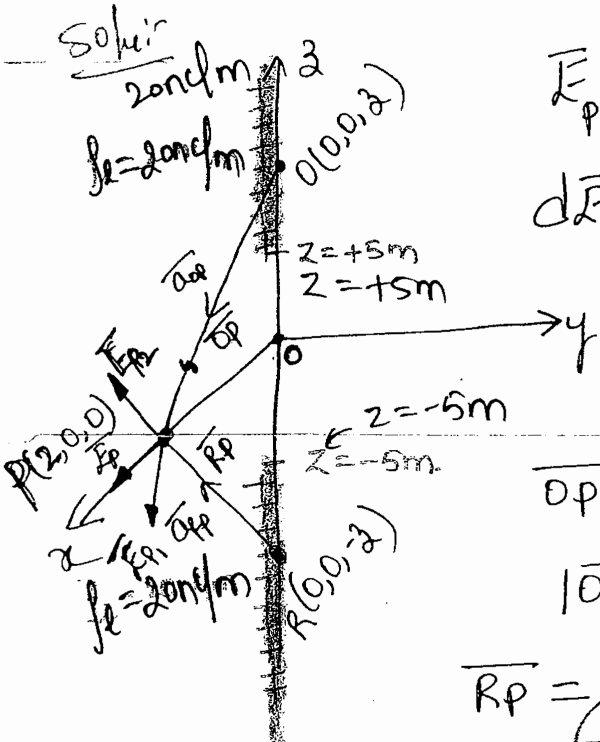
problems

The charge is distributed along the z-axis from Z= -5m to $-\infty$, and Z= +5m to $+\infty$ with a charge density of $\rho_l = 20n\ C/m$. find $\underline{E}$ at (2,0,0)m. also express the answer in cylindrical Co-ordinate.

Ans: E= 12.87 a_x V/m and E_{cyl}=13$a\rho$ V/m.

$\rho_l = 20nC/m$; $z = -5m$ to $-\infty$; $+5$ to $+\infty$; $\overline{E}$; $(2,0,0)m$.

Soln



$$\overline{E}_p = \overline{E}_{P_1} + \overline{E}_{P_2}\ V/m.$$

$$d\overline{E}_p = \frac{dQ}{4\pi\epsilon|\overline{OP}|^2}\overline{a}_{OP} + \frac{dQ}{4\pi\epsilon|\overline{RP}|^2}\overline{a}_{RP}\ V/m$$

$$dQ = \rho_l\, dz.$$

$$\overline{OP} = 2\overline{a}_x + (0-z)\overline{a}_z = 2\overline{a}_x - z\overline{a}_z$$

$$|\overline{OP}| = \sqrt{4+z^2}\ m = \sqrt{4+z^2}\ m.$$

$$\overline{RP} = 2\overline{a}_x + z\,\overline{a}_z \ ; \ |\overline{RP}| = \sqrt{2^2+z^2}$$

$$\cancel{2\overline{a}_x + z\overline{a}_z} ;\ |\overline{RP}| = \sqrt{4+z^2}\ .\ m$$

net field along 'z' direction is zero.

$$d\overline{E}_p = \frac{\rho_l\, dz}{4\pi\epsilon(4+z^2)^{3/2}}\left[2\overline{a}_x - \cancel{z\overline{a}_z}\right] + \frac{\rho_l\, dz}{4\pi\epsilon[4+z^2]^{3/2}}\left[2\overline{a}_x + \cancel{z\overline{a}_z}\right]$$

$$\overline{E}_p = \frac{\rho_l}{4\pi\epsilon}\int_{z=5}^{\infty}\frac{dz}{(4+z^2)^{3/2}}(2\overline{a}_x) + \frac{\rho_l}{4\pi\epsilon}\int_{z=-\infty}^{-5}\frac{dz}{[4+z^2]^{3/2}}[2\overline{a}_x]$$

$$\overline{E}_p = \frac{\rho_l}{4\pi\epsilon}(2\overline{a}_x)\left[\int_{z=5}^{\infty}\frac{dz}{(4+z^2)^{3/2}} + \int_{z=-\infty}^{-5}\frac{dz}{[4+z^2]^{3/2}}\right] \leftarrow (1)$$

$$= 20n \times 9 \times 10^9 \times 2\overline{a}_x\left[\ \text{---}\,||\,\text{---}\ \right]$$

$$(150)\quad = 360\overline{a}_x\left[\ \text{---}\,||\,\text{---}\ \right]$$

put $z = 2\tan\theta$; $z = 5 \Rightarrow \theta = 68.198$

$dz = 2\sec^2\theta \, d\theta$; $z = +\infty \Rightarrow \theta = +\pi/2$.

$$\int_{z=5}^{\infty} \frac{dz}{(4+z^2)^{3/2}} = \int_{\theta=68.198}^{+\pi/2} \frac{2\sec^2\theta \, d\theta}{[4+4\tan^2\theta]^{3/2}}$$

$$= \int_{\theta=68.198}^{+90^\circ} \frac{2\sec^2\theta \, d\theta}{(4\sec^2\theta)^{3/2}} = \int_{\theta=68.198}^{+90^\circ} \frac{\cancel{2\sec^2\theta}}{(2^{3}\sec^{3}\theta)} d\theta$$

$$= \int_{\theta=68.198}^{90^\circ} \frac{1}{2^2\sec\theta} d\theta = \frac{1}{4}\int_{\theta=68.198}^{90^\circ} \cos\theta \, d\theta = \frac{1}{4}\left[\sin\theta\Big|_{68.19}^{90^\circ}\right]$$

$$= \frac{1}{4}[1-0.928]$$

$$= 0.01789 \leftarrow (a)$$

III^{ly} $$\int_{z=-\infty}^{-5} \frac{dz}{(4^2+z^2)^{3/2}} = 0.01789 \text{ (due to symmetry i.e even function.}$$

$$= \int_{z=5}^{+\infty} \frac{dz}{(4^2+z^2)^{3/2}} = 0.01789 \leftarrow (b)$$

using (a) and (b) in eq^n (1)

$$\bar{E}_p = 360\,\bar{a}_x[0.01789 + 0.01789]$$

$$\boxed{\bar{E}_p = 12.884\,\bar{a}_x} \text{ V/m} \Rightarrow E_x = 12.884 \text{ V/m}, \quad E_y = E_z = 0 \text{ V/m}.$$

ii) $E_\rho = E_x\cos\phi \Rightarrow \phi = \tan^{-1}(y/x) = \tan^{-1}(0/2) = 0^\circ$

$\therefore E_\rho = E_x\cos(0) = E_x = 12.884$ V/m.

$E_\phi = -E_x\sin(\phi) = -E_x\sin(0) = 0$ and $E_z = 0$ V/m. (Contd)

$\overline{E}_p$ in Cylindrical Co-ordinate System is $\boxed{\overline{E}_p = 12.88\overline{a}_\rho \text{ V/m}}$

problem 16.

A line charge density $\rho_l = 24nC/m$ is located in free space on the line y=1m and z=2m.

i. Find **E** at the point P(6,-1,3).

ii. What point charge Q should be Located at A(-3,4,1) to make y-component of total E is zero at P.

Ans: i. E= -172.56 a_y+86.28 a_z V/m; ii. Q= -4.4311μC

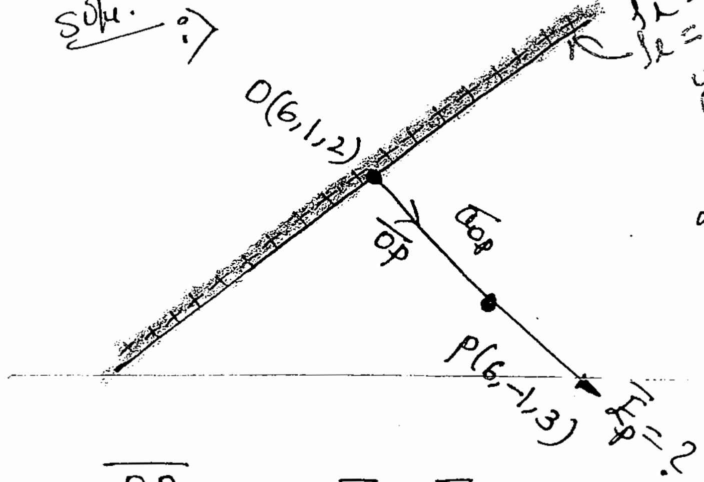
Sol:- i)

$\rho_l = 24nC/m$

$\rho_l = 24 nC/m.$

y=1m and z=2m.

y=1m, z=2m



$$\overline{E}_p = \frac{\rho_l}{2\pi\epsilon|\overline{OP}|}\overline{a}_{op} \text{ V/m.}$$

$$\overline{E}_p = \frac{\rho_l}{2\pi\epsilon|\overline{OP}|^2}\overline{OP} \text{ V/m.}$$

$\overline{OP} = -2\overline{a}_y + \overline{a}_z$;

$|\overline{OP}| = \sqrt{4+1} = \sqrt{5}\text{ m}$

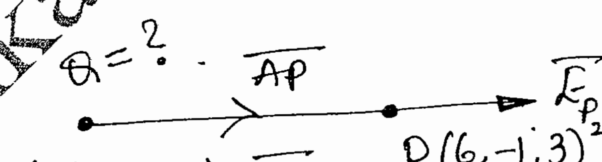
$$\overline{E}_p = \frac{24n \times 18\times10^9}{(\sqrt{5})^2}[-2\overline{a}_y + \overline{a}_z]$$

$$\overline{E}_p = 86.4[-2\overline{a}_y + \overline{a}_z]$$

$$\boxed{\overline{E}_{p_1} = -172.8\overline{a}_y + 86.4\overline{a}_z} \text{ V/m} \leftarrow (1)$$

$\overline{E}_{net} = \overline{E}_{p_1} + \overline{E}_{p_2}$ V/m.

ii)



$\Rightarrow E_{y_{net}} = 0$ V/m.

A(-3,4,1)

$$\overline{E}_{p_2} = \frac{Q}{4\pi\epsilon|\overline{AP}|^2}\overline{a}_{AP} \text{ V/m}$$

$\overline{AP} = 9\overline{a}_x - 5\overline{a}_y + 2\overline{a}_z$; $|\overline{AP}| = \sqrt{81+25+4}$

$|\overline{AP}| = \sqrt{110}$ m.

$$\bar{E}_{P_2} = \frac{Q \times 9 \times 10^9}{(\sqrt{110})^3}\left[3\bar{a}_x - 5\bar{a}_y + 2\bar{a}_z\right] \text{ V/m} \quad \leftarrow (2)$$

given

$$\boxed{E_{y_{net}} = E_{P_{1y}} + E_{P_{2\text{-}y}}} = 0 \Rightarrow E_{y_{net}} = E_{P_{1\text{-}y}} + E_{P_{2\text{-}y}} = 0; \text{ V/m}$$

i.e Sum of 'y' component in both eq (1) and (2) is given $\bar{E}_{y_{net}} = 0$

$$E_{y_{net}} = -172.8 - \frac{Q \times 9 \times 10^9}{(\sqrt{110})^3}(5) = 0$$

$$-172.8 = \frac{Q \times 9 \times 10^9}{(\sqrt{110})^3} \times 5$$

$$\Rightarrow -172.8 = \frac{Q \times 9 \times 10^9}{(\sqrt{110})^3} \times 5$$

$$Q = \frac{-172.8 \times (\sqrt{110})^3}{9 \times 10^9 \times 5}$$

$$Q = -4.430 \times 10^{-6} \text{ C}$$

$$\boxed{Q = -4.430\,\mu} \text{ Coulombs}$$

problem 17

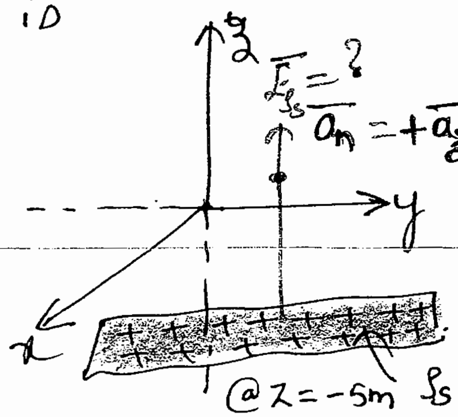
Two infinite sheets of uniform charge densities $\rho_s = \frac{1}{6\pi} n\,C/m^2$ are located at z= -5m and y= -5m. Determine the uniform line charge density ρ_l necessary to produce same value of **E** at (4,2,2)m if the line charge is at y=0 and z=0m.

Ans: $\rho_l = \frac{2}{3}$ nC/m.

$\rho_s = \frac{1}{6\pi} nC/m^2$; $z = -5m$; $y = -5m$; $(4,2,2)m$; $\bar{E}$; $y=0$; $z=0m$; ρ_l

soln: $\bar{E}$ due $\rho_s = \frac{1}{6\pi} nC/m^2$ @ $z=-5$ and $y=-5m$

i)

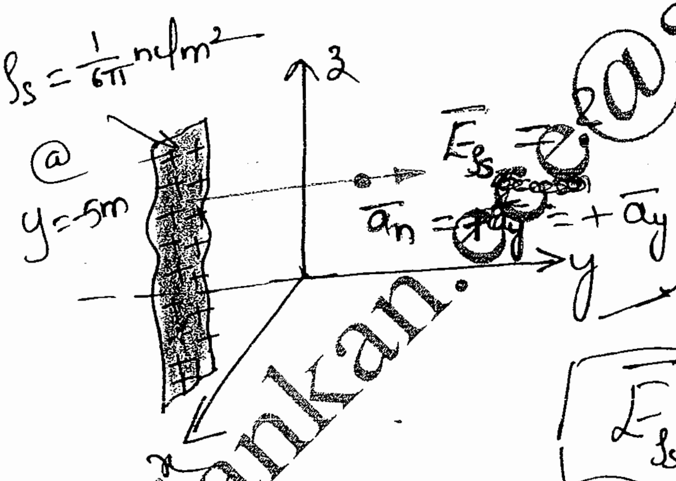


$$\bar{E}_{\rho_{s_{z=-3}}} = \frac{\rho_s}{2\epsilon}\bar{a}_n \; V/m$$

$$= \frac{1}{6\pi} n \times 18\pi \times 10^{9} (+\bar{a}_z)$$

$$\boxed{\bar{E}_{\rho_{s_{z=-3m}}} = 3\bar{a}_z} \; V/m$$

ii)



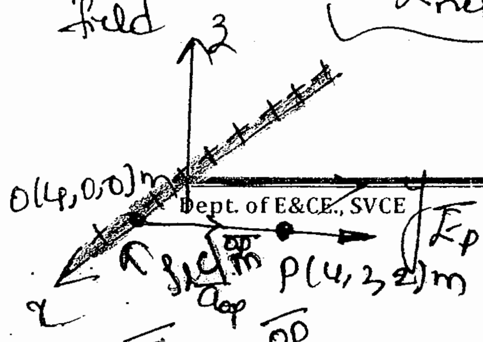
$$\bar{E}_{\rho_{s_{y=-5m}}} = \frac{\rho_s}{2\epsilon}\bar{a}_n^{(+\bar{a}_y)} \; V/m$$

$$\boxed{\bar{E}_{\rho_{s_{y=-5m}}} = 3\bar{a}_y} \; V/m$$

$$\bar{E}_{net} = \bar{E}_{\rho_{s_{z=-3m}}} + \bar{E}_{\rho_{s_{y=-5m}}} = 3\bar{a}_z + 3\bar{a}_y \; V/m$$

Find ρ_l C/m to produce same field

$$\boxed{\bar{E}_{net} = 3\bar{a}_y + 3\bar{a}_z} \; V/m \; ; \; |\bar{E}_{net}| = \sqrt{18} \; V/m.$$



$$\bar{E}_p = \frac{\rho_l}{2\pi\epsilon|\bar{op}|}\bar{a}_{op} \; ; \; \bar{op} = 2\bar{a}_y + 2\bar{a}_z$$

$$|\bar{op}| = \sqrt{8}\,m.$$

$$|\bar{E}_p| = \sqrt{18} = \frac{\rho_l}{2\pi\epsilon\sqrt{4+4}}$$

$$\rho_l = \sqrt{18} \times \sqrt{8} \times [18 \times 10^9]^{-1} = \underline{0.666\,nC/m}$$

$\boxed{\rho_l = \frac{2}{3}\, nC/m}$ (or) $666.66\, pC/m$ $x=2, y=5m$

problem 18

A line charge density $\rho_l = 50nC/m$ is located along the line x=2, y=5m in free space. $\rho_s = 18nC/m^2$

$\rho_l = 50nC/m$

i. Find $\bar{E}$ at P(1,3,-4) $P(1,3,-4)$

ii. If the surface x=4m contains a uniform surface charge density of $\rho_s = 18n\ C/m^2$. At what point in the z= 0 plane is total E=0 v/m?

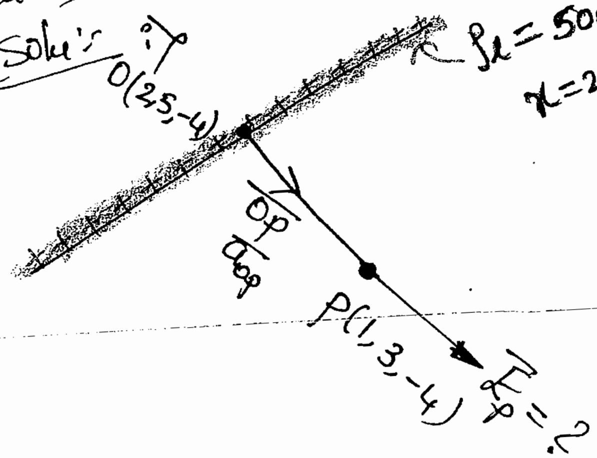
$\bar{E}$

$x=4m$

Ans: i. $E\rho_l$= -179.75 a_x -359.51 a_y V/m; $E\rho_s$ = -1016.489 a_x V/m

ii. x=2.88, y=5, z=0 i.e (2.88,5,0)

Soln: i)



$$\bar{E}_p = \frac{\rho_l}{2\pi\epsilon|\overline{OP}|}\,\overline{a_{op}}\ v/m$$

$$\bar{E}_p = \frac{\rho_l}{2\pi\epsilon|\overline{OP}|^2}\,\overline{OP}\ v/m$$

$$\overline{OP} = -\bar{a}_x - 2\bar{a}_y\ ;$$

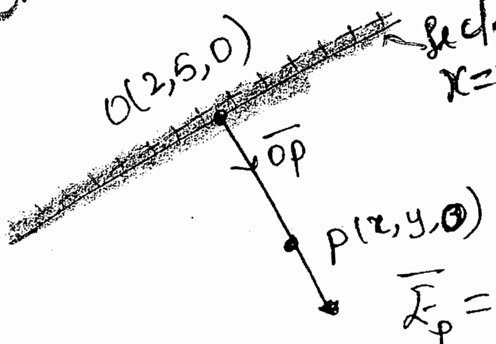
$$|\overline{OP}| = \sqrt{1+4} = \sqrt{5}\ m.$$

$$\bar{E}_p = \frac{50 \times 10^{-9} \times 18\times10^{9}}{(\sqrt{5})^2}\left[-\bar{a}_x - 2\bar{a}_y\right]$$

$$\boxed{\bar{E}_p = -180\bar{a}_x - 360\bar{a}_y}\ v/m$$

ii) Consider the point on z=0 plane is p(x,y,0).

the field E_{p_1} due to a line charge density is



$$\bar{E}_{p_1} = \frac{\rho_l}{2\pi\epsilon|\overline{OP}|}\,\overline{a_{op}}$$

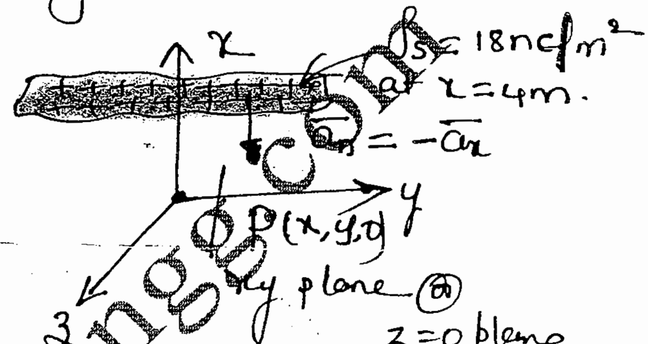
$$\overline{OP} = (x-2)\bar{a}_x + (y-5)\bar{a}_y\ .$$

$$\bar{E}_{p_1} = \frac{\rho_l}{2\pi\epsilon|\overline{OP}|^2}\,\overline{OP}\ v/m.$$

$$|\overline{OP}|^2 = (x-2)^2 + (y-5)^2$$

$$E_{P_1} = \frac{50\cancel{n} \times 18\times 10^{\cancel{9}}}{(x-2)^2+(y-2)^2}\left[(x-2)\,\bar{a}_x + (y-5)\,\bar{a}_y\right] \text{ V/m.} \quad \longleftarrow (1)$$

the field E_{P_2} due to sheet charge of $\rho_s = 18\,nC/m^2$ at $x = 4m$ is



$$E_{P_2} = \frac{\rho_s}{2\epsilon}\,\bar{a}_n$$

$$\bar{E}_{P_2} = 18\cancel{n} \times 18\pi \times 10^{\cancel{9}}\,(-\bar{a}_x)$$

$$\bar{E}_{P_2} = 18\times 18\pi\,(-\bar{a}_x) \text{ V/m.}$$

the total field is equal to zero.

$$\Rightarrow \text{ ie } \bar{E}_{net} = \bar{E}_{P_1} + \bar{E}_{P_2}$$

$$= \frac{50\times 18}{(x-2)^2+(y-5)^2}\left[(x-2)\,\bar{a}_x + (y-5)\,\bar{a}_y\right] - 18\times 18\pi\,\bar{a}_x \text{ V/m}$$

$$= E_x\,\bar{a}_x + E_y\,\bar{a}_y$$

the E_x component of net field

$$E_x = \left[\frac{50\times 18\,(x-2)}{(x-2)^2+(y-5)^2} - 18\times 18\pi\right] = 0 \Rightarrow \frac{900\,\cancel{(x-2)}}{(x-2)^{\cancel{2}}} - 324\pi = 0$$

solve for 'x'

$$\boxed{x = 2.88419}$$

lly.

$$E_y = \frac{50\times 18\,(y-5)}{(x-2)^2+(y-5)^2} = 0 \Rightarrow \boxed{y = 5}$$

∴ the point on a 'xy' plane i.e z=0 plane at which the total field E is zero is

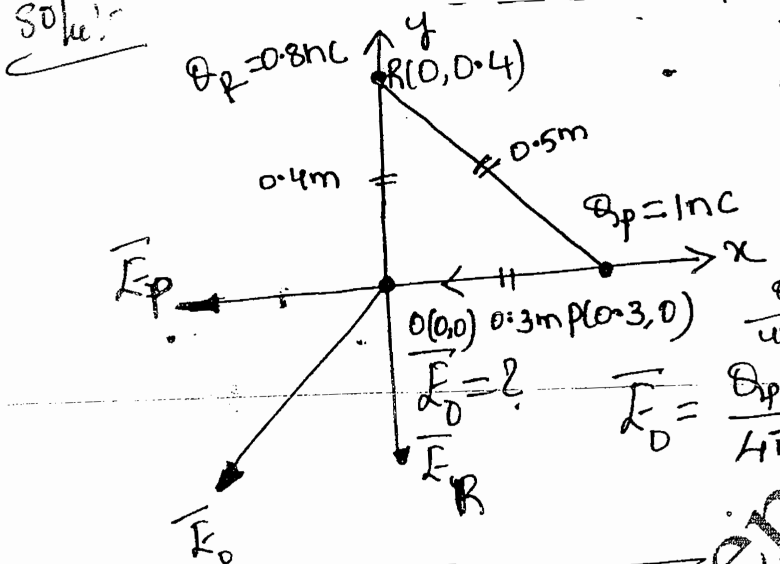
$$\underline{P(2.88419, 5, 0)}$$

problem 19.

Determine the E at a point A Located at a distance of 0.3m and 0.4m respectively from the charges Q_1 and Q_2 spaced 0.5m apart. Q_1 has charge of 1nC while Q_2=0.8nC.
Ans: $E = -99.7a_x - 44.9a_y$ V/m; $|E| = 109.417$V/m

E, 0.5m, Q_1, 0.3m, 0.4m, 1nC, $Q_2 = 0.8nC$

soln:



$$\bar{E}_O = \bar{E}_P + \bar{E}_R$$

$\overline{PO} = -0.3\bar{a}_x$; $|\overline{PO}| = 0.3m$

$\overline{RO} = -0.4\bar{a}_y$; $|\overline{RO}| = 0.4m.$

$$\frac{Q_P}{4\pi\epsilon|\overline{PO}|^2}\bar{a}_{OP} + \frac{Q_R}{4\pi\epsilon|\overline{RO}|^2}\bar{a}_{RO} \text{ V/m}$$

$$\bar{E}_O = \frac{Q_P}{4\pi\epsilon|\overline{PO}|^2}\bar{a}_{OP} + \frac{Q_R}{4\pi\epsilon|\overline{RO}|^2}\bar{a}_{RO} \text{ V/m}$$

$$\bar{E}_O = \frac{Q_P}{4\pi\epsilon|\overline{PO}|^3}\overline{PO} + \frac{Q_R\ \overline{RO}}{4\pi\epsilon|\overline{RO}|^3} \text{ V/m.}$$

$$\bar{E}_O = \frac{1n \times 9\times10^9}{(0.3)^3}[-0.3\bar{a}_x] + \frac{0.8n\times9\times10^9}{(0.4)^3}[-0.4\bar{a}_y]$$

$$\bar{E}_O = -100\,\bar{a}_x - 45\,\bar{a}_y \text{ V/m}$$

$$\boxed{\bar{E}_O = -100\,\bar{a}_x - 45\,\bar{a}_y} \text{ V/m}$$

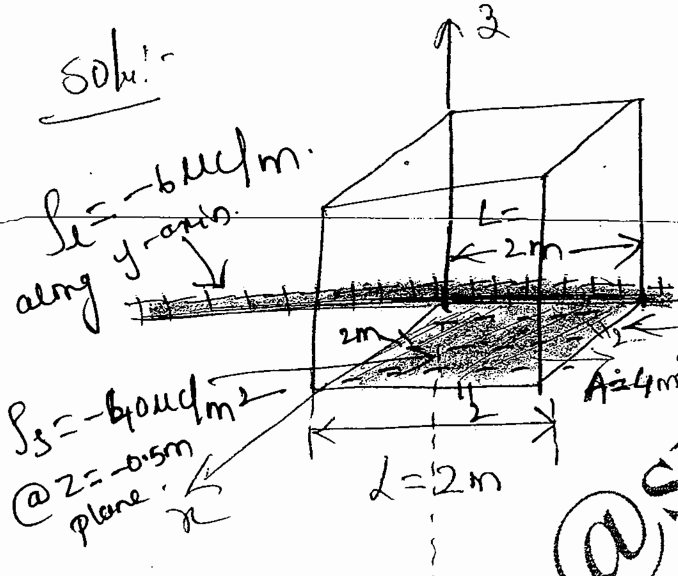
$$\boxed{|\bar{E}_O| = 109.658} \text{ V/m}$$

Problem 20 $\rho_s = -40\mu C/m^2$

Sheet of charge with $\rho_s = -40\mu C/m^2$ Located at $Z = -0.5m$, line charge of $\rho_l = -6\mu C/m$ line along y-axis, what is the net flux crossing the surface of a cube of 2m on an edge of centered at origin. (04M). 06 J/J 2014.

from Gauss Law Concept.

Soln:-



from Gauss's Law

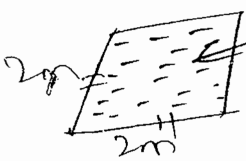
$$\boxed{\Psi_{total} = Q_{enclosed}} \text{ Coulomb's}$$

Flux due to line charge

$$\Psi' = \rho_l \times L = Q_{enclosed}$$

$$= -6\mu C/m \times 2m$$

$$\boxed{\Psi' = -12\mu} \text{ Coulomb's}$$



$A = 4m^2$.

Case ii Flux (Ψ'') due Sheet charge.

$$\Psi'' = \rho_s \times A = -40\mu C/m^2 \times 4m^2 = -160\mu C$$

$$\boxed{\Psi'' = -160\mu} \text{ Coulomb's}$$

The net Flux crossing the cube is

$$\Psi_{total} = \Psi' + \Psi'' = -12\mu - 160\mu$$

$$\boxed{\Psi_{total} = -172\mu} \text{ Coulomb's}$$

problem 21

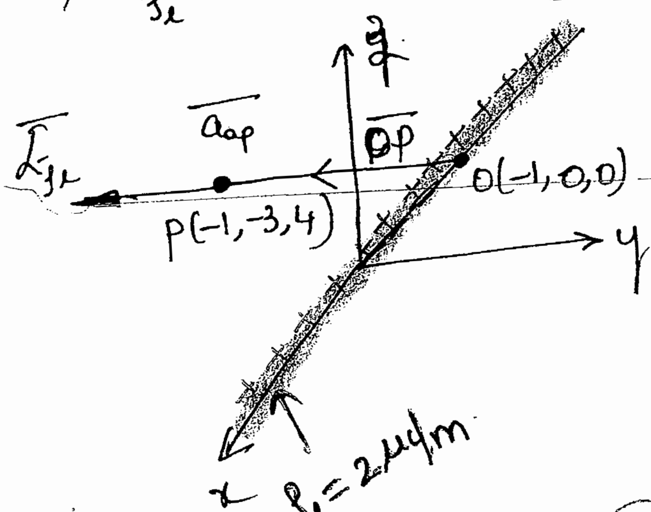
$\rho_l = 2\mu C/m$ $\quad$ $0.2\mu C/m^2$

Uniform line charge density of $\rho_l = 2\mu$C/m placed along x-axis and sheet charges of $0.2\mu C/m^2$ and $0.4\mu C/m^2$ placed at $y = \pm 2m$ plane. Find **E** at P(-1,-3,4) $\quad$ $P(-1,-3,4)m$

Sol:- $0.4\mu C/m^2$ $\quad$ $y = \pm 2m$ $\quad$ $\bar{E}$

$$\bar{E}_{net} = \bar{E}_{\rho_l} + \bar{E}_{\rho_{s_1}} + \bar{E}_{\rho_{s_2}} \ V/m.$$

→ $\bar{E}_{\rho_l}$ due to line charge density placed along x-axis



$$\bar{E}_{\rho_l} = \frac{\rho_l}{2\pi\epsilon|\overline{OP}|}\bar{a}_{op} \ V/m$$

$$\bar{E}_{\rho_l} = \frac{\rho_l}{2\pi\epsilon|\overline{OP}|^2}\overline{OP} \ V/m.$$

$$\overline{OP} = -3\bar{a}_y + 4\bar{a}_z$$

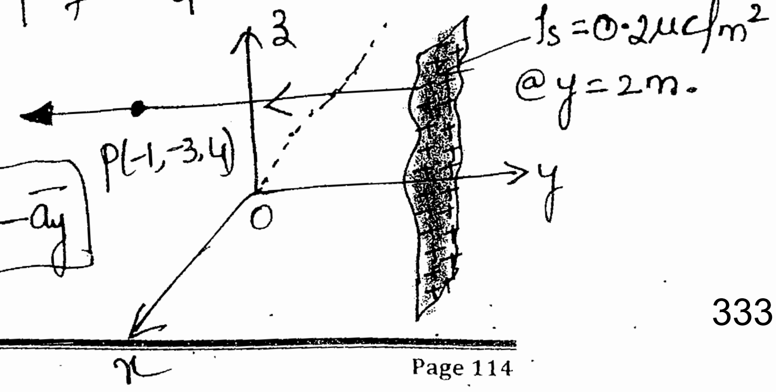
$$|\overline{OP}| = \sqrt{9+16} = 5m.$$

$$\bar{E}_{\rho_l} = \frac{2\mu \times 18\times10^9}{(5)^2}\left[-3\bar{a}_y + 4\bar{a}_z\right]$$

$$\bar{E}_{\rho_l} = -4320\bar{a}_x + 5760\bar{a}_y \ V/m$$

i.e $\boxed{\bar{E}_{\rho_l} = -4320\,\bar{a}_x + 5760\,\bar{a}_y}$ V/m

→ $\bar{E}_{\rho_{s_1}}$ due to sheet charge density of $\rho_{s_1} = 0.2\mu C/m^2$ placed at $y = +2m$.



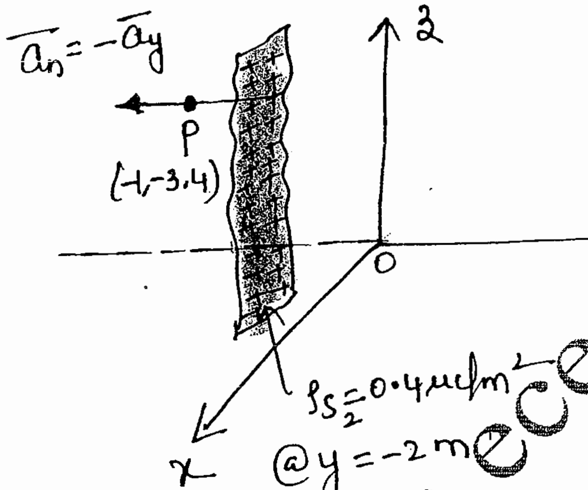
$\boxed{\bar{a}_n = -\bar{a}_y}$

$$\bar{E}_{\rho_{S_1}} = \frac{\rho_{S_1}}{2\epsilon}\bar{a}_n$$

$$= 0.2\mu \times 18\pi \times 10^9 (-\bar{a}_y)$$

$$\boxed{\bar{E}_{\rho_{S_1}} = -11309.733\bar{a}_y} \text{ V/m}$$

→ $\bar{E}_{\rho_{S_2}}$ due to sheet charge density of $\rho_{S_2} = 0.4\mu C/m^2$ placed at $y = -2m$. $P(-1, -3, 4)$.



$$\bar{E}_{\rho_{S_2}} = \frac{\rho_{S_2}}{2\epsilon}\bar{a}_n \text{ V/m}$$

$$\bar{E}_{\rho_{S_2}} = 0.4\mu \times 18\pi \times 10^9 (-\bar{a}_y)$$

$$\boxed{\bar{E}_{\rho_{S_2}} = -22619.467\bar{a}_y} \text{ V/m}$$

$\bar{E}_{net}$ at point $P(-1, -3, 4)$ is

$$\bar{E}_{net} = \bar{E}_{\rho_L} + \bar{E}_{\rho_{S_1}} + \bar{E}_{\rho_{S_2}}$$

$$= -4320\bar{a}_x + 5760\bar{a}_y - 11309.733\bar{a}_y - 22619.467\bar{a}_y$$

$$\boxed{\bar{E}_{net} = -4.320\bar{a}_x - 28.1692\bar{a}_y} \text{ kV/m}$$

problem 22.

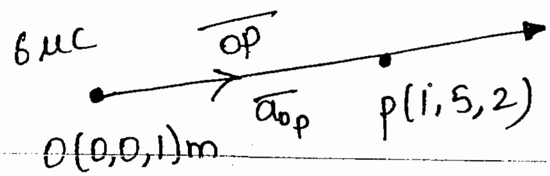
A point charge of 6 μC is located at (0,0,1)m the uniform line charge density of ρ_l = 180nC/m is along x-axis and uniform sheet charge with $\rho_s = 25n\ C/m^2$ over the plane z= -1. Find the combined electric field intensity at P(1,5,2) due to all the charges. EEE-J/J-2015

6μC; (0,0,1)m; $\rho_s = 25nC/m^2$; $\rho_l = 180nC/m$; -x-axis; P(1,5,2)m; z=-1m.

Sol:

$$\bar{E}_p = \bar{E}_{point} + \bar{E}_{line} + \bar{E}_{sheet} \quad V/m$$

→ $\bar{E}_{point} = ?$



$$\bar{E}_{point} = \frac{6\mu}{4\pi\epsilon|\overline{op}|^2}\bar{a}_{op} \ V/m.$$

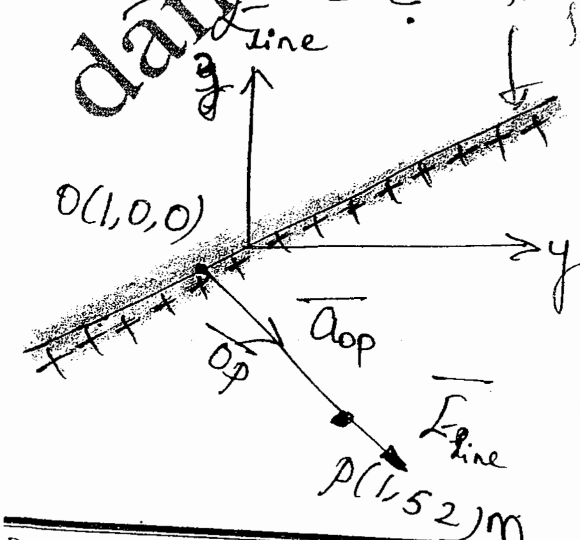
$$\overline{op} = \bar{a}_x + 5\bar{a}_y + \bar{a}_z \ ; \quad |\overline{op}| = \sqrt{1+25+1} = \sqrt{27}\ m$$

$$\bar{E}_{point} = \frac{6\mu}{4\pi\epsilon|\overline{op}|^3}|\overline{op}| \ V/m$$

$$\bar{E}_{point} = \frac{6\mu\times 9\times10^9}{(\sqrt{27})^3}\left[\bar{a}_x + 5\bar{a}_y + \bar{a}_z\right] \quad (384.9)$$

$$\boxed{\bar{E}_{point} = 384.9\,\bar{a}_x + 1924.5\,\bar{a}_y + 384.9\,\bar{a}_z} \ V/m$$

→ $\bar{E}_{line} = ?$ $\quad \rho_l = 180nC/m$



$$\bar{E}_{line} = \frac{\rho_l}{2\pi\epsilon|\overline{op}|}\bar{a}_{op} \ V/m$$

$$\overline{op} = 5\bar{a}_y + 2\bar{a}_z \ ; \quad |\overline{op}| = \sqrt{25+4}$$

$$|\overline{op}| = \sqrt{29}\ m$$

$$\bar{E}_{line} = \frac{\rho_l}{2\pi\epsilon|\overline{op}|^2}\overline{op} \ V/m$$

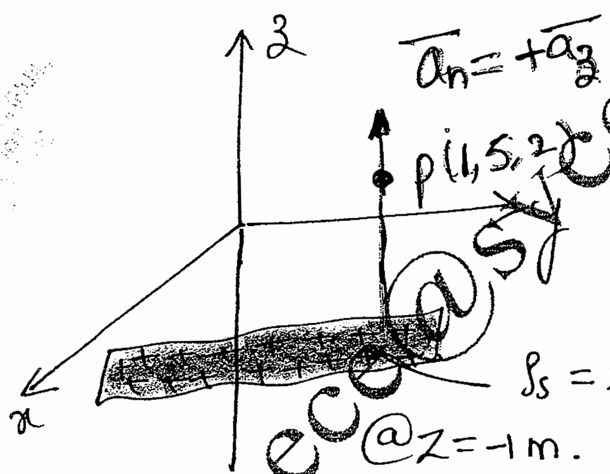
$$\bar{E}_{line} = \frac{180n\times18\times10^9}{(\sqrt{29})^2}\left[5\bar{a}_y + 2\bar{a}_z\right]$$

$$\bar{E}_{line} = 111.724\left[5\bar{a}_y + 2\bar{a}_z\right]$$

$$\bar{E}_{line} = 558.62\,\bar{a}_y + 223.448\,\bar{a}_z \text{ V/m.}$$

$$\boxed{\bar{E}_{line} = 558.62\,\bar{a}_y + 223.448\,\bar{a}_z} \text{ V/m}$$

$\rightarrow$ $\bar{E}_{Sheet} = ?$



$$\bar{E}_{Sheet} = \frac{\rho_s}{2\epsilon}\,\bar{a}_n \text{ V/m}$$

$$= 25n \times 18\pi \times 10^9 (+\bar{a}_z)$$

$$\boxed{\bar{E}_{Sheet} = 1413.7166\,\bar{a}_z} \text{ V/m}$$

$$\bar{E}_{net} = \bar{E}_{point} + \bar{E}_{line} + \bar{E}_{sheet} \text{ V/m}$$

$$= 384.9\,\bar{a}_x + 1924.5\,\bar{a}_y + 384.9\,\bar{a}_z + 558.62\,\bar{a}_y + 223.448\,\bar{a}_z + 1413.7166\,\bar{a}_z$$

$$\bar{E}_{net} = 384.9\,\bar{a}_x + 2483.12\,\bar{a}_y + 2022.064\,\bar{a}_z \text{ V/m.}$$

$$\boxed{\bar{E}_{net} = 0.3849\,\bar{a}_x + 2.483\,\bar{a}_y + 2.02206\,\bar{a}_z} \text{ kV/m}$$

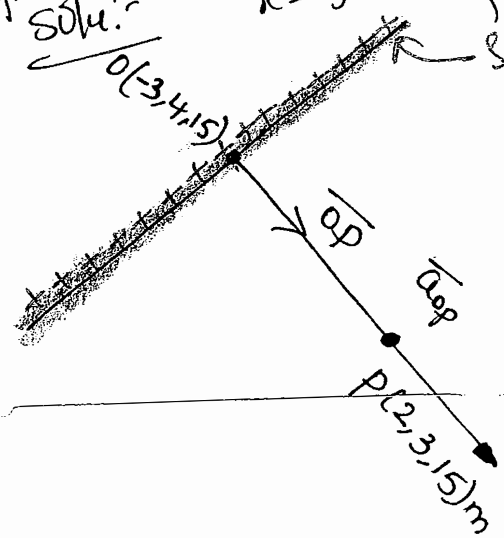
problem 23.

(2, 3,15)m $\rho_l = 25nC/m$.

Find the electric field intensity at a point (2,3,15)m due to a uniform line charge density of ρ_l = 25nC/m is lies along x= -3m and y=4m in free space. EEE J/J 2016.

25nC/m

Soln:- x=-3m y=4m $\rho_l = 25nC/m$



x=-3m and y=4m.

x=-3m and y=4m

$$\bar{E}_{\rho l} = \frac{\rho_l}{2\pi\epsilon|\overline{OP}|}\bar{a}_{op} \ V/m.$$

$$\bar{E}_{\rho l} = \frac{\rho_l}{2\pi\epsilon|\overline{OP}|^2}\overline{OP} \ V/m.$$

$\overline{OP} = 5\bar{a}_x - \bar{a}_y$; $|\overline{OP}| = \sqrt{25+1}$

$= \sqrt{26}$ m

$$\bar{E}_{\rho l} = \frac{25\cancel{n} \times \cancel{18\times10^9}}{(\sqrt{26})^2}[5\bar{a}_x - \bar{a}_y]$$

$$\bar{E}_{\rho l} = 17.3076[5\bar{a}_x - \bar{a}_y] \ V/m.$$

$$\boxed{\bar{E}_{\rho l} = 86.538\bar{a}_x - 17.307\bar{a}_y} \ V/m$$

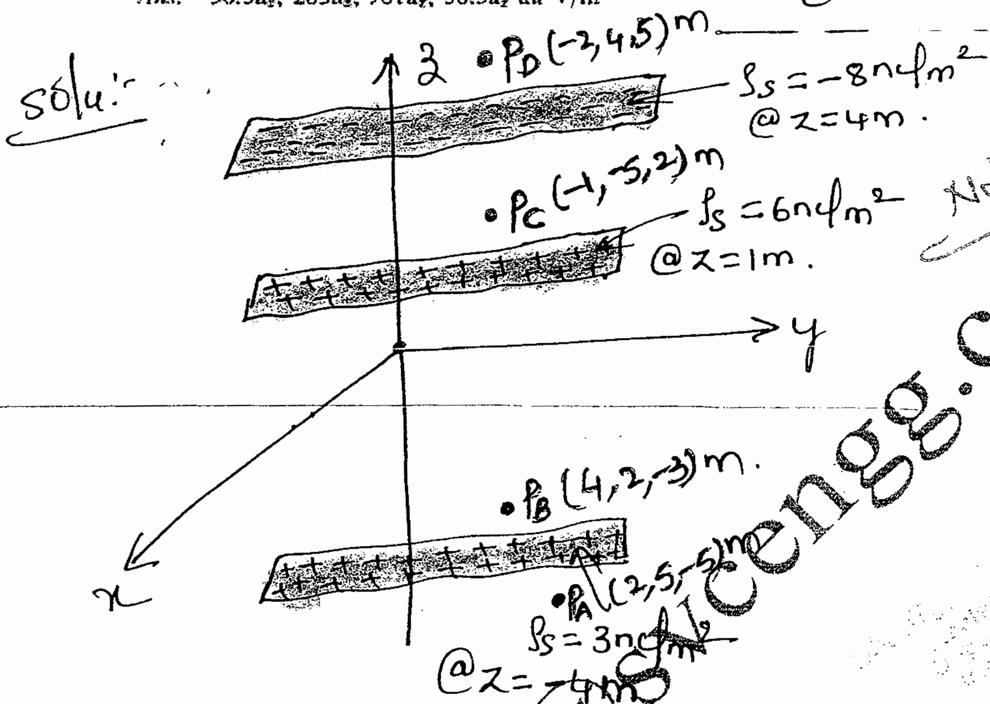
$$\boxed{|\bar{E}_{\rho l}| = 88.252} \ V/m$$

problem 24. $z=-4$, $6\,nC/m^2$ at $z=1$, $-8\,nC/m^2$ $z=4m$

Three infinite uniform sheets of charge are located in free space E i.e $3\,nC/m^2$ at $z = -4$, $6\,nC/m^2$ at $z = 1$, and $-8\,nC/m^2$ at $z = 4$. Find $\mathbf{E}$ at the point: (*a*) $P_A(2,5,-5)$; (*b*) $P_B(4,2,-3)$; (*c*) $P_C(-1,-5,2)$; (*d*) $P_D(-2,4,5)$.

Ans. $-56.5\mathbf{a}_z$; $283\mathbf{a}_z$; $961\mathbf{a}_z$; $56.5\mathbf{a}_z$ all V/m [W. H. Hayt]

soln:



Note: point one located approximately.

a) $\bar{E}_A = \bar{E}_{z=-4m} + \bar{E}_{z=1m} + \bar{E}_{z=4m}$ V/m.

$$\bar{E}_A = \frac{\rho_{s1}}{2\epsilon}(-\bar{a}_z) + \frac{\rho_{s2}}{2\epsilon}(-\bar{a}_z) + \frac{\rho_{s3}}{2\epsilon}(-\bar{a}_z)$$

$$= 3\cancel{n} \times 18\pi \times 10^9 (-\bar{a}_z) + (6\cancel{n})\, 18\pi \times 10^9 (-\bar{a}_z) + (-8\cancel{n})(18\pi \times 10^9)(-\bar{a}_z)$$

$$\bar{E}_A = [-3-6+8]\, 18\pi\, \bar{a}_z = -18\pi\, \bar{a}_z \text{ V/m}$$

$$\boxed{\bar{E}_A = -56.548\, \bar{a}_z} \text{ V/m}$$

b) $\bar{E}_B = 3\cancel{n} \times 18\pi \times 10^{\cancel{9}} (+\bar{a}_z) + 6\cancel{n} \times 18\pi \times 10^{\cancel{9}} (-\bar{a}_z) - 8\cancel{n}(18\pi \times 10^{\cancel{9}})(-\bar{a}_z)$

$$\bar{E}_B = 5 \times 18\pi\, (\bar{a}_z) = 282.743\, \bar{a}_z \text{ V/m}$$

$$\boxed{\bar{E}_B = 282.743\, \bar{a}_z} \text{ V/m}$$

c) $\bar{E}_C = 3\cancel{n} \times 18\pi \times 1\cancel{0}^{9}(+\bar{a}_z) + 6\cancel{n} \times 18\pi \times 1\cancel{0}^{9}(+\bar{a}_z)$
$- 8\cancel{n} \times 1\cancel{0}^{9}(-\bar{a}_z) \times 18\pi$

$\bar{E}_C = 17 \times 18\pi\, \bar{a}_z = 961.327\, \bar{a}_z$ V/m

$\boxed{\bar{E}_C = 961.327\, \bar{a}_z}$ V/m

d) $\bar{E}_D = 3\cancel{n} \times 18\pi \times 1\cancel{0}^{9}(+\bar{a}_z) + 6\cancel{n} \times 18\pi \times 1\cancel{0}^{9}(+\bar{a}_z)$
$- 8\cancel{n} \times 18\pi \times 1\cancel{0}^{9}(+\bar{a}_z)$

$= [9-8] \times 18\pi(+\bar{a}_z)$

$\bar{E}_D = 18\pi\, \bar{a}_z$ V/m

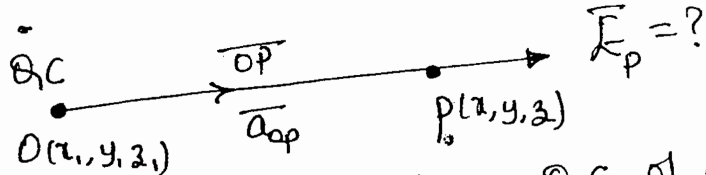
$\boxed{\bar{E}_D = 56.548\, \bar{a}_z}$ V/m

Question. $\bar{E}$ due to various charge distribution.

Define electric field intensity. Obtain an expression for electric field intensity $\bar{E}$, due to several charges, such as point charge, linear charge, surface charge and volume charge distribution. (06 Marks)

[02 - June/July 2011]

- Soln: a. $\Rightarrow \bar{E}$ due to point charge.



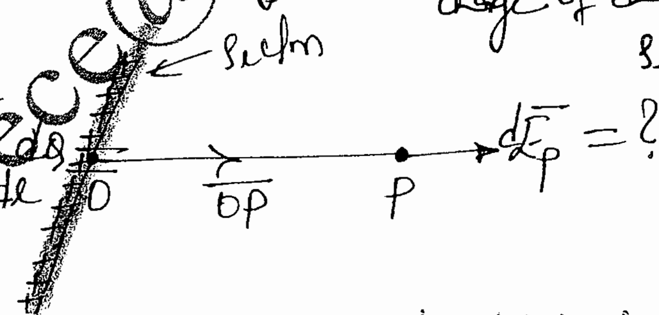
the field at a point 'p' due to QC of charge at point 'O' is

$$\bar{E}_p = \frac{Q}{4\pi\epsilon|\overline{OP}|^2}\bar{a}_{op} \text{ V/m.}$$

(or) $$\boxed{\bar{E}_p = \frac{Q}{4\pi\epsilon|\overline{OP}|^3}\overline{OP}} \text{ V/m}$$

b. $\bar{E}$ due to Line charge. Consider a infinite line charge of charge density ρ_L C/m.

dQ - differential charge
dL - differential Length



the $d\bar{E}_p$ due to dQ at point 'O' is

$$d\bar{E}_p = \frac{dQ}{4\pi\epsilon|\overline{OP}|^2}\bar{a}_{op} \text{ V/m.}$$

from defn of Charge density $\rho_L = \frac{dQ}{dL}$ C/m

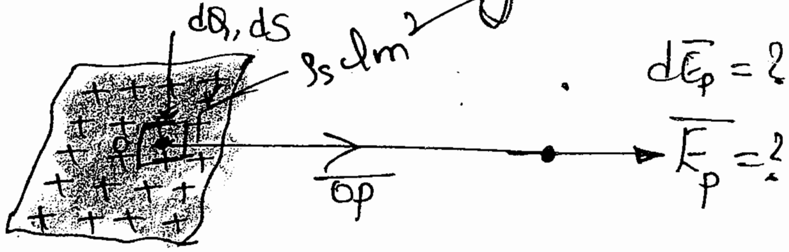
$$\Rightarrow dQ = \rho_L \cdot dL$$

$$d\bar{E}_p = \frac{\rho_L \cdot dL}{4\pi\epsilon|\overline{OP}|^2}\bar{a}_{op}$$

$$\boxed{\overline{E}_p = \int_{\langle l\rangle} \frac{\rho_l\, dl}{4\pi\epsilon|\overline{op}|^2}\,\overline{a}_{op}}\ \text{V/m}$$

c. $\overline{E}$ due to Sheet charge

Consider a sheet charge of charge density ρ_s C/m²



the $d\overline{E}_p$ due to dQ at 'O' is

$$d\overline{E}_p = \frac{dQ}{4\pi\epsilon|\overline{op}|^2}\,\overline{a}_{op}\ \text{V/m.}$$

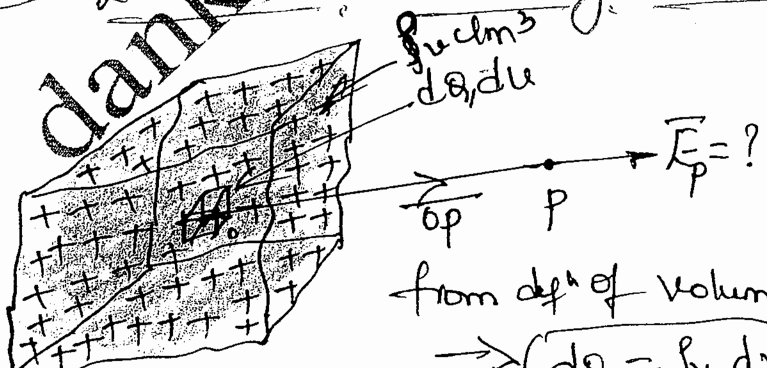
from defn of Surface charge density $\rho_s = \frac{dQ}{ds}$ C/m²

$$\Rightarrow \boxed{dQ = \rho_s\, ds}\ \text{Coulomb's}$$

$$d\overline{E}_p = \frac{\rho_s\, ds}{4\pi\epsilon|\overline{op}|^2}\,\overline{a}_{op}\ \text{V/m}$$

$$\boxed{\overline{E}_p = \int_{\langle S\rangle} \frac{\rho_s\, ds}{4\pi\epsilon|\overline{op}|^2}\,\overline{a}_{op}}\ \text{V/m.}$$

d. $\overline{E}$ due to Volume charge distribution :- Consider a volume charge of charge density ρ_v C/m³.



the $d\overline{E}_p$ due to dQ at 'O' is

$$d\overline{E}_p = \frac{dQ}{4\pi\epsilon|\overline{op}|^2}\,\overline{a}_{op}\ \text{V/m}$$

from defn of volume charge density $\rho_v = \frac{dQ}{dv}$ C/m³

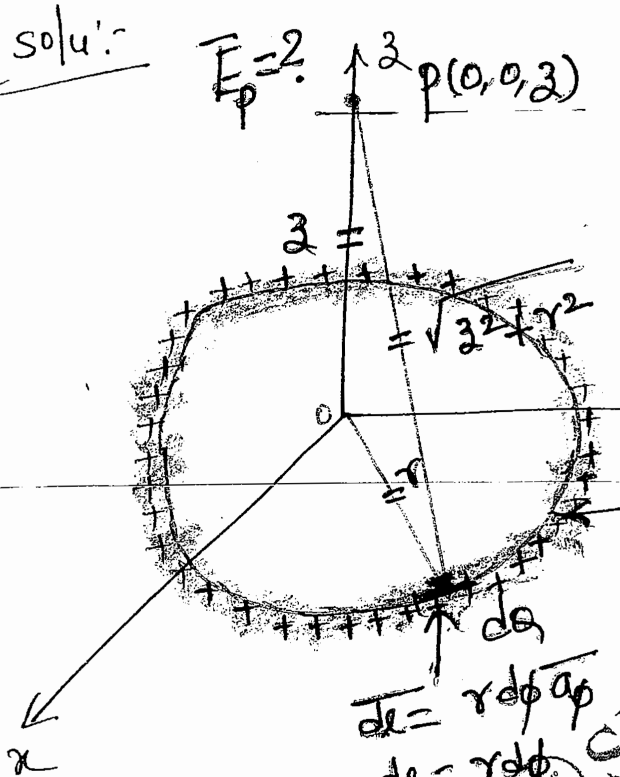
$$\Rightarrow \boxed{dQ = \rho_v\, dv}\ \text{Coulomb's}$$

$$\therefore d\overline{E}_p = \frac{\rho_v\, dv}{4\pi\epsilon|\overline{op}|^2}\,\overline{a}_{op}\ \text{V/m}\quad \text{or}\quad \boxed{\overline{E}_p = \int_{\langle vol\rangle} \frac{\rho_v\, dv}{4\pi\epsilon|\overline{op}|^2}\,\overline{a}_{op}}\ \text{V/m}$$

problem 25

26 Derive an expression for the electric field intensity at a point on the axis of a charged circular ring. Hence show that the electric field at the center of the ring is zero. (08 Marks)

solu':



Consider a uniform Line charge density of ρ_L c/m. in the form of a circular ring placed on xy axis.

The field at point 'p' can be calculated by first finding potential at Point P.

circular ring of uniform line charge density ρ_L c/m & radius r m.

The potential at a point p(0,0,z) is given

$$V_p = \frac{dQ}{4\pi\epsilon\sqrt{z^2+r^2}}$$

$$dQ = \rho_L \, dl$$

$$dV_p = \frac{\rho_L \, dl}{4\pi\epsilon\sqrt{z^2+r^2}} \text{ volt's and } dl = r d\phi$$

$$dV_p = \frac{\rho_L \, r d\phi}{4\pi\epsilon\sqrt{z^2+r^2}} \Rightarrow V_p = \int_{\phi=0}^{2\pi} \frac{\rho_L r}{4\pi\epsilon\sqrt{z^2+r^2}} d\phi$$

$$V_p = \frac{\rho_L r}{\cancel{4\pi}_{2}\epsilon\sqrt{z^2+r^2}} \times \cancel{2\pi} = \frac{\rho_L r}{2\epsilon\sqrt{z^2+r^2}} \text{ volt's}$$

∴ the potential at a point 'p' is

$$\boxed{V_p = \frac{\rho_l r}{2\epsilon\sqrt{z^2+r^2}}} \text{ volt/n} \leftarrow (a)$$

from eqn (a) V_p is a function of 'z' ∴

using concept of Gradient $\bar{E} = -\nabla V$ v/m

$$\bar{E} = -\frac{\partial V}{\partial z}\bar{a}_z \text{ v/m}$$

$$= -\frac{\partial}{\partial z}\left[\frac{\rho_l r}{2\epsilon(z^2+r^2)^{\frac{1}{2}}}\right]\bar{a}_z$$

$$= -\frac{\rho_l r}{\cancel{2}\epsilon} \times -\frac{1}{2}(z^2+r^2)^{-\frac{1}{2}-1} \times \cancel{2}z \quad \bar{a}_z$$

$$\boxed{\bar{E} = +\frac{\rho_l r z}{2\epsilon(z^2+r^2)^{3/2}}\bar{a}_z} \text{ v/m} \leftarrow (b)$$

the field at Center of the ring i.e when $z \to 0$

$$\therefore \boxed{\bar{E} = 0} \text{ V/m}$$

∴ Electric field at Center of the ring is zero.

problem5. A uniform line charge of 16 nC/m is Located along the line defined by $y=-2$, $z=5$. if $\epsilon = \epsilon_0$; (a) find $\bar{E}$ at $p(1,2,3)$

(b) Find $\bar{E}$ at that point in the $z=0$ plane where the direction of $\bar{E}$ is given by

$$\frac{1}{2}\bar{a}_y - \frac{2}{3}\bar{a}_z.$$

problem6. An uniform line charge $\rho_L = 2$ nC/m lies along the x-axis in free space, while point charges of 8nC each one Located at (0,0,1)m and (0,0,-1)m find

i. Find $\bar{E}$ at $(2,3,-4)$.

ii. To what value should ρ_L be changed to cause $\bar{E}$ to be zero at (0,0,3)?

problem7.

A uniform line charge of 2 μC/m is Located on the z-axis. Find $\bar{E}$ in Cartesian co-ordinate at $P(1,2,3)$. if the charge extends from

a> $z=-\infty$ to $z=+\infty$; b> $z=-4$ to $z=+4$.

Problem 1. A uniform line charge of infinite length with $\rho_L = 40\,nC/m$, lies along the z-axis. Find $\bar{E}$ at $(-2, 2, 8)$ in air.

Problem 2. A line charge of $2\,nC/m$ lies along y-axis while surface charge densities of $0.1\,nC/m^2$ and $-0.1\,nC/m^2$ exist on the plane $z=3$ and $z=-4\,m$ respectively. Find the $\bar{E}$ at $P(1, 7, -2)$.

Problem 3. A line charge density $\rho_L = 50\,nC/m$ is located along the line $x=0, y=5$ in free space. Find the magnitude and direction of the electric field intensity at a point $P(1, 3, -4)$.

Problem 4. Infinite uniform line charges of $5\,nC/m$ lie along the (positive and negative) x and y axes in free space. Find $\bar{E}$ at a) $P_A(0,0,4)$ b) $P_B(0,3,4)$.

Topic 1.3 d/e/f problem

Problem 11

A uniform Line charge density $\rho_L = 25$ nC/m, lies on the line $x = -3$, $z = 4$m in Space. Find $\bar{E}$ in Cartesian Components at

i. origin ii. P(2, 15, 3) iii. Q($\rho = 4, \phi = 60°, z = 2$).

Problem 14.

Find $\bar{E}$ at P(1, 5, 2)m in free space if a point charge of 6μC is Located at (0, 0, 1)m, the uniform line charge density $\rho_L = 180$ nC/m along x-axis and uniform Sheet charge with $\rho_S = 25$ nC/m² over the plane $z = -1$m.

Problem 24. three Infinite Uniform sheets of charge are Located in free Space i.e 3nC/m² at $z = -4$m, 6nC/m² at $z = 1$m and -8nC/m² at $z = 4$m. Find $\bar{E}$ at the point

a) $P_A(2, 5, -5)$; b) $P_B(4, 2, -3)$

c) $P_C(-1, -5, 2)$ and d) $P_D(-2, 4, 5)$.

(172)

Topic 1.4 : Electric Flux Density (D).

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Topic-4

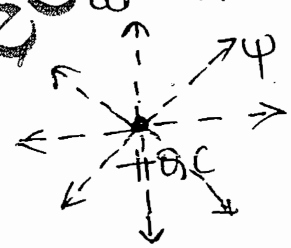
Electric Flux Density:

- Definition of Electric Flux and its properties
- Definition of Electric Flux density
- Relationship b/w flux electric field intensity and electric flux density.
- Flux density due to various charge distributions

1.4a. → Definition of Electric Flux (ψ) and its properties

* By defn Electric Flux (ψ) originates on positive charge and terminates on negative charge.

* In the absence of negative charge the Flux (ψ) terminates at infinity.



* By definition one Coulomb of electric flux gives rise to one Coulomb of Charge.

i.e the amount of total flux lines crossing of any closed surface is equal to the total charge enclosed. (173)

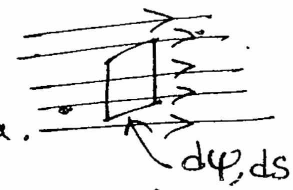
mathematically $\boxed{\psi_{total} = Q_{enclosed}}$ Coulomb's.

* Flux is a "Scalar quantity" and Measured in Coulomb's. 347

Flux lines are imaginary line that are radial in nature.

1.4b Electric flux density ($\overline{D}$) :-

Defn:- Electric flux density ($\overline{D}$) indicates an amount of flux ($d\psi$) crosses the differential area ds, which is normal to the surface.



i.e $\overline{D}$ is flux crossing per unit area.

$$\boxed{\overline{D} = \frac{d\psi}{ds} \; C/m^2} \quad \text{or} \quad \overline{D} = \frac{d\psi}{ds}\,\overline{a_n}\; C/m^2$$

where $\overline{a_n}$ - unit vector normal to the surface.

1.4C

$\overline{D}$ due to point charge :-

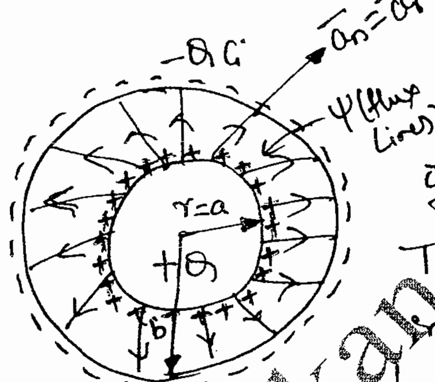


fig. Concentric spheres of radii 'a' & 'b' m

Consider a inner sphere of radius 'a' m and an outer sphere of radius 'b' m with charges of Q and -Q respectively.

The path of electric flux (ψ) extending from the inner sphere to the outer sphere are indicated by the symmetrically distributed stream lines drawn radially from one sphere to the other.

$$D\Big|_{r=a} = \frac{Q}{4\pi a^2}\overline{a_r} \;(\text{inner Sphere}) \qquad D\Big|_{r=bm} = \frac{Q}{4\pi b^2}\overline{a_r} \;(\text{outer Sphere})$$

at radial distance 'r' m where $a \le r \le bm$

$$\boxed{\overline{D} = \frac{Q}{4\pi r^2}\overline{a_r} \; C/m^2}$$

if inner radius $a \to 0$ ∴ it forms a point charge 'Q' C the $\overline{D}$ at any radial distance 'r' is given by

$$\boxed{\overline{D} = \frac{Q}{4\pi r^2}\overline{a_r}} \; C/m^2$$

Note:- $\varphi \rightarrow$ Scalar quantity, $\overline{D} \rightarrow$ vector quantity

1 ~~Define : i) Electric field intensity ii) Displacement flux density.~~ (04 Marks)

2 ~~Calculate D in rectangular coordinates at point P(2, −3, 6) produced by a uniform line charge ρ_L = 20 mc/m, on the x-axis.~~ (06 Marks)

Topic 1.4d Refer page no - 131 → Relationship b/w D & E.

Sol^n: Question: Find out the relation b/w $\overline{D}$ and $\overline{E}$ [10 Dec/Jan 2016] (4)

W.k.t the $\overline{E}$ due to point charge is given by

$$\overline{E} = \frac{Q}{4\pi\epsilon r^2}\overline{a_r} \text{ V/m} \leftarrow (1)$$

and $\overline{D}$ due to point charge Q given by

$$\overline{D} = \frac{Q}{4\pi r^2}\overline{a_r} \text{ C/m}^2 \leftarrow (2)$$

eq^n (2) / eq^n (1) $\Rightarrow \frac{\overline{D}}{\overline{E}} = \epsilon$

$$\Rightarrow \boxed{\overline{D} = \epsilon\overline{E}} \text{ C/m}^2$$

1.4e

Sol: $\overline{D}$ due to Infinite line charge density (ρ_L C/m)

W.k.t $\overline{E}$ due to infinite line charge is

$$\overline{E} = \frac{\rho_L}{2\pi\epsilon\rho}\overline{a_\rho} \text{ V/m.} \leftarrow (3)$$

using the relation $\overline{D} = \epsilon\overline{E}$ C/m²; eq^n (3) becomes

$$\therefore \boxed{\overline{D} = \epsilon\overline{E} = \frac{\rho_L}{2\pi\rho}\overline{a_\rho}} \text{ C/m}^2$$

Problem) Given the $\bar{D} = 0.3r^2 \bar{a}_r\, nC/m^2$ $\leftarrow E$

Given the electric flux density, $\bar{D} = 0.3r^2\hat{a}_r$ nc/m² in free space, find $\bar{E}$ at point P(r = 2m, θ = 25°, ϕ = 90°). (08 Marks)

$P(r=2m, \theta=25^\circ, \phi=90^\circ)$ [02-J/J 2010].

Solu:- $\bar{D} = 0.3r^2\, \bar{a}_r\ nC/m^2$.

in free space $\epsilon = \epsilon_0$ F/m.

$$\bar{E} = \frac{\bar{D}}{\epsilon_0} = \frac{0.3r^2}{\epsilon_0}\, \bar{a}_r\ nV/m$$

@ $r = 2m$

$$\bar{E} = \frac{0.3(2)^2}{\epsilon_0}\, \bar{a}_r\ nV/m.$$

$$\boxed{\bar{E} = 135.53\,\bar{a}_r}\ V/m$$

104 e

ii. → $\bar{D}$ due to Infinite sheet charge density ρ_s C/m²:-

$$\bar{E} = \frac{\rho_s}{2\epsilon}\, \bar{a}_n\ V/m$$

$$\Rightarrow \bar{D} = \epsilon\bar{E}\ C/m^2$$

$$\therefore \boxed{\bar{D} = \frac{\rho_s}{2}\, \bar{a}_n}\ C/m^2.$$

key note points.

Note:- from relation $\bar{D} = \epsilon\bar{E}$ C/m²

1. the $\bar{D}$ and $\bar{E}$ have same unit vectors i.e both are acts in the same direction.

2. $|\bar{D}| = \epsilon|\bar{E}| = \rho_s$ C/m²

3. ψ - electric flux scalar quantity & $\bar{D}$ - vector quantity.

Problem 2. Calculate $\bar{D}$ in rectangular Co-ordinates at point $P(2,-3,6)$ produced by a uniform Line charge $\rho_L = 20\,mC/m$ on the x-axis.
(6m) [02 J/J 2010]

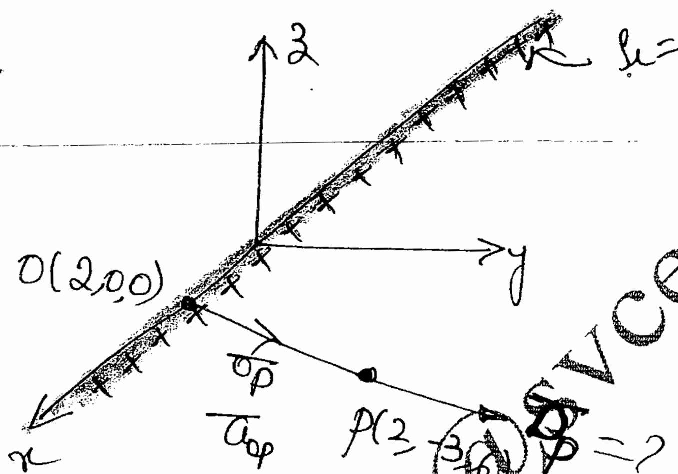
4 ~~State and explain Gauss Law. Find out the relation between D and E.~~ ~~(06 Marks)~~

soln:- Refer Pag NO-128.

problem 2

Q) Calculate $\overline{D}$ in rectangular Co-ordinates at point P(9, -3, 6) produced by a uniform Line charge $\rho_L = 20\, mC/m$ on the x-axis. (6m).

soln:



$$\overline{D_p} = \frac{\rho_L}{2\pi\rho}\,\overline{a_\rho}\ C/m^2$$

$$\overline{D_p} = \frac{\rho_L}{2\pi|\overline{OP}|}\,\overline{a_{OP}}\ C/m^2$$

$$= \frac{\rho_L}{2\pi|\overline{OP}|^2}\,\overline{OP}\ C/m^2$$

$$\overline{OP} = -3\overline{a_y} + 6\overline{a_z}\ ; \quad |\overline{OP}| = \sqrt{9+36} = \sqrt{45}\, m$$

$$\overline{D_p} = \frac{20m}{2\pi(45)}\left[-3\overline{a_y} + 6\overline{a_z}\right]$$

$$\overline{D_p} = -2.122\times10^{-4}\,\overline{a_y} + 4.244\times10^{-4}\,\overline{a_z}$$

$$\boxed{\overline{D_p} = -212.20\overline{a_y} + 424.413\overline{a_z}}\ \mu C/m^2$$

$$|\overline{D_p}| = 474.505\,\mu C/m^2$$

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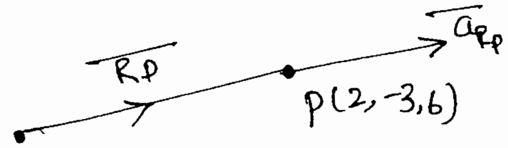
Problem 3 Calculate $\overline{D}$ in rectangular co-ordinates at P(2, -3, 6) produced by a point charge Q = 55 mC located at (-2, 3, 6)

Calculate $\vec{D}$ in rectangular co-ordinates at P(2, -3, 6) produced by a point charge Q = 55mc located at (-2, 3, 6). (04 Marks)

(4m) [06-Dec/Jan 2008]

Solu:



Q = 55mC
R(-2, 3, 6)

$$\overline{D} = \frac{Q}{4\pi|\overline{Rp}|^2}\overline{a}_{Rp} \ C/m^2$$

$$\overline{Rp} = 4\overline{a}_x - 6\overline{a}_y \ ; \ |\overline{Rp}| = \sqrt{16+36} = \sqrt{52} \ m.$$

$$\overline{D} = \frac{Q}{4\pi|\overline{Rp}|^3}\overline{Rp}$$

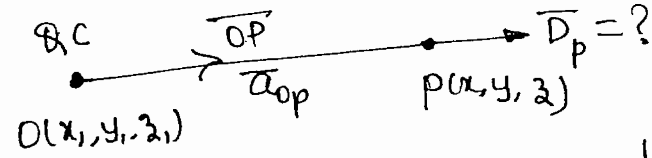
$$\overline{D} = \frac{55m}{(\sqrt{52})^3 \times 4\pi}\left[4\overline{a}_x - 6\overline{a}_y\right]$$

$$\boxed{\overline{D} = 46.688\,\overline{a}_x - 70.032\,\overline{a}_y} \ \mu C/m^2$$

$$|\overline{D}| = 84.1679 \ \mu C/m^2$$

1.4f
Qustion: Derive an expression for Electric Flux density due Various charge Distribution.

Sol^n: (a) $\overline{D}$ due to point charge.



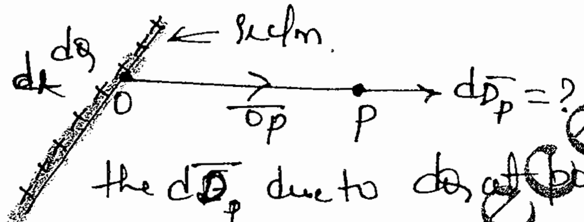
the Flux density at a point 'p' due to QC of charge at 'O' is

$$\boxed{\overline{D_p} = \frac{Q}{4\pi|\overline{op}|^2}\overline{a_{op}}} \ C/m^2$$

(or)

$$\boxed{\overline{D_p} = \frac{Q}{4\pi|\overline{op}|^3}\overline{op}}$$

(b) $\overline{D}$ due to Line charge distribution: Consider a line charge of ρ_L C/m.



the $d\overline{D_p}$ due to dQ at point 'O' is

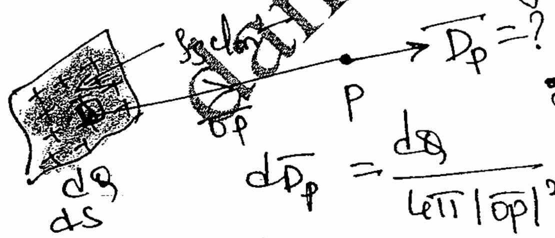
$$d\overline{D_p} = \frac{dQ}{4\pi|\overline{op}|^2}\overline{a_{op}} \ C/m^2$$

$$\rho_L = dQ/dL \ \ C/m$$

$$\Rightarrow d\overline{D_p} = \frac{\rho_L \, dL}{4\pi|\overline{op}|^2}\overline{a_{op}}$$

$$\boxed{\overline{D_p} = \int_{\langle L\rangle} \frac{\rho_L \, dL}{4\pi|\overline{op}|^2}\overline{a_{op}}} \ C/m^2$$

(c) $\overline{D}$ due to sheet charge distribution



Consider a sheet charge of ρ_s C/m²

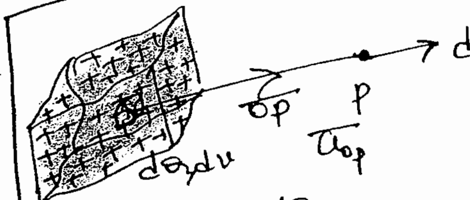
$$d\overline{D_p} = \frac{dQ}{4\pi|\overline{op}|^2}\overline{a_{op}}$$

$$\rho_s = dQ/dS \Rightarrow dQ = \rho_s \, dS$$

$$d\overline{D_p} = \frac{\rho_s \, dS}{4\pi|\overline{op}|^2}\overline{a_{op}} \ C/m^2$$

$$\boxed{\overline{D_p} = \int_{\langle S\rangle} \frac{\rho_s \, dS}{4\pi|\overline{op}|^2}\overline{a_{op}}} \ C/m^2$$

(d) $\overline{D}$ due to volume charge distribution.



Consider a volume charge density of ρ_v C/m³.

$$d\overline{D_p} = \frac{dQ}{4\pi|\overline{op}|^2}\overline{a_{op}} \ C/m^2$$

$$\rho_v = \frac{dQ}{dv} \ C/m^3 \Rightarrow dQ = \rho_v \, dv$$

$$d\overline{D_p} = \frac{\rho_v \, dv}{4\pi|\overline{op}|^2}\overline{a_{op}} \ C/m^2$$

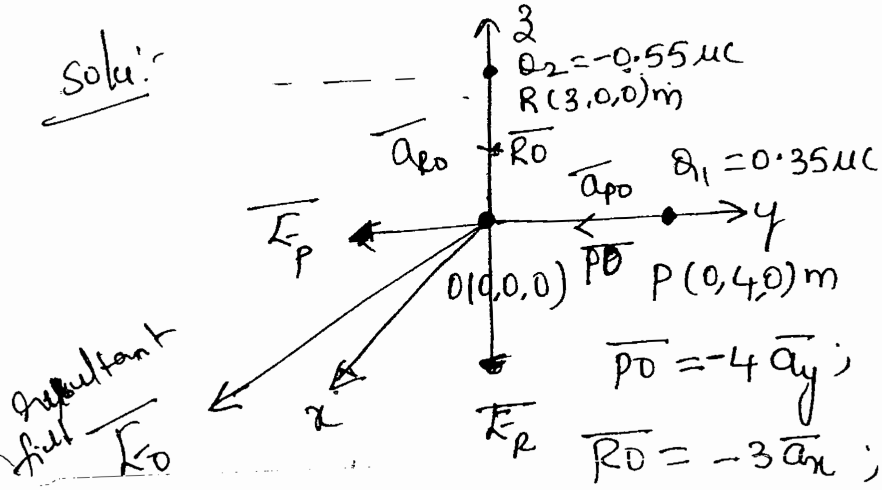
$$\boxed{\overline{D_p} = \int_{\langle vol\rangle} \frac{\rho_v \, dv}{4\pi|\overline{op}|^2}\overline{a_{op}}} \ C/m^2$$

problem 4.

Find $\overline{E}$ and $\overline{D}$ at the origin due $Q_1 = 0.35\mu C$ at $(0,4,0)$m and $Q_2 = -0.55\mu C$ at $(3,0,0)$m.

Find i. Electric field intensity and ii. Electric Flux density at the origin due to Q_1=0.35μC at(0,4,0)m and Q_2=-0.55μC at (3,0,0)m

soln:



$$\overline{E_O} = \overline{E_P} + \overline{E_R} \text{ V/m}$$

$$\overline{PO} = -4\overline{a_y}\ ; \quad |\overline{PO}| = 4\text{m}.$$

$$\overline{RO} = -3\overline{a_x}\ ; \quad |\overline{RO}| = 3\text{m}.$$

$$\overline{E_O} = \frac{Q_1}{4\pi\epsilon|\overline{PO}|^3}\overline{PO} + \frac{Q_2}{4\pi\epsilon|\overline{RO}|^3}\overline{RO} \text{ V/m}$$

$$= \frac{0.35\mu \times 9\times10^9}{(4)^3}\left[-4\overline{a_y}\right] - \frac{0.55\mu \times 9\times10^9}{(3)^3}\left[-3\overline{a_x}\right]$$

$$= -196.875\overline{a_y} + 550\overline{a_x}$$

$$\boxed{\overline{E_O} = 550\overline{a_x} - 196.875\overline{a_y}} \text{ V/m}$$

$$|\overline{E_O}| = 584.1744 \text{ V/m}.$$

$$\overline{D_O} = \epsilon\overline{E_O} = 8.854\left[550\overline{a_x} - 196.875\overline{a_y}\right] \text{ pC/m}^2$$

$$= 4869.7\overline{a_x} - 1743.13\overline{a_y} \text{ pC/m}^2$$

$$\boxed{\overline{D_O} = 4.8697\overline{a_x} - 1.74313\overline{a_y}} \text{ nC/m}^2$$

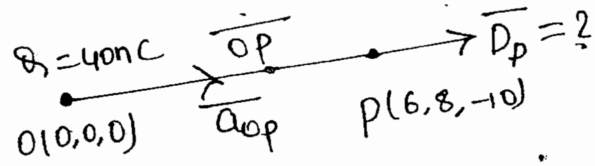
$$|\overline{D_O}| = 5.1722 \text{ nC/m}^2$$

problem 5

- Find **D** in Cartesian co-ordinate system at point p(6,8,-10) due to
  - i. point charge of 40nC at the origin
  - ii. A uniform line charge density of $40\mu C/m$ on the z-axis
  - iii. A uniform sheet charge density of $57.2\mu C/m^2$ on the plane x=12m.

Ans: i. $D = 6.7a_x + 9.0a_y - 11.25a_z\ pC/m^2$; ii. $D = 0.38a_x + 0.5a_y\ \mu C/m^2$;
iii. $D = -28.6\ a_x\ \mu C/m^2$

sol^n: i.



$$\overline{op} = 6\overline{a_x} + 8\overline{a_y} - 10\overline{a_z}\ ;\ |\overline{op}| = \sqrt{200}\ m$$

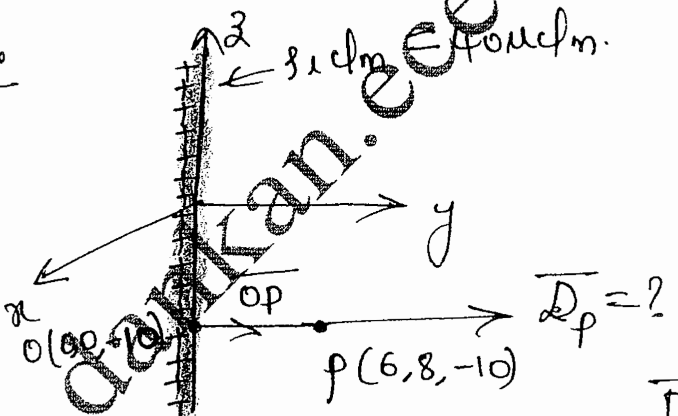
$$\overline{D_p} = \frac{Q}{4\pi|\overline{op}|^3}\cdot\overline{op}\ C/m^2$$

$$= \frac{40n \times}{4\pi[\sqrt{200}]^3}[6\overline{a_x} + 8\overline{a_y} - 10\overline{a_z}]$$

9.0031

$$\boxed{\overline{D_p} = 6.752\overline{a_x} + 9.0031\overline{a_y} - 11.2539\overline{a_z}}\ pC/m^2.$$

ii.



$$\overline{D_p} = \frac{\rho_L}{2\pi|\overline{op}|}\overline{a_{op}}\ C/m^2$$

$$= \frac{\rho_L}{2\pi|\overline{op}|^2}\overline{op}\ C/m^2$$

$$\overline{op} = 6\overline{a_x} + 8\overline{a_y}$$

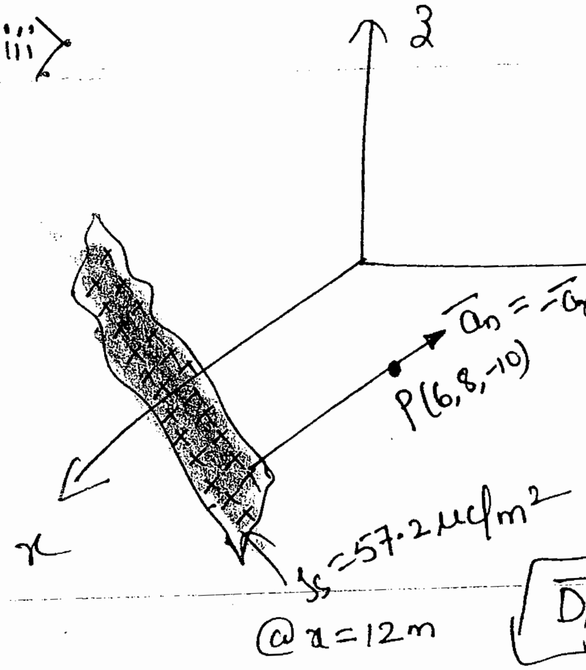
$$|\overline{op}| = \sqrt{36+64} = \sqrt{100}\ m.$$

$$\overline{D_p} = \frac{40\mu}{2\pi(100)}[6\overline{a_x} + 8\overline{a_y}]$$

$$\overline{D_p} = 0.3819\overline{a_x} + 0.5092\overline{a_y}\ \mu C$$

$$\boxed{\overline{D_p} = 0.3819\overline{a_x} + 0.5092\overline{a_y}}\ \mu C/m^2$$

iii)



$$\overline{D_P} = \frac{\rho_S}{2}\,\overline{a_n}\ C/m^2$$

$$\overline{a_n} = -\overline{a_x}$$

$$\therefore\ \overline{D_P} = \frac{57.2\mu}{2}(-\overline{a_x})$$

$$\boxed{\overline{D_P} = -28.6\,\overline{a_x}}\ \mu C/m^2$$

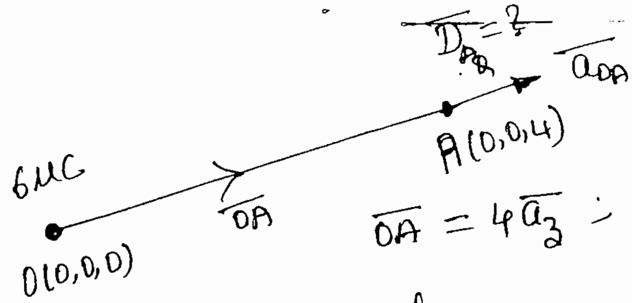
$$\boxed{\overline{D_P} = -28.6\,\overline{a_x}}\ \mu C/m^2$$

problem 6

$\leftarrow 180\,nC/m \quad \leftarrow x\text{-axis}$

- A point charge of $6\mu C$ is located at the origin, a uniform line charge density of 180nC/m lies along x-axis and uniform sheet of charge equal to $25nC/m^2$ lies in the z=0 plane. Find
  - i. D at A(0,0,4) Ans: $\mathbf{D}=49.5\ a_z\ nC/m^2$
  - ii. D at B(1,2,4) and Ans: $D=4.96a_x+12.7a_y+38.07a_z\ nC/m^2$
  - iii. Total electric flux leaving the surface of the sphere of 4m radius centered at origin. **Ans: $Q=8.69\mu C$**

solu: i)



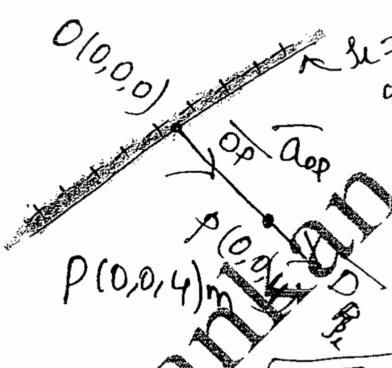
$\overline{OA} = 4\overline{a}_z \; ; \; |\overline{OA}| = 4m.$

$$\overline{D}_{A_Q} = \frac{Q}{4\pi|\overline{OA}|^2}\overline{a}_{OA}\ C/m^2$$

$$\overline{D}_{A_Q} = \frac{Q}{4\pi|\overline{OA}|^3}\overline{OA}\ C/m^2.$$

$$\overline{D}_{A_Q} = \frac{6\mu}{4\pi(4)^3}[4\overline{a}_z] = 0.02984\overline{a}_z\ \mu C/m^2.$$

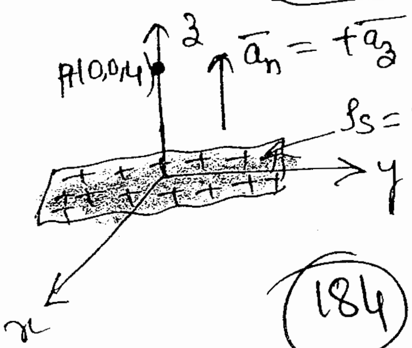
$$\boxed{\overline{D}_{A_Q} = 29.84\overline{a}_z}\ nC/m^2$$



$$\overline{D}_{A_{\rho_L}} = \frac{\rho_L}{2\pi|\overline{OP}|}\overline{a}_{OP} = \frac{\rho_L}{2\pi|\overline{OP}|^2}\overline{OP}\ C/m^2$$

$\overline{OP} = 4\overline{a}_z \; ; \; |\overline{OP}|^2 = 16.$

$$\boxed{\overline{D}_{A_{\rho_L}} = \frac{180n}{2\pi(16)} \times 4\overline{a}_z = 7.1619\,\overline{a}_z}\ nC/m^2$$



$$\overline{D}_{A_{\rho_s}} = \frac{\rho_s}{2}\overline{a}_n = \frac{\rho_s}{2}\overline{a}_z\ C/m^2$$

$$\overline{D}_{A_{\rho_s}} = \frac{25n}{2}\overline{a}_z\ C/m^2$$

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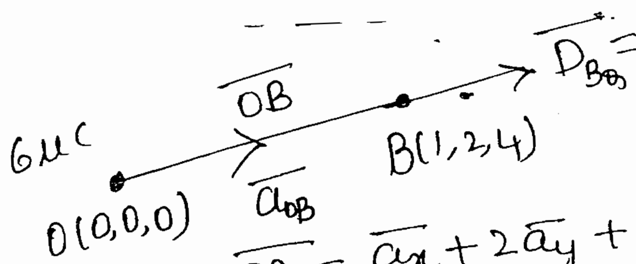
$$\boxed{\overline{D}_{A_{\rho_s}} = 12.5\,\overline{a}_z}\ nC/m^2$$

$$\overline{D}_A = \overline{D}_{A_Q} + \overline{D}_{A_{\rho_L}} + \overline{D}_{A_{\rho_s}} = [29.84\overline{a}_z + 7.16\overline{a}_z + 12.5\overline{a}_z]\ nC/m^2$$

$$\therefore \boxed{\overline{D_A} = 49.5\,\overline{a_z}}\ nC/m^2$$

ii) $\overline{D}$ at $B(1,2,4)$

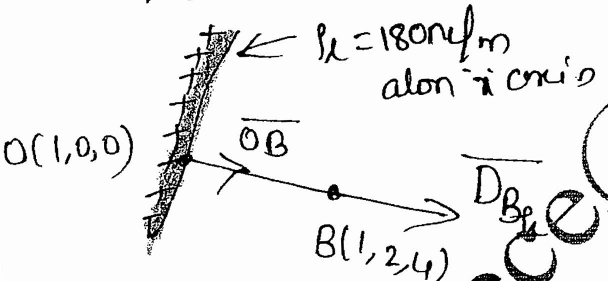
$\overline{D_{BQ}} = ?$ $\qquad \overline{D_{BQ}} = \dfrac{Q}{4\pi|\overline{OB}|^3}\,\overline{OB}$



$\overline{OB} = \overline{a_x} + 2\overline{a_y} + 4\overline{a_z}$; $|\overline{OB}| = \sqrt{21}$ m

$$\overline{D_{BQ}} = \frac{6\mu}{4\pi(\sqrt{21})^3}\left[\overline{a_x} + 2\overline{a_y} + 4\overline{a_z}\right]$$

$$\boxed{\overline{D_{BQ}} = 4.961\,\overline{a_x} + 9.9229\,\overline{a_y} + 19.845\,\overline{a_z}}\ nC/m^2$$



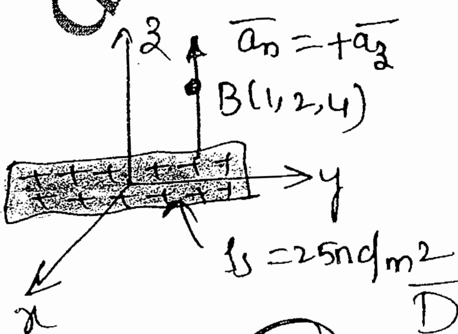
$\overline{OB} = 2\overline{a_y} + 4\overline{a_z}$

$|\overline{OB}| = \sqrt{4+16} = \sqrt{20}$ m.

$$\overline{D_{B\rho_l}} = \frac{\rho_l}{2\pi|\overline{OB}|^2}\,\overline{OB}\ C/m^2$$

$$\overline{D_{B\rho_l}} = \frac{180n}{2\pi(20)}\left[2\overline{a_y} + 4\overline{a_z}\right]$$

$$\boxed{\overline{D_{B\rho_l}} = 2.8647\,\overline{a_y} + 5.729\,\overline{a_z}}\ nC/m^2$$



$$\boxed{\overline{D_{B\rho_s}} = \frac{\rho_s}{2}\,\overline{a_n} = \frac{\rho_s}{2}\,\overline{a_z} = 12.5\,\overline{a_z}\ nC/m^2}$$

$$\overline{D_B} = \overline{D_{BQ}} + \overline{D_{B\rho_l}} + \overline{D_{B\rho_s}}$$

$$\overline{D_B} = 4.961\,\overline{a_x} + 9.9229\,\overline{a_y} + 19.845\,\overline{a_z} + 2.864\,\overline{a_y} + 5.729\,\overline{a_z} + 12.5\,\overline{a_z}$$

185

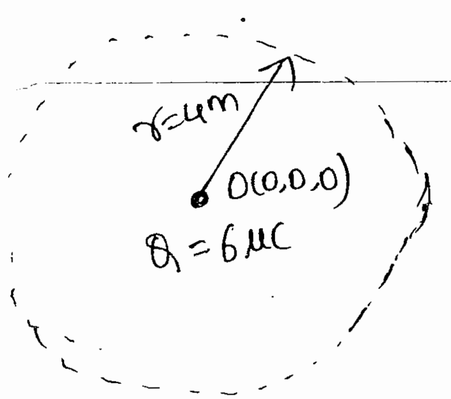
xii) $$\boxed{\overline{D}_B = 4.961\,\overline{a}_x + 12.7869\,\overline{a}_y + 38.074\,\overline{a}_z}\ nC/m^2$$

xii) V.imp iii) The Electric flux Leaving the Surface of the Sphere of radius Centered at origin.

from Gauss's Law $\boxed{\psi_{total} = Q_{enclosed}}$ Coulomb's

→ Case i. Flux Leaving due to charge at origin

$$\boxed{\psi'_{Leaving} = Q_{enc} = 6\,\mu C}$$

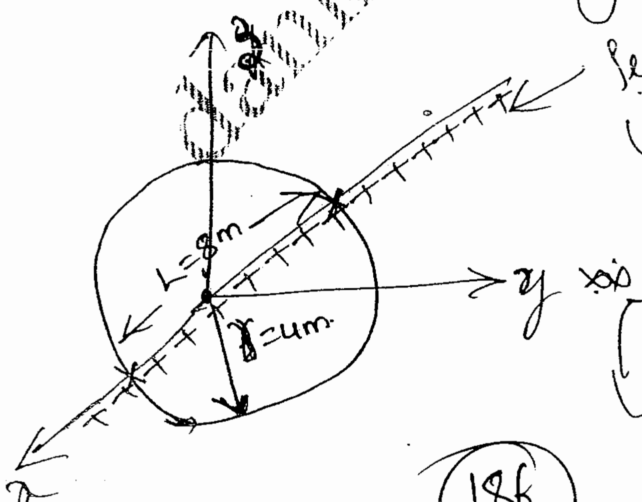


r = 4m Sphere.

Case ii. Flux Leaving due to a line charge of $\rho_L = 180\,nC/m$ placed along x axis

$L = 2r = 8m$

$\rho_L = 180\,nC/m$.



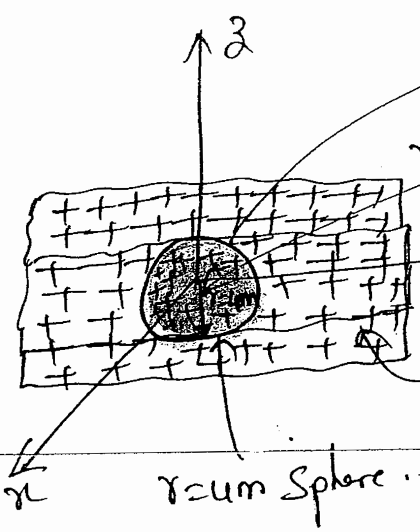
$$\psi''_{Leaving} = Q_{enclo} = \rho_L \times L$$
$$= 180\,nC/m \times 8m$$
$$\boxed{\psi''_{Leaving} = 1.44\,\mu C}$$

(186)

Case iii: Flux leaving due to a sheet charge of $\rho_s = 25\,nC/m^2$ at $z = 0\,m$ plane.

charge density covers only with $\odot$ having radius $r \leq 4m$.

$\therefore A = \pi r^2 = \pi(4)^2 = 16\pi\,m^2$



Area

$\boxed{A = 16\pi m^2}$

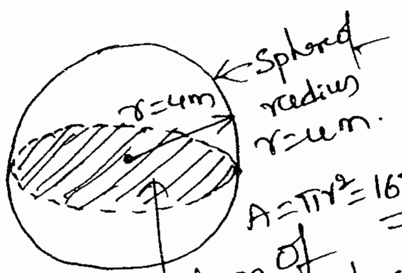
Flux leaving

$$\psi'''_{Leaving} = \rho_s \times A$$

$$= 25\,nC/m^2 \times 16\pi\,m^2$$

$$= 1.2566\,\mu\,\text{Coulomb's}$$

$$\boxed{\psi'''_{Leaving} = 1.25\mu\,C}$$



intersection of sheet charge with sphere

$\Rightarrow \odot = A = \pi r^2 m^2$

$\therefore$ the total flux leaving the surface of the sphere of 4m radius centered at origin is

$$\psi_{total} = Q_{enclosed} = \psi' + \psi'' + \psi'''$$

$$= 6\mu + 1.44\mu + 1.25\mu$$

$$\boxed{\psi_{total} = 8.7006\,\mu}\ \text{Coulomb's}$$

problem 7

Find D in Cartesian co-ordinate system at point p(6,8,-10) due to

i. point charge of 30mC at the origin

ii. A uniform line charge density of $40\mu C/m$ on the z-axis

iii. A uniform sheet charge density of $57.2\mu C/m^2$ on the plane x=9m.

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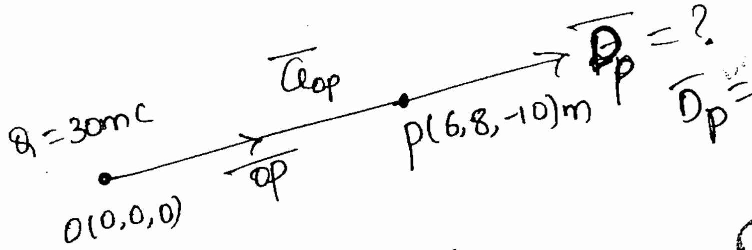
30mC ← p(6,8,-10) ← 40μC/m. ↑ 57.2μC/m² ← x=9m.

Sol^n: i.



$\overline{D}_p = ?$

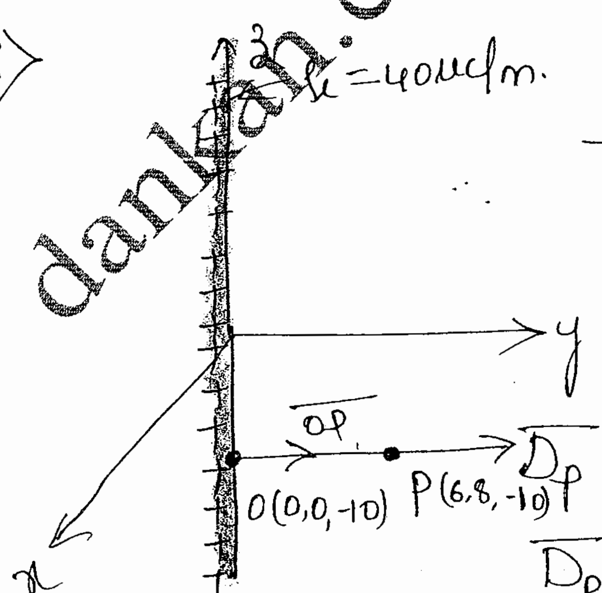
$$\overline{D}_p = \frac{Q}{4\pi|\overline{op}|^3}\,\overline{op} \quad C/m^2$$

$$\overline{op} = 6\overline{a}_x + 8\overline{a}_y - 10\overline{a}_z \; ; \; |\overline{op}| = \sqrt{36+64+100} = \sqrt{200}\,m$$

$$\overline{D}_p = \frac{30m}{4\pi(\sqrt{200})^3}\left[6\overline{a}_x + 8\overline{a}_y - 10\overline{a}_z\right]$$

$$\boxed{\overline{D}_p = 5.064\overline{a}_x + 6.7523\overline{a}_y - 8.44\overline{a}_z} \; \mu C/m^2$$

ii)



$$\overline{D}_{p_{\rho l}} = \frac{\rho_l}{2\pi|\overline{op}|^2}\,\overline{op}$$

$$\overline{op} = 6\overline{a}_x + 8\overline{a}_y$$

$$|\overline{op}| = \sqrt{36+64} = \sqrt{100}\,m$$

$$\overline{D}_{p_{\rho l}} = \frac{40\mu}{2\pi(100)}\left[6\overline{a}_x + 8\overline{a}_y\right]$$

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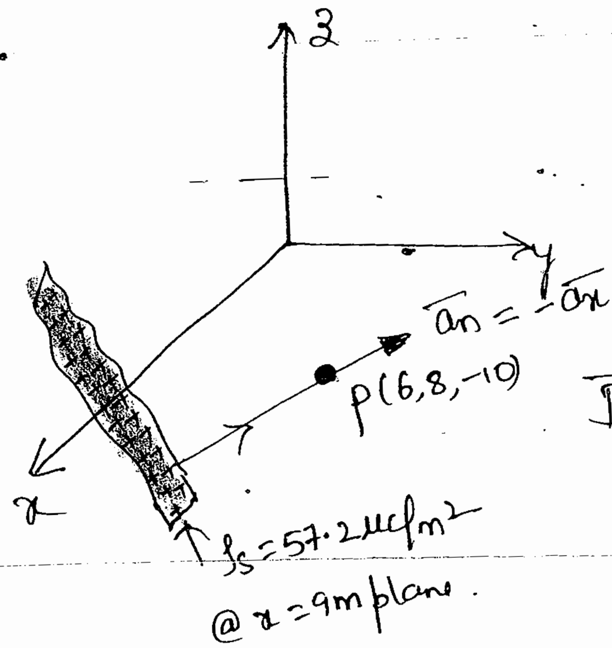
$$\boxed{\overline{D}_{p_{\rho l}} = 381.971\,\overline{a}_x + 509.29\,\overline{a}_y} \; nC/m^2$$

or

$$\boxed{\overline{D}_{p_{\rho l}} = 0.3819\overline{a}_x + 0.5092\overline{a}_y} \; \mu C/m^2$$

iii)



$$\bar{D}_P = \frac{\rho_s}{2}\bar{a}_n$$

$$\bar{D}_P = \frac{57.2\mu}{2}(-\bar{a}_x)$$

$$\boxed{\bar{D}_P = -28.6\,\bar{a}_x}\ \mu C/m^2 \rightarrow \boxed{\bar{D}_P = -28.6\,\bar{a}_x\ \mu C/m^2}$$

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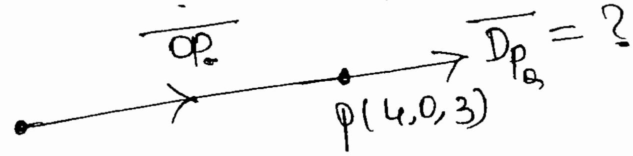
problem 8 (4,0,3)m −15.734mC (4,0,0) m 9.427mC/m y-axis

- Find $\overline{D}$ at (4,0,3) due to a point charge -15.734mC at (4,0,0) and a line charge 9.427mC/m along y-axis.
  Ans $D = 240a_x + 42a_z \; \mu C/m^2$

Solu:- $\overline{D_0} = \overline{D_Q} + \overline{D_{\rho_L}}$ C/m²

Case i



$Q = -15.734mC$

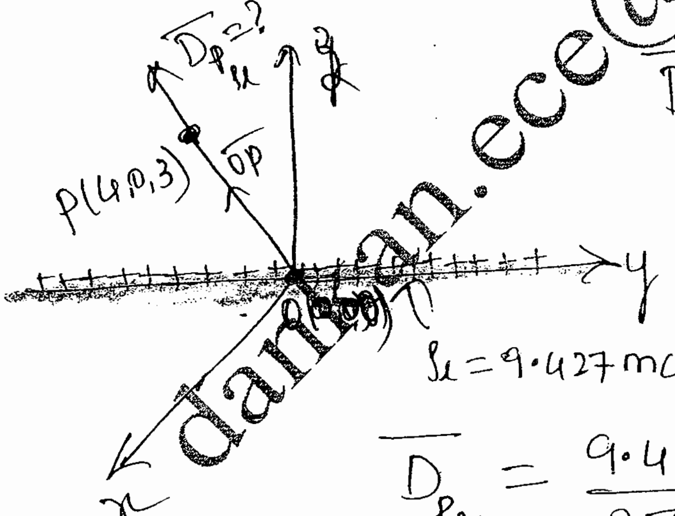
$O(4,0,0)m$ $\overline{OP} = 3\overline{a_z}$; $|\overline{OP}| = 3m.$

$$\overline{D_{P_Q}} = \frac{Q}{4\pi|\overline{OP}|^3}\overline{OP} = \frac{-15.734m}{4\pi(3)^3} \cdot 3\overline{a_z}$$

$$\boxed{\overline{D_{P_Q}} = -139.11\overline{a_z}} \; \mu C/m^2$$

Case ii $\overline{D_{P_{\rho L}}}$ i.e. $\overline{D}$ due line charge density ρ_L C/m.

$$\overline{D} = \frac{\rho_L}{2\pi|\overline{OP}|}\overline{a_{OP}} \; C/m^2$$



$$\overline{D_{P_{\rho L}}} = \frac{\rho_L}{2\pi|\overline{OP}|^2}\overline{OP} \; C/m^2$$

$\overline{OP} = 4\overline{a_x} + 3\overline{a_z}$

$\rho_L = 9.427 mC/m.$ $|\overline{OP}| = \sqrt{16+9} = 5m$

$$\overline{D_{P_{\rho L}}} = \frac{9.427m}{2\pi(5)^2}\left[4\overline{a_x} + 3\overline{a_z}\right]$$

6.0014×10^{-5}

$$\boxed{\overline{D_{P_{\rho L}}} = 240.056\overline{a_x} + 180.042\overline{a_z}} \; \mu C/m^2$$

the net Fluxdensity $(\overline{D})$ at point 'p' is

$$\overline{D_p} = \overline{D_Q} + \overline{D_{\rho_L}}$$

$$= -139.11\,\overline{a_z} + 240.056\,\overline{a_x} + 180.42\,\overline{a_z}\ \mu C/m^2$$

$$\boxed{\overline{D_p} = 240.056\,\overline{a_x} + 41.31\,\overline{a_z}}\ \mu C/m^2$$

$$\boxed{|\overline{D_p}| = 243.584}\ \mu C/m^2$$

(191)

Topic 1.4 problems

problem 5. Find $\bar{D}$ in cartesian Co-ordinate System at point $p(6,8,-10)$ due to

i. point charge of 40nC at the origin.

ii. A uniform line charge density of 40 μC/m on the z-axis

iii. A uniform Sheet charge density of 57.2 μC/m² on the plane x=12m.

problem 6.

A point charge of 6μC is Located at the origin, a uniform Line charge density of 180nC/m Lies along x-axis and uniform Sheet charge equal to 25nC/m² lies in the z=0 plane. Find

i. $\bar{D}$ at A(0,0,4).

ii. $\bar{D}$ at B(1,2,4).

iii. Total electric flux Leaving the Surface of the sphere of 4m radius Centred at origin.

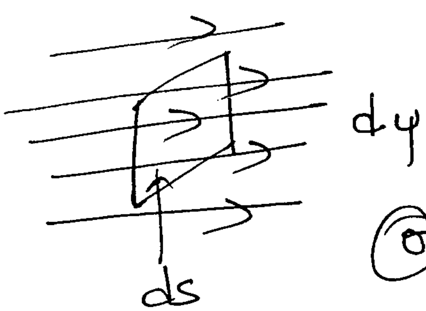
(192)

2b.) Define electric flux density. Find $\vec{D}$ in Cartesian co-ordinate system at a point p(6, 8, −10) due to a point charge of 40mC at the origin and a uniform line charge of $\rho_L = 40\mu C/m$ on the z-axis. **(10 Marks)**

Soln:- Definition of Electric Flux density ($\bar{D}$):-

Electric flux density ($\bar{D}$) indicates an amount of flux ($d\psi$) crosses the differential area ds, which is normal to the surface.

i.e $\bar{D}$ is flux crossing per unit area.



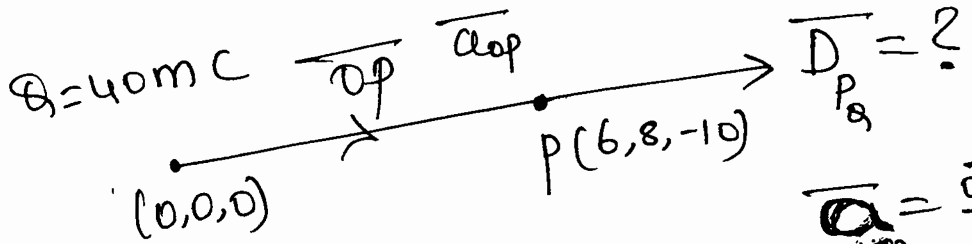
$$\boxed{\bar{D} = \frac{d\psi}{ds}} \ C/m^2$$

(or)

$$\boxed{\bar{D} = \frac{d\psi}{ds}\,\bar{a_n}} \ C/m^2$$

where $\bar{a_n}$ - unit vector normal to the surface

$\overline{D}$ due to point charge



$$\overline{a_{op}} = \frac{\overline{OP}}{|\overline{OP}|}.$$

$$\overline{OP} = 6\overline{a_x} + 8\overline{a_y} - 10\overline{a_z};$$

$$|\overline{OP}| = \sqrt{200}\ m.$$

$$\overline{D_{P_Q}} = \frac{Q}{4\pi |\overline{OP}|^2}\overline{a_{op}} \quad C/m^2$$

$$\overline{D_{P_Q}} = \frac{Q}{4\pi |\overline{OP}|^3}\overline{OP} \quad C/m^2$$

$$\overline{D_{P_Q}} = \frac{40\times 10^{-3}}{4\pi(\sqrt{200})^3}\left[6\overline{a_x} + 8\overline{a_y} - 10\overline{a_z}\right]$$

$$\overline{D_{P_Q}} = \left[1.12539\times 10^{-6}\right]\left[6\overline{a_x} + 8\overline{a_y} - 10\overline{a_z}\right]$$

$$\boxed{\overline{D_{P_Q}} = 6.7523\overline{a_x} + 9.0031\overline{a_y} - 11.2539\overline{a_z}}\ \mu C/m^2$$

$$= 6.752\overline{a_x} + 9.003\overline{a_y} - 11.2539\overline{a_z};\ \mu C/m^2$$

the net $\overline{D}$ at a point $P(6, 8, -10)$ m due to point and line charge is given by

$$\overline{D}_{P_{net}} = \overline{D}_{P_{Q}} + \overline{D}_{P_{\rho L}} \quad C/m^2$$

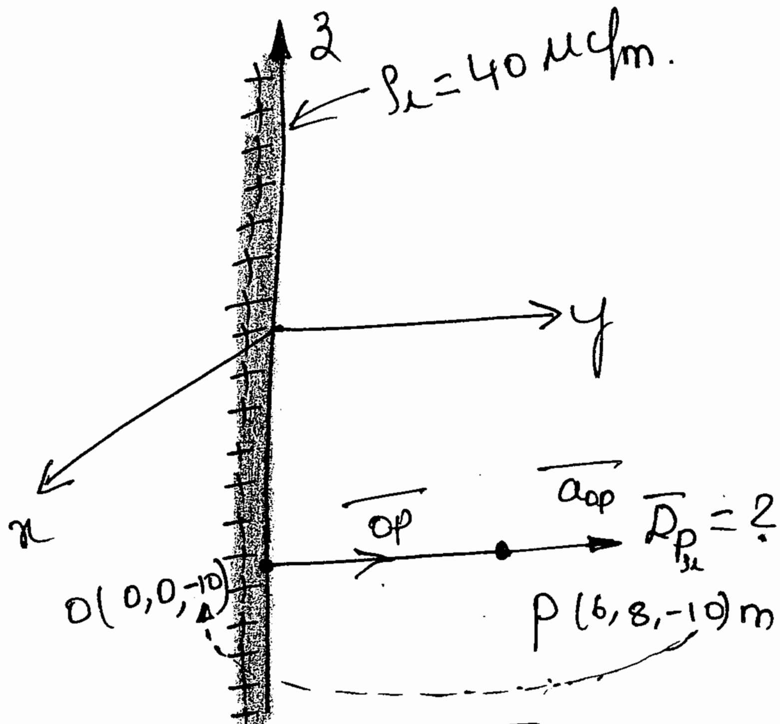
$$= [6.7523\,\overline{a}_x + 9.0031\,\overline{a}_y - 11.2539\,\overline{a}_z]\ \mu C/m^2$$

$$+ [0.3819\,\overline{a}_x + 0.5092\,\overline{a}_y]\ \mu C/m^2$$

$$\boxed{\overline{D}_{P_{net}} = 7.1342\,\overline{a}_x + 9.5123\,\overline{a}_y - 11.2539\,\overline{a}_z}\ \mu C/m^2$$

$$|\overline{D}_{P_{net}}| = \sqrt{7.1342^2 + 9.5123^2 + 11.2539^2}\ \mu C/m^2$$

$$\boxed{|\overline{D}_{P_{net}}| = 16.3716}\ \mu C/m^2$$



$$\overline{D}_{\rho_l} = \frac{\rho_l}{2\pi|\overline{op}|}\,\overline{a}_{op} \ \ C/m^2$$

$$\boxed{\overline{D}_{\rho_l} = \frac{\rho_l}{2\pi|\overline{op}|^2}\,\overline{op}} \ \ C/m^2$$

$$\boxed{\overline{a}_{op} = \frac{\overline{op}}{|\overline{op}|}}$$

$$\overline{op} = 6\overline{a}_x + 8\overline{a}_y$$

$$|\overline{op}| = \sqrt{36+64}$$

$$\boxed{|\overline{op}| = \sqrt{100}\ m}$$

$$\overline{D}_{\rho_l} = \frac{40\mu}{2\pi\,(100)}\left[6\overline{a}_x + 8\overline{a}_y\right] C/m^2$$

$$= 6.36619\left[6\overline{a}_x + 8\overline{a}_y\right]\times 10^{-8}\ C/m^2$$

$$\boxed{\overline{D}_{\rho_l} = 0.3819\,\overline{a}_x + 0.5092\,\overline{a}_y} \ \ \mu C/m^2$$

Module-1 Summary

a. List of Symbols

unit.

1. Charge (Q) ⟶ Coulombs (C)
2. Force (F) ⟶ Newton's (N)
3. distance (r) ⟶ meter (m).
4. Line charge density (ρ_L) ⟶ C/m
5. Surface charge density (ρ_s) ⟶ C/m^2
6. Volume charge density (ρ_v) ⟶ C/m^3
7. Electric field intensity ($\bar{E}$) ⟶ V/m or N/C
8. Electric flux (ψ) ⟶ Coulombs (C)
9. Electric flux density ($\bar{D}$) ⟶ C/m^2.
10. $\boxed{\nabla \cdot \bar{D} = \rho_v}$ C/m^3 ... point form of Gauss's Law (or) Maxwell's first equation. (electro static).
11. relationship b/w $\bar{D}$ and $\bar{E}$

$$\boxed{\bar{D} = \epsilon \bar{E}} \; C/m^2$$

12. $|\bar{D}| = \rho_s$ C/m^2 ... Surface charge density.

13. Gauss's Law

$$\boxed{\Psi_{total} = \oint_{\langle S \rangle} \overline{D} \cdot \overline{ds} = Q_{enclosed}}$$ Coulomb's

14. Divergence theorem.

$$\boxed{\oint_{\langle S \rangle} \overline{D} \cdot \overline{ds} = \int_{\langle vol \rangle} \nabla \cdot \overline{D} \; dv}$$ Coulomb's

15. $\epsilon = \epsilon_0 \epsilon_r$ F/m ... permittivity of the medium.

$\epsilon_0 = 8.854 \times 10^{-12} \simeq \frac{10^{-9}}{36\pi}$ F/m ....

$\epsilon_r = 1$; ... relative permittivity (in air or Vaccum medium)

B. List of Formulae:-

1. Experimental Law of Coulomb.

The Force of attraction (or) repulsion between any two point charges Q_1 and Q_2 is proportional to the product of the charges and inversely proportional to the square of the distance b/w them.

i.e $F = \frac{Q_1 Q_2}{r^2}$: N (or) $\boxed{F = \frac{k Q_1 Q_2}{r^2}}$ N

$$\boxed{\overline{F} = \frac{Q_1 Q_2}{4\pi \epsilon r^2} \overline{a_r}} \text{ N} \quad \text{.... vector form.}$$

Note:- $\boxed{\frac{1}{4\pi \epsilon_0} = 9 \times 10^9}$ m/F-rad

$\epsilon = \epsilon_0 = 8.854 \times 10^{-12}$ F/m ... in freespace (or) Vaccum medium.

2. Force on a point Charge due to n-multiple point charges

$$\boxed{\overline{F} = \frac{Q^2}{4\pi \epsilon_0} \sum_{i=1}^{n} \frac{\overline{a_{i0}}}{|R_{i0}|^2}} \text{ N} ; \text{ if } Q_1 = Q_2 = Q \text{ C}$$

3. Electric Field intensity (E) → V/m

$\overline{E}$ is the Force per unit charge.

$$\boxed{\overline{E} = \frac{\overline{F_t}}{\cancel{Q_t} \to 1C} = \frac{Q}{4\pi \epsilon r^2}\,\overline{a_r}}$$ V/m or N/C

$\overline{E}$ due to multiple point charges

$$\boxed{\overline{E} = \frac{Q}{4\pi\epsilon_0} \sum_{i=1}^{n} \frac{\overline{a_{ip}}}{|\overline{R_{ip}}|^2}}$$ V/m or N/C

4. Charge distribution Techniques

→ line charge density (ρ_L) $= \dfrac{\text{total charge spread}}{\text{Length of the Line}}$ C/m.

$$\boxed{\rho_L = \frac{dQ}{dL}} \text{ C/m} \quad \text{or} \quad \boxed{Q = \int_{\langle L\rangle} \rho_L \cdot dL} \text{ C}$$

or $\boxed{\rho_L = \dfrac{Q}{L}}$ C/m and $\boxed{Q = \rho_L L}$ C

→ Surface charge density (ρ_S) $= \dfrac{\text{total charge spread}}{\text{Area of the Surface}}$ C/m²

$$\boxed{\rho_S = \frac{dQ}{dS}} \text{ C/m}^2 \quad \text{or} \quad \boxed{dQ = \rho_S \cdot dS} \text{ C}$$

$$\boxed{Q = \int_{\langle S\rangle} \rho_S \cdot dS} \text{ Coulombs}$$

$$\boxed{\rho_S = \frac{Q}{S}}\ C/m^2 \quad \text{and} \quad \boxed{Q = \rho_S \cdot S}\ \text{Coulomb's}$$

→ Volume charge density $(\rho_v) = \dfrac{\text{total charge spread in the volume}}{\text{total volume}}$

$$\boxed{\rho_v = \frac{dQ}{dv}}\ C/m^3 \quad \text{and} \quad \boxed{dQ = \rho_v\, dv}\ \text{Coulomb's}$$

$$\boxed{Q = \int_{\langle vol \rangle} \rho_v \cdot dv}\ \text{Coulomb's}$$

5. Field $(\overline{E})$ due to infinite line charge density

$$\boxed{\overline{E} = \frac{\rho_l}{2\pi\epsilon\rho}\,\overline{a_\rho}}\ V/m \text{ (or) } N/C$$

where ρ - $\perp^r$ distance from infinite line charge to the desired point where we measure the field Intensity.

Note:- $\boxed{\dfrac{1}{2\pi\epsilon_0} = 18 \times 10^9}$ m/farad

6. $\overline{E}$ due to infinite sheet charges:-

$$\boxed{\overline{E} = \frac{\rho_s}{2\epsilon}\,\overline{a_n}}\ V/m \text{ (or) } N/C$$

where $\overline{a_n}$ - unit normal vector which is $\perp^r$ to plane containing a infinite sheet charge.

7. Electric Flux (ψ):-

Electric flux (ψ) is a scalar quantity, by defn Electric flux originates at positive charge and terminates at negative charge.

$$\boxed{\psi = Q} \text{ Coulombs.}$$

8. Electric Flux density:- ($\overline{D}$)

$$\boxed{\overline{D} = \frac{d\psi}{ds}\overline{a_n}} \ C/m^2$$

ie an amount of flux ($d\psi$) crosses the differential area ds, which is normal to $\overline{a_n}$.

- Relationship between $\overline{D}$ and $\overline{E}$ is given by

$$\boxed{\overline{D} = \epsilon\overline{E}} \ C/m^2$$

- $\overline{D}$ due to point charge $\boxed{\overline{D} = \frac{Q}{4\pi r^2}\overline{a_r}} \ C/m^2$

- $\overline{D}$ due to infinite line charge $\boxed{\overline{D} = \frac{\rho_l}{2\pi\rho}\overline{a_\rho}} \ C/m^2$

- $\overline{D}$ due to infinite sheet charge

$$\boxed{\overline{D} = \frac{\rho_s}{2}\overline{a_n}} \ C/m^2$$

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Module -2

Part-A

Gauss's law and Divergence: Gauss' law, Divergence. Maxwell's First equation (Electrostatics), Vector Operator ▼ and divergence theorem.

Part-B

Energy, Potential and Conductors: Energy expended in moving a point charge in an electric field, The line integral, Definition of potential difference and potential, The potential field of point charge, Current and Current density, Continuity of current.

Part A

Topics:

2.1 Gauss's Law
2.2 Gaussian Surface and its characteristics
2.3 Applications of Gauss's Law
2.4 Limitations of Gauss's Law
✓ Solved Problems
2.5 Vector operator and concept of Divergence & Divergence in all 3 co-ordinate systems
2.6 Maxwell's First Equation(Electrostatics)/[point form of Gauss's Law]
✓ Solved Problems
2.7 Divergence Theorem
✓ Solved Problems

Part B

Topics:

2.8 Energy expended in moving a point charge in an electric field.
✓ Solved Problems
2.9 The line integral
✓ Solved Problems
2.10 Definition of potential difference and potential
2.11 The potential field of point and system of charge
a. Potential due to point charge
b. Potential due to system of charges
c. Potential field of a ring of uniform line charge density
d. Potential due to infinite line charge
✓ Solved Problems
2.12 Current and Current density

2.13 Continuity of current

✓ Solved Problems

Miscellaneous Topics
2.14 Potential Gradient
2.15 Gradient in all 3 coordinate systems
✓ Solved Problems
2.16 Energy density in an electrostatic field
✓ Solved Problems

Summary
- List of Symbols
- List of Formulae

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Topic 2.1 Gauss's Law.

1. Gauss's Law

1. Gauss's Law

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06-DEC2010

1 State and prove the Gauss's law. (06 Marks)

02-DEC2010

2 State and prove the Gauss theorem (08 Marks)

02-DEC2008/Jan 2009

3 State and prove Gauss theorem. (07 Marks)

06-DEC2011/Jan 2012

4 State and prove Gauss's law for point charge (06 Marks)

10-DEC2011/Jan 2012

5 State and prove Gauss's law. (06 Marks)

06 - June /July 2011

02 - June /July 2010

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9 State and prove Gauss's law. (06 Marks)

10-Dec/Jan 2015

10 State and prove Gauss law for point charge (06 Marks)

06-Jan 2008

10 - June /July 2015

02 J/J 2010,

11 State and prove Gauss law. (04 Marks)

10 - June /July 2014

06 J/J-2011,

06 –Dec/Jan 2008

10-Jan 2015

Question: ① State and prove Gauss's Law / Gauss theorem.

(or)

② State and prove Gauss's Law for point charge. 10-J/J 2015

[06-Dec 2010, 02-Dec-2010, 02-Jan 2009, 06-Jan 2012, 10-Jan 2012, 10-J/J 2014]

Statement:- "The Electric Flux (ψ) passing through any Closed Surface is equal to the total charge enclosed by that Surface."

i.e.
$$\boxed{\psi_{total} = \oint_{\langle S \rangle} \bar{D} \cdot \bar{dS} = Q_{enclosed}}$$ Coulomb's

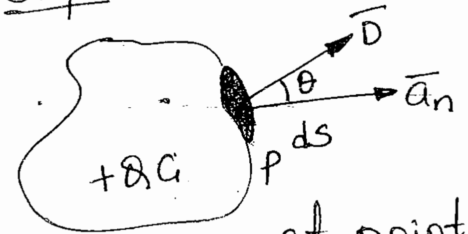
Step-1 Step2

proof:- proof is Carried out in two step's

Step1. $\psi_{total} = \oint_{\langle S \rangle} \bar{D} \cdot \bar{dS}$ C

Step2. $\oint_{\langle S \rangle} \bar{D} \cdot \bar{dS} = Q_{enclosed}$ Coulomb's

Step 1.



Let a positive charge Q C is enclosed by a closed surface of any shape. at point 'p' consider an differential element of surface ds and let $\bar{D}$ makes an angle θ with $\bar{ds}$ as shown in fig.

$$\bar{ds} = ds\,\bar{a}_n$$

the differential flux ($d\psi$) coming out of differential surface (ds) is given by

$$d\psi = \text{Flux crossing } ds$$

$$= D_{normal}\, ds$$

$$d\psi = D\cos\theta\, ds$$

$$d\psi = D\, ds\cos\theta \quad \ldots \text{using dot product concept } \bar{A}\cdot\bar{B} = AB\cos\theta$$

(or) $$\boxed{d\psi = \bar{D}\cdot\bar{ds}} \text{ Coulombs}$$

the total flux (ψ_{total}) crossing the closed surface is

$$\boxed{\psi_{total} = \oint_{\langle S\rangle} \bar{D}\cdot\bar{ds}} \text{ Coulombs}$$

Note:- $\oint_{\langle S\rangle}$ means closed surface integral.

Step2. Consider a positive charge of 'Q' C situated at the center of an imaginary sphere of radius 'r'.

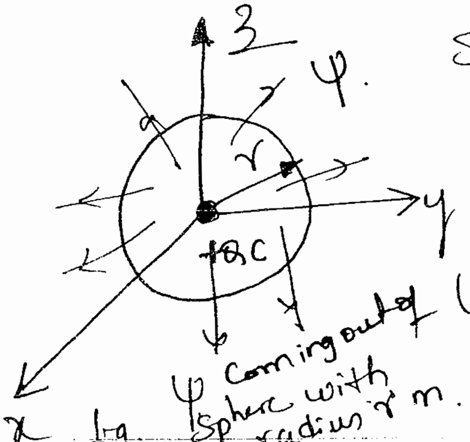


fig.

Since charge 'Q' @ origin, the total flux crossing the sphere is given by

$$\psi_{total} = \oint_{\langle S \rangle} \bar{D} \cdot \bar{dS} \quad \leftarrow (1)$$

w.k.t $\bar{D}$ due to point charge

i.e. $\bar{D} = \dfrac{Q}{4\pi r^2}\,\bar{a_r}$ C/m^2.

and the differential surface vector ($\bar{dS}$) for $r = k$ sphere

P(r, θ, φ): dr, $r\,d\theta$, $r\sin\theta\, d\phi$

$$\bar{dS} = r^2 \sin\theta\, d\theta\, d\phi\, \bar{a_r}$$

$r = k$: sphere.

$$\therefore \psi_{total} = \oint_{\langle S \rangle} \frac{Q}{4\pi r^2}\,\bar{a_r} \cdot r^2 \sin\theta\, d\theta\, d\phi\, \bar{a_r}$$

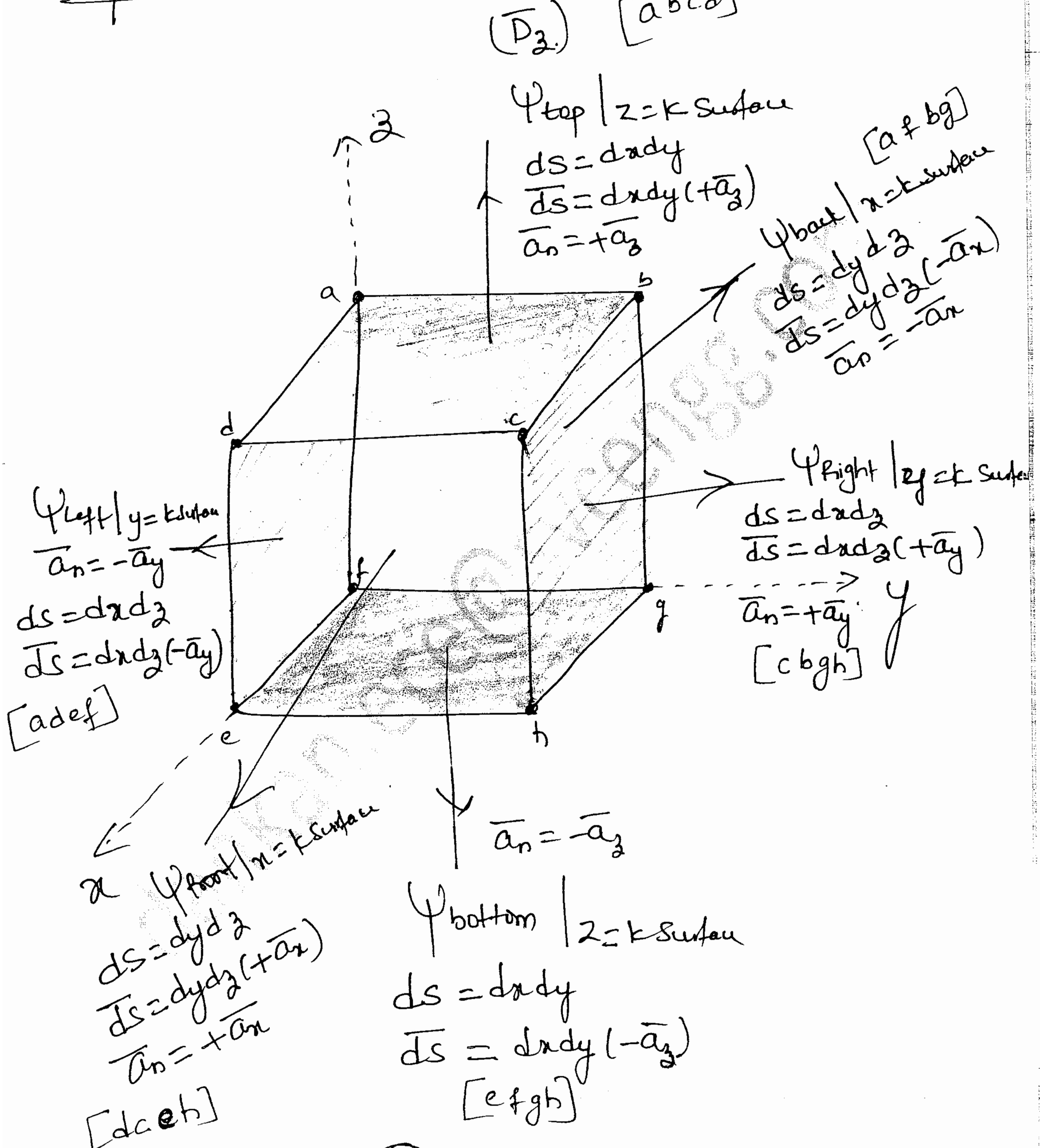
$$= \oint_{\langle S \rangle} \frac{Q}{4\pi \cancel{r^2}} \cdot \cancel{r^2} \sin\theta\, d\theta\, d\phi\, \cancel{\bar{a_r}\cdot\bar{a_r}}^{\,1}$$

$$= \frac{Q}{4\pi} \underbrace{\int_{\theta=0}^{\pi} \sin\theta\, d\theta}_{2} \underbrace{\int_{0}^{2\pi} d\phi}_{2\pi} = \frac{Q}{\cancel{4\pi}} \times \cancel{4\pi} = Q \text{ C}$$

$$\boxed{\psi_{total} = Q_{enclosed}} \quad \text{Coulomb's} \quad (3)$$

This means that Q C of electric flux (ψ) are crossing the surface if the total or net charge enclosed.

Key note:-


($\bar{D}_z$) [abcd]
$\psi_{top}|_{z=k \text{ Surface}}$
$ds = dx dy$
$\overline{ds} = dx dy (+\bar{a}_z)$
$\bar{a}_n = +\bar{a}_z$
[afbg]
$\psi_{back}|_{x=k \text{ surface}}$
$ds = dy dz$
$\overline{ds} = dy dz (-\bar{a}_x)$
$\bar{a}_n = -\bar{a}_x$
$\psi_{Right}|_{y=k \text{ Surface}}$
$ds = dx dz$
$\overline{ds} = dx dz (+\bar{a}_y)$
$\bar{a}_n = +\bar{a}_y$
[cbgh]
$\psi_{Left}|_{y=k \text{ surface}}$
$\bar{a}_n = -\bar{a}_y$
$ds = dx dz$
$\overline{ds} = dx dz (-\bar{a}_y)$
[adef]
z
y
x
a
b
c
d
e
f
g
h
$\bar{a}_n = -\bar{a}_z$
$\psi_{front}|_{x=k \text{ Surface}}$
$ds = dy dz$
$\overline{ds} = dy dz (+\bar{a}_x)$
$\bar{a}_n = +\bar{a}_x$
[dceh]
$\psi_{bottom}|_{z=k \text{ Surface}}$
$ds = dx dy$
$\overline{ds} = dx dy (-\bar{a}_z)$
[efgh]

4

Topic 2.3 Application of Gauss Law :- 15 - June/July 2017 (6M) CBCS- Scheme

Questions (Application problem)

6 State and explain Gauss law as applied to an electric field. (05 Marks)
02 - June /July 2011

(or)

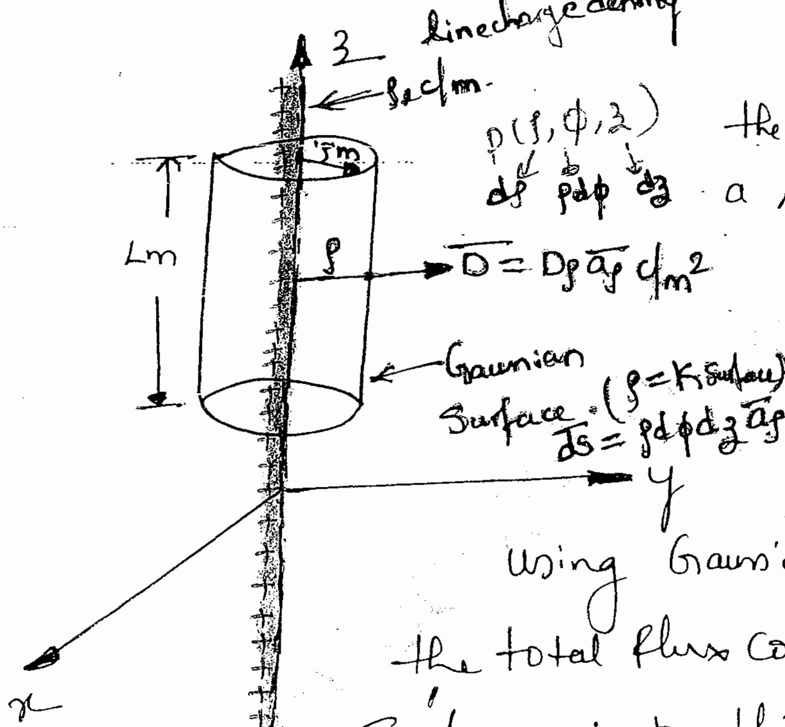
7 State Gauss's law. Using Gauss's law obtain an expression for electric field intensity E due to an infinite line charge along Z – axis having a uniform charge density ρL c/m. (08 Marks)
06 - June /July 2012

(or)

10 – Dec/Jan 2014

8 State Gauss law and use it to determine electric field intensity due to an infinitely long line charge. (08 Marks)

Soln: $\bar{E}$ due to infinite Line charge using Gauss's Law.



the total charge enclosed by a Length 'L' m is

$$Q = \rho_L \times L \quad C$$

$$\boxed{Q = \rho_L L} \quad C$$

Using Gauss's Law

the total flux coming out of a Gaussian Surface is nothing but the charge enclosed by that Surface

i.e $$\Psi_{total} = \oint_{\langle S \rangle} \bar{D} \cdot \bar{ds} = Q_{enclosed} = \rho_L L \text{ Coulombs}$$

$$\rho_L \cdot L = \oint_{\langle S \rangle} \bar{D} \cdot \bar{ds} = \oint_{\langle S \rangle} \bar{D}_\rho \cdot \bar{ds}$$

$$= \oint_{\langle S \rangle} D_\rho \bar{a}_\rho \cdot \rho \, d\phi \, dz \, \bar{a}_\rho$$

$$\rho_L L = D_\rho \oint_{\langle S \rangle} \rho \, d\phi \, dz \; \cancel{\overline{a_\rho} \cdot \overline{a_\rho}}^{\,1}$$

$$= \rho D_\rho \int_{\phi=0}^{2\pi} d\phi \int_{z=0}^{L} dz$$

$$\rho_L \cancel{L} = \rho D_\rho \times 2\pi \times \cancel{L} \quad \text{Coulombs}$$

$$\boxed{D_\rho = \frac{\rho_L}{2\pi\rho}} \; C/m^2 \longleftarrow (1)$$

the Flux density $\overline{D}$ is normal to the Gaussian Surface

$$\overline{D} = D_\rho \overline{a_\rho} \quad C/m^2$$

using eqn (1)

$$\boxed{\overline{D} = \frac{\rho_L}{2\pi\rho} \overline{a_\rho}} \; C/m^2$$

and Electric field intensity $(\overline{E})$ at any point is given by $\overline{E} = \frac{\overline{D}}{\epsilon}$ V/m.

$$\boxed{\overline{E} = \frac{\rho_L}{2\pi\epsilon\rho} \overline{a_\rho}} \; V/m$$

$$\boxed{\overline{D} = \epsilon \overline{E}} \; C/m^2$$

Gaussian Surface and its Characteristics :- EEE J/J 2016 (6M)

Gausian Surface :- The Surface over which the Gauss's Law is applied is called "Gaussian Surface".

$$\boxed{\psi_{total} = \oint_{\langle S \rangle} \bar{D} \cdot \bar{dS} = Q_{enclosed}}$$ C

Characteristics :-

1. The Surface must be closed.
2. at Each point of the surface $\bar{D}$ must be normal ($\perp^r$) to the Surface.
3. The Electric flux density D is ~~Constant over the Surface~~ at which $\bar{D}$ is normal.
4. the Surface may be irregular but must be closed.

Topic 2.4

Limitations of Gauss's Laws :-

1. Valid only for Closed Surfaces.
2. valid only for where $\bar{D}$ must be $\perp^r$ to the differential (Gaussian) Surface.
3. valid for only Gaussian Surfaces where D is Constant throughout the Surface.
4. it can be applied only if the Surface encloses the Volume Completely.

Solve problem

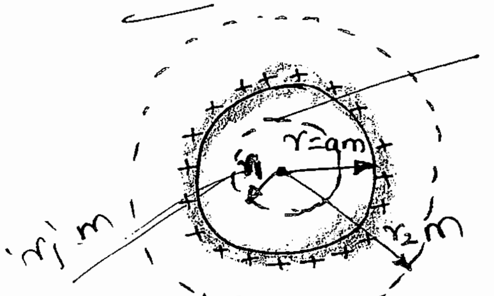
Problem 2 (Application 2)

Spherical radius 'a'.

10 - June /July 2014

12 A charge is uniformly distributed over a spherical surface of radius 'a'. Determine electric field intensity everywhere in space. Use Gauss law. (06 Marks)

Sol^n:



Sphere with radius r=am.

A charge of Q C is uniformly distributed.

the Field intensity $(\bar{E})$ Every where in space

$\Rightarrow$ case i. $r > a$ m.

case ii. $r < a$ m

case iii. $r = a$ m

case i. $\bar{E}$ outside the sphere i.e $r > a$m.

$$\psi_{total} = \oint_{<S>} \bar{D} \cdot \bar{ds} = Q_{enc} \Rightarrow \text{using Gauss Law}$$

$\bar{D}$ – in radially out $\therefore \bar{D} = D_r \bar{a_r}$ C/m^2.

$\bar{ds}$ .. $r = k$ Surface i.e $r = a$m $\Big\}$ $\bar{ds} = r^2 \sin\theta d\theta d\phi \, \bar{a_r}$.

$$Q_{enc} = \int_{<S>} D_r \bar{a_r} \cdot r^2 \sin\theta d\theta d\phi \, \bar{a_r}$$

$$= D_r r^2 \int_{\theta=0}^{\pi} \sin\theta \, d\theta \int_{0}^{2\pi} d\phi \; \overbrace{\bar{a_r} \cdot \bar{a_r}}^{1}$$

$$Q = D_r r^2 \times 2 \times 2\pi \Rightarrow \boxed{D_r = \frac{Q}{4\pi r^2}} \text{ C/m}^2$$

$$\bar{D} = D_r \bar{a_r} \text{ C/m}^2$$

$$\therefore \boxed{\bar{D} = \frac{Q}{4\pi r^2} \bar{a_r}} \text{ C/m}^2$$

and $\bar{E} = \frac{\bar{D}}{\epsilon} = \frac{Q}{4\pi\epsilon r^2}\bar{a_r}$ V/m ....r > a m.

$$Q = \int_{\langle vol \rangle} \rho_v dv = \int_{\langle vol \rangle} \rho_v r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \rho_v \int_{r=0}^{a} r^2 dr \times \int_{\theta=0}^{\pi} \sin\theta \, d\theta \int_{\phi=0}^{2\pi} d\phi = \rho_v \frac{4}{3}\pi a^3 \, C$$

$$\therefore \boxed{\bar{E} = \frac{\rho_v \frac{4}{3}\pi a^3}{4\pi\epsilon r^2} = \frac{a^3 \rho_v}{3\epsilon r^2}\bar{a_r}} \text{ V/m}$$

and $\boxed{\bar{D} = \epsilon\bar{E} = \frac{a^3\rho_v}{3r^2}\bar{a_r}}$ C/m² ← ⓐ

Case ii. $\bar{E}$ and $\bar{D}$ at $r = a$ m.

put $r = a$ in the above set of eq^n's

$\therefore \boxed{\bar{E} = \frac{\rho_v a}{3\epsilon}\bar{a_r}}$ V/m and $\boxed{\bar{D} = \epsilon\bar{E} = \frac{\rho_v a}{3}\bar{a_r}}$ C/m²

Case iii. Field Inside the sphere i.e $r < a$ m.

$$Q_{in} = \oint_{\langle S \rangle} \bar{D} \cdot \bar{dS} = \int_{\langle S \rangle} D_r \bar{a_r} \cdot r^2 \sin\theta \, d\theta \, d\phi = D_r \int_{\theta=0}^{\pi} \sin\theta \, d\theta \times \int_{\phi=0}^{2\pi} d\phi \times r^2$$

$$= D_r \times 2 \times 2\pi \times r^2 = D_r (4\pi r^2)$$

$$D_r = \frac{Q}{4\pi r^2} \text{ C/m}^2 \text{ ---- } r < a \text{ m}$$

$\int_{\langle vol \rangle} \rho_v dv = \rho_v \times \frac{4}{3}\pi r^3$

$$\therefore \bar{D} = D_r \bar{a_r} = \frac{Q}{4\pi r^2}\bar{a_r} \text{ C/m}^2 = \frac{\int_{\langle vol \rangle} \rho_v dv}{4\pi r^2}\bar{a_r} \text{ C/m}^2$$

$$= \frac{\rho_v \times \frac{4}{3}\pi r^3}{4\pi r^2}\bar{a_r} = \frac{\rho_v r}{3}\bar{a_r} \text{ C/m}^2$$

i.e $\boxed{\bar{D} = \frac{\rho_v r}{3}\bar{a_r}}$ C/m² and $\boxed{\bar{E} = \frac{\bar{D}}{\epsilon} \text{ V/m} = \frac{\rho_v r}{3\epsilon}\bar{a_r}}$ V/m

Contd in next

$$\bar{E} = \begin{cases} \frac{a^3 \rho_v}{3\epsilon r^2} \bar{a}_r & ; r > a\,m \\ \frac{\rho_v a}{3\epsilon} \bar{a}_r & ; r = a\,m \end{cases} \quad \text{and} \quad \frac{\rho_v r}{3\epsilon} \bar{a}_r \quad r > a\,m.$$

02-DEC2008/Jan 2009

problem 3.

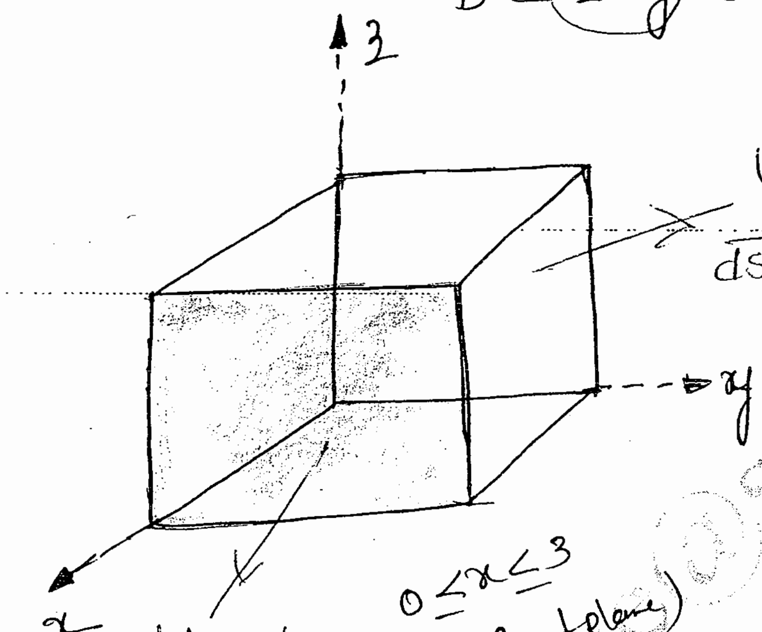
In a certain region of space, $\vec{D} = 2xy\hat{a}_x + 3yz\hat{a}_y + 4zx\hat{a}_z$ $\quad \bar{D} = 2xy\bar{a}_x + 3yz\bar{a}_y + 4zx\bar{a}_z$ C/m^2

Evaluate the amount of electric flux that passes through the portion bounded by $-1 \le y \le 2$ and $0 \le z \le 4$ in the $x = 3$ plane. (06 Marks)

Soln: $0 \le z \le 4$ — $x = 3$ plane — $-1 \le y \le 2$

given

$$\bar{D} = \underbrace{2xy}_{D_x}\bar{a}_x + \underbrace{3yz}_{D_y}\bar{a}_y + \underbrace{4zx}_{D_z}\bar{a}_z \ \text{C/m}^2$$



$\Psi_{back}|_{x=0\,plane}$.

$\bar{dS} = dy\,dz\,(-\bar{a}_x)$.

$-1 \le y \le 2$ and $0 \le z \le 4$ $x = 3$ plane.

$0 \le x \le 3$

$\Psi_{front}|_{x=3\,plane}$ (front plane)

$\bar{dS} = dy\,dz\,(+\bar{a}_x)$.

D_x.

The Electric flux that passes through $x = 3$ plane nothing but Front Surface

i.e $\Psi_{front} = ?$

using Gauss's Law $\Psi = \oint_{\langle S \rangle} \bar{D}\cdot\bar{dS}$ Coulombs

$$\Psi_{front} = \int_{\langle S \rangle} \bar{D}_x \cdot \bar{dS} = \int_{\langle S \rangle} 2xy\,\bar{a}_x \cdot dy\,dz\,\bar{a}_x$$

$$\Psi_{front}\Big|_{x=3\,plane} = 2x\int_{y=-1}^{2} y\,dy \int_{z=0}^{4} dz\ \overbrace{\bar{a}_x\cdot\bar{a}_x}^{1}$$

put $x = 3$

$$= \cancel{2}(3)\cdot\frac{y^2}{\cancel{2}}\Big|_{-1}^{2}\times 4$$

$$= 3\left[(2)^2-(-1)^2\right]\times 4$$

$$\Psi_{front} = 3[4-1]\times 4 = 3\times 3\times 4$$

$$\boxed{\Psi_{front} = 36}\ \text{Coulomb's}$$

Electric Flux that passes through the portion bounded by $-1 \le y \le 2$ and $0 \le z \le 4$ in the $x=3$ plane is

$$\boxed{\Psi_{front} = 36}\ \text{Coulomb's}$$

Key Note:- 1) $\Psi_{top} = \Psi_{bottom} = 0$; when $D_z = 0\ C/m^2$.

(&) $\Psi_{top} + \Psi_{bottom} = 0$; when $D_z \neq f^n(z)$

2) $\Psi_{front} = \Psi_{back} = 0$; when $D_x = 0\ C/m^2$

& $\Psi_{front} + \Psi_{back} = 0$; when $D_x \neq f^n(x)$.

3) $\Psi_{left} = \Psi_{right} = 0$; when $D_y = 0\ C/m^2$

& $\Psi_{left} + \Psi_{right} = 0$; when $D_y \neq f^n(y)$.

11

Note:- Illy the Same kind of results are also valid in Cylindrical and Spherical C.S. (coordinate Syste...)

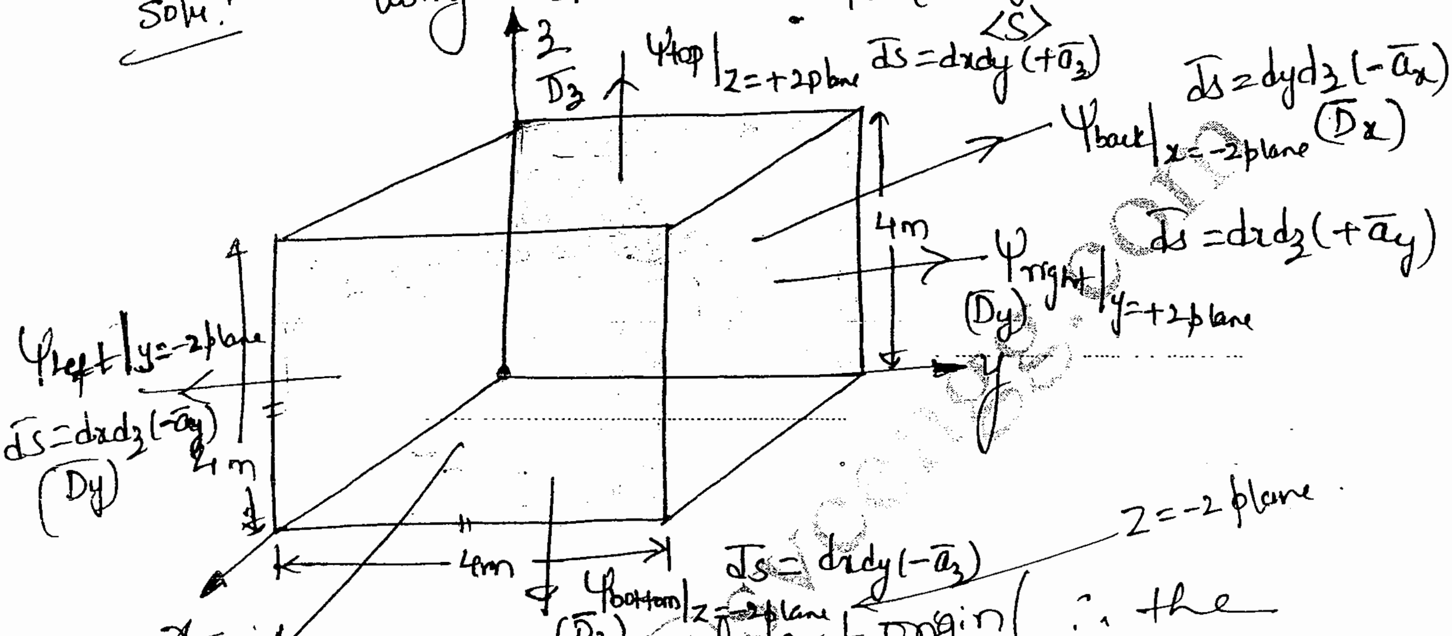
problem 4

$\frac{20x^5}{5}\bar{a}_x \, C/m^2$

06 - June /July 2012

A cube of 4m centered at origin with edges parallel to the coordinate axes of Cartesian co-ord system. If $\bar{D}$ (electric flux density) $= \frac{20x^5}{5}\hat{a}x$ C/m^2, what is the total charge contained in the cube. (06 Marks)

Soln:- using Gauss Law $\Psi_{total} = \oint_{\langle S \rangle} \bar{D}\cdot\bar{dS}$ Coulomb's.



Cube is Symmetrical w.r.t origin and side of 4m ∴ the range of x, y and z is

$$\boxed{-2 \leq x, y, z \leq +2}$$

Given $\bar{D} = \frac{20x^2}{5}\bar{a}_x \, C/m^2$.

$$\Psi_{total} = \cancel{\Psi_{top}}^{0} + \cancel{\Psi_{bottom}}^{0} + \cancel{\Psi_{Left}}^{0} + \cancel{\Psi_{right}}^{0} + \Psi_{front} + \Psi_{back} \; C$$

Since in given $\bar{D}$, $D_y = D_z = 0$

$$\Psi_{Left} = \Psi_{right} = \Psi_{top} = \Psi_{bottom} = 0.$$

$$\psi_{front}\Big|_{x=+2\,plane} = \int_{\langle S \rangle} \bar{D}_x \cdot \bar{ds} = \int_{\langle S \rangle} 4x^5\, \bar{a}_x \cdot dy\,dz\, \bar{a}_x$$

$$= 4x^5 \int_{y=-2}^{2} dy \int_{z=-2}^{2} dz\ \cancel{\bar{a}_x \cdot \bar{a}_x}^{\,1} \Big|_{x=2\,plane}$$

$$= 4(2)^5 \times 4 \times 4 = 2048 \text{ Coulomb's}$$

$$\boxed{\psi_{front} = 2048} \text{ Coulomb's}$$

$$\psi_{back}\Big|_{x=-2\,plane} = \int_{\langle S \rangle} \bar{D}_x \cdot \bar{ds} = \int_{\langle S \rangle} 4x^5\, \bar{a}_x \cdot dy\,dz\,(-\bar{a}_x)$$

$$= -4x^5 \int_{y=-2}^{2} dy \int_{z=-2}^{2} dz\ \cancel{\bar{a}_x \cdot \bar{a}_x}^{\,1} \Big|_{x=-2\,plane}$$

$$= -4(-2)^5 \times 4 \times 4 = +2048 \text{ Coulomb's}$$

$$\boxed{\psi_{back} = 2048} \text{ Coulomb's}$$

$$\psi_{total} = \psi_{front} + \psi_{back} \Rightarrow \psi_{total} = 2048 + 2048$$

$$\boxed{\psi_{total} = 4096} \text{ Coulomb's}$$

or

2nd Method :- using divergence theorem ; $\bar{D} = 4x^5\, \bar{a}_x\ C/m^2$

$$\psi_{total} = \oint_{\langle S \rangle} \bar{D} \cdot \bar{ds} = \int_{\langle vol \rangle} (\nabla \cdot \bar{D})\, dv \text{ Coulomb's}$$

$$\nabla \cdot \bar{D} = 20x^4\ C/m^3 \Rightarrow \psi_{total} = \int_{\langle v \rangle} 20x^4\, dx\,dy\,dz$$

$$= 20 \int_{-2}^{2} x^4\, dx \int_{-2}^{2} dy \int_{-2}^{2} dz = 20 \times 12.8 \times 4 \times 4 = 4096\, C$$

i.e. $\boxed{\psi_{total} = 4096}$ Coulomb's

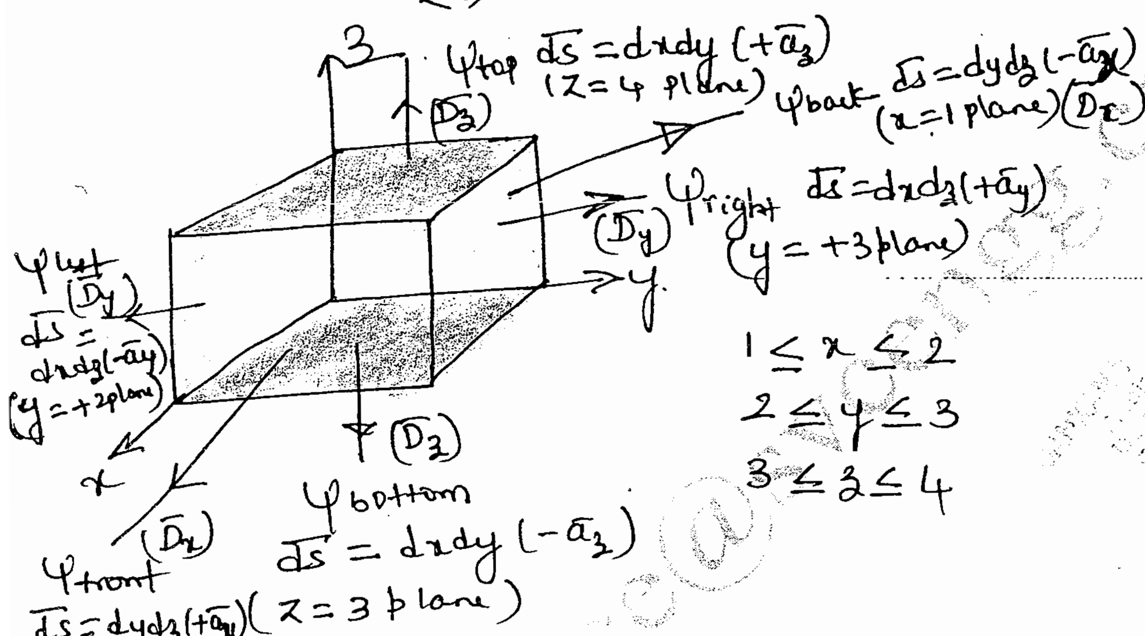
Problems

02 - June /July 2012

Find the total charge in a volume defined by six planes for which $1 \le x \le 2$, $2 \le y \le 3$, $3 \le z \le 4$, if $\vec{D} = (4x\hat{a}_x + 3y^2\hat{a}_y + 2z^2\hat{a}_z)$ Coulomb/m². (06 Marks)

Sol'n: Method-I :- using Gauss's Law

$$\Psi_{total} = \oint_{\langle S \rangle} \bar{D} \cdot \bar{dS} = \Psi_{top} + \Psi_{bottom} + \Psi_{left} + \Psi_{right} + \Psi_{front} + \Psi_{back}.$$



$1 \le x \le 2$
$2 \le y \le 3$
$3 \le z \le 4$

$$\Psi_{top}\Big|_{Z=4\,plane} = \int_{\langle S \rangle} \bar{D_z} \cdot \bar{dS} = \int_{\langle S \rangle} 2z^2 \bar{a}_z \cdot dxdy\,\bar{a}_z$$

$$= 2z^2 \int_{x=1}^{2} dx \int_{y=2}^{3} dy\; \cancel{\bar{a}_z \cdot \bar{a}_z}^{\,1} = 2(4)^2(1)(1) = \underline{32 \text{ Coulombs}}$$

$$\boxed{\Psi_{top} = 32}\ C$$

$$\Psi_{bottom}\Big|_{z=3\,plane} = \int_{\langle S \rangle} \bar{D_z} \cdot \bar{dS} = \int_{\langle S \rangle} 2z^2\,\bar{a}_z \cdot (-dxdy\,\bar{a}_z)$$

$$= -2z^2 \int_{x=1}^{2} dx \int_{y=2}^{3} dy\; \cancel{\bar{a}_z \cdot \bar{a}_z}^{\,1} = -2(3)^2(1)(1) = \underline{-18 \text{ Coulomb's}}$$

$$\boxed{\Psi_{bottom} = -18}\ C$$

$$\Psi_{Left}\Big|_{y=2\,plane} = \int_{\langle S \rangle} \bar{D_y} \cdot \bar{dS} = \int_{\langle S \rangle} 3y^2\,\bar{a}_y \cdot dxdz\,(-\bar{a}_y)$$

$$= -3y^2 \int_{x=1}^{2} dx \int_{z=3}^{4} dz\; \cancel{\bar{a}_y \cdot \bar{a}_y}^{\,1} = -3(2)^2(1)(1) = \underline{-12 \text{ Coulomb's}}$$

$$\boxed{\Psi_{Left} = -12}\ C$$

$$\Psi_{right}\Big|_{y=3\,plane} = \int_{\langle s\rangle} \overline{D_y}\cdot\overline{ds} = \int_{\langle s\rangle} 3y^2\,\overline{a_y}\cdot dx\,dz\,(+\overline{a_y})$$

$$= 3y^2 \int_{x=1}^{2} dx \int_{z=3}^{4} dz \;\; \cancel{\overline{a_y}\cdot\overline{a_y}}^{\,1} = 3(3)^2(1)(1) = \underline{27 \text{ Coulomb's}}$$

$$\boxed{\Psi_{right} = 27}\ C$$

$$\Psi_{front}\Big|_{x=2\,plane} = \int_{\langle s\rangle} \overline{D_x}\cdot\overline{ds} = \int_{\langle s\rangle} 4x\,\overline{a_x}\cdot dy\,dz\,\overline{a_x}$$

$$= 4x \int_{y=2}^{3} dy \int_{z=3}^{4} dz \;\; \cancel{\overline{a_x}\cdot\overline{a_x}}^{\,1} = 4(2)(1)(1) = \underline{8 \text{ Coulomb's}}$$

$$\boxed{\Psi_{front} = 8}\ C$$

$$\Psi_{back}\Big|_{x=1\,plane} = \int_{\langle s\rangle} \overline{D_x}\cdot\overline{ds} = \int_{\langle s\rangle} 4x\,\overline{a_x}\cdot dy\,dz\,(-\overline{a_x})$$

$$= -4x \int_{y=2}^{3} dy \int_{z=3}^{4} dz \;\; \cancel{\overline{a_x}\cdot\overline{a_x}}^{\,1} = -4(1)(1)(1) = \underline{-4 \text{ Coulomb's}}$$

$$\boxed{\Psi_{back} = -4}\ C$$

$$\Psi_{total} = 32 - 18 - 12 + 27 + 8 - 4 = \underline{33 \text{ Coulomb's}}$$

(or) $\boxed{\Psi_{total} = 33}$ Coulomb's

(or) 2nd Method:- using divergence theorem i.e $\oint_{\langle s\rangle} \overline{D}\cdot\overline{ds} = \int_{\langle vol\rangle} \nabla\cdot\overline{D}\, dv$

$$\nabla\cdot\overline{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = [4 + 6y + 4z]$$

$$\int_{\langle vol\rangle} [4 + 6y + 4z]\, dv = \int_{\langle vol\rangle} 4\,dv + \int_{\langle vol\rangle} 6y\, dv + \int_{\langle vol\rangle} 4z\, dv$$

$$= 4\int_{x=1}^{2} dx \int_{y=2}^{3} dy \int_{z=3}^{4} dz + 6\int_{x=1}^{2} dx \int_{y=2}^{3} y\,dy \int_{z=3}^{4} dz + 4\int_{x=1}^{2} dx \int_{y=2}^{3} dy \int_{z=3}^{4} z\,dz$$

$$= 4(1)(1)(1) + 6(1)(2.5)(1) + 4(1)(1)(3.5)$$

$$= 4 + 15 + 14 = 33 \text{ Coulomb's}$$

$$\therefore \boxed{\Psi_{total} = \oint_{\langle s\rangle} \overline{D}\cdot\overline{ds} = \int_{\langle vol\rangle} \nabla\cdot\overline{D}\, dv = 33}\ \text{Coulomb's}$$

problem 6 $\bar{D} = (2y^2z - 8xy)\bar{a}_x + (4xyz - 4x^2)\bar{a}_y + (2xy^2 - 4z)\bar{a}_z$ C/m²

10 June/July 2016

$[10^{-14} m^3]$ Let $\bar{D} = (2y^2z - 8xy)\hat{a}_x + (4xyz - 4x^2)\hat{a}_y + (2xy^2 - 4z)\hat{a}_z$. Determine the total charge within a volume of 10^{-14} m³ located at P(1, −2, 3). (05 Marks) $P(1, -2, 3)$

Sol^n: $Q_{total} = ?$ $v = 10^{-14} m^3$; $P(1, -2, 3)$

Maxwell's first eq^n

Note: this problem must come under maxwell's first eq^n TOP: (i.e. [2.6])

$\nabla \cdot \bar{D} = \rho_v$ C/m³

total charge $\boxed{Q_{total} = \rho_v \cdot (\text{Volume})}$ Coulomb's

$$\nabla \cdot \bar{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} \quad C/m^3$$

$$\bar{D} = (2y^2z - 8xy)\bar{a}_x + (4xyz - 4x^2)\bar{a}_y + (2xy^2 - 4z)\bar{a}_z \ C/m^2$$

$$D_x = 2y^2z - 8xy \ ; \ D_y = 4xyz - 4x^2 \ ; \ D_z = 2xy^2 - 4z$$

$$\frac{\partial D_x}{\partial x} = -8y \ ; \ \frac{\partial D_y}{\partial y} = 4xz \ ; \ \frac{\partial D_z}{\partial z} = -4$$

$$\nabla \cdot \bar{D} = [-8y + 4xz - 4] \ C/m^3$$

$$\nabla \cdot \bar{D}\Big|_{@P(1,-2,3)} = -8(-2) + 4(1)(3) - 4 = 24 \ C/m^3.$$

$\therefore$ $\rho_{v@P} = 24$ C/m³.

the total charge (Q_t) within the volume $V = 10^{-14}$ m³

is $Q_t = \rho_v \times \text{volume} = 24 \times 10^{-14}$ Coulomb's

$$\text{so} \quad \boxed{Q_t = 240 \times 10^{-15} = 240 \text{ fC}}$$

$[f = \text{femto} = 1 \times 10^{-15}]$

Topic 2.5

Vector operator and concept of Divergence & Divergence in all 3 co-ordinate systems

Vector operator (∇) :-

* ∇ operator is a vector operator it can be operate on scalar aswell as vector.

∇

operate on → Scalar f^n (ϕ) → $\boxed{\nabla\phi = \text{Gradient}}$ ⇒ "vector"

operate on → vector f^n ($\bar{A}$)

- dot → $\boxed{\nabla\cdot\bar{A} = \text{Divergence}}$ ⇒ "Scaler"
- Cross product → $\boxed{\nabla\times\bar{A} = \text{Curl}}$ ⇒ vector.

* → ∇ in all three co-ordinates system.

Cartesian C.S :- $P(x, y, z)$ → dx, dy, dz.

$$\boxed{\nabla = \frac{\partial}{\partial x}\bar{a}_x + \frac{\partial}{\partial y}\bar{a}_y + \frac{\partial}{\partial z}\bar{a}_z} \quad m^{-1}$$

* → ∇ in Cylindrical C.S :- $P(\rho, \phi, z)$ → $d\rho$, $\rho\, d\phi$, dz

$$\boxed{\nabla = \frac{\partial}{\partial \rho}\bar{a}_\rho + \frac{1}{\rho}\frac{\partial}{\partial \phi}\bar{a}_\phi + \frac{\partial}{\partial z}\bar{a}_z} \quad m^{-1}$$

* → ∇ in Spherical Co-System :- $P(r, \theta, \phi)$ → dr, $r\,d\theta$, $r\sin\theta\, d\phi$

$$\boxed{\nabla = \frac{\partial}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\bar{a}_\phi} \quad m^{-1}$$

2.5a. Concept of Divergence :-

Let us Consider Electric Flux density $(\bar{D})$ in general form

$$\bar{D} = D_x \bar{a}_x + D_y \bar{a}_y + D_z \bar{a}_z \; C/m^2$$

w.k.t from Gauss's Law

$$\oint_{\langle S \rangle} \bar{D}\cdot \bar{dS} = Q = \left[\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right] \Delta x \Delta y \Delta z$$

$$\oint_{\langle S \rangle} \bar{D}\cdot \bar{dS} = Q = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) \Delta v \quad \text{(a)}$$

Charge enclosed in volume $\Delta v = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) \times$ volume Δv

from eqn (a)

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \frac{\oint_{\langle S \rangle} \bar{D}\cdot \bar{dS}}{\Delta v} = Q/\Delta v \quad C/m^3$$

when volume shrinks to zero i.e $\lim_{\Delta v \to 0}$

$$\boxed{\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \lim_{\Delta v \to 0} \frac{\oint_{\langle S \rangle} \bar{D}\cdot \bar{dS}}{\Delta v} = \rho_v} \; C/m^3$$

$\therefore$ The Divergence of $\bar{D}$ is defined as

$$\boxed{\text{Divergence of } \bar{D} = \operatorname{div} D = \lim_{\Delta v \to 0} \frac{\oint_{\langle S \rangle} \bar{D}\cdot \bar{dS}}{\Delta v}} \; C/m^3 = \rho_v \; C/m^3$$

subtopic under (2.5a)

Physical meaning of Divergence :-

(i) The Divergence of the vector Flux density $(\bar{D})$ is the outflow of flux from a small closed surface per unit volume as the volume shrinks to zero.

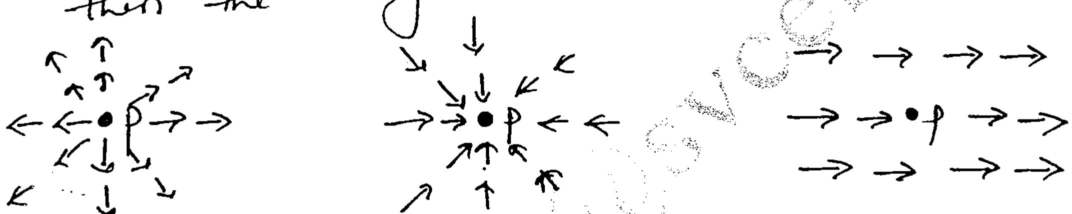
Physical meaning of a Divergence (contd) :-

the Divergence of $\overline{D}$ at a given point is a measure of how much the field represented by $\overline{D}$ diverges (or) converges from that point.

1. $\nabla\cdot\overline{D} > 0$ ie $\nabla\cdot\overline{D} = +ve$
if the field is diverging at point P of vector field $\overline{D}$ as shown in the fig a. then divergence of $\overline{D}$ at point P is positive.
the field is spreading out from point P.

2. $\nabla\cdot\overline{D} < 0$ i.e $\nabla\cdot\overline{D} = -ve$.
if the field is converging at the point P as shown in the fig b; then the divergence of $\overline{D}$ at the point P is negative.

3. $\nabla\cdot\overline{D} = 0$. whatever field is converging, same is diverging then the divergence of $\overline{D}$ at point P is zero.



$\nabla\cdot\overline{D} > 0$ +ve. Diverging. | $\nabla\cdot\overline{D} < 0$ −ve. Converging | $\nabla\cdot\overline{D} = 0$. field flow.

(ii) Divergence in all three Co-ordinate Systems :-

1. Cartesian Co-ordinate System.

$$P(x, y, z) \quad dx,\ dy,\ dz.$$

$$\overline{D} = D_x\overline{a_x} + D_y\overline{a_y} + D_z\overline{a_z} \ C/m^2$$

$$\boxed{\nabla\cdot\overline{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}} \ C/m^3$$

~~3. Maxwell's First Equation(Electrostatics)[point form of Gauss's Law]~~ / Gauss's Law in Differential form

02-DEC2008/Jan 2009

~~1 What is Divergence of a vector? Obtain point-form of Gauss law. (07 Marks)~~

02 - June /July 2011

~~2 a. With usual notations obtain differential form of Gauss's law, i.e, $\nabla \cdot \overline{D} = \rho_v$. (08 Marks)~~

10-DEC/Jan 2016

~~3 b. Derive Maxwell's first equation in electrostatics. (04 Marks)~~ soln — refer Page NO-163

2. Cylindrical Co-ordinate System.

$$P(\rho, \phi, z) \quad d\rho,\ \rho d\phi,\ dz \qquad dv = \rho\, d\rho\, d\phi\, dz$$

$$\overline{D} = D_\rho \overline{a}_\rho + D_\phi \overline{a}_\phi + D_z \overline{a}_z \ C/m^2$$

$$\boxed{\nabla \cdot \overline{D} = \frac{1}{\rho}\frac{\partial(\rho \cdot D_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}} \ C/m^3$$

3. Spherical / polar Co-ordinate System :-

$$P'(r, \theta, \phi) \quad dr,\ r d\theta,\ r\sin\theta\, d\phi \qquad dv = r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$\overline{D} = D_r \overline{a}_r + D_\theta \overline{a}_\theta + D_\phi \overline{a}_\phi \ C/m^2$$

$$\boxed{\nabla \cdot \overline{D} = \frac{1}{r^2}\frac{\partial(r^2 D_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta\, D_\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial \phi}} \ C/m^3$$

Maxwell's first equation (Electrostatics) | point form of Gauss law:-

(Q) $\nabla \cdot \overline{D} = \rho_v \; C/m^3$ [02-Jan 2009 | 02-Jan-2011, 10-Dec/Jan 2016, 15-Dec/Jan-2017 (CBCS scheme)]

w.k.t from concept of divergence

$$\text{div} D = \nabla \cdot \overline{D} = \lim_{\Delta v \to 0} \frac{\oint_{\langle S \rangle} \overline{D} \cdot d\overline{S}}{\Delta v} \; C/m^3.$$

$$\nabla \cdot \overline{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \lim_{\Delta v \to 0} \frac{\oint_{\langle S \rangle} \overline{D} \cdot d\overline{S}}{\Delta v} \quad \leftarrow (1) \; C/m^3$$

from Gauss law $\oint_{\langle S \rangle} \overline{D} \cdot d\overline{S} = Q$ Coulomb's.

per unit volume

$$\frac{\oint \overline{D} \cdot d\overline{S}}{\Delta v} = Q/\Delta v$$

as volume shrinks to zero $\lim_{\Delta v \to 0} \frac{\oint \overline{D} \cdot d\overline{S}}{\Delta v} = \lim_{\Delta v \to 0} Q/\Delta v \leftarrow (2).$

using eqn (2) in eqn (1)

$$\nabla \cdot \overline{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \rho_v = \lim_{\Delta v \to 0} Q/\Delta v \; C/m^3$$

from defn of volume chargedensity $\rho_v = dQ/dv = \lim_{\Delta v \to 0} Q/\Delta v \; C/m^3$

$$\nabla \cdot \overline{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \lim_{\Delta v \to 0} \frac{\oint_{\langle S \rangle} \overline{D} \cdot d\overline{S}}{\Delta v} = \lim_{\Delta v \to 0} Q/\Delta v = \rho_v \; C/m^3$$

$$\text{div} D = \boxed{\nabla \cdot \overline{D} = \rho_v} \; C/m^3 \quad \leftarrow \text{Maxwell's first equation (electrostatic)}$$

"it states that the Electric flux per unit volume leaving a vanishingly small volume unit is exactly equal to the volume charge density (ρ_v)."

i.e $\boxed{\nabla \cdot \overline{D} = \rho_v} \; C/m^3$

Contd (next page)

Gauss Law relates the flux leaving any closed surface to the charge enclosed and Maxwell's first equation makes an identical statement on a per unit volume basis for a vanishingly small volume.

c. Given ~~D = z Sinϕ aρ + PSinϕ az c/m²~~ compute the volume charge density at (1, 30° 2). (04 Marks)

problem 7.

$\bar{D} = z\sin\phi\,\bar{a}_\rho + \rho\sin\phi\,\bar{a}_z\ C/m^2$. (10-Dec-Jan 2016)

Soln:- $\bar{D} = z\sin\phi\,\bar{a}_\rho + \rho\sin\phi\,\bar{a}_z\ C/m^2$

$p(1, 30°, 2)$

given $\bar{D}$ is in Cylindrical Co-ordinate System

$$\therefore\ \nabla\cdot\bar{D} = \frac{1}{\rho}\frac{\partial(\rho\cdot D_\rho)}{\partial\rho} + \cancel{\frac{1}{\rho}\frac{\partial D_\phi}{\partial\phi}}^{\,0} + \frac{\partial D_z}{\partial z}\ C/m^3 = \rho_v.$$

$D_\rho = z\sin\phi$, $\quad D_\phi = 0$. $\quad D_z = \rho\sin\phi$

$$\nabla\cdot\bar{D} = \rho_v = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left[\rho\cdot z\sin\phi\right] + \frac{\partial}{\partial z}\left[\rho\sin\phi\right]$$

$$= \frac{z\sin\phi}{\rho}\cancel{\frac{\partial(\rho)}{\partial\rho}}^{\,1} + 0$$

$$\nabla\cdot\bar{D} = \rho_v = \frac{z\sin\phi}{\rho}$$

$\nabla\cdot\bar{D}$ @ $p(1, 30°, 2)$ $\quad \rho = 1,\ \phi = 30°,\ z = 2$

$$\nabla\cdot\bar{D} = \rho_{v_p} = \frac{(2)\sin(30°)}{(1)} = 2\times\tfrac{1}{2} = 1\ C/m^3$$

$\boxed{\rho_{v_p} = 1}\ C/m^3$ volume charge density at $p(1, 30°, 2)$

$\bar{D} = \frac{10\cos\theta \sin\phi}{r} \bar{a}_r \; C/m^2$

problem 8

06-DEC2010

Determine the volume charge density, if the field is $\bar{D} = \frac{10\cos\theta\sin\phi}{r}\hat{a}_r \; c/m^2$ (04 Marks)

Soln:- $\bar{D} = \frac{10\cos\theta \sin\phi}{r} \bar{a}_r \; C/m^2$

$D_r = \frac{10\cos\theta \sin\phi}{r}$; $D_\phi = 0$, $D_\theta = 0 \; C/m^2$

$\nabla \cdot \bar{D}$ in Spherical coordinate System is

$$\rho_v = \nabla\cdot\bar{D} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 D_r\right] + \cancel{\frac{1}{r\sin\theta}\frac{\partial(\sin\theta D_\theta)}{\partial\theta}} \xrightarrow{} 0 \text{ bcz } D_\theta = 0 + \cancel{\frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi}} \xrightarrow{} 0 \; D_\phi = 0$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^{\cancel{2}} \cdot \frac{10\cos\theta\sin\phi}{\cancel{r}}\right]$$

$$\rho_v = \nabla\cdot\bar{D} = \frac{1}{r^2} 10\cos\theta\sin\phi \cancel{\frac{\partial r}{\partial r}} \rightarrow 1$$

Volume charge density ρ_v

$$\boxed{\rho_v = \nabla\cdot\bar{D} = \frac{10\cos\theta\sin\phi}{r^2}} \; C/m^3$$

Problems Hayt

i) $\overline{D} = \frac{1}{z^2}[10xyz\,\overline{a_x} + 5x^2z\,\overline{a_y} + (2z^3 - 5x^2y)\overline{a_z}]$ c/m^2

$\overline{D} = 5z^2\,\overline{a_\rho} + 10\rho z\,\overline{a_z}$ at $P(3, -45°, 5)$.

Calculate the divergence of vector D at the points specified using cartesian, cylindrical and spherical coordinates:

i) $D = \frac{1}{z^2}[10xyz.a_x + 5x^2za_y + (2z^3 - 5x^2y)a_z]$ c/m^2 at point P(2, 3, 5) [10-Jan 2012]

ii) $D = 5z^2.a_\rho + 10\rho z\,.\,a_z$ at $\rho(3, -45°, 5)$ (08 Marks) [W. H. Hayt]

iii) $\overline{D} = 2r\sin\theta\sin\phi\,\overline{a_r} + r\cos\theta\sin\phi\,\overline{a_\theta} + r\cos\phi\,\overline{a_\phi}$ c/m^2 at $P(3, -45°, -45°)$

Soln: i. $\overline{D} = \frac{1}{z^2}[10xyz\,\overline{a_x} + 5x^2z\,\overline{a_y} + (2z^3 - 5x^2y)\overline{a_z}]$ c/m^2

$D_x = 10\frac{xy}{z}$; $D_y = \frac{5x^2}{z}$; $D_z = 2z - \frac{5x^2y}{z^2}$

$$\nabla\cdot\overline{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \rho_v \text{ c/m}^3$$

$\frac{\partial D_x}{\partial x} = 10y/z$, $\frac{\partial D_y}{\partial y} = 0$; $\frac{\partial D_z}{\partial z} = 2 + \frac{10x^2y}{z^3}$

$$\nabla\cdot\overline{D} = \rho_v = \frac{10y}{z} + 0 + 2 + \frac{10x^2y}{z^3}$$

ρ_v @ $P(2, 3, 5)$ $x = 2, y = 3$ and $z = 5$.

$$\nabla\cdot\overline{D} = \rho_{v_p} = \frac{10(3)}{5} + 2 + \frac{10(2)^2(3)}{(5)^3} = 6 + 2 + 0.96$$

$$\boxed{\nabla\cdot\overline{D} = \rho_{v_p} = 8.96} \text{ c/m}^3$$

ii) $\overline{D} = 5z^2\,\overline{a_\rho} + 10\rho z\,\overline{a_z}$ at $P(3, -45°, 5)$

$D_\rho = 5z^2$, $D_\phi = 0$. $D_z = 10\rho z$

$$\nabla\cdot\overline{D} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho D_\rho) + \cancel{\frac{1}{\rho}\frac{\partial D_\phi}{\partial\phi}}^{\,0} + \frac{\partial D_z}{\partial z}$$

$$\nabla\cdot\bar{D} = \rho_v = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\cdot 5z^2) + 0 + \frac{\partial}{\partial z}(10\rho z)$$

$$= \frac{5z^2}{\rho} + 10\rho \quad ; \quad P(3, -45°, 5)$$

$$\nabla\cdot\bar{D} = \rho_{v_P} = \frac{5(5)^2}{3} + 10(3) = 71.8666 \text{ C/m}^3$$

$$\boxed{\nabla\cdot\bar{D} = \rho_{v_P} = 71.67} \text{ C/m}^3$$

iii) $\bar{D} = 2r\sin\theta\sin\phi\,\bar{a}_r + r\cos\theta\sin\phi\,\bar{a}_\theta + r\cos\phi\,\bar{a}_\phi$ C/m².

at $P(3, -45°, -45°)$

$D_r = 2r\sin\theta\sin\phi$; $D_\theta = r\cos\theta\sin\phi$; $D_\phi = r\cos\phi$.

$$\nabla\cdot\bar{D} = \rho_v = \frac{1}{r^2}\frac{\partial(r^2 D_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta D_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi} \text{ C/m}^3$$

$$\frac{\partial(r^2 D_r)}{\partial r} = \frac{\partial}{\partial r}\left[r^2\cdot 2r\sin\theta\sin\phi\right] = 6r^2\sin\theta\sin\phi.$$

$$\frac{\partial(\sin\theta D_\theta)}{\partial\theta} = \frac{\partial}{\partial\theta}\left[\sin\theta\, r\sin\phi\cos\theta\right] = \frac{1}{2}r\sin\phi\frac{\partial}{\partial\theta}\left[\sin(2\theta)\right]$$

$$= \frac{r\sin\phi}{\cancel{2}}\cdot\cos(2\theta)\cdot\cancel{2} = r\sin\phi\cos(2\theta)$$

$$\frac{\partial D_\phi}{\partial\phi} = \frac{\partial}{\partial\phi}\left[r\cos\phi\right] = -r\sin\phi.$$

$$\nabla\cdot\bar{D} = \rho_v = \frac{1}{\cancel{r^2}}6\cancel{r^2}\sin\theta\sin\phi + \frac{1}{\cancel{r}\sin\theta}\left[\cancel{r}\sin\phi\cos(2\theta)\right] + \frac{1}{\cancel{r}\sin\theta}\left[-\cancel{r}\sin\phi\right]$$

$$= 6\sin\theta\sin\phi + \frac{\cos(2\theta)}{\sin\theta}\sin\phi - \frac{\sin\phi}{\sin\theta}$$

$$\nabla\cdot\bar{D} = \rho_{v_{P(3,-45°,-45°)}} = 6\sin(-45°)\sin(-45°) + \frac{\overset{0}{\cos(90)}\cancel{\sin(-45°)}}{\cancel{\sin(-45°)}} - \frac{\overset{1}{\cancel{\sin(-45°)}}}{\cancel{\sin(-45°)}}$$

$$= 6(1/\sqrt{2})(1/\sqrt{2}) + 0 - 1 = 6(1/2) - 1 = 3 - 1 = 2 \text{ C/m}^3$$

$$= 2 \text{ C/m}^3$$

$$\boxed{\nabla\cdot\overline{D} = \rho_{v_p} = 2}\ C/m^3$$

$\overline{D} = 5\sin\theta\,\overline{a}_\theta + 5\sin\phi\,\overline{a}_\phi\ C/m^2$

problem 10

10 - June /July 2012

17 Given $\overline{D} = 5\sin\theta\,\overline{a}_\theta + 5\sin\phi\,\overline{a}_\phi$ find the charge density at (0.5m, π/4, π/4). (04 Marks)

Sol'n:- $\overline{D} = 5\sin\theta\,\overline{a}_\theta + 5\sin\phi\,\overline{a}_\phi\ C/m^2$ (0.5m, π/4, π/4).

$$\nabla\cdot\overline{D} = \cancel{\frac{1}{r^2}\frac{\partial(r^2 D_r)}{\partial r}}^{\,0} + \frac{1}{r\sin\theta}\frac{\partial(D_\theta\sin\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi}\ C/m^3$$

$$D_r = 0\ ;\quad D_\theta = 5\sin\theta\ ;\quad D_\phi = 5\sin\phi$$

$$\nabla\cdot\overline{D} = \rho_v = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left[5\sin\theta\sin\theta\right] + \frac{1}{r\sin\theta}\frac{\partial(5\sin\phi)}{\partial\phi}$$

$$= \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(5\sin^2\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(5\sin\phi)$$

$$= \frac{5}{r\sin\theta}\times 2\sin\theta\cos\theta + \frac{1}{r\sin\theta}5\cos\phi$$

$\nabla\cdot\overline{D}$ @ P(0.5m, π/4, π/4)

$$\theta = \phi = \pi/4.$$

$$\nabla\cdot\overline{D} = \rho_{v_p} = \frac{5}{(0.5)\cancel{\sin\pi/4}}\times 2\cancel{\sin\pi/4}\cos\pi/4 + \frac{5}{(0.5)}\frac{\cos(\pi/4)}{\sin(\pi/4)}$$

$$= 20\,(1/\sqrt{2}) + 10\,\frac{\cancel{(1/\sqrt{2})}}{\cancel{(1/\sqrt{2})}} = 24.142\ C/m^3$$

$$\boxed{\nabla\cdot\overline{D} = \rho_{v_p} = 24.142}\ C/m^3$$

~~Let $\bar{D} = 5r^2 a_r$ mc/m² for $r \leq 0.08$m and $\bar{D} = \frac{0.205}{r^2} a_r$ μc/m² for $r \geq 0.08$m. Find ρ_v for i) $r = 0.06$m; ii) $r = 0.1$m (08 Marks)~~ [06-J|J 2009]

problem 11

Let $\bar{D} = 5r^2 a_r$ mc / mt² for $r < 0.08$mt and $\bar{D} = \frac{0.1}{r^2} a_r$ mc/mt² for $r > 0.08$ mt. Find ρ_v for i) $r = 0.06$ mt ii) $r = 0.1$ mt. (08 Marks)

[06-Jan 2013]

Soln:
$$\bar{D} = 5r^2 \bar{a_r} \ mc/m^2 \ ; \ r \leq 0.08m$$

$$\bar{D} = \frac{0.205}{r^2} \bar{a_r} \ \mu c/m^2 \ \text{for} \ r \geq 0.08m$$

$$\rho_v = \nabla\cdot\bar{D} = \frac{1}{r^2}\frac{\partial}{\partial r}[r^2 D_r] + \frac{1}{r\sin\theta}\cancel{\frac{\partial}{\partial\theta}[D_\theta \sin\theta]} \rightarrow 0 \text{ bcz } D_\theta = 0$$

$$+ \frac{1}{r\sin\theta}\cancel{\frac{\partial D_\phi}{\partial\phi}} \rightarrow 0 \text{ bcz } D_\phi = 0.$$

$$\boxed{\rho_v = \nabla\cdot\bar{D} \ \frac{1}{r^2}\frac{\partial}{\partial r}[r^2 D_r]} \ c/m^3$$

Casei.
$$D_r = 5r^2 \ mc/m^2 \ \text{for} \ r \leq 0.08$$

$$\rho_v = \nabla\cdot\bar{D} = \frac{1}{r^2}\frac{\partial}{\partial r}[r^2 . 5r^2] \quad mc/m^3$$

$$= \frac{5}{r^2}\frac{\partial}{\partial r}[r^4] = \frac{5}{r^2}.4r^3 \ mc/m^3$$

$$\rho_v = \nabla\cdot\bar{D} = 20r \ mc/m^3 \ ; \ r \leq 0.08m$$

$$\boxed{\rho_{v\,@r=0.08} = 20(0.06) = 1.2 \ mc/m^3} \ ; \ r = 0.06m$$

Caseii.
$$D_r = \frac{0.205}{r^2} \ mc/m^2 \ \text{for} \ r \geq 0.08$$

$$\rho_v = \nabla \cdot \bar{D} = \frac{1}{r^2} \frac{\partial}{\partial r}\left[\cancel{r^2} \cdot \frac{0.205}{\cancel{r^2}}\right] \quad mC/m^3$$

$$= \frac{1}{r^2} \cancel{\frac{\partial}{\partial r}[0.205]}^{\,0} \quad mC/m^3$$

$$\rho_v = \nabla \cdot \bar{D} = 0 \quad mC/m^3 \quad ; r > 0.08m$$

ρ_v @ $r = 0.1m$ is

$$\boxed{\rho_v\Big|_{r=0.1} = \nabla \cdot \bar{D} = 0} \; mC/m^3$$

Q27) Sol^n:-

$\nabla \cdot \bar{D} = \rho_v = 20r \; ; r \leq 0.08m$

$$\boxed{\rho_{v\,@\,r=0.06m} = 1.2 \, mC/m^3}$$

$\nabla \cdot \bar{D} = \rho_v = 0 \; ; r > 0.08m.$

and

$$\boxed{\rho_{v\,@\,r=0.1m} = \nabla \cdot \bar{D} = 0} \; mC/m^3$$

problem 12 $\rightarrow \overline{D} = r\overline{a_r} + \sin\theta\,\overline{a_\theta} + \sin\theta\cos\phi\,\overline{a_\phi}$ C/m^2

$\leftarrow (4m, 45°, 60°)$ 10 - June /July 2015

Find the volume charge density at (4m, 45°, 60°). If the electric flux density is given by,
$\vec{D} = (r\hat{a}_r + \sin\theta\hat{a}_\theta + \sin\theta\cos\phi\hat{a}_\phi)$ C/m^2. (06 Marks)

Sol^n:- $\overline{D} = r\,\overline{a_r} + \sin\theta\,\overline{a_\theta} + \sin\theta\cos\phi\,\overline{a_\phi}$ C/m^2.

$P(4, 45°, 60°)$

$D_r = r$ C/m^2 ; $D_\theta = \sin\theta$ C/m^2 ; $D_\phi = \sin\theta\cos\phi$ C/m^2.

$$\rho_v = \nabla\cdot\overline{D} = \frac{1}{r^2}\frac{\partial}{\partial r}[r^2 \cdot D_r] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}[\sin\theta\cdot D_\theta] + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi}$$

$$\rho_v = \nabla\cdot\overline{D} = \frac{1}{r^2}\frac{\partial}{\partial r}[\underbrace{r^2\cdot r}_{r^3}] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}[\underbrace{\sin\theta\cdot\sin\theta}_{\sin^2\theta}] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}[\sin\theta\cos\phi]$$

$$\rho_v = \nabla\cdot\overline{D} = \frac{1}{\cancel{r^2}}\cdot 3\cancel{r^2} + \frac{1}{r\cancel{\sin\theta}}2\cancel{\sin\theta}\cos\theta + \frac{1}{r\cancel{\sin\theta}}\cancel{\sin\theta}(-\sin\phi)$$

$$\rho_v = \nabla\cdot\overline{D} = 3 + \frac{2\cos\theta}{r} + \frac{(-\sin\phi)}{r}$$

$r = 4m$; $\theta = 45°$, $\phi = 60°$

$$= 3 + \frac{2\cos(45°)}{4} - \frac{\sin(60°)}{4}$$

$$= 3 + 0.3535 - 0.2165 = 3.1369 \text{ C/m}^3$$

$$\boxed{\rho_{v_P} = \nabla\cdot\overline{D} = 3.1369} \text{ C/m}^3$$

Topic 2.7 Divergence theorem

Questions

4 Divergence Theorem [06 Dec/Jan 2010, 06 - Jan 2014, 02 - June/July - 2012, 06 - J/J 2009, 10 - J/J 2014, 06 - J/J 2013, 06 Jan 2013]

06-DEC2009/Jan 2010

State and prove divergence theorem. —(05 Marks)

(or)

06-DEC 2013/Jan 2014

~~State and prove divergence theorem.~~ ~~(06 Marks)~~

State and prove Gauss's Divergence theorem. (06 Marks)

(or)

02 - June /July 2012

State and prove Gauss divergence theorem. hence arrive at Poisson's equation. (08 Marks)

06- June /July 2009

20 ~~State and prove Divergence theorem~~ ~~(Marks)~~

~~10 June/July 2014~~

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State and ~~prove divergence theorem.~~ ~~(05 Marks)~~

~~06 June/July 2013~~

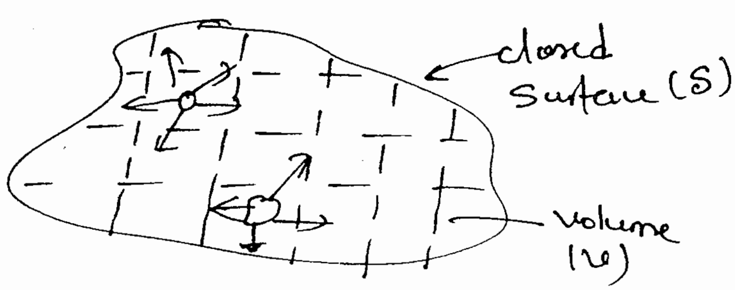
~~State and prove Gauss's divergence theorem.~~ ~~(07 Marks)~~

~~06 Jan 2013~~

State the ~~prove divergence theorem.~~ ~~(06 Marks)~~

Statement:- "The Divergence theorem states that the total flux crossing the closed surface is equal to the volume integral of the divergence of the flux density throughout the enclosed volume."

proof:-



from Gauss's Law

$$\psi_{total} = \oint_{\langle S \rangle} \overline{D} \cdot \overline{dS} = Q_{enc} \quad \text{Coulomb's} \leftarrow (1)$$

the volume charge density $\rho_v = \frac{dQ}{dv}$ C/m³

$$\Rightarrow dQ = \rho_v \, dv$$

$$Q = \int_{\langle vol \rangle} \rho_v \, dv \quad C \leftarrow (2)$$

equating eqⁿ (1) and eqⁿ (2)

$$\oint_{\langle S \rangle} \overline{D} \cdot \overline{dS} = Q = \int_{\langle vol \rangle} \rho_v \, dv \quad \text{Coulomb's}$$

using Maxwell's first equation (electrostatic)

i.e $\rho_v = \nabla \cdot \overline{D}$ C/m³

$$\oint_{\langle S \rangle} \overline{D} \cdot \overline{dS} = Q = \int_{\langle vol \rangle} \rho_v \, dv = \int_{\langle vol \rangle} (\nabla \cdot \overline{D}) \, dv \quad \text{Coulomb's.}$$

$$\Rightarrow \boxed{\oint_{\langle S \rangle} \overline{D} \cdot \overline{dS} = \int_{\langle vol \rangle} (\nabla \cdot \overline{D}) \, dv} \Leftarrow \text{Gauss's Divergence theorem. Coulomb's}$$

Note;-

This is relation is true for any general vector $\overline{A}$. (31)

i.e $\oint_{\langle S \rangle} \overline{A} \cdot \overline{dS} = \int_{\langle vol \rangle} (\nabla \cdot \overline{A}) \, dv.$

Cntd

poisson's eqn:

from Maxwell's first eqn

$$\nabla \cdot \bar{D} = \rho_v \ \ C/m^3.$$

$$\nabla \cdot (\epsilon \bar{E}) = \rho_v.$$

$$\epsilon \nabla \cdot \bar{E} = \rho_v$$

$$\nabla \cdot \bar{E} = \rho_v / \epsilon \ \ V/m^2$$

using potential gradient

ie $\bar{E} = -\nabla V \ \ V/m.$

$$\nabla \cdot (-\nabla V) = \rho_v / \epsilon$$

$$\boxed{\nabla^2 V = -\rho_v / \epsilon} \ \ V/m^2 \leftarrow (a)$$

eqn (a) called poisson's eqn ~~from~~ derived from point form of Gauss Law.

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Hayt problem 13 $\bar{D} = 2xy\bar{a}_x + x^2\,\bar{a}_y\ c/m^2$ 10- Dec/Jan 2016.

XV.

4. Verify both sides of Gauss Divergence theorem if $\bar{D} = 2xy\,\bar{a}_x + x^2\,\bar{a}_y$ c/m² present in the region bounded by $0 \le x \le 1$, $0 \le y \le 2$, $0 \le z \le 3$ (08 Marks)

$0 \le x \le 1,\ 0 \le y \le 2,\ 0 \le z \le 3$ 15 - Dec/Jan 2017 (8M)

Gauss's Divergence theorem

$$\oint_{\langle S \rangle} \bar{D}\cdot \overline{ds} = \int_{\langle vol \rangle} (\nabla\cdot\bar{D})\, dv \leftarrow (a) \quad \text{(CBCS Scheme)}.$$

R.H.S. $\displaystyle\int_{\langle vol \rangle} \nabla\cdot\bar{D}\, dv = ?$

$$\bar{D} = 2xy\,\bar{a}_x + x^2\,\bar{a}_y\ c/m^2$$

$$D_x = 2xy\ ; \quad D_y = x^2\ ; \quad D_z = 0.$$

$$\nabla\cdot\bar{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \cancel{\frac{\partial D_z}{\partial z}}^{\,0}$$

$$= \frac{\partial}{\partial x}(2xy) + \frac{\partial}{\partial y}(x^2) = 2y + 0 = 2y\ c/m^3.$$

$$\boxed{\nabla\cdot\bar{D} = 2y}\ c/m^3$$

$$\int_{\langle vol \rangle} (\nabla\cdot\bar{D})\, dv = \int_{\langle vol \rangle} 2y\, dx\, dy\, dz$$

$$= 2\int_{x=0}^{1} dx \int_{y=0}^{2} y\, dy \int_{z=0}^{3} dz = 2\times 1\times 2\times 3 = 12\,C$$

$$\therefore \int_{\langle vol \rangle} (\nabla\cdot\bar{D})\, dv = 12 \text{ Coulombs} \leftarrow (1).$$

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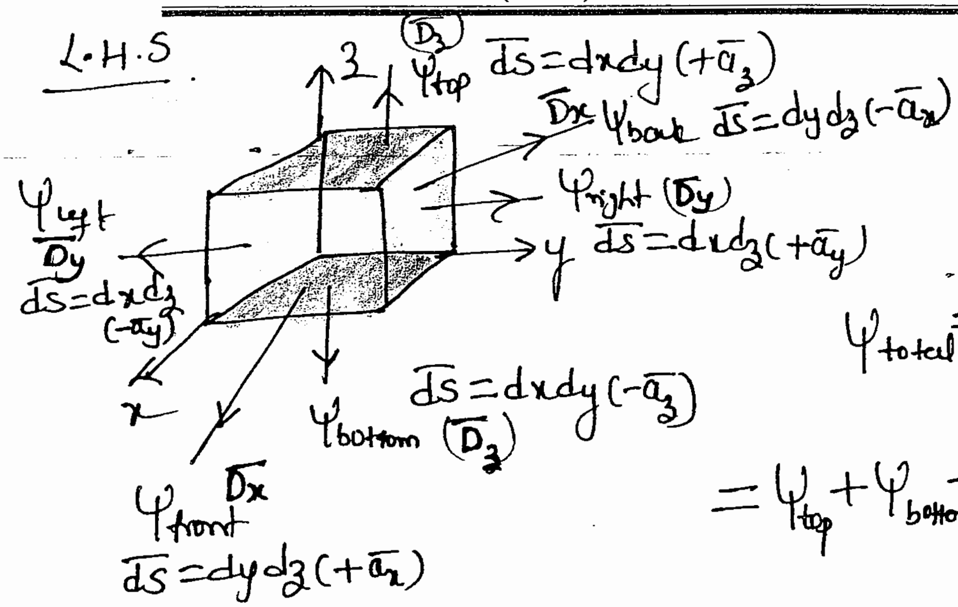
L.H.S



$0 \le x \le 1$
$0 \le y \le 2$
$0 \le z \le 3$

$$\Psi_{total} = \oint_{\langle S \rangle} \bar{D} \cdot \bar{ds} \quad \text{Coulombs}$$

$$= \Psi_{top} + \Psi_{bottom} + \Psi_{Left} + \Psi_{right} + \Psi_{front} + \Psi_{back}.$$

$\boxed{\Psi_{top} = 0\ C}$; bcz $D_z = 0\ C/m^2$ and $\boxed{\Psi_{bottom} = 0}$; bcz $D_z = 0$.

$$\Psi_{Left}\Big|_{y=0} = \int_{\langle S \rangle} \bar{D}_y \cdot \bar{ds} = \int_{\langle S \rangle} x^2 \bar{a}_y \cdot dx\,dz\,(-\bar{a}_y) = -\int_{x=0}^{1} x^2 dx \int_{z=0}^{3} dz$$

$$= -\tfrac{1}{\cancel{3}} \cdot \cancel{3} = -1\ C \;;\; \boxed{\Psi_{Left} = -1}\ C$$

$$\Psi_{right}\Big|_{y=2} = \int_{\langle S \rangle} x^2 \bar{a}_y \cdot dx\,dz\,(+\bar{a}_y) = \int_{x=0}^{1} \cancel{x^2}^{\,1/3} dx \cdot \int_{z=0}^{3} \cancel{dz}^{\,3} = +1\ C. \quad \boxed{\Psi_{right} = +1}\ C$$

$$\Psi_{front}\Big|_{x=1} = \int_{\langle S \rangle} \bar{D}_x \cdot \bar{ds} = \int_{\langle S \rangle} 2xy\,\bar{a}_x \cdot dy\,dz\,(+\bar{a}_x)$$

$$= 2\cancel{x}^{\,1} \int_{y=0}^{2} y\,dy \int_{z=0}^{3} dz = 2 \times 2 \times 3 = 12\ C. \quad \boxed{\Psi_{front} = 12}\ C$$

$$\Psi_{back}\Big|_{x=0} = \int_{\langle S \rangle} \bar{D}_x \cdot \bar{ds} = \int_{\langle S \rangle} 2xy\,\bar{a}_x \cdot dy\,dz\,(-\bar{a}_x)$$

$$= -2\cancel{x}^{\,0} \int_{y=0}^{2} y\,dy \int_{z=0}^{3} dz = -2(0) \times 2 \times 3 = 0\ C. \quad \boxed{\Psi_{back} = 0}\ C$$

$$\therefore \Psi_{total} = 0 + 0 + (\cancel{-1}) + \cancel{1} + 12 + 0 = 12\ C$$

L.H.S $\boxed{\Psi_{total} = \oint_{\langle S \rangle} \bar{D} \cdot \bar{ds} = 12\ C}$ ← (2)

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eq (1) = eq (2) ∴ Divergence theorem is Verified.

Problem 14 $\bar{D} = 2xyz\,\bar{a}_x + 3y^2z\,\bar{a}_y + x\,\bar{a}_z$ C/m^2

06-DEC2008/Jan 2009

Evaluate both sides of gauss – divergence theorem for the field $\vec{D}\ 2xyz\vec{a}_x + 3y^2z\vec{a}_y + x\vec{a}_z (c/m^2)$. the region is defined by $-1 \le x, y, z \le 1$(m). (07 Marks)

Soln:- Divergence theorem $-1 \le x, y, z \le 1$m.

$$\oint_{S} \bar{D}\cdot \bar{dS} = \int_{vol} \nabla\cdot\bar{D}\, dv$$

RHS. $\bar{D} = 2xyz\,\bar{a}_x + 3y^2z\,\bar{a}_y + x\,\bar{a}_z$ C/m^2

$$\nabla\cdot\bar{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$= 2yz + 6yz + 0$$

$$\boxed{\nabla\cdot\bar{D} = 8yz}\ \text{C/m}^3$$

$$\int_{vol} \nabla\cdot\bar{D}\, dv = \int_{vol} 8yz\, dx\, dy\, dz$$

$$= 8\int_{-1}^{+1} dx \int_{-1}^{+1} y\, dy \int_{-1}^{+1} z\, dz$$

$$= 8(2)(0)(0) = 0 \text{ Coulomb's}$$

Note:- $\int_{-a}^{a} f(x)\, dx = 0$ if $f(x)$ is an odd fn i.e $f(-x) = -f(x)$

$$\boxed{\int_{vol} (\nabla\cdot\bar{D})\, dv = 0 \text{ Coulomb's}} \longleftarrow (a)$$

LHS Gauss Law $\oint_{S} \bar{D}\cdot\bar{dS} = \Psi_{total}$

$$\Psi_{total} = \Psi_{top} + \Psi_{bottom} + \Psi_{Left} + \Psi_{right} + \Psi_{front} + \Psi_{back} \text{ Coulomb's}$$

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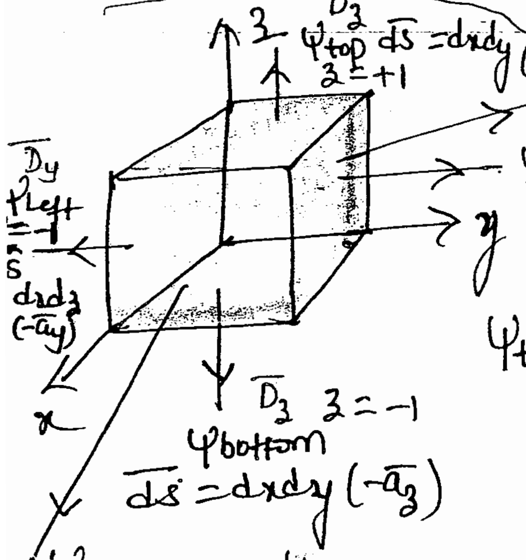
$$\bar{D} = 2xyz\,\bar{a}_x + 3y^2z\,\bar{a}_y + x\,\bar{a}_z \ \text{C/m}^2.$$

$-1 \le x, y, z \le +1$. L.H.S



ψ_{front} $x=+1$ plane. $\bar{ds} = dydz(+\bar{a}_x)$ D_x

$$\psi_{total} = \oint_{\langle S\rangle} \bar{D}\cdot\bar{ds} \ \text{Coulombs}$$

$$\psi_{top} = \int_{\langle S\rangle} \bar{D}_z\cdot\bar{ds} = \int_{\langle S\rangle} x\bar{a}_z \cdot dxdy\,\bar{a}_z = \int_{x=-1}^{+1} x\,dx \int_{y=-1}^{+1} dy \ \cancel{\bar{a}_z\cdot\bar{a}_z}^{\,1}$$

$$= (1)(0) = 0 \ \text{Coulombs}$$

$\boxed{\psi_{top} = 0}$ C

$$\psi_{bottom} = \int_{\langle S\rangle} \bar{D}_z\cdot\bar{ds} = \int_{\langle S\rangle} x\bar{a}_z\cdot dxdy(-\bar{a}_z) = -\int_{x=-1}^{+1} x\,dx \int_{y=-1}^{+1} dy \ \cancel{\bar{a}_z\cdot\bar{a}_z}^{\,1}$$

$$= -(1)(0) = 0 \ \text{Coulombs}$$

$\boxed{\psi_{bottom} = 0}$ C

$$\psi_{Left}\Big|_{y=-1\text{ plane}} = \int_{\langle S\rangle} \bar{D}_y\cdot\bar{ds} = \int_{\langle S\rangle} 3y^2z\,\bar{a}_y\cdot dxdz(-\bar{a}_y) = -3y^2\int_{x=-1}^{+1} dx \int_{z=-1}^{+1} z\,dz \ \cancel{\bar{a}_y\cdot\bar{a}_y}^{\,1}$$

$$= -3(-1)^2 \times 2\times 0\times 1 = 0 \ \text{Coulombs}$$

$\boxed{\psi_{Left} = 0}$ C

$$\psi_{right}\Big|_{y=+1\text{ plane}} = \int_{\langle S\rangle} \bar{D}_y\cdot\bar{ds} = \int_{\langle S\rangle} 3y^2z\,\bar{a}_y\cdot dxdz(+\bar{a}_y) = 3y^2\int_{x=-1}^{+1} dx \int_{z=-1}^{+1} z\,dz \ \cancel{\bar{a}_y\cdot\bar{a}_y}^{\,1}$$

$$= +3(1)^2 \times 2\times 0\times 1 = 0 \ \text{Coulombs.}$$

$\boxed{\psi_{right} = 0}$ C

$$\psi_{front}\Big|_{x=+1\text{ plane}} = \int_{\langle S\rangle} \bar{D}_x\cdot\bar{ds} = \int_{\langle S\rangle} 2xyz\,\bar{a}_x\cdot dydz\,\bar{a}_x = 2x\int_{y=-1}^{+1} y\,dy \int_{z=-1}^{+1} z\,dz \ \cancel{\bar{a}_x\cdot\bar{a}_x}^{\,1}$$

$$= 2(1)\times 0\times 0\times 1 = 0\ \text{C}, \quad \boxed{\psi_{front} = 0}\ \text{C}$$

$$\psi_{back}\Big|_{x=-1\text{ plane}} = \int_{\langle S\rangle} \bar{D}_x\cdot\bar{ds} = \int_{\langle S\rangle} 2xyz\,\bar{a}_x\cdot dydz(-\bar{a}_x) = -2x\int_{y=-1}^{+1} y\,dy \int_{z=-1}^{+1} z\,dz \ \cancel{\bar{a}_x\cdot\bar{a}_x}^{\,1}$$

$$= -2(-1)\times 0\times 0\times 1 = 0\ \text{C}$$

$\boxed{\psi_{back} = 0}$ C

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contd. next page.

$\therefore \boxed{\Psi_{total} = 0\,C} = \oint_{\langle S \rangle} \bar{D} \cdot \bar{dS} \leftarrow (b)$

$eq^n (a) = eq^n (b)$ ie $LHS = RHS$ $\therefore$ Divergence theorem is verified.

10-DEC2011/Jan 2012

problem 15

Given $\vec{D} = 30e^{-r}\hat{a}_r - 2z\hat{a}_z$ c/mt². Verify divergence theorem for the volume enclosed by $r = 2, z = 5$. (08 Marks)

$r=2, z=5m$. (or) $\bar{D} = 30e^{-r}\bar{a}_r - 2z\bar{a}_z$ c/m²

Given that $\bar{A} = 30\,e^{-r}\,\hat{a}_r - 2z\,\hat{a}_z$. Evaluate both sides of the divergence theorem for the volume enclosed by $r = 2$, $z = 0$ and $z = 5$. (08 Marks)

(or) $A(r,\phi,z) = 30e^{-r}\bar{a}_r - 2z\bar{a}_z$ c/m²

06 –Dec/Jan 2008

A vector field is given by. $A(r,\phi,z) = 30e^{-r}a_r - 2za_z$. Verify the divergence theorem for the volume enclosed by. $r = 2, z = 5$. $r=2, z=5$ m. (08 Marks)

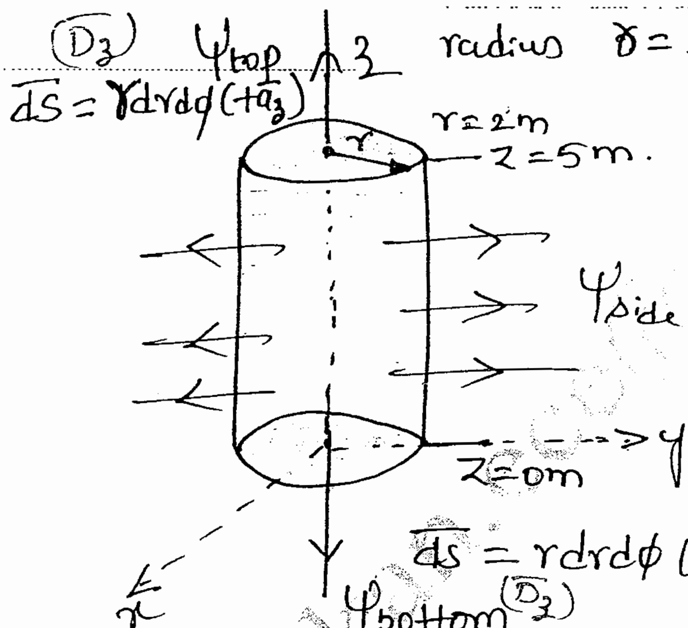
15-June/July 2017 [10M - CBCS-scheme]

Soln: Given $\bar{D} = 30e^{-r}\bar{a}_r - 2z\bar{a}_z$ c/m²

radius $r = 2$m and height $z = 5$m.

(D_z) Ψ_{top}

$\bar{dS} = r\,dr\,d\phi(+\bar{a}_z)$



$P(r, \phi, z)$

$dr \quad r\,d\phi \quad dz.$

Ψ_{side} $\bar{dS} = r\,d\phi\,dz(+\bar{a}_r)$

($r = k$ surface).

(D_r)

$\bar{dS} = r\,dr\,d\phi\,(-\bar{a}_z)$

Ψ_{bottom} (D_z)

@ $z = 0$m.

$\bar{dS} = ds\,\bar{a}_n$

Divergence theorem

$$\boxed{\oint_{\langle S \rangle} \bar{D} \cdot \bar{ds} = \int_{\langle vol \rangle} \nabla \cdot \bar{D}\, dv}$$ Coulombs

L.H.S

$$\Psi_{total} = \oint_{\langle S \rangle} \bar{D} \cdot \bar{ds} = \Psi_{top}\Big|_{z=5m} + \Psi_{bottom}\Big|_{z=0m} + \Psi_{side}\Big|_{r=2m}$$

(37)

$\bar{D} = 30e^{-r}\bar{a}_r - 2z\,\bar{a}_z \; C/m^2$. $0 < r \leq 2m$. $0 < z \leq 5m$

$$\Psi_{top}\Big|_{z=+5} = \int_{\langle S\rangle} \bar{D}_z \cdot \bar{dS} = \int_{\langle S\rangle} -2z\,\bar{a}_z \cdot r\,dr\,d\phi\,(+\bar{a}_z)$$

$$= -2z \int_{r=0}^{2} r\,dr \int_{0}^{2\pi} d\phi \;\cancel{\bar{a}_z \cdot \bar{a}_z}^{\,1}\Big|_{z=5m \text{ surface}}$$

$$= -2(5)(2)(2\pi)(1) = -40\pi \text{ Coulomb's}$$

$$\boxed{\Psi_{top} = -40\pi} \; C$$

(or) Fluxdensity $D = \frac{\Psi}{Area} \; C/m^2$.

$$\Rightarrow \boxed{\Psi = D \cdot A} \; C$$

$$\Psi_{top} = D_z \cdot A = -2z \times \pi r^2 = -2(5)\pi(2)^2 = \underline{-40\pi \; C}$$

A - area of top circle.

$$\Psi_{bottom}\Big|_{z=0m} = \int_{\langle S\rangle} \bar{D}_z \cdot \bar{dS} = \int_{\langle S\rangle} -2z\,\bar{a}_z \cdot r\,dr\,d\phi\,(-\bar{a}_z)$$

$$= +2z \int_{r=0}^{2} r\,dr \int_{0}^{2\pi} d\phi \;\cancel{\bar{a}_z \cdot \bar{a}_z}^{\,1}\Big|_{z=0m \text{ surface}}$$

$$= 2(0)(2)(2\pi)(1) = 0 \text{ Coulomb's}$$

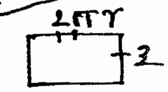
$$\boxed{\Psi_{bottom} = 0} \; C$$

(or) $\Psi_{bottom} = D_z \cdot A = -2z \times \pi r^2 = -2(0) \times \pi (2)^2 = \underline{0C}$

$z = 0$ on bottom circle.

(38)

$$\Psi_{side}\Big|_{r=2m} = \int_{\langle S\rangle} \overline{D_r}\cdot\overline{dS} = \int_{\langle S\rangle} 30e^{-r}\bar{a}_r \cdot r\,d\phi\,dz\,\bar{a}_r = 30e^{-r} r \int_{\phi=0}^{2\pi} d\phi \int_{z=0}^{5} dz\ \bar{a}_r\cdot\bar{a}_r$$



$$= 30e^{-r}(2)\times 2\pi\times 5 = 600e^{-2}\pi \text{ Coulomb's;}$$

$$\boxed{\Psi_{side} = 600e^{-2}\pi} \text{ C}$$

(or)

$$\Psi_{side} = D_r\cdot A\Big|_{r=2m}$$
$$= 30e^{-r}\,2\pi r z$$
$$= 30e^{-2}\,2\pi(2)\times 5$$
$$= 600e^{-2}\pi \text{ C}$$

$$\Psi_{total} = \oint_{\langle S\rangle} \overline{D}\cdot\overline{dS} = \Psi_{top} + \Psi_{bottom} + \Psi_{side}$$

$$= -40\pi + 0 + 600e^{-2}\pi = 41.201\pi \text{ C}$$

$$\boxed{\Psi_{total} = 129.437} \text{ Coulomb's} \leftarrow (a)$$

R.H.S

$$\int_{\langle v\rangle} \nabla\cdot\overline{D}\,dv = ? \qquad \overline{D} = 30e^{-r}\bar{a}_r - 2z\bar{a}_z \text{ C/m}^2$$

$D_r = 30e^{-r}$, $D_z = -2z$

divergence in cylindrical coordinate system.

$$\nabla\cdot\overline{D} = \frac{1}{r}\frac{\partial(r\cdot D_r)}{\partial r} + \cancel{\frac{1}{r}\frac{\partial D_\phi}{\partial\phi}} + \frac{\partial D_z}{\partial z}$$

be 0, $D_\phi = 0$ $\quad D_\phi = 0$

$$= \frac{1}{r}\frac{\partial}{\partial r}(r\cdot 30e^{-r}) + \frac{\partial}{\partial z}(-2z) = \frac{30}{r}\frac{\partial}{\partial r}(re^{-r}) - 2(1).$$

$$= \frac{30}{r}\left[-re^{-r} + e^{-r}\right] - 2 = \left[-30e^{-r} + \frac{30e^{-r}}{r} - 2\right]$$

$$\boxed{\nabla\cdot\overline{D} = 30e^{-r}\left[\frac{1}{r} - 1\right] - 2} \text{ C/m}^3 \qquad dv = r\,dr\,d\phi\,dz$$

$$\int_{\langle vol\rangle} \nabla\cdot\overline{D}\,dv = \int_{\langle vol\rangle} \frac{30e^{-r}}{\cancel{r}}\cancel{r}\,dr\,d\phi\,dz - \int_{\langle vol\rangle} 30e^{-r} r\,dr\,d\phi\,dz - 2\int_{\langle vol\rangle} r\,dr\,d\phi\,dz$$

$$= \left[\int_{r=0}^{2} 30e^{-r}dr - 30\int_{r=0}^{2} re^{-r}dr - 2\int_{r=0}^{2} r\,dr\right]\int_{\phi=0}^{2\pi} d\phi \int_{z=0}^{5} dz$$

$$= \left[(25.94) - 17.82 - 4\right](2\pi)(5) = 41.2\pi \text{ Coulomb's}$$

$$\boxed{\int_{\langle vol\rangle} (\nabla\cdot\overline{D})\,dv = 41.2\pi \text{ C} = 129.437} \text{ Coulomb's} \leftarrow (b)$$

eq^n (a) = eq^n (b) $\therefore$ Divergence theorem is Verified.

(39)

problem 16 $\bar{D} = 5\bar{a}_r \, C/m^2$

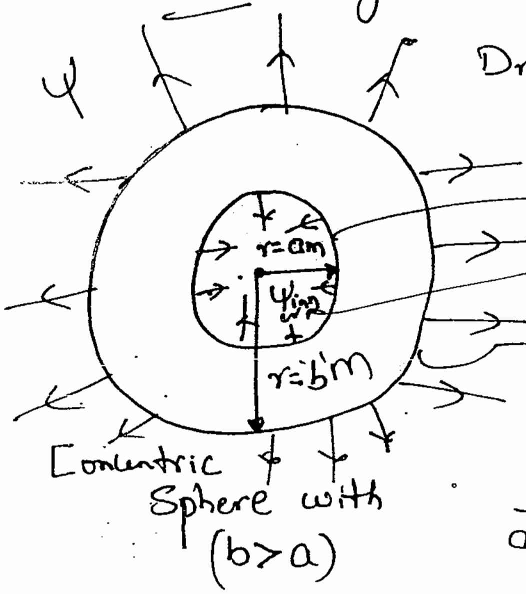
10 Jan 2013

Given $\bar{D} = 5\bar{a}_r$ c/m², prove divergence theorem for a shell region enclosed by spherical surfaces at r = a and r = b (b > a) and centred at the origin. (08 Marks)

$r = a, \quad r = b \; (b > a)$

soln: given $\bar{D} = 5\,\bar{a}_r \, C/m^2$.

$D_r = 5 \, C/m^2$.



$P(r, \theta, \phi)$ → dr, $r\,d\theta$, $r\sin\theta\,d\phi$

inner sphere

Ψ_{inner}

$\Psi_{inner}\Big|_{r=a\,m}$ constant sphere

$\bar{ds} = r^2 \sin\theta\, d\theta\, d\phi\, (-\bar{a}_r)$

$\Psi_{out}\Big|_{r=b\,m}$ constant sphere

$\bar{ds} = r^2 \sin\theta\, d\theta\, d\phi\, (+\bar{a}_r)$

Gauss divergence theorem

$$\boxed{\oint_{\langle S \rangle} \bar{D} \cdot \bar{ds} = \int_{\langle vol \rangle} (\nabla \cdot \bar{D})\, dv} \longleftarrow (1)$$

L.H.S

$$\Psi_{total} = \oint_{\langle S \rangle} \bar{D} \cdot \bar{ds} = \Psi_{outer\ sphere}\Big|_{r=b\,m} + \Psi_{inner\ sphere}\Big|_{r=a\,m}.$$

$$\Psi_{outer\ sphere}\Big|_{r=b\,m} = \int_{\langle S \rangle} \bar{D}_r \cdot \bar{ds} = \int_{\langle S \rangle} 5\,\bar{a}_r \cdot r^2 \sin\theta\, d\phi\, d\theta\, \bar{a}_r \Bigg|_{r=b\,m}$$

$$= 5 r^2 \int_{\theta=0}^{\pi} \sin\theta\, d\theta \int_{\phi=0}^{2\pi} d\phi \; \cancel{\bar{a}_r \cdot \bar{a}_r}^{\,1} = 5(b)^2 (2)(2\pi)$$

$$= \underline{20\pi b^2 \; C}.$$

$$= 20\pi b^2 \text{ Coulomb's}$$

shortcut

$$\boxed{\begin{aligned} \Psi_{outer} &= D_r \cdot A \\ &= 5 \times 4\pi r^2 \big|_{r=b} \\ \Psi_{outer} &= 20\pi b^2 \; C \end{aligned}}$$

r - area of outer sphere i.e r = bm.

$$\boxed{\Psi_{outer} = 20\pi b^2} \; C \quad (40)$$

$$\Psi_{\substack{inner \\ sphere}}\Big|_{r=a\,m} = \int_{\langle S\rangle} \overline{D_r}\cdot\overline{dS} = \int_{\langle S\rangle} 5\,\overline{a_r}\cdot\left[r^2\sin\theta\, d\phi\, d\theta\,(-\overline{a_r})\right]\Big|_{r=a\,m}$$

$$= -5r^2 \int_{\theta=0}^{\pi} \sin\theta\, d\theta \int_{\phi=0}^{2\pi} d\phi \;\; \cancel{\overline{a_r}\cdot\overline{a_r}}^{\,1}\Big|_{r=a\,m}$$

$$= -5a^2(2)(2\pi)(1) = -20\pi a^2 \text{ Coulombs}$$

$$\boxed{\Psi_{\substack{inner \\ sphere}} = -20\pi a^2} \text{ C}$$

Short cut: $\Psi_{inner} = D_r \times A$; A - area of inner sphere

$= -5\times 4\pi r^2\big|_{r=a\,m}$; $D_r = -5C$

$\boxed{\Psi_{inner} = -20\pi a^2}$ Coulombs.

$$\Psi_{total} = \oint_{\langle S\rangle} \overline{D}\cdot\overline{dS} = \Psi_{\substack{outer \\ sphere}} + \Psi_{\substack{inner \\ sphere}} = \left[20\pi b^2 - 20\pi a^2\right] \text{C}$$

$$\boxed{\Psi_{total} = \oint_{\langle S\rangle} \overline{D}\cdot\overline{dS} = 20\pi(b^2-a^2)} \text{ Coulombs} \leftarrow (a)$$

RHS: $\int_{\langle vol\rangle} \nabla\cdot\overline{D}\, dv = ?$ $\quad dv = r^2\sin\theta\, dr\, d\theta\, d\phi$

divergence in Spherical Coordinate System is

$$\nabla\cdot\overline{D} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 D_r\right] + \frac{1}{r\sin\theta}\cancel{\frac{\partial}{\partial\theta}\left[\sin\theta\, D_\theta\right]}^{\,0} + \frac{1}{r\sin\theta}\cancel{\frac{\partial D_\phi}{\partial\phi}}^{\,0}$$

$$\nabla\cdot\overline{D} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[5r^2\right] = \frac{5}{r^{\cancel{2}}}\times 2\cancel{r} = \underline{10/r} \text{ C/m}^3$$

$$\int_{\langle vol\rangle} \nabla\cdot\overline{D}\, dv = \int_{\langle vol\rangle} \frac{10}{\cancel{r}}\cdot r^{\cancel{2}}\sin\theta\, dr\, d\theta\, d\phi$$

$$= 10\int_{r=a}^{b} r\cdot dr \int_{\theta=0}^{\pi} \sin\theta\, d\theta \int_{\phi=0}^{2\pi} d\phi$$

$$= \cancel{10}^{\,5}\, \frac{r^2}{\cancel{2}}\Big|_a^b \times 2 \times 2\pi = 5\left[b^2-a^2\right]\times 4\pi = 20\pi(b^2-a^2)$$

$$\boxed{\int_{\langle vol\rangle} (\nabla\cdot\overline{D})\, dv = 20\pi(b^2-a^2)} \text{ Coulombs} \leftarrow (b)$$

(41) L.H.S = R.H.S ∴ divergence theorem is verified.

problem 7

$\bar{D} = \frac{5r}{3}\bar{a_r}$ $r \le a$

Prove that the divergence theorem for the given region r ≤ a (spherical coordinate system), having flux density, $\bar{D} = \frac{5r}{3}\bar{a_r}$.

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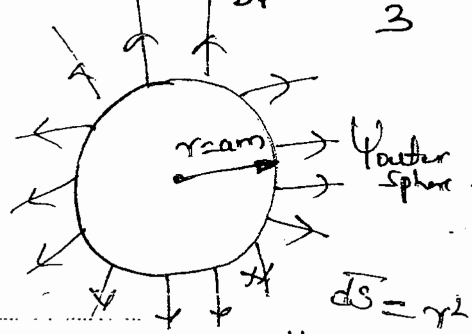
(08 Marks)

10-June/July 2013

Sol:- $\bar{D} = \frac{5r}{3}\bar{a_r}$ C/m² $P(r, \theta, \phi)$ dr $rd\theta$ $r\sin\theta d\phi$

$D_r = \frac{5r}{3}$ C/m².

Gauss divergence theorem.



$$\boxed{\oint_{\langle S\rangle} \bar{D}\cdot\bar{ds} = \int_{\langle vol\rangle} (\nabla\cdot\bar{D})\, dv} \text{ C} \leftarrow (1)$$

$\bar{ds} = r^2 \sin\theta d\theta d\phi(+\bar{a_r})$

fig. sphere with radius 'a' m. L.H.S

$$\Psi_{total} = \oint_{\langle S\rangle} \bar{D}\cdot\bar{ds} = \Psi_{outer\ sphere}\Big|_{r=a\ sphere} \text{ Coulomb's}$$

$$= \int_{\langle S\rangle} \bar{D_r}\cdot\bar{ds} = \int_{\langle S\rangle} \frac{5r}{3}\bar{a_r}\cdot r^2\sin\theta d\theta d\phi(+\bar{a_r})$$

$$= \frac{5r^3}{3}\int_{\theta=0}^{\pi} \sin\theta\, d\theta \int_{\phi=0}^{2\pi} d\phi\ \bar{a_r}\cdot\bar{a_r}\Big|_{\substack{r=a m\\ \text{Sphere}}}$$

$$= \frac{5(a)^3}{3}\times 2\times 2\pi\times 1 = \frac{20}{3}\pi a^3 \text{ Coulomb's}$$

$$\boxed{\Psi_{total} = \oint_{\langle S\rangle} \bar{D}\cdot\bar{dS} = \frac{20}{3}\pi a^3} \text{ Coulomb's} \leftarrow (a)$$

(or) $\Psi_{outer} = D_r\cdot A$; A Area of outer sphere.

$$= \frac{5r}{3}\cdot 4\pi r^2 = \frac{20}{3}\pi r^3\Big|_{r=am} = \frac{20}{3}\pi a^3 \text{ Coulomb's}$$

$$\boxed{\Psi_{outer} = \frac{20}{3}\pi a^3} \text{ C} \quad (42)$$

RHS $\int_{\langle vol \rangle} (\nabla \cdot \bar{D})\, dv = ?$ $\quad$ bcz $D_\theta = 0$ $\quad$ bcz $D_\phi = 0$

$$\nabla \cdot \bar{D} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \cdot D_r\right] + \frac{1}{r\sin\theta}\cancel{\frac{\partial}{\partial \theta}\left[\sin\theta D_\theta\right]} + \frac{1}{r\sin\theta}\cancel{\frac{\partial D_\phi}{\partial \phi}} \ C/m^3$$

$$D_r = \frac{5r}{3} \cdot C/m^2$$

$$\nabla \cdot \bar{D} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \cdot \frac{5r}{3}\right] = \frac{5}{3r^2}\frac{\partial}{\partial r}(r^3)$$

$$= \frac{5}{\cancel{3r^2}} \cdot \cancel{3r^2} = \underline{\underline{5}} \ C/m^3$$

$$\int_{\langle vol \rangle} \nabla \cdot \bar{D}\, dv = \int_{\langle vol \rangle} 5 \ r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$= 5 \int_{r=0}^{a} r^2 dr \int_{\theta=0}^{\pi} \sin\theta\, d\theta \int_{\phi=0}^{2\pi} d\phi$$

$$= 5 \cdot \frac{r^3}{3}\Big|_0^a \times 2 \times 2\pi$$

$$= \frac{5}{3}\left[a^3 - 0\right] \times 4\pi = \frac{20\pi}{3}a^3 \text{ Coulomb's}$$

$$\therefore \boxed{\int_{\langle vol \rangle} \nabla \cdot \bar{D}\, dv = \frac{20}{3}\pi a^3} \text{ Coulomb's} \longleftarrow (b)$$

eq^n (a) = eq^n (b) $\therefore$ divergence theorem is verified.

problem 18

$\Rightarrow \overline{D} = \frac{10r^3}{4} \overline{a_r}\ C/m^2$

23 Given $\vec{D} = \frac{10r^3}{4}\hat{a}_r$ in cylindrical co-ordinates, evaluate both sides of the divergence theorem for the volume enclosed by the cylinder with r = 2 m, z = 0 to 10 m. **(10 Marks)**

Sol^n:

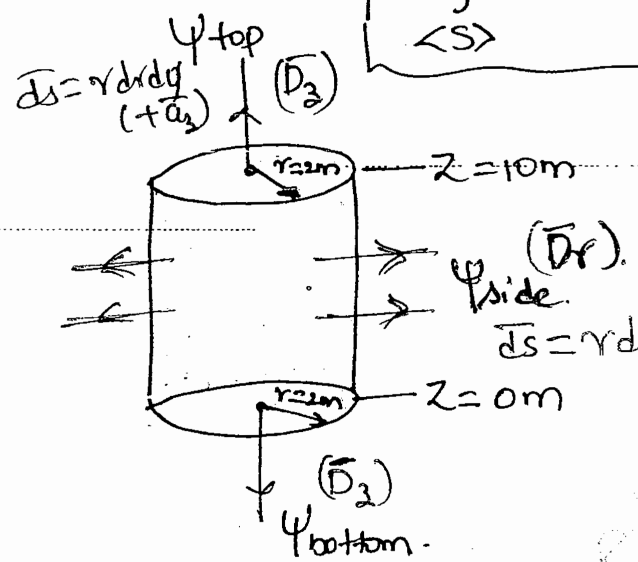
$r = 2m,\ z = 0 \text{ to } 10m.$

$\overline{D} = \frac{10r^3}{4}\overline{a_r}\ C/m^2.$

$P(r, \phi, z)$ → dr, $rd\phi$, dz

Gauss divergence theorem

$$\boxed{\oint_{\langle S \rangle} \overline{D}\cdot\overline{ds} = \int_{\langle vol \rangle} (\nabla\cdot\overline{D})\, dv} \leftarrow (1)$$



$\overline{ds} = r\,dr\,d\phi\,(-\overline{a_z})$

L.H.S

$\Psi_{total} = \oint_{\langle S \rangle} \overline{D}\cdot\overline{ds}$

$= \underbrace{\Psi_{top}}_{0} + \underbrace{\Psi_{bottom}}_{0} + \Psi_{side}$

bc, $D_z = 0$.

$$\Rightarrow \Psi_{total} = \oint_{\langle S \rangle} \overline{D}\cdot\overline{ds} = \Psi_{side}\Big|_{r=2m}$$

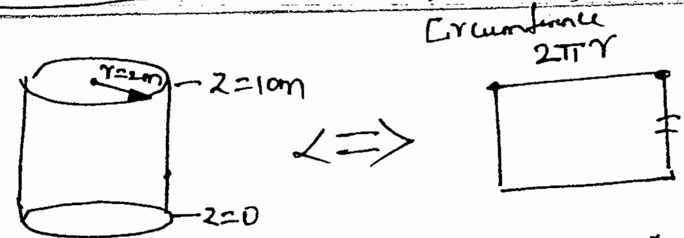
$$\Psi_{side}\Big|_{r=2m} = \int_{\langle S \rangle} \overline{D_r}\cdot\overline{ds} = \int_{\langle S \rangle} \frac{10r^3}{4}\overline{a_r}\cdot r\,d\phi\,dz\,(+\overline{a_r})\Bigg|_{r=2m}$$

$$= \frac{10r^3}{4}\cdot r \underbrace{\int_{\phi=0}^{2\pi} d\phi}_{2\pi} \times \underbrace{\int_{z=0}^{10} dz}_{10}\ \underbrace{\overline{a_r}\cdot\overline{a_r}}_{1}\Bigg|_{r=2m}.$$

$$= \frac{10(2)^3 \times 2}{4} \times 2\pi \times 10 \times 1 = \underline{800\pi \text{ Coulomb's}}$$

$$\boxed{\Psi_{total} = \oint_{\langle S \rangle} \overline{D}\cdot\overline{ds} = \Psi_{side} = 800\pi}$$ Coulomb's ← (a)

Shortcut



$\Rightarrow A = 2\pi r z \; m^2$

r = 2m ; constant cylinder.

$$\Psi_{side} = D_r \cdot A = \frac{10r^3}{4} \times 2\pi r z \Big|_{\substack{r=2m \\ \& \, z=10 \text{ height}}}$$

$$= \frac{10r^4}{4} \times 2\pi z = \frac{10(2)^4 \times 2\pi \times 10}{4}$$

$$\therefore \boxed{\Psi_{side} = 800\pi \; C}$$

R.H.S :- $\int_{\langle vol \rangle} \nabla\cdot\overline{D} \, dv = ?$

divergence $(\nabla\cdot\overline{D})$ in cylindrical Co-ordinate System

$$\nabla\cdot\overline{D} = \frac{1}{r}\frac{\partial}{\partial r}(r\cdot D_r) + \cancel{\frac{1}{r}\frac{\partial D_\phi}{\partial \phi}}^{\to 0 \text{ bcz } D_\phi = 0} + \cancel{\frac{\partial D_z}{\partial z}}^{\to 0 \text{ bcz } D_z = 0}$$

$$D_r = \frac{10r^3}{4} \; C/m^2$$

$$\nabla\cdot\overline{D} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot\frac{10r^3}{4}\right] = \frac{10}{4r}\frac{\partial}{\partial r}[r^4] = \frac{10}{\cancel{4}r}\cdot\cancel{4}r^{\cancel{3}\,2}$$

$$\boxed{\nabla\cdot\overline{D} = 10r^2} \; C/m^3$$

$$\int_{\langle vol \rangle} \nabla\cdot\overline{D}\, dv = \int_{\langle vol \rangle} 10r^2 \cdot r\,dr\,d\phi\,dz = 10\int_{r=0}^{2} r^3 dr \int_{\phi=0}^{2\pi} d\phi \int_{z=0}^{10} dz$$

$$= 10 \times \frac{r^4}{4}\Big|_0^2 \times 2\pi \times 10 = \frac{10}{4}[2^4] \times 20\pi$$
$$= \underline{800\pi \; C}$$

$$\boxed{\int_{\langle vol \rangle} (\nabla\cdot\overline{D})\, dv = 800\pi} \; C \leftarrow (b)$$

eq^n (a) = eq^n (b) ∴ divergence theorem is Verified. Contd next page

problem 19

Given $\bar{D} = 5r\,\bar{a}_r\ C/m^2$, prove divergence theorem for a shell region enclosed by spherical surface at $r = a$ and $r = b$ ($b > a$) and centered at the origin.

46

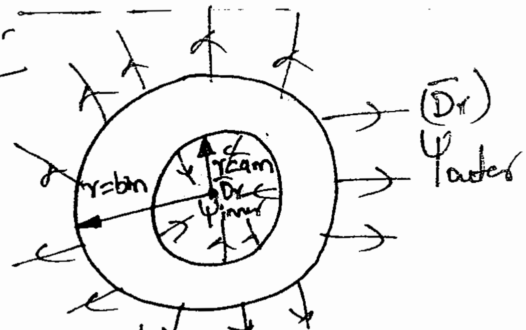
$$\boxed{\Psi_{total} = \oint_{\langle S \rangle} \overline{D} \cdot \overline{ds} = \int_{\langle vol \rangle} (\nabla \cdot \overline{D})\, dv = 800\pi} \text{ Coulomb's}$$

06 – May/June 2010

problem 19

Given $\overline{D} = 5r\hat{a}_r$ c/m², prove divergence theorem for a shell region enclosed by spherical surfaces at r = a and r = b (b>a) and centered at the origin. (08 Marks)

Solu:



Concentric spheres with (b>a).

(D_r) $\overline{D} = 5r\overline{a}_r$ c/m²

$$\Psi_{outer\ sphere}\Big|_{r=bm} \Rightarrow \overline{ds} = r^2 \sin\theta\, d\theta\, d\phi\, (+\overline{a}_r)$$

$p(r, \theta, \phi)$

$dr \quad r d\theta \quad r\sin\theta\, d\phi$

$(\overline{D}_r)$

$$\Psi_{inner\ sphere}\Big|_{r=am} \Rightarrow \overline{ds} = r^2 \sin\theta\, d\theta\, d\phi\, (-\overline{a}_r)$$

divergence theorem $\oint_{\langle S \rangle} \overline{D} \cdot \overline{ds} = \int_{\langle vol \rangle} (\nabla \cdot \overline{D})\, dv \leftarrow (1)$

L.H.S $\Psi_{total} = \oint_{\langle S \rangle} \overline{D} \cdot \overline{ds} = \Psi_{outer\ sphere}\Big|_{r=bm} + \Psi_{inner\ sphere}\Big|_{r=am}$.

$$\Psi_{outer\ sphere}\Big|_{r=bm} = \int_{\langle S \rangle} \overline{D_r} \cdot \overline{ds} = \int_{\langle S \rangle} 5r\overline{a}_r \cdot r^2 \sin\theta\, d\theta\, d\phi (+\overline{a}_r)$$

$$= 5r^3 \int_{\theta=0}^{\pi} \sin\theta\, d\theta \int_{\phi=0}^{2\pi} d\phi\ \overline{a}_r \cdot \overline{a}_r \Big|_{r=bm}$$

$$= 5b^3 \times 2 \times 2\pi = 20\pi b^3 \text{ Coulomb's}$$

$$\boxed{\Psi_{outer\ sphere} = 20\pi b^3}\ C \quad \text{or} \quad \Psi_{outer} = D_r \cdot A\Big|_{r=bm}$$

$$= 5r \times 4\pi r^2 = 20\pi r^3 \Big|_{r=bm}$$

$$\therefore \boxed{\Psi_{outer} = 20\pi b^3}\ C$$

$$\Psi_{\substack{inner\\Sphere}}\Big|_{r=am} = \int_{\langle S\rangle} \bar{D_r}\cdot\bar{ds} = \int_{\langle S\rangle} 5r\,\bar{a_r}\cdot r^2\sin\theta\,d\theta\,d\phi\,(-\bar{a_r})$$

$$= -5r^3 \int_{\theta=0}^{\pi}\sin\theta\,d\theta \int_{\phi=0}^{2\pi} d\phi\;\;\bar{a_r}\cdot\bar{a_r}\Big|_{r=am}$$

$$= -5a^3\times 2\times 2\pi\times 1 = -20\pi a^3 \text{ Coulomb's}$$

$$\boxed{\Psi_{\substack{inner\\Sphere}} = -20\pi a^3}\; C \quad \text{or} \quad \Psi_{inner} = -D_r\cdot A\Big|_{r=am}$$

$$= -5r\times 4\pi r^2 = -20\pi r^3$$

$$\boxed{\Psi_{inner} = -20\pi a^3}\; C.$$

$$\Psi_{total} = \oint_{\langle S\rangle}\bar{D}\cdot\bar{ds} = \Psi_{outer}+\Psi_{inner} = 20\pi b^3 - 20\pi a^3 \; C$$

$$\boxed{\Psi_{total} = \oint_{\langle S\rangle}\bar{D}\cdot\bar{ds} = 20\pi(b^3-a^3)} \text{ Coulomb's} \leftarrow (a)$$

R.H.S: $\int_{\langle V\rangle}(\nabla\cdot\bar{D})\cdot dv = ?$ $D_r = 5r$, $D_\theta = 0$, $D_\phi = 0$.

$$\nabla\cdot\bar{D} = \frac{1}{r^2}\frac{\partial}{\partial r}[r^2 D_r] + \cancel{\frac{1}{r\sin\theta}\frac{\partial[\sin\theta D_\theta]}{\partial\theta}} + \cancel{\frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi}}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}[r^2\cdot 5r] = \frac{5}{\cancel{r^2}}\times 3\cancel{r^2} = 15\; C/m^3$$

$$\boxed{\nabla\cdot\bar{D} = 15}\; C/m^3$$

$$\int_{\langle vol\rangle}(\nabla\cdot\bar{D})\,dv = \int_{\langle vol\rangle} 15r^2\sin\theta\,dr\,d\theta\,d\phi$$

$$= 15\int_{r=a}^{b} r^2\,dr \int_{\theta=0}^{\pi}\sin\theta\,d\theta \int_{\phi=0}^{2\pi} d\phi$$

$$= \frac{\cancel{15}^{\,5}}{\cancel{3}}[b^3-a^3]\times 2\times 2\pi = 20\pi(b^3-a^3) \text{ Coulomb's}$$

$$\boxed{\int_{\langle vol\rangle}\nabla\cdot\bar{D}\,dv = 20\pi(b^3-a^3)}\; C \leftarrow (b)$$

eq (a) = eq (b) theorem
∴ Divergence is verified

. Given that $\overline{D} = \frac{5r^2}{4}\,\overline{a_r}$ C/m² in Spherical Co-ordinates. Evaluate both sides of the divergence theorem for the volume enclosed by $r=4$m and $\theta_0 = \pi/4$. [Schaum's outline].

$P(r, \theta, \phi)$ → dr, $r\,d\theta$, $r\sin\theta\, d\phi$

Sol:- $\overline{D} = \frac{5r^2}{4}\,\overline{a_r}$ C/m².

$r = 4$ m and $\theta = \pi/4$.

$D_r = \frac{5r^2}{4}$ C/m², $D_\theta = D_\phi = 0$ C/m².

Divergence theorem $\oint_{\langle S\rangle} \overline{D}\cdot\overline{dS} = \int_{\langle vol\rangle} (\nabla\cdot\overline{D})\, dv$

L.H.S

$$\Psi_{total} = \oint_{\langle S\rangle} \overline{D}\cdot\overline{dS} = \Psi_{\substack{outer\\sphere}}\Big|_{r=4m} + \cancelto{0}{\Psi_{side}\Big|_{\theta=\pi/4}}$$ bcz $D_\theta = 0$.

$$\Psi_{\substack{outer\\sphere}}\Big|_{r=4m} = \int_{\langle S\rangle} \overline{D_r}\cdot\overline{dS} = \int_{\langle S\rangle} \frac{5r^2}{4}\,\overline{a_r}\cdot r^2\sin\theta\, d\theta\, d\phi\, \overline{a_r}\Big|_{r=4m}$$

$$= \frac{5r^4}{4}\int_{\theta=0}^{\pi/4} \sin\theta\, d\theta \int_{\phi=0}^{2\pi} d\phi\ \cancelto{1}{\overline{a_r}\cdot\overline{a_r}}\Big|_{r=4m.}$$

$$= \frac{5(4)^4}{4} \times 0.29289 \times 2\pi \times 1 = 588.8902 \text{ Coulombs}$$

$$\boxed{\Psi_{\substack{outer\\sphere}}\Big|_{r=4m} = \Psi_{total} = \oint_{\langle S\rangle} \overline{D}\cdot\overline{dS} = 588.8902} \text{ C} \leftarrow (a)$$

R.H.S $\int_{\langle vol\rangle} (\nabla\cdot\overline{D})\, dv = ?$ $dv = r^2\sin\theta\, dr\, d\theta\, d\phi$

divergence in spherical co-ordinate system is

$$\nabla\cdot\bar{D} = \frac{1}{r^2}\frac{\partial}{\partial r}[r^2 D_r] + \cancel{\frac{1}{r\sin\theta}\frac{\partial(\sin\theta D_\theta)}{\partial\theta}}^{\,0} + \cancel{\frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi}}^{\,0} \; C/m^3$$

$D_\theta = 0$ $D_\phi = 0$

$$D_r = \frac{5r^2}{4} \; C/m^2$$

$$\nabla\cdot\bar{D} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \cdot \frac{5r^2}{4}\right] = \frac{5}{4r^2}\frac{\partial}{\partial r}[r^4]$$

$$= \frac{5}{\cancel{4r^2}} \times \cancel{4}r^{\cancel{3}} = 5r \; C/m^3$$

$$\boxed{\nabla\cdot\bar{D} = 5r} \; C/m^3$$

$$\int_{vol} \nabla\cdot\bar{D}\, dv = \int_{vol} 5r \cdot r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$= 5\int_{r=0}^{4} r^3 dr \int_{\theta=0}^{\pi/4} \sin\theta\, d\theta \int_{\phi=0}^{2\pi} d\phi$$

$$= 5 \times 64 \times 0.29289 \times 2\pi = \underline{588.89028} \text{ Coulomb's}$$

$$\boxed{\int_{vol} (\nabla\cdot\bar{D})\, dv = 588.89028} \text{ Coulomb's} \longleftarrow (b)$$

eq^n (a) $= eq^n$ (b) $\therefore$ divergence theorem is verified.

Problem 2) Given that $\overline{D} = 10r^3/4 \ \overline{a_r}$ C/m^2 in cylindrical Co-ordinate System. Evaluate bothsides of the divergence theorem for the volume enclosed by $r = 1m$, $r = 2m$, $z = 0$ and $z = 10m$. [Schaum's outline]

Sol^n: $\overline{D} = \frac{10r^3}{4} \overline{a_r}$ C/m^2 .... in cylindrical C.S

$D_r = \frac{10r^3}{4}$ C/m^2, $D_\phi = D_z = 0$. $P(r, \phi, z)$ dr $rd\phi$ dz

Divergence theorem

$$\boxed{\oint_{\langle S \rangle} \overline{D} \cdot \overline{ds} = \int_{\langle vol \rangle} (\nabla \cdot \overline{D}) dv} \quad \text{(1)}$$

b_{12} $D_z = 0$

L.H.S

$$\oint_{\langle S \rangle} \overline{D} \cdot \overline{ds} = \Psi_{total} = \Psi_{outer}\Big|_{r=2m} + \Psi_{inner}\Big|_{r=1m} + \cancel{\Psi_{top}\Big|_{z=10m}}^{0} + \cancel{\Psi_{bottom}\Big|_{z=0m}}^{0}$$

$$\Psi_{outer}\Big|_{r=2m} = \int_{\langle S \rangle} \overline{D_r} \cdot \overline{ds} = \int_{\langle S \rangle} \frac{10r^3}{4} \overline{a_r} \cdot r d\phi dz \overline{a_r}\Big|_{r=2m}$$

$\overline{ds} = r d\phi dz (+\overline{a_r})$

$$= \frac{10r^3}{4} \cdot r \int_{\phi=0}^{2\pi} d\phi \int_{z=0}^{10} dz \ \cancel{\overline{a_r} \cdot \overline{a_r}}^{1} \Big|_{r=2m}$$

$$= \frac{10(2)^4}{4} \times 2\pi \times 10 = 800\pi \text{ Coulomb's}$$

$$\boxed{\Psi_{outer}\Big|_{r=2m} = 800\pi} \ C$$

$$\Psi_{inner}\Big|_{r=1m} = \int_{\langle S \rangle} \overline{D_r} \cdot \overline{ds}$$

$\overline{ds} = r d\phi dz (-\overline{a_r})$

$$\Psi_{inner}\Big|_{r=1m} = \int_{\langle S\rangle} \frac{10 r^3}{4}\,\bar{a}_r \cdot r\,d\phi\,dz\,(-\bar{a}_r)\Big|_{r=1m}$$

$$= -\frac{10 r^4}{4} \int_{\phi=0}^{2\pi} d\phi \int_{z=0}^{10} dz\; \bar{a}_r \cdot \bar{a}_r \Big|_{r=1m}$$

$$= -\frac{10(1)^4}{4} \times 2\pi \times 10 \times 1 = \underline{-50\pi \text{ Coulomb's}}$$

$$\boxed{\Psi_{inner}\Big|_{r=1m} = -50\pi}\ C$$

$$\Psi_{total} = \oint_{\langle S\rangle} \bar{D}\cdot\bar{ds} = \Psi_{outer}\Big|_{r=2m} + \Psi_{inner}\Big|_{r=1m} = 800\pi - 50\pi = 750$$

i.e. $\boxed{\oint_{\langle S\rangle} \bar{D}\cdot\bar{ds} = 750\pi}\ C \longleftarrow$ ⓐ

R.H.S $\int_{\langle vol\rangle} \nabla\cdot\bar{D}\ dv = ?$ $\quad dv = r\,dr\,d\phi\,dz$

divergence $\nabla\cdot\bar{D}$ in cylindrical Co-ordinate system is by $D_\phi = D_z = 0$.

$$\therefore \nabla\cdot\bar{D} = \frac{1}{r}\frac{\partial(r\cdot D_r)}{\partial r} + \frac{1}{r}\frac{\partial D_\phi}{\partial\phi}^{\nearrow 0} + \frac{\partial D_z}{\partial z}^{\nearrow 0}$$

$$\nabla\cdot\bar{D} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot 10r^3/4\right] = \frac{10}{4r}\times 4r^3 = \underline{10r^2\ C/m^3}$$

$$\int_{\langle vol\rangle} (\nabla\cdot\bar{D})\,dv = \int_{\langle vol\rangle} 10r^2 \times r\,dr\,d\phi\,dz = \int_{\langle vol\rangle} 10r^3\,dr\,d\phi\,dz$$

$$= 10\int_{r=1}^{2} r^3\,dr \int_{\phi=0}^{2\pi} d\phi \int_{z=0}^{10} dz \qquad \boxed{\int_{\langle vol\rangle}(\nabla\cdot\bar{D})\,dv = 750\pi\ C} \longleftarrow ⓑ$$

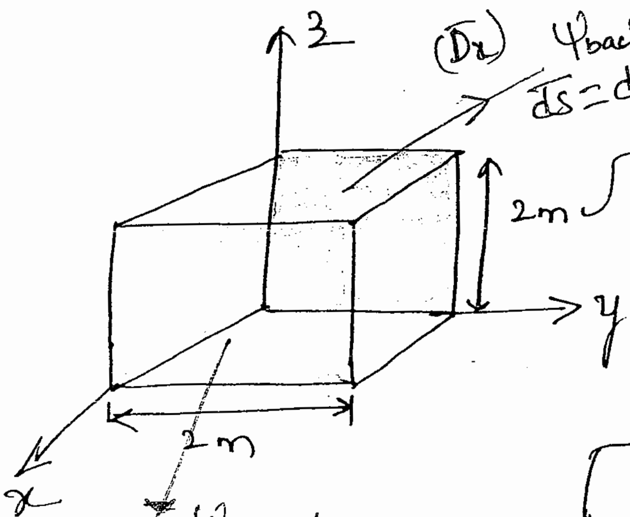
$$= 10\times 3.75\times 2\pi\times 10 = \underline{750\pi \text{ Coulomb's}}$$

Eq ⓐ = eq ⓑ ∴ divergence theorem is verified.

Problem 22: Given that $\overline{D} = \frac{10x^3}{3}\,\overline{a_x}$ C/m², Evaluate both sides of the divergence theorem for the volume of a cube 2m on an edge centered at the origin and with edges parallel to the axes.

Soln: given $\overline{D} = \frac{10x^3}{3}\,\overline{a_x}$ C/m².

$D_x = \frac{10x^3}{3}$ C/m², $D_y = D_z = 0$ C/m²



(D_x) $\psi_{back}|_{x=-1 \text{ plane}}$

$\overline{ds} = dy\,dz(-\overline{a_x})$

$-1 \le x, y, z \le +1$ bcz cube is centered at origin. (Symmetrical w.r.t origin).

$\psi_{front}|_{x=+1 \text{ plane}}$

$\overline{ds} = dy\,dz(+\overline{a_x})$

Divergence theorem

$$\boxed{\oint_{\langle S \rangle} \overline{D}\cdot\overline{ds} = \int_{\langle vol \rangle} (\nabla\cdot\overline{D})\,dv} \longleftarrow (1)$$

L.H.S

bcz $D_z = 0$ bcz $D_y = 0$.

$$\psi_{total} = \oint_{\langle S \rangle} \overline{D}\cdot\overline{ds} = \cancelto{0}{\psi_{top}}\Big|_{z=+1} + \cancelto{0}{\psi_{bottom}}\Big|_{z=-1} + \cancelto{0}{\psi_{Left}}\Big|_{y=-1} + \cancelto{0}{\psi_{Right}}\Big|_{y=+1}$$

$$+ \psi_{front}\Big|_{x=+1} + \psi_{back}\Big|_{x=-1}$$

$$\psi_{front}\Big|_{x=+1 \text{ plane}} = \int_{\langle S \rangle} \overline{D_x}\cdot\overline{ds} = \int_{\langle S \rangle} \frac{10x^3}{3}\overline{a_x}\cdot dy\,dz(+\overline{a_x})\Big|_{x=+1}$$

$$= \frac{10x^3}{3}\int_{y=-1}^{+1} dy \int_{z=-1}^{+1} dz \;\cancelto{1}{\overline{a_x}\cdot\overline{a_x}}\Big|_{x=+1 \text{ plane}}$$

$$= \frac{10(1)^3}{3} \times 2 \times 2 \times 1 = \frac{40}{3} \text{ Coulombs}$$

06 - June /July 2011

8 ~~State and explain ... (06 Marks)~~

10-Dec/Jan 2015

11 ~~State vector form of Coulomb's law of force between two point charges and indicate the units of the quantities in the equation (06 Marks)~~

10 - June /July 2014

13 ~~State and explain Coulomb's law in vector form (04 Marks)~~

06 - June /July 2013

14 ~~State and explain Coulomb's law in vector form (05 Marks)~~

$$\Psi_{back}\Big|_{x=-1\,plane} = \int_{\langle S\rangle} \bar{D}_x \cdot \bar{ds} = \int_{\langle S\rangle} \frac{10x^3}{3}\bar{a}_x \cdot dy\,dz\,(-\bar{a}_x)\Big|_{x=-1\,plane}$$

$$= -\frac{10x^3}{3}\int_{y=-1}^{+1} dy \int_{z=-1}^{+1} dz\; \bar{a}_x \cdot \bar{a}_x \Big|_{x=-1}$$

$$= -\frac{10(-1)^3}{3} \times 2 \times 2 = +\frac{40}{3}\ \text{Coulomb's}$$

$$\Psi_{total} = \oint_{\langle S\rangle} \bar{D}\cdot\bar{ds} = \Psi_{front} + \Psi_{back}$$

$$= 40/3 + 40/3 = 80/3\ C$$

$$\boxed{\Psi_{total} = \oint_{\langle S\rangle} \bar{D}\cdot\bar{ds} = 80/3}\ \text{Coulomb's} \longleftarrow (a)$$

R.H.S $\int_{\langle vol\rangle} (\nabla\cdot\bar{D})\,dv = ?$ $\quad dv = dx\,dy\,dz$

divergence in Cartesian C.S $\quad$ bcz $D_y = D_z = 0$

$$\nabla \cdot \bar{D} = \frac{\partial D_x}{\partial x} + \cancel{\frac{\partial D_y}{\partial y}}^{0} + \cancel{\frac{\partial D_z}{\partial z}}^{0}$$

$$= \frac{\partial}{\partial x}\left[\frac{10x^3}{3}\right] = \frac{10}{\cancel{3}} \times \cancel{3} x^2 = 10x^2 \; c/m^3$$

$$\int_{\langle vol \rangle} (\nabla \cdot \bar{D})\, dv = \int_{\langle vol \rangle} 10x^2 \, dx\, dy\, dz$$

$$= 10 \int_{x=-1}^{+1} x^2 dx \int_{y=-1}^{+1} dy \int_{z=-1}^{+1} dz$$

$$= 10 \times 2/3 \times 2 \times 2 = 80/3 \text{ Coulomb's}$$

i.e $\boxed{\int_{\langle vol \rangle} (\nabla \cdot \bar{D})\, dv = \frac{80}{3}}$ Coulomb's $\leftarrow$ (b)

problem 23 $\quad$ eq^n (a) = eq^n (b), $\therefore$ Divergence theorem is Verified.

Show that $\nabla \cdot \bar{E} = 0$ for the field of

i) point charge.

ii) a uniform Line charge.

Soln: i. $\nabla \cdot \bar{E} = 0$ due to point charge

w.k.t $\bar{E}$ due to point charge in spherical C.S is

$$\bar{E} = \frac{Q}{4\pi\epsilon r^2} \bar{a}_r \;\; v/m \leftarrow (1)$$

$$E_r = \frac{Q}{4\pi\epsilon r^2} \; v/m; \; E_\theta = 0; \; E_\phi = 0 \; v/m$$

∴ Divergence $\nabla \cdot \bar{E}$ in Spherical Co-ordinate System is

by $E_\theta = E_\phi = 0$.

$$\nabla \cdot \bar{E} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 E_r\right] + \frac{1}{r\sin\theta}\cancel{\frac{\partial(\sin\theta E_\theta)}{\partial\theta}}^{0} + \frac{1}{r\sin\theta}\cancel{\frac{\partial E_\phi}{\partial\phi}}^{0}$$

$$\therefore \nabla \cdot \bar{E} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[\cancel{r^2}\cdot\frac{Q}{4\pi\epsilon \cancel{r^2}}\right]$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left[\overset{\text{Const}}{\cancel{\frac{Q}{4\pi\epsilon}}}\right] = \frac{1}{r^2}\cancel{\frac{\partial}{\partial r}(\text{constant})}^{0}$$

$$\therefore \boxed{\nabla \cdot \bar{E} = 0} \text{ V/m}^3$$

ii. W.K.T $\bar{E}$ due to infinite Line charge is

$$\bar{E} = \overset{E_\rho}{\cancel{\frac{\rho_L}{2\pi\epsilon\rho}}}\,\bar{a}_\rho \text{ V/m}$$ which is in Cylindrical C.S.

$E_\phi = E_z = 0$ V/m.

∴ $\nabla \cdot \bar{E}$ in Cylindrical C.S is

$E_\phi = E_z = 0$.

$$\nabla \cdot \bar{E} = \frac{1}{\rho}\frac{\partial(E_\rho\cdot\rho)}{\partial\rho} + \frac{1}{\rho}\cancel{\frac{\partial E_\phi}{\partial\phi}}^{0} + \cancel{\frac{\partial E_z}{\partial z}}^{0}$$

$$\nabla \cdot \bar{E} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left[\cancel{\rho}\cdot\frac{\rho_L}{2\pi\epsilon\cancel{\rho}}\right] = \frac{1}{\rho}\cancel{\frac{\partial}{\partial\rho}[\text{constant}]}^{0}$$

$\left| \frac{\rho_L}{2\pi\epsilon} = \text{constant}\right.$

$$\therefore \nabla \cdot \bar{E} = \frac{1}{\rho}\times 0 = 0$$

$$\boxed{\nabla \cdot \bar{E} = 0} \text{ V/m}^3$$

The Divergence of $\bar{E}$ for this charge configuration is zero everywhere except at $\rho = 0$, where the expression is indeterminate.

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Problem 24. Show that $\nabla \cdot \bar{D} = 0$ for the field of a

i> point charge.

ii> uniform line charge.

Soln:- i> point charge

$\bar{D}$ due to point charge is $\bar{D} = \frac{Q}{4\pi r^2}\bar{a}_r$ C/m². ... in spherical C.S

$$\nabla \cdot \bar{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \cdot D_r)$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left[\cancel{r^2}\cdot \frac{Q}{4\pi \cancel{r^2}}\right] = \frac{1}{r^2}\cancel{\frac{\partial}{\partial r}}\left[Q/4\pi\right]$$

$$\therefore \boxed{\nabla \cdot \bar{D} = 0} \text{ C/m}^3$$

ii> $\bar{D}$ due to uniform line charge density is

$$\bar{D} = \frac{\rho_L}{2\pi \rho}\bar{a}_\rho \text{ C/m}^2 \text{ ... in Cylindrical C.S.}$$

$$\nabla \cdot \bar{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}[\rho \cdot D_\rho] = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\cancel{\rho}\cdot \frac{\rho_L}{2\pi \cancel{\rho}}\right]$$

$$\nabla \cdot \bar{D} = \frac{1}{\rho}\cancel{\frac{\partial}{\partial \rho}}[\text{Constant}] = 0 \text{ C/m}^3$$

$$\boxed{\nabla \cdot \bar{D} = 0} \text{ C/m}^3$$

proved

Problem 25 Given $\bar{D} = e^{-y}[\cos x\,\bar{a}_x - \sin x\,\bar{a}_y]$ C/m² find $\nabla\cdot\bar{D} = ?$

$$\nabla\cdot\bar{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \cancel{\frac{\partial D_z}{\partial z}}^{\,0} \quad \text{bcz } D_z = 0.$$

$$D_x = e^{-y}\cos x \text{ C/m}^2 \; ; \; D_y = -e^{-y}\sin x \text{ C/m}^2$$

$$\frac{\partial D_x}{\partial x} = -e^{-y}\sin x \text{ C/m}^3 \; ; \; \frac{\partial D_y}{\partial y} = +e^{-y}\sin x \text{ C/m}^3$$

$$\nabla\cdot\bar{D} = -e^{-y}\cancel{\sin x} + e^{-y}\cancel{\sin x} = 0 \text{ C/m}^3$$

$$\boxed{\nabla\cdot\bar{D} = 0} \text{ C/m}^3$$

Problem 26

Given $\bar{D} = x^2\,\bar{a}_x + yz\,\bar{a}_y + xy\,\bar{a}_z$ C/m² find $\nabla\cdot\bar{D}$.

$$\nabla\cdot\bar{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} \text{ C/m}^3$$

$$= \frac{\partial}{\partial x}(x^2) + \frac{\partial}{\partial y}(yz) + \cancel{\frac{\partial}{\partial z}(xy)}^{\,0}$$

$$= 2x + z \text{ C/m}^3$$

$$\boxed{\nabla\cdot\bar{D} = 2x + z} \text{ C/m}^3.$$

Problem 27

$\bar{D} = (x^2+y^2)^{-1/2}\,\bar{a}_x$ find $\nabla\cdot\bar{D}$ at $(2, 2, 0)$

$$D_y = D_z = 0.$$

$$\therefore \nabla\cdot\bar{D} = \frac{\partial D_x}{\partial x} = \frac{\partial}{\partial x}[x^2+y^2]^{-1/2}$$

$$= -\frac{1}{2}[x^2+y^2]^{-3/2} \times 2x$$

@ $(2, 2, 0)$ i.e $x = 2, y = 2$ & $z = 0$

$$\nabla\cdot\bar{D} = -88.388 \times 10^{-3} \text{ C/m}^3$$

$$\boxed{\nabla\cdot\bar{D}\Big|_{(2,2,0)} = -8.84} \text{ mC/m}^3$$

problem 28

∴ Given $\overline{D} = r\sin\phi\, \overline{a_r} + 2r\cos\phi\, \overline{a_\phi} + 2z^2\, \overline{a_z}$ C/m² find $\nabla\cdot\overline{D}$

Soln:- given $\overline{D}$ is in cylindrical C.S

$$\therefore \nabla\cdot\overline{D} = \frac{1}{r}\frac{\partial}{\partial r}[r\cdot D_r] + \frac{1}{r}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z} \text{ C/m}^3$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot r\sin\phi\right] + \frac{1}{r}\frac{\partial}{\partial\phi}(2r\cos\phi) + \frac{\partial}{\partial z}(2z^2) \text{ C/m}^3$$

$$= \frac{\sin\phi}{\cancel{r}}\cdot 2\cancel{r} + \frac{2\cancel{r}}{\cancel{r}} \times -\sin\phi + 2\times 2z$$

$$= \cancel{2\sin\phi} - \cancel{2\sin\phi} + 4z = 4z \text{ C/m}^3$$

$$\boxed{\nabla\cdot\overline{D} = 4z} \text{ C/m}^3$$

problem 29

∴ Given $\overline{D} = 10\sin^2\phi\, \overline{a_r} + r\overline{a_\phi} + \frac{z^2}{r}\cos^2\phi\, \overline{a_z}$ C/m²

find $\nabla\cdot\overline{D}$ at $(2, \phi, 5)$.

Soln: $$\nabla\cdot\overline{D} = \frac{1}{r}\frac{\partial}{\partial r}[r\cdot D_r] + \frac{1}{r}\frac{\partial D_\phi}{\partial\phi} + \frac{\partial D_z}{\partial z} \text{ C/m}^3$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot 10\sin^2\phi\right] + \frac{1}{r}\cancel{\frac{\partial}{\partial\phi}}[r]^{\to 0} + \frac{\partial}{\partial z}\left[\frac{z^2}{r}\cos^2\phi\right]$$

$$\nabla\cdot\overline{D} = \frac{10}{r}\sin^2\phi + \frac{\cos^2\phi}{r}\times 2z$$

@ $P(2, \phi, 5)$ $r = 2$, $z = 5$

$$= \frac{10\sin^2\phi}{2} + \frac{\cos^2\phi}{2}\times 10 = 5\left[\cancel{\sin^2\phi + \cos^2\phi}^{\,1}\right] = 5 \text{ C/m}^3$$

$$\therefore \boxed{\nabla\cdot\overline{D} = 5} \text{ C/m}^3$$

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Problem 30

∴ Given $\overline{D} = \frac{5}{r^2}\overline{a_r} + \frac{10}{\sin\theta}\overline{a_\theta} - r^2\phi\sin\theta\,\overline{a_\phi}$ Find $\nabla\cdot\overline{D}$.

Sol^n: given $\overline{D}$ ... in Spherical C.S

$$\therefore \nabla\cdot\overline{D} = \frac{1}{r^2}\frac{\partial}{\partial r}[r^2 D_r] + \frac{1}{r\sin\theta}\frac{\partial[\sin\theta D_\theta]}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi}\ C/m^3$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left[\cancel{r^2}\cdot 5/\cancel{r^2}\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left[\cancel{\sin\theta}\times\frac{10}{\cancel{\sin\theta}}\right]$$

$$+ \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left[-r^2\phi\sin\theta\right]$$

$$= 0 + 0 + \frac{1}{r\cancel{\sin\theta}}\times -r^2\cancel{\sin\theta} = -r\ C/m^3$$

$$\boxed{\nabla\cdot\overline{D} = -r}\ C/m^3$$

problem 31

∴ Given that $\overline{D} = \rho_0 z\,\overline{a_z}$ in the region $-1 \le z \le +1$ in Cartesian Co-ordinates and $\overline{D} = \frac{\rho_0 z}{|z|}\overline{a_z}$ elsewhere. Find Charge density. [Schaum's outline]

Sol^n: $\overline{D} = \rho_0 z\,\overline{a_z}$ C/m^2 ... $-1 \le z \le +1$

$$\nabla\cdot\overline{D} = \rho_v = \frac{\partial D_z}{\partial z}\ C/m^3$$ by $\overline{D}$ has only D_z component

$$= \frac{\partial}{\partial z}(\rho_0 z) = \rho_0(1) = \rho_0\ C/m^3$$

$$\boxed{\nabla\cdot\overline{D} = \rho_v = \rho_0}\ C/m^3$$ in the region $-1 \le z \le +1$.

and $\overline{D} = \frac{\rho_0 z}{|z|} \overline{a_z}$ ... elsewhere

$$|z| = \begin{cases} +z \; ; & z \geq 0 \\ -z \; ; & z < 0 \end{cases} \quad \therefore \overline{D} = \begin{cases} +\rho_0 \overline{a_z} \; ; & z \geq 0 \\ -\rho_0 \overline{a_z} \; ; & z < 0 \end{cases}$$

In general $\overline{D} = \pm \rho_0 \overline{a_z}$ C/m²

$$\rho_v = \nabla \cdot \overline{D} = \frac{\partial D_z}{\partial z} = \frac{\partial}{\partial z}(\pm \rho_0) = 0 \text{ C/m}^3$$

$$\boxed{\rho_v = 0} \text{ C/m}^3$$

problem 32

∴ Given that $\overline{D} = \frac{10r^3}{4} \overline{a_r}$ C/m² in the region $0 \leq r \leq 3m$ in cylindrical Co-ordinates and $\overline{D} = \frac{810}{4r} \overline{a_r}$ C/m² elsewhere,

Find the charge density.

Soln: $\overline{D}$ is fn of only D_r i.e $D_\phi = D_z = 0$.

$$\rho_v = \nabla \cdot \overline{D} = \frac{1}{r}\frac{\partial}{\partial r}[D_r \cdot r] = \frac{1}{r}\frac{\partial}{\partial r}\left[r \cdot \frac{10r^3}{4}\right]$$

$$= \frac{10}{4r} \times 4r^{3} = 10r^2 \text{ C/m}^3.$$

$$\boxed{\rho_v = 10r^2 \text{ C/m}^3} \; ..... \; 0 \leq r \leq 3m$$

and $\rho_v = \nabla \cdot \overline{D} = \frac{1}{r}\frac{\partial}{\partial r}\left[\cancel{r}\frac{810}{4\cancel{r}}\right] = \frac{810}{4r} \times 0 = 0$ C/m³

$$\boxed{\rho_v = \nabla \cdot \overline{D} = 0} \text{ C/m}^3 \; .... \text{ for } r > 3m$$
(elsewhere).

problem 33

Given that $\overline{D} = \frac{Q}{\pi r^2}[1-\cos 3r]\,\overline{a_r}$ in Spherical Co-ordinates.

Find the charge density.

Soln: $\overline{D}$ has only D_r Component. $D_\theta = D_\phi = 0$.

$$\nabla\cdot\overline{D} = \rho_v = \frac{1}{r^2}\frac{\partial}{\partial r}[r^2 \cdot D_r] \quad C/m^3$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left[\cancel{r^2}\cdot\frac{Q}{\pi \cancel{r^2}}[1-\cos 3r]\right]$$

$$= \frac{1}{r^2}\left[\cancel{\frac{\partial}{\partial r}}\left(\frac{Q}{\pi}\right)^{0} - \frac{Q}{\pi}\frac{\partial}{\partial r}[\cos 3r]\right]$$

$$= \frac{1}{r^2}\left[\frac{-Q}{\pi}\times -\sin(3r)\times 3\right] = +\frac{3\sin(3r)}{r^2}Q/\pi$$

$$\boxed{\rho_v = \nabla\cdot\overline{D} = \frac{3Q}{\pi r^2}\sin(3r)} \quad C/m^3$$

problem 34

In the region $0 < r \le 1m$, $\overline{D} = -\frac{2\times10^{-4}}{r}\overline{a_r}\ C/m^2$ and for $r > 1m$, $\overline{D} = \frac{-4\times10^{-4}}{r^2}\overline{a_r}\ C/m^2$, in spherical Co-ordinates.

Find the Charge density in both regions.

Soln: $D_\theta = D_\phi = 0\ C/m^2$.

$\therefore \rho_v$ for $0 < r \le 1m$ is

$$\rho_v = \nabla\cdot\overline{D} = \frac{1}{r^2}\frac{\partial}{\partial r}[r^2\cdot D_r] = \frac{1}{r^2}\frac{\partial}{\partial r}\left[\cancel{r^2}\cdot\frac{-2\times10^{-4}}{\cancel{r}}\right]$$

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$$\rho_v = \frac{-2\times10^{-4}}{r^2}\ C/m^3 \ ; \ 0<r\le 1m$$

and ρ_v for $r > 1m$

$$\rho_v = \nabla\cdot\bar{D} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 D_r\right] = \frac{1}{r^2}\frac{\partial}{\partial r}\left[\cancel{r^2} \times \frac{-4\times10^{-4}}{\cancel{r^2}}\right]$$

$$= \frac{1}{r^2}\times 0 = 0\ C/m^3$$

$$\boxed{\rho_v = \nabla\cdot\bar{D} = 0}\ C/m^3 \quad \ldots \text{for region } r > 1m$$

problem35

In the region $r \leq 2$, $\bar{D} = \frac{5r^2}{4}\bar{a_r}$ and for $r > 2$, $\bar{D} = \frac{20}{r^2}\bar{a_r}$ in Spherical Coordinates. Find the charge density. [Schaum's outline]

$D_\theta = D_\phi = 0$

Sol'n: $\rho_v = \nabla\cdot\bar{D} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 D_r\right]$ C/m^3

i. $\rho_v = ?$ for $r \leq 2m$

$$\rho_v = \nabla\cdot\bar{D} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\cdot\frac{5r^2}{4}\right]$$

$$= \frac{1}{\cancel{4}r^2}\times 5\times\cancel{4}r^3 = 5r\ C/m^3$$

ρ_v for $r > 2m$ is

$$\rho_v = \nabla\cdot\bar{D} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[\cancel{r^2}\cdot\frac{20}{\cancel{r^2}}\right] = \frac{1}{r^2}\times 0 = 0\ C/m^3$$

$$\therefore\ \boxed{\rho_v = \nabla\cdot\bar{D} = \begin{cases} 5r\ C/m^3 & ; r \leq 2m \\ 0\ C/m^3 & ; r > 2m. \end{cases}}$$

Hayt

Problem 36. Given a 60 μC point charge located at the origin, find the total electric flux passing through

i. the portion of the sphere $r = 26$ cm bounded by $0 < \theta < \pi/2$ and $0 < \phi < \pi/2$.

ii. the closed surface defined by $\rho = 26$ cm and $z = \pm 26$ cm.

iii. the plane $z = 26$ cm.

[W.H. Hayt].

Sol^n: given $Q = 60\,\mu C$ at origin.

using Gauss's law $\Psi_{total} = \oint_{\langle S \rangle} \bar{D} \cdot \bar{ds}$ Coulomb's.

i. W.K.T $\bar{D}$ due to point charge of Q C in spherical C.S is

$$\bar{D} = \frac{Q}{4\pi r^2}\,\bar{a}_r \ \ C/m^2$$

$P(r, \theta, \phi)$: dr, $r\,d\theta$, $r\sin\theta\,d\phi$

given $r = 26$ cm i.e $r = k$ surface

$$\bar{ds} = r^2 \sin\theta\, d\theta\, d\phi\ \bar{a}_r.$$

$$\Psi_{total} = \oint_{\langle S \rangle} \bar{D} \cdot \bar{ds} = \oint_{\langle S \rangle} \frac{Q}{4\pi \cancel{r^2}}\,\bar{a}_r \cdot \cancel{r^2} \sin\theta\, d\theta\, d\phi\ \bar{a}_r$$

$$= \frac{Q}{4\pi} \int_{\theta=0}^{\pi/2} \sin\theta\, d\theta \times \int_{\phi=0}^{\pi/2} d\phi \times \underbrace{\bar{a}_r \cdot \bar{a}_r}_{1}$$

$$= \frac{60\,\mu}{4\cancel{\pi}} \times 1 \times \frac{\cancel{\pi}}{2} \times 1 = 7.5\,\mu \text{ Coulomb's}$$

Ans $\boxed{\Psi_{total} = 7.5\,\mu}$ Coulomb's

ii. given $\rho = 26\,cm$ and $z = \pm 26\,cm$

$\overline{D}$ due to point charge in Spherical C.S is

$$\overline{D} = \frac{Q}{4\pi\rho^2}\overline{a}_\rho \; C/m^2$$ and $\overline{ds}$ .. for $\rho = k$ Surface

$p(\rho, \phi, z)$ — $\overline{ds} = \rho\, d\phi\, dz\, (+\overline{a}_\rho)$

$d\rho \quad \rho d\phi \quad dz$

using Gauss's Law

$$\psi_{total} = \oint_{\langle S \rangle} \overline{D}\cdot\overline{ds} = \oint_{\langle S \rangle} \frac{Q}{4\pi\rho^{\cancel{2}}}\overline{a}_\rho \cdot \cancel{\rho}\, d\phi\, dz\, \overline{a}_\rho \Bigg|_{\rho = 0.26\,m.}$$

$$= \frac{Q}{4\pi\rho}\int_{\phi=0}^{2\pi} d\phi \int_{z=-0.26}^{+0.26} dz \; \cancel{\overline{a}_\rho \cdot \overline{a}_\rho}^{\,1}$$

$$\psi_{total} = \frac{60\mu}{4\cancel{\pi}\times 0.26}\times \cancel{2\pi} \times 0.52 \times 1 = \underline{60\mu C}$$

$$\therefore \boxed{\psi_{total} = 60\mu}\, C$$

iii) $z = 26\,cm$ plane i.e $z = 0.26\,m$

$$\psi = \frac{Q}{4\pi\rho}\int_{\phi=0}^{2\pi} d\phi \int_{z=0}^{0.26} dz \; \cancel{\overline{a}_\rho \cdot \overline{a}_\rho}^{\,1} \Bigg|_{\rho = 0.26\,m.}$$

$$= \frac{60\mu}{\underset{2}{\cancel{4}}\cancel{\pi}\times \cancel{0.26}}\times \cancel{2\pi}\times \cancel{0.26} = 30\mu C$$

$$\boxed{\psi_{total} = 30}\,\mu C$$

Given the Electric Flux density $\bar{D} = 0.3r^2 \bar{a}_r \; nC/m^2$ in free Space.

i) Find $\bar{E}$ at point $p(r=2, \theta=25°, \phi=90°)$;

ii) Find the total Charge within the Sphere $r=3m$.

iii) Find the total Electric Flux Leaving the Sphere $r=4m$.

Sol^n: given $\bar{D} = 0.3r^2 \bar{a}_r \; nC/m^2$.

i. $\bar{D} = \epsilon \bar{E} \; C/m^2$

$$\bar{E} = \frac{\bar{D}}{\epsilon} = \frac{0.3r^2 \times 10^{-9}}{\epsilon} \bar{a}_r \; V/m$$

$$\bar{E}_p = \frac{0.3(2)^2 \times 10^{-9}}{8.854 \times 10^{-12}} \bar{a}_r \; V/m = 135.53 \bar{a}_r \; V/m.$$

$$\boxed{\bar{E}_p = 135.53 \bar{a}_r} \; V/m$$

ii. the total Charge (Q_{total}) within the Sphere $r=3m$.

using Gaussian Law

$r=k$ sphere

$\bar{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \bar{a}_r$.

$$Q_{total} = \oint_{\langle S \rangle} \bar{D} \cdot \bar{ds}$$

shortcut: or $Q_{total} = \psi_{total}$

$= D_r \cdot A \big|_{r=3m}$

$= 0.3r^2 \times 4\pi r^2 \big|_{r=3m}$

$= 0.3 \times 4\pi \times (3)^4$

$= 0.3 \times 4\pi \times 81$

$= 305.36$

$$= \oint_{\langle S \rangle} 0.3r^2 \bar{a}_r \cdot r^2 \sin\theta \, d\theta \, d\phi \, \bar{a}_r \Big|_{r=3m}$$

$$= 0.3r^4 \int_{\theta=0}^{\pi} \sin\theta \, d\theta \int_{\phi=0}^{2\pi} d\phi \; \bar{a}_r \cdot \bar{a}_r \Big|_{r=3m}$$

$$= 0.3(3)^4 \times 2 \times 2\pi \times 1 = 305.362 \text{ Coulomb's}$$

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$$\boxed{Q_{total} = 305.362} \text{ Coulomb's}$$

iii. the total Flux Leaving the sphere $r = 4m$

using Gauss's Law $\qquad \overline{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \overline{a_r}$ ---- $r=4$ sphere.

$$\Psi_{total} = \oint_{\langle S \rangle} \overline{D} \cdot \overline{ds} \quad \text{Coulomb's}$$

$$= \int_{\langle S \rangle} 0.3 r^2 \, \overline{a_r} \cdot r^2 \sin\theta \, d\theta \, d\phi \, \overline{a_r} \Big|_{r=4m}$$

$$= 0.3 r^4 \underbrace{\int_{\theta=0}^{\pi} \sin\theta \, d\theta}_{2} \underbrace{\int_{\phi=0}^{2\pi} d\phi}_{2\pi} \underbrace{\overline{a_r} \cdot \overline{a_r}}_{1} \Big|_{r=4m}$$

$$= 0.3(4)^4 \times 2 \times 2\pi \times 1 = 965.09 \text{ Coulomb's}$$

$$\therefore \boxed{\Psi_{total} = 965.09} \text{ Coulomb's}$$

Shortcut:- (or) $\Psi_{total} = D_r A \Big|_{r=4m}$

$$= 0.3 r^2 \times 4\pi r^2 \Big|_{r=4m}$$

$$= 0.3(4)^2 \times 4\pi (4)^2 = 965.09 \text{ C}$$

problem 38

- In Each of the following parts, Find the numerical value of div D ~~at the~~ point Specified.

i. $\overline{D} = (2xyz - y^2)\overline{a_x} + (x^2 z - 2xy)\overline{a_y} + x^2 y \overline{a_z} \; C/m^2$ at $P(2, 3, -1)$.

ii. $\overline{D} = 2\rho z^2 \sin^2\phi \, \overline{a_\rho} + \rho z^2 \sin 2\phi \, \overline{a_\phi} + 2\rho^2 z \sin^2\phi \, \overline{a_z} \; C/m^2$ at $P(2, 110°, -1)$.

iii. $\overline{D} = 2r \sin\theta \cos\phi \, \overline{a_r} + r\cos\theta \cos\phi \, \overline{a_\theta} - r\sin\phi \, \overline{a_\phi} \; C/m^2$ at $P(1.5, 30°, 50°)$

Sol:- i. $\nabla\cdot\bar{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$ C/m³

$$\nabla\cdot\bar{D} = \frac{\partial}{\partial x}(2xyz - y^2) + \frac{\partial}{\partial y}(x^2 z - 2xy) + \frac{\partial}{\partial z}(x^2 y)$$

$$= 2yz + (-2x) + 0 = 2yz - 2x$$

$\nabla\cdot\bar{D}$ @ P(2, 3, −1) i.e $x = 2, y = 3, z = -1$

$$\nabla\cdot\bar{D}\Big|_{@P} = 2(3)(-1) - 2(2) = -6 - 4 = -10 \text{ C/m}^3$$

$$\boxed{\nabla\cdot\bar{D}_P = -10} \text{ C/m}^3$$

ii. $\nabla\cdot\bar{D} = \frac{1}{\rho}\frac{\partial}{\partial\rho}[\rho\cdot D_\rho] + \frac{1}{\rho}\frac{\partial D_\phi}{\partial\phi} + \frac{\partial D_z}{\partial z}$

$$= \frac{1}{\rho}\frac{\partial}{\partial\rho}\left[\rho\cdot 2\rho z^2 \sin^2\phi\right] + \frac{1}{\rho}\frac{\partial}{\partial\phi}(\rho z^2 \sin 2\phi) + \frac{\partial}{\partial z}(2\rho^2 z \sin^2\phi)$$

$$= \frac{2z^2\sin^2\phi}{\rho}\times 2\rho + \frac{\cancel{\rho} z^2}{\cancel{\rho}}\times\cos 2\phi\times 2 + 2\rho^2\sin^2\phi\,(1)$$

$\nabla\cdot\bar{D}$ at P($\rho = 2, \phi = 110°, z = -1$)

$$\nabla\cdot\bar{D} = \frac{\cancel{2}(-1)^2\sin^2(110)}{\cancel{2}}\times 2\times 2 + (-1)^2\cos(220°)\times 2 + 2(2)^2\sin^2(110)$$

$$\boxed{\nabla\cdot\bar{D}_{@P} = 9.06417 \text{ C/m}^3}$$

iii. $\overline{D} = 2r\sin\theta\cos\phi\,\overline{a_r} + r\cos\theta\cos\phi\,\overline{a_\theta} - r\sin\phi\,\overline{a_\phi}$ C/m^2

at $P(r=1.5,\ \theta=30^\circ,\ \phi=50^\circ)$

$$\nabla\cdot\overline{D} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 D_r\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\, D_\theta\right] + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi}\ \text{C/m}^3$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\cdot 2r\sin\theta\cos\phi\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\cdot r\cos\theta\cos\phi\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left[-r\sin\phi\right]\ \text{C/m}^3$$

$$= 6\sin\theta\cos\phi + \frac{\cos 2\theta}{\sin\theta}\cos\phi - \frac{\cos\phi}{\sin\theta}$$

at $P(r=1.5,\ \theta=30^\circ,\ \phi=50^\circ)$

$$\nabla\cdot\overline{D}\Big|_{@P} = 6\sin(30^\circ)\cos(50^\circ) + \frac{\cos(60^\circ)}{\sin 30^\circ}\cos(50^\circ) - \frac{\cos 50^\circ}{\sin 30^\circ}$$

$$= 1.9283 + 0.64278 - 1.2857$$

$$\boxed{\nabla\cdot\overline{D}\Big|_{@P} = 1.2852}\ \text{C/m}^3$$

Problem 39

i. Determine an expression for the volume charge density associated with each $\overline{D}$ field following

i. $\overline{D} = \frac{4xy}{z}\overline{a_x} + \frac{2x^2}{z}\overline{a_y} - \frac{2x^2y}{z^2}\overline{a_z}$ C/m²

ii. $\overline{D} = z\sin\phi\,\overline{a_\rho} + z\cos\phi\,\overline{a_\phi} + \rho\sin\phi\,\overline{a_z}$ C/m²

iii. $\overline{D} = \sin\theta\sin\phi\,\overline{a_r} + \cos\theta\sin\phi\,\overline{a_\theta} + \cos\phi\,\overline{a_\phi}$ C/m²

Soln:- i. $\nabla\cdot\overline{D} = \rho_v = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$ C/m³

$$\rho_v = \frac{\partial}{\partial x}\left(\frac{4xy}{z}\right) + \frac{\partial}{\partial y}\left(\frac{2x^2}{z}\right) + \frac{\partial}{\partial z}\left(\frac{-2x^2y}{z^2}\right)$$

$$\nabla\cdot\overline{D} = \rho_v = \frac{4y}{z} + 0 - 2x^2y \times -2/z^3 = \frac{4y}{z} + \frac{4x^2y}{z^3}$$

$$\boxed{\rho_v = \nabla\cdot\overline{D} = \frac{4y}{z}\left[1 + x^2/z^2\right]} \text{ C/m}^3$$

ii. $\nabla\cdot\overline{D} = \rho_v = \frac{1}{\rho}\frac{\partial}{\partial\rho}[\rho\cdot D_\rho] + \frac{1}{\rho}\frac{\partial D_\phi}{\partial\phi} + \frac{\partial D_z}{\partial z}$ C/m³

$$= \frac{1}{\rho}\frac{\partial}{\partial\rho}[\rho\cdot z\sin\phi] + \frac{1}{\rho}\frac{\partial}{\partial\phi}[z\cos\phi] + \cancel{\frac{\partial}{\partial z}[\rho\sin\phi]}^{\,0}$$

$$= \cancel{\frac{z\sin\phi}{\rho}} + \cancel{\frac{1}{\rho}z \times -\sin\phi} + 0 = 0 \text{ C/m}^3$$

$$\boxed{\rho_v = \nabla\cdot\overline{D} = 0} \text{ C/m}^3$$

iii. $\nabla\cdot\overline{D} = \rho_v = \frac{1}{r^2}\frac{\partial}{\partial r}[r^2 \cdot D_r] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}[\sin\theta\cdot D_\theta] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}D_\phi \quad C/m^3$

$$\nabla\cdot\overline{D} = \rho_v = \frac{1}{r^2}\frac{\partial}{\partial r}[r^2 \cdot \sin\theta\sin\phi] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}[\sin\theta\cos\theta\sin\phi] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}[\cos\phi]$$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$$= \frac{2\cancel{r}\sin\theta\sin\phi}{r^{\cancel{2}}} + \frac{\sin\phi}{\cancel{2}r\sin\theta}\cos 2\theta\times\cancel{2} - \frac{\sin\phi}{r\sin\theta}$$

$$= \frac{2}{r}\sin\theta\sin\phi + \frac{\sin\phi}{r\sin\theta}[\underbrace{\cos 2\theta - 1}_{-2\sin^2\theta}]$$

$$= \frac{2}{r}\sin\theta\sin\phi + \frac{\sin\phi}{r\cancel{\sin\theta}}\times -2\sin^{\cancel{2}}\theta$$

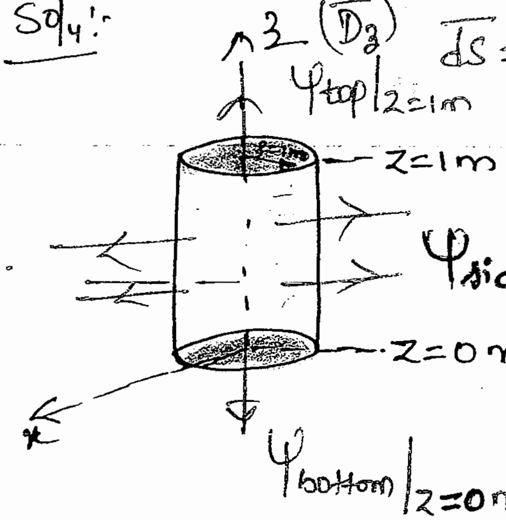
$$= \frac{2}{r}\sin\theta\sin\phi - \frac{2}{r}\sin\theta\sin\phi = 0\ C/m^3$$

$$\boxed{\nabla\cdot\overline{D} = \rho_v = 0}\ C/m^3$$

problem 40

1. if $\overline{D} = 10e^{-2z}(\rho\,\overline{a}_\rho + \overline{a}_z)\ C/m^2$. Determine the total flux of $\overline{D}$ out of the entire surface of the cylinder $\rho = 1m$, $0 \le z \le 1$. Confirm the result by using the divergence theorem.

Sol$_4$:



$P(\rho, \phi, z)$ → $d\rho$, $\rho d\phi$, dz

$\bar{D} = 10e^{-2z}\rho\,\bar{a}_\rho + 10e^{-2z}\bar{a}_z\ C/m^2$

$\bar{ds} = \rho\,d\rho\,d\phi\,(+\bar{a}_z)$

$\bar{ds} = \rho\,d\phi\,dz\,(+\bar{a}_\rho)$

using Gauss Law

$\Psi_{total} = \oint_{\langle S \rangle} \bar{D}\cdot\bar{ds}$ Coulomb's

$\bar{ds} = \rho\,d\rho\,d\phi\,(-\bar{a}_z)$

$\Psi_{total} = \Psi_{top}\Big|_{z=1m} + \Psi_{bottom}\Big|_{z=0m} + \Psi_{side}\Big|_{\rho=1m}$

$$\Psi_{top}\Big|_{z=1m} = \int_{\langle S \rangle} \bar{D}_z \cdot \bar{ds} = \int_{\langle S \rangle} 10e^{-2z}\bar{a}_z \cdot \rho\,d\rho\,d\phi\,\bar{a}_z \Big|_{z=1m}.$$

$$= 10e^{-2z} \int_{\rho=0}^{1} \rho\,d\rho \int_{\phi=0}^{2\pi} d\phi\ \bar{a}_z\cdot\bar{a}_z \Big|_{z=1m}$$

$$\Psi_{top}\Big|_{z=1m} = 10e^{-2} \times 0.5 \times 2\pi \times 1 = \underline{\underline{10e^{-2}\pi}}\ \text{Coulomb's}$$

$$\Psi_{bottom}\Big|_{z=0m} = \int_{\langle S \rangle} 10e^{-2z}\bar{a}_z \cdot \rho\,d\rho\,d\phi\,(-\bar{a}_z)\Big|_{z=0m}$$

$$= -10e^{-2z}\int_{\rho=0}^{1m} \rho\,d\rho \times \int_{\phi=0}^{2\pi} d\phi\ \bar{a}_z\cdot\bar{a}_z \Big|_{z=0m}$$

$$= -10 \times \cancel{e^{-2(0)}} \times 0.5 \times 2\pi \times 1 = \underline{\underline{-10\pi}}\ \text{Coulomb's}$$

$$\Psi_{side}\Big|_{\rho=1m} = \int_{\langle S \rangle} \bar{D}_\rho \cdot \bar{ds} = \int_{\langle S \rangle} 10e^{-2z}\rho\,\bar{a}_\rho \cdot \rho\,d\phi\,dz\,\bar{a}_\rho \Big|_{\rho=1m}$$

$$\Psi_{side}\Big|_{\rho=1m} = 10\rho \int_{\phi=0}^{2\pi} d\phi \int_{z=0}^{1} e^{-2z}\, dz \ \bar{a}_\rho / \bar{a}_\rho \Big|_{\rho=1m}$$

$$= 10 \times 1 \times \cancel{2}\pi \times -\frac{1}{\cancel{2}}\left[e^{-2}-1\right] \times 1 = \underline{10\pi\left[1-e^{-2}\right]}\ C$$

$$\Psi_{total} = \Psi_{top} + \Psi_{bottom} + \Psi_{side}$$

$$= \cancel{10e^{-2}\pi} - \cancel{10\pi} + \cancel{10\pi} - \cancel{10\pi e^{-2}} = 0\ C$$

(or) $\boxed{\Psi_{total} = \oint_{\langle S\rangle} \bar{D}\cdot \bar{ds} = 0}$ Coulomb's ← (a)

Method-2 Verification using divergence theorem

i.e $\int_{\langle vol\rangle} (\nabla\cdot\bar{D})\,dv = ?$ $\quad dv = \rho\, d\rho\, d\phi\, dz$.

but $D_\phi = 0$. ($D_\phi = 0$).

$$\nabla\cdot\bar{D} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left[\rho\cdot D_\rho\right] + \frac{1}{\rho}\cancel{\frac{\partial D_\phi}{\partial\phi}}^{0} + \frac{\partial D_z}{\partial z}\ C/m^3$$

$$= \frac{1}{\rho}\frac{\partial}{\partial\rho}\left[\rho\cdot 10e^{-2z}\rho\right] + \frac{\partial}{\partial z}\left[10e^{-2z}\right]$$

$$= \frac{1}{\cancel{\rho}} \times 10e^{-2z} \times 2\cancel{\rho} + 10e^{-2z}\times(-2)$$

$$\nabla\cdot\bar{D} = \cancel{20}e^{-2z} - \cancel{20}e^{-2z} = 0\ C/m^3$$

$$\therefore \boxed{\int_{\langle vol\rangle} (\nabla\cdot\bar{D})\,dv = \int_{\langle vol\rangle} 0\cdot dv = 0}\ C/m^3 \leftarrow (b)$$

eq^n (a) = eq^n (b) ∴ Ψ_{total} is verified using divergence theorem.

it indicates that $\bar{D}$ has no outward flux.

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Problem 41. Given the Field $\overline{D} = 6\rho\sin(0.5\phi)\,\overline{a}_\rho + 1.5\rho\cos(0.5\phi)\,\overline{a}_\phi$ C/m². Evaluate bothsides of the divergence theorem for the region bounded by $\rho = 2m$, $\phi = 0$ to $\phi = \pi$ and $z = 0$ & $z = 5m$.

Sol/ⁿ: Divergence theorem

$D_\rho = 6\rho\sin(0.5\phi)$ C/m²

$D_\phi = 1.5\rho\cos(0.5\phi)$ C/m² & $D_z = 0$ C/m²

$$\oint_{\langle S\rangle} \overline{D}\cdot\overline{ds} = \int_{\langle vol\rangle} (\nabla\cdot\overline{D})\,dv \quad \text{Coulomb's} \leftarrow (1)$$

$P(\rho, \phi, z)$: $d\rho$, $\rho\,d\phi$, dz

bcz $D_z = 0$

L.H.S

$$\psi_{total} = \oint_{\langle S\rangle} \overline{D}\cdot\overline{ds} = \cancel{\psi_{top}}\Big|_{z=5m}^{\to 0} + \cancel{\psi_{bottom}}\Big|_{z=0m}^{\to 0} + \psi_{side}\Big|_{\rho=2m} + \psi_{\phi=0} + \psi_{\phi=\pi^c}$$

ψ_{side}: $\overline{ds} = \rho\,d\phi\,dz\,(+\overline{a}_\rho)$; Coulomb's

$\psi_{\phi=\pi^c}$: $\overline{ds} = d\rho\,dz\,(+\overline{a}_\phi)$

$\psi_{\phi=0}$: $\overline{ds} = d\rho\,dz\,(-\overline{a}_\phi)$

$$\psi_{side}\Big|_{\rho=2m} = \int_{\langle S\rangle} \overline{D_\rho}\cdot\overline{ds}$$

$$= \int_{\langle S\rangle} 6\rho\sin(0.5\phi)\,\overline{a}_\rho\cdot\rho\,d\phi\,dz\,\overline{a}_\rho\Big|_{\rho=2m}$$

$$= 6\rho^2 \int_{\phi=0}^{\pi} \sin(0.5\phi)\,d\phi \int_{z=0}^{5} dz \times \cancel{\overline{a}_\rho\cdot\overline{a}_\rho}^{\,1}\Big|_{\rho=2m}$$

$$= 6(2)^2 \times 2 \times 5 \times 1 = \underline{\underline{240 \text{ Coulomb's}}}$$

$$\psi\Big|_{\phi=0^c} = \int_{\langle S\rangle} \overline{D_\phi}\cdot\overline{ds} = \int_{\langle S\rangle} 1.5\rho\cos(0.5\phi)\,\overline{a}_\phi\cdot d\rho\,dz\,(-\overline{a}_\phi)\Big|_{\phi=0^c}$$

$$= -1.5\cos(0.5\phi)\int_{\rho=0}^{2}\rho\, d\rho \int_{z=0}^{5} dz\ \overline{a}_\phi \cdot \overline{a}_\phi \Big|_{\phi=0^c}$$

$$= -1.5 \times \cos(0) \times 2 \times 5 \times 1 = \underline{\underline{-15 \text{ Coulomb's}}}$$

$$\Psi_{\phi=\pi^c} = \int_{\langle S\rangle} \overline{D}_\phi \cdot \overline{ds} = \int_{\langle S\rangle} 1.5\rho\cos(0.5\phi)\overline{a}_\phi \cdot d\rho\, dz\,(\overline{a}_\phi)\Big|_{\phi=\pi}$$

$$= 1.5\cos(0.5\phi)\int_{\rho=0}^{2}\rho\, d\rho \times \int_{z=0}^{5} dz \times \overline{a}_\phi \cdot \overline{a}_\phi \Big|_{\phi=\pi^c}$$

$$= 1.5 \times \cos(\pi/2) \times 2 \times 5 \times 1 = \underline{\underline{0\ C}}$$

$$\therefore \Psi_{total} = \Psi\big|_{\rho=2m} + \Psi\big|_{\phi=0^c} + \Psi\big|_{\phi=\pi^c}$$

$$= 240 - 15 + 0 = \underline{\underline{225}} \text{ Coulomb's}$$

$$\boxed{\Psi_{total} = \oint_{\langle S\rangle} \overline{D}\cdot\overline{ds} = 225} \text{ Coulomb's} \leftarrow (a)$$

(or)

<u>R.H.S</u> $\overline{D} = 6\rho\sin(0.5\phi)\,\overline{a}_\rho + 1.5\rho\cos(0.5\phi)\,\overline{a}_\phi\ C/m^2.$

$$\int_{\langle vol\rangle} (\nabla\cdot\overline{D})\, dv = ? \qquad dv = \rho\, d\rho\, d\phi\, dz.$$

$$\nabla\cdot\overline{D} = \frac{1}{\rho}\frac{\partial}{\partial\rho}[\rho D_\rho] + \frac{1}{\rho}\frac{\partial D_\phi}{\partial\phi} + \cancel{\frac{\partial D_z}{\partial z}}^{0} \ C/m^3$$

bcz $D_z = 0$

$$\nabla\cdot\bar{D} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left[\rho\cdot 6\rho\sin(0.5\phi)\right] + \frac{1}{\rho}\frac{\partial}{\partial\phi}\left[1.5\rho\cos(0.5\phi)\right]$$

$$= \frac{6\sin(0.5\phi)}{\cancel{\rho}}\times 2\cancel{\rho} + \frac{1.5\cancel{\rho}}{\cancel{\rho}}\times -\sin(0.5\phi)\times 0.5$$

$$\boxed{\nabla\cdot\bar{D} = 12\sin(0.5\phi) - 0.75\sin(0.5\phi) = 11.25\sin(0.5\phi)\ C/m^3}$$

$$\nabla\cdot\bar{D} = 11.25\sin(0.5\phi)\ C/m^3$$

$$\int_{\langle vol\rangle}(\nabla\cdot\bar{D})\,dv = \int_{\langle vol\rangle} 11.25\sin(0.5\phi)\ \rho\,d\rho\,d\phi\,dz$$

$$= 11.25\int_{\rho=0}^{2}\rho\,d\rho \times \int_{\phi=0}^{\pi}\sin(0.5\phi)\,d\phi \times \int_{z=0}^{5} dz$$

$$= 11.25\times 2\times 2\times 5 = \underline{225\ \text{Coulombs}}$$

$$\boxed{\int_{\langle vol\rangle}(\nabla\cdot\bar{D})\,dv = 225\ \text{Coulombs}} \longleftarrow (b)$$

$\therefore$ eq^n (a) = eq^n (b) Divergence theorem is verified.

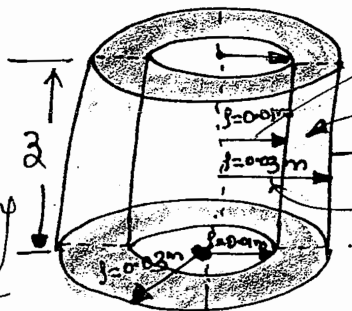
problem 42

Page No-16 SB

Volume charge density $\rho_v = 0$ for $\rho < 0.01m$ and also $\rho > 0.03m$. In the region $0.01 < \rho < 0.03m$ $\rho_v = 10^{-8}\cos(50\pi\rho)$ C/m³. Find Electric flux density $\overline{D}$ every where. --- 06-DEC/Jan 2009. (7 Marks).

Sol'n:

$$\rho_v = \begin{cases} 10^{-8}\cos(50\pi\rho)\ C/m^3 & ; 0.01 < \rho < 0.03m \\ 0 ; & o.w. \end{cases}$$



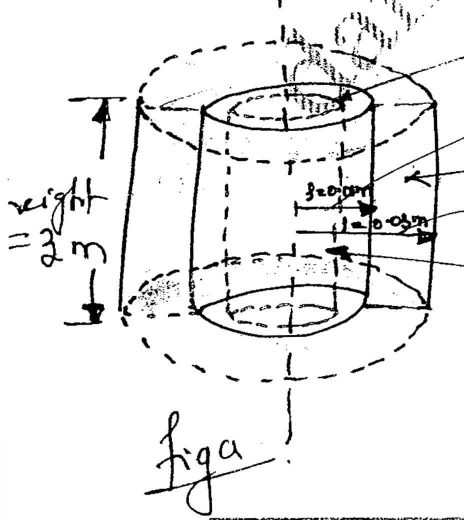
the flux density $\overline{D}$ must be radially out.

i.e $\boxed{\overline{D} = D_\rho \overline{a}_\rho\ C/m^2}$

$\overline{D}$, i.e electric flux density every where is defined by the following cases

i) $\rho < 0.01m$ ii) $0.01 < \rho < 0.03m$ and iii) $\rho > 0.03m$

Case i. when $\rho < 0.01m$. $\overline{D} = D_\rho \overline{a}_\rho$ C/m².



Since Gaussian surface under $\rho_v = 0$ C/m3

$\therefore Q_{enc} = 0$ C

$D_\rho = \psi / Area = \dfrac{Q_{enclosed}}{Area}$.

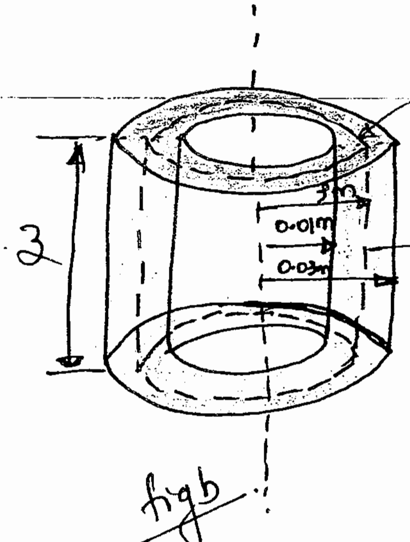
Since $\rho_v = 0$

$\therefore Q_{en} / \rho_v \langle 0.01 \rangle = 0$ C

$\therefore D_\rho = 0$ C/m²

(77) $\Rightarrow \boxed{\overline{D} = 0\ C/m^2}$; $\rho < 0.01m$

Case ii $\quad 0.01 < \rho < 0.03\,m.$



$$D_\rho = \frac{\Psi_{total}}{A}$$

Area of the Gaussian Surface $\Rightarrow$ $2\pi\rho \times z$

$$\boxed{A = 2\pi\rho z}\ m^2$$

and $\Psi_{total} = Q_{encl} = \int_{\langle vol \rangle} \rho_v \, dv$

$\rho_v = 10^{-8}\cos(50\pi\rho)\ C/m^3$ and $dv = \rho\, d\rho\, d\phi\, dz$.

$$\Psi_{total} = Q_{encl} = \int_{\langle vol \rangle} 10^{-8}\cos(50\pi\rho) \times \rho\, d\rho\, d\phi\, dz \quad C$$

$$= 10^{-8}\int_{\rho=0.01}^{\rho} \rho\cos(50\pi\rho)\, d\rho \times \int_{\phi=0}^{2\pi} d\phi \times \int_{z=0}^{z} dz \quad C$$

$$= 10^{-8}\left[\rho \times \frac{+\sin(50\pi\rho)}{50\pi}\Big|_{0.01}^{\rho} - \times \frac{-\cos(50\pi\rho)}{(50\pi)^2} \times 1 \Big|_{0.01}^{\rho}\right] \times 2\pi \times z$$

$$= 10^{-8}\left[\frac{1}{50\pi}\left[\rho\sin(50\pi\rho) - 0.01\sin(50\pi \times 0.01)\right] + \frac{1}{(50\pi)^2}\left[\cos(50\pi\rho) - \cos[50\pi(0.01)]\right] \times 2\pi z\right]$$

$$= 10^{-8}\left[\frac{\rho\sin(50\pi\rho)}{50\pi} - \frac{0.01}{50\pi} + \frac{\cos(50\pi\rho)}{(50\pi)^2} - 0\right] 2\pi z$$

$$D_\rho = \frac{\Psi_{total}}{A} = \frac{10^{-8}\left[\frac{\rho\sin(50\pi\rho)}{50\pi} - \frac{0.01}{50\pi} + \frac{\cos(50\pi\rho)}{(50\pi)^2}\right]\cancel{2\pi}\cancel{3}}{\cancel{2\pi}\rho\cancel{3}}$$

$$= \frac{10^{-8}}{\rho}\left[\frac{\rho\sin(50\pi\rho)}{50\pi} - \frac{0.01}{50\pi} + \frac{\cos(50\pi\rho)}{(50\pi)^2}\right]$$

$$D_\rho = 10^{-8}\left[\frac{\sin(50\pi\rho)}{50\pi} - \frac{0.01}{50\pi\rho} + \frac{\cos(50\pi\rho)}{(50\pi)^2\rho}\right] C/m^2$$

$$= \left[\frac{\sin(50\pi\rho)}{50\pi} + \frac{\cos(50\pi\rho)}{(50\pi)^2\rho} - \frac{0.01}{50\pi\rho}\right] \times \frac{10^{-8}\times 10^{-4}}{10^{-4}} \; c/m^2$$

10^{-12}

$$\therefore D_\rho = \left[\frac{1}{50\times10^{-4}}\cdot\frac{\sin(50\pi\rho)}{\pi} + \frac{1}{50^2\times10^{-4}}\frac{\cos(50\pi\rho)}{\pi^2\rho} - \frac{0.01}{10^{-4}\times 50\pi\rho}\right]\times 10^{-12} c/m^2$$

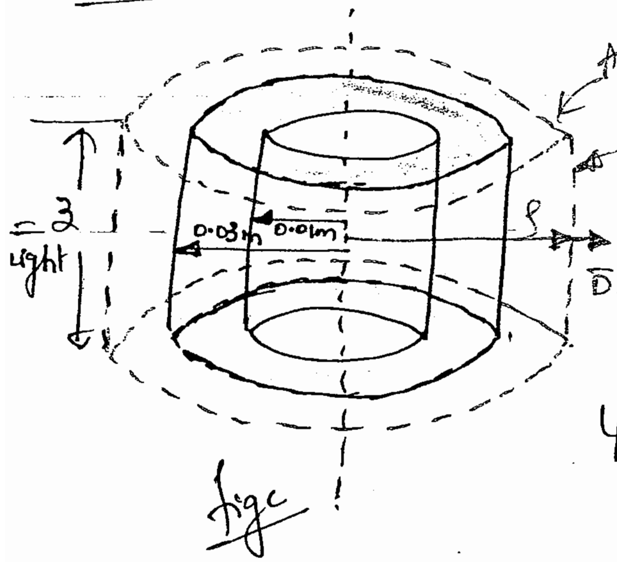
$$D_\rho = \left[\frac{200\sin(50\pi\rho)}{\pi} + \frac{4\cos(50\pi\rho)}{\pi^2\rho} - \frac{2}{\pi\rho}\right] pc/m^2$$

$$\therefore \overline{D} = D_\rho\,\overline{a_\rho} \; c/m^2$$

$$\boxed{\overline{D} = \left[\frac{200\sin(50\pi\rho)}{\pi} + \frac{4\cos(50\pi\rho)}{\pi^2\rho} - \frac{2}{\pi\rho}\right]\overline{a_\rho}\; pc/m^2}$$

$0.01 < \rho < 0.03m$

Case iii. $\rho > 0.03m$.



and $\rho_v \neq 0$ b/w $0.01 < \rho < 0.03m$

$\bar{D} = D_\rho \bar{a}_\rho$ C/m²

$D_\rho = \frac{\Psi_{total}}{A}$ C/m².

$\boxed{Area = 2\pi\rho z}$ m²

$\Psi_{total} = Q_{enclosed} = \int_{\langle vol \rangle} \rho_v dv$

$\Psi_{total} = Q_{encl} = \int_{\langle vol \rangle} \rho_v dv = \int_{\langle vol \rangle} 10^{-8} \cos(50\pi\rho) \rho \, d\rho \, d\phi \, dz$

$= 10^{-8} \int_{\rho=0.01}^{0.03} \rho \cos(50\pi\rho) \, d\rho \times \int_{\phi=0}^{2\pi} d\phi \times \int_{z=0}^{z} dz$

$= 10^{-8} \times -2.5465 \times 10^{-4} \times 2\pi \times z = \underline{-2.5465 \times 10^{-12} \times 2\pi z}$ C

$D_\rho = \frac{\Psi_{total}}{A} = \frac{-2.5465 \times 10^{-12} \times \cancel{2\pi z}}{\cancel{2\pi}\rho \cancel{z}} = -\frac{2.5465}{\rho}$ pC/m²

$\boxed{\bar{D} = D_\rho \bar{a}_\rho = -\frac{2.5465}{\rho} \bar{a}_\rho}$ pC/m²

$$\bar{D} = \begin{cases} 0 \; ; & \rho < 0.01m \\ \left[\frac{200\sin(50\pi\rho)}{\pi} + \frac{4\cos(50\pi\rho)}{\pi^2\rho} - \frac{2}{\pi\rho}\right] \bar{a}_\rho \text{ pC/m}^2 ; & 0.01 < \rho < 0.03 \\ -\frac{2.5465}{\rho} \bar{a}_\rho \text{ pC/m}^2 \; ; & \rho > 0.03m \end{cases}$$

(a) **2nd method** :- using Maxwell's first equation.

given $\rho_v = \begin{cases} 0 ; & \rho \le 0.01m \\ 10^{-8}\cos(50\pi\rho)\ C/m^3 ; & 0.01 < \rho < 0.03m \\ 0 ; & ow. \end{cases}$

$\bar{D} = D_\rho \bar{a}_\rho\ C/m^2$.

Case i. ie when $\rho < 0.01m$. $\rho_v = 0\ C/m^3$ given

from Maxwell's first eq^n $\nabla \cdot \bar{D} = \rho_v\ C/m^3$

$$\nabla \cdot \bar{D} = 0$$

is valid only when $\bar{D} = 0 \Rightarrow \boxed{D_\rho = 0}\ C/m^2$

$\therefore \boxed{\bar{D} = 0}\ C/m^2$; when $\rho < 0.01m$

Case ii. when $0.01 < \rho < 0.03m$.

$$\nabla \cdot \bar{D} = \rho_v\ C/m^3$$

Since $\bar{D}$ is fn of only D_ρ component.

$$\therefore \nabla \cdot \bar{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho \cdot D_\rho) = \rho_v$$

$$\Rightarrow \frac{\partial}{\partial \rho}[\rho \cdot D_\rho] = \rho \cdot \rho_v$$

$$\rho \cdot D_\rho = \int_{\langle \rho \rangle} \rho \cdot \rho_v\, d\rho.$$

$$\rho \cdot D_\rho = \int_{\rho=0}^{\rho} (\rho \cdot \rho_v)\, d\rho = \cancel{\int_{\rho=0}^{0.01m} \rho\, \rho_v\, d\rho} + \int_{\rho=0.01}^{\rho} \rho \cdot \rho_v\, d\rho$$

$\rightarrow 0$ bcz $\rho_v = 0$, for $\rho < 0.01m$

refer fig b.

$$\rho \cdot D_\rho = \int_{\rho=0.01}^{\rho} \rho \cdot \times 10^{-8} \cos(50\pi\rho)\, d\rho$$

$$= 10^{-8}\left[\rho \times \frac{\sin(50\pi\rho)}{50\pi}\Bigg|_{0.01}^{\rho} - \int_{\rho=0.01}^{\rho} \left(\frac{\sin(50\pi\rho)}{50\pi}\right) \times 1\, d\rho \right]$$

$$= 10^{-8}\left[\frac{\rho \sin(50\pi\rho)}{50\pi} - \frac{0.01 \sin(50\pi \times 0.01)}{50\pi} + \frac{\cos(50\pi\rho)}{(50\pi)^2}\Bigg|_{0.01}^{\rho} \right]$$

$$= 10^{-8}\left[\frac{\rho \sin(50\pi\rho)}{50\pi} - \frac{0.01 \cancel{\sin(\pi/2)}^{1}}{50\pi} + \frac{\cos(50\pi\rho)}{(50\pi)^2} - \frac{\cancel{\cos(50\pi \times 0.01)}^{0}}{(50\pi)^2} \right]$$

bq $\cos\pi/2 = 0$

$$= 10^{-8}\left[\frac{\rho \sin(50\pi\rho)}{50\pi} - \frac{0.01}{(50\pi)} + \frac{\cos(50\pi\rho)}{50^2 \times \pi^2} \right]$$

$$D_\rho = 10^{-8}\left[\frac{\sin(50\pi\rho)}{50\pi} - \frac{0.01}{\rho\,(50\pi)} + \frac{\cos(50\pi\rho)}{50^2 \times \pi^2 \rho} \right] C/m^2$$

$$D_\rho = \left[\frac{200 \sin(50\pi\rho)}{\pi} - \frac{2}{\pi\rho} + \frac{4\cos(50\pi\rho)}{\pi^2 \rho} \right] \times 10^{-12}\, C/m^2$$

$$\therefore \quad \overline{D} = D_\rho\, \overline{a_\rho} \quad C/m^2$$

$$\boxed{\overline{D} = \left[\frac{200 \sin(50\pi\rho)}{\pi} - \frac{2}{\pi\rho} + \frac{4\cos(50\pi\rho)}{\pi^2 \rho} \right] p\, C/m^2}$$

$\longrightarrow 0.01 < \rho < 0.03\,m.$

and $\overline{D} = 0\, C/m^2$ when $\rho = 0.01\,m$.

Case iii ie when $\rho > 0.03\,m$. --- follow the same procedure as case ii.

$$\Rightarrow \rho \cdot D_\rho = \int \rho \cdot \rho_v \, d\rho$$

$\rho < 0.01$; $\rho_v = 0$; $\rho > 0.03$, $\rho_v = 0$

$$\rho \cdot D_\rho = \cancel{\int_{\rho=0}^{0.01} \rho \cdot \rho_v \, d\rho} + \int_{0.01}^{0.03} \rho \cdot \rho_v \, d\rho + \cancel{\int_{\rho=0.03}^{\rho} \rho \cdot \rho_v \, d\rho}$$

$$\rho \cdot D_\rho = \int_{0.01}^{0.03} \rho \times 10^{-8} \cos(50\pi\rho)\, d\rho$$

use calci.

$$\rho \cdot D_\rho = -2.5465 \times 10^{-4} \times 10^{-8}$$

$$D_\rho = \frac{-2.5465}{\rho} \times 10^{-12}\ C/m^2$$

$$\bar{D} = D_\rho \bar{a}_\rho \ C/m^2$$

$$\therefore \boxed{\bar{D} = -\frac{2.5465}{\rho}\bar{a}_\rho} \ pC/m^2$$

$$\bar{D} = \begin{cases} 0; & \rho \le 0.01\,m \\ \left[\frac{200\sin(50\pi\rho)}{\pi} + \frac{4\cos(50\pi\rho)}{\pi^2\rho} - \frac{2}{\pi\rho}\right]\bar{a}_\rho\ pC/m^2; & 0.01 < \rho < 0.03 \\ -84.8933\,\bar{a}_\rho\ pC/m^2; & \rho = 0.03\,m \\ -\frac{2.5465}{\rho}\bar{a}_\rho\ pC/m^2; & \rho > 0.03\,m \end{cases}$$

$-84.8933\,\bar{a}_\rho\ pC/m^2$; $\rho = 0.03\,m$.

Module-2 Part B

Topics: 2.8

- Energy expended in moving a point charge in an electric field
- ~~The line integral~~

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2.8. Energy expended in Moving a point charge in an Electric field (E)

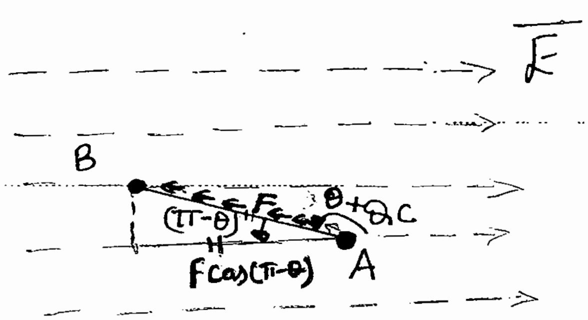

fig a. Moving a charge in an Uniform field (E)

Consider a charge +QC at a point A in a uniform electric field $\bar{E}$.

the Force acting on a charge Q is

$$\boxed{\bar{F} = Q\bar{E}} \ N$$

the direction of force acts in the direction of the field.

the magnitude of force $F = QE \leftarrow (a)$.

Let us Consider that charge is moving through a distance ΔL A to B, which is along an arbitrary direction, say along inclined at an angle θ to the direction of the field.

Since the charge is moving against the field. for such a Movement the force acting on charge 'Q' is

Force acting on charge $= F\cos(\pi - \theta)$ Newton's

using eq^n (a)

$$= QE\cos(\pi - \theta) \quad (\cos(\pi-\theta) = -\cos\theta)$$

$$\therefore \boxed{\text{Force acting} = -QE\cos\theta}$$

$\Rightarrow$ workdone = Force acting $\times$ displacement

$$= -QE\cos\theta \times \Delta L$$

$$W = -QE\cos\theta(\Delta L)$$

$$\boxed{W = -Q\,\overline{E}\cdot\overline{\Delta L}} \text{ Joules.}$$

Def^{n} of dot product

$\overline{E}\cdot\overline{\Delta L} = E\,\Delta L\cos\theta$

$\overline{A}\cdot\overline{B} = AB\cos\theta$

If the charge is moved through a differential distance $\overline{dl}$ in the field $\overline{E}$, then the differential workdone

$$dw = -Q\,\overline{E}\cdot\overline{dl}$$

the total workdone is given by

$$W = -Q\int_A^B \overline{E}\cdot dl = -Q\int_{\text{initial}}^{\text{final}} \overline{E}\cdot \overline{dl} \text{ Joules.}$$

$$\Rightarrow \boxed{W = -Q\int_{\text{initial}}^{\text{final}} \overline{E}\cdot\overline{dl}} \text{ Jouls (or) N.m}$$

Definition

Def^{n} of Workdone :- Work is said to be done when the test charge is moved against the electric field.

Key Note points :-

1. NO Workdone is required to move a point charge along the direction perpendicular to the field ($\bar{E}$).

i.e $\bar{E}\cdot\bar{dl} = 0$ when $\bar{E}$ and $\bar{dl}$ are $\perp^{r}$ Eachother.

2. $\bar{E}\cdot\bar{dl} \neq 0$; when $\bar{E}$ and $\bar{dl}$ are ~~not~~ perpendicular.

3. Differential length vectors in all three-coordinate Systems :-

problem 43

Q7. Write the Expression for $\bar{E}$ and $\bar{dl}$ with units in i) Cartesian System ii) Cylindrical System and iii) Spherical System. (06 marks) 02-J/J-2010.

Sol:- i) Cartesian Coordinate System

$$\bar{E} = E_x \bar{a}_x + E_y \bar{a}_y + E_z \bar{a}_z \ \ V/m.$$

$$P(x, y, z) \quad x \to dx, \; y \to dy, \; z \to dz$$

$$\bar{dl} = dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z \; ; \; m$$

$$\boxed{\bar{E}\cdot\bar{dl} = E_x dx + E_y dy + E_z dz} \ \text{Volts}$$

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ii) Cylindrical Co-ordinate System.

$$P(\rho, \phi, z) \rightarrow d\rho,\ \rho\, d\phi,\ dz$$

$$\overline{E} = E_\rho \overline{a_\rho} + E_\phi \overline{a_\phi} + E_z \overline{a_z} \ ; \ V/m$$

$$\overline{dl} = d\rho\, \overline{a_\rho} + \rho\, d\phi\, \overline{a_\phi} + dz\, \overline{a_z} \ ; \ m.$$

$$\boxed{\overline{E} \cdot \overline{dl} = E_\rho\, d\rho + \rho E_\phi\, d\phi + E_z\, dz} \text{ volts}$$

iii) Spherical Co-ordinate System.

$$\overline{E} = E_r \overline{a_r} + E_\theta \overline{a_\theta} + E_\phi \overline{a_\phi} \ ; \ V/m$$

$$P(r, \theta, \phi) \rightarrow dr,\ r\, d\theta,\ r\sin\theta\, d\phi$$

$$\overline{dl} = dr\, \overline{a_r} + r\, d\theta\, \overline{a_\theta} + r\sin\theta\, d\phi\, \overline{a_\phi} \ ; \ m$$

$$\boxed{\overline{E} \cdot \overline{dl} = E_r\, dr + r E_\theta\, d\theta + r\sin\theta E_\phi\, d\phi} \text{ volts.}$$

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Problem 44 An Electric Field is given by $\bar{E} = (\frac{x}{2} + 2y)\bar{a}_x + 2x\bar{a}_y$ N/C. Find the work done in Moving a point charge $Q = -20\mu C$.

i) from the origin to (4,0,0) m.

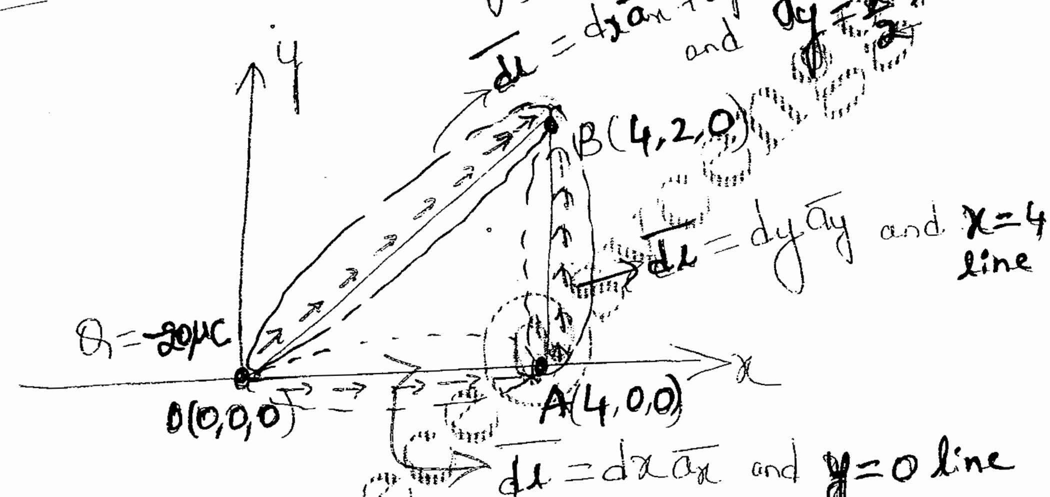
ii) from (4,0,0) to (4,2,0) m.

iii) from origin to (4,2,0) m.

iv) from (4,2,0) to origin.

Solu:

$$\bar{E} = \left(\frac{x}{2} + 2y\right)\bar{a}_x + 2x\bar{a}_y \text{ N/C}$$



$$\boxed{W = -Q\int_{\text{initial}}^{\text{final}} \bar{E}\cdot\overline{dL} \text{ Joules}}$$

i) $W_{OA} = -Q\int_O^A \bar{E}\cdot\overline{dL} = -(-20\mu)\int_{O(0,0,0)}^{A(4,0,0)} \left(\frac{x}{2} + 2y\right)dx$

but on path OA the value of y is zero $\therefore y = 0$

$$W_{OA} = +20\mu\int_{x=0}^{4}\left(\frac{x}{2}\right)dx = 20\mu \times 4 = 80\mu \text{ Joules}$$

$$\boxed{W_{OA} = 80\mu} \text{ Joules}$$

ii) $$W_{AB} = -Q\int_A^B \overline{E}\cdot\overline{dl} = -(-20\mu)\int_{(4,0,0)}^{(4,2,0)} 2x\,dy \Big|_{x=4}$$ on path A to B

$$= +20\mu \int_{y=0}^{2} 2(4)\,dy = 20\mu \times 8 \times 2$$

$$\boxed{W_{AB} = 320\mu}$$ Joules

iii) $$W_{OB} = -Q\int_O^B \overline{E}\cdot\overline{dl} = -(-20\mu)\int_{O(0,0,0)}^{B(4,2,0)} [E_x dx + E_y dy]$$

$$= +20\mu \int_O^B \left\{\left[\frac{x}{2}+2y\right]dx + 2x\,dy\right\} \Big|_{y = x/2}$$ eq^n of line b/w O to B.

$$= +20\mu\left[\int_{x=0}^{4}\left(\frac{x}{2}+\cancel{2}\cdot\frac{x}{\cancel{2}}\right)dx + \int_{y=0}^{2} 2(2y)\,dy\right]$$

$$= +20\mu\left[\int_{x=0}^{4} 1.5x\,dx + \int_{y=0}^{2} 4y\,dy\right]$$

$$= +20\mu\left[12 + 8\right] = 400\mu \text{ Joules}$$

$$\boxed{W_{OB} = 400\mu}$$ Joules

iv) $$W_{BO} = -Q\int_B^O \overline{E}\cdot\overline{dl} = -W_{OB} = -400\mu \text{ Joules}$$

$$\boxed{W_{BO} = -400\mu}$$ Joules

Imp. Observations :- $\Rightarrow$ $\boxed{W = -Q \int_{initial}^{final} \bar{E} \cdot \bar{dl}}$ Joules ← ⓐ

1. −ve Sign eqⁿ ⓐ indicates that point charge is moved against the field $\bar{E}$.
2. Work done is Scalar in nature.
3. 'W' is +ve when charge 'Q' is moved against the direction of $\bar{E}$. i.e external Source is required.
4. 'W' is −ve when charge 'Q' is moved in the direction of field $\bar{E}$. i.e NO External Source is required.
5. Workdone is independent of the path selected from O to B but it depends on end points O and B. [Ref. problem 44]

i.e $\underset{\uparrow \text{path-2}}{W_{OB}} = \underbrace{W_{AB} + W_{OA}}_{\text{path-1}} = 320\mu + 80\mu = 400\mu J.$

6. NO Work done (i.e W = 0 Joules) is required to move a point charge of Q C over a closed path i.e starting point and ending points both are same.

i.e $W_{OA} + W_{AB} + W_{BO}$ $\Rightarrow$ closed path

$80\mu + 320\mu - 400\mu = 0$ Joules.

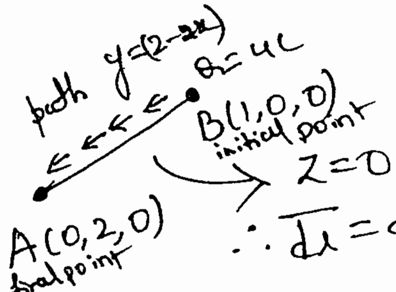
7. W = 0 when the path selected is $\perp^{r}$ to the field $\bar{E}$.

Q) Calculate the Workdone in moving a 4C charge from B(1,0,0) to A(0,2,0) along the path $y = 2-2x$, $z=0$ in the field i) $\bar{E} = 5\bar{a}_x$ V/m ii) $\bar{E} = 5x\bar{a}_x$ V/m

iii) $\bar{E} = 5x\bar{a}_x + 5y\bar{a}_y$ V/m.

Sol:-

$$W = -Q\int_{initial}^{final} \bar{E}\cdot\bar{dl}$$



$\therefore \bar{dl} = dx\,\bar{a}_x + dy\,\bar{a}_y$

$$W = -Q\int_B^A \bar{E}\cdot\bar{dl}$$

i) $\bar{E} = 5\bar{a}_x$ V/m and $\bar{dl} = dx\,\bar{a}_x + dy\,\bar{a}_y$ m

$\bar{E}\cdot\bar{dl} = 5dx$

$$W_{AB} = -(4)\int_{x=1}^{0} 5dx = -4\times -5 = +20 \text{ Joules}$$

$$\boxed{W_{AB} = 20} \text{ joules}$$

ii) When $\bar{E} = 5x\bar{a}_x$ V/m ; $\bar{dl} = dx\,\bar{a}_x + dy\,\bar{a}_y$ m

$\bar{E}\cdot\bar{dl} = 5x\,dx$

$$W_{AB} = -4\int_{x=1}^{0} 5x\,dx = -4(-2.5) = \underline{10 \text{ Joules}}$$

$$\boxed{W_{AB} = 10} \text{ Joules}$$

iii) When $\bar{E} = 5x\bar{a}_x + 5y\bar{a}_y$ V/m

$\bar{E}\cdot\bar{dl} = 5x\,dx + 5y\,dy$

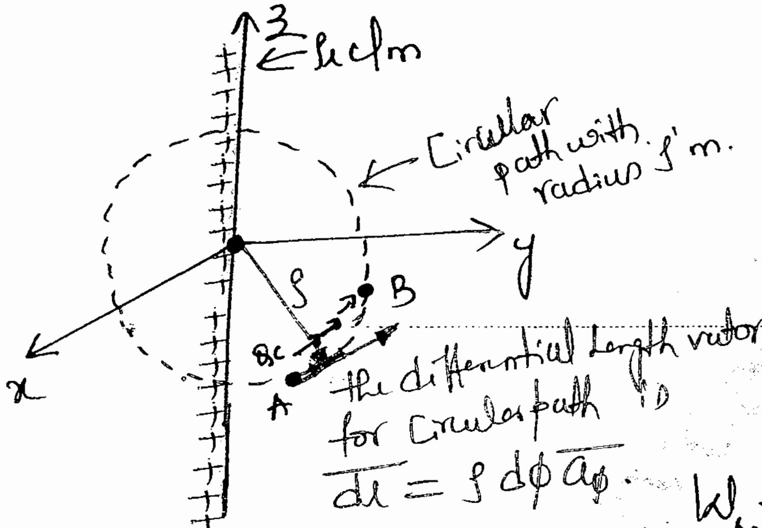
$$W_{AB} = -4\left[\underbrace{\int_{x=1}^{0} 5x\,dx}_{-2.5} + \underbrace{\int_{y=0}^{2} 5y\,dy}_{10}\right] = -4\left[-2.5+10\right] = -30 \text{ joules}$$

$$\boxed{W_{AB} = -30} \text{ Joules}$$

Case Study

Case i. Show that No workdone is required to Move a point charge of Q C along Circular path with an Electric field Intensity (E) due to Infinite Line charge.

Soln:- Consider a Infinite Line charge placed along z axis



$\overline{dl} = \rho\, d\phi\, \overline{a_\phi}$

fig. Circular path.

w.k.t the field E due to an infinite Line charge is

$$\overline{E} = \frac{\rho_L}{2\pi\epsilon\rho}\overline{a_\rho} \text{ V/m} \quad \leftarrow (a)$$

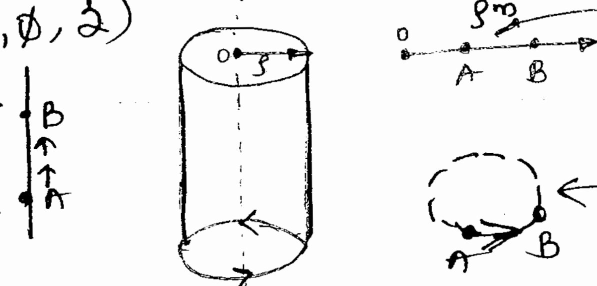
$$W_{AB} = -Q\int_A^B \overline{E}\cdot\overline{dl}$$

$$W_{AB} = -Q\int_A^B \frac{\rho_L}{2\pi\epsilon\rho}\overline{a_\rho}\cdot\rho\, d\phi\, \overline{a_\phi}$$

$$W_{AB} = -Q\int_A^B \frac{\rho_L}{2\pi\epsilon\rho}\times\rho\, d\phi\; \cancel{\overline{a_\rho}\cdot\overline{a_\phi}}^{\,0} = 0 \quad \left[\overline{a_\rho}\cdot\overline{a_\phi} = 0 \text{ (dot product concept)}\right.$$

$\boxed{W_{AB} = 0}$ Joules this shows that the workdone is zero when charge Q C is moving in a Circular path [i.e path Selected is perpendicular to the field $\overline{E}$].

Note:- $P(\rho, \phi, z)$



Vertical path $\overline{dl} = dz\,\overline{a_z}$

radial path $\overline{dl} = d\rho\,\overline{a_\rho}$ m.

Circular path $\overline{dl} = \rho\, d\phi\, \overline{a_\phi}$

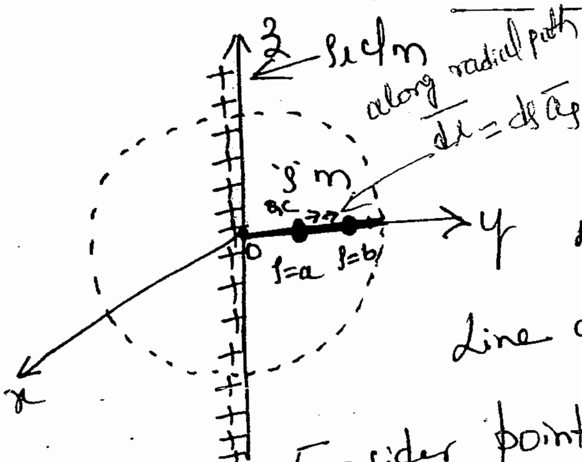
$P(\rho, \phi, z)$ $d\rho$ $\rho d\phi$ dz

Case ii

Question

06 - June /July 2011

Obtain the expression for the work done in bringing a charge 'Q' from one point to another point along the radial path in an electric field due to an infinite line charge. Hence find the potential difference between that two points. (06 Marks)



Consider an infinite line charge placed along 'z' axis w.r.t the field due to infinite line charge $\bar{E} = \frac{\rho_L}{2\pi\epsilon\rho}\bar{a}_\rho$ V/m. ← (a)

Consider point charge of Q C, which is moved from $\rho = $ 'a' m to $\rho = $ 'b' m radially along a path radially the workdone required is $\boxed{W = -Q\int_a^b \bar{E}\cdot\bar{dl}}$ Joules ← (b)

$$\therefore W_{ab} = -Q\int_{\rho=a}^{b} \frac{\rho_L}{2\pi\epsilon\rho}\bar{a}_\rho \cdot d\rho\,\bar{a}_\rho = -Q\int_{\rho=a}^{b}\frac{\rho_L}{2\pi\epsilon\rho}\, d\rho\, \cancel{\bar{a}_\rho\cdot\bar{a}_\rho}^{\,1}$$

$$\therefore W_{ab} = -Q \times \frac{\rho_L}{2\pi\epsilon}\int_{\rho=a}^{b} \frac{1}{\rho}\, d\rho = -Q\frac{\rho_L}{2\pi\epsilon}\ln\rho\Big|_a^b$$

note
$\boxed{\int \frac{1}{x}dx = \ln x}$
$\log m/n = \log m - \log n$

$$W_{ab} = -Q\frac{\rho_L}{2\pi\epsilon}[\ln b - \ln a]$$

$$\boxed{W_{ab} = -Q\frac{\rho_L}{2\pi\epsilon}\ln(b/a)} \text{ Joules}$$

$$\Rightarrow W_{ba} = -Q\,\rho_L/2\pi\epsilon\,\ln(a/b) = +Q\frac{\rho_L}{2\pi\epsilon}\ln(b/a) = -W_{ab}.$$

the potential difference b/w points a & b is V_{ab}

$$\boxed{V_{ab} = \frac{W_{ba}}{Q \to 1C} = \frac{\rho_L}{2\pi\epsilon}\ln(b/a)} \text{ Volts}$$

i.e $$\boxed{V_{ab} = \frac{W_{ba}}{\cancel{Q}1C} = \frac{\rho_L}{2\pi\epsilon}\ln(b/a)}$$ Volt's

Problem 45. Find the workdone in moving a charge +2C from (2,0,0)m to (0,2,0)m along the straight line path joining two points, if the $\bar{E} = 12x\,\bar{a}_x - 4y\,\bar{a}_y$ V/m. Aug 05 (6M).

Sol^n:

$Q = 2C$, $A(2,0,0)$, $B(0,2,0)$m

$$W_{AB} = -Q\int_A^B \bar{E}\cdot\bar{dl}$$

$$\bar{dl} = dx\,\bar{a}_x + dy\,\bar{a}_y$$

$$\bar{E}\cdot\bar{dl} = 12x\,dx - 4y\,dy \quad \text{Volt's}$$

$$W_{AB} = -2\left[\int_{x=2}^{0} 12x\,dx - \int_{y=0}^{2} 4y\,dy\right]$$

$$W_{AB} = -2(-24 - 8) = +64 \text{ Joules}$$

$$\boxed{W_{AB} = +64} \text{ Joules}$$

(94)

Topic 2.9 The Line Integral :-

Question Prove that the work done, in moving a charge Q from initial position B to final position A, in uniform electric field $\bar{E}$, does not depend upon the path. (06 Marks)

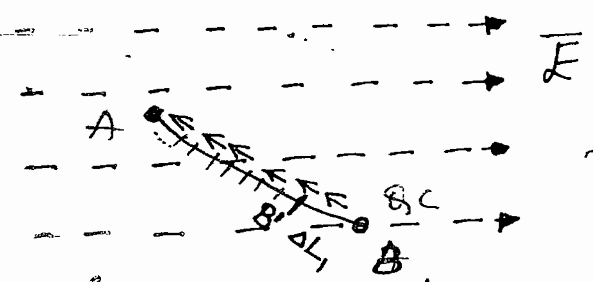


fig. Line integral of $\bar{E}$

Consider a point charge at a point B in a uniform field Intensity $\bar{E}$. it is required to find the workdone in moving the charge from B to A along an arbitrary path as shown in fig.

W.k.t the workdone ΔW is moving the charge through a small length ΔL from B to B' is given by

$$\Delta W = -Q(\bar{E}\cdot\overline{\Delta L}) \text{ Joules} \leftarrow (a)$$

if the total path from B to A is divided into Large No. of segments. Let their Lengths be $\overline{\Delta L_1}, \overline{\Delta L_2}, \ldots$ etc and let the field over the respective lengths be $\bar{E}_1, \bar{E}_2, \bar{E}_3 \ldots\ldots$ etc

∴ the total workdone W in moving the charge 'Q' from B to A is

$$W = \sum \Delta W = -Q\left[\bar{E}_1\cdot\overline{\Delta L_1} + \bar{E}_2\cdot\overline{\Delta L_2} + \cdots\right]$$

$$= -Q\sum_i \bar{E}_i\cdot\overline{\Delta L_i} \text{ Joules.}$$

if the lengths of the segments are made infinitely small then $\Delta L \rightarrow dl$ and $\sum \rightarrow \int$

$$\therefore \boxed{W = -Q\int_B^A \bar{E}\cdot\overline{dl}} \text{ Joules} \leftarrow (b)$$

Since the applied field $\bar{E}$ is uniform $\therefore$ eq^n (b) becomes

$$W = -Q\,\bar{E} \cdot \int_B^A \overline{dL} = -Q\,\bar{E} \cdot \bar{L}_{BA} \text{ Joules}$$

$$\boxed{W = -Q\,\bar{E} \cdot \bar{L}_{BA}} \text{ Joules} \leftarrow (c).$$

in a uniform $\bar{E}$.

Conclusion:-

from eq^n (c). the workdone involved in moving the charge depends only on Q, $\bar{E}$ and a vector drawn from initial (B) to final (A) point of the path choosen ($\bar{L}_{BA}$). it doesnot depends on the particular path we have selected along which to carry the charge.

problem 46

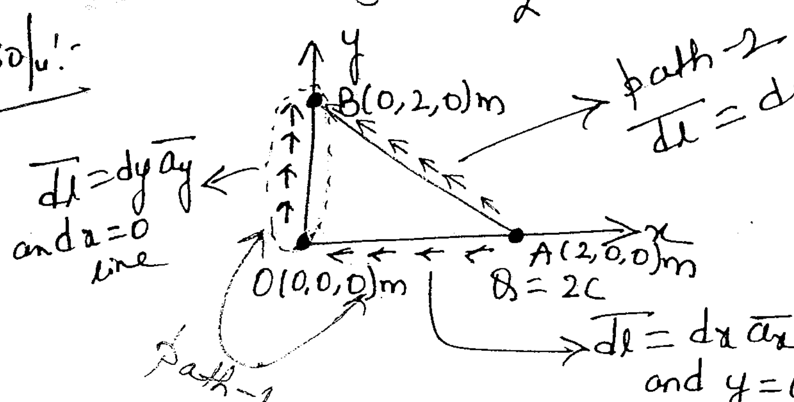
8) $\bar{E} = 2x\,\bar{a}_x - 4y\,\bar{a}_y$ V/m. Find the workdone in moving a point charge of +2C

i) (2,0,0)m to (0,0,0)m and then from (0,0,0) to (0,2,0)m.

ii) from (2,0,0)m to (0,2,0) along the straight line path joining the two points. [15-Dec/Jan 2017 CBCS-scheme] (6m)

$\bar{E} = 2x\,\bar{a}_x - 4y\,\bar{a}_y$ V/m

Sol^n:-



$\boxed{y = -x + 2}$ eq^n of straight line joining points A and B.

7 ~~Write the expression for E and dL with units in i) Cartesian system ii) Cylindrical system and iii) Spherical system.~~ (06 Marks)

Sol^n ~~refer~~ Page NO. $\boxed{192-a}$

path-1

i) $W_{AO} = -Q\int_A^O \bar{E}\cdot\overline{dl} = -2\int_{x=2}^{0} 2x\,dx = -2(-4) = +8$

$\boxed{W_{AO} = 8}$ Joules

and $W_{OB} = -Q\int_O^B \bar{E}\cdot\overline{dl} = -2\int_{y=0}^{2}(-4y)\,dy = +16$ Joules

$\boxed{W_{OB} = +16}$ Joules

$\Rightarrow W_{AB} = W_{AO} + W_{OB} = 8 + 16 = 24$ Joules

ii) Second path $\Rightarrow$ Straight line joining b/w A to B.

$W_{AB} = -Q\int_A^B \bar{E}\cdot\overline{dl} = -Q\int_{A(2,0,0)}^{(0,2,0)}[2x\,dx - 4y\,dy]$

$= -2\left[\int_{x=2}^{0} 2x\,dx - \int_{y=0}^{2} 4y\,dy\right]$

$= -2\left[-4 - 8\right] = -2(-12) = +24$ joules

$\boxed{W_{AB} = +24}$ joules

from (i) and (ii) it is observed that work done is independent of path selected.

problem 47

$\bar{E} = 2x\bar{a}_x - 3y^2\bar{a}_y + 4\bar{a}_z$ V/m.

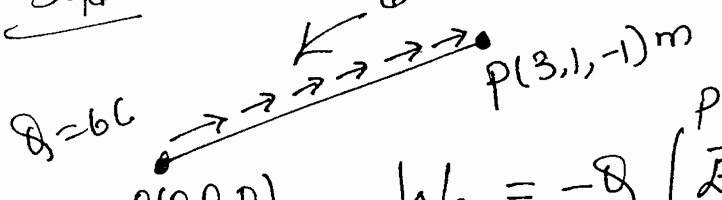
10 - June /July 2015

8 Find the amount of energy required to move a 6 coulomb of point charge from the origin to P(3, 1, -1) m in the field $\vec{E} = (2x\,a_x - 3y^2\,a_y + 4\,a_z)$ V/m along the straight line path, $x = -3z$, $y = x + 2z$ (05 Marks)

$x = -3z$

Soln:- $y = x + 2z$ $\quad \bar{dl} = dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z$

$y = x + 2(-x/3)$
$= x - \frac{2}{3}x = x/3$
$\boxed{y = x/3}$



$W_{op} = -Q \int_O^P \bar{E} \cdot \bar{dl}$ Joules

path $x = -3z$ and $y = x + 2z$

path
$x = -3z$
$dx = -3dz$
$y = x + 2z$
$= -3z + 2z$
$y = -z$
$dy = -dz$

$\bar{E} = 2x\,\bar{a}_x - 3y^2\,\bar{a}_y + 4\bar{a}_z$ V/m.

$\bar{E} \cdot \bar{dl} = 2x\,dx - 3y^2\,dy + 4\,dz.$

$W_{op} = -6 \int_{O(0,0,0)}^{(3,1,-1)} \left[2x\,dx - 3y^2\,dy + 4\,dz \right]$

$-6\left[\int_{z=0}^{-1} 2(-3z)(-3dz) - \int_{z=0}^{-1} 3(-z)^2(-dz) - 4 \right]$

using given path ie $x = -3z$ and $y = x + 2z$

$= -6 \left[\int_{x=0}^{3} 2x\,dx - \int_{y=0}^{1} 3y^2\,dy + 4\int_{z=0}^{-1} dz \right]$

$= -6\left[9 - 1 + 4(-1) \right] = -6[4]$

$= -24$ joules

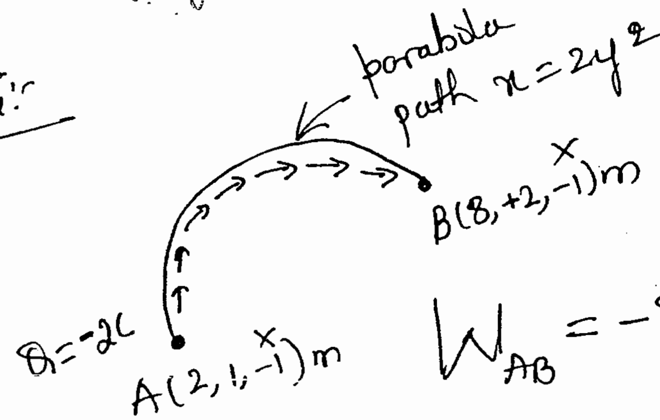
$\boxed{W_{op} = -24}$ joules

98

problem 48 Determine workdone in carrying a charge of $-2C$ from $(2, 1, -1)$ to $(8, 2, -1)$ in the Electric field $\bar{E} = y\bar{a}_x + x\bar{a}_y$ V/m Considering the path along the parabola $x = 2y^2$.

Aug-07 (7M) | Feb-2010 (8M)

Soln:



bz 'z' is not vary

$$\bar{dl} = dx\,\bar{a}_x + dy\,\bar{a}_y + \cancel{dz\,\bar{a}_z}^{\,0}$$

$$W_{AB} = -Q\int_A^B \bar{E}\cdot\bar{dl}$$

$$\bar{E}\cdot\bar{dl} = y\,dx + x\,dy \quad \text{volts}$$

$$W_{AB} = -(-2C)\int_{A(2,1,-1)}^{B(8,2,-1)} [y\,dx + x\,dy]$$

$$= +2\left[\int_{x=2}^{8} y\,dx + \int_{y=1}^{2} x\,dy\right]$$

put $x = 2$

path $x = 2y^2 \Rightarrow y^2 = x/2$

$y = \pm\sqrt{x/2}$ → $+\sqrt{x/2} = +1$ valid ; → $-\sqrt{x/2} = -1$ X invalid

the valid eqn must satisfy both the points ie A and B.

$$W_{AB} = 2\left[\int_{x=2}^{8}(\sqrt{x/2})\,dx + \int_{y=1}^{2} 2y^2\,dy\right]$$

$$= 2[(9.333) + 4.66667] = 2(14) = 28 \text{ Joules}$$

$$\boxed{W_{AB} = 28 \text{ Joules}}$$

(99)

problem 49

$$\overline{E} = -8xy\,\overline{a_x} - 4x^2\overline{a_y} + \overline{a_z} \text{ V/m}.$$

06 –Dec/Jan 2008

~~9 If $\vec{E}$ ... Find the work done in carrying ... charge from A(1, 8, 5) to B(2, 18, ...) along the path ...~~ (08 Marks)

(or) $6C$ $A(1,8,5)$

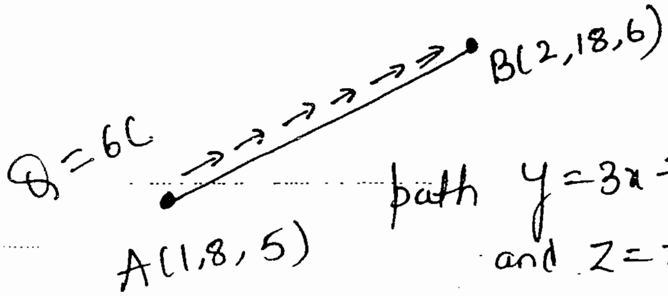
1 If $\vec{E} = -8xy\hat{a}_x - 4x^2\hat{a}_y + \hat{a}_z$ v/m, find the work done in carrying a 6C charge from A(1, 8, 5) to B(2, 18, 6) along the path $y = 3x + 2$, $z = x + 4$. (08 Marks)

[06-Dec/Jan 2008]

Sol:- $B(2,18,6)$ $y = 3x+2,\ z = x+4.$

$$\overline{E} = -8xy\,\overline{a_x} - 4x^2\,\overline{a_y} + \overline{a_z} \text{ V/m}.$$

$$\overline{dl} = dx\,\overline{a_x} + dy\,\overline{a_y} + dz\,\overline{a_z} \text{ m}.$$



path $y = 3x+2$ and $z = x+4$

Note:- its invalid path bcz both the points are not satisfying eq^n $y=3x+2$

$$\overline{E}\cdot\overline{dl} = -8xy\,dx - 4x^2\,dy + dz \text{ ; Volt's}$$

$$W_{AB} = -Q\int_A^B \overline{E}\cdot\overline{dl} = -Q\int_{A(1,8,5)}^{B(2,18,6)} \left[-8xy\,dx - 4x^2\,dy + dz\right]$$

$$= -6\left[\int_{x=1}^{2} -8xy\,dx - \int_{y=8}^{18} 4x^2\,dy + \int_{z=5}^{6} dz\right]$$

use $y = 3x+2$ use $x = \left(\frac{y-2}{3}\right)$ 1

$$= -6\left[\int_{x=1}^{2} -8x(3x+2)\,dx - \int_{y=8}^{18} 4\left(\frac{y-2}{3}\right)^2 dy + 1\right]$$

$$= -6\left[-80 - 574.814 + 1\right] = -6(-653.81)$$

$$= +3922.88 \text{ Joules}$$

$$W_{AB} = 3922.88 = 3.9228\text{ K Joules}$$

Problem 50

Q) An $\bar{E} = -8xy\,\bar{a}_x - 4x^2\bar{a}_y + \bar{a}_z$ V/m. the charge of 6C is to be moved from B(1,8,5) to A(2,18,6). Find the work done in each of the following cases

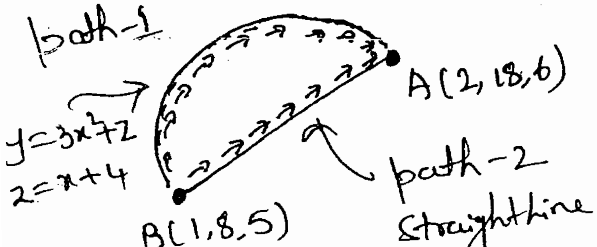
i) ~~the path selected~~ is $y = 3x^2 + z$ and $z = x + 4$.

ii) The straight line from B to A.

Jan 2012 (8M).

Soln: $\bar{E} = -8xy\,\bar{a}_x - 4x^2\,\bar{a}_y + \bar{a}_z$ V/m.

$Q = 6C \qquad \overline{dl} = dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z$: m.



$W = -Q\int_B^A \bar{E}\cdot\overline{dl}$ Joules

$\frac{y_2-y_1}{x_2-x_1} = \frac{y-y_1}{x-x_1}$

$\frac{18-8}{2-1} = \frac{y-8}{x-1} \Rightarrow \frac{y-8}{x-1} = 10$

$y - 8 = 10(x-1) \Rightarrow y = 10x - 10 + 8 \quad \therefore \boxed{y = 10x - 2}$

and $x = \left(\frac{y+2}{10}\right)$

case i. path $y = 3x^2 + z$ and $z = x+4$

$\Rightarrow y = 3x^2 + \cancel{z}^{\,x+4} = 3x^2 + x + 4$

$\therefore \boxed{y = 3x^2 + x + 4}$

$W = -6\int_B^A [-8xy\,dx - 4x^2dy + dz]$ Joules ← (a)

$W = -6\left[\int_{x=1}^{2} -8xy\,dx - \int_{y=8}^{18} 4x^2dy + \int_{z=5}^{6} dz\right]$

eliminate 'y' interms of x

Range ~~specification~~ to 'y'

1

(101)

$$W = -6\left[\int_{x=1}^{2} -8x(3x^2+x+4)\,dx - \int_{y=8}^{18} 4x^2\,dy + 1\right].$$

w.r.t $y = 3x^2 + x + 4$

$dy = 6x\,dx + dx + 0 \Rightarrow dy = (6x+1)dx$

L.h limit $y=8 \Rightarrow x=1$

$y = 18 \Rightarrow x = 2.$

$$W = -6\left[\int_{x=1}^{2} -8x(3x^2+x+4)\,dx - \int_{x=1}^{2} 4x^2(6x+1)\,dx + 1\right]$$

$$= -6\left[-156.666 - 99.33 + 1\right]$$

$$W = -6(-254.999) \simeq +1530 \text{ Joules}$$

$$\boxed{W_{BA} \simeq 1530 \text{ Joules} \simeq 1.53\text{k}} \text{ Joules} \leftarrow (1)$$

Case ii. path selected is Straight Line. eqn of Line joining b/w B to A is $\boxed{y = 10x - 2}$ and $x = \left(\frac{y+2}{10}\right)$

from eqn (a) $W = -6\int_B^A [-8xy\,dx - 4x^2\,dy + dz]$ Joules

$$W = -6\left[\int_{x=1}^{2} -8x(10x-2)\,dx - \int_{y=8}^{18} 4\left(\frac{y+2}{10}\right)^2 dy + \int_{z=5}^{6} dz\right]$$

$$= -6\left[-162.666 - 93.333 + 1\right] \simeq 1530 \text{ Joules}$$

$$\boxed{W_{BA} \simeq 1530 \text{ Joules (or) } 1.53\text{k}} \text{ Joules} \leftarrow (2)$$

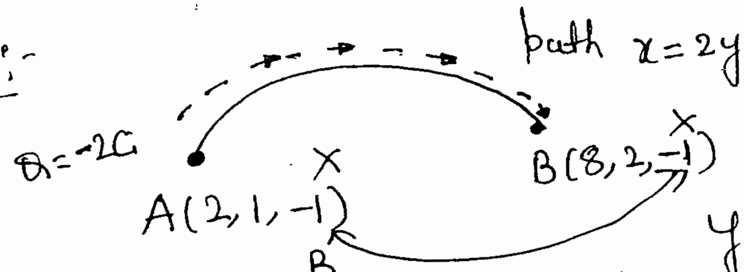
from eqn (1) and (2) it is observed that workdone is independent of path chosen.

(102)

Problem 51. Determine workdone in Carrying a charge of $-2C$ from $(2,1,-1)$ to $(8,2,-1)$ in an electric field $\bar{E} = y\bar{a}_x + x\bar{a}_y$ V/m along the path $x = 2y^2$. (7M)

EEE · J/J 2016.

Soln:-



and $y^2 = x/2$

$$y = \pm\sqrt{x/2}$$

$$W_{AB} = -Q\int_A^B \bar{E}\cdot\bar{dl}$$

$\boxed{y = +\sqrt{\frac{x}{2}}}$ valid and $\boxed{y = -\sqrt{\frac{x}{2}}}$ not valid

put $x = 2$, $y = 1$ ✓ put $x = 2$, $y = -1$ ✗

$$\bar{dl} = dx\,\bar{a}_x + dy\,\bar{a}_y \text{ m}$$

$$\bar{E} = y\bar{a}_x + x\bar{a}_y \text{ V/m}$$

$$\bar{E}\cdot\bar{dl} = y\,dx + x\,dy \text{ Volts}$$

$$W_{AB} = -(-2)\int_A^B [y\,dx + x\,dy]$$

$$= +2\left[\int_{x=2}^{8} y\,dx + \int_{y=1}^{2} x\,dy\right]$$

$$W_{AB} = 2\left[\int_{x=2}^{8}\sqrt{\frac{x}{2}}\,dx + \int_{y=1}^{2} 2y^2\,dy\right]$$

$$= 2\left[9.333 + 4.6666\right]$$

$$= 2[14] = 28 \text{ joules}$$

$$\boxed{W_{AB} = 28} \text{ Joules}$$

problem 52

$V = -2xy + 3$ volts.

02 - June/July 2011

5 The electric potential at an arbitrary point in free space is given as $V = -2xy + 3$ volts. Show that $\oint E.dl = 0$ for the closed contour shown in Fig.Q.2(b). (06 Marks)

$\oint_{\langle c \rangle} \bar{E} \cdot \bar{dl} = 0.$

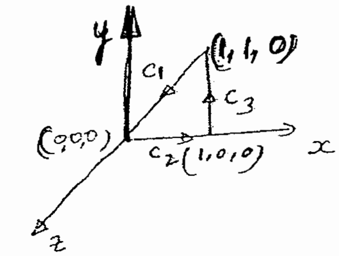


Fig.Q.2(b).

soln:- From ~~Concept of~~ Gradient

$$\boxed{\bar{E} = -\nabla V} \; V/m.$$

$$\therefore \bar{E} = -\left[\frac{\partial V}{\partial x}\bar{a_x} + \frac{\partial V}{\partial y}\bar{a_y} + \frac{\partial V}{\partial z}\bar{a_z}\right] V/m.$$

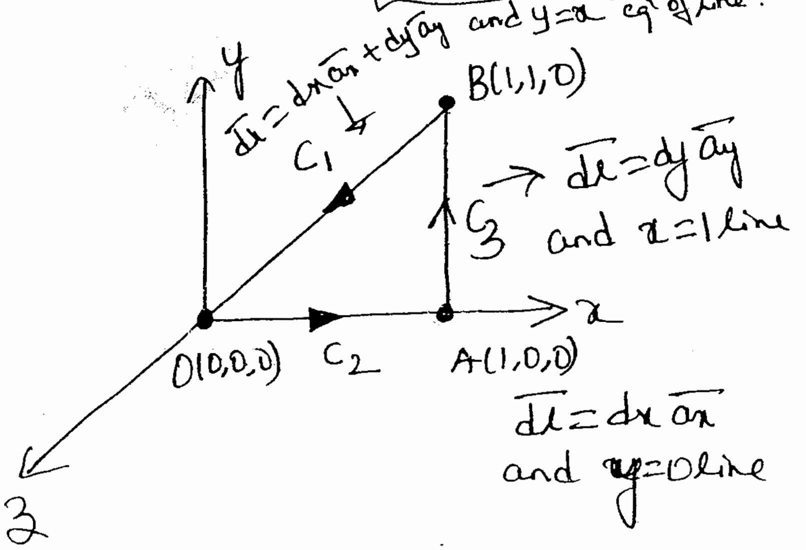
given $V = -2xy + 3$

$$\frac{\partial V}{\partial x} = -2y; \quad \frac{\partial V}{\partial y} = -2x; \quad \frac{\partial V}{\partial z} = 0.$$

$$\bar{E} = -\left[-2y\,\bar{a_x} - 2x\,\bar{a_y} + 0\right] = 2y\,\bar{a_x} + 2x\,\bar{a_y}\; V/m$$

$$\therefore \boxed{\bar{E} = 2y\,\bar{a_x} + 2x\,\bar{a_y}} \; V/m$$

$\bar{E} \cdot \bar{dl} = 2y\,dx + 2x\,dy$



C_2:-

$$\int_0^A \bar{E} \cdot \bar{dl} = \int_0^A 2y\,dx \Big|_{y=0\,\text{line}}$$

$$\therefore \boxed{\int_0^A \bar{E} \cdot \bar{dl} = 0}$$

$\underline{\underline{C_3}}$

$$\int_A^B \bar{E}\cdot\bar{dl} = \int_A^B 2x\,dy \Big|_{x=1 \text{ line.}}$$

$$= \int_{y=0}^{1} 2(1)\,dy = 2\int_{y=0}^{1} dy = \underline{\underline{2 \text{ volts}}}.$$

$\underline{\underline{C_1}}$

$$\int_B^O \bar{E}\cdot\bar{dl} = \int_B^O [2y\,dx + 2x\,dy] \Big|_{\text{eqn of line } y=x}$$

put $y=x$ put $x=y$

$$= \int_{x=1}^{0} 2x\,dx + \int_{y=1}^{0} 2y\,dy$$

$$= \left(2\int_{x=1}^{0} x\,dx\right) + \left(2\int_{y=1}^{0} y\,dy\right) = -1 - 1 = -2 \text{ volts}$$

$$C_1 + C_2 + C_3 \Rightarrow \oint_C \bar{E}\cdot\bar{dl} = \int_O^A \bar{E}\cdot\bar{dl} + \int_A^B \bar{E}\cdot\bar{dl} + \int_B^O \bar{E}\cdot\bar{dl}$$

$$= 0 + 2 - 2 = 0.$$

XX.

$$\Rightarrow \boxed{\oint_C \bar{E}\cdot\bar{dl} = 0}$$

analogous to KVL i.e
KVL — the total voltage in a closed loop is zero.

(105)

problem 53

$\bar{E} = y\bar{a}_x + x\bar{a}_y + 2\bar{a}_z$ V/m.

2C $B(1,0,1)$ $A(0.8, 0.6, 1)$

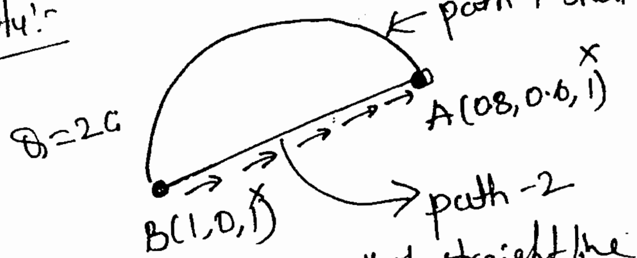
10-DEC 2013/Jan 2014

3 Determine the work done in carrying a charge of 2C from B(1, 0, 1) to A(0.8, 0.6, 1) in an electric field $\bar{E} = y\hat{a}_x + x\hat{a}_y + 2\hat{a}_z$ V/mt along the (i) short arc of circle $x^2 + y^2 = 1$, $Z = 1$. (06 Marks)

$x^2 + y^2 = 1$, $z = 1$.

(ii) straight line joining the point B to A.

Sol'n:



obs

'z' is not varying

$\overline{dl} = dx\overline{a_x} + dy\overline{a_y} + dz\overline{a_z}$

eq^n of straight line

$$\frac{y_2-y_1}{x_2-x_1} = \frac{y-y_1}{x-x_1} \Rightarrow \frac{0.6-0}{0.8-1} = \frac{y-0}{x-1}$$

$$y = 3(1-x) \Rightarrow \boxed{y = 3-3x}$$

and $\boxed{x = (3-y)/3 = 1 - y/3}$

$$\bar{E}\cdot\overline{dl} = y\,dx + x\,dy + 2dz \text{ volt's}$$

Case i. short arc of O^2 $x^2+y^2=1$, $z=1$.

$$W_{BA} = -Q\int_B^A \bar{E}\cdot\overline{dl} \text{ Joules}$$

$$= -Q\int_B^A [y\,dx + x\,dy + 2dz]$$

$$= -2\left[\int_{x=1}^{0.8} y\,dx + \int_{y=0}^{0.6} x\,dy + 2\int_{z=1}^{1} dz\right] \rightarrow (a)$$

write 'y' in terms of x — write x in terms of 'y' — using eq^n $x^2+y^2=1$

$y = \pm\sqrt{1-x^2}$ $x = \pm\sqrt{1-y^2}$

$\therefore y = \begin{cases} +\sqrt{1-x^2} \checkmark \text{ valid eq}^n = +0.6 \\ -\sqrt{1-x^2} \times = -0.6 \end{cases}$ when $x = 0.8$

$x = \begin{cases} +\sqrt{1-y^2} \checkmark \text{ Valid eq}^n = +0.8 \checkmark \\ -\sqrt{1-y^2} \times \; -0.8 \end{cases}$ when $y = 0.6$

Valid eq^n in the one in which ~~both the~~ points has to be satisfied.

i.e when $x = 0.8 \Rightarrow y = +0.6 \checkmark$ | IIIy when $y = 0.6 \Rightarrow x = +0.8 \checkmark$

$$\therefore W_{BA} = -2\left[\int_{x=1}^{0.8} \sqrt{1-x^2}\,dx + \int_{y=0}^{0.6} \sqrt{1-y^2}\,dy + 0\right]$$

$$= -2\left[-0.08175 + 0.56175\right] = \underline{-0.96 \text{ Joules}}$$

$$\boxed{W_{BA} = -0.96} \text{ Joules} \leftarrow (1)$$

case ii. straight line path $y = 3 - 3x$ (or) $x = 1 - y/3$.

from eq^n (a) $W_{BA} = -2\left[\int_{x=1}^{0.8} y\,dx + \int_{y=0}^{0.6} x\,dy + 2\int_{z=1}^{1} dz\right]$

$$W_{BA} = -2\left[\int_{x=1}^{0.8} (3-3x)\,dx + \int_{y=0}^{0.6} (1-y/3)\,dy + 0\right]$$

$$= -2\left[-0.06 + 0.54 + 0\right] = \underline{-0.96 \text{ Joules}}$$

$$\boxed{W_{BA} = -0.96} \text{ Joules} \leftarrow (2)$$

from eq (1) and (2) it is observed that work done is independent of path choosen.

problem 55

$$\overline{E} = 5e^{-r/4}\,\overline{a_r} + \frac{10}{r\sin\theta}\,\overline{a_\phi} \text{ V/m}.$$

06-DEC 2013/Jan 2014

$Q = -5\mu C$ $\quad (2, \pi/4, \pi/2)$ m.

Find the work done in moving a point charge Q = −5 μc from the origin to (2, π/4, π/2) in spherical coordinate system

$$\overline{E} = 5e^{-r/4}\,a_r + \frac{10}{r\sin\theta}\,a_\phi$$

(06 Marks)

and $E_\theta = 0$.

Soln:
$$\overline{E} = \underbrace{5e^{-r/4}}_{E_r}\,\overline{a_r} + \underbrace{\frac{10}{r\sin\theta}}_{E_\phi}\,\overline{a_\phi} \text{ V/m}.$$

the given field $\overline{E}$ is in Spherical C.S.

$P(r, \theta, \phi)$ → dr, $r\,d\theta$, $r\sin\theta\, d\phi$

$A(2, \pi/4, \pi/2)$

$Q = -5\mu$

$O(0,0,0)$

$$\overline{dl} = dr\,\overline{a_r} + r\,d\theta\,\overline{a_\theta} + r\sin\theta\, d\phi\,\overline{a_\phi} \text{ m}.$$

$$\overline{E}\cdot\overline{dl} = E_r\,dr + \cancel{rE_\theta\,d\theta}^{\,0 \text{ bcz } E_\theta = 0} + r\sin\theta\, E_\phi\, d\phi \text{ Volts}$$

$$W_{OA} = -Q\int_O^A \overline{E}\cdot\overline{dl} = -Q\int_O^A\left[5e^{-r/4}dr + \frac{10}{\cancel{r\sin\theta}} \times \cancel{r\sin\theta}\, d\phi\right]$$

$$= -(-5\mu)\left[\int_{r=0}^{2} 5e^{-0.25r}\,dr + 10\int_{\phi=0}^{\pi/2} d\phi\right]$$

$$= +5\mu\left[7.86939 + \cancel{10}^{\,5} \times \pi/\cancel{2}\right] = 117.88\mu \text{ Joules}$$

$$\therefore \boxed{W_{OA} = 117.88\mu} \text{ Joules}$$

Note:- if $Q = +5\mu C$ then $W_{OA} = -117.88\mu$ Joules

Problem 54

Given the field $\bar{E} = \frac{k}{r}\bar{a_r}$ V/m in cylindrical co-ordinate system. Show that the work needed to move a point charge Q from any radial distance 'r' to a point at twice that radial distance is independent of r.

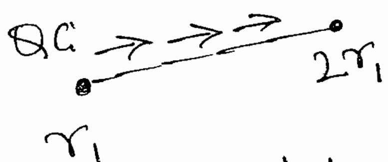
Sol'n:- $\bar{E} = \frac{k}{r}\bar{a_r}$ V/m.

$$W = -Q\int_{initial}^{final}\bar{E}\cdot\bar{dl} \quad \text{Joules}$$

Since point charge moving along radial path

$\therefore \bar{dl} = dr\,\bar{a_r}$

$$W = -Q\int_{r_1}^{2r_1}\frac{k}{r}\bar{a_r}\cdot dr\,\bar{a_r}$$



$$W = -Qk\int_{r_1}^{2r_1}\frac{1}{r}\,dr\,\bar{a_r}\cdot\bar{a_r}$$

$$W = -Qk\ln(r)\Big|_{r_1}^{2r_1} \quad \text{Joules}$$

$$= -Qk\left[\ln(2r_1) - \ln(r_1)\right] = -Qk\left[\ln(2r_1) - \ln(r_1)\right]$$

$$= -Qk\ln\left[\frac{2r_1}{r_1}\right] = -Qk\ln(2)$$

$$\therefore \boxed{W = -Qk\ln 2}$$

this shows W is independent of 'r'.

Topics: 2. to

- Definition of potential difference and potential
- ~~The potential field of point charge~~

Questions

1) obtain an Equation for the electric Scalar potential. (6M) 02 Dec-10, Dec 2011 | Jan 2012.

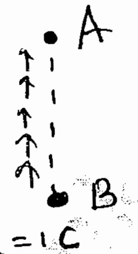
→ Define electric Scalar potential.

→ Define potential difference and absolute potential. (4M) 10-J/J 2014.

→ Determine the potential difference b/w two points due to a point charge 'q' at origin. (4M) 10-Dec/Jan 2016. [15-June/July 2017 (6M) CBCS] → 10 - Dec 2015 (04)

Q8. potential difference :- The potential of a point A with respect to point B is defined as the work done in moving a unit positive point charge from point B to A against to the Electric field $\bar{E}$.

i.e



$$V_{AB} = \frac{W}{Q_u \to 1C} = -\int_B^A \bar{E}\cdot\bar{dl} \quad \text{Joules | Coulombs (or) Volts}$$

(or) In general potential at a point A w.r.t B

$Q_u = 1C$

$$\boxed{V_{AB} = -\int_B^A \bar{E}\cdot\bar{dl} = -\int_{\text{initial}}^{\text{final}} \bar{E}\cdot\bar{dl}} \quad \text{volts}$$

2011 Potential field due to point Charge. 10-J/J 2014.

Question Expression for Electric Scalar potential (V) :- (6m) 10-Dec/Jan 2013 [15 J/J 2017 CBCS]

Consider a point charge of 'Q C' which is placed at origin 'O'. the field $\bar{E}$ due to 'Q' C is given by

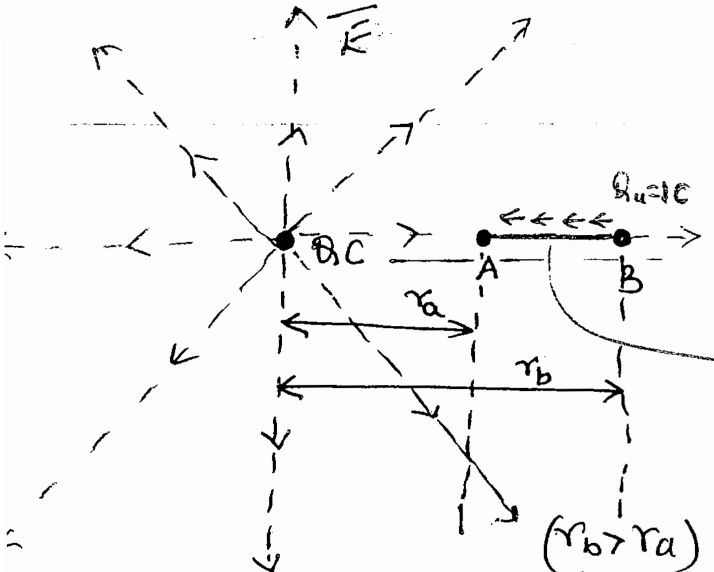


fig. concept of potential difference & absolute potential.

$$\overline{E} = \frac{Q}{4\pi\epsilon r^2}\overline{a_r} \quad V/m \leftarrow (1)$$

eq^n in Spherical C.S

from eq^n (1) it is observed that the field is in radial direction.

in radial path

$$\overline{dl} = dr\,\overline{a_r}$$

the potential difference b/w the points A and B is the workdone required to move a point charge of 1C from point B to point A along radial path against to the field $\overline{E}$.

i.e $$V_{AB} = \frac{W}{\cancel{Q_u}\,1C} = -\int_B^A \overline{E}\cdot\overline{dl} = -\int_{r_b}^{r_a} \overline{E}\cdot\overline{dl}$$

$$= -\int_{r_b}^{r_a} \frac{Q}{4\pi\epsilon r^2}\overline{a_r}\cdot dr\,\overline{a_r}$$

$$= -\frac{Q}{4\pi\epsilon}\int_{r_b}^{r_a}\frac{1}{r^2}\,dr\;\cancelto{1}{\overline{a_r}\cdot\overline{a_r}}$$

$$\int \frac{1}{x^2}\,dx = \frac{-1}{x}$$

$$= \frac{-Q}{4\pi\epsilon}\times\frac{-1}{r}\Bigg|_{r_b}^{r_a}$$

$$= + \frac{Q}{4\pi\epsilon}\left[\frac{1}{r_a} - \frac{1}{r_b}\right] \quad ; \quad r_b > r_b$$

xx.

$$\therefore \boxed{\hat{V}_{AB} = \frac{Q}{4\pi\epsilon}\left[\frac{1}{r_a} - \frac{1}{r_b}\right] \; ; \text{ volt's} \quad \text{where } r_b > r_a}$$

$$\hat{V}_{AB} = \frac{Q}{4\pi\epsilon r_a} - \frac{Q}{4\pi\epsilon r_b} \quad \text{volt's}$$

xx.

$$\boxed{\hat{V}_{AB} = V_A - V_B} \quad \text{volt's}$$

xx **ABSOLUTE POTENTIAL**:- Dec/Jan 2015 (4m).

Special case:- when $r_b \to \infty$ i.e the point B becomes infinite point (or) ground point (or) reference point.

the potential $\hat{V}_B \to 0$ bcz $r_b \to \infty$.

$$\therefore \hat{V}_{AB} = V_{A\infty} = \hat{V}_A = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_a} - \cancel{\frac{1}{r_b}}^{\,0}\right]$$

$$\boxed{\hat{V}_A = \frac{Q}{4\pi\epsilon r_a}} \quad \text{volt's}$$

In general

$$\boxed{V = \frac{Q}{4\pi\epsilon r}} \quad \text{volt's}$$

where r distance from point charge location to the desired point where we measure the potential.

Definition / Def^n:- Absolute potential (or) potential w.r.t ground is defined as the workdone required to move a point charge of 1C from infinite ~~path~~ (ground point) to a specific point along the radial path against to the field $\bar{E}$.

i.e $\boxed{V = \frac{Q}{4\pi\epsilon r}}$ volt's

(112)

Topic 2.11 (i) questions

02-DEC2010

1 Obtain an equation for the electric scalar potential. (06 Marks)

or

10 June /July 2014

9 Determine the potential difference between two points due to a point charge 'q' at the origin. (04 Marks)

Soln:- Refer Page No. 211.

Topic 2.11 (ii)

potential field of a System of charges:-

Question: Discuss with relevant equations the potential field of a system of charges. 10 J/J 2013 (8M).

Soln:-

Classification:-

i) potential due to point charge.

potential field due to Continuous distribution of charges:

ii) due to line charge density.

iii) due to Surface charge density.

iv) due to volume charge density.

i) potential due to point charge:- W.K.T the potential at a point 'p' w.r.t ground point is

$$V_p = \frac{Q}{4\pi\epsilon r} \text{ volt's} \quad \leftarrow (1)$$

eqn (1) is valid only when point charge 'Q' must be Located at the origin.

if 'Q' is Located otherthan origin, then eqn (1) is modified as

10-June/July 2013

3 Discuss with relevant equations the potential field of a system of charges and hence obtain the potential field of a ring of uniform line charge density. (08 Marks)

Solu:- Page NO - 214.

4 Define electric scalar potential. Derive an expression for potential due to several point charges. (06 Marks)

Solu:- refer page NO. 215.

10-Dec/Jan 2016

10 a. Derive an equation for potential due to infinite line charge. (04 Marks)

06-DEC2010

13 Derive an equation for the potential at a point, due to an infinite line charge. (06 Marks)

Solu:- refer Page NO-

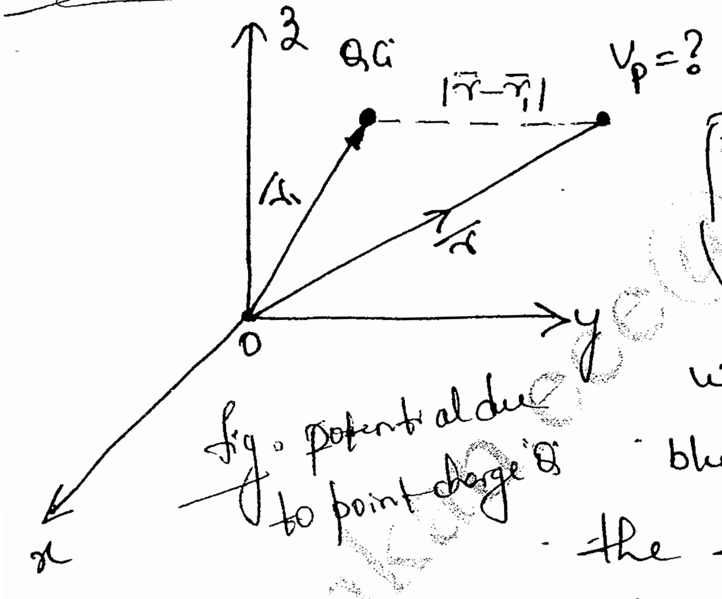


fig. potential due to point charge Q

$$\boxed{V_p = \frac{Q}{4\pi\epsilon|\bar{r}-\bar{r}_1|}} \text{ volts}$$

where $|\bar{r}-\bar{r}_1|$ is the distance b/w point charge (Q) location to the specific point (P) where we measure the potential.

3) Special case:- potential due to several point charges:-

Consider a point charges of $Q_1, Q_2, \ldots Q_n$ Coulombs located at a points 1, 2, 3 ... n respectively. the potential at a point 'p' is measured due to all n-point charges using principle of Superposition

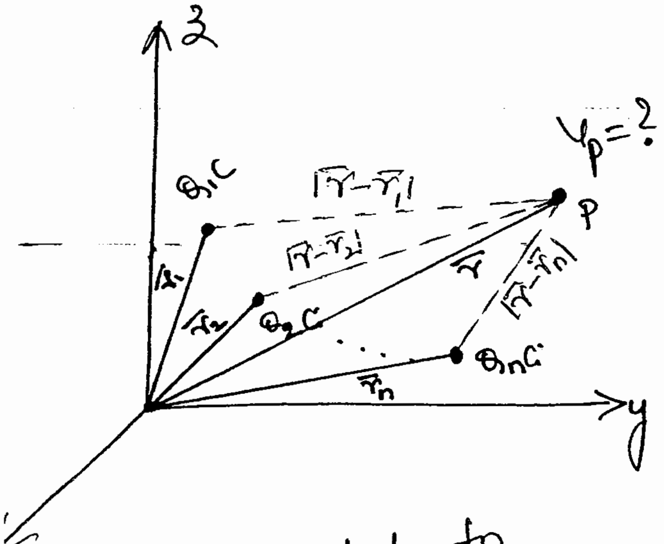


fig. potential due to several point charges.

$$\bar{V}_p = V_{p_1} + V_{p_2} + \cdots + V_{p_n} \text{ volt's}$$

$$V_p = \frac{Q_1}{4\pi\epsilon|\bar{r}-\bar{r}_1|} + \frac{Q_2}{4\pi\epsilon|\bar{r}-\bar{r}_2|} + \cdots + \frac{Q_n}{4\pi\epsilon|\bar{r}-\bar{r}_n|} \text{ volt's}$$

$$V_p = \frac{1}{4\pi\epsilon}\left[\frac{Q_1}{|\bar{r}-\bar{r}_1|} + \frac{Q_2}{|\bar{r}-\bar{r}_2|} + \cdots + \frac{Q_n}{|\bar{r}-\bar{r}_n|}\right]$$

$$\boxed{\bar{V}_p = \frac{1}{4\pi\epsilon}\sum_{i=1}^{n}\frac{Q_i}{|\bar{r}-\bar{r}_i|}} \text{ volt's}$$

if $Q_1 = Q_2 = \cdots = Q_n = Q$ C

then $$\boxed{V_p = \frac{Q}{4\pi\epsilon}\sum_{i=1}^{n}\frac{1}{|\bar{r}-\bar{r}_i|}} \text{ volt's}$$

ii) potential due to Line charge distribution :-

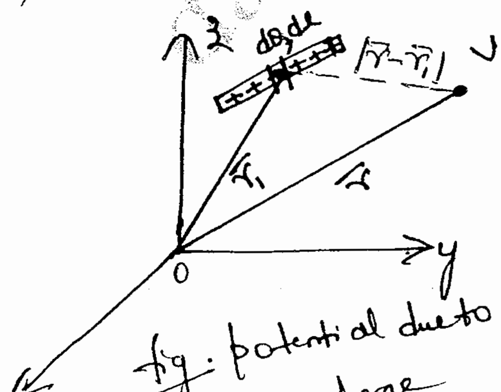


fig: potential due to Line charge distribution.

from defn of Line charge density ρ_L C/m

$$\rho_L = \frac{dQ}{dl} \text{ C/m}$$

$$\Rightarrow \boxed{dQ = \rho_L\, dl} \text{ Coulomb's}$$

The differential potential due to dQ is given by $$dV_p = \frac{dQ}{4\pi\epsilon|\bar{r}-\bar{r}_1|} \text{ volt's}$$

the total potential V_p due to the entire Line charge is given by $V_p = \int_{<l>} \frac{dQ}{4\pi\epsilon|\bar{r}-\bar{r}_1|}$ volt's

i.e $$\boxed{V_p = \int_{<l>} \frac{\rho_l \, dl}{4\pi\epsilon|\bar{r}-\bar{r}_1|}} \text{ volt's}$$

iii) potential due to Surface charge distribution :-

Surface charge density $\rho_s = dQ/ds$ C/m²

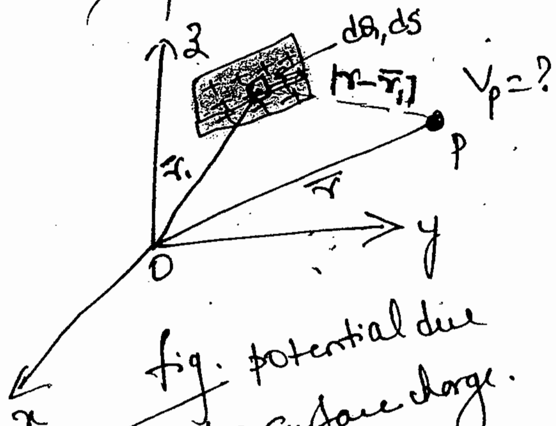


fig. potential due to Surface charge density

$$\boxed{dQ = \rho_s \cdot dS} \text{ Coulomb's}$$

$$dV_p = \frac{dQ}{4\pi\epsilon|\bar{r}-\bar{r}_1|} \text{ volt's}$$

$$V_p = \int_{<S>} \frac{dQ}{4\pi\epsilon|\bar{r}-\bar{r}_1|} \text{ volt's}$$

the total potential $$\boxed{V_p = \int_{<S>} \frac{\rho_s \, dS}{4\pi\epsilon|\bar{r}-\bar{r}_1|}} \text{ volt's}$$

iv) potential due to volume charge distribution :-

$$\rho_v = \frac{dQ}{dv} \text{ C/m}^3 \Rightarrow dQ = \rho_v \, dv \text{ C}$$

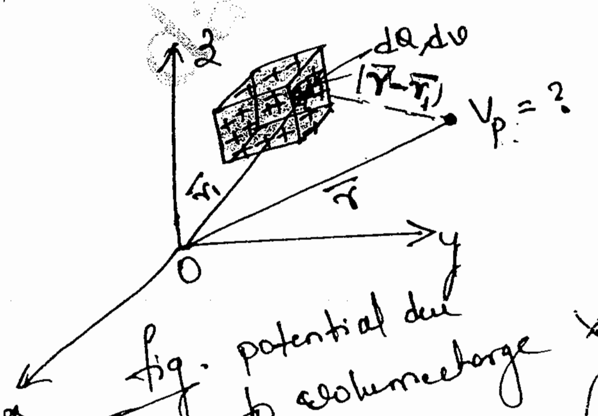


fig. potential due to volume charge density.

$$dV_p = \frac{dQ}{4\pi\epsilon|\bar{r}-\bar{r}_1|} \text{ volt's}$$

$$dV_p = \frac{\rho_v \, dv}{4\pi\epsilon|\bar{r}-\bar{r}_1|} \text{ volt's}$$

$$\boxed{V_p = \int_{<v>} \frac{\rho_v \, dv}{4\pi\epsilon|\bar{r}-\bar{r}_1|}} \text{ volt's}$$

problem 55 ← 15nC V_1

10-DEC2011/Jan 2012

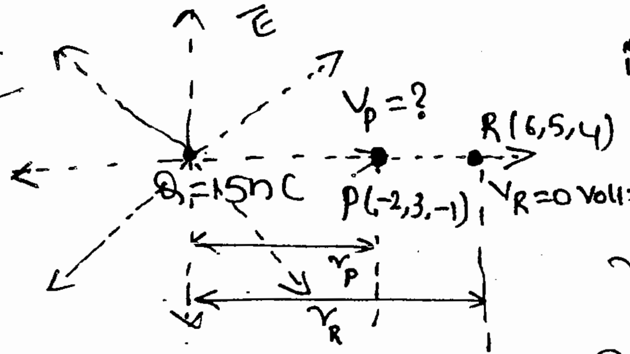
2 A 15 nc point charge is at the origin in free space. Calculate v_1 if point P is located at P(−2, 3, −1) and : i) V = 0 at (6, 5, 4) ii) V = 0 at infinity. (08 Marks)

P(−2,3,−1) (or) — iii) V = 5V at (2,0,4).

06- June /July 2009

7 ~~A 15 nc point charge is at the origin in free space. Calculate V_1 if point P is located at (−2, 3, −1). Also calculate V_1 at P if V = 0 at (6, 5, 4).~~ (08 Marks)

soln:



i) V_{PR}

$$V_{PR} = \frac{Q}{4\pi\epsilon}\left[\frac{1}{r_p} - \frac{1}{r_R}\right]$$

$$r_p = \sqrt{(-2)^2+3^2+(-1)^2} = \sqrt{14}\ m$$

$$r_R = \sqrt{6^2+5^2+4^2} = \sqrt{77}\ m.$$

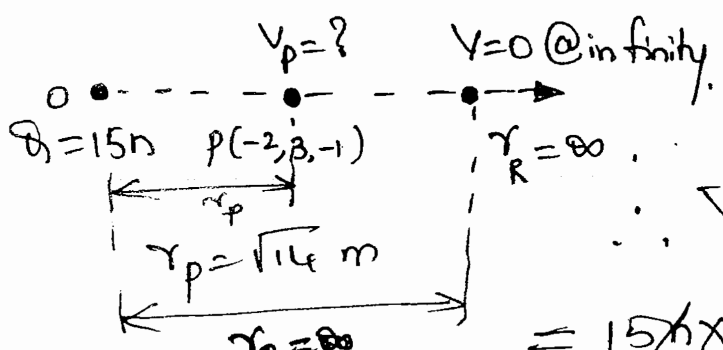
$$V_{PR} = 15\cancel{n}\times 9\times 10^{\cancel{9}}\left[\frac{1}{\sqrt{14}} - \frac{1}{\sqrt{77}}\right] = 20.695 \text{ volts}$$

$$V_{PR} = V_p - \cancel{V_R}^{\,0\,(\text{given})} = V_p = 20.695 \text{ volts}$$

∴ the potential at a point P w.r.t V=0V @ (6,5,4) is $\boxed{V_p = 20.695}$ volts

ii)

$$V_{PR} = \frac{Q}{4\pi\epsilon}\left[\frac{1}{r_p} - \frac{1}{r_R}\right]$$



$r_R \to \infty$; $\frac{1}{r_R} \to 0$

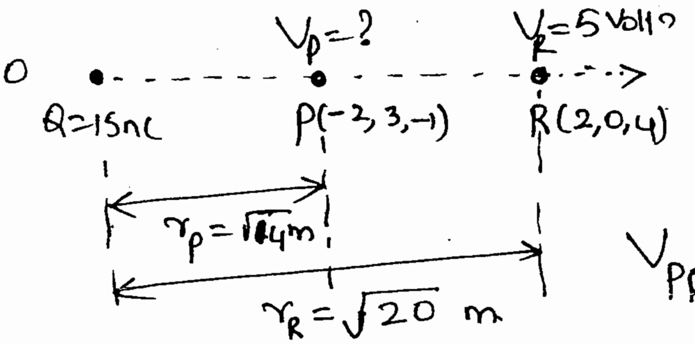
$$\therefore V_{PR} = V_p = \frac{Q}{4\pi\epsilon r_p}$$

$$= \frac{15\cancel{n}\times 9\times 10^{\cancel{9}}}{\sqrt{14}} = 36.080 \text{ volts}$$

∴ the potential at a point 'p' w.r.t $V = 0V$ @ infinity

is $\boxed{V_p = 36.080}$ volt's

iii)



$$V_{PR} = \frac{Q}{4\pi\epsilon}\left[\frac{1}{r_p} - \frac{1}{r_R}\right]$$

$$V_{PR} = 15\cancel{n} \times 9\times 10^{9}\left[\frac{1}{\sqrt{14}} - \frac{1}{\sqrt{20}}\right]$$

$$V_{PR} = 15 \times 9 \times 0.04365 = \underline{5.8933} \text{ volt's}$$

$$V_{PR} = V_p - V_R \text{ volt's}$$

$$V_p = V_{PR} + V_R = 5 + 5.893 = \underline{10.893 \text{ volt's}}$$

the potential at a point 'p' w.r.t $V = 5V$ at $R(2,0,4)$

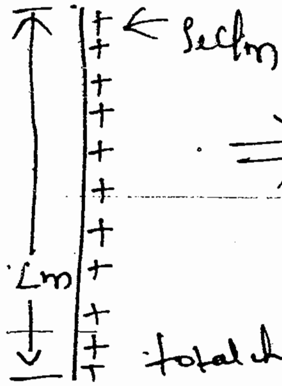
is $\boxed{V_p = 10.893}$ volt's

Topic 2.11 (c)

XX → Potential field of a ring of uniform Line charge density :-

Question 3 → obtain the potential field of a ring of uniform Line charge density. 10-J/J 2012. (8m).

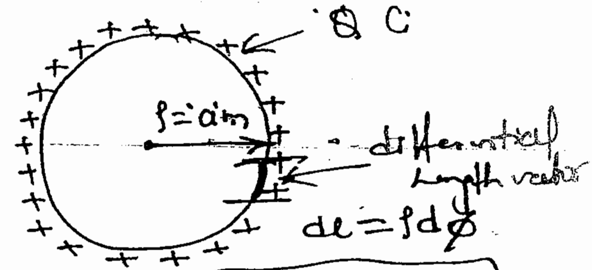
Note:-



$\Rightarrow$ in the form of Circular ring

total charge 'Q' is uniformly distributed

$\therefore \boxed{Q = \rho_\ell L}$ Coulomb's



$\boxed{L = 2\pi\rho = 2\pi a}$ m

i.e circumference

Derivation:-

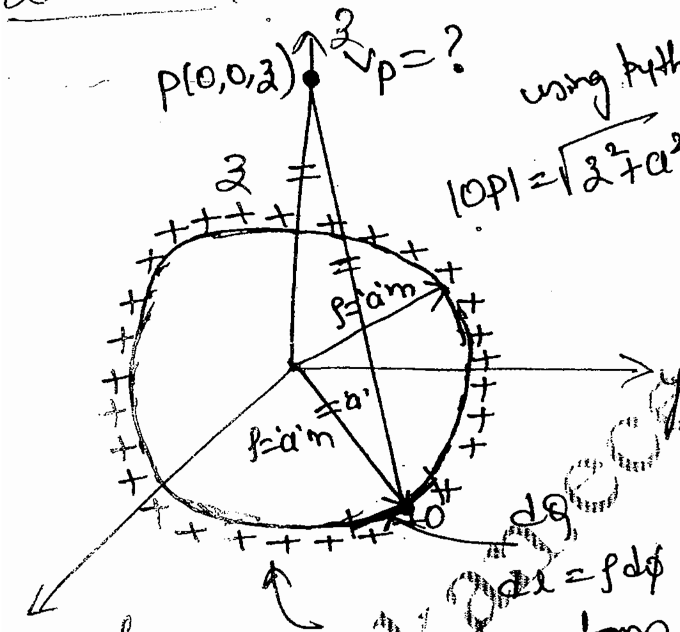


fig. Circular ring of uniform line charge density place on xy plane.

using Pythagorous theorem

$|OP| = \sqrt{z^2 + a^2}$ m

the potential at a point $P(0,0,z)$ is obtained by considering a differential charge 'dQ' over a length '$d\ell$'.

$$dV_p = \frac{dQ}{4\pi\epsilon |OP|}$$

$$dV_p = \frac{\rho_\ell \cdot d\ell}{4\pi\epsilon \left[\sqrt{z^2 + a^2}\right]}$$

$$V_p = \int_{\langle \ell \rangle} \frac{\rho_\ell d\ell}{4\pi\epsilon\sqrt{z^2+a^2}} = \int_{\phi=0}^{2\pi} \frac{\rho_\ell \times a d\phi}{4\pi\epsilon\sqrt{z^2+a^2}} = \frac{\rho_\ell a}{4\pi\epsilon\sqrt{z^2+a^2}} \int_0^{2\pi} d\phi$$

$$= \frac{\rho_\ell a}{4\pi\epsilon\sqrt{z^2+a^2}} \times 2\pi = \frac{\rho_\ell a}{2\epsilon\sqrt{z^2+a^2}}$$

$$\boxed{V_p = \frac{\rho_\ell a}{2\epsilon\sqrt{z^2+a^2}}} \text{ volts}$$

119

Special case:- if all the charge is Concentrated at origin (i.e in the form of a point charge) then the potential at a point 'p' is

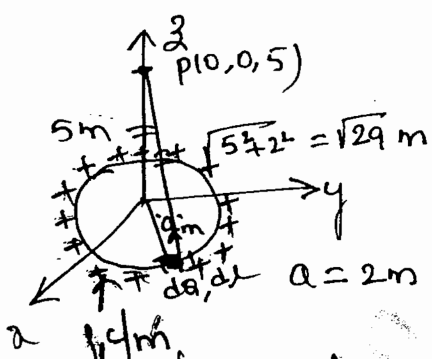
$$\boxed{V_p = \frac{Q}{4\pi\epsilon z}} \text{ volt's.}$$

Problem 56

Q) 40nC of charge is uniformly distributed around a Circular ring of radius 2m. Find the potential at a point on the axis 5m from the plane of the ring. Compare with the result where all the charge is at origin in the form of a point charge.

Soln:- $Q = 40nC$

$\rho_L = Q/L = \frac{Q}{2\pi a}$ C/m



L = Length of line charge density $= 2\pi a$.

$\therefore \rho_L = \frac{40n}{2\pi(2)} = \frac{40n}{4\pi} = \frac{10}{\pi}$ nC/m.

potential at point p

$$V_p = \frac{\rho_L a}{2\epsilon\sqrt{z^2+a^2}} = \frac{\frac{10}{\pi}n \times 2}{2\epsilon\sqrt{5^2+2^2}} = \underline{66.7 \text{ volt's}}$$

Case i. $\boxed{V_p = 66.7}$ volt's

if the charge is concentrated at the origin, then the potential at $(0,0,5)$m is $V_p = \frac{Q}{4\pi\epsilon z} = \frac{40\times10^{-9}\times9\times10^{9}}{5}$

Case ii. $\boxed{V_p = 72}$ volt's

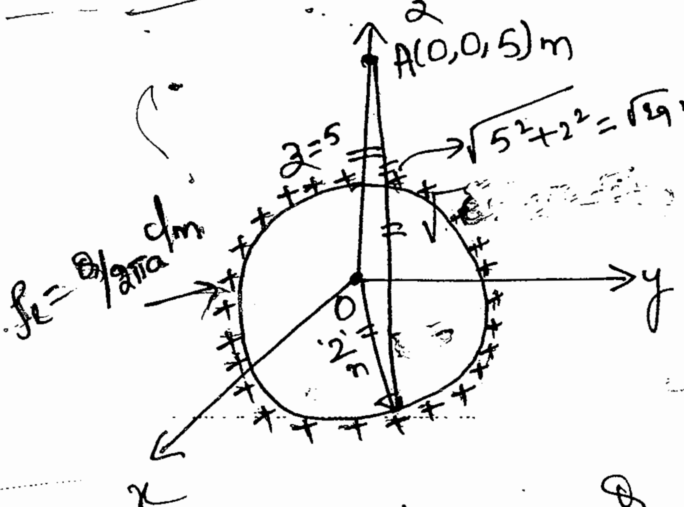
Cntd Next page

Obs:- $V_p \propto \frac{1}{\text{distance}(r)}$; $r\uparrow \Rightarrow V_p\downarrow$ (Case i) and $r\downarrow \Rightarrow V_p\uparrow$ (Case ii)

Problem 57: A total charge of 40/3 nc is uniformly distributed over a circular ring of radius 2m placed in z = 0 plane, with center as origin. Find the electric potential at A (0, 0, 5). (06 Marks)

40/3 nC, 2m, A (0,0,5) m

Sol^n:- $\rho_L = Q/L = Q/2\pi a$ C/m.



$$V_A = \frac{\rho_L a}{2\epsilon\sqrt{a^2+z^2}} \text{ volts}$$

$$V_A = \frac{\frac{Q}{2\pi a}\cdot a}{2\epsilon\sqrt{a^2+z^2}}$$

$$V_A = \frac{Q}{4\pi\epsilon\sqrt{a^2+z^2}} = \frac{40}{3}\cancel{n}\times 9\times 10^{\cancel{9}} \times \frac{1}{\sqrt{5^2+2^2}}$$

$$\boxed{V_A = 22.283} \text{ volts}$$

if all the charge is concentrate at origin then potential at point 'p' is $V_p = \frac{Q}{4\pi\epsilon z}$

$$V_p = \frac{40}{3}\cancel{n}\times 9\times\cancel{10^9}\times\frac{1}{5} = \underline{24 \text{ volts}}$$

$$\boxed{V_p = 24} \text{ volts}$$

potential due to uniform ring. potential due to point charge

$a \to 0 \Rightarrow$ point charge @ origin

$$V_p = \frac{\rho_L \times a}{2\epsilon\sqrt{a^2+z^2}} = \frac{\rho_L\times 2\pi a}{2\epsilon(2\pi)\sqrt{a^2+z^2}} = \frac{\rho_L\times L}{4\pi\epsilon\sqrt{a^2+z^2}} = \frac{Q}{4\pi\epsilon z} \text{ volts}$$

xix.x

2.11d → potential due to Infinite Line charge :-

xv Q10> Derive an equation for the potential at a point due to
Q13> an Infinite Line charge. (6M) 10 J/J 2013, 06-Dec 2010.

soln:

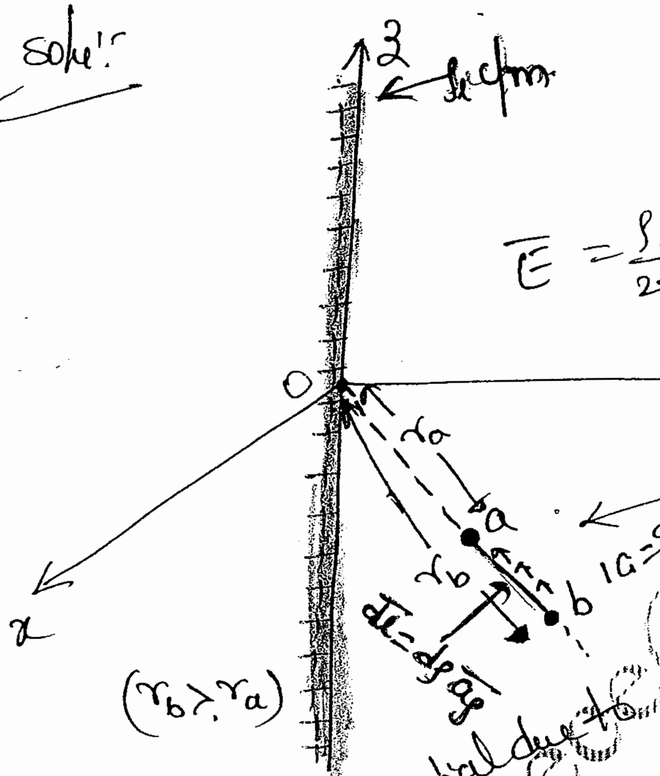


fig. potential due to infinite line charge.

w.k.t the field $\bar{E}$ due to infinite Line charge is

$$\bar{E} = \frac{\rho_L}{2\pi\epsilon\rho}\bar{a}_\rho \text{ V/m}$$

The direction of field is act along 'a_ρ' i.e radial direction.

the potential difference b/w the points 'a' and 'b' is V_{ab} is nothing but the workdone required to carry a unit +ve charge from point b to point a along radial path against to the field $\bar{E}$.

$$\text{i.e } V_{ab} = -\int_b^a \bar{E}\cdot\bar{dL} = \int_a^b \bar{E}\cdot\bar{dL} \text{ volts}$$

$$\bar{V}_{ab} = \int_{r_a}^{r_b} \frac{\rho_l}{2\pi\epsilon\rho} \bar{a}_\rho \cdot d\rho\, \bar{a}_\rho \quad \text{Volt's}$$

$$= \frac{\rho_l}{2\pi\epsilon} \int_{r_a}^{r_b} \frac{1}{\rho} d\rho\, \cancel{\bar{a}_\rho \cdot \bar{a}_\rho}^{\,1}$$

$$= \frac{\rho_l}{2\pi\epsilon} \ln\rho \Big|_{r_a}^{r_b}$$

$$\bar{V}_{ab} = \frac{\rho_l}{2\pi\epsilon}\left[\ln r_b - \ln r_a\right]$$

xi) $$\boxed{\bar{V}_{ab} = \frac{\rho_l}{2\pi\epsilon} \ln\left[r_b / r_a\right]}$$ volt's ; $r_b > r_a$

key note, point ~~summary~~

→ $W = -Q \int_{\text{initial}}^{\text{final}} \bar{E} \cdot \bar{dl}$

→ $V_{AB} = \frac{W}{\cancel{Q_u} \to 1C} = -\int_B^A \bar{E} \cdot \bar{dl}$ volt's

→ $\bar{V}_{AB} = \frac{Q}{4\pi\epsilon}\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$ volt's ; $(r_b > r_a)$

→ potential at a point w.r.t ground i.e $r_b \to \infty$

$\bar{V}_A = \frac{Q}{4\pi\epsilon r_a}$ volt's

→ potential due to circular ring of charge density

$\bar{V}_P = \frac{\rho_l a}{2\epsilon\sqrt{a^2+z^2}}$ volt's

→ potential due to infinite line charge $V_{ab} = \frac{\rho_l}{2\pi\epsilon} \ln(r_b/r_a)$

problem 58 $\overline{E} = 40xy\overline{a_x} + 20x^2\overline{a_y} + 2\overline{a_z}$ V/m

10-June/July 2013

Given the field E = 40 xya_x+20x^2a_y+2a_z V/m, calculate the potential between the two points P(1, -1, 0) and Q(2, 1, 3). (06 Marks)

Soln:- P(1,-1,0) Q(2,1,3).

$$\overline{E} = 40xy\overline{a_x} + 20x^2\overline{a_y} + 2\overline{a_z} \text{ V/m.}$$

$$\overline{dl} = dx\overline{a_x} + dy\overline{a_y} + dz\overline{a_z}$$

P(1,-1,0) (x_1 y_1) Q(2, 1, 3) (x_2 y_2)

$$V_{PQ} = -\int_Q^P \overline{E}\cdot\overline{dl} = +\int_P^Q \overline{E}\cdot\overline{dl} \text{ volts}$$

Path b/w P and Q $\dfrac{y_2-y_1}{x_2-x_1} = \dfrac{y-y_1}{x-x_1}$

$$\frac{1+1}{2-1} = \frac{y+1}{x-1} \Rightarrow 2(x-1) = y+1$$

$$y = 2x-2-1 \Rightarrow \boxed{y = 2x-3} \text{ (or) } x = \left(\frac{y+3}{2}\right)$$

$$V_{PQ} = \int_P^Q \left[40xy\,dx + 20x^2dy + 2dz\right]$$

$$V_{PQ} = \int_{x=1}^{2} 40xy\,dx + \int_{y=-1}^{1} 20x^2dy + \int_{z=0}^{3} 2dz$$

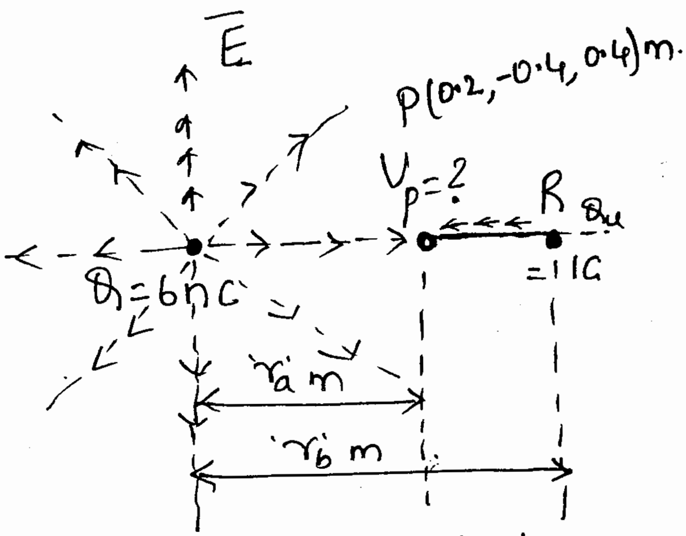
$$= \int_{x=1}^{2} 40x(2x-3)\,dx + \int_{y=-1}^{1} 20\left(\frac{y+3}{2}\right)^2 dy + 2\int_{z=0}^{3} dz$$

$$V_{PQ} = 6.6667 + 93.3333 + 6 = \underline{106 \text{ volts}}$$

$$\boxed{V_{PQ} = 106} \text{ volts}$$

4b. A point charge of 6nC is located at the origin in free space find potential of point P if **P** is located at (0.2, −0.4, 0.4) and i) V =0 at infinity ii) V = 0 at (1, 0, 0) iii) V = 20V at (−0.5, 1, −1). **(10 Marks)**

Soln:-



i) $V = 0$ at infinity

i.e $r_b \to \infty$.

$$\therefore V_p = \frac{Q}{4\pi\epsilon r_a} \text{ volt's}$$

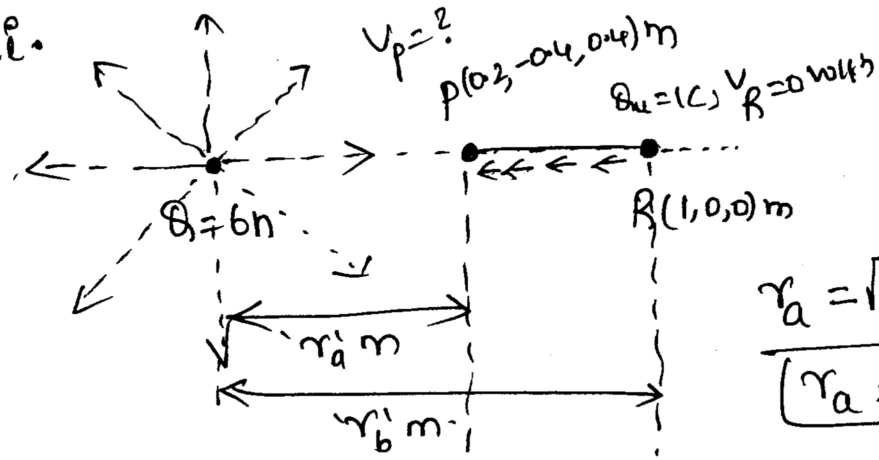
$$r_a = \sqrt{0.2^2 + (-0.4)^2 + (0.4)^2} = 0.6 \text{ m}$$

$$\boxed{r_a = 0.6 \text{ m}}$$

$$V_p = \frac{(6n)\, 9\times10^9}{0.6} = 90 \text{ volt's}$$

$$\boxed{V_p = 90} \text{ volt's}$$

ii.



$r_a = \sqrt{0.2^2 + 0.4^2 + 0.4^2}$

$\boxed{r_a = 0.6}$ m

$r_b = \sqrt{1^2 + 0 + 0} = \sqrt{1} \Rightarrow \boxed{r_b = 1 m}$

$$V_{PR} = \frac{Q}{4\pi\epsilon}\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$

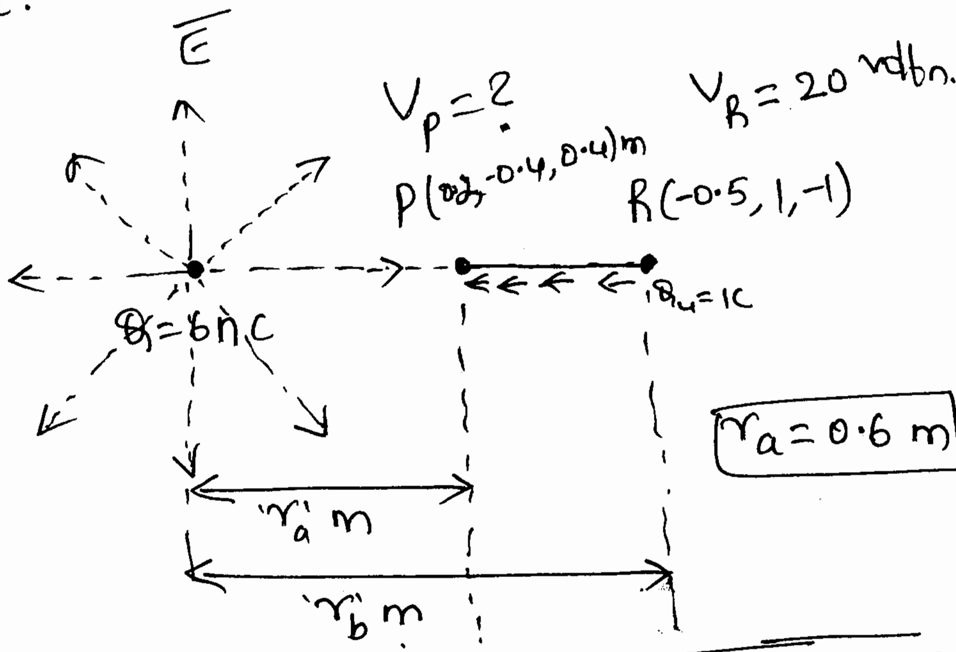
$$V_{PR} = (6n)(9\times10^{+9})\left[\frac{1}{0.6} - 1\right] = 36 \text{ volts}$$

$\boxed{V_{PR} = 36}$ volts

$\Rightarrow V_{PR} = V_p - \cancelto{0}{V_R} \Rightarrow$ given $V_R = 0$ volts

$\boxed{V_p = V_{PR} = 36}$ volts

iii.



$$\boxed{r_a = 0.6\ m}$$

$$r_b = \sqrt{(-0.5)^2 + (1)^2 + (-1)^2}$$

$$\boxed{r_b = 1.5\ m}$$

$$V_{PR} = \frac{Q}{4\pi\epsilon}\left[\frac{1}{r_a} - \frac{1}{r_b}\right] ; \text{ volts}$$

$$= (6n)(9\times10^9)\left[\frac{1}{0.6} - \frac{1}{1.5}\right]$$

$$\boxed{V_{PR} = 54} \text{ volts}$$

$$V_{PB} = V_P - V_B \text{ : volts}$$

$$V_P = V_{PB} + V_B$$

$$V_P = 54 + 20$$

$$\boxed{V_P = 74} \text{ volts}$$

10-Dec/Jan 2015

8 Define potential difference and absolute potential. (04 Marks)

refer page NO- 211 and 213.

problem 59

An Electric field is expressed in rectangular Co-ordinates by $\bar{E} = 6x^2\,\bar{a}_x + 6y\,\bar{a}_y + 4\,\bar{a}_z$ V/m.

Find

i) V_{MN} if points M and N are specified by M(2, 6, -1) and N(-3, -3, 2).

ii) V_M if V = 0 volts at Q(4, -2, -35).

iii) V_N if V = 2 volts at P(1, 2, -4).

Soln:

$$\bar{E} = 6x^2\,\bar{a}_x + 6y\,\bar{a}_y + 4\,\bar{a}_z \text{ V/m}$$

$$\overline{dl} = dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z \text{ m}$$

$$\bar{E}\cdot\overline{dl} = 6x^2 dx + 6y\,dy + 4dz \text{ volts}$$

i) $$V_{MN} = -\int_N^M \bar{E}\cdot\overline{dl} = +\int_M^N \bar{E}\cdot\overline{dl}$$

$$= \int_{x=2}^{-3} 6x^2 dx + \int_{y=6}^{-3} 6y\,dy + \int_{z=-1}^{2} 4\,dz$$

$$= -70 - 81 + 12 = -139 \text{ volts}$$

$$\boxed{V_{MN} = -139} \text{ volts}$$

ii) V_M if $V_Q = 0$ volts at $Q(4,-2,-35)$

$$V_{MQ} = V_M - \cancel{V_Q}^{\,0 \text{ (given)}} = V_M = -\int_Q^M \bar{E}\cdot\bar{dl}$$

$M(2,6,-1)$ $Q(4,-2,-35)$

$$V_{MQ} = V_M = \int_M^Q \bar{E}\cdot\bar{dl} = \int_{x=2}^{4} 6x^2 dx + \int_{y=6}^{-2} 6y\,dy + \int_{z=-1}^{-35} 4\,dz$$

$$= 112 - 96 - 34(4) = -120 \text{ volts}$$

$$\boxed{V_M = V_{MQ} = -120} \text{ volts}$$

iii) $V_N = ?$ $V_P = 2V$

$N(-3,-3,2)$ $P(1,2,-4)$

$$V_{NP} = -\int_P^N \bar{E}\cdot\bar{dl} = \int_N^P \bar{E}\cdot\bar{dl} = V_N - V_P$$

$$\Rightarrow V_N = V_{NP} + V_P = \int_N^P \bar{E}\cdot\bar{dl} + V_P$$

$$V_N = \left[\int_{x=-3}^{1} 6x^2 dx + \int_{y=-3}^{2} 6y\,dy + \int_{z=2}^{-4} 4\,dz\right] + 2$$

$$V_N = 56 + (-15) - 6(4) + 2$$

$$V_N = 56 - 15 - 24 + 2 = 19 \text{ volts}$$

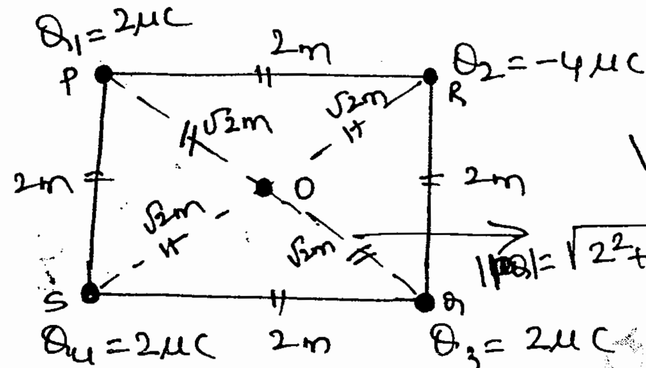
$$\boxed{V_N = 19} \text{ volts}$$

problem 60

02 - June /July 2012

Calculate the potential at the center of a square of side 2m. while charges 2μc, −4μc and 2μc are located at its four corners. (05 Marks)

Soln:



$V_0 = 2$

$|\overline{RQ}| = \sqrt{2^2+2^2} = \sqrt{8} = 2\sqrt{2}$ using pythagorous theorem.

(assume)

the potential at point 'O' is

$$V_0 = V_1 + V_2 + V_3 + V_4 \text{ volts}$$

$$V_0 = \frac{Q_1}{4\pi\epsilon|\overline{PO}|} + \frac{Q_2}{4\pi\epsilon|\overline{RO}|} + \frac{Q_3}{4\pi\epsilon|\overline{QO}|} + \frac{Q_4}{4\pi\epsilon|\overline{SO}|} \text{ volts}$$

$$|\overline{PO}| = |\overline{RO}| = |\overline{QO}| = |\overline{SO}| = \sqrt{2} \text{ m}.$$

$$\therefore V_0 = \frac{1}{4\pi\epsilon(\sqrt{2})}\left[Q_1 + Q_2 + Q_3 + Q_4\right]$$

$$= \frac{9\times10^9}{\sqrt{2}}\left[2\mu - \cancel{4\mu} + \cancel{2\mu} + \cancel{2\mu}\right]$$

$$= 12.727\times10^3 \text{ volts}$$

$$\boxed{V_0 = 12.727 \text{ kvolts}}$$

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problem 61

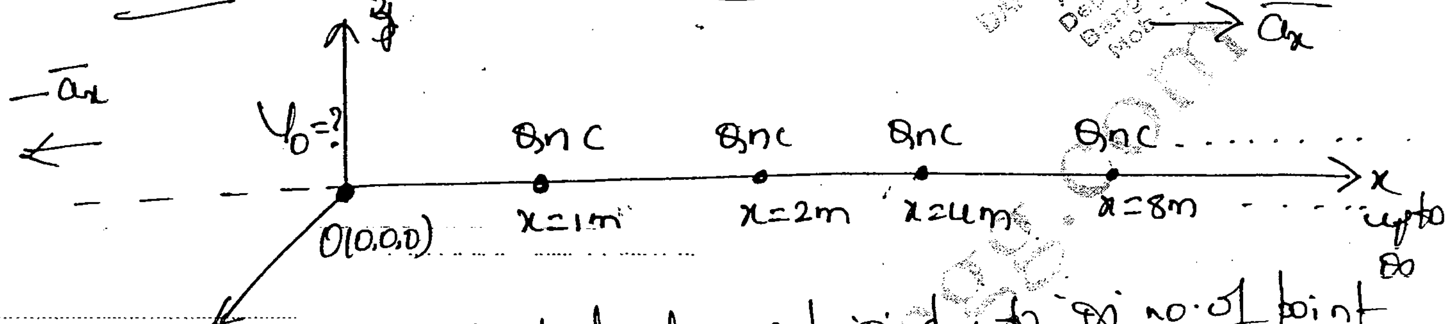
10-June/July 2016

a. Infinite number of charges each of Qnc are placed along x axis at x = 1, 2, 4, 8,........∞. Find the electric potential and electric field intensity at a point x = 0 due to the all charges. (06 Marks)

soln:

$$\boxed{V = \frac{Q}{4\pi\epsilon r}} \text{ volt's}$$

case i.



the potential at point 'O' due to '∞' no. of point charges placed along 'x' axis is

$$V_0 = V_1 + V_2 + V_4 + V_8 + \cdots\cdots$$

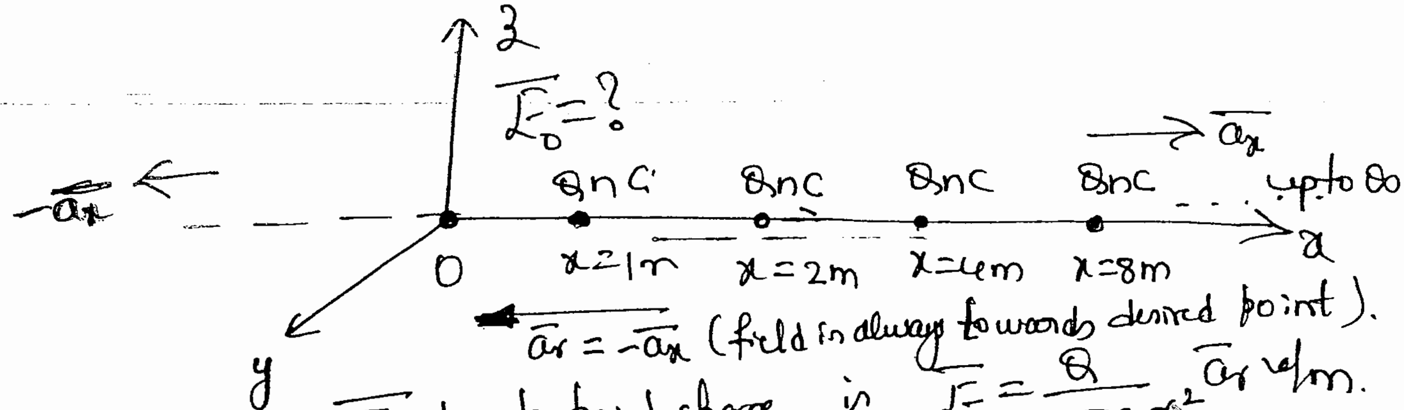
$$V_0 = \frac{Qn}{4\pi\epsilon(1)} + \frac{Qn}{4\pi\epsilon(2)} + \frac{Qn}{4\pi\epsilon(4)} + \frac{Qn}{4\pi\epsilon(8)} + \cdots\cdots$$

$$V_0 = Q\cancel{n} \times 9 \times \cancel{10^{9}} \left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots\cdots\right]$$

$$V_0 = 9Q \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n \qquad \Bigg| \sum_{n=0}^{\infty} a^n = \frac{1}{(1-a)} ;|a|<1$$

$$V_0 = 9Q \times \frac{1}{1-\frac{1}{2}} = 9Q \times 2 = 18Q \text{ volt's}$$

$$\boxed{V_0 = 18Q} \text{ volt's}$$



$\bar{a}_r = -\bar{a}_x$ (field is always towards desired point).

$\bar{E}$ due to point charge is $\bar{E} = \frac{Q}{4\pi\epsilon r^2}\bar{a}_r$ V/m.

Since field acts towards the desired point. In this case all the point charges are placed along 'x' axis and the desired point is at origin $\therefore \bar{a}_r \Rightarrow -\bar{a}_x$

i.e field direction.

$$\therefore \bar{E}_0 = \bar{E}_1 + \bar{E}_2 + \bar{E}_4 + \bar{E}_8 + \cdots \text{ V/m}$$

$$\bar{E}_0 = \left[\frac{8n}{4\pi\epsilon(1)^2} + \frac{8n}{4\pi\epsilon(2)^2} + \frac{8n}{4\pi\epsilon(4)^2} + \frac{8n}{4\pi\epsilon(8)^2} + \cdots\right](-\bar{a}_x)$$

$$\bar{E}_0 = \frac{8n}{4\pi\epsilon}\left[1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots\right](-\bar{a}_x)$$

$$\bar{E}_0 = \left[8\cancel{n} \times 9 \times \cancel{10^9} \sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n = 9 \cdot 8 \frac{1}{1-1/4} = \cancel{9}^{3} \cdot 8 \times \frac{4}{\cancel{3}}\right](-\bar{a}_x)$$

Ans.

$$\boxed{\bar{E}_0 = 128(-\bar{a}_x) = -128\,\bar{a}_x} \text{ V/m.}$$

Problem 62: For a line charge $\rho_L = \left(\frac{10^{-9}}{2}\right)$ c/m on the z-axis. Find V_{AB} where A is $(2m, \pi/2, 0)$ and $B(4m, \pi, 5m)$.

Sol'n: $$V_{AB} = -\int_B^A \bar{E} \cdot \bar{dl}$$

where $\bar{E} = \frac{\rho_L}{2\pi\epsilon\rho}\bar{a_\rho}$ and $\bar{dl} = d\rho\,\bar{a_\rho}$

$$V_{AB} = -\int_B^A \frac{\rho_L}{2\pi\epsilon\rho}\bar{a_\rho} \cdot d\rho\,\bar{a_\rho}$$

$$= -\int_{\rho=4}^{2} \frac{\rho_L}{2\pi\epsilon\rho} d\rho \; \bar{a_\rho}\cdot\bar{a_\rho}$$

$$= +\frac{\rho_L}{2\pi\epsilon}\int_{\rho=2}^{4} \frac{1}{\rho} d\rho$$

$$= \frac{\rho_L}{2\pi\epsilon} \ln\rho \Big|_2^4$$

$$V_{AB} = \frac{\rho_L}{2\pi\epsilon}\ln(4/2) = \frac{\rho_L}{2\pi\epsilon}\ln(2).$$

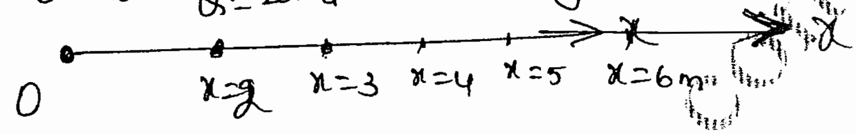
$$V_{AB} = \frac{10^{-9}}{2} \times 18\times10^{9} \ln(2) = 6.238 \text{ volts}$$

$$\boxed{V_{AB} = 6.238} \text{ volts}$$

problem 63 Five equal point charges $Q = 20nC$ are located at $x = 2, 3, 4, 5, 6$ m. Find the potential at the origin.

soln:- — — — Potential due to many point charges.

$V_0 = ?$ $Q = 20nC$ Each.



$$V = \frac{Q}{4\pi\epsilon r} \text{ volts.}$$

Since $Q_2 = Q_3 = Q_4 = Q_5 = Q_6 = 20nC$

using Superposition principle.

$$V_0 = V_2 + V_3 + V_4 + V_5 + V_6$$

$$= \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4} + \frac{1}{r_5} + \frac{1}{r_6}\right]$$

$$V_0 = 20\cancel{n} \times 9 \times 10^{+9}\left[\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}\right]$$

$$= 18 \times 1.45$$

$$\boxed{V_0 = 26.1} \text{ volts}$$

Topic 2012

Topics:

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9. Current and Current density
10. Continuity of current

02-DEC2008/Jan 2009

1 Obtain an expression for the equation of continuity. (05 Marks)

10-DEC 2013/Jan 2014

2 Derive an expression for continuity equation in point form. (04 Marks)

10-June/July 2013

3 Discuss current and current density and derive the expression for continuity equation. (06 Marks)

06- June /July 2009

7 Derive point form of continuity equation. (06 Marks)

10 - June /July 2015

9 With usual notations, prove point form of continuity equation, $\nabla.J = \frac{\partial\rho_v}{\partial t}$. (05 Marks)

10 - June /July 2014

10 Derive point form of continuity equation. (04 Marks) (05 Marks)

06 – Jan 2013

11 Derive the integral and point form of continuity equation. (06 Marks)

06 –Dec/Jan 2008

12 Starting with principle of charge conservation ... (06 Marks)

Topic 2012

Current and Current density :-

Current (I) :- The current is defined as the rate of flow of charge per unit time.

i.e $\boxed{I = \frac{dQ}{dt}}$ C/sec or Ampere

Note :- One Ampere (1A) of Current is nothing but One Coulomb of charge passing across the Surface in one Second.

Current density ($\bar{J}$) :- The Current density ($\bar{J}$) is defined as the Current passing through the unit Surface area when Surface is at normal to the direction of Flow of Current (I).

i.e $$\boxed{\bar{J} = \frac{dI}{dS}} \quad A/m^2.$$

(or) $$\bar{J} = \frac{dI}{dS}\,\bar{a}_n \quad A/m^2$$

where $\bar{a}_n$ is the unit vector normal to the direction of flow of current.

Note: i. Current (I) is Scalar quantity.

ii. Current density ($\bar{J}$) is Vector quantity.

→ Relation b/w Current (I) and Current density ($\bar{J}$) :-

Question

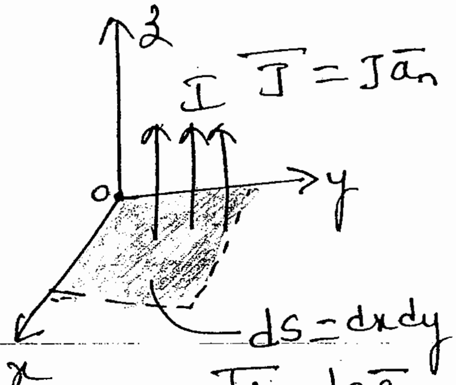
Q5) prove that total Current Flowing through the Surface S is given by $I = \int_{\langle S \rangle} \bar{J}\cdot d\bar{S}$ Amp (04 M)

02 J/J 2011.

Consider a Surface S and the Current flows through the Surface (I).

the direction of Current is normal to the Surface (S).

∴ the direction of $\bar{J}$ is also normal to the Surface.



$\bar{ds} = ds\,\bar{a}_n$

$\bar{ds} = dxdy\,\bar{a}_z$

the differential Current dI passing through the differential Surface ds is given by

$dI = \bar{J} \cdot \bar{ds}$

$\bar{J} = \frac{dI}{ds}$ A/m^2.

$\Rightarrow \quad dI = \bar{J} \cdot \bar{ds}$ Ampere's

$$\boxed{I = \int_{S} \bar{J} \cdot \bar{ds}} \text{ Ampere's}$$

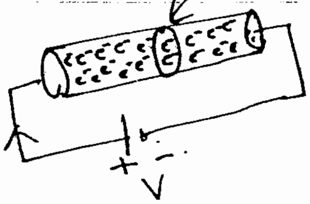
if Considered Surface is to be closed then

$$\boxed{I = \oint_{S} \bar{J} \cdot \bar{ds}} \text{ Ampere's.}$$

2013 XXX V.Emp XX Continuity of Current eq^n :- Topic 2013
02 Jan 2009, 10-Jan 2014, 10 J/J 2013, 06 J/J 2009
XX Q1.2.3.7.9.10.11.12. 10 J/J 2015, 10 J/J-2014, 06-Jan 2008
CBCS scheme [15-Dec/Jan 2017] (6m) 06 Jan 20..

$-dQ/dt$

the Current Flow through the any Closed Surface is given by



$$I = \oint_{\langle S\rangle} \bar{J}\cdot\overline{dS} \quad \text{Ampere's} \leftarrow (1)$$

The charge inside the closed Surface is depicted by Q_i, then the rate of decrease is $\frac{-dQ_i}{dt}$ (i.e. defn of KCL)

by principle of Conservation of charge

$$I = \oint_{\langle S\rangle} \bar{J}\cdot\overline{dS} = -\frac{dQ_i}{dt} \leftarrow (2)$$

i.e current @ any node is equal to zero.

using divergence theorem and $Q_i = \int_{\langle v\rangle} \rho_v \, dv$ Coulomb's

$$\oint_{\langle S\rangle} \bar{J}\cdot\overline{dS} = \int_{\langle vol\rangle} \nabla\cdot\bar{J}\, dv = I = -\frac{d}{dt}\int_{\langle vol\rangle} \rho_v \, dv \; ; A$$

$$\boxed{\int_{\langle vol\rangle} (\nabla\cdot\bar{J})\, dv = \int_{\langle vol\rangle} \left(-\frac{\partial \rho_v}{\partial t}\right) dv} \leftarrow (3)$$

integral form of Continuity Ampere's Current eq^n.

XX, X

$$\boxed{\nabla\cdot\bar{J} = -\frac{\partial \rho_v}{\partial t}} \quad A/m^3 \leftarrow (4)$$

point form (or) differential form of Continuity current eq^n.

The equation states that the current (or) the charge per second, diverging from a small volume per unit volume is equal to the time rate of decrease of charge per unit volume at every point.

Note:- for a Steady Current ρ_v = Constant (t) $\therefore \frac{\partial \rho_v}{\partial t} = 0$ $\therefore$

$$\boxed{\nabla \cdot \bar{J} = 0}$$

problem 64 06 - June /July 2011

08/09 Given the vector current density 02 J/J 2010, 06 J/J - 2011.

$\vec{J} = 10\rho^2 z \vec{a}_\rho - 4\rho\cos^2\phi\,\vec{a}_\phi$ mA/m²

v. Imp i) Find the current flowing outward through the circular band $\rho = 3$, $0 < \phi < 2\pi$, $2 < z < 2.8$.

ii) Find Current density at $P(\rho = 3, \phi = 30°, z = 2m)$; (06 Marks)

Sol:- ii) $\bar{J} = 10\rho^2 z\,\bar{a}_\rho - 4\rho\cos^2\phi\,\bar{a}_\phi$ mA/m².

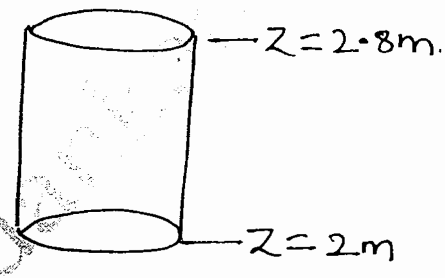
the Current density at $P(3, 30°, 2)$ is

$\rho = 3m$; $\phi = 30°$ and $z = 2m$.

$$\bar{J} = 10(3)^2(2)\,\bar{a}_\rho - 4(3)\cos^2(30°)\,\bar{a}_\phi \text{ mA/m}^2$$

$$\boxed{\bar{J} = 180\,\bar{a}_\rho - 9\bar{a}_\phi} \text{ mA/m}^2$$

i) given $\rho = 3m$, $0 < \phi < 2\pi$, $2 < z < 2.8m$



$P(\rho, \phi, z)$
$d\rho$ $\rho d\phi$ dz

$$\bar{J} = \underbrace{10\rho^2 z}_{J_\rho}\,\bar{a}_\rho \underbrace{-4\rho\cos^2\phi}_{J_\phi}\,\bar{a}_\phi \text{ mA/m}^2.$$

$$\therefore I = \oint_{S} \bar{J} \cdot \bar{dS} \text{ Amperes.}$$

not a valid constant surfaces

$$\because J_z = 0$$

$$I = I'\Big|_{\rho=3m} + I''\Big|_{\phi=0^c} + I'''\Big|_{\phi=2\pi^c} + \cancel{I^{iv}}\Big|_{z=2m} + \cancel{I^{v}}\Big|_{z=2.81m}$$

$$\overline{ds} = \rho\, d\phi\, dz\,(+\bar{a}_\rho) \qquad \overline{ds} = d\rho\, dz\,(-\bar{a}_\phi) \qquad \overline{ds} = d\rho\, dz\,(+\bar{a}_\phi)$$

$$I'\Big|_{\rho=3m} = \int_{\langle S\rangle} \overline{J_\rho}\cdot\overline{ds} = \int_{\langle S\rangle} 10\rho^2 z\,\bar{a}_\rho \cdot \rho\, d\phi\, dz\,(\bar{a}_\rho)\Big|_{\rho=3m} \times 1m$$

$$= 10\rho^3 \int_{\phi=0}^{2\pi} d\phi \int_{z=2}^{2.8} z\, dz \;\; \overbrace{\bar{a}_\rho\cdot\bar{a}_\rho}^{1} \times 1m$$

$$I'\Big|_{\rho=3m} = 10(3)^3 \times 2\pi \times 1.92 \times 1m = 3.2572\,A$$

$$\boxed{I'\Big|_{\rho=3m} = 3.2572}\ \text{Amperes}$$

$$I''\Big|_{\phi=0^c} = \int_{\langle S\rangle} \overline{J_\phi}\cdot\overline{ds} = \int_{\langle S\rangle} -4\rho\cos^2\phi\,\overline{a_\phi}\cdot d\rho\, dz\,(-\bar{a}_\phi)\Big|_{\phi=0^c} \times 1m$$

$$= +4m\cos^2(0) \times \int_{z=2}^{2.8} dz \times \int_{\rho=0}^{3} \rho\, d\rho = 0.0144\,A$$

$$I'''\Big|_{\phi=2\pi^c} = \int_{\langle S\rangle} \overline{J_\phi}\cdot\overline{ds} = \int_{\langle S\rangle} -4\rho\cos^2\phi\,\overline{a_\phi}\cdot d\rho\, dz\,(+\bar{a}_\phi) \times 1m\Big|_{\phi=2\pi^c}$$

$$= -4m\cos^2(2\pi) \times \int_{z=2}^{2.8} dz \times \int_{\rho=0}^{3} \rho\, d\rho = -0.0144\,A$$

$$I = I' + I'' + I''' = 3.2572 + \cancel{0.0144} - \cancel{0.0144} = 3.257A$$

$\therefore$ $\boxed{I \simeq 3.26}$ Ampere's

06 - June /July 2012

problem 65

The current density due flow of charges in a very small region in the viscinity of the origin is given by $J = J_o\left[x^2\hat{a}_x + y^2\hat{a}_y + z^2\hat{a}_z\right]$ A/m², where J_o is a constant. Find the time rate of increase of charge density at each of the following points (all in meters):
i) (0.02, 0.01, 0.01) ii) (0.02, -0.01, -0.01). (06 Marks)

soln:-

given $\bar{J} = J_o\left[x^2\bar{a}_x + y^2\bar{a}_y + z^2\bar{a}_z\right] A/m^2$.

i) (0.02, 0.01, 0.01)
ii) (0.02, -0.01, -0.01).

$\dfrac{\partial \rho_v}{\partial t} = ?$

$J_x = J_o x^2$; $J_y = J_o y^2$; $J_z = J_o z^2$ A/m²

using Continuity Current eqⁿ

$$\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t} \; A/m^3$$

$$\Rightarrow \frac{\partial \rho_v}{\partial t} = -\nabla \cdot \bar{J} = -\left[\frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z}\right] A/m^3$$

$$\frac{\partial J_x}{\partial x} = J_o(2x) = 2x J_o \; A/m^3$$

$$\frac{\partial J_y}{\partial y} = J_o(2y) = 2y J_o \; A/m^3$$

$$\frac{\partial J_z}{\partial z} = J_o(2z) = 2z J_o \; A/m^3 \therefore 2z J_o \, A/m^3.$$

$$\boxed{\frac{\partial \rho_v}{\partial t} = -2J_o\left[x + y + z\right]} \; A/m^3$$

case i) (0.02, 0.01, 0.01)

$$\frac{\partial \rho_v}{\partial t} = -2J_o\left[0.02 + 0.01 + 0.01\right] = -0.08 J_o A/m^3$$

ii) (0.02, -0.01, -0.01)

$$\frac{\partial \rho_v}{\partial t} = -2J_0\left[0.02-0.01-0.01\right] = 0\ A/m^3$$

02 - June /July 2011

5 Prove the total current flowing through the surface, S is given by $I = \int_s J.ds$ AMM. (04 Marks)

problem 66

soln — refer. Page No. 228.

Q) Find the Current Crossing the portion of the y=0 plane defined by $-0.1 \le x \le 0.1m$ and $-0.002m \le z \le 0.002m$ if $\overline{J} = 10^2|x|\ \overline{a_y}\ A/m^2$.

soln

$$I = \oint_{\langle S\rangle} \overline{J}\cdot\overline{ds}$$

$$\overline{J} = 10^2|x|\ \overline{a_y}\ A/m^2$$

$$\overline{ds} = dx\,dz\,(\overline{a_y}) \quad \because y=0 \text{ plane or } xz \text{ plane.}$$

$$I = \oint_{\langle S\rangle} \overline{J}\cdot\overline{ds} = \int_{\langle S\rangle} 10^2|x|\,\overline{a_y}\cdot dx\,dz\,\overline{a_y}\ \Big|_{y=0\text{ plane}}$$

$$= 10^2 \times \int_{x=-0.1}^{0.1} |x|\,dx \times \int_{z=-0.002}^{0.002} dz \times \overline{a_y}\cdot\overline{a_y}^{\,1}$$

$$= 10^2 \times 0.004 \times \left[\int_0^{0.1} x\,dx + \int_{-0.1}^{0} -x\,dx\right]$$

$$\left[\because |x| = \begin{cases} x ; & x \ge 0 \\ -x ; & x < 0\end{cases} \qquad \int |x|\,dx = \begin{cases} \frac{x^2}{2} ; & x \ge 0 \\ -\frac{x^2}{2} ; & x < 0\end{cases}\right.$$

$$= 0.4\left[5\times10^{-3} + 5m\right]$$

Ans.

$$\boxed{I = 4mA}$$

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problem 67

Q) Find the Current Crossing the portion of the $x=0$ plane defined by $-\pi/4 \le y \le \pi/4$ m and $-0.01 \le z \le 0.01$ m. if $\overline{J} = 100\cos(2y)\,\overline{a_x}$ A/m^2.

$$I = \int_{\langle S \rangle} \overline{J}\cdot\overline{ds} = \int_{\langle S \rangle} 100\cos(2y)\,\overline{a_x}\cdot dy\,dz\,(+\overline{a_x}).$$

$$= \int_{y=-\pi/4}^{\pi/4} 100\cos(2y)\,dy \times \int_{z=-0.01}^{0.01} dz \times \cancel{\overline{a_x}\cdot\overline{a_x}}^{\,1}$$

$$I = 100 \times 0.02 \times 1 = 2.0\,A \Rightarrow \boxed{I = 2}\,A$$

problem 68

Q) Given $\overline{J} = 10^3 \sin\theta\,\overline{a_r}$ A/m^2 in spherical Co-ordinates. Find the Current Crossing the spherical shell $r = 0.02$m.

Sol^n: $I = \int_{\langle S \rangle} \overline{J}\cdot\overline{ds}\Big|_{r=k\ \text{Sphere}}$.

$P(r, \theta, \phi)$ → dr, $r\,d\theta$, $r\sin\theta\,d\phi$

$$\overline{ds} = r^2\sin\theta\,d\theta\,d\phi\,\overline{a_r}$$

$$I = \int_{\langle S \rangle} 10^3\sin\theta\,\overline{a_r}\cdot r^2\sin\theta\,d\theta\,d\phi\,\overline{a_r}\Big|_{r=0.02m}.$$

$$= 10^3 r^2 \int_{\theta=0}^{\pi} \sin^2\theta\,d\theta \times \int_{\phi=0}^{2\pi} d\phi\ \ \cancel{\overline{a_r}\cdot\overline{a_r}}^{\,1}$$

$$= 10^3(0.02)^2 \times 1.5707 \times 2\pi \times 1 = \underline{3.9478A}$$

$$\boxed{I = 3.95}\ \text{Amperes}$$

problem 68a.

→ A Conductor Carries steady Current of I amperes. The components of Current density vector $\bar{J}$ are $J_x = 2ax$ and $J_y = 2ay$. Find the third component J_z. Derive any relation employed. June - 2006 (10M).

Note:- Module-5A Question.

Sol:- using Continuity eq^n

$$\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t} \; A/m^3.$$

if Conductor Carries Steady Current then

$$\rho_v = \text{constant} \Rightarrow \frac{\partial \rho_v}{\partial t} = 0 \; C/m^3\text{-sec}.$$

$$\Rightarrow \nabla \cdot \bar{J} = 0$$

$$\frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} = 0$$

$$\frac{\partial}{\partial x}(2ax) + \frac{\partial}{\partial y}(2ay) + \frac{\partial J_z}{\partial z} = 0$$

$$2a + 2a + \frac{\partial J_z}{\partial z} = 0$$

$$\frac{\partial J_z}{\partial z} = -4a.$$

Integrating w.r.t z

$$\boxed{J_z = -4az + K} \; A/m^2$$

where K - integral Constant.

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→ soln refer - Page No - 229. 02 - June /July 2010

8 Given the vector current density $\bar{J} = 10\rho^2 z\hat{a}_\rho - 4\rho\cos^2\phi\hat{a}_\phi$ mA/m². find the total current flowing outward through the circular band $\rho = 3$m, $0 < \phi < 2\pi^c$, $2m < z < 2.8$m. (06 Marks) (repeated).

Miscellaneous Topics

Topic 2.14 Potential Gradient.

Questions

Show that the electric field intensity is a negative of the gradient of the electric Scalar potential.
ie $\bar{E} = -\nabla V$ V/m. (5m)

(or).

Show that $\bar{E} = -\nabla V$ V/m. (6m)

(or).

prove, using Cartesian Co-ordinate System, that $\bar{E} = -\nabla V$ V/m where E and V have their respective names of field intensity and potential. (7m).

[02-Dec 2010, 02-Jan 2009, 06-Jan 2009, 06-Jan 2010, 06-Jan 2019, 06-Jan 2014, 10-Jan 2014, 02-June/July 2012, 02-June/July 2010]

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Soln:- Method I :- using Spherical Co-ordinate System.

w.k.t the Electric field Intensity $\overline{E}$ due to point charge is given by

$$\overline{E} = \frac{Q}{4\pi\epsilon r^2}\,\overline{a_r} \text{ V/m} \quad \leftarrow (1)$$

the potential at a point due to point charge is given by

$$V = \frac{Q}{4\pi\epsilon r} \text{ volts} \quad \leftarrow (2)$$

$\Rightarrow V = f^n(r)$ only = Scalar f^n.

∇ in Spherical Co-ordinate System

$$\nabla = \frac{\partial}{\partial r}\overline{a_r} + \frac{1}{r}\frac{\partial}{\partial \theta}\overline{a_\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\overline{a_\phi} \text{ m}$$

$$\nabla V = \text{Gradient} = \frac{\partial V}{\partial r}\overline{a_r} + \frac{1}{r}\cancel{\frac{\partial V}{\partial \theta}}^{0}\overline{a_\theta} + \frac{1}{r\sin\theta}\cancel{\frac{\partial V}{\partial\phi}}^{0}\overline{a_\phi} \text{ V/m}$$

Since $V = f^n(r)$ only $\therefore$

$$\nabla V = \frac{\partial V}{\partial r}\overline{a_r} \quad \leftarrow (3)$$

from eq^n (2) $\frac{\partial V}{\partial r} = \frac{-Q}{4\pi\epsilon r^2} \quad \leftarrow (4)$

using eq^n (4) in (3)

$$\nabla V = -\frac{Q}{4\pi\epsilon r^2}\bar{a}_r = \bar{E}$$

using eqn (1)

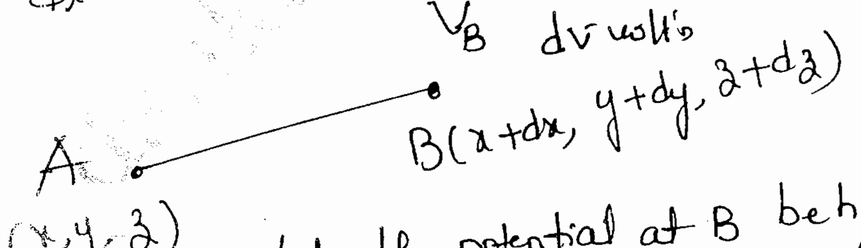
$$\Rightarrow \nabla V = -\bar{E}$$

(a) $$\boxed{\bar{E} = -\nabla V} \quad V/m$$

i.e $\bar{E}$ is a negative potential gradient of the potential V.

Method-II Using Rectangular Co-ordinate System

Consider two neighboring points $A(x, y, z)$ and $B(x+dx, y+dy, z+dz)$, separated by a small distance $\bar{dl}$ in an Electric field $\bar{E}$.



Let the potential at B be higher than that at A by an amount of dv

$\therefore$ the workdone in moving a charge Q_c from A to B is

$$dw = -Q\,\bar{E}\cdot\bar{dl} \text{ Joules.}$$

if $Q = 1C$ i.e unit charge, the potential $dw = dv$

i.e $dV = \frac{dw}{Q \to 1C} = -\bar{E} \cdot \bar{dl}$ Volts

$$dV = -\bar{E} \cdot [dx\, \bar{a}_x + dy\, \bar{a}_y + dz\, \bar{a}_z] \leftarrow (1)$$

where $\bar{a}_x$, $\bar{a}_y$ and $\bar{a}_z$ are the unit vectors along x, y and z directions.

The potential difference dV can be considered as the change in the potential V as we move from A to B,

which is $$dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$$

i.e
$$dV = \left[\frac{\partial V}{\partial x} \bar{a}_x + \frac{\partial V}{\partial y} \bar{a}_y + \frac{\partial V}{\partial z} \bar{a}_z\right] \cdot [dx\, \bar{a}_x + dy\, \bar{a}_y + dz\, \bar{a}_z] \leftarrow (2)$$

eq^n (1) in eq^n (2)

$$-\bar{E} \cdot [dx\, \bar{a}_x + \cancel{dy\, \bar{a}_y} + dz\, \bar{a}_z] = \left[\frac{\partial V}{\partial x} \bar{a}_x + \frac{\partial V}{\partial y} \bar{a}_y + \frac{\partial V}{\partial z} \bar{a}_z\right] \cdot [dx\, \bar{a}_x + \cancel{dy\, \bar{a}_y} + dz\, \bar{a}_z]$$

$$-\bar{E} = \frac{\partial V}{\partial x} \bar{a}_x + \frac{\partial V}{\partial y} \bar{a}_y + \frac{\partial V}{\partial z} \bar{a}_z$$

$$-\bar{E} = \left[\frac{\partial}{\partial x} \bar{a}_x + \frac{\partial}{\partial y} \bar{a}_y + \frac{\partial}{\partial z} \bar{a}_z\right] \cdot V$$

$$-\bar{E} = \nabla V$$

$$\Rightarrow \boxed{\bar{E} = -\nabla V} \text{ V/m}$$

Thus the $\bar{E}$ at any point is given by the negative of the gradient of potential at that point.

Solved problems

problem 68

2 . ~~Given the potential field, $V = 50x^2yz + 20y^2$ volts in free space~~

Find: i) ~~Voltage at a point P(1, 2, 3) ii) Field strength at P iii) $\vec{a}_r$ at P.~~ ~~(07 Marks)~~

$\Rightarrow V = 50x^2yz + 20y^2$ volts ~~06-DEC2010~~

7 Given the potential field $V = 50x^2yz + 20y^2$ volts in free space, find

i) Potential V at P(1, 2, 3) , V at P(1, 2, 3)

ii) $|E_P|$ (Magnitude of electric potential) $\rightarrow |\bar{E}_p|$ (7m) 06-Dec 2010,

iii) $\hat{a}_r$ at P. $\rightarrow \bar{a}_r$ at P (05 Marks) Jan 2012

soln: given $\bar{V} = 50x^2yz + 20y^2$ volts

i) $V_p = 50(1)^2(2)(+3) + 20(2)^2$

$$\boxed{V_p = 380} \text{ volts}$$

ii) $\bar{E} = -\nabla V = -\left[\frac{\partial V}{\partial x}\bar{a}_x + \frac{\partial V}{\partial y}\bar{a}_y + \frac{\partial V}{\partial z}\bar{a}_z\right]$ V/m.

$\frac{\partial V}{\partial x} = 100xyz$; $\frac{\partial V}{\partial y} = 50x^2z$; $\frac{\partial V}{\partial z} = 50x^2y$.

$$\bar{E} = -100xyz\,\bar{a}_x - 50x^2z\,\bar{a}_y - 50x^2y\,\bar{a}_z \text{ V/m}.$$

@ P(1, 2, 3)

so $$\boxed{\bar{E}_p = -\left[600\,\bar{a}_x + 230\,\bar{a}_y + 100\,\bar{a}_z\right]} \text{ V/m}$$

iii) $\bar{a}_r$ at P i.e $\bar{a}_r = \frac{\bar{E}_p}{|\bar{E}_p|}$

$|\bar{E}_p| = 650.307$ V/m

$$\boxed{\bar{a}_r = \text{dirc}^{y} \text{ of } \bar{E} = -\left[0.92\,\bar{a}_x + 0.35\,\bar{a}_y + 0.153\,\bar{a}_z\right]}$$

Topic 2.15

Gradient in all three Co-ordinate Systems

→ Cartesian Co-ordinate System.

$P(x, y, z)$ → dx, dy, dz

$$\nabla = \frac{\partial}{\partial x}\overline{a_x} + \frac{\partial}{\partial y}\overline{a_y} + \frac{\partial}{\partial z}\overline{a_z} \quad m^{-1}$$

V → Scalar f^n.

$$\boxed{\nabla V = \frac{\partial V}{\partial x}\overline{a_x} + \frac{\partial V}{\partial y}\overline{a_y} + \frac{\partial V}{\partial z}\overline{a_z}} \quad V/m.$$

→ Cylindrical C.S $P(\rho, \phi, z)$ → $d\rho$, $\rho d\phi$, dz

$$\nabla = \frac{\partial}{\partial \rho}\overline{a_\rho} + \frac{1}{\rho}\frac{\partial}{\partial \phi}\overline{a_\phi} + \frac{\partial}{\partial z}\overline{a_z} \quad m^{-1}$$

$$\boxed{\nabla V = \frac{\partial V}{\partial \rho}\overline{a_\rho} + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\overline{a_\phi} + \frac{\partial V}{\partial z}\overline{a_z}} \quad V/m.$$

→ Spherical C.S $P(r, \theta, \phi)$ → dr, $r d\theta$, $r\sin\theta d\phi$

$$\nabla = \frac{\partial}{\partial r}\overline{a_r} + \frac{1}{r}\frac{\partial}{\partial \theta}\overline{a_\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\overline{a_\phi}$$

$$\boxed{\nabla V = \frac{\partial V}{\partial r}\overline{a_r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\overline{a_\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\overline{a_\phi}} \quad V/m.$$

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Key Notes:- TO Solve problem's

if given a potential field

$V = f^n$ (Spatial Variables)

$P(x, y, z)$ and $P(r, \theta, \phi)$

$P(\rho, \phi, z)$

① V_P

② $\bar{E} = -\nabla V$ V/m.

③ $\bar{E}_P$ (or) ④ $|\bar{E}_P|$ (or) $\frac{dV}{dN}$

$\frac{dV}{dN} \rightarrow$ nothing but Mag of $\bar{E}$.

⑤ $\bar{a}_E$ = direction of $\bar{E}$ = $\frac{\bar{E}}{|\bar{E}|} = \bar{a}_r$

⑥ $\bar{D} = \epsilon \bar{E}$ C/m²

⑦ $|\bar{D}_P| = \rho_S$ C/m²

Maxwell's first eq^n

⑧ $\boxed{\nabla \cdot \bar{D} = \rho_v}$ C/m³

@ P

volume charge density.

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problem 69

$\rightarrow V = \frac{\cos(2\phi)}{r}$

Let $V = \frac{\cos 2\phi}{r}$ in the free space, in cylindrical system. Find :

i) $\bar{E}$ at A(2, 30°, 1)

ii) ρ_v at B(0.5, 60°, 1)

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(08 Marks)

10- Jan 2013

$\bar{E}$ at $A(2, 30°, 1)$

ρ_v at $B(0.5, 60°, 1)$

soln:

$$\nabla V = \frac{\partial V}{\partial r}\bar{a_r} + \frac{1}{r}\frac{\partial V}{\partial \phi}\bar{a_\phi} + \cancel{\frac{\partial V}{\partial z}}^{0}\bar{a_z} \quad \text{bcz } V \neq f^n(z)$$

$$\text{i)} \quad \bar{E} = -\nabla V = -\left[\frac{\partial V}{\partial r}\bar{a_r} + \frac{1}{r}\frac{\partial V}{\partial \phi}\bar{a_\phi}\right] \text{ V/m}$$

$$\frac{\partial V}{\partial r} = -\frac{\cos 2\phi}{r^2} \quad ; \quad \frac{\partial V}{\partial \phi} = -\frac{2\sin(2\phi)}{r^2}$$

$$\bar{E} = \underbrace{+\frac{\cos 2\phi}{r^2}}_{E_r}\bar{a_r} + \underbrace{\frac{2\sin(2\phi)}{r^2}}_{E_\phi}\bar{a_\phi}$$

$\bar{E}$ at $A(2, 30°, 1)$ put $r = 2, \phi = 30°$

$$\bar{E}_A = \frac{\cos(60)}{2^2}\bar{a_r} + \frac{2\sin(60°)}{2^2}\bar{a_\phi} \text{ V/m.}$$

$$\boxed{\bar{E}_A = 0.125\,\bar{a_r} + 0.43301\,\bar{a_\phi}} \text{ V/m}$$

$$\text{ii)} \quad \rho_v = \nabla\cdot\bar{D} = \frac{1}{r}\frac{\partial}{\partial r}[rD_r] + \frac{1}{r}\frac{\partial}{\partial \phi}[D_\phi] + \frac{\partial D_z}{\partial z}$$

$$\rho_v = \nabla\cdot\bar{D} = \epsilon\,\nabla\cdot\bar{E} = \frac{1}{r}\frac{\partial}{\partial r}[rE_r] + \frac{1}{r}\frac{\partial E_\phi}{\partial \phi} + \cancel{\frac{\partial E_z}{\partial z}}^{0}$$

$$\rho_v = \epsilon\left[\frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot\frac{\cos 2\phi}{r^2}\right] + \frac{1}{r}\frac{\partial}{\partial \phi}\left[\frac{2\sin(2\phi)}{r^2}\right]\right]$$

$$= \epsilon\left[-\frac{\cos 2\phi}{r^3} + \frac{1}{r^3}4\cos(2\phi)\right] \quad ; \quad \rho_v @ B(0.5, 60°, 1)$$

$$\rho_v = \epsilon\left[-\frac{\cos(120°)}{0.5^3} + \frac{4\cos(120°)}{0.5^3}\right] = -12\epsilon = -12\times 8.854\times 10^{-12}$$

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(or) $\rho_v = -0.10624 \text{ nC/m}^3$

problem 70

$x(1,2,-1)$ $V = 3x^2y + 2y^2z + 3xyz$.

Find the electric field intensity at point x(1, 2, -1) given the potential $V = 3x^2y + 2y^2z + 3xyz$.
(05 Marks)

Dec 2014

Soln:

$$\bar{E} = -\nabla V$$

$$= -\left[\frac{\partial V}{\partial x}\bar{a_x} + \frac{\partial V}{\partial y}\bar{a_y} + \frac{\partial V}{\partial z}\bar{a_z}\right] V/m.$$

$$\frac{\partial V}{\partial x} = 6xy + 3yz. \quad \frac{\partial V}{\partial y} = 3x^2 + 4yz + 3xz.$$

$$\frac{\partial V}{\partial z} = 2y^2 + 3xy.$$

$\bar{E}$ @ x (1, 2, -1) put x = 1, y = 2, z = -1.

$$\frac{\partial V}{\partial x} = 12 - 6 = 6. \quad \frac{\partial V}{\partial y} = \cancel{3} - 8 - \cancel{3} = -8$$

$$\frac{\partial V}{\partial z} = 2(4) - 6 = 2$$

$$= -\left[6\bar{a_x} - 8\bar{a_y} + 2\bar{a_z}\right] V/m.$$

$$\boxed{\bar{E}_x = -6\bar{a_x} + 8\bar{a_y} - 2\bar{a_z}} \; V/m$$

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problem 71 $V = 100(x^2-y^2)$ volts $P(2,-1,3)$m $V, \bar{E}, \bar{D}$ and ρ_s at P

Given V = 100 ($x^2 - y^2$) volts, and pt. on the surface, P(2, -1, 3)$_m$, find V, $\bar{E}$, $\bar{D}$ and ρ_s at P, and the equation of the conductor surface. (06 Marks)

Soln:- $V = 100(x^2-y^2)$ volts

$P(2,-1,3)$ i.e $x=2,\ y=-1,\ z=3.$

i) $V_p = 100[2^2-(-1)^2] = 100[4-1] = \underline{300\text{ volts}}$.

ii) $\bar{E} = -\nabla V = -\left[\frac{\partial V}{\partial x}\bar{a}_x + \frac{\partial V}{\partial y}\bar{a}_y + \frac{\partial V}{\partial z}\bar{a}_z\right]$ V/m.

$\frac{\partial V}{\partial x} = 100(2x)$ $\quad \frac{\partial V}{\partial y} = 100(-2y)$ $\quad \frac{\partial V}{\partial z} = 0$.

$\frac{\partial V}{\partial x} = 200x$; $\frac{\partial V}{\partial y} = -200y$

$\bar{E} = -200x\,\bar{a}_x + 200y\,\bar{a}_y$ V/m.

$\bar{E}_p = -200(2)\bar{a}_x + 200(-1)\bar{a}_y$ V/m

$\boxed{\bar{E}_p = -400\bar{a}_x - 200\bar{a}_y}$ V/m

iii) $\bar{D}_p = \epsilon_0 \bar{E}_p = 8.854[-400\bar{a}_x - 200\bar{a}_y]$ pC/m^2.
$= [-3.5416\,\bar{a}_x - 1.770\,\bar{a}_y]$ nC/m^2.

iv) ρ_s at P $\rho_s = |\bar{D}_p| = \underline{3.95927}$ nC/m^2

v) eqn of Conductor Surface

$V = 100(x^2-y^2)$ volts

put $V = 300$ volts

$300 = 100(x^2-y^2)$

$\Rightarrow \boxed{x^2-y^2=3}$ ← eqn of conductor Surface.

problem 72

$V = 100\sinh(5x)\sin(5y)$ volts

Given the potential field in free space, $V = 100\sinh(5x)\sin(5y)$ Volts and point $P(0.1, 0.2, 0.3)_m$, find (i) V at P (ii) $\bar{E}$ at P (iii) $|\bar{E}|$ at P (iv) $|\rho_s|$ at P, assuming P lies on the conductor surface. (08 Marks)

$p(0.1, 0.2, 0.3)m$

$V = 100\sinh(5x)\sin(5y)$ $\quad P(0.1, 0.2, 0.3)$

i) $V_p = 100\sinh(5\times0.1)\sin[5(0.2)] = 0.90943$ volts

$$\boxed{V_p = 0.90943} \text{ volts}$$

ii) $\bar{E} = -\nabla V = -\frac{\partial V}{\partial x}\bar{a}_x - \frac{\partial V}{\partial y}\bar{a}_y$

$\frac{\partial V}{\partial x} = 100\cosh(5x)\times5\sin(5y) = 500\cosh(5x)\sin(5y)$.

$\frac{\partial V}{\partial y} = 500\sinh(5x)\cos(5y)$.

$\bar{E} = -500\cosh(5x)\sin(5y)\bar{a}_x - 500\sinh(5x)\cos(5y)\bar{a}_y$ V/m.

$$\boxed{\bar{E}_p = -9.8398\,\bar{a}_x - 260.50\,\bar{a}_y} \text{ V/m.}$$

iii) $|\bar{E}_p| = 260.693$ V/m.

iv) $\rho_s = |\bar{D}| = \epsilon|\bar{E}_p| = 8.854\times 260.693$ pC/m²

$$\boxed{|\rho_s| = 2.30818} \text{ nC/m}^2$$

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problem 73 $V = (x+1)^2 + (y+2)^2 + (z+3)^2$ volts

Electrical potential at an arbitrary point in free – space is given as : $V = (x+1)^2 + (y+2)^2 + (z+3)^2$ volts. at P(2, 1, 0) find $P(2,1,0)m$.

i) V ii) $\vec{E}$ iii) $|\vec{E}|$ iv) $\vec{D}$ v) $|\vec{D}|$ vi) $P_v \rightarrow \rho_v$. (10 Marks)

V, $\bar{E}$, $|\bar{E}|$, $\bar{D}$, $|\bar{D}|$, ρ_v.

Solu: i) $V = (x+1)^2 + (y+2)^2 + (z+3)^2$

$$V_p = 3^2 + 3^2 + 3^2 = 27 \text{ Volts}$$

$$\boxed{V_p = 27} \text{ Volts}$$

ii) $$\bar{E} = -\frac{\partial V}{\partial x}\bar{a}_x - \frac{\partial V}{\partial y}\bar{a}_y - \frac{\partial V}{\partial z}\bar{a}_z \text{ V/m}.$$

$$\frac{\partial V}{\partial x} = 2(x+1) \; ; \; \frac{\partial V}{\partial y} = 2(y+2) \; ; \; \frac{\partial V}{\partial z} = 2(z+3)$$

$$\bar{E} = -2(x+1)\bar{a}_x - 2(y+2)\bar{a}_y - 2(z+3)\bar{a}_z \text{ V/m}.$$

$$\boxed{\bar{E}_p = -6\bar{a}_x - 6\bar{a}_y - 6\bar{a}_z} \text{ V/m}$$

iii) $$|\bar{E}_p| = \sqrt{108} = 10.3923 \text{ V/m}.$$

iv) $$\bar{D} = \epsilon\bar{E} = 8.854\,\bar{E} \text{ pC/m}^2.$$

$$\bar{D} = -17.708(x+1)\bar{a}_x - 17.708(y+2)\bar{a}_y - 17.708(z+3)\bar{a}_z \text{ pC/m}^2.$$

v) $$\boxed{|\bar{D}_p| = \epsilon|\bar{E}_p| = 92.0134 \text{ pC/m}^2}$$

vi) $$\rho_v = \nabla\cdot\bar{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} \text{ C/m}^3$$

$$\rho_v = \nabla\cdot\bar{D} = [-17.708 \quad -17.708 \quad -17.708] \text{ pC/m}^3$$

$$\boxed{\rho_{v_p} = -53.124 \text{ pC/m}^3}$$

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8) if the potential field $V = 3x^2 + 3y^2 + 2z^3$ Volts, Find i) V ii) $\bar{E}$ and iii) $\bar{D}$ at $P(-4,5,4)$. 14-Jan 2015. EEE (6 Marks)

Soln: $V = 3x^2 + 3y^2 + 2z^3$ volt's

$V_p = 3(-4)^2 + 3(5)^2 + 2(4)^3$

$\boxed{V_p = 251}$ volt's

ii) $\bar{E} = -\nabla V = -\frac{\partial V}{\partial x}\bar{a}_x - \frac{\partial V}{\partial y}\bar{a}_y - \frac{\partial V}{\partial z}\bar{a}_z$ V/m.

$\frac{\partial V}{\partial x} = 6x$; $\frac{\partial V}{\partial y} = 6y$; $\frac{\partial V}{\partial z} = 6z^2$.

$\bar{E} = -6x\,\bar{a}_x - 6y\,\bar{a}_y - 6z^2\,\bar{a}_z$ V/m.

$\bar{E}_p = -6(-4)\,\bar{a}_x - 6(5)\,\bar{a}_y - 6(4)^2\,\bar{a}_z$ V/m

$\boxed{\bar{E}_p = +24\,\bar{a}_x - 30\,\bar{a}_y - 96\,\bar{a}_z}$ V/m.

iii) $\bar{D}_p = \epsilon \bar{E}_p = 8.854[24\,\bar{a}_x - 30\,\bar{a}_y - 96\,\bar{a}_z]$ pC/m²

$\boxed{\bar{D}_p = 0.21249\,\bar{a}_x - 0.2656\,\bar{a}_y - 0.8499\,\bar{a}_z}$ nC/m²

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problem 75

$V = \frac{2\cos\phi}{r^2}$ $P(0.5, 45°, 60°)$.

Find V and the volume charge density in free space, if $V = \frac{2\cos\phi}{r^2}$ at P(0.5, 45°, 60°).

(07 Marks)

soln:- $\bar{V} = \frac{2\cos\phi}{r^2}$ ---- in spherical C.S

$r = 0.5, \ \theta = 45°$
$\phi = 60°$

$$V_p = \frac{2\cos(60°)}{(0.5)^2}$$

$$\boxed{V_p = 4} \text{ volts}$$

$\bar{E} = -\nabla V \ \text{V/m}$

$$\rho_v = \nabla\cdot\bar{D} = \epsilon\nabla\cdot\bar{E} = +\epsilon\nabla\cdot(-\nabla V) = -\epsilon\nabla^2 V$$

$$\rho_v = -\epsilon\nabla^2 V \ \text{C/m}^3$$

$V \neq f^n(\theta)$ 0

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] + \cancel{\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial V}{\partial\theta}\right]} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$$

$$V = \frac{2\cos\phi}{r^2} \text{ volts}$$

$$\frac{\partial V}{\partial r} = -\frac{4\cos\phi}{r^3} \text{ volts}$$

$$\frac{\partial V}{\partial\phi} = \frac{-2\sin\phi}{r^2}$$

$$\frac{\partial^2 V}{\partial\phi^2} = \frac{-2\cos\phi}{r^2}$$

$$\rho_v = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\cdot\frac{(-4\cos\phi)}{r^3}\right] + \frac{1}{r^2\sin^2\theta}\times\frac{-2\cos\phi}{r^2}$$

$$= \frac{1}{r^2}\times\frac{+4\cos\phi}{r^2} - \frac{2\cos\phi}{r^4\sin^2\theta} = \frac{4\cos\phi}{r^4} - \frac{2\cos\phi}{r^4\sin^2\theta}$$

$$\rho_{v_p} = \frac{4\cos(60°)}{(0.5)^4} - \frac{2\cos(60°)}{(0.5)^4\sin^2(45)} = 32 - 32 = 0 \ \text{C/m}^3$$

$$\rho_v = 0 \ \text{C/m}^3$$

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problem 76

$\boxed{\rho_v = 0}$ C/m³ (E) $V = 2x^2y - 5z$. $P(-4,3,6)$

Find the potential, electric field intensity and volume charge density at a point P(– 4, 3, 6) provided the potential field V = $2x^2y - 5z$. (08 Marks) 10-Dec-2014

(or)

Given potential field V = $2x^2y - 5z$ and at a point P(-4, 3, 6), obtain

i) V, ii) $\vec{E}$, iii) Direction of $\vec{E}$. (05 Marks) Jan 2015

(or)

Given $V = 2x^2y - 5z$ at point P(-4, 3, 6). Find the potential, electric field intensity and volume charge density. (08 Marks)

soln:- given $V = 2x^2y - 5z$ volts. Jul-2013 at $P(-4,3,6)$.

i. $V_p = 2(-4)^2(3) - 5(6) = 66$ volts

$\frac{\partial V}{\partial x} = 4xy$; $\frac{\partial V}{\partial y} = 2x^2$; $\frac{\partial V}{\partial z} = -5$. $\boxed{V_p = 66}$ volts

ii. $\bar{E} = -\frac{\partial V}{\partial x}\bar{a}_x - \frac{\partial V}{\partial y}\bar{a}_y - \frac{\partial V}{\partial z}\bar{a}_z$ V/m.

$\bar{E} = -4xy\,\bar{a}_x - 2x^2\bar{a}_y + 5\bar{a}_z$ V/m.

$\bar{E}_p = -4(-4)(3)\bar{a}_x - 2(-4)^2\bar{a}_y + 5\bar{a}_z$ V/m.

$\boxed{\bar{E}_p = +48\bar{a}_x - 32\bar{a}_y + 5\bar{a}_z}$ V/m.

iii. $\boxed{|\bar{E}_p| = 57.905}$ V/m.

iv. $\bar{D}_p = \epsilon\bar{E}_p = 8.854[+48\bar{a}_x - 32\bar{a}_y + 5\bar{a}_z]$ pC/m²

$\boxed{\bar{D}_p = +0.4249\bar{a}_x - 0.2833\bar{a}_y + 0.04427\bar{a}_z}$ nC/m²

v. $|\bar{D}_p| = \epsilon_0|\bar{E}_p| = 512.69$ pC/m²

(or) $\boxed{|\bar{D}_p| = 0.51269}$ nC/m²

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vi. Volume charge density (ρ_v)

$$\rho_v = \nabla\cdot\bar{D} = \epsilon\,\nabla\cdot\bar{E} \quad C/m^3$$

P(-4, 3, 6)

$$= \epsilon\left[\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right] \quad C/m^3$$

$E_x = -4xy$ V/m ; $E_y = -2x^2$ V/m ; $E_z = 5$ V/m

$$\frac{\partial E_x}{\partial x} = -4y \;;\; \frac{\partial E_y}{\partial y} = 0 \;;\; \frac{\partial E_z}{\partial z} = 0$$

$$\rho_v = \epsilon(-4y) \quad C/m^3$$

$y = 3$

$$\rho_{v_p} = \epsilon(-4\times 3) \; C/m^3 = -12\epsilon \; C/m^3$$

$$\boxed{\rho_{v_p} = -106.248} \; pC/m^3$$

2nd Method (or)

$$\rho_{v_p} = -\epsilon\nabla^2 V = -\epsilon\left[\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}\right]$$

$$V = 2x^2y - 5z$$

$$\frac{\partial V}{\partial x} = 4xy \;;\; \frac{\partial^2 V}{\partial x^2} = 4y$$

$$\frac{\partial V}{\partial y} = 2x^2 \;;\; \frac{\partial^2 V}{\partial y^2} = 0$$

$$\frac{\partial V}{\partial z} = -5 \;;\; \frac{\partial^2 V}{\partial z^2} = 0$$

$$\rho_{v_p} = -\epsilon(4y) = -4y\epsilon \; C/m^3 = -12\epsilon \; C/m^3$$

$$= -106.248 \; pC/m^3 \text{ (or)}$$

$$= -0.10624 \; nC/m^3$$

problem 77

$V = \frac{60\sin\theta}{r^2}$ (3, 60°, 25°)

An electric potential is given by $V = \frac{60\sin\theta}{r^2}$. Find v and $\bar{E}$ at (3, 60°, 25°). (08 Marks)

(or) Dec/Jan 2014.

Dec/Jan 2016

b. If $V = \frac{60\sin\theta}{r^2}$ V find V and $\bar{E}$ at P (3,60,25) (05 Marks)

soln:

$$V = \frac{60\sin\theta}{r^2} \text{ volts}$$

P(3, 60°, 25°)

$$V_p = \frac{60\sin(60)}{3^2} = 5.773 \text{ volts}$$

$$\boxed{V_p = 5.7735} \text{ volts}$$

$$\bar{E} = -\nabla V = -\frac{\partial V}{\partial r}\bar{a_r} - \frac{1}{r}\frac{\partial V}{\partial \theta}\bar{a_\theta} - \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\bar{a_\phi} \text{ V/m.}$$

$$\frac{\partial V}{\partial r} = \frac{-2\times 60\sin\theta}{r^3} = \frac{-120\sin\theta}{r^3}$$

$$\frac{\partial V}{\partial \theta} = \frac{60\cos\theta}{r^2}; \quad \frac{\partial V}{\partial \phi} = 0.$$

$$\bar{E} = +\frac{120\sin\theta}{r^3}\bar{a_r} - \frac{60\cos\theta}{r^3}\bar{a_\theta} \text{ V/m.}$$

$$\bar{E_p} = \frac{120\sin(60)}{3^3}\bar{a_r} - \frac{60\cos(60)}{3^3}\bar{a_\theta} \text{ V/m}$$

$$\boxed{\bar{E_p} = 3.849\bar{a_r} - 1.111\bar{a_\theta}} \text{ V/m}$$

$$\boxed{|\bar{E_p}| = 4.00616} \text{ V/m}$$

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problem 78 Find the Electric Field Intensity at point $x(1,2,-1)$ given the potential $V = 3x^2y + 2y^2z + 3xyz$.

[EEE-(5M) Jan 2014.]

Sol^n: i. $V = 3x^2y + 2y^2z + 3xyz$.

$$V_p = 3(1)^2(2) + 2(2)^2(-1) + 3(1)(2)(-1)$$

$$= \cancel{6} + 8(-1) - \cancel{6} = -8$$

$$\boxed{V_p = -8} \text{ volts}$$

ii. $$\bar{E} = -\frac{\partial V}{\partial x}\bar{a_x} - \frac{\partial V}{\partial y}\bar{a_y} - \frac{\partial V}{\partial z}\bar{a_z} \text{ V/m.}$$

$$\frac{\partial V}{\partial x} = 6xy + 3yz \; ; \; \frac{\partial V}{\partial y} = 3x^2 + 4yz + 3xz.$$

$$\frac{\partial V}{\partial z} = 2y^2 + 3xy.$$

$$\bar{E} = -(6xy + 3yz)\bar{a_x} - (3x^2 + 4yz + 3xz)\bar{a_y} - (2y^2 + 3xy)\bar{a_z} \text{ V/m.}$$

$$\bar{E_p} = -[6(1)(2) + 3(2)(-1)]\bar{a_x} - [3(1)^2 + 4(2)(-1) + 3(1)(-1)]\bar{a_y} - [2(2)^2 + 3(1)(2)]\bar{a_z} \text{ V/m}$$

$$\boxed{\bar{E_p} = -6\bar{a_x} - 8\bar{a_y} - 14\bar{a_z}} \text{ V/m}$$

$$|\bar{E_p}| = 17.2046 \text{ V/m}$$

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problem 79

A potential field in free space is expressed as $V = \frac{60\sin\theta}{r^2}$ v. Find the electric flux density at the point (3, 60°, 25°) in spherical co-ordinates. (06 Marks)

15 - Dec/Jan 2017 CBCS-Scheme.

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P(3, 60°, 25°)

Solu:- given

freespace medium $V = \frac{60\sin\theta}{r^2}$ volt's

$\boxed{\epsilon = \epsilon_0}$ F/m $\qquad \bar{D} = ?$ @ P(3, 60°, 25°)

Gradient in Spherical Co-ordinate System is given by

$$\nabla V = \frac{\partial V}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\bar{a}_\phi \quad V/m$$

$$\frac{\partial V}{\partial r} = \frac{-2\times 60\sin\theta}{r^3} = -\frac{120\sin\theta}{r^3} \ V/m.$$

$$\frac{\partial V}{\partial \theta} = \frac{+60\cos\theta}{r^2} \ V/m.$$

$$\frac{\partial V}{\partial \phi} = 0 \quad \text{Since } V \neq f^n(\phi).$$

$$\nabla V = -\frac{120\sin\theta}{r^3}\bar{a}_r + \frac{1}{r}\cdot\frac{60\cos\theta}{r^2}\bar{a}_\theta \ V/m.$$

$$\nabla V = -\frac{120\sin\theta}{r^3}\bar{a}_r + \frac{60\cos\theta}{r^3}\bar{a}_\theta \ V/m.$$

from concept of potential gradient

$$\bar{E} = -\nabla V = +\frac{120\sin\theta}{r^3}\bar{a_r} - \frac{60\cos\theta}{r^3}\bar{a_\theta} \text{ V/m}$$

$\bar{E}$ @ $P(3, 60°, 25°)$

i.e $r = 3m$ and $\theta = 60°$.

$$\bar{E_p} = \frac{120\sin(60°)}{(3)^3}\bar{a_r} - \frac{60\cos(60°)}{(3)^3}\bar{a_\theta}$$

$$\boxed{\bar{E_p} = 3.849\,\bar{a_r} - 1.111\,\bar{a_\theta}} \text{ V/m.}$$

the Electric flux density $\bar{D_p}$ is given by

$$\bar{D_p} = \epsilon_0 \bar{E_p} \text{ C/m}^2$$

$$= 8.854\left[3.849\,\bar{a_r} - 1.111\,\bar{a_\theta}\right] \text{ pC/m}^2$$

$$\boxed{\bar{D_p} = 34.079\,\bar{a_r} - 9.8376\,\bar{a_\theta}} \text{ pC/m}^2$$

$$\boxed{|\bar{D_p}| = 35.4705} \text{ pC/m}^2$$

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topic

Topic: 2.16

Energy density in an electrostatic field.

Questions

02-DEC2010

27 Derive equations of energy stored and energy density in an electrostatic field. (06 Marks)

(or)

06-DEC2008/Jan 2009

29 ~~Derive an expression for energy and energy density in an electrostatic field.~~ (04 Marks)

02 - June /July 2011

30 ~~Derive an expression for energy density in an electric~~ field. (06 Marks)

02 - June /July 2012

31 Prove that the energy density in an electrostatic field is $\frac{1}{2}\vec{D}\cdot\vec{E}$ where D and E are the electric flux density and the electric field intensity respectively. (08 Marks)

06- June /July 2009

(or).

Derive

32 Derive the expression for the energy stored in Electrostatic field having electric field intensity $\vec{E}$. (06 Marks)

intensity $\vec{E}$

(or)

$e = \frac{1}{2}\epsilon E^2$ J/m3

34 Prove that the energy density in an electrostatic field is given by $\frac{1}{2}\epsilon\vec{E}^2$ J/m³. (08 Marks)

06 –Dec/Jan 2008

(or)

10–Dec/Jan 2016

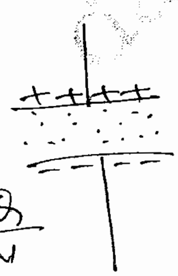
36 c. Derive an equitation for energy stored in terms of $\bar{E}$ and $\bar{D}$ (05 Marks)

solu:

Energy density :- Energy stored per unit volume (J/m³).

w.k.t. Energy stored in a capacitor

$e = \frac{1}{2}CV^2$ Joules ← (1).



and $Q = CV$ ← (2)

$C = \frac{Q}{V}$

eq (2) in eq (1)

$e = \frac{1}{2}QV$ ← (3)

V - potential b/w the plates (volts)

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the total charge Q in a volume is given by

$$Q = \int_{\langle vol \rangle} \rho_v \, dv \leftarrow (4)$$

ρ_v – volume charge density (C/m^3)
dv – differential volume (m^3).

using eq^n (4) in eq^n (3)

$$e = \frac{1}{2} \int_{\langle vol \rangle} \rho_v V \, dv \leftarrow (5)$$

using Maxwell's first equation

$$\nabla \cdot \bar{D} = \rho_v \ C/m^3$$

$$e = \frac{1}{2} \int_{\langle vol \rangle} \nabla \cdot \bar{D} \ V \, dv \leftarrow (6)$$

using a vector identity

$$\nabla \cdot (\phi \bar{A}) = \phi \, \nabla \cdot \bar{A} + \bar{A} \nabla \phi$$

Scalar fn, vector

i.e. $\nabla (V\bar{D}) = V \, \nabla \cdot \bar{D} + \bar{D} \nabla V$

$$\Rightarrow V \, \nabla \cdot \bar{D} = \nabla (V\bar{D}) - \bar{D} \nabla V \leftarrow (7)$$

using eq^n (7) in (6).

$$e = \frac{1}{2} \int_{\langle vol \rangle} \nabla (V \cdot \bar{D}) \, dv - \frac{1}{2} \int_{\langle vol \rangle} \bar{D} \cdot \nabla V \, dv$$

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using Divergence theorem

$$\int_{\langle vol \rangle} \nabla(V\bar{D})\, dv = \oint_{\langle S \rangle} V\bar{D}\cdot \bar{dS}$$

$$\Rightarrow e = \frac{1}{2}\cancel{\oint_{\langle S \rangle} V\bar{D}\cdot\bar{dS}} \to 0 - \frac{1}{2}\int_{\langle vol \rangle} \bar{D}\cdot\nabla V\, dv \leftarrow (8)$$

w.k.t $\bar{E}$ due to point charge

$$\bar{E} = \frac{Q}{4\pi\epsilon r^2}\bar{a_r}$$

as $r \to \infty$ $\bar{E} \to 0 \Rightarrow \bar{D} = 0$

$$\therefore \oint_{\langle S \rangle} \bar{D}\cdot\bar{dS} = 0$$

As Surface increases (or) approaches to ∞ approaches to zero.

$\therefore$ first term in eqn (8)

$$\Rightarrow e = -\frac{1}{2}\int_{\langle vol \rangle} \bar{D}\cdot[\nabla V]\, dv = \frac{1}{2}\int_{\langle vol \rangle} \bar{D}\cdot[-\nabla V]\, dv$$

using Gradient Concept $\bar{E} = -\nabla V$ V/m.

$$\boxed{e = \frac{1}{2}\int_{\langle vol \rangle} \bar{D}\cdot\bar{E}\, dv} \text{ Joules}$$

$\Rightarrow$ Expression of Energy stored.

$$de = \frac{1}{2}\bar{D}\cdot\bar{E}\, dv$$

Note:- $\bar{A}\cdot\bar{A} = A^2$

and Energy density

$$\Rightarrow \boxed{\frac{de}{dv} = \frac{1}{2}\bar{D}\cdot\bar{E}} \text{ J/m}^3$$

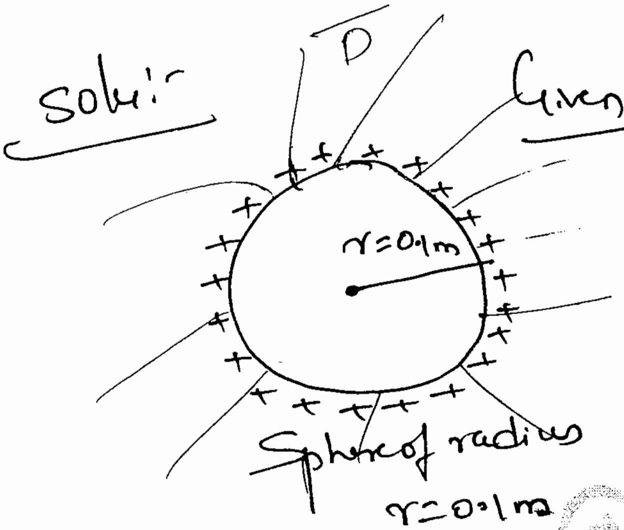
(or) ***

$$\boxed{\frac{de}{dv} = \frac{1}{2}\bar{D}\cdot\bar{E} = \frac{1}{2}\epsilon\bar{E}\cdot\bar{E} = \frac{1}{2}\epsilon E^2} \text{ J/m}^3$$

problem 80

A metallic sphere of radius 10cm has a surface charge density of 10n C/m². calculate electric Energy Stored in the system. (6m)/(7m).

10 June/July 2014
06 - Jan 2009.



Solⁿ:

Given:-

$r = 10cm = 0.1m$

$\rho_s = 10nC/m^2$

the total charge 'Q' enclosed by the sphere is given by $Q = \rho_s \times$ Area of Sphere.

$Q = \rho_s \times 4\pi r^2$

$Q = 10n \times 4\pi (0.1)^2 = \underline{1.2566 nC}$

the Flux density $\bar{D} = \frac{Q}{4\pi r^2} \bar{a}_r$ C/m².

$|\bar{D}| = \frac{Q}{4\pi r^2}$ C/m².

Energy stored in the System $e = \int_{\langle vol \rangle} \frac{1}{2\epsilon_0} |\bar{D}|^2 dv$.

$e = \int_{\langle vol \rangle} \frac{1}{2} \bar{D} \cdot \bar{E} \, dv$

$$e = \frac{1}{2\epsilon_0}\int_{\langle vol \rangle} |\bar{D}|^2 \cdot dv$$

$$e = \frac{1}{2\epsilon_0}\int_{\langle vol \rangle} \frac{Q^2}{(4\pi)^2 r^4} \cdot r^2 \sin\theta\, dr\, d\theta\, d\phi.$$

$$e = \frac{Q^2}{(4\pi)^2 (2\epsilon_0)} \int_{r=0.1}^{\infty} \frac{1}{r^2} dr \int_{\theta=0}^{\pi} \sin\theta\, d\theta \int_{\phi=0}^{2\pi} d\phi$$

$$e = \frac{Q^2}{32 \cdot \pi^2 \epsilon_0} \cdot \left. \frac{r^{-2+1}}{-2+1} \right|_{0.1}^{\infty} \times 2 \times 2\pi.$$

$$e = \frac{-Q^2}{32 \pi^2 \epsilon_0} \left[\frac{1}{\infty} - \frac{1}{0.1} \right] \times 4\pi.$$

$$e = \frac{-Q^2}{32 \pi^2 \epsilon_0} [0 - 10] \times 4\pi$$

$$e = \frac{+40\pi Q^2}{32 \pi^2 \epsilon_0} = \frac{40 (1.2566 \times 10^{-9})^2}{32\pi \times 8.854 \times 10^{-12}}$$

$$e = 7.096 \times 10^{-8} \text{ Joules} = 70.96\,\text{nJoules}$$

problem 81 A potential function is $V = 2x + 4y$ volts in in free space. Find the stored energy in freespace in the 1m³ volume centered at origin. 06-Jan2008 (6m).

06 – Dec/Jan 2008

~~A potential function is $v = 2x + 4y$ volts in free space. Find the stored energy in free space in the 1 m³ volume centered at origin.~~ (06 Marks)

Solu: $\therefore V = 2x + 4y$ volts.

Energy stored in the system

$$e = \frac{1}{2}\int_{\langle vol \rangle} \bar{E}\cdot\bar{D}\, dv \text{ Joules}$$

$$e = \frac{1}{2}\int_{\langle vol \rangle} \bar{E}\cdot\epsilon\bar{E}\, dv$$

$$= \frac{1}{2}\epsilon E^2 \underbrace{\cancel{\int_{\langle vol \rangle} dv}}_{\rightarrow 1m^3 \text{ (given)}} = \underline{\underline{\frac{1}{2}\epsilon E^2 \text{ jouls}}}$$

$$\bar{E} = -\nabla V = -\left[\frac{\partial V}{\partial x}\bar{a}_x + \frac{\partial V}{\partial y}\bar{a}_y\right]$$

$$\frac{\partial V}{\partial x} = 2; \quad \frac{\partial V}{\partial y} = 4.$$

$$\bar{E} = -2\bar{a}_x - 4\bar{a}_y \text{ V/m} \therefore |\bar{E}| = \sqrt{4+16}$$

$$|\bar{E}|^2 = E^2 = 4 + 16 = 20 \text{ V/m}.$$

$$e = \frac{1}{2}\times 8.854\times 10^{-12}\times(20) = 88.54\times 10^{-12} \text{ Jouls}.$$

$$\boxed{e = 88.54\,p} \text{ Joules}$$

$$= \frac{10^{-8}}{36\pi} \text{ jouls}$$

problem 82 Find the energy stored in free space for the region $2mm < r < 3mm$, $0 < \theta < 90°$, $0 < \phi < 90°$ given the potential field

(i) $V = \frac{200}{r}$ volts (ii) $V = \frac{300}{r^2}\cos\theta$ Volts.

Sol'n: $V = \frac{200}{r}$ volts.

$0.002 < r < 0.003$, $0 < \theta < 90°$ and $0 < \phi < 90°$

$$\bar{E} = -\nabla V = -\frac{\partial V}{\partial r}\bar{a}_r = +\frac{200}{r^2}\bar{a}_r \ \text{V/m}.$$

$$\bar{E}^2 = \bar{E}\cdot\bar{E} = E^2 = \left(\frac{200}{r^2}\right)^2$$

$$e = \frac{1}{2}\int_{\langle vol\rangle} \epsilon_0 E^2 dv = \frac{\epsilon_0}{2}\int_{\langle vol\rangle}\left(\frac{200}{r^2}\right)^2 dv$$

$$dv = r^2 \sin\theta \, dr\, d\theta\, d\phi$$

$$= \frac{\epsilon_0 (200)^2}{2}\int_{\langle vol\rangle} \frac{1}{r^4}\cdot r^2 \sin\theta \, dr\, d\theta\, d\phi$$

$$e = \frac{\epsilon_0(200)^2}{2}\left[\int_{r=0.002}^{0.003}\frac{1}{r^2}dr \int_{\theta=0}^{\pi/2}\sin\theta\, d\theta \int_{\phi=0}^{\pi/2} d\phi\right]$$

$$e = \frac{(200)^2\epsilon_0}{2}\left[166.667 \times 1 \times \pi/2\right]$$

$$e = 46.359 \times 10^{-6} \text{ Joules}$$

(or) $\boxed{e = 46.359\,\mu}$ Joules

(167)

ii) $V = \dfrac{300\cos\theta}{r^2}$ volts

$$\bar{E} = -\left[\frac{\partial V}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\bar{a}_\theta\right]$$

$$\bar{E} = \frac{600}{r^3}\cos\theta\,\bar{a}_r + \frac{300}{r^3}\sin\theta\,\bar{a}_\theta \ \ V/m.$$

$$\bar{E}^2 = \left(\frac{600}{r^3}\cos\theta\right)^2 + \left(\frac{300}{r^3}\sin\theta\right)^2$$

(or) $$E^2 = \frac{600^2}{r^6}\cos^2\theta + \frac{300^2}{r^6}\sin^2\theta.$$

$$e = \frac{1}{2}\int_{vol} \epsilon E^2 \, dv \ \text{Joules} \qquad dv = r^2\sin\theta\, dr\, d\theta\, d\phi$$

$$= \frac{1}{2}\epsilon\left[600^2\int_{r=0.002}^{0.003}\frac{1}{r^4}dr\int_{\theta=0}^{\pi/2}\cos^2\theta\sin\theta\, d\theta\int_{\phi=0}^{\pi/2}d\phi + 300^2\int_{r=0.002}^{0.003}\frac{1}{r^4}dr\int_{\theta=0}^{\pi/2}\sin^3\theta\, d\theta\int_{\phi=0}^{\pi/2}d\phi\right]$$

$$= \frac{1}{2}\epsilon\left[600^2(2932100\dot{0})(0.333)(0.5\pi) + 300^2\times(2932100\dot{0})(0.6667)(0.5\pi)\right]$$

$$e = 36.698 \ \text{Joules}$$

$$\boxed{e \simeq 36.7} \ \text{Joules} = \underline{36.7 \ \text{Joules}}$$

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Module 2 problems

problem 2

A charge in uniformly distributed over a spherical surface of radius 'a' m. Determine electric field intensity everywhere in space. Use Gauss's Law.

problem 3

In a certain region of space $\bar{D} = 2xy\,\bar{a}_x + 3yz\,\bar{a}_y + 4zx\,\bar{a}_x$ C/m². Evaluate the amount of Electric flux that pass through the portion bounded by $-1 \le y \le 2$ and $0 \le z \le 4$ in the $x = 3$ plane.

problem 4

A cube of 4m centered at origin with edges parallel to the co-ordinate axes of cartesian co-ordinate system. If $\bar{D}$ (electric flux density) $= \frac{20x^5}{5}\,\bar{a}_x$ C/m², what is the total charge contained in the cube.

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problem 5

Find the total charge in a volume defined by six planes for which $1 \le x \le 2$, $2 \le y \le 3$, $3 \le z \le 4$. if $\overline{D} = 4x\,\overline{a_x} + 3y^2\,\overline{a_y} + 2z^2\,\overline{a_z}$ C/m²

problem 6

Let $\overline{D} = (2y^2 z - 8xy)\,\overline{a_x} + (4xyz - 4x^2)\,\overline{a_y} + (2xy^2 - 4z)\,\overline{a_z}$. Determine the total charge within a volume of 10^{-14} m³ located at $P(1,-2,3)$.

problem 7

Given $\overline{D} = z \sin\phi\,\overline{a_\rho} + \rho \sin\phi\,\overline{a_z}$ C/m². Compute the volume charge density at $(1, 30°, 2)$.

problem 9

Calculate the divergence of vector $\overline{D}$ at the points specified using cartesian, cylindrical and spherical coordinates.

i) $\overline{D} = \frac{1}{z^2}\left[10xyz\,\overline{a_x} + 5x^2 z\,\overline{a_y} + (2z^3 - 5x^2 y)\,\overline{a_z}\right]$ C/m² at $P(2,3,5)$.

ii) $\overline{D} = 5z^2\,\overline{a_\rho} + 10\rho z\,\overline{a_z}$ at $P(3,-45°,5)$.

iii) $\overline{D} = 2r \sin\theta \sin\phi\,\overline{a_r} + r\cos\theta \sin\phi\,\overline{a_\theta} + r\cos\phi\,\overline{a_\phi}$ C/m² at $P(3,-45°,-45°)$.

Module 2 problems

problem 10

Given $\overline{D} = 5\sin\theta\,\overline{a_\theta} + 5\sin\phi\,\overline{a_\phi}$. Find the charge density at $(0.5m, \pi/4, \pi/4)$:

problem 11

Let $\overline{D} = 5r^2\,\overline{a_r}$ mC/m^2 for $r < 0.08m$.
and $\overline{D} = \frac{0.1}{r^2}\,\overline{a_r}$ mC/m^2 for $r > 0.08m$.
Find ρ_v for i) $r = 0.06m$ ii) $r = 0.1m$.

problem 13

Verify both sides of Gauss Divergence theorem if $\overline{D} = 2xy\,\overline{a_x} + x^2\,\overline{a_y}$ C/m^2 present in the region bounded by $0 \le x \le 1$, $0 \le y \le 2$, $0 \le z \le 3$.

Problem 19

Given $\overline{D} = 5r\,\overline{a_r}$ C/m^2, prove divergence theorem for a shell region enclosed by spherical surface at $r = a$ and $r = b$ $(b > a)$ and centered at the origin.

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Module - 2 "Summary"

a. List of Symbols :

1. Workdone (or) Energy (W) ⟶ Joules (J)
2. potential difference (V) ⟶ J/C (or) volt.
3. Energy density (e) ⟶ J/m^3.
4. Current (I) ⟶ Ampere (A)
5. Current density ($\bar{J}$) ⟶ A/m^2.
6. Conductivity (σ) ⟶ v/m (or) S/m.
7. Resistance (R) ⟶ Ω (ohm).
8. drift velocity ($\bar{v}_d$) ⟶ m/sec.
9. mobility (μ_e) ⟶ m^2/V-sec.

I b. Formulae:

1. Gauss's Law: The electric flux passing through any closed surface is equal to the total charge enclosed by that surface.

i.e $$\boxed{\psi = \oint_{\langle S \rangle} \overline{D} \cdot \overline{dS} = Q_{enclosed}}$$ Coulomb's

2. $|\overline{D}| = \rho_s \; C/m^2$ and $\overline{D} = \epsilon_0 \overline{E} \; C/m^2$

$$\boxed{|\overline{D}| = \epsilon_0 |\overline{E}|} \; C/m^2$$

3. Del (∇) or Vector Operator

a. Cartesian Co-ordinate System $p(x, y, z) \to dx, dy, dz$

$$\boxed{\nabla = \frac{\partial}{\partial x} \overline{a_x} + \frac{\partial}{\partial y} \overline{a_y} + \frac{\partial}{\partial z} \overline{a_z}} \; m^{-1}$$

b. Cylindrical Co-ordinate System $p(\rho, \phi, z) \to d\rho, \rho\, d\phi, dz$

$$\boxed{\nabla = \frac{\partial}{\partial \rho} \overline{a_\rho} + \frac{1}{\rho} \frac{\partial}{\partial \phi} \overline{a_\phi} + \frac{\partial}{\partial z} \overline{a_z}} \; m^{-1}$$

c. Spherical Co-ordinate System $P(r, \theta, \phi)$

dr $r\,d\theta$ $r\sin\theta\,d\phi$

$$\boxed{\nabla = \frac{\partial}{\partial r}\overline{a_r} + \frac{1}{r}\frac{\partial}{\partial \theta}\overline{a_\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\overline{a_\phi}} \; m^{-1}.$$

4. Divergence $(\nabla\cdot\overline{D})$.

$P(x, y, z)$

dx dy dz.

a. Cartesian Co-ordinate System.

$$\overline{D} = D_x\overline{a_x} + D_y\overline{a_y} + D_z\overline{a_z} \; C/m^2.$$

$$\boxed{\nabla\cdot\overline{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}} \; C/m^2 \Rightarrow \text{Scalar}$$

b. Cylindrical Co-ordinate System.

$P(\rho, \phi, z)$ $dv = \rho\, d\rho\, d\phi\, dz$.

$d\rho$ $\rho\, d\phi$ dz

$$\overline{D} = D_\rho\overline{a_\rho} + D_\phi\overline{a_\phi} + D_z\overline{a_z} \; C/m^2$$

$$\boxed{\nabla\cdot\overline{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}[\rho\cdot D_\rho] + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}} \; C/m^2$$

c. Divergence in Spherical coordinate system.

$P(r, \theta, \phi)$ $dv = r^2 \sin\theta \, dr \, d\theta \, d\phi$

dr $r d\theta$ $r\sin\theta d\phi$

$$\overline{D} = D_r \overline{a_r} + D_\theta \overline{a_\theta} + D_\phi \overline{a_\phi} \ C/m^2$$

$$\boxed{\nabla \cdot \overline{D} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 D_r\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left[\sin\theta D_\theta\right] + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial \phi}} \ C/m^2$$

4. Maxwells first equation or point form of Gauss's Law

it states that the Electric flux per unit volume Leaving a vanishingly Small volume unit is exactly equal to the volume charge density there.

i.e $\boxed{\nabla \cdot \overline{D} = \rho_v}$ C/m^3

5. Divergence theorem:- The divergence theorem states that the total Electric Flux crossing the closed Surface is equal to the integral of the divergence of the Flux density throughout the enclosed volume.

i.e $\psi = \boxed{\oint_{\langle s \rangle} \overline{D} \cdot \overline{ds} = \int_{\langle vol \rangle} (\nabla \cdot \overline{D}) \, dv}$ = Qenclosed Coulombs.

6. Energy expanded in moving a point charge in an Electric field.

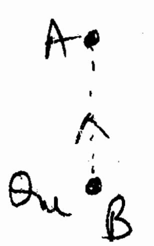
Work done $\boxed{W = -Q \int_{initial}^{final} \overline{E} \cdot \overline{dl}}$ Joules

7. The Line integral

* Work done is independent of the path choosen in any electrostatic field (uniform / non-uniform).
* if the path choosen to be $\perp^{r}$ to the $\overline{E}$ then workdone is Zero and also if the path choosen to be a closed path then also workdone is to be zero.

8. Definition of potential difference and potential :-

The potential of point A with respect to point B is defined as the workdone in moving a unit positive charge Q_u from point B to A against to the field $\overline{E}$.



$$\boxed{V_{AB} = \frac{W}{Q_u} = -\int_{B}^{A} \overline{E} \cdot \overline{dl}} \ J/c \text{ (or) Volt.}$$

$$= -\int_{initial}^{final} \overline{E} \cdot \overline{dl} \text{, volts}$$

* if Charge Q is at origin then potential difference b/w points A and B is

$$\boxed{V_{AB} = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_a} - \frac{1}{r_b}\right]} \text{ volts.}$$

9. Absolute potential (V_a) :-

The workdone in moving a unit test charge from the infinity (∞) to the Specific point against field $\bar{E}$ is called absolute potential.

$$\boxed{V = \frac{Q}{4\pi\epsilon_0 r}} \text{ volts} = -\int_{+\infty}^{final} \bar{E} \cdot \bar{dl} \text{ volts}$$

10. potential of a point Charge and System of charges.

→ point charge

$$\boxed{V = \frac{Q}{4\pi\epsilon_0 r}} \text{ volts}$$

→ line charge

$$\boxed{V = \int_{\langle l \rangle} \frac{\rho_l}{4\pi\epsilon_0 r} dl} \text{ volts}$$

→ Surface charge

$$\boxed{V = \int_{\langle s \rangle} \frac{\rho_s \, ds}{4\pi\epsilon_0 r}} \text{ volts}$$

→ volume charge

$$\boxed{V = \int_{\langle vol \rangle} \frac{\rho_v \, dv}{4\pi\epsilon_0 r}} \text{ volts}$$

11. Potential Gradient :-

$$\boxed{\bar{E} = -\nabla V} \ \text{V/m}$$

a. Cartesian Co-ordinate System

$$\boxed{\nabla V = \frac{\partial V}{\partial x}\bar{a}_x + \frac{\partial V}{\partial y}\bar{a}_y + \frac{\partial V}{\partial z}\bar{a}_z} \ \text{V/m}$$

b. Cylindrical Co-ordinate System.

$$\boxed{\nabla V = \frac{\partial V}{\partial \rho}\bar{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\bar{a}_\phi + \frac{\partial V}{\partial z}\bar{a}_z} \ \text{V/m.}$$

c. Spherical Co-ordinate System.

$$\boxed{\nabla V = \frac{\partial V}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\bar{a}_\phi} \ \text{V/m}$$

Note :- Gradient results in vector.

12. The potential field of a circular ring of uniform line charge density is given by

$$\boxed{V = \frac{\rho_L \cdot a}{2\epsilon_0\sqrt{a^2+z^2}}} \ \text{volts.}$$

$$\rho_L = \frac{Q}{\text{length}} = \frac{Q}{\text{Circumference}}$$

$$\boxed{\rho_L = \frac{Q}{2\pi a}} \ \text{C/m}$$

where a - radius of ring.

13. Energy density in a electrostatic field:-

$$\boxed{\text{Energy density } (e) = \frac{1}{2}\overline{D}\cdot\overline{E} = \frac{1}{2}\epsilon E^2} \text{ Joules/m}^3$$

Note:- $\overline{E}\cdot\overline{E} = E^2$

and $\overline{D} = \epsilon\overline{E}$ C/m².

14. Current (I):- rate of flow of charge per unit time.

$$\boxed{I = \frac{dQ}{dt}} \text{ C/sec or Amperes}$$

15. Current density ($\overline{J}$)

$$\boxed{\overline{J} = \frac{dI}{ds} \text{ A/m}^2} \text{ or } \boxed{\overline{J} = \frac{dI}{ds}\overline{a_n}} \text{ A/m}^2$$

i.e Current passing through the unit surface area, when surface is at normal to the direction of flow of current (I).

and $$\boxed{I = \oint_{\langle S\rangle} \overline{J}\cdot\overline{ds}} \text{ Amperes}$$

16. Continuity of current equation:-

- $$\boxed{\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t}} \ A/m^3$$ ...... point form.

- $$I = \oint_{\langle S \rangle} \bar{J} \cdot \overline{dS} = -\frac{dQ}{dt} = -\int_{\langle vol \rangle} \left(\frac{\partial \rho_v}{\partial t}\right) \cdot dv$$

.. Integral form.

- $$\boxed{\oint_{\langle S \rangle} \bar{J} \cdot \overline{dS} = \int_{\langle vol \rangle} -\left(\frac{\partial \rho_v}{\partial t}\right) \cdot dv}$$ Ampere's.

- relationship between $\bar{J}$, ρ_v and $\bar{v}$

$$\boxed{\vec{J} = \rho_v \bar{v}} \ A/m^2$$

where $\bar{v}$ - velocity vector.

17. point form of ohm's Law.

$$\boxed{\bar{J} = \sigma \bar{E}} \ A/m^2$$

18. Drift velocity (v_d)

$$\boxed{v_d = -\mu \bar{E}} \ m/sec.$$

where μ - mobility of electrons $(m^2/V\text{-}sec)$.

Module -3(Part-A)

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Part-A : Poisson's and Laplace's Equations

Derivation of Poisson's and Laplace's Equations, Uniqueness theorem, Examples of the solution of Laplace's equation.

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Module -3

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Part-A : Poisson's and Laplace's Equations

Derivation of Poisson's and Laplace's Equations. Uniqueness theorem. Examples of the solution of Laplace's equation.

Topics:

1. Derivation of Poisson's and Laplace's Equations.
   - ➢ Poisson's and Laplace's Equations in all three co-ordinate Systems.
2. Uniqueness theorem
3. Applications: Examples of the solution of Laplace's equation

Topics: 1

1. **Derivation of Poisson's and Laplace's Equations.**
   - ➢ **Poisson's and Laplace's Equations in all three co-ordinate Systems.**

02-DEC2010

1 Derive Poisson's and Laplace equations (04 Marks)

06-DEC2008/Jan 2009

2 Derive Poisson's and Laplace's equations. Write laplace's equation in CCS and SCS. **(06 Marks)**

06-DEC2009/Jan 2010

3 With usual notations, deduce the Poisson's equation and Laplace equation from Maxwell's first equation. Express $\nabla^2 V$ in different co-ordinate systems. **(10 Marks)**

06-DEC2011/Jan 2012

4 Derive the expressions for Poisson's and Laplace's equation. **(04 Marks)**

10-DEC2011/Jan 2012

5 Starting with point form of Gauss law deduce Poisson's and Laplace's equations. **(06 Marks)**

10-Jan 2013

6 With usual representations derive Poisson's equation. **(05 Marks)**

06-DEC 2013/Jan 2014

7 Derive Laplace's equation **(06 Marks)**

10-DEC 2013/Jan 2014

8 Derive Poisson's and Laplace's equation. **(06 Marks)**

10-June/July 2013

9 Derive Poisson's and Laplace's equations. **(05 Marks)**

Topic 3.1

Derivation of Poisson's and Laplace Equations.

Questions:-

Derive Poisson's and Laplace's equations. (5m).
(or)
Starting from Gauss's Law in point form, derive Poisson's and Laplace equation. (6m)

[02 Dec 2010, 06-Jan 2009, 06- Jan 2010, 06-Jan 2012, 10-Jan 2012, 10-Jan 2013, 06-Jan 2014, 10-Jan 2014, 10-J|J 2013, 06-J|J 2011, 02 J|J-2011, 06-J|J 2012, 10-June/July 2012, 06 June/July 2009, 06-June/July 2009, 10-Dec/Jan 2015, 06 Jan 2013, 06 J|J 2013, 06 Dec/Jan 2008, 06-J|J 2016, 10-Jan 2016, 10 J|J -2016, 06-Dec 2010].

06 - June /July 2011

10 Starting with point form of Gauss law deduce Poisson's and Laplace's equations. (04 Marks)

02 - June /July 2011

11 From point form of Gauss's law, ~~obtain~~ Poisson's and Laplace's equation. (06 Marks)

06 - June /July 2012

12 Starting from Gauss' law in point form, derive Poisson and Laplace equations. (04 Marks)

10 - June /July 2012

06- June /July 2009

13 Starting from Gauss's law in integral form, derive Laplace's and Poisson's equations. Write Laplace's equation in all the coordinate systems. (06 Marks)

06- June /July 2009

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14 Derive Laplace's equations. (05 Marks)

010-Dec/Jan 2015

15 Derive the expression for Poisson's and Laplace's equation. (04 Marks)

16 Write the expression for Laplace's equation in cylindrical and spherical coordinates. (04 Marks)

06 – Jan 2013

17 Obtain Poisson's and Laplace's equations from Maxwell's first equation. (06 Marks)

06 - June /July 2013

18 Starting with point form of Gauss law, deduce Poisson's & Laplace equations (05 Marks)

06 –Dec/Jan 2008

19 [illegible]

06 –June/July 2014

20 Derive Poisson's and Laplace's equations and write Laplace's equation in cylindrical and polar coordinates. (06 Marks)

10 –Dec/Jan 2016

21 a. Expand ∇^2 operation in different co-ordinate system. (03 Marks)

10–June/July 2016

22 a. Explain Poisson's and Laplace's equations (06 Marks)

** [15-J/J 2017 (3m) CBCS]

06-DEC2010

soln:

w.k.t from point form of Gauss's Law / Maxwell's first equation

$$\nabla \cdot \bar{D} = \rho_v \ C/m^3 \leftarrow (1).$$

using relation b/w $\bar{D}$ and $\bar{E}$

$$\bar{D} = \epsilon \bar{E} \ C/m^2 \leftarrow (2)$$

eq^n (2) in eq^n (1)

$$\nabla \cdot (\epsilon \bar{E}) = \rho_v \ C/m^3 \Rightarrow \nabla \cdot \bar{E} = \rho_v/\epsilon \leftarrow (3)$$

using gradient relationship
$\bar{E}$ is a -ve of gradient of potential

i.e $\bar{E} = -\nabla V \ V/m \leftarrow (4)$

eq^n (4) in (3).

$$\nabla \cdot (-\nabla V) = \rho_v/\epsilon \quad \frac{C/m^3}{F/m}$$

$$-\nabla^2 V = \rho_v/\epsilon \quad V/m^2$$

$$\Rightarrow \boxed{\nabla^2 V = -\rho_v/\epsilon} \ V/m^2 \leftarrow (a)$$

⇒ for a homogenous region 'ε' constant eq^n (a) Called poisson's equation.

In a charged free region $\rho_v = 0$ ∴ eq^n (a) becomes

$$\boxed{\nabla^2 V = 0} \ V/m^2 \leftarrow (b)$$

and eq^n (b) Called Laplace's equation.

Note:

→ $\boxed{\nabla^2 V = -\rho_v/\epsilon}$ V/m^2 ... poisson's equation.

→ $\boxed{\nabla^2 V = 0}$ V/m^2 Laplace's equation.

3.1a Laplace's and Poisson's equation in all three coordinate Systems:-

→ Cartesian Coordinate System:- $P(x, y, z)$

Laplace's equation $\nabla^2 V = 0$

i.e $$\boxed{\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0} \; V/m^2$$

Poisson's equation $\nabla^2 V = -\rho_v/\epsilon$ V/m^2

$$\boxed{\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\rho_v/\epsilon} \; V/m^2$$

→ Cylindrical Coordinate System:- $P(\rho, \phi, z)$

Laplace's equation $\nabla^2 V = 0$ V/m^2

i.e $$\boxed{\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho \frac{\partial V}{\partial \rho}\right] + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0} \; V/m^2$$

Poisson's equation $\nabla^2 V = -\rho_v/\epsilon$ V/m^2

$$\boxed{\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho \frac{\partial V}{\partial \rho}\right] + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = -\rho_v/\epsilon} \; V/m^2$$

→ Spherical Coordinate System:- $P(r, \theta, \phi)$

Laplace's equation $\nabla^2 V = 0$ V/m^2

$$\boxed{\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{\partial V}{\partial r}\right] + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left[\sin\theta \frac{\partial V}{\partial \theta}\right] + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0} \; V/m^2$$

Poisson's equation $\nabla^2 V = -\rho_v/\epsilon \quad V/m^2$.

$$\boxed{\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial V}{\partial\theta}\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2} = -\rho_v/\epsilon}$$

V/m^2.

3.1b Important Vector operations

Cartesian co / a) Rectangular Coordinate System:

$P(x, y, z)$ dx dy dz $dv = dx\,dy\,dz \; m^3$

operation | Expression | Remark

→ DEL operator $\boxed{\nabla = \frac{\partial}{\partial x}\bar{a}_x + \frac{\partial}{\partial y}\bar{a}_y + \frac{\partial}{\partial z}\bar{a}_z}$ m^{-1} ← vector operator

if V – Scalar fn

→ Gradient ∇V V/m $\boxed{\nabla V = \frac{\partial V}{\partial x}\bar{a}_x + \frac{\partial V}{\partial y}\bar{a}_y + \frac{\partial V}{\partial z}\bar{a}_z}$ V/m results in ← vector.

let vector $\bar{A} = A_x\bar{a}_x + A_y\bar{a}_y + A_z\bar{a}_z$

→ Divergence $\nabla\cdot D$ C/m³ $\boxed{\nabla\cdot\bar{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}}$ C/m³ ← results in scalar.

→ Laplace's eqn $\nabla^2 V = 0$ V/m² $\nabla\cdot(\nabla V) = \nabla^2 V = \frac{\partial}{\partial x}\left(\frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial V}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial V}{\partial z}\right) = 0$

$$\boxed{\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0}$$ V/m² ⇐ results in scalar

Divergence of gradient ⇒ Laplacian.

→ Poisson's eqn $\nabla^2 V = -\rho_v/\epsilon$ V/m² $\boxed{\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\rho_v/\epsilon}$ V/m² ⇐ results in Scalar.

i Cylindrical Co-ordinate System :-

$P(\rho, \phi, z)$ $dv = \rho\, d\rho\, d\phi\, dz$ m^3

$d\rho$ $\rho d\phi$ dz

operation	Expression		Remark
1. Del operator	$\nabla = \frac{\partial}{\partial \rho}\bar{a}_\rho + \frac{1}{\rho}\frac{\partial}{\partial \phi}\bar{a}_\phi + \frac{\partial}{\partial z}\bar{a}_z$	m^{-1}	Vector operator
2. Gradient $\boxed{\nabla V \ V/m}$	$\nabla V = \frac{\partial V}{\partial \rho}\bar{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\bar{a}_\phi + \frac{\partial V}{\partial z}\bar{a}_z$	V/m	Vector
	$\bar{A} = A_\rho \bar{a}_\rho + A_\phi \bar{a}_\phi + A_z \bar{a}_z$		
3. Divergence $\nabla\cdot\bar{D}$ C/m^3	$\nabla\cdot\bar{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}[\rho D_\rho] + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$	C/m^3	Scalar
4. Laplace's eq^n $\nabla^2 V = 0$ V/m^2	$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho\frac{\partial V}{\partial \rho}\right] + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$	V/m^2	Scalar
5. Poisson's eq^n $\boxed{\nabla^2 V = -\rho_v/\epsilon}$ V/m^2	$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho\frac{\partial V}{\partial \rho}\right] + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = -\rho_v/\epsilon$	V/m^2	Scalar

ii Spherical / polar Coordinate System

$P(r, \theta, \phi)$ $dv = r^2 \sin\theta\, dr\, d\theta\, d\phi$ m^3

dr $r d\theta$ $r\sin\theta d\phi$

operation	Expression		Remark
1. DEL operator	$\nabla = \frac{\partial}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\bar{a}_\phi$	m^{-1}	Vector operator
2. Gradient $\boxed{\nabla V}$ V/m	$\nabla V = \frac{\partial V}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\bar{a}_\phi$	V/m	Vector
	$\bar{A} = A_r \bar{a}_r + A_\theta \bar{a}_\theta + A_\phi \bar{a}_\phi$		
3. Divergence $\nabla\cdot\bar{D}$ C/m^3	$\nabla\cdot\bar{D} = \frac{1}{r^2}\frac{\partial}{\partial r}[r^2 D_r] + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}[\sin\theta \cdot D_\theta] + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial \phi}$: C/m^3		Scalar
4. Laplace's eq^n $\nabla^2 V = 0$ V/m^2	$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left[\sin\theta\frac{\partial V}{\partial \theta}\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0$	V/m^2	Scalar
5. Poisson's eq^n $\nabla^2 V = -\rho_v/\epsilon$ V/m^2	$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left[\sin\theta\frac{\partial V}{\partial \theta}\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = -\rho_v/\epsilon$	V/m^2	Scalar

23 Determine whether or not the potential equations i) $V = 2x^2 - 4y^2 + z^2$ and ii) $V = r^2\cos\phi + \theta$ satisfy the Laplace's equation. (04 Marks)

02-DEC2010

24 Check whether the following potential equations are satisfying Laplace's equation or not
i) $V = 20x^2yz + 10xy^2z^2$
ii) $V = 15x^2 + 10y^2 - 25z^2$ (05 Marks)

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02-DEC2008/Jan 2009

25 **Derive Laplace's equation, verify whether the potential field given below satisfies Laplace's equation $V = 2x^2 - 3y^2 + z^2$.** **(07 Marks)**

10-DEC2011/Jan 2012

26 Determine whether or not the potential equations :
(23 i) $V = 2x^2 - 4y^2 + z^2$ ii) $V = r^2 \cos\phi + \theta$ iii) $V = r\cos\phi + z$
satisfy the Laplace's equation. (23 ii) (06 Marks)

10-Jan 2013

27 Verify that the potential field given below satisfies the Laplace's equation $V = 2x^2 - 3y^2 + z^2$. (Q25)
(05 Marks)

06-DEC 2013/Jan 2014

28 Verify whether the potential field given below satisfies Laplace's equation.
(i) $V = x^2 - y^2 + z^2$ (ii) $V = 2x^2 - 3y^2 + z^2$ (Q25) (06 Marks)

10-June/July 2013

06 – Jan 2013

34 Verify that the potential field given below satisfy Laplace's equation
$V = 2x^2 - 3y^2 + z^2$ (Q25) (06 Marks)

06 –June/July 2014

35 Determine whether the following potential fields satisfies Laplace's equation or not:
i) $V = x^2 + y^2 - z^2$; ii) $V = \rho\cos\phi + z$ (06 Marks)

EEE-10–June/July 2016

10 –Dec/Jan 2016

37 b. Verify that the potential field given below satisfies the Laplace equation
$V = 2x^2 - 3y^2 + z^2$ ← (Q25)
$V = [Ar^4 + Br^{-4}] \sin 4\phi$ → refer Q No-39 (i) (page No-289) (08 Marks)

problem

i) $V = 2x^2 - 4y^2 + z^2$

Laplace eqⁿ $\nabla^2 V = 0$

i.e. $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$.

$\frac{\partial V}{\partial x} = 2(2x) = 4x$; $\frac{\partial^2 V}{\partial x^2} = 4$.

$\frac{\partial V}{\partial y} = -8y$; $\frac{\partial^2 V}{\partial y^2} = -8$.

$\frac{\partial V}{\partial z} = +2z$; $\frac{\partial^2 V}{\partial z^2} = 2$

$\Rightarrow 4 - 8 + 2 = -2 \neq 0$ i.e. $\boxed{\nabla^2 V \neq 0}$

∴ given potential field not satisfying Laplace's eqⁿ.

ii) $V = r^2 \cos\phi + \theta$

given potential field is in Spherical C.S

$\nabla^2 V = 0$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{\partial V}{\partial r}\right] + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left[\sin\theta \frac{\partial V}{\partial \theta}\right] + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0$$

$\frac{\partial V}{\partial r} = 2r\cos\phi$; $\frac{\partial V}{\partial \theta} = 1$

$\frac{\partial V}{\partial \phi} = -r^2 \sin\phi$; $\frac{\partial^2 V}{\partial \phi^2} = -r^2 \cos\phi$

$$\nabla^2 V = \frac{1}{r^2}\left[r^2 \times 2r\cos\phi\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\right] + \frac{1}{\cancel{r^2}\sin^2\theta} \times -\cancel{r^2}\cos\phi$$

$$\nabla^2 V = \frac{2\cos\phi}{\cancel{r^2}} \times 3\cancel{r^2} + \frac{1}{\cancel{r^2}\sin\theta}\left[\cos\theta\right] - \frac{\cos\phi}{\sin^2\theta}$$

$$= 6\cos\phi + \frac{\cot\theta}{r^2} - \frac{\cos\phi}{\sin^2\theta} \neq 0 \quad \text{i.e.} \boxed{\nabla^2 V \neq 0}$$

∴ the given potential field $V = r^2\cos\phi + \theta$ volts is not satisfying the Laplace's equation.

iii) $V = 20x^2yz + 10xy^2z^2$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\frac{\partial V}{\partial x} = 40xyz + 10y^2z^2 ; \quad \frac{\partial^2 V}{\partial x^2} = 40yz$$

$$\frac{\partial V}{\partial y} = 20x^2z + 20xyz^2 ; \quad \frac{\partial^2 V}{\partial y^2} = 20xz^2$$

$$\frac{\partial V}{\partial z} = 20x^2y + 20xy^2z ; \quad \frac{\partial^2 V}{\partial z^2} = 20xy^2$$

$$40yz + 20xz^2 + 20xy^2 \neq 0$$

i.e. $\boxed{\nabla^2 V \neq 0}$ (10)

∴ given potential field $V = 20x^2yz + 10xy^2z^2$ not satisfying the Laplace equation.

iv) $V = 15x^2 + 10y^2 - 25z^2$

$\frac{\partial V}{\partial x} = 30x$; $\frac{\partial^2 V}{\partial x^2} = 30$.

$\frac{\partial V}{\partial y} = 20y$; $\frac{\partial^2 V}{\partial y^2} = 20$

$\frac{\partial V}{\partial z} = -50z$; $\frac{\partial^2 V}{\partial z^2} = -50$

Laplace eqn $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$

i.e $30 + 20 - 50 = 0$

$\nabla^2 V = 0$

∴ the given potential field $V = 15x^2 + 10y^2 - 25z^2$ Volt Satisfying the Laplace equation.

v) $V = 2x^2 - 3y^2 + z^2$

$\frac{\partial V}{\partial x} = 4x$; $\frac{\partial^2 V}{\partial x^2} = 4$.

$\frac{\partial V}{\partial y} = -6y$; $\frac{\partial^2 V}{\partial y^2} = -6$.

$\frac{\partial V}{\partial z} = 2z$; $\frac{\partial^2 V}{\partial z^2} = 2$.

$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$

i.e $4 - 6 + 2 = 0$

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∴ $\boxed{\nabla^2 V = 0}$

the given potential field $V = 2x^2 - 3y^2 + z^2$ volts satisfying the Laplace's eq^n.

Vi) (ii) $V = r\cos\phi + z$.

the given potential field is in Cylindrical C.S

$$\nabla^2 V = \frac{1}{r}\left[\frac{\partial}{\partial r}\left[r\frac{\partial V}{\partial r}\right]\right] + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}.$$

$$\frac{\partial V}{\partial r} = \cos\phi, \quad \frac{\partial V}{\partial \phi} = -r\sin\phi \ ; \ \frac{\partial^2 V}{\partial \phi^2} = -r\cos\phi$$

$$\frac{\partial V}{\partial z} = 1 \ ; \ \frac{\partial^2 V}{\partial z^2} = 0.$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}[r \cdot \cos\phi] + \frac{1}{r^{\cancel{2}}}[-\cancel{r}\cos\phi] + 0$$

$$= \frac{\cos\phi}{r} - \frac{\cos\phi}{r} + 0 = 0$$

i.e $\boxed{\nabla^2 V = 0}$

∴ the given potential field $V = r\cos\phi + z$ volts satisfying the Laplace equation.

Q28) i) $V = x^2 - y^2 + z^2$ volt's

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0.$$

$$\frac{\partial V}{\partial x} = 2x; \quad \frac{\partial^2 V}{\partial x^2} = 2.$$

$$\frac{\partial^2 V}{\partial y^2} = -2 \quad ; \quad \frac{\partial^2 V}{\partial z^2} = 2.$$

$$\cancel{2} - \cancel{2} + 2 = 2 \neq 0 \Rightarrow \boxed{\nabla^2 V \neq 0}$$

∴ the given potential field doesnot satisfying the Laplace's equation.

ii) $V = x^2 + y^2 + z^2$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\frac{\partial V}{\partial x} = 2x \quad ; \quad \frac{\partial^2 V}{\partial x^2} = 2$$

$$\frac{\partial V}{\partial y} = 2y \quad ; \quad \frac{\partial^2 V}{\partial y^2} = 2$$

$$\frac{\partial V}{\partial z} = 2z \quad ; \quad \frac{\partial^2 V}{\partial z^2} = 2$$

$$2 + 2 + 2 = 6 \neq 0$$

∴ $\boxed{\nabla^2 V \neq 0}$

The given potential field $V = x^2 + y^2 + z^2$ volt's not satisfying the Laplace's eq^n

viii) $V = r\cos\theta + \phi$ ix) $V = \rho^2 + z^2$

viii) Soln

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial V}{\partial\theta}\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$$

$$\frac{\partial V}{\partial r} = \cos\theta \;;\; \frac{\partial V}{\partial\theta} = -r\sin\theta \quad \frac{\partial^2 V}{\partial\phi^2} = 0.$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\cos\theta\right] + \frac{1}{r^2\sin\theta}\left[-r\sin^2\theta\right] + 0$$

$$= \frac{\cos\theta}{r^{\cancel{2}}}(2\cancel{r}) + \frac{1}{r^{\cancel{2}}\cancel{\sin\theta}} - \cancel{r}\times 2\cancel{\sin\theta}\cos\theta$$

$$= \frac{2\cos\theta}{r} - \frac{2\cos\theta}{r} = 0 \quad \text{i.e } \boxed{\nabla^2 V = 0}\ V/m^2$$

$\therefore$ the given potential field $\boxed{V = r\cos\theta + \phi}$ volt's satisfying the Laplace's eqn.

ix) $V = \rho^2 + z^2$

Soln

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left[\rho\frac{\partial V}{\partial\rho}\right] + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial\phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\frac{\partial V}{\partial\rho} = 2\rho \;;\quad \frac{\partial V}{\partial z} = 2z \;;\quad \frac{\partial^2 V}{\partial z^2} = 2 \;;\quad \frac{\partial^2 V}{\partial\phi} = 0$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left[\rho\cdot 2\rho\right] + 0 + 2$$

$$= \frac{2\times 2\cancel{\rho}}{\cancel{\rho}} + 2 = 4 + 2 = 6 \neq 0$$

i.e $\boxed{\nabla^2 V \neq 0}$

$\therefore$ given potential field $V = \rho^2 + z^2$ volt's does not satisfying the Laplace's eqn.

problem 2

10 J/J 2013 (7m)

✔ Calculate numerical values for V and ρ_v at point P in free space if: (a) $V = \frac{4yz}{x^2+1}$, at $P(1, 2, 3)$; (b) $V = 5\rho^2 \cos 2\phi$, at $P(\rho = 3, \phi = \frac{\pi}{3}, z = 2)$; (c) $V = \frac{2\cos\phi}{r^2}$, at $P(r = 0.5, \theta = 45°, \phi = 60°)$.

~~Ans~~

$V = \frac{4yz}{x^2+1}$

$P(\rho=3, \phi=\frac{\pi}{2}, z=2)$, $V = 5\rho^2 \cos 2\phi$

$V = \frac{2\cos\phi}{r^2}$ at $P(r=0.5, \theta=45°, \phi=60°)$

10 June/July 2013

29 ~~Calculate numerical values for V and ρ_v at point P in free space if $V = \frac{4yz}{x^2+1}$ at P(1, 2, 3).~~

(07 Marks)

(a) Soln:- $V = \frac{4yz}{(x^2+1)}$ volts

i) $V_p = \frac{4(2)(3)}{(1)^2+1} = 12$ volts $\boxed{V_p = 12}$ volts

ii) to find ρ_v ie volume charge density

using poisson's eqn $\nabla^2 V = -\rho_v/\epsilon_0$ c/m²

$$\Rightarrow \rho_v = -\nabla^2 V (\epsilon_0) \ c/m^3$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$V = 4yz(x^2+1)^{-1}$$

$$\frac{\partial V}{\partial x} = -4yz(x^2+1)^{-2}(2x) = -8xyz(x^2+1)^{-2}$$

$$\frac{\partial^2 V}{\partial x^2} = -8xyz\left[-2(x^2+1)^{-3}(2x)\right] + (x^2+1)^{-2}(-8yz)$$

$$= +32x^2yz(x^2+1)^{-3} - 8yz(x^2+1)^{-2}$$

$$\frac{\partial^2 V}{\partial y^2} = 0 \text{ and } \frac{\partial^2 V}{\partial z^2} = 0$$

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$$\therefore \quad \nabla^2 V = \frac{\partial^2 V}{\partial x^2} = 32x^2yz(x^2+1)^{-3} - 8yz(x^2+1)^{-2}$$

$$\rho_v = -\epsilon \nabla^2 V \quad @ \; P(1,2,3)$$

$$\nabla^2 V_p = 32(1)^2(2)(3)(1+1)^{-3} - 8(2)(3)(1+1)^{-2}$$

$$= 192(2)^{-3} - 48(2)^{-2}$$

$$= 24 - 12 = \underline{\underline{12}}$$

$$\boxed{\nabla^2 V_p = 12} \; V/m^2$$

$$\rho_{v_p} = -\epsilon \nabla^2 V_p = -12\epsilon \quad C/m^3$$

$$\boxed{\rho_{v_p} = -106.248} \; pC/m^3$$

b) $V = 5\rho^2 \cos(2\phi)$ at $P(\rho=3, \phi=\pi/3, z=2)$

$$V_p = 5(3)^2 \cos(2\pi/3) = -22.5 \text{ volts}$$

$$\boxed{V_p = -22.5} \text{ volts}$$

using ~~Poisson's~~ eq^n $\nabla^2 V = -\rho_v/\epsilon$

$$\boxed{\rho_v = -\epsilon \nabla^2 V}\ C/m^3$$

$\nabla^2 V$ in Cylindrical C.S

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho \frac{\partial V}{\partial \rho}\right] + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \cancel{\frac{\partial^2 V}{\partial z^2}}^{\,0}$$ $V \neq f^n(z)$

$$V = 5\rho^2 \cos(2\phi)$$

$$\frac{\partial V}{\partial \rho} = 10\rho \cos(2\phi) \quad \text{and} \quad \frac{\partial V}{\partial \phi} = -10\rho^2 \sin(2\phi)$$

$$\frac{\partial^2 V}{\partial \phi^2} = -20\rho^2 \cos(2\phi)$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho \times 10\rho \cos(2\phi)\right] - \frac{20\cancel{\rho^2}\cos(2\phi)}{\cancel{\rho^2}} + 0$$

$$= \frac{10\cos(2\phi)}{\cancel{\rho}} \times 2\cancel{\rho} - 20\cos(2\phi)$$

$$= 20\cancel{\cos(2\phi)} - 20\cancel{\cos(2\phi)}$$

$$\nabla^2 V = 0\ V/m^2 \qquad \Rightarrow \nabla^2 V = 0\ V/m^2$$

$$= 0\ V/m^2$$

$\nabla^2 V_p \Rightarrow$

$$\therefore \rho_{v_p} = -\epsilon \nabla^2 V_p = -\epsilon(0)\ C/m^3$$

$$\rho_{v_p} = 0\ pC/m^3$$

(or) $\boxed{\rho_{v_p} = 0}\ C/m^3 \Rightarrow 0\ C/m^3$

(17) $\boxed{\rho_v = 0}\ C/m^3$

c) $V = \frac{2\cos\phi}{r^2}$ at $p(r=0.5,\ \theta=45^\circ,\ \phi=60^\circ)$

$$V_p = \frac{2\cos(60^\circ)}{(0.5)^2} = \underline{4 \text{ volt's}}$$

$$\boxed{V_p = 4} \text{ volt's} \qquad \boxed{V_p = 4} \text{ volt's}$$

$$\boxed{\rho_v = -\epsilon \nabla^2 V} \ C/m^3$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{\partial V}{\partial r}\right] + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\cancel{\left[\sin\theta \frac{\partial V}{\partial \theta}\right]}^{0} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

$$\frac{\partial V}{\partial r} = \frac{-4\cos\phi}{r^3} \ ; \ \frac{\partial V}{\partial \theta} = 0$$

and $\frac{\partial V}{\partial \phi} = \frac{-2\sin\phi}{r^2}$; $\frac{\partial^2 V}{\partial \phi^2} = \frac{-2\cos\phi}{r^2}$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[\cancel{r^2}\frac{(-4\cos\phi)}{r^{\cancel{3}}}\right] + 0 - \frac{2\cos\phi}{r^2} \times \frac{1}{r^2\sin^2\theta}$$

$$= \frac{-4\cos\phi}{r^2} \times \frac{-1}{r^2} - \frac{2\cos\phi}{r^4\sin^2\theta} = +\frac{4\cos\phi}{r^4} - \frac{2\cos\phi}{r^4\sin^2\theta}$$

$$\nabla^2 V_p = \frac{4\cos(60^\circ)}{0.5^4} - \frac{2\cos(60^\circ)}{0.5^4\sin^2(45)} = 32 - 32 = \underline{\underline{0}} \ V/m^2$$

zero

$$\rho_{v_p} = -\epsilon\nabla^2 V_p = -\epsilon(0) C/m^3 = \ C/m^3$$

$$= 0 \ C/m^3$$

$$\boxed{\rho_{v_p} = \underline{0}} \ C/m^3$$

problem 3 $\bar{E} = 5\cos(z)\,\bar{a}_z$ V/m

$\bar{E} = (12yx^2 - 6z^2x)\bar{a}_x + (4x^3 + 18zy^2)\bar{a}_y + (6y^3 - 6zx^2)\bar{a}_z$

10 - June /July 2012

06-J/J 2013 (7m)

30 Determine whether or not the following vectors represent a possible electric field

i) $\vec{E} = 5\cos z\,\hat{a}_z$ V/m

ii) $\vec{E} = (12yx^2 - 6z^2x)\,\hat{a}_x + (4x^3 + 18zy^2)\,\hat{a}_y + (6y^3 - 6zx^2)\,\hat{a}_z$ (06 Marks)

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06 - June /July 2013

33 ~~Given vector $\vec{E} = (12yx^2 - 6z^2x)\hat{a}_x + (4x^3 - 18zy^2)\hat{a}_y + (6y^3 - 6zx^2)\hat{a}_z$. Check whether it represents a possible electric field.~~ (07 Marks)

Soln:-

Note:- $\bar{E}$ is said to be possible Electric field only when it doesnot arrived from charged free region.

i.e $\nabla^2 V \neq 0$

$\Rightarrow \nabla\cdot(\nabla V) \neq 0$

$\nabla\cdot(-\bar{E}) \neq 0$

(or) $\Rightarrow \boxed{\nabla\cdot\bar{E} \neq 0}$

i) $\bar{E} = 5\cos(z)\,\bar{a}_z$ V/m.

$E_z = 5\cos(z)$

$$\nabla\cdot\bar{E} = \cancelto{0}{\frac{\partial E_x}{\partial x}} + \cancelto{0}{\frac{\partial E_y}{\partial y}} + \frac{\partial E_z}{\partial z} = \frac{\partial}{\partial z}[5\cos z]$$

$= -5\sin(z) \neq 0$ i.e $\boxed{\nabla\cdot\bar{E} \neq 0}$

$\therefore$ the given field $\bar{E} = 5\cos(z)\,\bar{a}_z$ V/m is a possible electric field.

ii) $\bar{E} = (12yx^2 - 6z^2x)\bar{a}_x + (4x^3 + 18zy^2)\bar{a}_y + (6y^3 - 6zx^2)\bar{a}_z$ V/m.

$E_x = 12yx^2 - 6z^2x$ V/m ; $E_y = 4x^3 + 18zy^2$.

$\frac{\partial E_x}{\partial x} = 24yx - 6z^2$; $\frac{\partial E_y}{\partial y} = 36zy$

$E_z = 6y^3 - 6zx^2$

$\frac{\partial E_z}{\partial z} = -6x^2$

$\Rightarrow \nabla \cdot \bar{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$

$24yx - 6z^2 + 36zy - 6x^2 \neq 0$

i.e $\boxed{\nabla \cdot \bar{E} \neq 0}$

$\therefore$ the given field $\bar{E} = (12yx^2 - 6z^2x)\bar{a}_x + (4x^3 + 18zy^2)\bar{a}_y + (6y^3 - 6zx^2)\bar{a}_z$ V/m doesnot arrived from charged free region $\therefore$ it is a possible Electric field.

problem 4

$V = 3x^2yz + ky^3z$ volts

010-Dec/Jan 2015

Given the potential field $V = 3x^2yz + ky^3z$ volts :
i) Find k if potential field satisfies Laplace's equation → Find k if potential field satisfies Laplace equation
ii) Find $\vec{E}$ at (1, 2, 3). → $\vec{E}$ at (1, 2, 3) (06 Marks)

Sol'n: (i) $V = 3x^2yz + ky^3z$ volts

given $\nabla^2 V = 0$

i.e $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$

$\frac{\partial V}{\partial x} = 6xyz$; $\frac{\partial^2 V}{\partial x^2} = 6yz$

$\frac{\partial V}{\partial y} = 3x^2z + 3ky^2z$; $\frac{\partial^2 V}{\partial y^2} = 6kyz$.

$\frac{\partial V}{\partial z} = 3x^2y + ky^3$; $\frac{\partial^2 V}{\partial z^2} = 0$.

$\Rightarrow \nabla^2 V = 0$

$6yz + 6kyz = 0$

$\boxed{k = -1}$

the value of $\boxed{k = -1}$ such that the potential field $V = 3x^2yz + ky^3z$ satisfies the Laplace eqn.

ii) $\bar{E} = -\nabla V = -\frac{\partial V}{\partial x}\bar{a}_x - \frac{\partial V}{\partial y}\bar{a}_y - \frac{\partial V}{\partial z}\bar{a}_z$

$= -6xyz\,\bar{a}_x - [3x^2z + 3(-1)y^2z]\,\bar{a}_y - [3x^2y - y^3]\,\bar{a}_z$

$\bar{E}_p = -36\,\bar{a}_x - [9 - 36]\,\bar{a}_y - [6 - 8]\,\bar{a}_z$

$\boxed{\bar{E}_p = -36\bar{a}_x + 27\bar{a}_y + 2\bar{a}_z}$ V/m ; $|\bar{E}_p| = 45.044$ V/m problem5

$V = x^2 yz + Ay^3 z$ volts

EEE-10–June/July 2016

b. A potential field is given by $v = x^2yz + Ay^3z$ volts determine of 'A' such that v satisfies Laplace equation and hence find electric field E at p(2, 1, −1). (06 Marks)

$\bar{E} \longrightarrow P(2,1,-1)$ (6m)

Soln:

$$V = x^2yz + Ay^3z \text{ volts}$$

given $\nabla^2 V = 0$.

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0.$$

$$\frac{\partial V}{\partial x} = 2xyz \; ; \; \frac{\partial^2 V}{\partial x^2} = 2yz.$$

$$\frac{\partial V}{\partial y} = x^2z + 3Ay^2z \; ; \; \frac{\partial^2 V}{\partial y^2} = 6Ayz.$$

$$\frac{\partial V}{\partial z} = x^2y + Ay^3 \; ; \; \frac{\partial^2 V}{\partial z^2} = 0.$$

$$\nabla^2 V = 2yz + 6Ayz + 0 = 0$$

$$2\cancel{yz} = -6A\cancel{yz}$$

$$A = 2/-6 = -1/3 \quad \boxed{A = -1/3}$$

∴ the value of A such that given V satisfies the Laplace's eq^n is $\boxed{A = -1/3}$

$$\rightarrow \bar{E} = -\nabla V = -\frac{\partial V}{\partial x}\bar{a}_x - \frac{\partial V}{\partial y}\bar{a}_y - \frac{\partial V}{\partial z}\bar{a}_z \text{ V/m.}$$

$$= -2xyz\,\bar{a}_x - [x^2z - y^2z]\,\bar{a}_y - [x^2y - \tfrac{1}{3}y^3]\,\bar{a}_z \text{ V/m}$$

$$\boxed{\bar{E}_p = +4\,\bar{a}_x + 3\,\bar{a}_y - \tfrac{11}{3}\,\bar{a}_z} \text{ V/m}$$

$$\boxed{|\bar{E}_p| = 6.20035} \text{ V/m}$$

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$$V = A \ln\left[\frac{B(1-\cos\theta)}{1+\cos\theta}\right] \text{volts}$$

06-DEC2009/Jan 2010

problmb

Given $V = A \ln\left[B\frac{(1-\cos\theta)}{1+\cos\theta}\right]$ volts — V

i) Show that V satisfies Laplace equation in spherical coordinates. ← $r = 5m$, $\theta = 90°$, $\phi = 60°$

ii) Find A and B so that $V = 100V$, $|E| = 500$ V/m at $r = 5$mt, $\theta = 90°$ and $\phi = 60°$. (10 Marks)

A B $V=100V$ $|E| = 500V/m$

(10m)

sol^n: using vector identity

$$\boxed{\frac{1-\cos\theta}{1+\cos\theta} = \tan^2(\theta/2)}$$

i) $V = A\ln\left[B\frac{(1-\cos\theta)}{(1+\cos\theta)}\right]$ volt's $= A\ln[B\tan^2\theta/2]$ volts

$\nabla^2 V$ in Spherical co-ordinate system is

$$\nabla^2 V = \cancel{\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right]} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial V}{\partial\theta}\right] + \cancel{\frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}}$$

$\to 0$ bcz $V \neq f^n(r)$

$\to 0$ bcz $V \neq f^n(\phi)$

$$\nabla^2 V = \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial V}{\partial\theta}\right]$$

$$\frac{\partial V}{\partial\theta} = A\frac{1}{\cancel{B}\tan^{\cancel{2}}\theta/2} \times \cancel{2}\cancel{B}\tan\theta/2 \cdot \sec^2\theta/2 \times 1/\cancel{2}$$

$$\frac{\partial V}{\partial\theta} = A \times \frac{\cos\theta/2}{\sin\theta/2} \times \frac{1}{\cos^{\cancel{2}}\theta/2} = \frac{A}{\tfrac{1}{2}\sin(\theta)}$$

$$\boxed{\frac{\partial V}{\partial\theta} = \frac{2A}{\sin\theta}}$$

$$\nabla^2 V = \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\cancel{\sin\theta}\,\frac{2A}{\cancel{\sin\theta}}\right] = \frac{1}{r^2\sin\theta}\frac{\partial(2A)}{\partial\theta} \to 0$$

i.e

$$\therefore \boxed{\nabla^2 V = 0}$$

$\therefore$ given potential field $V = A\ln\left[B\frac{(1-\cos\theta)}{(1+\cos\theta)}\right]$ satisfying the Laplace eq^n.

ii) $V = A\ln\left[B\frac{(1-\cos\theta)}{(1+\cos\theta)}\right]$ volt's

given $V = 100$ volt's, $|E| = 500$ V/m @ $p(5, 90°, 60°)$

$A = ?$ $B = ?$

$$100 = A\ln\left[B\frac{1-\cos 90}{1+\cos 90}\right] = A\ln(B)$$

$$100 = A\ln(B) \longrightarrow (a).$$

$$\bar{E} = -\nabla V = -\frac{1}{r}\frac{\partial V}{\partial\theta}\bar{a}_\theta \text{ V/m} \Rightarrow \text{Since } V \text{ f}^n \text{ of only } \theta$$

$$\bar{E} = -\frac{1}{r}\cdot\frac{2A}{\sin\theta}\bar{a}_\theta \text{ V/m}.$$

$$|E_p| = \frac{1}{r}\frac{2A}{\sin(\theta)} = 500 \text{ V/m}$$

i.e $A = \frac{500(5)\sin(90)}{2} = \frac{2500}{2} = 1250$

$$\therefore \boxed{A = 1250}$$

using eq^n (a) $B = e^{100/A} = e^{100/1250} = 1.08328$

$$\therefore \boxed{B = 1.0832}$$

$$V = [Ar^4 + Br^{-4}]\sin(4\phi)$$

problm7

06-DEC2011/Jan 2012

Given the potential field $V = [Ar^4 + Br^{-4}]\sin 4\phi$:

i) Show that $\nabla^2 V = 0$. $\nabla^2 V = 0$. $V = 100$ volts, $|E| = 500$ V/m

ii) Find A and B such that $V = 100$V and $|\bar{E}| = 500$ V/m at $P(r = 1, \phi = 22.5°, z = 2)$. $P(r=1, \phi = 22.5°, z=2)$.

(08 Marks)

solu: - from bit (ii) it can be concluded that the given potential field is in cylindrical co-ordinate system. $P(r, \phi, z)$.

i) Laplace eq^n: $\nabla^2 V = 0$ in cylindrical C.S is

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial V}{\partial r}\right] + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \underbrace{\frac{\partial^2 V}{\partial z^2}}_{0 \text{ bcz } V \neq f^n(z)} = 0$$

$$\frac{\partial V}{\partial r} = [4Ar^3 - 4Br^{-5}]\sin(4\phi)$$

$$\frac{\partial V}{\partial \phi} = [Ar^4 + Br^{-4}]\cos(4\phi)(4)$$

$$\frac{\partial^2 V}{\partial \phi^2} = [Ar^4 + Br^{-4}][-\sin(4\phi)]\,4\times 4.$$

$$= -16\underbrace{[Ar^4 + Br^{-4}]\sin(4\phi)}_{V \text{ (given)}}$$

$$\boxed{\frac{\partial^2 V}{\partial \phi^2} = -16V}$$

$$\Rightarrow \nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot[4Ar^3 - 4Br^{-5}]\sin(4\phi)\right] - 16V/r^2$$

$$= \frac{1}{r}\left[\frac{\partial}{\partial r}[4Ar^4 - 4Br^{-4}]\sin 4\phi\right] - \frac{16V}{r^2}$$

$$= \frac{1}{r}\left[(16Ar^3 + 16Br^{-5})\sin(4\phi)\right] - \frac{16V}{r^2}$$

$\times \frac{r}{r}$

$$= \frac{16}{r^2}\underbrace{\left[(Ar^4 + Br^{-4})\sin(4\phi)\right]}_{V \text{ (given)}} - \frac{16V}{r^2}$$

$$= \frac{16V}{r^2} - \frac{16V}{r^2} = 0 \Rightarrow \text{i.e. } \boxed{\nabla^2 V = 0}$$

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∴ given potential $V = [Ar^4 + Br^{-4}] \sin(4\phi)$ volts

satisfying the Laplace's eqⁿ. $V \neq f^n(z)$.

ii) $\bar{E} = -\nabla V = -\left[\frac{\partial V}{\partial r}\bar{a_r} + \frac{1}{r}\frac{\partial V}{\partial \phi}\bar{a_\phi} + \cancel{\frac{\partial V}{\partial z}\bar{a_z}}\right]$ V/m.

$\frac{\partial V}{\partial r} = [4Ar^3 - 4Br^{-5}]\sin(4\phi)$ and

$\frac{\partial V}{\partial \phi} = 4[Ar^4 - Br^{-4}]\cos(4\phi)$.

$\bar{E} = -[4Ar^3 - 4Br^{-5}]\sin(4\phi)\bar{a_r} - \frac{4}{r}[Ar^4 - Br^{-4}]\cos(4\phi)\bar{a_\phi}$ V/m

given $P(1, 22.5°, 2) \Rightarrow r = 1m$ $\phi = 22.5°$, $z = 2$.

$\bar{E_p} = -[4A - 4B]\sin(90°)\bar{a_r} - \frac{4}{r}[A - B]\cos(90°)\bar{a_\phi}$ V/m

$\bar{E_p} = -[4A - 4B]\bar{a_r}$ V/m.

$|\bar{E_p}| = [4A - 4B]$ V/m. given $|\bar{E_p}| = 500$ V/m.

∴ $4A - 4B = 500 \leftarrow (a)$. or $A - B = 125 \leftarrow (a')$.

2nd condⁿ. $V_p = 100$ volts

$100 = [A + B]\sin(90°) \Rightarrow A + B = 100 \leftarrow (b)$

Solve a' & b $\Rightarrow$ $\boxed{A = 112.5}$ and $\boxed{B = -12.5}$

or alternatively ie $\bar{E_p} = -[4A - 4B]\bar{a_r}$ or $\bar{E_p} = [4B - 4A]\bar{a_r}$

$|\bar{E_p}| = 500 = 4B - 4A \Rightarrow B - A = 125 \leftarrow (a'')$

Solving (a'') and (b) $\Rightarrow$ $\boxed{B = 112.5}$ and $A = -12.5$

Note:- Both the solⁿ of Ans are valid here.

~~6 a. Find the potential and volume charge density at P(0.5, 1.5, 1)m in free space given the~~ potential field $V = 6\rho\phi Z$ volts. (08 Marks)

15 - Dec / Jan 2017
(CBCS Scheme)

$V = 6\rho\phi z$ volts

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Soln:- given potential field

$V = 6\rho\phi z$ volts ... in Cylindrical Coordinate System.

the point $p(0.5, 1.5, 1)m$ ... in Cartesian Coordinate System.

$$P(0.5, 1.5, 1) \Longleftrightarrow P(\rho, \phi, z)$$

$$\rho = \sqrt{x^2+y^2} = \sqrt{0.5^2+1.5^2} = \sqrt{2.5}\ m.$$

$$\phi = \tan^{-1}(y/x) = \tan^{-1}\left(\frac{1.5}{0.5}\right) = 71.56^\circ$$

$$\boxed{z=1}\ m$$

$$P(0.5, 1.5, 1) \Longleftrightarrow P(\sqrt{2.5}, 71.56^\circ, 1).$$

given medium in free space $\boxed{\epsilon = \epsilon_0}$ F/m.

$V = 6\rho\phi z$

@ $p(\sqrt{2.5}, 71.56^\circ, 1)$

$\pi^c = 180^\circ$

$1^\circ = \left(\frac{\pi}{180}\right)^c$

$$\boxed{71.56^\circ = \frac{\pi}{180} \times 71.56 = 1.249^c}$$

$V_p = 6(\sqrt{2.5})(1.249)(1)$

$\boxed{V_p = 11.8494}$ volt's

The volume charge density $\rho_v = ?$

using poisson's eq^n ie $\nabla^2 V = -\rho_v/\epsilon_0$

$\Rightarrow \boxed{\rho_v = -\nabla^2 V(\epsilon_0)}$ c/m³. ← (1)

$\nabla^2 V$ equation in Cylindrical Co-ordinate system is given by:

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho \frac{\partial V}{\partial \rho}\right] + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} \; V/m^2$$

$V = 6\rho\phi z$ volts

$\frac{\partial V}{\partial \rho} = 6\phi z$ | $\frac{\partial V}{\partial \phi} = 6\rho z$, $\frac{\partial^2 V}{\partial \phi^2} = 0$ | $\frac{\partial V}{\partial z} = 6\rho\phi$, $\frac{\partial^2 V}{\partial z^2} = 0$.

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho \cdot 6\phi z\right]$$

$$\boxed{\nabla^2 V = \frac{6\phi z}{\rho}} \; V/m^2 \longleftarrow (2)$$

using eq^n (2) in eq^n (1)

$$\rho_v = -\frac{6\phi z}{\rho} \cdot \epsilon_0 \quad C/m^3$$

ρ_v @ $P(\sqrt{2.5}, 1.249^c, 1)$

$$\rho_{v_p} = \frac{-6(1.249)(1)}{\sqrt{2.5}} \times 8.854 \times 10^{-12}\ C/m^3$$

$$\boxed{\rho_{v_p} = -41.964}\ pC/m^3$$

Topic 3.2 [06-Dec 2010, 06 Jan 2014, 10-J/J 2013, 02 J/J 2011, 10-Jan 2015, 10-J/J 2015, 06-June 2010, 06 Jan 2008, 10-J/J 2014, 06, June/July 2013, 15-Dec/Jan 2017] CBCS scheme.

V.imp

Uniqueness theorem

V.V. Question. Type (i)

06-DEC2010

40 State and prove the uniqueness theorem. (06 Marks)

(or)

06-DEC 2013/Jan 2014

~~41 State and prove uniqueness theorem. (08 Marks)~~

[15 June/July 2017 (5m) CBCS]

10-June/July 2013

42 State and prove the uniqueness theorem. (08 Marks)

02 - June /July 2011

43 State and prove uniqueness theorem. (06 Marks)

010-Dec/Jan 2015

44 State and prove uniqueness theorem. (06 Marks)

10 - June /July 2015

45 State and prove uniqueness theorem. (05 Marks)

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10 - June /July 2014

46 State and prove uniqueness theorem. (10 Marks)

06 – May/June 2010

47 State and prove uniqueness theorem. (10 Marks)

06 –Dec/Jan 2008

48 State and explain uniqueness theorem. (06 Marks)

06 - June /July 2013

(or)

Type (ii) 49 State uniqueness theorem and prove two solutions V_1 and V_2 are equal using Laplace's equation. (05 Marks)

Statement :- Any solution of Laplace's equation that satisfies the same boundary conditions must be the only solution regardless of the method used.

i.e Uniqueness theorem states that Laplace's equation (and also poisson's eq^n) has one and only one solution.

soln → Refer Next Page.

Module-3

15-Dec/Jan 2017 (CBCS) Scheme.

5 a. State and ~~explain~~ uniqueness theorem. (08 Marks)

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Soln:-

Statement:- Any solution of Laplace's equation that satisfies the same boundary conditions must be the only solution regardless of the method used.

i.e Uniqueness theorem states that Laplace's equation (and also poisson's eqn) has one and only one solution.

proof. The theorem is proved by contradiction. assume that there are two solutions V_1 and V_2 of Laplaces equation, both of which satisfy the prescribed boundary conditions.

Thus $\nabla^2 V = 0$ ..... Laplace's eqn

if V_1 and V_2 are the two solution's

then $\nabla^2 V_1 = 0$ and $\nabla^2 V_2 = 0 \longleftarrow (1)$

On boundary the solution's are equal

i.e $V_1 = V_2 \longleftarrow (2)$

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~~Consider the difference in Solution~~

i.e $V_2 - V_1 = V_d \leftarrow (3)$

which obeys $\nabla^2 V_d = \nabla^2 V_2 - \nabla^2 V_1 = 0$

on boundary $V_d = 0$ i.e $V_2 = V_1$ $\Big\} \leftarrow (4)$

$$\Rightarrow \boxed{\nabla^2 V_d = 0} \leftarrow (5)$$

using divergence theorem

$$\int_{vol} (\nabla \cdot \bar{A})\, dv = \oint_{S} \bar{A} \cdot \bar{dS} \leftarrow (6)$$

where 'S' is the Surface Sorrounding volume v.

Let vector field $\bar{A} = V_d \nabla V_d$ and using the vector identity

$$\nabla \cdot \bar{A} = \nabla \cdot (V_d \nabla V_d) = V_d \cancel{\nabla^2 V_d}^{\,0 \text{ (from eqn (5))}} + \nabla V_d \cdot \nabla V_d$$

$$\Rightarrow \boxed{\nabla V_d \cdot \nabla V_d = \nabla \cdot \bar{A}} \leftarrow (7)$$

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~~using eq^n (7) in eq^n (6)~~

$$\int_{\langle vol \rangle} (\nabla V_d \cdot \nabla V_d)\, dv = \oint_{\langle S \rangle} V_d \nabla V_d \, ds \leftarrow (8)$$

from eq^n (1) and eq^n (4) it is Evident that the right hand side of eq^n (8) vanishes.

$$\text{i.e} \quad V_d \nabla V_d = \cancelto{0}{(V_2 - V_1)} \left[\nabla (V_2 - V_1)\right] \quad \text{(from eq}^n\text{ 2)}$$

$$\Rightarrow \boxed{V_d \nabla V_d = 0}$$

$\therefore$ eq^n (8) becomes

$$\int_{\langle vol \rangle} (\nabla V_d \cdot \nabla V_d)\, dv = 0$$

<u>Note</u>: $\bar{A} \cdot \bar{A} = |A|^2 = A^2$

lly $\nabla V_d \cdot \nabla V_d = |\nabla V_d|^2$

$$\Rightarrow \int_{\langle vol \rangle} |\nabla V_d|^2 \, dv = 0 \leftarrow (9)$$

~~Since the integration in always~~ positive and cannot be zero.

eqn (9) is true only when $\nabla V_d = 0$ ← (10)

and eqn (10) is true only when

$$V_d = 0 \quad \text{(or)} \quad V_d = k \text{ (constant)}$$

i.e $V_d = 0 \Rightarrow V_2 - V_1 = 0$

$$\Rightarrow \boxed{V_2 = V_1}$$

(or) $(V_2 - V_1) =$ constant everywhere.

⇒ shows everywhere showing that V_1 and V_2 cannot be two different solution's of the same problem.

problem 8

06-DEC2008/Jan 2009

It is known that V = XY is a solution of laplace's equation, where X is a function of x alone and Y is a function y alone. Determine which of the following potential functions are also solutions of laplace's equation i) V = 100X, ii) V = 80XY, iii) V = 3XY + x – by.

(07 Marks)

Sol:- given $V = XY$ is a sol^n of Laplace's eq^n.

i.e $\nabla^2 V = 0$ and $\boxed{\nabla^2(XY) = 0} \leftarrow (1)$.

and $X = f^n(x)$ alone & $Y = f^n(y)$ alone

i) $V = 100X$

the Laplace eq^n $\nabla^2 V = 0$

$$\nabla^2(100X) = 100\nabla^2 X = 100\frac{\partial^2 X}{\partial x^2}$$

$$= 100X'' \neq 0$$

bcz 'X' is unknown i.e it can by any degree.

i.e $\boxed{\nabla^2 X \neq 0}$

$\therefore$ $\boxed{V = 100X}$ is not a solution of a Laplace's equation.

ii) $V = 80XY$

$\nabla^2 V = 0$

$$\Rightarrow \nabla^2(80XY) = 80\,\underbrace{\nabla^2(XY)}_{0 \text{ (from eq}^n (1))}$$

$= 0$ i.e $\boxed{\nabla^2(80XY) = 0}$

$\therefore$ $\boxed{V = 80XY}$ is a Solution of Laplace eq^n!

iii) $V = 3xy + x - by$

$\nabla^2 V = 0$

$\Rightarrow \nabla^2 V = \nabla^2 [3xy + x - by]$

from eq^n (1)

$= 3\nabla^2(xy) + \nabla^2(x) - b\nabla^2(y)$

$= \frac{\partial^2}{\partial x^2}(x) - b\frac{\partial^2}{\partial y^2}(y) = 0 - b(0) = 0$

i.e

$\Rightarrow \boxed{\nabla^2(3xy + x - by) = 0}$

$\therefore$ given potential $V = 3xy + x - by$ volt's is a Solution of Laplace equation.

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~~$V_1 = V_2 \Rightarrow V_2 - V_1 = 0 \Rightarrow \boxed{V_2 = V_1}$ (or) $V_2 - V_1 =$ constant~~

~~Shown every where showing that V_1 and V_2 cannot be different solution's of the same problem, hence proved.~~

Applications: Examples of the solution of Laplace's equation

Topic 3.3 Questions

3.39. Capacitance of a parallel plate capacitor.

10-DEC 2013/Jan 2014

Using Laplace's equation derive an expression for capacitance of parallel plate capacitor. (06 Marks)

(or)

10 - June /July 2014

The two metal plates having an area 'A' and a separation 'd' form a parallel plate capacitor. The upper plate is held at a potential V_0 and lower plate is grounded. Determine:

i) Potential distribution
ii) The electric field intensity
iii) Capacitance of parallel plate capacitor (10 Marks)

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(or)

06 –June/July 2014

Using Laplace's equation, find capacitance of metallic parallel plates (08 Marks)

find capacitance of metallic parallel plates

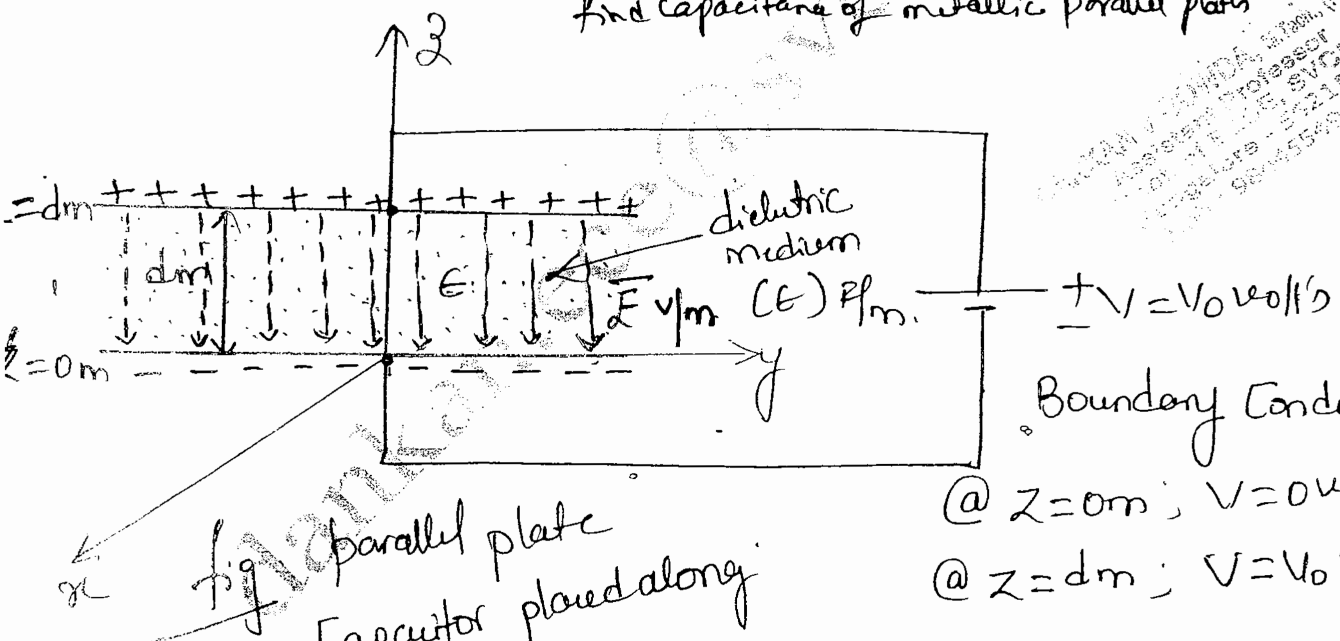


fig. parallel plate Capacitor placed along 'z' axis.

Boundary Condition's

@ $z = 0$ m ; $V = 0$ volt's

@ $z = d$ m ; $V = V_0$ volt's

Consider a parallel plate Capacitor placed along 'z' axis, and plates are separated by a distance of 'd' m. the potential $V = V_0$ volt's applied @ $z = d$ m plate and $V = 0$ volt's @ $z = 0$ m plate.

Consider a Laplace eqn

$$\nabla^2 V = 0 \ V/m.$$

$$\cancel{\frac{\partial^2 V}{\partial x^2}} + \cancel{\frac{\partial^2 V}{\partial y^2}} + \frac{\partial^2 V}{\partial z^2} = 0$$

0

Since Capacitor is placed along 'z' axis ∴ $V = f^n(z)$ alone.

$$\Rightarrow \nabla^2 V = \frac{\partial^2 V}{\partial z^2} = 0.$$

Solve for V

ie integrate w.r.t z

$$\frac{\partial V}{\partial z} = C_1 \leftarrow (1)$$

$$\Rightarrow V = C_1 z + C_2 \leftarrow (2)$$

using BC1 i.e V=0 volts @ z=0m.

$$0 = C_1(0) + C_2 \Rightarrow \boxed{C_2 = 0}$$

and BC2 ie V = V0 volts @ z = dm

$$V_0 = C_1(d) + 0 \Rightarrow \boxed{C_1 = V_0/d}$$

∴ eqn (1) becomes

$$\boxed{V = \frac{V_0}{d} z} \text{ volts} \leftarrow (a)$$

potential distribution b/w parallel plates.

using Gradient concept $\boxed{\bar{E} = -\nabla V}$ volts/m.

$$\bar{E} = -\frac{\partial V}{\partial z}\bar{a}_z \text{ V/m} \Rightarrow \text{ Since } V = f^n(z) \text{ only (from eq}^n \text{ (a))}$$

from eq^n (1)

$$\bar{E} = -c_1\bar{a}_z = -\frac{V_0}{d}\bar{a}_z$$

note:- -ve sign indicates field $\bar{E}$ acts towards $-z$ dir^n i.e $(-\bar{a}_z)$.

$$\therefore \boxed{\bar{E} = -\frac{V_0}{d}\bar{a}_z} \text{ V/m} \leftarrow (b)$$

$\bar{E}$ (field) distribution b/w parallel plates.

$$|\bar{E}| = \frac{V_0}{d} \text{ V/m}$$

$$\bar{D} = \epsilon\bar{E} \text{ C/m}^2 \Rightarrow |\bar{D}| = \epsilon|\bar{E}| = \rho_s = Q/A \text{ C/m}^2$$

$$\Rightarrow |\bar{D}| = \epsilon\frac{V_0}{d} = \rho_s = Q/A \text{ C/m}^2$$

$$\Rightarrow \epsilon\frac{V_0}{d} = Q/A$$

The Capacitance b/w parallel plates is $C = Q/V_0$

$$\therefore \Rightarrow \boxed{C = \frac{Q}{V_0} = \frac{\epsilon A}{d}} \text{ Farad's}$$

$$\Rightarrow \boxed{\rho_s = \pm\epsilon|\bar{E}|} \text{ C/m}^2 \text{ Surface charge density.}$$

*** procedure to Solve Laplace's eq^n (Application of Laplace's eq^n / Boundary value problem's).

Key Note.

Step 1. Consider the Laplace eq^n

$$\nabla^2 V = 0.$$

2nd order partial differential eq^n

Integrate twice Solve for V

V is in term's of two integral Constants Say C_1 and C_2.

Step 2. Using Boundary Condition's Solve for C_1 and C_2.

and Substitute C_1 and C_2 in the Result's Expression of V. ⟸ potential distribution

Step 3. Using Concept of gradient i.e

$$\boxed{E = -\nabla V} \text{ V/m}$$ find field distribution.

Step 4:

w.k.t $\bar{D} = \epsilon \bar{E}$ C/m²

(a)

$|\bar{D}| = \epsilon|\bar{E}| = \rho_s$ C/m² i.e Surface charge density

$\Rightarrow |\bar{D}| = \epsilon|\bar{E}| = \rho_s = Q/A$

Equating these two terms solve for an expression of Capacitance in the given region i.e $\boxed{C = Q/V_0 = \ ?}$ Farad's.

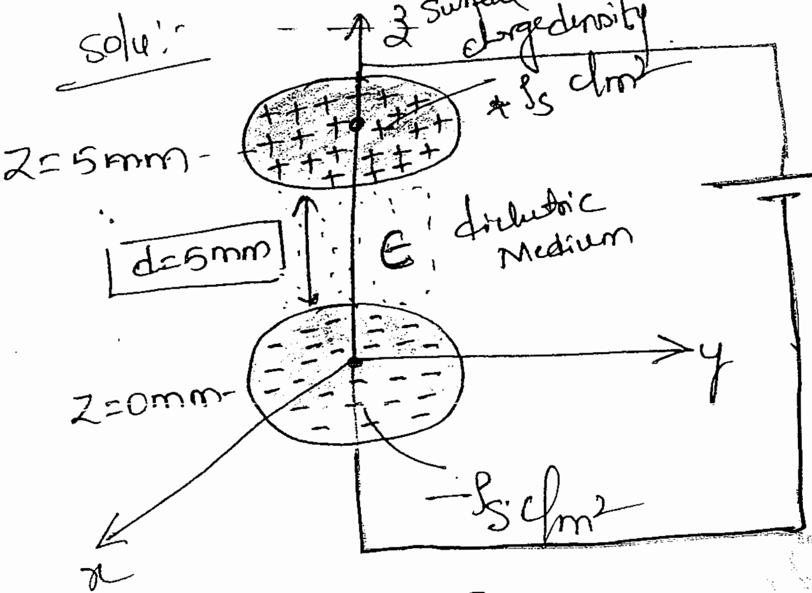
problem 9

06 - June /July 2012

50) Two parallel conducting discs are separated by distance 5mm at Z = 0; and Z = 5mm. V = 0 at Z = 0; and V = 100 volts at Z = 5mm and it is only in Z direction. Starting from Laplace equation find surface charge densities on the discs. [Take $\epsilon = \epsilon_o = 8.854 * 10^{-12}$ F/m]. (08 Marks)

$\epsilon = \epsilon_0 = 8.854 \times 10^{-12}$ F/m

solu:-



Boundary Condition's

BC's

@ Z=0mm V = 0 volts

@ Z = 5mm ; V = 100 volts

$|\rho_s| = ?$

Consider the Laplace eqn $\nabla^2 V = 0$. V/m²

$$\cancel{\frac{\partial^2 V}{\partial x^2}} + \cancel{\frac{\partial^2 V}{\partial y^2}} + \frac{\partial^2 V}{\partial z^2} = 0$$

0 0

Since parallel Conducting discs are placed along z axis

$$\Rightarrow \nabla^2 V = \frac{\partial^2 V}{\partial z^2} = 0$$

solve for V

$$\Rightarrow \frac{\partial V}{\partial z} = C_1$$

$$\Rightarrow \boxed{V = C_1 z + C_2}$$

using BC's @ $z = 0$ mm $V = 0$ volts

$\Rightarrow \boxed{C_2 = 0}$

and @ $z = 5$ mm $V = 100$ volts

$100 = C_1(5m) \Rightarrow \boxed{C_1 = 20K}$

$\therefore \boxed{V = 20\times10^3 z}$ volts

and $\bar{E} = -\nabla V = -\frac{\partial V}{\partial z}\bar{a}_z$ V/m. bcz disc placed along 'z' axis

but $\frac{\partial V}{\partial z} = C_1$

$\bar{E} = -[C_1]\bar{a}_z = -20k\,\bar{a}_z$ V/m

$\Rightarrow |\bar{E}| = 20k$ V/m.

$|\bar{D}| = \epsilon|\bar{E}| = |\rho_s| = \epsilon(20k)$ C/m²

$\boxed{|\rho_s| = 177.08}$ nC/m²

or $\rho_{s+} = 177.08$ nC/m² (upper disc)

and $\rho_{s-} = -177.08$ nC/m² (down disc).

or $\boxed{\rho_s = \pm 177.08}$ nC/m²

Topic 3.2b Capacitance of a co-axial Cable.

Questions

06-DEC2008/Jan 2009

5 Using laplace's equation, prove that the potential distribution at any point in the region between two concentric cylinders of radii A and B as $V = V_0 \frac{\ln(\rho/B)}{\ln(A/B)}$ (Volts) (07 Marks)

(or)

10-DEC2011/Jan 2012

Use Laplace's equation to find the capacitance per unit length of a co-axial cable of inner radius 'a' m and outer radius 'b' m. Assume $v = v_0$ at $r = a$ and $V = 0$ at $r = b$. (08 Marks)

(or)

02 - June /July 2012

Applying Laplace equation show that the potential in the space between the two conductors of a co-axial cable of infinite length is $V = V_0 \frac{\ln(R_2/r)}{\ln(R_2/R_1)}$, where $R_1 < r < R_2$. $R_1 < r < R_2$

$V_0 \rightarrow$ Potential on the inner conductor
$R_1 \rightarrow$ Radius of the inner conductor
$R_2 \rightarrow$ Radius of the outer conductor

$V = V_0 \frac{\ln(\cancel{r}/R_2)}{\ln(R_1/R_2)}$ (07 Marks)

(or)

06- June /July 2009

~~Using Laplace equations, derive the expression for the capacitance of co-axial cable.~~ (10 Marks)

include y carb

06 – Jan 2013

Derive the expression for capacitance of a co – axial cable using Laplace's equation. (08 Marks)

06 – May/June 2010

~~Derive the expression for capacitance of a coaxial cable using Laplace's equation. (10 Marks)~~

Sol:

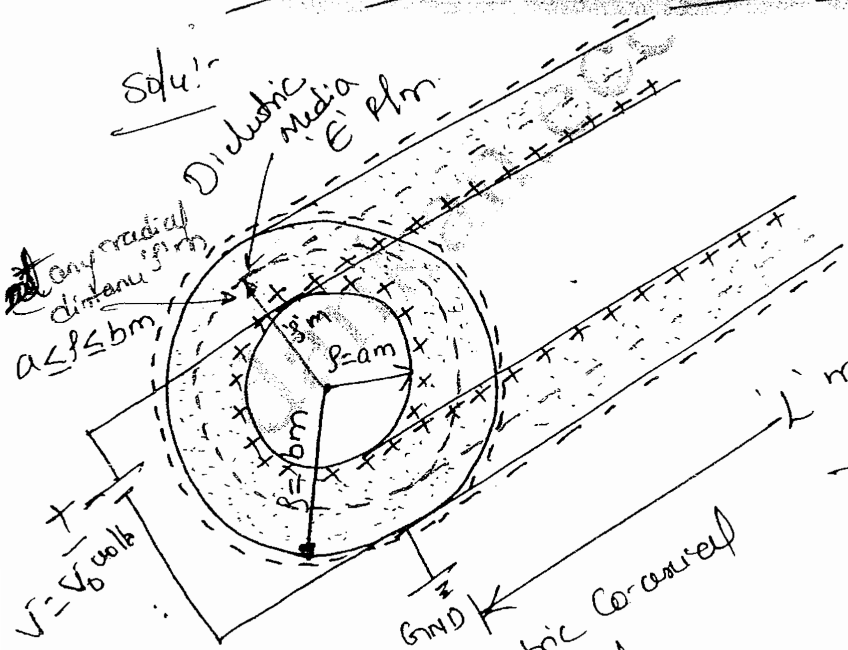


fig. Concentric Co-axial cable with b>am (a≤ρ≤bm)

Consider a Concentric Co-axial cable with inner radius of 'a' m and outer radius of 'b' m with (b>am). the potential applied at a inner Cylinder is $V = V_0$ volt's and $V = 0$ Volt's on outer Cylinder.
i.e @ $\rho = bm$; $V = 0$ volt's
@ $\rho = am$; $V = V_0$ volt's.

the given problem related to cylindrical Co-ordinate System.

$\therefore$ the Laplace's eqn in cylindrical Co-ordinate System is

--- i.e. $\nabla^2 V = 0$. by $V \neq f^n(\phi, z)$.

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho \frac{\partial V}{\partial \rho}\right] + \frac{1}{\rho^2}\cancelto{0}{\frac{\partial^2 V}{\partial \phi^2}} + \cancelto{0}{\frac{\partial^2 V}{\partial z^2}} = 0$$

ie V is f^n of only radial Component 'ρ' $V = f^n(\rho)$ alone.

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho \frac{\partial V}{\partial \rho}\right] = 0$$

$\Rightarrow$ $\rho \neq 0$ $\therefore \frac{1}{\rho} \neq 0$

$$\frac{\partial}{\partial \rho}\left[\rho \frac{\partial V}{\partial \rho}\right] = 0$$

Integrating once w.r.t 'ρ'

$$\rho \frac{\partial V}{\partial \rho} = C_1$$

$$\frac{\partial V}{\partial \rho} = \frac{C_1}{\rho}$$

$$\Rightarrow \boxed{V = C_1 \ln \rho + C_2} \text{ volt's}$$

using Boundary Condition's

i.e BC_1 @ $\rho = b\,m$; $V = 0$ volt's

$$0 = C_1 \ln(b) + C_2 \quad \leftarrow (1)$$

BC_2 @ $\rho = a\,m$; $V = V_0$ volt's

$$V_0 = C_1 \ln(a) + C_2 \quad \leftarrow (2)$$

eq^n (2) $- eq^n$ (1)

$$V_0 = C_1 \ln(a/b) \Rightarrow \boxed{C_1 = \frac{V_0}{\ln(a/b)}}$$

and using eq^n (1) i.e $C_2 = -C_1 \ln(b)$

$$\boxed{C_2 = -\frac{V_0}{\ln(a/b)} \ln(b)}$$

$\therefore$ the potential distribution b/w Concentric cylinder

is $V = \frac{V_0}{\ln(a/b)} \ln(\rho) - \frac{V_0}{\ln(a/b)} \ln(b)$

(or) $\boxed{V = \frac{V_0}{\ln(a/b)} \ln(\rho/b)}$ Volts ← (a)

← potential distribution b/w the Co-axial cable. $a \le \rho \le b$ m

the Electric field Intensity $\bar{E}$ V/m from Concept of gradient

$$\bar{E} = -\nabla V \text{ V/m} = -\frac{\partial V}{\partial \rho} \bar{a}_\rho \text{ V/m}$$

Since V f^n of ρ alone from eq^n (a).

$$\bar{E} = -\frac{C_1}{\rho} \bar{a}_\rho \text{ V/m} = -\frac{V_0}{\rho \ln(a/b)} \bar{a}_\rho \text{ V/m}.$$

(or) $\boxed{\bar{E} = +\frac{V_0}{\rho \ln(b/a)} \bar{a}_\rho}$ V/m ← (b)

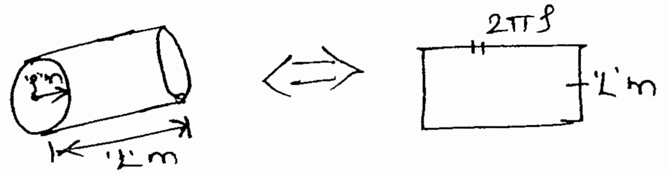
← field distribution b/w the Co-axial cable $a \le \rho \le b$

$\boxed{|\bar{E}| = \frac{V_0}{\rho \ln(b/a)}}$ V/m ← (i)

$\rightarrow \bar{D} = \epsilon \bar{E}$ C/m²

(or) $|\bar{D}| = \epsilon|\bar{E}| = \rho_s = \frac{Q}{A}$ C/m²

A - Surface area of the Co-axial cable with radius ρ m.



$$\boxed{A = 2\pi\rho L} \text{ m}^2$$

$$\epsilon|\bar{E}| = \frac{Q}{A} \text{ C/m}^2$$

using eqⁿ (i)

$$\epsilon\left[\frac{V_0}{\rho \ln(b/a)}\right] = \frac{Q}{2\pi\rho L} \text{ C/m}^2$$

$$\Rightarrow \frac{\epsilon V_0}{\cancel{\rho} \ln(b/a)} = \frac{Q}{2\pi\cancel{\rho} L}$$

$\Rightarrow$ the Capacitance b/w two concentric Co-axial cable is $C = Q/V_0$ Farad's

$$C = \frac{Q}{V_0} = \frac{2\pi\epsilon L}{\ln(b/a)}$$

$$\therefore \boxed{C = \frac{2\pi\epsilon L}{\ln(b/a)}} \text{ Farad's}$$

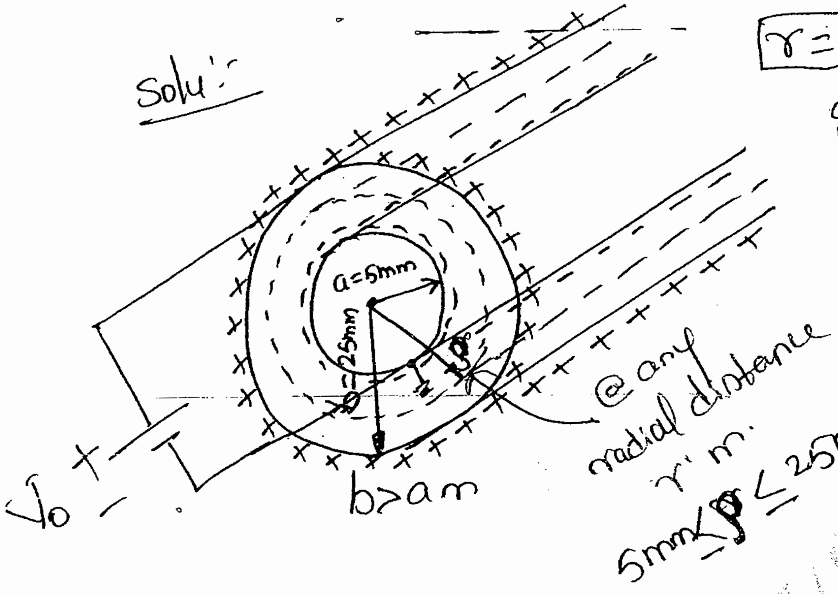
Capacitance per unit length $\boxed{C/L = \frac{2\pi\epsilon}{\ln(b/a)}}$ F/m problem 10 $\bar{E} = -8.28\times10^3\,\bar{a}_r$ V/m. $r=15$mm. $r=5$mm $r=25$mm

06 - June /July 2012

Long concentric and right conducting cylinders in free space at r = 5mm and r = 25mm in cylindrical co-ordinates have voltages of zero and V_0 respectively. If the electric field intensity $\bar{E} = -8.28 * 10^3\ \bar{a}r$ V/m at r = 15mm, starting from Laplace equation find V_0 and charge density on the outer conductor [Take $\epsilon = \epsilon_0 = 8.854 * 10^{-12}$ F/m]. (08 Marks)

$\boxed{r=\rho}$ and $\bar{a}_r = \bar{a}_\rho$.

Soln:-



given
@ $a=5$mm $V=0$ volt's
@ $b=25$mm $V=V_0$ volt's
and
$\bar{E} = -8.28\times10^3\,\bar{a}_\rho$ V/m
at $\rho = 15$mm.

using Laplace eq^n $\nabla^2 V = 0$ V/m^2

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left[\rho\frac{\partial V}{\partial\rho}\right] + \cancel{\frac{1}{\rho^2}\frac{\partial^2 V}{\partial\phi^2}} + \cancel{\frac{\partial^2 V}{\partial z^2}} = 0$$

0 bcz $V = f^n(\rho)$ only.

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left[\rho\frac{\partial V}{\partial\rho}\right] = 0$$

$\rho \neq 0 \quad \therefore \frac{1}{\rho} \neq 0$

$$\Rightarrow \frac{\partial}{\partial\rho}\left[\rho\frac{\partial V}{\partial\rho}\right] = 0$$

$$\rho\frac{\partial V}{\partial\rho} = C_1$$

$$\Rightarrow \boxed{\frac{\partial V}{\partial\rho} = \frac{C_1}{\rho}}$$

$$\boxed{V = C_1 \ln(\rho) + C_2}$$ Volt's

using Boundary Condition:

i.e @ $\rho = 5mm$; $V = 0$ volts

$$\therefore 0 = C_1 \ln(5m) + C_2 \leftarrow (1)$$

and @ $\rho = 25mm$; $V = V_0$ volts

$$V_0 = C_1 \ln(25m) + C_2 \leftarrow (2)$$

Solving (1) & (2)

$$V_0 = C_1 \ln(25/5)$$

$$V_0 = C_1 \ln(5) \Rightarrow \boxed{C_1 = \frac{V_0}{\ln(5)}}$$

and from eq^n (1) $C_2 = -C_1 \ln(5m)$

$$\boxed{C_2 = -\frac{V_0 \ln(5m)}{\ln(5)}}$$

$\therefore$ the potential

$$V = \frac{V_0}{\ln(5)} \ln(\rho) - \frac{V_0 \ln(5m)}{\ln(5)}$$

$$\boxed{V = \frac{V_0}{\ln(5)} \ln(\rho/5m)} \text{ Volts}$$

the field $\bar{E} = -\nabla V = -\frac{\partial V}{\partial \rho} \bar{a}_\rho$ V/m

$$\bar{E} = -\frac{C_1}{\rho} \bar{a}_\rho \text{ V/m} = \frac{-V_0}{\rho \ln(5)} \bar{a}_\rho \text{ V/m}$$

given $\bar{E} = -8.28\times10^{3}\,\bar{a}_\rho$ V/m @ $\rho = 15$mm.

$$\therefore \bar{E}_{@\rho=15mm} = \frac{-V_0}{(15m)\ln(5)}\bar{a}_\rho = -8.28\times10^{3}\,\bar{a}_\rho$$

$$V_0 = 15m(\ln 5)(8.28\times10^{3})$$

$\boxed{V_0 = 199.892}$ volt'n (or) $\boxed{V_0 = 199.9}$ volt'n

To find 'ρ_s' @ $\rho = 25$mm ie outer cylinder.

$$\rho_s = |\bar{D}| = \epsilon|\bar{E}| = \epsilon\left[\frac{V_0}{\rho\ln(5)}\right] C/m^2$$

$$\rho_s = 8.854\times10^{-12}\times\frac{199.9}{(25m)[\ln(5)]} = 43.986\,nC/m^2$$

$\boxed{\rho_s = +43.986}\,nC/m^2$

'ρ_s' at inner cylinder ie @ $\rho = 5$mm.

$$\rho_s = -8.854\times10^{-12}\times\frac{199.8}{(5m)(\ln 5)} = -219.83\,nC/m^2$$

$\boxed{\rho_s = -219.8}\,nC/m^2$

and $\bar{D} = \epsilon\bar{E}$ @ $\rho = 25$mm

ie $\boxed{\bar{D} = D\cdot\bar{a}_\rho = -43.986\,\bar{a}_\rho}\,nC/m^2$

Problem 11 $|E|$ $\leftarrow P(3,1,2)$ 06 - J/J 2011

✓ D6.3. Find $|\mathbf{E}|$ at $P(3, 1, 2)$ for the field of: (a) two coaxial conducting cylinders, $V = 50$ V at $\rho = 2$ m, and $V = 20$ V at $\rho = 3$ m; (b) two radial conducting planes, $V = 50$ V at $\phi = 10°$, and $V = 20$ V at $\phi = 30°$.

10 J/J 2016

Ans. 23.4 V/m; 27.2 V/m $\phi = 10°$ $V = 20\text{V}$ at $\phi = 30°$

06 - June /July 2011

57 ~~Find E at P(3, 1, 2) for the field of two co-axial conducting cylinders V = 50 V at ρ = 2m, V = 20 V at ρ = 3 m.~~ **(08 Marks)**

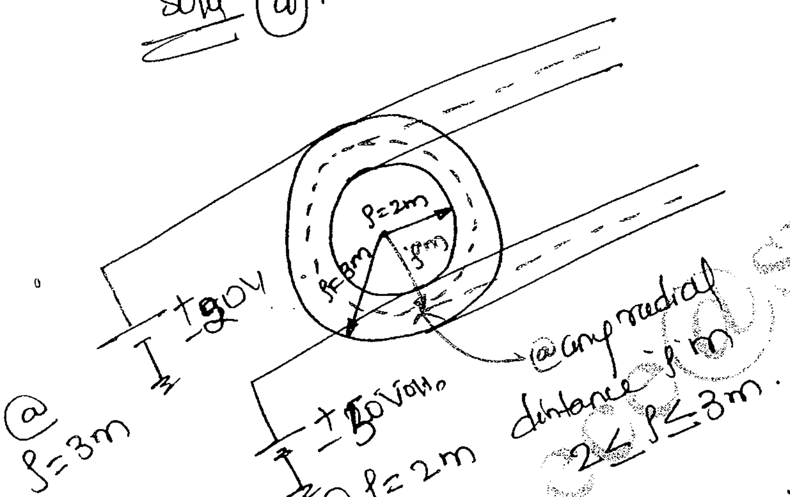
$\bar{E}$ $\leftarrow P(3,1,2)$ 06 - J/J 2011

10-June/July 2016

62 b. Find $\bar{E}$ at P(3, 1, 2) for the field of two co-axial conducting cylinders V = 50 V at ρ = 2 m and V = 20 V at ρ = 3m. (08 Marks) $\rho = 2$m.

$V = 20$ V $\leftarrow \rho = 3$m $V = 50$ V

Soln (a).



Boundary Conditions (BC's)

@ $\rho = 2$m $V = 50$ volts

and

@ $\rho = 3$m $V = 20$ volts.

@ any radial distance ρ m $2 \le \rho \le 3$m.

using Laplace's eqn $\nabla^2 V = 0$ V/m²

Since V is a fn of radial component ρ alone

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho \frac{\partial V}{\partial \rho}\right] = 0.$$

$$\rho \neq 0 \ \& \ \frac{1}{\rho} \neq 0$$

$$\Rightarrow$$

$$\therefore \frac{\partial}{\partial \rho}\left[\rho \frac{\partial V}{\partial \rho}\right] = 0$$

$$\Rightarrow \rho \frac{\partial V}{\partial \rho} = C_1 \quad \text{and} \quad \boxed{V = C_1 \ln(\rho) + C_2} \text{ volts}$$

using Boundary Condition's

@ $\rho = 2m$; $V = 50$ volt's

$$50 = C_1 \ln(2) + C_2 \leftarrow (1)$$

@ $\rho = 3m$; $V = 20$ volt's

$$20 = C_1 \ln(3) + C_2 \leftarrow (2)$$

Solving (1) and (2)

$$30 = C_1 \ln(2/3) \Rightarrow \boxed{C_1 = \frac{30}{\ln(2/3)}} = -73.989$$

and from eqⁿ (2) $C_2 = 20 - C_1 \ln(3)$

$$C_2 = 20 - \frac{30}{\ln(2/3)} \times \ln(3)$$

$$\boxed{C_2 = -61.2853}$$

$$\therefore \boxed{V = -73.989 \ln(\rho) - 61.28} \text{ volt's}$$

$2m \leq \rho \leq 3m$.

The field distribution

$$\bar{E} = -\nabla V = -\frac{\partial V}{\partial \rho} \bar{a}_\rho \text{ V/m}.$$

$$\bar{E} = -\frac{C_1}{\rho} \bar{a}_\rho = +\frac{73.989}{\rho} \bar{a}_\rho \text{ V/m}.$$

$\bar{E}$ @ $P(3, 1, 2)$ (x, y, z) $\Rightarrow$ $\rho = \sqrt{x^2+y^2} = \sqrt{9+1} = \sqrt{10}$ m

$\phi = \tan^{-1}(y/x) = 18.43°$

$z = 2m$

$$P(3,1,2) \Longleftrightarrow P(\sqrt{10}, 18.43^\circ, 2).$$

and $\boxed{\rho = \sqrt{10}}$ m.

$$\bar{E} = \frac{73.989}{\sqrt{10}} \bar{a}_\rho \text{ V/m} = 23.397 \bar{a}_\rho \text{ V/m}$$

$$\boxed{\bar{E}_P = 23.397 \bar{a}_\rho} \text{ V/m in Cylindrical C.S.}$$

$E_\rho = 23.397$ V/m.

$E_\phi = 0$ V/m & $E_z = 0$ V/m

$\bar{E}_P$ in Cartesian C.S

$$\bar{E}_P = E_x \bar{a}_x + E_y \bar{a}_y + \cancel{E_z \bar{a}_z}^{\,0} \text{ V/m}.$$

$$E_x = E_\rho \cos(\phi) = 23.397 \cos(18.43^\circ) = \underline{22.1969} \text{ V/m}$$

$$E_y = E_\rho \sin(\phi) = 23.397 \sin(18.43^\circ) = \underline{7.396} \text{ V/m}$$

$$\boxed{\bar{E}_P = 22.1969 \bar{a}_x + 7.396 \bar{a}_y} \text{ V/m - In Cartesian Coordinate System.}$$

and $|\bar{E}_P| = \sqrt{22.1969^2 + 7.396^2}$

$$= 23.397 \simeq 23.4 \text{ volt's}$$

$$\boxed{|\bar{E}_P| = 23.4} \text{ volt's}$$

b) xxx

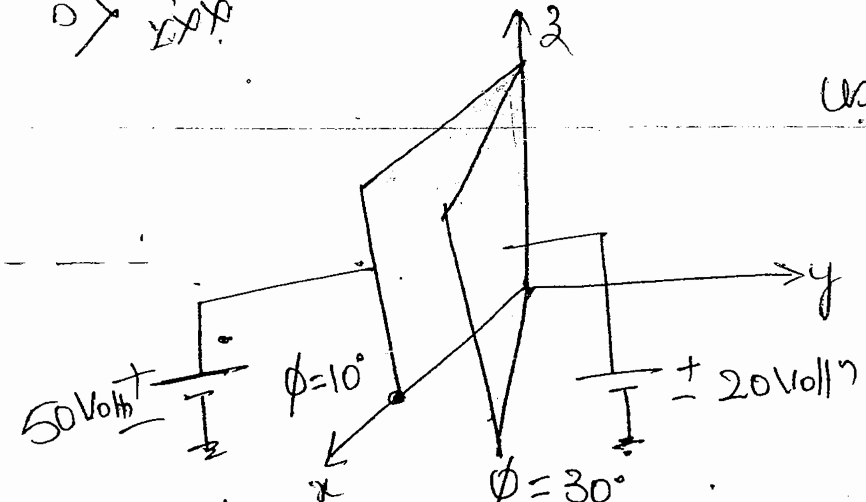


fig. radial Conducting planes

using Laplace eq^n

$$\nabla^2 V = 0$$

BC's

@ $\phi = 10°$; $V = 50$ volts

@ $\phi = 30°$; $V = 20$ volts

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho \frac{\partial V}{\partial \rho}\right] + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho \frac{\partial V}{\partial \rho}\right] \to 0$ bcz $V = f^n(\phi)$ alone

$\frac{\partial^2 V}{\partial z^2} \to 0$ bcz $V = f^n(\phi)$ alone

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial^2 V}{\partial \phi^2} = 0$$

$$\rho \neq 0 \quad \therefore \frac{1}{\rho} \neq 0$$

$$\Rightarrow \frac{\partial^2 V}{\partial \phi^2} = 0$$

Integrating wrt ϕ

$$\frac{\partial V}{\partial \phi} = C_1$$

$$\boxed{V = C_1 \phi + C_2}$$

$V = C_1\phi + C_2$ Volts

using boundary cond^n @ $\phi = 10^\circ = \left(\frac{\pi}{18}\right)^c$ $V = 50$ Volts

$\pi^c = 180^\circ$

$1^\circ = \left(\frac{\pi}{180}\right)^c$

$10^\circ = \left(\frac{\pi}{18}\right)^c$

$30^\circ = \left(\frac{3\pi}{18}\right)^c$

$$50 = \frac{\pi}{18}C_1 + C_2 \leftarrow (1)$$

@ $\phi = 30^\circ$ @ $(3\pi/18)^c$; $V = 20$ Volts

$$20 = \frac{3\pi}{18}C_1 + C_2 \leftarrow (2)$$

Solving eq^n (1) and (2) $\quad 30 = -\frac{2\pi}{18}C_1 \Rightarrow \boxed{C_1 = -85.943}$

from eq^n (2) $\quad C_2 = 20 - \frac{3\pi}{18}C_1$

$$\boxed{C_2 = 20 - \frac{3\pi}{18}(-85.943) = 64.999} \simeq 65$$

$$\boxed{V = -85.94\phi + 65} \text{ Volts}$$ ← potential distribution b/w planes $\phi = 10^\circ$ to $\phi = 30^\circ$.

$$\bar{E} = -\frac{\partial V}{\rho\,\partial\phi}\bar{a}_\rho \text{ V/m} = -\frac{C_1}{\rho}\bar{a}_\rho$$

$$= +\frac{85.943}{\rho}\bar{a}_\rho \text{ V/m.}$$

$$\therefore \boxed{\bar{E} = \frac{85.94}{\rho}\bar{a}_\rho} \text{ V/m}$$

Part

$P(3,1,2) \Longleftrightarrow$ cylindrical $P(\sqrt{10}, 18.43^\circ, 2)$

$$\bar{E}_P = \frac{85.94}{(\sqrt{10})}\bar{a}_\rho = 27.1766\,\bar{a}_\rho \text{ V/m}$$ ← In cylindrical CS

$$\boxed{\bar{E}_P = 27.1766\,\bar{a}_\rho} \text{ V/m} ; \quad E_\rho = 27.1766 \text{ V/m}$$

$$E_\phi = E_z = 0 \text{ V/m}$$

$$\bar{E}_P = E_x\bar{a}_x + E_y\bar{a}_y = E_\rho\cos\phi\,\bar{a}_x + E_\rho\sin\phi\,\bar{a}_y \text{ V/m}$$

$$= 27.176\cos(18.43^\circ) + 27.176\sin(18.43^\circ)\bar{a}_y$$

XX

$$\boxed{\bar{E}_P = 25.782\,\bar{a}_x + 8.5915\,\bar{a}_y} \text{ V/m}$$ ← in rectangular C.S $\quad \boxed{|\bar{E}_P| = 27.176} \text{ V/m}$

$\therefore \bar{E}_p = 25.782\,\bar{a}_x + 8.5915\,\bar{a}_y$ V/m. In rectangular C.S

(or) $\boxed{|\bar{E}_p| = 27.176}$ V/m.

Question

Topic 3.3C

Capacitance of a Concentric Sphere.

Find the capacitance between the two concentric spheres of radii r = b and r = a, such that b > a, if the potential V = 0 at r = b, using the Laplace's equation. (10 Marks) 06-Dec 2010.

(or)

06-DEC2011/Jan 2012

By applying Laplace's equation, find the expression for capacitance between the two concentric spheres. Make suitable assumptions. (08 Marks)

10-Jan 2013

Dankan V Gowda MTech.,(Ph.D)
Assistant Professor, Dept. of E&CE
Email:dankan.ece@svcengg.com

(or)

10-Dec/Jan 2016

c. Solve the Laplace equation for the potential field and find the capacitance in homogeneous region between two concentric conducting spheres with radii a and b such that b > a if V = 0 at r = b, V = Vo at r = a. (09 Marks)

Sol:n

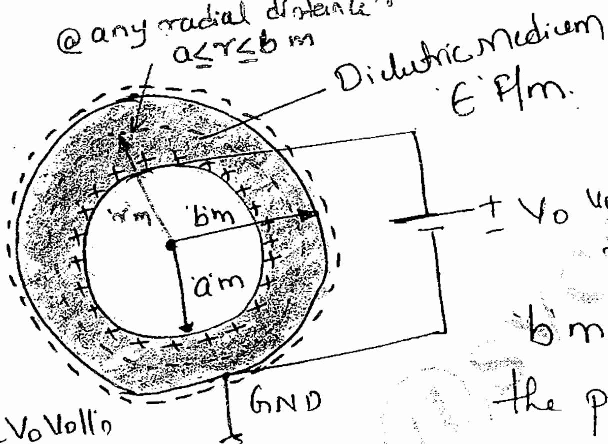


Fig. Concentric Sphere (b>a)m
$a \le r \le b$ m

BC's:
@ r = a m V = Vo Volt's
@ r = b m V = 0 Volt's

Consider a Concentric Sphere of inner radius 'a' m and outer radius 'b' m. where b > a m. the potential V = Vo volt's applied at r = a m and V = 0 volt's at r = b m.

the given problem related to spherical C.S ∴ the Laplace's equation in S.C.S is

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\,\cancel{\frac{\partial V}{\partial\theta}}\right] + \frac{1}{r^2\sin^2\theta}\cancel{\frac{\partial^2 V}{\partial\phi^2}} = 0$$

Since $V = f^n(r)$ → 0 ; Since $V = f^n(r)$ → 0

W.K.T the potential V is a function of radial component 'r' only ∴ the Laplace eqn becomes

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] = 0$$

$$r^2 \neq 0 \quad \therefore \frac{1}{r^2} \neq 0$$

$$\Rightarrow \frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] = 0$$

Integrating w.r.t 'r'

$$r^2\frac{\partial V}{\partial r} = C_1$$

$$\frac{\partial V}{\partial r} = \frac{C_1}{r^2}$$

again Integrating w.r.t r

$$\boxed{V = -\frac{C_1}{r} + C_2} \text{ volt's}$$

using Boundary Condition's

BC'n @ $r = a$ m $V = V_0$ volt's

$$V_0 = -\frac{C_1}{a} + C_2 \longleftarrow (1)$$

@ $r = b$ m $V = 0$ volt's

$$0 = -\frac{C_1}{b} + C_2 \longleftarrow (2)$$

eq'n (1) − eq'n (2)

$$V_0 = C_1\left[\frac{1}{b} - \frac{1}{a}\right] \quad \text{and} \quad \text{from eq'n (2)} \quad C_2 = +C_1/b$$

$$\Rightarrow \boxed{C_1 = \frac{V_0(ab)}{a-b}} \qquad \boxed{C_2 = \frac{V_0 a}{(a-b)}}$$

$\therefore$ the potential V becomes

$$V = -\frac{V_0(ab)}{(a-b)} + \frac{V_0 a}{(a-b)}$$

(or) $$\boxed{V = +\frac{V_0(ab)}{(b-a)} - \frac{V_0 a}{(b-a)}}$$ Volt's ← potential distribution b/w $a \le r \le b$ m.

The field $\bar{E} = -\frac{\partial V}{\partial r}\bar{a}_r = -\nabla V$ V/m

bcz V is $f^n(r)$ alone.

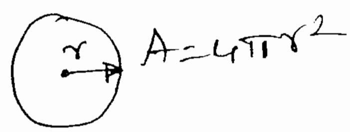
$$\therefore \bar{E} = -\frac{C_1}{r^2}\bar{a}_r = -\frac{V_0(ab)}{(a-b)r^2}\bar{a}_r = \frac{V_0(ab)}{(b-a)r^2}\bar{a}_r \text{ V/m}$$

$$\boxed{\bar{E} = \frac{V_0(ab)}{(b-a)r^2}\bar{a}_r} \text{ V/m}$$ ← field distribution b/w $a \le r \le b$ m.

$\rightarrow$ $\bar{D} = \epsilon\bar{E}$ C/m²

$\Rightarrow$ $|\bar{D}| = \epsilon|\bar{E}| = \rho_s = Q/A$ C/m²

$$\epsilon\frac{V_0(ab)}{(b-a)r^2} = \frac{Q}{A}$$



Area of sphere with radius 'r' $a<r<b$m

$$\epsilon\frac{V_0(ab)}{(b-a)\cancel{r^2}} = \frac{Q}{4\pi\cancel{r^2}}$$

$\Rightarrow$ Capacitance b/w Concentric sphere's

$$C = \frac{Q}{V_0} = \frac{4\pi\epsilon(ab)}{(b-a)} = \frac{4\pi\epsilon}{\frac{(b-a)}{ab}} = \frac{4\pi\epsilon}{\frac{\cancel{b}}{a\cancel{b}} - \frac{\cancel{a}}{\cancel{a}b}}$$

$$= 4\pi\epsilon / \left[\frac{1}{a} - \frac{1}{b}\right] \text{ Farad's}$$

(or) $$\boxed{C = \frac{4\pi\epsilon}{\left[\frac{1}{a} - \frac{1}{b}\right]}}$$ Farad's $b > a$ m where $a \le r \le b$ m

$$\boxed{C = \frac{4\pi\epsilon}{\left[\frac{1}{a} - \frac{1}{b}\right]}}$$ Farad's

where $b > a$ m

10 - June /July 2012

problem 12:

Conducting spherical shells with radii a = 10 cm and b = 30 cm are maintained at a potential difference of 100V such that V = 0 at r = b and V = 100V at r = a. Determine V and E in the region between the shells. If ϵ_r = 2.5 in the region, determine the total charge induced on the shells and the capacitance there on. (08 Marks)

Soln:

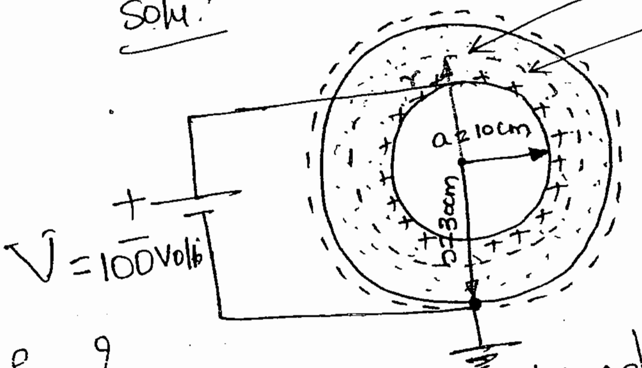


fig. Concentric sphere's with 10cm < r < 30cm

b > a m.

$\rho_{s_+} = ?$

$\rho_{s_-} = ?$

$C = Q/V_0 = ?$

and $Q = ?$

Boundary Condition's

@ $a = 0.1$ m $V = 100$ Volts.

and

@ $b = 0.3$ m ; $V = 0$ Volt's

$\epsilon_r = 2.5$ F/m

$\epsilon = \epsilon_0 \epsilon_r$ F/m $= 2.5\,\epsilon_0$ F/m.

Laplace eqn $\nabla^2 V = 0$ $\quad V = f^n(r)$ alone

i.e $$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\,\cancel{\frac{\partial V}{\partial\theta}}\right] + \frac{1}{r^2\sin^2\theta}\cancel{\frac{\partial^2 V}{\partial\phi^2}} = 0$$

0 bcz $V = f^n(r)$ alone

$r^2 \neq 0 \; \therefore \; \frac{1}{r^2} \neq 0$.

$$\Rightarrow \nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] = 0$$

$$\Rightarrow r^2\frac{\partial V}{\partial r} = C_1 \; ;$$

$$\Rightarrow \frac{\partial V}{\partial r} = \frac{C_1}{r^2}$$

$$\boxed{V = -\frac{C_1}{r} + C_2}$$

using BC's @ $a = r = 0.1\,m$ $V = 100$ volt's

$$100 = -\frac{C_1}{0.1} + C_2 \leftarrow (1)$$

@ $r = b = 0.3m$; $V = 0$ volt's

$$0 = -\frac{C_1}{0.3} + C_2 \leftarrow (2)$$

$$C_1 = \frac{100(0.1 \times 0.3)}{(0.1 - 0.3)} = -15 \Rightarrow \boxed{C_1 = -15}$$

$$C_2 = \frac{100(0.1)}{(0.1 - 0.3)} = -50 \Rightarrow \boxed{C_2 = -50}$$

$$\boxed{V = +\frac{15}{r} - 50}$$ volt's potential distribution b/w $0.1 \leq r \leq 0.3m$.

$$\bar{E} = -\nabla V = -\frac{\partial V}{\partial r}\bar{a}_r = -\frac{C_1}{r^2}\bar{a}_r = \frac{+15}{r^2}\bar{a}_r \text{ V/m}$$

$$\boxed{\bar{E} = \frac{15}{r^2}\bar{a}_r} \text{ V/m} ; \quad |\bar{E}| = \frac{15}{r^2} \text{ V/m}.$$

$\Rightarrow$ $\bar{D} = \epsilon \bar{E}$ C/m²

$\Rightarrow$ $|\bar{D}| = \epsilon|\bar{E}| = \rho_s = Q/A$ C/m²

$$\rho_s = \epsilon|\bar{E}| = \frac{15}{r^2}\epsilon \Rightarrow \boxed{\rho_s = \frac{15}{r^2}\epsilon} \text{ C/m}^2$$

$\epsilon = \epsilon_0\epsilon_r$ F/m

@ outer sphere $r = 0.3m$ ∴ $\boxed{\rho_{s-} = \frac{-15}{0.3^2}\epsilon_0\epsilon_r}$ C/m²

ρ_s -ve charged $= -3.689$ nC/m²

$$\boxed{\rho_{s-} = -3.689 \text{ nC/m}^2}$$

ρ_S @ inner sphere that is at $r = 0.1$ m (ρ_S +ve charged)

$$\rho_{S+} = +\frac{15}{r^2}\epsilon \ C/m^2 = \frac{15}{r^2}\epsilon_0\epsilon_r \ C/m^2$$

$$\rho_{S+} = \frac{15}{0.1^2} \times 8.854 \times 10^{-12} \times 2.5 \ C/m^2$$

$$\boxed{\rho_{S+} = 33.20} \ nC/m^2 = +33.2025 \ nC/m^2$$

and Capacitance b/w Concentric Sphere's

$$C = Q/V_0 = \frac{4\pi\epsilon}{\left[\frac{1}{a} - \frac{1}{b}\right]} = \frac{4\pi \times \epsilon_0\epsilon_r}{\left[\frac{1}{a} - \frac{1}{b}\right]}$$

$$C = \frac{4\pi \times 8.854 \times 10^{-12} \times 2.5}{\left[\frac{1}{0.1} - \frac{1}{0.3}\right]} = 41.7234 \ p\text{Farad's}$$

$$\boxed{C = 41.7234 \ pF}$$

→ ~~the~~ total Charge induced on the shells as

$$|Q| = CV_0 \Rightarrow |Q| = 100 \times 41.723p$$

$$= 4.1723 \ n \ Coulomb's$$

(or) $\boxed{Q = \pm 4.1723} \ nC$

$Q = +4.1723$ nC @ $a = 0.1$ m innersphere

$Q = -4.1723$ nC @ $a = 0.3$ m outer sphere.

5c. Find V at (2, 1, 3) for the field of

i) 2 co-axial conducting cylinders V = 20V at ρ = 3m

ii) 2 concentric conducting spheres V = 50V at r = 3m and V = 20V at r = 5m. **(08 Marks)**

Soln:- i) given Boundary conditions V = 20V at ρ = 3m.

Note:- in the given problem only one Boundary condition is given, with one boundary condition finding two unknowns is not possible.

assume another Boundary condition

Say V = 50V at ρ = 2m.

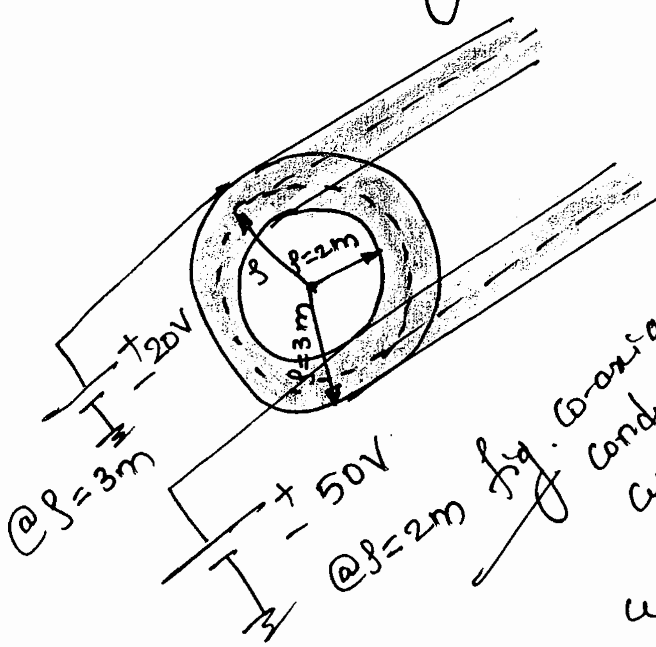


fig. co-axial conducting cylinder

Boundary conditions

at ρ = 2m, V = 50 volts

and

at ρ = 3m, V = 20 volts

using Laplace equation

$\nabla^2 V = 0$ V/m2

Since V is a function of radial component 'ρ' alone.

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho \frac{\partial V}{\partial \rho}\right] = 0$$

$$\Rightarrow \rho \neq 0 \text{ and } \frac{1}{\rho} \neq 0$$

$$\therefore \frac{\partial}{\partial \rho}\left[\rho \frac{\partial V}{\partial \rho}\right] = 0$$

$\Rightarrow$ Integrating w.r.t 'ρ'

$$\rho \frac{\partial V}{\partial \rho} = C_1$$

and $\frac{\partial V}{\partial \rho} = \frac{C_1}{\rho}$

again integrating w.r.t 'ρ'

$$\boxed{V = C_1 \ln(\rho) + C_2} \text{ volt's}$$

using Boundary condition's

i.e @ $\rho = 2m$, $V = 50$ volts

$$50 = C_1 \ln(2) + C_2 \leftarrow (1)$$

@ $\rho = 3m$, $V = 20$ volts

$$20 = C_1 \ln(3) + C_2 \longleftarrow (2)$$

Solving eqn (1) and eqn (2)

$$\boxed{C_1 = \frac{30}{\ln(2/3)} = -73.989}$$

and $C_2 = 20 - C_1 \ln(3) = 20 - \frac{30}{\ln(2/3)} \ln(3)$

$$\boxed{C_2 = 101.2853}$$

$$\therefore \boxed{V = -73.989 \ln(\rho) + 101.285} \text{ volts}$$

$3m < \rho < 5m$

potential at a point $P(2, 1, 3)$ is
$x\ y\ z$

$$\rho = \sqrt{x^2 + y^2}\ m \Rightarrow \rho = \sqrt{4+1} = \sqrt{5}\ m.$$

$$V_p = [-73.989 \ln(\sqrt{5}) + 101.285] \text{ Volts}$$

$$\boxed{V_p = 41.744} \text{ volts}$$

$V_p = 41.7446$ Volts

62-c

ii. given V at (2, 1, 3) in .. Cartesian C.S.

$x = 2,\ y = 1,\ z = 3.$ $\quad p(x, y, z) \Longleftrightarrow p(r, \theta, \phi)$

$$r = \sqrt{x^2+y^2+z^2} = \sqrt{2^2+1^2+3^2} = \sqrt{4+1+9}$$

$$\boxed{r = \sqrt{14}\ m}$$

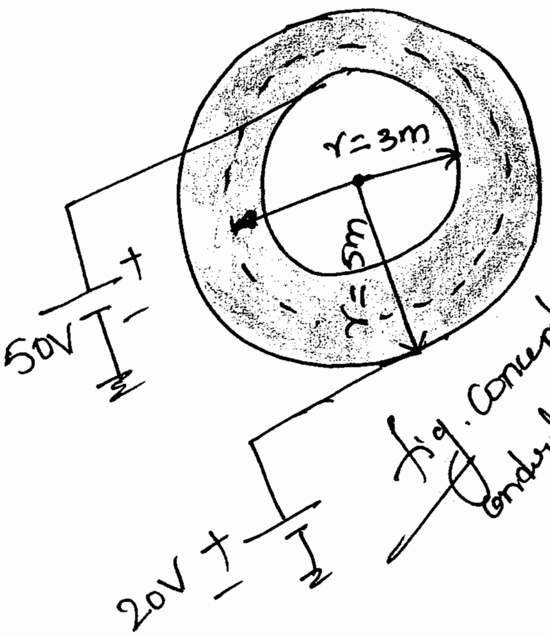


fig. concentric conducting spheres.

Boundary conditions

at $r = 3m$; $V = 50V$.

and

at $r = 5m$; $V = 20V$.

using Laplace equation

$$\nabla^2 V = 0$$

Since 'V' is a function of radial component 'r' only.

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{\partial V}{\partial r}\right] = 0$$

.. in spherical C.S

$r^2 \neq 0 \quad \therefore \frac{1}{r^2} \neq 0$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{\partial V}{\partial r}\right] = 0$$

Integrating w.r.t 'r'

$$r^2 \frac{\partial V}{\partial r} = C_1$$

$$\Rightarrow \frac{\partial V}{\partial r} = \frac{C_1}{r^2}$$

again Integrating w.r.t 'r'

$$\boxed{V = -\frac{C_1}{r} + C_2} \longleftarrow (a)$$

using Boundary conditions i.e

@ $r = 3m$; $V = 50V$.

$$50 = -\frac{C_1}{3} + C_2 \longleftarrow (1)$$

@ $r = 5m$; $V = 20V$

$$20 = -\frac{C_1}{5} + C_2 \longleftarrow (2)$$

Solving eqn (1) and eqn (2)

$\boxed{C_1 = -225}$ and $\boxed{C_2 = -25}$.

62-e

eqⁿ ⓐ becomes

$$\boxed{V = +\frac{225}{r} - 25} \text{ volts}$$

$3m < r < 5m$

the potential at a point $P(2,1,3)$ i.e

$r = \sqrt{14}$ m.

$$V_p = \frac{225}{\sqrt{14}} - 25$$

$$\boxed{V_p = 35.133} \text{ volts}$$

problem 13

$54pF$

10-June/July 2016

c A spherical capacitor has a capacitance of 54 pF. It consists of two concentric spheres with inner and outer radii differing by 4 cm. Dielectric in between is air. Determine inner and outer radii. (08 Marks)

$4cm$.

Sol^n

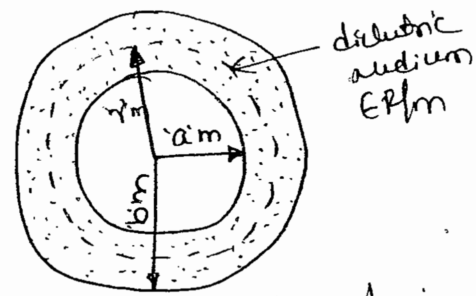


fig. Concentric spheres $b > a$

the Capacitance b/w two Concentric spheres is

$$C = \frac{4\pi\epsilon}{\left[\frac{1}{a} - \frac{1}{b}\right]} F. \leftarrow (1)$$

and given $b - a = 4cm$

$$b - a = 0.04 \leftarrow (2)$$

given $C = 54pF$

$$\therefore eq^n (1) \quad 54pF = \frac{4\pi\epsilon}{[a^{-1} - b^{-1}]} F \leftarrow (3)$$

using eq^n (2) $\boxed{b = 0.04 + a} \leftarrow (4)$

$\Rightarrow$ eq^n (4) in (3)

$$54pF\left[a^{-1} - (0.04 + a)^{-1}\right] = 4\pi\epsilon$$

$$54pF\left[a^{-1} - (0.04 + a)^{-1}\right] - 4\pi \times 8.854 \times 10^{-12} = 0$$

$$54p\left[a^{-1} - (0.04 + a)^{-1}\right]$$

using Calculator: Solve eq^n in Calci

$\boxed{a = 0.12076}$ m (or) $\boxed{a = 12.076 \, cm}$.

from eq^n (4) $b = 0.04 + 0.12076 = 0.16076m$

63 $\therefore$ $\boxed{b = 0.1607m}$ (or) $\boxed{b = 16.076}$ cm $= 16.0763 cm$

Topic 3.4 Applications of Poisson's Equation.

$\rho_v = \frac{200\epsilon_0}{r^{2.4}}$ C/m^3 10-DEC 2013/Jan 2014

problem 14

i) In free space the volume charge density $\rho_v = \frac{200\epsilon_0}{r^{2.4}}$ C/m^3, use Poisson's equation to find the potential V as a function of r ie V(r), if it is assumed that $r^2 E_r \to 0$ as $r \to 0$ and $V \to 0$ as $r \to \infty$. (08 Marks) [W.H. Hayt]

Use Spherical Co-ordinate System.

ii) Find potential V as a function of r using Gauss's Law and Line Integral.

soln:- i) Method-I using poisson's equation.

ie $\nabla^2 V = -\rho_v/\epsilon$ V/m^2

given $\rho_v = \frac{200\epsilon_0}{r^{2.4}}$ C/m^3

in free space $\epsilon = \epsilon_0$.

$\therefore \Rightarrow \nabla^2 V = -\rho_v/\epsilon_0$

$$\boxed{\nabla^2 V = \frac{-200}{r^{2.4}}} \text{ V/m}^2 \leftarrow (a)$$

the poisson's eqn in Spherical C.S

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\cancel{\frac{\partial V}{\partial\theta}}\right] + \frac{1}{r^2\sin^2\theta}\cancel{\frac{\partial^2 V}{\partial\phi^2}}$$

$\qquad\qquad 0 \qquad\qquad 0$

bcz $V = f^n(r)$ alone.

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right]$$

using eqn (a)

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] = \frac{-200}{r^{2.4}}$$

$$\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] = -200r^{-0.4}$$

Integrating w.r.t 'r'

$$r^2\frac{\partial V}{\partial r} = -\frac{200r^{0.6}}{0.6} + C_1$$

$$r^2\frac{\partial V}{\partial r} = -333.33r^{0.6} + C_1 \leftarrow (1)$$

the electric field in spherical C.S

$\bar{E} = -\nabla V = -\frac{\partial V}{\partial r}\bar{a}_r$ V/m.

$E_r = -\frac{\partial V}{\partial r}$

$-r^2 E_r = -333.33r^{0.6} + C_1$

given condn

as $r \to 0$; $r^2 E_r \to 0$

$0 = 0 + C_1 \Rightarrow \boxed{C_1 = 0}$

$\therefore$ eqn (1) becomes

$$\frac{\partial V}{\partial r} = -333.33r^{-1.4}$$

Integrating w.r.t r

$$V = +\frac{833.32}{r^{0.4}} + C_2$$

using 2nd condn $V \to 0$ as $r \to \infty$

$0 = 0 + C_2 \Rightarrow \boxed{C_2 = 0}$

∴ the potential field $\boxed{V(r) = \frac{833.32}{r^{0.4}}}$ volts

or

ii) Method-II :- Verification using Gauss's Law and Line Integral.

w.k.t from Maxwell's first eqn

$\nabla \cdot \bar{D} = \rho_v \ C/m^3$

$\nabla \cdot (\epsilon_0 \bar{E}) = \rho_v$

$\nabla \cdot \bar{E} = \rho_v / \epsilon_0 = \frac{200}{r^{2.4}}$

the divergence of $\bar{E}$ in spherical Co-ordinate System is

i.e $\nabla \cdot \bar{E} = \frac{1}{r^2}\frac{\partial}{\partial r}[r^2 E_r] = \rho_v/\epsilon_0$

bcz $\bar{E}$ in fn of 'r' only

$\frac{1}{r^2}\frac{\partial}{\partial r}[r^2 E_r] = \frac{200}{r^{2.4}}$

$\frac{\partial}{\partial r}[r^2 E_r] = \frac{200}{r^{0.4}}$

Integrating w.r.t 'r'

$r^2 E_r = 200\frac{r^{-0.4+1}}{0.6} + C_1$

$r^2 E_r = 333.33 r^{0.6} + C_1$

as $r^2 E_r \to 0$ as $r \to 0$

∴ $\boxed{C_1 = 0}$

∴ $r^2 E_r = 333.33 r^{0.6}$

(or) $\boxed{E_r = 333.33 r^{-1.4}}$ V/m

$\bar{E} = E_r \bar{a}_r$ V/m $= 333.33 r^{-1.4} \bar{a}_r$ V/m

and potential field

$V = -\int \bar{E} \cdot \bar{dl}$

$= -\int 333.33 r^{-1.4} \bar{a}_r \cdot dr \bar{a}_r$

$= -333.33 \int r^{-1.4} dr \ \bar{a}_r \cdot \bar{a}_r$

$V = -333.33 \frac{r^{-0.4}}{-0.4} + C_2$

as $V \to 0$ when $r \to \infty$

$\Rightarrow 0 = 0 + C_2$

∴ $\boxed{C_2 = 0}$

∴ $\boxed{V(r) = +\frac{833.33}{r^{0.4}}}$ Volts

In both the method's the potential field $V(r)$ is same

i.e $\boxed{V(r) = \frac{833.33}{r^{0.4}}}$ Volts

Problem 15

$\rho_v = -2\times10^7 \epsilon_0 \sqrt{x}\ C/m^3$.

Given the volume charge density $\rho_v = -2\times10^7\epsilon_0\sqrt{x}\,C/m^3$ in free space, let $V = 0$ at $x = 0$ and $V = 2$ V at $x = 2.5$ mm. At $x = 1$ mm, find: (a) V; (b) E_x.

$V=0$ at $x=0$ $V=2V$ at $x=2.5mm$. at $x=1mm$ V & E_x

Ans. 0.302 V; −555 V/m

Sol^n: given $\rho_v = -2\times10^7 \epsilon_0 \sqrt{x}\ C/m^3$

using poisson's eq^n $\nabla^2 V = -\rho_v/\epsilon_0\ V/m^2$.

$\Rightarrow \rho_v/\epsilon_0 = 2\times10^7\sqrt{x}$

$$\nabla^2 V = -\rho_v/\epsilon_0 = +2\times10^7\sqrt{x}\ V/m^2$$

Since V is a f^n (x) alone

$$\therefore \nabla^2 V = \frac{\partial^2 V}{\partial x^2}\ V/m^2$$

$$\Rightarrow \nabla^2 V = \frac{\partial^2 V}{\partial x^2} = +2\times10^7\sqrt{x}$$

Integrating w.r.t 'x'

$$\frac{\partial V}{\partial x} = +2\times10^7 \frac{x^{1/2+1}}{(1/2+1)} + C_1$$

$$\frac{\partial V}{\partial x} = +\frac{2\times10^7}{(3/2)} x^{3/2} + C_1 \leftarrow (a)$$

again Integrating w.r.t 'x'

$$V = \frac{+2\times10^7}{(3/2)} \frac{x^{3/2+1}}{(3/2+1)} + C_1 x + C_2$$

$$V = +\frac{8\times10^7}{15} x^{3/2+1} + C_1 x + C_2$$

$$\boxed{V = +\frac{8\times10^7}{15} x^{5/2} + C_1 x + C_2}\ \text{volts}$$ (66)

i.e $V = +\frac{8\times10^7}{15} x^{5/2} + C_1 x + C_2$

using Boundary Condition's

i.e @ $x = 0m$ $V = 0$ volt's

$0 = 0 + C_1(0) + C_2 \Rightarrow \boxed{C_2 = 0}$

@ $x = 2.5mm$ $V = 2$ volt's

$2 = \frac{+8\times10^7}{15}(2.5m)^{5/2} + C_1(2.5m) + 0$

$2 = +1.6667 + C_1(2.5m)$

$\Rightarrow C_1(2.5m) = 0.3333$ ⓐ $\boxed{C_1 = 133.332}$

$\therefore \boxed{V(x) = +\frac{8\times10^7}{15} x^{5/2} + 133.332x}$ volt's

V @ $x = 1mm$ is

$V = +\frac{8\times10^7}{15}(1m)^{5/2} + 133.332 \times (1m)$

$V = 0.168654 + 0.133332 = 0.301986$

ⓐ $\boxed{V_{@x=1mm} = 0.302 \text{ Volt's}}$

from eqⁿ ⓐ

and $\bar{E} = -\frac{\partial V}{\partial x}\bar{a_x} = -\left[\frac{4\times10^7}{3}x^{1.5} + 133.332\right]\bar{a_x}$

$\bar{E}\Big|_{x=1mm} = -\left[\frac{4\times10^7}{3}(1m)^{1.5} + 133.332\right]\bar{a_x}$ V/m

$= -554.957\,\bar{a_x}$ V/m

ⓐ $\boxed{\bar{E}\Big|_{x=1mm} \simeq -555\,\bar{a_x}}$ V/m $= E_x \bar{a_x}$ V/m

ⓑ $\therefore \boxed{E_x = -555}$ V/m

xxx. problem 16 A Capacitor has square plates each of side 'a' m. The plates make an angle θ with each other. Show that for small θ, the capacitance is JTJ 2008 (7M).

$$C = \frac{\varepsilon_0 a^2}{d}\left[1 - \frac{a\theta}{2d}\right] \text{ Farad's}$$

Sol:- W.K.T. Capacitance b/w parallel plates

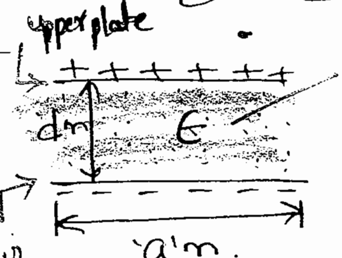
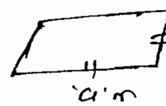


fig: parallel plate capacitor

with $\theta = 0$ $\boxed{C = \frac{\varepsilon A}{d} = \frac{\varepsilon a^2}{d}}$ F

side of each plate is 'a' m



Area $A = a^2\, m^2$.

Note:- for a small angle θ, $\tan\theta \simeq \theta$.
assume capacitor placed along x axis.

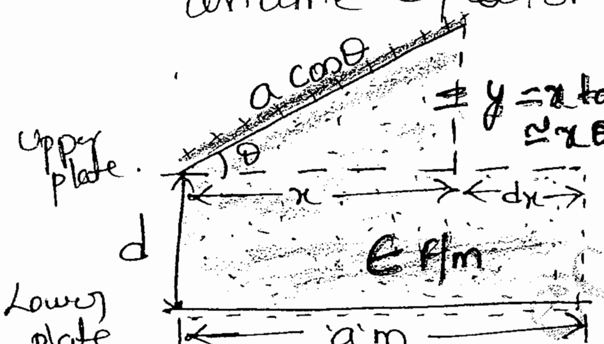


fig. parallel plate capacitor with small 'θ'

The incremental capacitance due to angle 'θ' is given by $dc = \frac{\epsilon a}{d'}dx$.

$$dc = \frac{\epsilon a}{(d + x\theta)}\cdot dx$$

Now the distance b/w the plates

$$\boxed{d' = d + x\theta}$$

the total capacitance

$$C = \int_{x=0}^{x=a} \frac{\varepsilon a}{(d + x\theta)}dx$$

$$\Rightarrow C = \frac{\varepsilon a}{\theta}\int_{x=0}^{a} \frac{1}{(x + d/\theta)}dx$$

$$= \frac{\varepsilon a}{\theta}\log(x + d/\theta)\Big|_0^a$$

$$G = \frac{\varepsilon a}{\theta}\left[\log(a + d/\theta) - \log(d/\theta)\right]$$

$$G = \frac{\varepsilon a}{\theta}\log\left[\frac{a + d/\theta}{d/\theta}\right] = \frac{\varepsilon a}{\theta}\log\left[\frac{a\theta + d}{d}\right]$$

$$\boxed{G = \frac{\varepsilon a}{\theta}\log\left[1 + \frac{a\theta}{d}\right]} \text{ farads} \leftarrow (1)$$

using Taylor's series

$$\log[1 + x] = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

$$\log\left[1 + \frac{a\theta}{d}\right]' = \frac{a\theta}{d} - \frac{(a\theta/d)^2}{2} + \frac{(a\theta/d)^3}{3} - \cdots$$

$$\simeq \left[\frac{a\theta}{d} - \frac{(a\theta/d)^2}{2}\right] \leftarrow (2)$$

eq (2) in eq (1)

$$G = \frac{\varepsilon a}{\theta}\left[\frac{a\theta}{d} - \frac{(a\theta/d)^2}{2}\right]$$

$$G = \frac{\varepsilon a}{\cancel{\theta}} \cdot \frac{a\cancel{\theta}}{d}\left[1 - \frac{a\theta}{2d}\right]$$

$$G = \frac{\varepsilon a^2}{d}\left[1 - \frac{a\theta}{2d}\right] \text{ farads}$$

$$\therefore \boxed{G = \frac{\varepsilon a^2}{d}\left[1 - \frac{a\theta}{2d}\right]} \text{ farads}$$

problem 17

8) A parallel plate Capacitor is filled with a dielectric of 0·03 power factor and $\epsilon_r = 10$. The plates have an area of $250 mm^2$ and the distance b/w them is 10mm. If 5000V (rms) at 1 MHz is applied to the capacitor find the power dissipated as ~~heat~~. (10M). 10-J/J 2015.

Solu:- given $pf = \cos\phi = 0{\cdot}03$

$\epsilon_r = 10$ F/m, Area(A) = $250 mm^2$

$= 250 \times (10^{-3})^2 m^2$

$A = 250\mu\ m^2$

$d = 10mm = 10 \times 10^{-3} m$

$V_{rms} = 5000$ volts $f = 1MHz$.

the Capacitance 'C' of a parallel plate is

$$C = \frac{\epsilon A}{d} = \frac{\epsilon_0 \epsilon_r A}{d}$$

$$C = \frac{8{\cdot}854 \times 10^{-12} \times 10 \times 250 \times 10^{-6}}{10 \times 10^{-3}}$$

$$C = 2{\cdot}2135 \times 10^{-3} F$$

$$\boxed{C = 2{\cdot}2135\ pF}$$

the power dissipation in a AC ckt is given by

$$P = V_{rms} I_{rms} \cos\phi \text{ Watt's}$$

$$P = V_{rms} \frac{V_{rms}}{Z} \cos\phi$$

$$\boxed{P = \frac{V_{rms}^2}{Z} \cos\phi} \text{ Watt's}$$

$Z = R - jX_C = -jX_C$ ($R = 0\Omega$)

$C = C$ $= |X_C| \angle -90^\circ$

(a) $P = \frac{V_{rms}^2}{|X_C|} \cos\phi$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 1 \times 10^6 \times 2{\cdot}2135 \times 10^{-12}}$$

$$\boxed{Z = X_C = 71{\cdot}90193\ k}\ \Omega$$

∴ power dissipated

$$P = \frac{(5000)^2}{71{\cdot}9019 \times 10^3} (0{\cdot}03)$$

$$\boxed{P = 10{\cdot}43087} \text{ Watt's}$$

problem 18

~~Miscellaneous Topics [out of syllabus]~~

~~Applications of Poisson's Equation~~

06 - June /July 2011

71 Using Poisson's equation obtain the expression for the junction potential in a P – n junction. (08 Marks)

10 - June /July 2016

~~72 c. Using Poisson's equation obtain the expression for the potential in a p-n junction. (06 Marks)~~

Solu:- Consider a pn junction which is placed along x-axis.

V_S ρ_v c/m3 V_j - Junction potential.

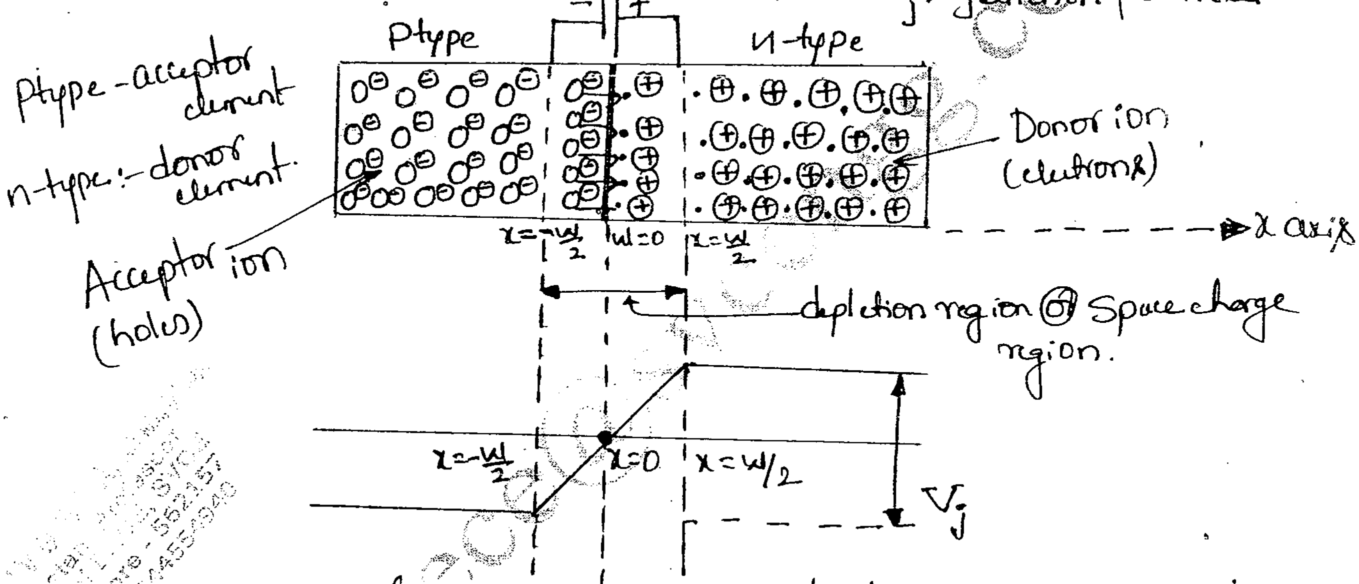


fig. pN junction diode.

the Barrier potential (or) junction potential $\boxed{V_j = -\int E\,dx}$ volts

* Consider the Concentration of holes in p-Section and electrons in n-Section. this concentration is uniform. i.e the charge density ρ_v C/m 3 is Constant almost entirely over the respective Sections.
* But in depletion region charge concentration is Subjected to Variation.

* Let us Consider the width of the depletion region to be 'w'.

Boundary Conditions :-

from fig. @ w = 0 = x ; $V_j = 0$ volts

@ $x = \pm w/2$; V_j = Constant.

$\therefore \frac{\partial V}{\partial x} = 0$ @ $x = w/2$.

the junction potential i) V_j = ? and ii) Electric field across the junction E = ?

Let potential $V_j = V_1$ @ $x = +w/2$

and $V_j = V_2$ @ $x = -w/2$

$\therefore$ the junction potential

$V_j = \begin{pmatrix}\text{potential} \\ @\ x = w/2\end{pmatrix} - \begin{pmatrix}\text{potential} \\ @\ x = -w/2\end{pmatrix}$.

i.e $\boxed{V_j = V_1 - V_2}$

using poisson's equation

$\nabla^2 V = -\rho_v/\epsilon$ V/m²

Since p-n junction placed along x-axis $\therefore \nabla^2 V = \frac{\partial^2 V}{\partial x^2}$

$\Rightarrow \frac{\partial^2 V}{\partial x^2} = -\rho_v/\epsilon$

Integrating w.r.t 'x'

$\frac{\partial V}{\partial x} = -\frac{\rho_v}{\epsilon} x + C_1 \leftarrow (1)$

using BC, ie $\frac{\partial V}{\partial x} = 0$ @ $x = w/2$. 72

$$0 = -\frac{\rho_v}{\epsilon}\frac{w}{2} + C_1$$

$$\Rightarrow \boxed{C_1 = \frac{\rho_v w}{2\epsilon}} \leftarrow (2)$$

again integrating eqⁿ (1) w.r.t x

$$V = -\frac{\rho_v}{\epsilon}\frac{x^2}{2} + C_1 x + C_2$$

$$V = \frac{-\rho_v x^2}{2\epsilon} + C_1 x + C_2$$

@ $x = w = 0 \Rightarrow \boxed{V=0}$ volt's

$$0 + 0 + 0 + C_2$$

$$\Rightarrow \boxed{C_2 = 0}$$

$$\therefore V = -\frac{\rho_v x^2}{2\epsilon} + \frac{\rho_v w}{2\epsilon}x$$

$$\boxed{V = \frac{\rho_v w}{2\epsilon}x - \frac{\rho_v}{2\epsilon}x^2} \text{ Volt's}$$

@ $x = \frac{w}{2}$; $v = v_1$

$$\therefore v_1 = \frac{\rho_v w}{2\epsilon}\frac{w}{2} - \frac{\rho_v}{2\epsilon}\frac{w^2}{4}$$

$$V_1 = \frac{\rho_v}{4\epsilon}w^2 - \frac{\rho_v}{8\epsilon}w^2$$

$$\boxed{V_1 = \frac{\rho_v}{8\epsilon}w^2} \text{ volt's.}$$

@ $x = -w/2$; $v = v_2$

$$V_2 = \frac{\rho_v(w)}{2\epsilon}\left(\frac{-w}{2}\right) - \frac{\rho_v}{2\epsilon}\frac{w^2}{4}$$

$$= \frac{-\rho_v w^2}{4\epsilon} - \frac{\rho_v}{8\epsilon}w^2$$

$$\boxed{V_2 = -\frac{3}{8}\frac{\rho_v}{\epsilon}w^2} \text{ volt's}$$

a) i) $\therefore$ the junction pontential V_j

@ $V_j = V_1 - V_2$

$$= \frac{\rho_v}{8\epsilon}w^2 + \frac{3}{8}\frac{\rho_v}{\epsilon}w^2$$

$$\boxed{V_j = \frac{\rho_v w^2}{2\epsilon}} \text{ volt's}$$

ii) the Electric field across the junction

$$\bar{E} = -\nabla V = -\frac{\partial V}{\partial x}\bar{a}_x \text{ V/m.}$$

using eqⁿ (1) and (2)

$$\bar{E} = -\left[\frac{-\rho_v x}{\epsilon} + C_1\right]\bar{a}_x \text{ V/m}$$

$$\bar{E} = \left(\frac{\rho_v x}{\epsilon} - \frac{\rho_v w}{2\epsilon}\right)\bar{a}_x \text{ V/m}$$

$$\boxed{\bar{E} = \frac{\rho_v}{\epsilon}(x - w/2)\,\bar{a}_x} \text{ V/m}$$

Summary:

(i) Junction potential

$$\boxed{V_j = \frac{\rho_v w^2}{2\epsilon}} \text{ volt's}$$

(ii) Electric field Intensity ($\bar{E}$)

$$\boxed{\bar{E} = \frac{\rho_v}{\epsilon}\left(x - \frac{w}{2}\right)\bar{a}_x} \text{ V/m.}$$

problem 19

10-Jan 2013

- A large spherical cloud of radius 'b' has a uniform volume charge distribution of ρ_v c/m³. find the potential distribution and electric field intensity at any point in space using Laplace. (10 Marks)

ρ_v C/m³

Solu: assume Boundary Cond^n's

as $r \to \infty$; $\boxed{V = 0}$

and $r \to 0$; $r^2 E_r \to 0$

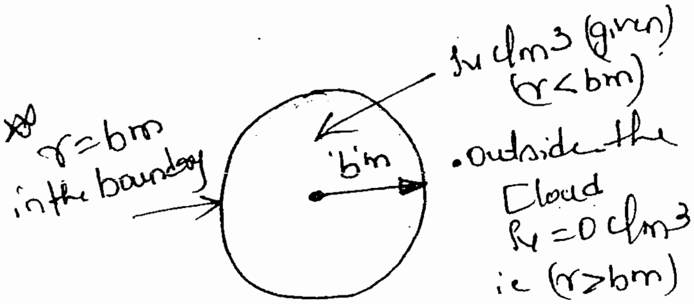


fig. Spherical cloud of radius 'b' m.

→ for $r<b$m use $\nabla^2 V = -\rho_v/\epsilon_0$ bcz $\rho_v \neq 0$: Poisson's eq^n

→ for $r>b$m use $\nabla^2 V = 0$ bcz $\rho_v = 0$ ie Laplace's eq^n.

Case ii. the potential V_0 outside the Cloud (ie $r>b$m).

$$\nabla^2 V_0 = 0 \quad V/m^2$$

Since V_0 f^n(r) only

$$\therefore \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{\partial V_0}{\partial r}\right] = 0.$$

$r \neq 0 \therefore \frac{1}{r^2} \neq 0$

$$\frac{\partial}{\partial r}\left[r^2 \frac{\partial V_0}{\partial r}\right] = 0$$

Integrating w.r.t 'r'

$$r^2 \frac{\partial V_0}{\partial r} = C_1$$

$$\frac{\partial V_0}{\partial r} = \frac{C_1}{r^2}$$

Integrating w.r.t 'r'

$$\boxed{V_0 = \frac{-C_1}{r} + C_2} \text{ volt's}$$

using Boundary condition's

BC1. as $r \to \infty$ $V_0 \to 0$

$\therefore 0 = 0 + C_2 \Rightarrow \boxed{C_2 = 0}$

$$\therefore \boxed{V_0 = -C_1/r} \text{ volt's} \leftarrow (1)$$

$$\bar{E}_0 = -\nabla V_0 = -\frac{\partial V_0}{\partial r}\bar{a}_r \text{ V/m}$$

$$\boxed{\bar{E}_0 = -\frac{C_1}{r^2}\bar{a}_r} \text{ V/m}$$

Case i. potential and field inside the Cloud ie ($r<b$m).

$$\nabla^2 V_i = -\rho_v/\epsilon_0 \quad V/m^2$$

Since V_i f^n(r) only

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{\partial V_i}{\partial r}\right] = -\rho_v/\epsilon_0$$

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$$\frac{\partial}{\partial r}\left[r^2 \frac{\partial V_i}{\partial r}\right] = -\frac{\rho_v}{\epsilon_0} r^2$$

Integrating w.r.t r

$$r^2 \frac{\partial V_i}{\partial r} = \frac{-\rho_v}{\epsilon_0}\frac{r^3}{3} + C_3$$

$$\boxed{\frac{\partial V_i}{\partial r} = -\frac{\rho_v}{\epsilon_0}\frac{r}{3} + C_3 r^{-2}} \text{ V/m} \quad \leftarrow (a)$$

the Field $\bar{E}_i = -\nabla V_i$ V/m

$$\bar{E}_i = -\frac{\partial V_i}{\partial r}\bar{a}_r \text{ V/m}$$

$$\bar{E}_i = \left(\frac{\rho_v r}{3\epsilon_0} - \frac{C_3}{r^2}\right)\bar{a}_r \text{ V/m}$$

$$\bar{E}_i = E_r \bar{a}_r$$

$$\therefore E_r = \frac{\rho_v r}{3\epsilon_0} - \frac{C_3}{r^2}$$

$$r^2 E_r = \frac{\rho_v r^3}{3\epsilon_0} - C_3$$

as $r \to 0$; $r^2 E_r \to 0$

$$\therefore 0 = 0 - C_3$$

$$\Rightarrow \boxed{C_3 = 0}$$

$$\therefore \boxed{\bar{E}_i = \frac{\rho_v r}{3\epsilon_0}\bar{a}_r} \text{ V/m}$$

$\therefore$ eq^n (a) becomes

$$\boxed{\frac{\partial V_i}{\partial r} = -\frac{\rho_v r}{3\epsilon_0}} \text{ V/m} \quad \leftarrow (b)$$

@ Boundary of interface

i.e $r = b$m

the potential $V_i = V_0$ volts and Normal components of $\bar{D}$ are equal

i.e $\bar{D}_i = \bar{D}_0$ @ $r = b$m

$$\cancel{\epsilon_0}\bar{E}_i = \cancel{\epsilon_0}\bar{E}_0$$

$$\frac{\rho_v b}{3\epsilon_0} = -C_1/b^2$$

$$\Rightarrow \boxed{C_1 = \frac{-\rho_v b^3}{3\epsilon_0}}$$

$\therefore$ eq^n (1) becomes

$$V_0 = -\frac{C_1}{r} \text{ volts}$$

$$V_0 = +\frac{\rho_v b^3}{3\epsilon_0 r}$$

@ $r = b$m

$$V_0 = \frac{\rho_v b^3}{3\epsilon_0 b} = \frac{\rho_v b^2}{3\epsilon_0}$$

$$\therefore \boxed{V_0 = \frac{\rho_v b^2}{3\epsilon_0}} \text{ volts} \quad \leftarrow (c)$$

and from eqn (6)

$$\frac{\partial V_i}{\partial r} = \frac{-\rho_v r}{3\epsilon_0}$$

Integrating w.r.t 'r'

$$V_i = \frac{-\rho_v}{3\epsilon_0}\frac{r^2}{2} + C_4$$

$$V_i = \frac{-\rho_v}{6\epsilon_0} r^2 + C_4$$

@ Boundary $V_i = V_0$

i.e $r = b$ m.

$$\frac{-\rho_v r^2}{6\epsilon_0} + C_4 = \frac{\rho_v b^2}{3\epsilon_0}$$

$$C_4 = \frac{\rho_v b^2}{3\epsilon_0} + \frac{\rho_v b^2}{6\epsilon_0}$$

$$\Rightarrow \boxed{C_4 = \frac{\rho_v b^2}{2\epsilon_0}}$$

$$\therefore \boxed{V_i = \frac{-\rho_v r^2}{6\epsilon_0} + \frac{\rho_v b^2}{2\epsilon_0}}$$

Summary:-

I.

$$\boxed{V_i = \frac{\rho_v}{2\epsilon_0}\left[b^2 - r^2/3\right] \qquad \bar{E}_i = \frac{\rho_v r}{3\epsilon_0}\,\bar{a}_r \text{ V/m}}$$

potential and field inside the Cloud i.e $r < b$ m.

II.

$$\boxed{V_0 = \frac{\rho_v b^2}{3\epsilon_0}} \text{ volts}$$

$$\bar{E}_0 = \frac{-C_1}{r^2}\,\bar{a}_r \text{ V/m}$$

i.e $\boxed{\bar{E}_0 = +\frac{\rho_v b^3}{3\epsilon_0 r^2}\,\bar{a}_r}$ V/m

potential and field outside the Cloud i.e $r > b$ m.

III @ the Boundary i.e $r = b$ m

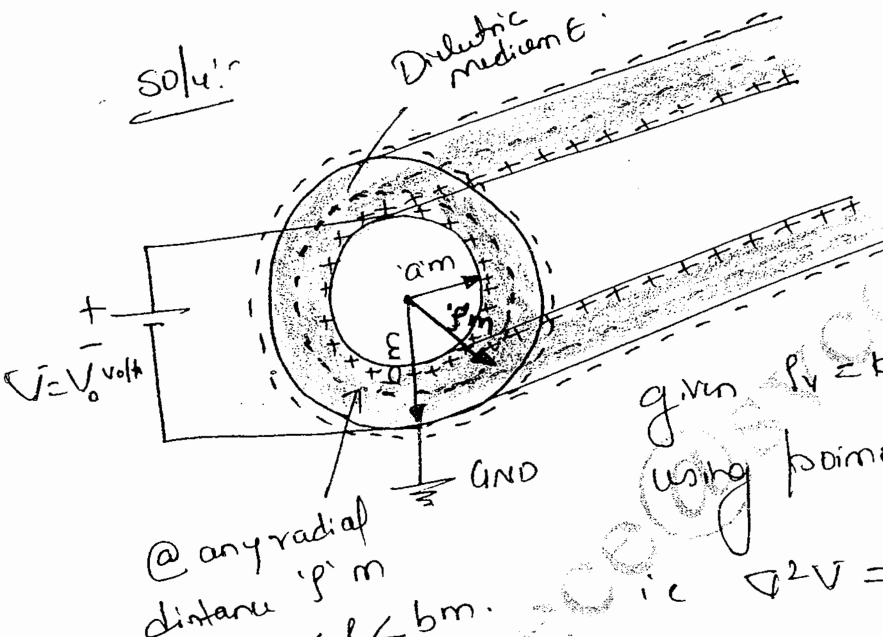
$$\boxed{V_i = V_0 \text{ and } \bar{E}_i = \bar{E}_0}$$

ii) Problem 2.10

a) The annular Space b/w inner and outer Conductors of a Long Co-axial Cylindrical structure is filled with an electron cloud having a volume charge density $\rho_v = K/\rho$ C/m3 for $a<\rho<b$, where 'a' and 'b' are radii of inner and outer Conductors respectively. assume that, the inner Conductor is maintained at a potential V_0 and the outer Conductor is grounded. Determine the potential distribution in the region $a<\rho<b$.

Dec/Jan 2009 (10M).

solu:



given $\rho_v = K/\rho$ C/m3 for $a<\rho<b$ m.

using poisson's eqn (bcz $\rho_v \neq 0$).

i.e $\nabla^2 V = -\rho_v/\epsilon$ V/m2

Since V is fn of radial component i.e $V = f^n(\rho)$ only

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho \frac{\partial V}{\partial \rho}\right] + \cancel{\frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2}}^{0} + \cancel{\frac{\partial^2 V}{\partial z^2}}^{0}$$

$$\therefore \nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho \frac{\partial V}{\partial \rho}\right] = -\rho_v/\epsilon \text{ V/m}^2.$$

given $\rho_v = K/\rho$ C/m3

$$\therefore \cancel{\frac{1}{\rho}}\frac{\partial}{\partial \rho}\left[\rho \frac{\partial V}{\partial \rho}\right] = -\frac{K}{\cancel{\rho}}\cdot \frac{1}{\epsilon}$$

$$\Rightarrow \frac{\partial}{\partial \rho}\left[\rho \frac{\partial V}{\partial \rho}\right] = -k/\epsilon$$

Integrating w.r.t 'ρ'

$$\rho \frac{\partial V}{\partial \rho} = -k/\epsilon \, \rho + C_1$$

$$\frac{\partial V}{\partial \rho} = -k/\epsilon + C_1/\rho$$

again Integrating w.r.t. 'ρ'

$$\boxed{V = -k/\epsilon \, \rho + C_1 \ln \rho + C_2} \leftarrow (1)$$

using BC's

i.e @ BC₁

1. @ $\rho = a$ m ; $V = V_0$ volt's
2. @ $\rho = b$ m ; $V = 0$ volt's

BC₂ $0 = -k/\epsilon \, b + C_1 \ln b + C_2$

$$C_1 \ln b + C_2 = \frac{kb}{\epsilon} \leftarrow (2)$$

BC₁ using $\rho = a$ m ; $V = V_0$

$$V_0 = -\frac{k}{\epsilon} a + C_1 \ln a + C_2 \leftarrow (3)$$

solving eq^n (2) and eq^n (3)

eq^n (2) − eq^n (3)

$$C_1 \ln(b/a) = \frac{k}{\epsilon}(b-a) - V_0$$

$$C_1 = \left[\frac{k}{\epsilon}(b-a) - V_0\right] \Big/ \ln(b/a) \leftarrow (4)$$

and

$$C_2 = \frac{kb}{\epsilon} - C_1 \ln b$$

$$C_2 = \frac{V_0 \ln b + \frac{k}{\epsilon}[a \ln b - b \ln a]}{\ln(b/a)} \leftarrow (5)$$

using eq^n (4) and (5) in eq^n (1)

$$\boxed{V = -\frac{k}{\epsilon}\rho + \left[\frac{\frac{k}{\epsilon}(b-a) - V_0}{\ln(b/a)}\right]\ln(\rho) + \frac{V_0 \ln b + \frac{k}{\epsilon}[a \ln b - b \ln a]}{\ln(b/a)}}$$

volt's

potential distribution b/w $a < \rho < b$.

valid $a < \rho < b$.

ie

$$V = -\frac{k}{\epsilon}\rho + \left[\frac{\frac{k}{\epsilon}(b-a) - V_0}{\ln(b/a)}\right]\ln(\rho) + \frac{V_0 \ln(b) + \frac{k}{\epsilon}[a \ln b - b \ln a]}{\ln(b/a)} \text{ volts}$$

Problem 21

~~Q320~~. The annular space b/w inner and outer conductors of long co-axial cylindrical structure is filled with a an electron cloud having a volume charge density $\rho_v = 1/\rho$ for $a < \rho < b$ m, where 'a' and 'b' are radii of inner and outer conductors respectively. assume that the inner conductor is maintained at a potential of V_0 volts and outer conductor is grounded. Determine the potential distribution in the region $a < \rho < b$ m.

Soln:- given $\rho_v = 1/\rho \ C/m^3 \ ; a < \rho < b$ m.

Note:- put $K = 1$ in the previous problem 20 i.e

~~Q2. page No. 334~~ $\Rightarrow \boxed{Q_1 = Q_2}$

Ans. the potential distribution b/w $a < \rho < b$ m

is

$$\boxed{V = -\frac{1}{\epsilon}\rho + \left[\frac{\frac{1}{\epsilon}(b-a) - V_0}{\ln(b/a)}\right]\ln(\rho) + \left[\frac{V_0 \ln(b) + \frac{1}{\epsilon}\left[a\ln(b) - b\ln(a)\right]}{\ln(b/a)}\right]} \text{ volts}$$

Module 3 Part A

problem 1. Determine whether (or) not the potential equation's satisfying Laplace's equation.

i) $V = 2x^2 - 4y^2 + z^2$.

ii) $V = r^2 \cos\phi + \theta$.

iii) $V = 20x^2yz + 10xy^2z^2$.

iv) $V = 15x^2 + 10y^2 - 25z^2$.

v) $V = 2x^2 - 3y^2 + z^2$

vi) $V = r\cos\phi + z$.

vii) $V = x^2 + y^2 + z^2$

viii) $V = r\cos\theta + \phi$

ix) $V = \rho^2 + z^2$.

problem 2. Calculate numerical values for V and ρ_v at point P in free space if:

a) $V = \frac{4yz}{x^2+1}$ at $P(1, 2, 3)$

b) $V = 5\rho^2 \cos(2\phi)$ at $P(\rho = 3, \phi = \frac{\pi}{3}, z = 2)$.

c) $V = \frac{2\cos\phi}{r^2}$ at $P(r = 0.5, \theta = 45°, \phi = 60°)$

problem 9

Two parallel conducting discs are separated by distance 5mm at $z=0$ and $z=5mm$. $V=0$ volts at $z=0$; and $V=100$ volts at $z=5mm$ and it is only in z direction. Starting from Laplace equation find surface charge densities on the discs [take $\epsilon = \epsilon_0 = 8.854 \times 10^{-12}$ F/m].

problem 10

Long concentric and right conducting cylinders in free space at $r=5mm$ and $r=25mm$ in cylindrical co-ordinates have voltages of zero and V_0 respectively. if the electric field intensity $\bar{E} = -8.28 \times 10^3 \, \bar{a}_r$ V/m at $r=15mm$, starting from Laplace equation find V_0 and charge density on the outer conductor [Take $\epsilon = \epsilon_0 = 8.854 \times 10^{-12}$ F/m] F/m..

problem 11 Find $|E|$ at $P(3,1,2)$ for the field of

a) two coaxial conducting cylinders, $V=50V$ at $\rho = 2m$ and $V=20V$ at $\rho = 3m$.

b) two radial conducting planes, $V=50V$ at $\phi = 10°$ and $V=20V$ at $\phi = 30°$.

problem 12.

Conducting spherical shells with radii a = 10cm and b = 30cm are maintained at a potential difference of 100V such that V = 0 at r = b and V = 100V at r = a. Determine V and E in the region between the shells, if $\epsilon_r = 2.5$ in the region, determine the total charge induced on the shells and the capacitance there on.

problem 13.

A spherical capacitor has a capacitance of 54 pF. it consist of two concentric spheres with inner and outer radii differing by 4 cm. Dielectric in between is air, Determine inner and outer radii.

Module -3 [Summary]
(part-A)

List of formulaes :-

1. poisson's Equation

$$\boxed{\nabla^2 V = -\rho_v/\epsilon} \quad V/m^2$$

2. Laplaces equation

$$\boxed{\nabla^2 V = 0} \quad V/m^2.$$

3. Laplace's Equation in all three co-ordinate system.

a. Cartesian Co-ordinate system. $P(x, y, z)$ dx dy dz

$$\boxed{\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0}$$

b. Cylindrical Co-ordinate System $P(\rho, \phi, z)$ $d\rho$ $\rho d\phi$ dz

$$dv = \rho\, d\rho\, d\phi\, dz$$

$$\boxed{\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho \frac{\partial V}{\partial \rho}\right] + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0} \quad V/m^2$$

c. Spherical Co-ordinate System $P(r, \theta, \phi)$

dr $\quad r d\theta$ $\quad r\sin\theta\, d\phi$

$$dv = r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$\boxed{\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial V}{\partial\theta}\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2} = 0} \quad V/m^2$$

4. <u>Uniqueness theorem</u>

"Any solution of Laplace equation that satisfies the same boundary conditions must be the only solution regardless of the method used."

i.e $\nabla^2 V = 0$

$\Rightarrow \boxed{V_1 = V_2}$

* Poisson's equation $\nabla^2 V = -\rho_v/\epsilon$ V/m^2 is used to find $V, \bar{E}, \bar{D}$, $\rho_s = |\bar{D}|$ and capacitance (C), C/L, total charge (Q) etc, within a region where $\rho_v \neq 0$.

* lly the Laplace's equation $\nabla^2 V = 0$ V/m² is used to find $V \rightarrow \bar{E} \rightarrow \bar{D} \rightarrow |\bar{D}| = \rho_s$ C/m² $\rightarrow Q = \rho_s \cdot A$ $\rightarrow C \rightarrow C/L$ etc. within a region where $\rho_v = 0$ [i.e charge free region].

<u>Note:-</u> for a charged free region $\boxed{\rho_v = 0}$ C/m³

* Applications of Laplace equation.

* Capacitance of a parallel plate capacitor

$$\boxed{C = \frac{\epsilon A}{d}} \text{ Farads.}$$

* Capacitance of a Co-axial cable using Laplace's equation.

$$\boxed{C = \frac{2\pi\epsilon L}{\ln(b/a)}} \text{ F (or) } \boxed{\frac{C}{L} = \frac{2\pi\epsilon}{\ln(b/a)}} \text{ F/m}$$

$$\boxed{|\bar{D}| = \rho_s = \epsilon|\bar{E}|} \text{ C/m}^2$$

* Capacitance of a Concentric spheres :-

$$\boxed{C = \frac{4\pi\epsilon L}{\left[\frac{1}{a} - \frac{1}{b}\right]}} \text{ F ; } \boxed{\frac{C}{L} = \frac{4\pi\epsilon}{\left[\frac{1}{a} - \frac{1}{b}\right]}} \text{ F/m}$$

where. $b > a$

* Capacitance of a Isolated Sphere of radius 'a' meter.

$$\boxed{C = 4\pi \epsilon a} \text{ Farads.}$$

6. Procedure to solve Poisson's or Laplace's equation :-

using $\nabla^2 V = 0$

or $\nabla^2 V = -\rho_v/\epsilon$

↓

use boundary conditions — Solve for potential (V)

↓

find $\boxed{\bar{E} = -\nabla V}$ V/m

↓

$\boxed{\bar{D} = \epsilon \bar{E}}$ C/m²

↓ use

$\boxed{|\bar{D}| = \rho_s}$ C/m²

↓

$\boxed{Q = \rho_s \cdot A}$ Coulomb's

↓

$Q = CV$

or $\boxed{C = \frac{Q}{V}}$ Farads.

7. if the given vector ($\bar{E}$) represent a possible electric field only when $\boxed{\nabla^2 V \neq 0}$.

i.e given field should not be arise from charged free region. then $\bar{E}$ is a possible representation of electric field.

Procedure. given ($\bar{E}$) $\xrightarrow{\text{use}}$ $\bar{E} = -\nabla V$ V/m

↓ find

$\nabla^2 V = -\nabla \cdot \bar{E}$

↓ check ?

$\nabla^2 V = 0$

(or) $\nabla^2 V \neq 0$

* if $\nabla^2 V = 0$; then given field is not a possible electric field.

* if $\nabla^2 V \neq 0$; then given field is possible representation of electric field.

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Module -3 (Part-B)

Part-B : Steady Magnetic Field
Biot-Savart Law, Ampere's circuital law, Curl, Stokes' theorem, Magnetic flux and magnetic flux density, Scalar and Vector Magnetic Potentials.

Topics:
3.5 Biot-Savart Law
    Applications of Biot-Savart Law
    a. Magnetic Field Intensity due to Infinite Long Straight Filament
    b. Magnetic Field Intensity due to finite length Filament
    c. Magnetic Field Intensity on the axis of a Circular Loop.
    d. Magnetic Field Intensity at a point on the axis of a solenoid.
    e. Magnetic Field Intensity at center of a square current loop.
3.6 Ampere's circuital law
3.7 Applications of Ampere's Circuital Law
    a. Magnetic Field Intensity due to Infinite Long Straight Filament
    b. Magnetic Field Intensity of a Co-axial cable
    c. Magnetic Field Intensity of a Toroidal coil
3.8 Concept of Curl
    a. Point form of Ampere's Law
    b. Curl in all three co-ordinate systems
3.9 Stokes' theorem
3.10 Magnetic flux and magnetic flux density
3.11 Scalar and Vector Magnetic Potentials
Summary
- List of Symbols
- List of Formulae

Module - 3 (Part B)

Steady Magnetic Field.

Introduction :-

The source for electric field is charge. Similarly In addition to the electric field magnetic field is also present in the medium but the source for magnetic field is

a. permanent magnet.

b. Electric field changing with time.

[modified Ampere's Law $\nabla \times \bar{H} = \bar{J}_c + \frac{\partial D}{\partial t}$ A/m^2]

c. dc Current.

In this module we will discuss only magnetic field due to dc current carrying filament.

dc current carrying conductor result's steady magnetic field. Steady means constant (or) not changing with time.

Topic 3.5. Biot - Savart Law.

~~Topics:~~
~~1. Biot-Savart Law~~
~~1.1 Applications of Biot-Savarts Law~~
- ~~Magnetic Field Intensity due to Infinite Long Straight Filament~~
- ~~Magnetic Field Intensity due to finite length Filament~~
- ~~Magnetic Field Intensity on the axis of a Circular Loop.~~

~~**1.1Biot-Savart Law**~~ ~~02-DEC2008/Jan 2009~~

1. ~~a. **State and explain Biot-Savart's law.**~~ ~~**(04 Marks)**~~
2. ~~06-DEC2011/Jan 2012~~
   ~~State and explain Biot Savart law.~~ ~~(04 Marks)~~
3. ~~10 -Jan 2013~~
   ~~State and explain Biot Savart law.~~ ~~(06 Marks)~~
4. ~~06-DEC 2013/Jan 2014~~
   ~~State and explain Biot-Savart law.~~ ~~(06 Marks)~~
5. ~~10-DEC 2013/Jan 2014~~
   ~~State and explain Biot-Savart law.~~ ~~(06 Marks)~~
6. ~~02 - June /July 2010~~
   ~~State and explain vector form of Biot-Savart law. Explain units of all physical quantities involved.~~ ~~(06M)~~
7. ~~06 – May/June 2010~~
   ~~State and explain Biot-Savart law for a small differential current element.~~ ~~(04 Marks)~~

Question.

State and explain Biot-Savart Law. (4M)

(or)

State and explain, Vector form of Biot-Savart Law. Explain units of all physical quantities involved (6M)

(or)

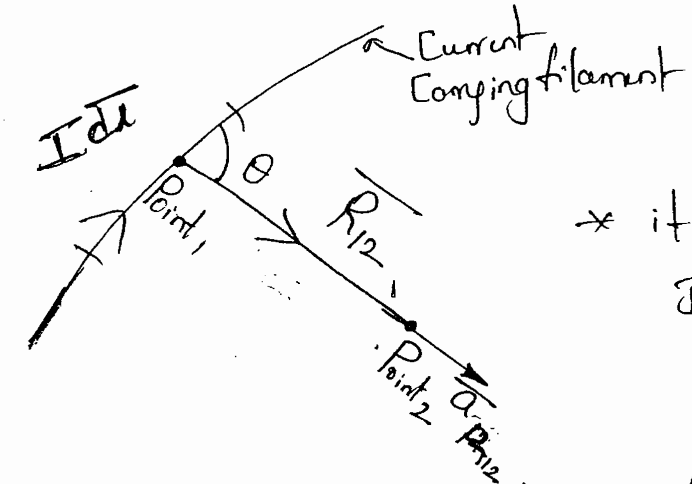
State and explain Biot-Savart Law for a Small differential Current Element. (4M)

[02-Jan 2009, 06-Jan 2012, 10-Jan 2013, 06-Jan 2014, 02-June/July 2010, 06-June 2010].

[15-June/July 2017 (4M) CBCS]

2

3.5. Biot - Savart Law :-



* This Law is also Called as Ampere's Law for the Current element.

* it gives differential magnetic field Intensity (dH) due to differential Current element.

* Consider a Filament through which Current of I amp is passing. to find Magnetic field intensity at point P_2. Consider a Small Section of filament of Length dl, the differential Current element is $I\overline{dl}$.

Statement :- Magnitude of dH at point P_2 is proportional to a) product of Current & differential Length dl. b) the Sine of the angle b/w the filament and Line connecting differential Length to the point of intrest P_2. c) And it is inversely proportional to the Square of the distance from filament to point P.

a. $dH \propto Idl$ b. $dH \propto \sin\theta$ c. $dH \propto \frac{1}{R_{12}^2}$

$\Rightarrow$ Combindly i.e $$dH \propto \frac{Idl \sin\theta}{R_{12}^2}$$

the Constant of proportionality is $1/4\pi$

$$\therefore \boxed{dH = \frac{Idl \sin\theta}{4\pi R_{12}^2}} \quad A/m \text{ or } N/wb.$$

The direction of dH is normal to the plane containing the differential element and the line drawn from the filament to the point P.

In Vector notation the differential Mag field at point P_2.

$$\boxed{d\bar{H}_2 = \frac{I\,\bar{dl} \times \bar{a}_{R_{12}}}{4\pi R_{12}^2}} \quad A/m$$

Where $\times$ indicates Cross product operation.

$\bar{a}_{R_{12}}$ - unit vector from differential current element to point P.

$I\,\bar{dl}$ - differential current element.

R_{12} - distance of differential current element from point P_2.

$$\bar{a}_{R_{12}} = \frac{\bar{R}_{12}}{|\bar{R}_{12}|}$$

$$\boxed{d\bar{H}_2 = \frac{I\,\bar{dl} \times \bar{R}_{12}}{4\pi R_{12}^3}} \quad A/m$$

The Integral form of Biot-Savart Law (ie the net field at point P_2) is

$$\boxed{\bar{H}_2 = \oint_{\langle l \rangle} \frac{I\,\bar{dl} \times \bar{R}_{12}}{4\pi R_{12}^3}} \quad A/m.$$

List the applications of Biot-Savart's Law. Explain any one with necessary mathematical representations. 10 - June /July 2014

List the applications of Biot-Savarts law. Explain any one with necessary mathematical representations. (06 Marks)

Applications of Biot-Savart's Law ... used to find

i. Magnetic field intensity at a point due to Infinite Long straight current carrying filament.

ii. Magnetic field intensity at a point due to finite length current carrying filament.

iii. Magnetic field intensity at a point due to axis of a circular current carrying loop.

iv. Magnetic field intensity at a point on the axis of a solenoid.

v. used to find $\vec{H}$ or $\vec{B}$ at center of current carrying square loop.

Note:- Derive any one application using Biot-savart's Law.

problem 1

02-DEC2008/Jan 2009

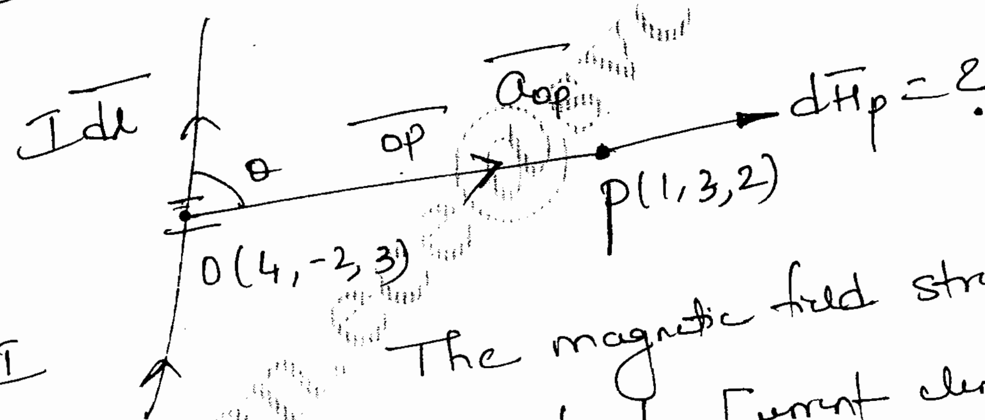
Find the magnetic field strength at the point (1, 3, 2) caused by a current element $2\pi(0.6u_x - 0.8u_y)\mu$ A/m situated at (4, -2, 3). **(04 Marks)**

Question

Find the Magnetic field Strength at the point P(1, 3, 2) Caused by a Current element a Current element $2\pi(0.6\bar{a}_x - 0.8\bar{a}_y)\,\mu A\text{-}m$ Situated at (4, −2, 3). (4M)

Solu:

$$I\bar{dl} = 2\pi(0.6\bar{a}_x - 0.8\bar{a}_y)\,\mu Am$$



The magnetic field strength at point P due to Current element is given by

$$d\bar{H}_p = \frac{I\bar{dl} \times \bar{a}_{op}}{4\pi|\bar{op}|^2}\ A/m.$$

$$d\bar{H}_p = \frac{I\bar{dl} \times \bar{op}}{4\pi|\bar{op}|^3}\ A/m$$

$$\overline{OP} = (1-4)\,\overline{a_x} + (3+2)\,\overline{a_y} + (2-3)\,\overline{a_z}$$

$$\boxed{\overline{OP} = -3\,\overline{a_x} + 5\,\overline{a_y} - \overline{a_z}}$$

$$|\overline{OP}| = \sqrt{9+25+1} = \sqrt{35}\ \text{m.}$$

$$d\overline{H} = \frac{I\overline{dl} \times \overline{OP}}{4\pi\left(\sqrt{35}\right)^3}\ \text{A/m}$$

$$I\overline{dl} = 2\pi(0{\cdot}6)\,\overline{a_x} - 2\pi(0{\cdot}8)\,\overline{a_y}\ \mu\text{Am}$$

$$\boxed{I\overline{dl} = 1{\cdot}2\pi\,\overline{a_x} - 1{\cdot}6\pi\,\overline{a_y}}\ \mu\text{A-m.}$$

$$I\overline{dl} \times \overline{OP} = \begin{vmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ 1{\cdot}2\pi\mu & -1{\cdot}6\pi\mu & 0 \\ -3 & 5 & -1 \end{vmatrix}$$

$$= \left[+1{\cdot}6\pi\mu - 0\right]\overline{a_x} - \left[-1{\cdot}2\pi\mu - 0\right]\overline{a_y} + \left[6\pi\mu - 4{\cdot}8\pi\mu\right]\overline{a_z}$$

$$I\,\overline{dl} \times \overline{OP} = 1.6\pi\,\overline{a_x} + 1.2\pi\,\overline{a_y} + 1.2\pi\,\overline{a_z}\ ;\ \mu Am^2$$

$$d\overline{H_p} = \frac{I\,\overline{dl} \times \overline{OP}}{4\pi\,|\overline{OP}|^3}$$

$$d\overline{H_p} = \frac{1.6\pi\,\overline{a_x} + 1.2\pi\,\overline{a_y} + 1.2\pi\,\overline{a_z}}{4\pi\,(\sqrt{35})^3} \cdot \mu A/m$$

$$d\overline{H_p} = 1.9317 \times 10^{-3}\,\overline{a_x} + 1.448 \times 10^{-3}\,\overline{a_y} + 1.448 \times 10^{-3}\,\overline{a_z} \cdot \mu A/m$$

$$\boxed{d\overline{H_p} = 1.9317\,\overline{a_x} + 1.448\,\overline{a_y} + 1.448\,\overline{a_z}\ \eta A/m.}$$

$$\boxed{|d\overline{H_p}| = 2.81511\,\eta A/m.}$$

(8)

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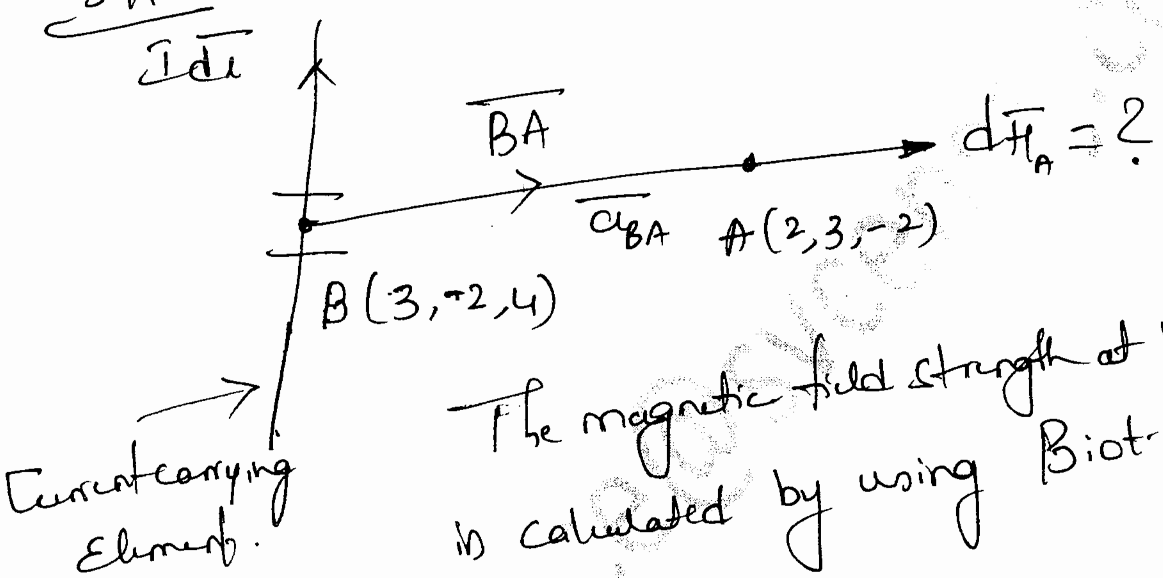
Problem 2

Find the magnitude of magnetic field at $A(2,3,-2)$m due to a current element $I\overline{dl} = \pi(0.5\bar{a}_x - 0.6\bar{a}_y + 0.8\bar{a}_z)$ μAm situated at $B(3,-2,4)$m.

(8m)

Dec/Jan 2005.

Soln:



The magnetic field strength at point A is calculated by using Biot-Savart's Law

$$d\overline{H}_A = \frac{I\overline{dl} \times \bar{a}_{BA}}{4\pi |\overline{BA}|^2} \text{ A/m.}$$

$$d\overline{H}_A = \frac{I\overline{dl} \times \overline{BA}}{4\pi |\overline{BA}|^3} \text{ A/m.}$$

$$I\overline{dl} = \pi(0.5\bar{a}_x - 0.6\bar{a}_y + 0.8\bar{a}_z) \ \mu\text{A-m.}$$

(9)

673

$$\overline{BA} = (2-3)\overline{a_x} + (3+2)\overline{a_y} + (-2-4)\overline{a_z}$$

$$\overline{BA} = -\overline{a_x} + 5\overline{a_y} - 6\overline{a_z}.$$

$$|\overline{BA}| = \sqrt{1+25+36} = \sqrt{62}\ m.$$

$$I\overline{dl} \times \overline{BA} = \begin{vmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ 0.5 & -0.6 & 0.8 \\ -1 & 5 & -6 \end{vmatrix} \times \pi(\mu).$$

$$= (\mu \times \pi)\left\{[+3.6-4]\overline{a_x} - [-3+0.8]\overline{a_y} + [2.5-0.6]\overline{a_z}\right.$$

$$I\overline{dl} \times \overline{BA} = \pi\left[-0.4\overline{a_x} + 2.2\overline{a_y} + 1.9\overline{a_z}\right]\mu A{\cdot}m^2.$$

$$d\overline{H_A} = \frac{I\overline{dl} \times \overline{BA}}{4\pi\ |\overline{BA}|^3}$$

$$= \frac{\cancel{\pi}\left[-0.4\overline{a_x} + 2.2\overline{a_y} + 1.9\overline{a_z}\right]\mu A\cancel{m^2}}{4\cancel{\pi}\,(\sqrt{62})^3\ \cancel{m^3}}$$

$$= -2.048\times10^{-4}\,\overline{a_x} + 1.1266\times10^{-3}\,\overline{a_y} + 9.729\times10^{-4}\,\mu A/m$$

674

$$d\bar{H}_A = -0.2048\,\bar{a}_x + 1.1266\,\bar{a}_y + 0.972\,\bar{a}_z \ \eta A/m.$$

$$\boxed{d\bar{H}_A = -0.2048\,\bar{a}_x + 1.1266\,\bar{a}_y + 0.972\,\bar{a}_z} \ \eta A/m.$$

the magnitude of Magnetic field strength at point A is given by

$$|d\bar{H}_A| = \sqrt{(-0.2048)^2 + (1.1266)^2 + (0.972)^2} \ \eta A/m$$

$$\boxed{|d\bar{H}_A| = 1.50198 \ \eta A/m.}$$

(11)

Topic 3.5a

Applications of Biot-Savarts Law

a. Magnetic Field Intensity due to Infinite Long Straight Filament

10-June/July 2013

Derive an expression for magnetic field intensity at a point P due to an infinitely long straight filament carrying a current I. Also obtain the magnetic field intensity caused by a finite length current filament on the z-axis. (08 Marks)

next Topic

(or)

06 - June /July 2012

State Biot-Savart law and use this to find magnetic field intensity at a point 'P' due to an infinite length filament carrying current I and placed on Z-axis. Point P is at a distance 'r' m from origin. (08 Marks)

(or)

02 - June /July 2012

On the basis of Biot-Savart law, obtain an expression for the magnetic field intensity at some distance due to a current carrying straight conductor of infinite length. (08 Marks)

(or)

06- June /July 2009

Derive the expression for field at a point P due to an infinitely long filament carrying direct current I. (08 Marks)

Question.

Derive an expression for magnetic field intensity at a point 'p' due to an infinitely long straight filament carrying a current I. (8M)

[10-J/J 2013, 06-J/J 2012, 02-J/J 2012, 06 J/J 2009].

Topic 3.5a
3.5a
Imp.

Magnetic Field Intensity ($\bar{H}$) due to Infinite Long Straight Filament :-

Consider a Infinite length long-straight filament placed along z-axis. assume that DC current of I ampere's flows in $+z$ direction.

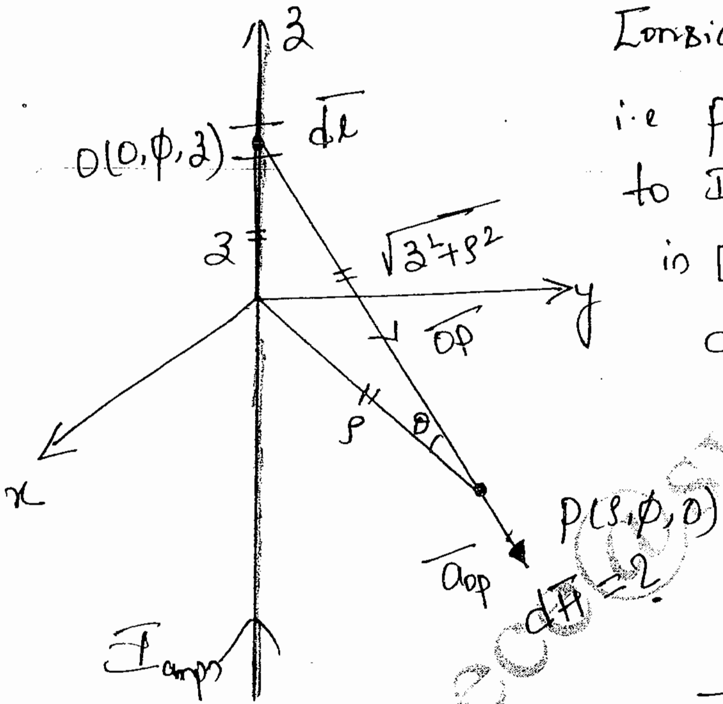


fig Infinite Length Current Carrying filament.

Consider a point 'p' on x-y plane i.e $p(\rho,\phi,0)$. the field ($\bar{H}$) due to Infinite Length Current Carrying filament is Calculated by Considering a differential Current element at point $O(0,\phi,z)$

i.e $dl = dz$ & $\overline{dl} = dz\,\bar{a}_z$

and

$$I\,\overline{dl} = I\,dz\,\bar{a}_z .$$

$$\overline{OP} = (\rho-0)\bar{a}_\rho + (\phi-\phi)\bar{a}_\phi + (0-z)\bar{a}_z$$

$$\overline{OP} = \rho\,\bar{a}_\rho - z\,\bar{a}_z$$

$$\boxed{|\overline{OP}| = \sqrt{\rho^2+z^2}}$$

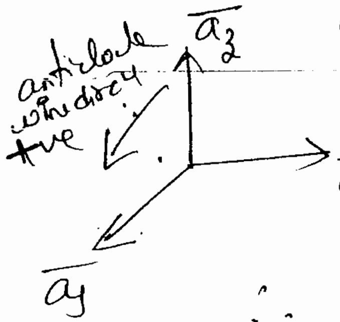
$$\bar{a}_{op} = \frac{\overline{OP}}{|\overline{OP}|} = \frac{\rho\,\bar{a}_\rho - z\,\bar{a}_z}{\sqrt{\rho^2+z^2}}$$

using Biot-Savart Law i.e the differential Magnetic field ($d\bar{H}$) at a point 'p' due to Current Carrying filament is

$$d\overline{H}_p = \frac{I\,\overline{dl} \times \overline{a_R}}{4\pi R^2} = \frac{I\,\overline{dz} \times \overline{a_{op}}}{4\pi |\overline{op}|^2}$$

$$d\overline{H_p} = \frac{I\,dz\,\overline{a_z}}{4\pi(\rho^2+z^2)} \times \left(\frac{\rho\,\overline{a_\rho} - z\,\overline{a_z}}{\sqrt{\rho^2+z^2}}\right)$$

from concept of Cross product



$$\overline{a_z} \times \overline{a_\rho} = \overline{a_\phi}$$

and

$$\overline{a_z} \times \overline{a_z} = 0.$$

$$\therefore\ d\overline{H_p} = \frac{I\rho\,dz\,\overline{a_\phi}}{4\pi(\rho^2+z^2)^{3/2}}$$

the net field at a point 'p' due to Infinite Length Current Carrying filament is

$$\overline{H_p} = \int_{z=-\infty}^{\infty} d\overline{H_p} = \frac{I\rho\,\overline{a_\phi}}{4\pi} \int_{z=-\infty}^{\infty} \frac{dz}{(\rho^2+z^2)^{3/2}}$$

put $z = \rho\tan\theta$; $dz = \rho\sec^2\theta\,d\theta$

$$\rho^2+z^2 = \rho^2+\rho^2\tan^2\theta = \rho^2\sec^2\theta$$

$$(\rho^2+z^2)^{3/2} = (\rho\sec\theta)^3 = \rho^3\sec^3\theta.$$

L.L $z = -\infty \Rightarrow \theta = -\pi/2$

U.L $z = +\infty \Rightarrow \theta = +\pi/2$

$\theta = \tan^{-1}(z/\rho)$

$$\overline{H_p} = \frac{I \rho \overline{a_\phi}}{4\pi} \int_{\theta=-\pi/2}^{\pi/2} \frac{\rho \sec^2\theta \, d\theta}{\rho^3 \sec^3\theta}$$

$$\overline{H_p} = \frac{I}{4\pi\rho} \overline{a_\phi} \int_{\theta=-\pi/2}^{\pi/2} \cos\theta \, d\theta \rightarrow 2$$

$$\overline{H_p} = \frac{I}{\cancel{4}\pi\rho} \overline{a_\phi} (\cancel{2})$$

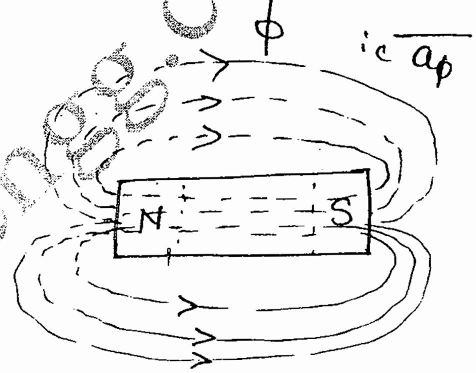
$$\therefore \boxed{\overline{H_p} = \frac{I}{2\pi\rho} \overline{a_\phi}} \text{ A/m or N/wb}$$

Obs:-

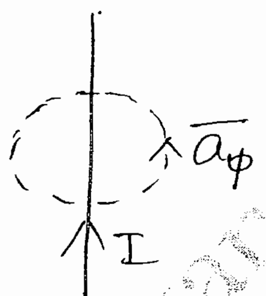
* where ρ is $\perp^r$ distance from point p to infinite length Current Carrying filament.
* the direction $\overline{a_\phi}$ is obtained by right hand rule. i.e if you grip the Current filament in right hand with thumb in the direction of Current, the direction of fingers around the Current filament gives the direction of $\overline{H}$.
* The Unit Vector $\overline{a_\phi}$ is perpendicular to the Current Carrying filament.

In general $\boxed{\overline{H} = \frac{I}{2\pi\rho} \overline{a_\phi}}$ A/m ← (a)

* from eq^n (a) Magnetic field is Circular in nature.



$$\boxed{\overline{H_P} = \frac{I}{2\pi\rho}\overline{a_\phi}}$$



Topic 3.5b

Applications of Biot-Savarts Law

b. Magnetic Field Intensity due to finite length Filament

10-DEC2011/Jan 2012

Starting from Biot-Savart law, derive an expression for the magnetic field intensity at a point due to finite length of current carrying conductor. **(06 Marks)**

02 - June /July 2010

For the Fig.Q5(b), use Biot-Savart law to find magnetic field H at point P. **(06 Marks)**

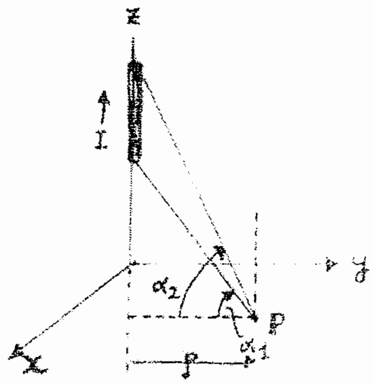


Fig.Q5(b)

10-Dec/Jan 2015

Starting form Biot-Savort's law, derive the expression for the magnetic field intensity at a point due to finite length current carrying conductor. **(08 Marks)**

Dec/Jan 2016

Derive expression for $\vec{H}$ due to straight conductor of finite length. **(08 Marks)**

10 - June /July 2012

State Biot-Savart law. Obtain an expression for magnetic field intensity due to straight conductor of finite length. **(07 Marks)**

Question.

Starting from Biot - Savart's Law, derive the Expression for the magnetic field intensity at a point due to finite Length Current Carrying conductor. (6m)

[10-Dec/Jan 2012, 02-J/J 2010, 10-Dec/Jan 2015, Dec/Jan 2016, 10-J/J 2012].

3.5 b Magnetic Field Intensity due to Finite Length Current Carrying Filament:-

$dl = dz$.

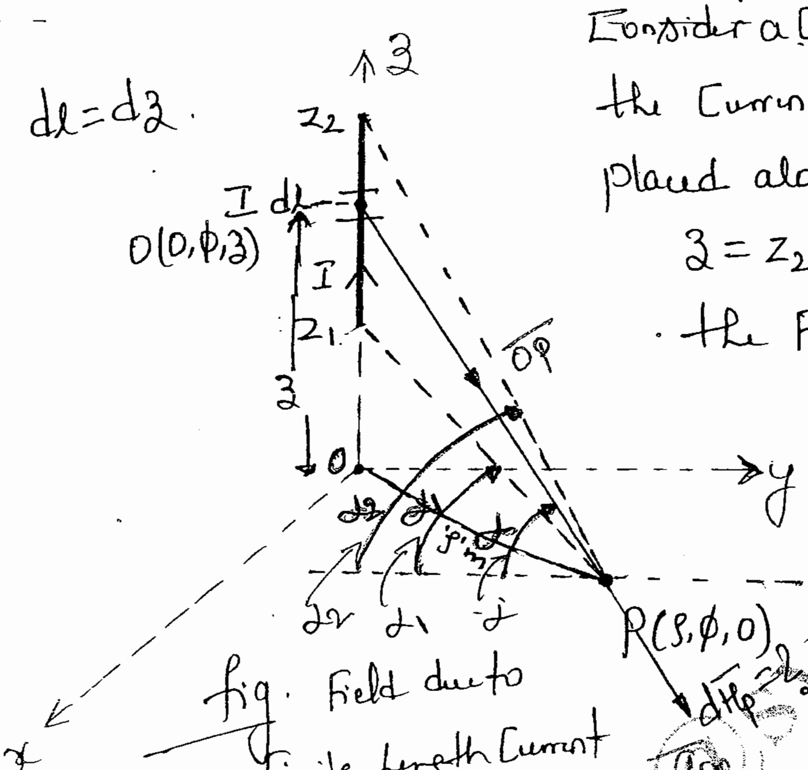


fig. Field due to Finite Length Current Carrying Filament.

Consider a current filament through which the current of I amp's is passing is placed along z-axis from $z = z_1$ to $z = z_2$.

The Field $\overline{H_p}$ at point p on xy plane is [i.e $p(\rho,\phi,0)$] to be calculated by considering the differential current element $(I\overline{dl})$.

$$dl = dz$$

$$\overline{dl} = dz\,\overline{a_z}.$$

The vector $\overline{OP} = \rho\overline{a_\rho} - z\overline{a_z}$;

$$|\overline{OP}| = \sqrt{\rho^2 + z^2} \quad ; \quad |\overline{OP}|^2 = \rho^2 + z^2$$

The differential current element is $I\overline{dl} = I\,dz\,\overline{a_z}$.

The differential field at point 'p' is $d\overline{H_p}$

$$d\overline{H_p} = \frac{I\overline{dl} \times \overline{a_R}}{4\pi R^2} = \frac{I\overline{dl} \times \overline{a_{op}}}{4\pi |\overline{OP}|^2}$$

$$d\overline{H_p} = \frac{I\,dz\,\overline{a_z}}{4\pi(\rho^2 + z^2)} \times \left[\frac{\rho\overline{a_\rho} - z\overline{a_z}}{\sqrt{\rho^2 + z^2}}\right]$$

using Cross product of unit vectors

$\bar{a}_z \times \bar{a}_\rho = \bar{a}_\phi$; $\bar{a}_z \times \bar{a}_z = 0$.

$$d\bar{H}_p = \frac{I\,dz}{4\pi(\rho^2+z^2)^{3/2}}\left[\rho\,\bar{a}_\phi\right] \quad A/m$$

the net Field at point 'p' is

$$\bar{H}_p = \int_{z_1}^{z_2} d\bar{H}_p = \int_{z_1}^{z_2} \frac{I\rho\,dz}{4\pi(\rho^2+z^2)^{3/2}}\,\bar{a}_\phi$$

$$\bar{H}_p = \frac{I\rho}{4\pi}\bar{a}_\phi \int_{z_1}^{z_2} \frac{dz}{(\rho^2+z^2)^{3/2}}$$

put $z = \rho\tan\alpha$; $(\rho^2+z^2)^{3/2} = \rho^3\sec^3\alpha$

$dz = \rho\sec^2\alpha\,d\alpha$

Limits: $\tan(\alpha_1) = \frac{z_1}{\rho} \Rightarrow z_1 = \rho\tan(\alpha_1)$.

$z \to z_1$; $\alpha \to \alpha_1$

and $\tan(\alpha_2) = \frac{z_2}{\rho} \Rightarrow z_2 = \rho\tan(\alpha_2)$

$z \to z_2$; $\alpha \to \alpha_2$

$$\bar{H}_p = \frac{I\cancel{\rho}}{4\pi}\bar{a}_\phi \int_{\alpha_1}^{\alpha_2} \frac{\cancel{\rho}\,\cancel{\sec^2\alpha}\,d\alpha}{\rho^{\cancel{3}}\,\sec^{\cancel{3}}\alpha}$$

$$\overline{H_p} = \frac{I\,\overline{a_\phi}}{4\pi\rho}\int_{\alpha_1}^{\alpha_2}\frac{1}{\sec\alpha}\,d\alpha$$

$$\overline{H_p} = \frac{I}{4\pi\rho}\,\overline{a_\phi}\int_{\alpha_1}^{\alpha_2}\cos(\alpha)\,d\alpha$$

$$\overline{H_p} = \frac{I}{4\pi\rho}\,\overline{a_\phi}\left[\sin(\alpha)\Big|_{\alpha_1}^{\alpha_2}\right]$$

$$\overline{H_p} = \frac{I}{4\pi\rho}\,\overline{a_\phi}\left[\sin(\alpha_2)-\sin(\alpha_1)\right]$$

$$\boxed{\overline{H_p} = \frac{I}{4\pi\rho}\left[\sin(\alpha_2)-\sin(\alpha_1)\right]\overline{a_\phi}}\quad A/m$$

Note: when $\alpha_2 \to \pi/2$ and $\alpha_1 \to -\pi/2$ $\Rightarrow$ Infinite Long Filament.

$$\overline{H_p} = \frac{I}{\cancel{4}\pi\rho}\left[\sin(\pi/2)-\sin(-\pi/2)\right]\overline{a_\phi}$$

$$= \frac{I}{2\cancel{4}\pi\rho}\times\cancel{2}\,\overline{a_\phi}$$

$$\boxed{\overline{H_p} = \frac{I}{2\pi\rho}\,\overline{a_\phi}}\quad A/m$$

$\bar{H}$ due to infinite length current carrying filament

Problem-3 P(0.4, 0.3, 0) m 8A

b. Determine the magnetic field intensity H at point P(0.4, 0.3, 0), if the 8A current in a conductor inward from infinity to origin on the x axis and outward to infinity along y axis. (08 Marks)

15-Dec/Jan 2017 (CBCS)

Solu

Dankan V Gowda MTech.,(Ph.D)
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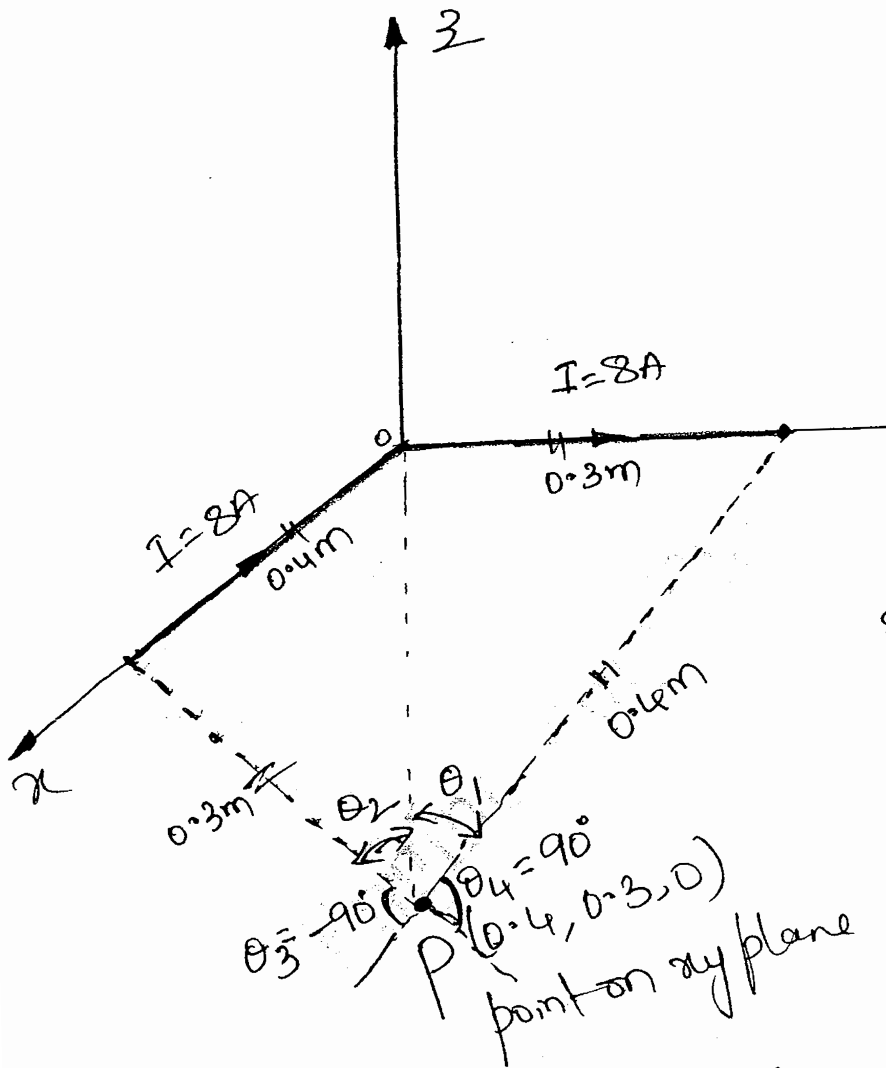
+919844554940

10-Dec/Jan 2014. (7m)



$\theta_1 = \tan^{-1}\left(\frac{-0.3}{0.4}\right) = -36.86$

$\boxed{\theta_1 = -36.86°}$

$\theta_2 = \tan^{-1}\left(\frac{0.4}{0.3}\right)$

$\boxed{\theta_2 = 53.13°}$

W.k.t $\bar{H}$ due to infinite Line charge is given by

$$\bar{H} = \frac{I}{4\pi\rho}\left[\sin\alpha_2 - \sin\alpha_1\right]\bar{a}_\phi \text{ A/m.}$$

21

ρ - radial distance from current filament to the ... point.

21

the field H_x is given by

$$\overline{H_x} = \frac{I}{4\pi\rho}\left[\sin\theta_2 - \sin\theta_3\right]\overline{a_\phi} \text{ A/m}$$

$\boxed{\rho = 0.3m}$

$$\overline{H_x} = \frac{8}{4\pi \times 0.3}\left[\sin(53.13^\circ) - \sin(-90^\circ)\right]\overline{a_\phi}$$

$$\boxed{\overline{H_x} = 3.819\,\overline{a_\phi}} \text{ A/m}$$

from the concept of right-hand screw rule

the unit vector $\boxed{\overline{a_\phi} = -\overline{a_z}}$

$$\Rightarrow \boxed{\overline{H_x} = -3.819\,\overline{a_z}} \text{ A/m}$$

My Field $\overline{H_y}$ due to current filament along y-dirⁿ.

$$\overline{H_y} = \frac{I}{4\pi\rho}\left[\sin\theta_4 - \sin\theta_1\right]\overline{a_\phi} \text{ A/m}$$

and $\rho = 0.4$ m

$$\overline{H_y} = \frac{8}{4\pi(0.4)}\left[\sin 90^\circ - \sin(-36.86)\right]\overline{a_\phi} \text{ A/m}$$

$\overline{H_y} = 2.546\,\overline{a_\phi}$ A/m.

from the concept of right hand screw rule, the unit vector $\overline{a_\phi}$ referred to y-axis is $-\overline{a_z}$

i.e $\boxed{\overline{a_\phi} = -\overline{a_z}}$

$\therefore\ \overline{H_y} = -2.546\,\overline{a_z}$ A/m.

the net field at point 'p' is given by

$\overline{H_p} = \overline{H_x} + \overline{H_y}$ A/m

$\overline{H_p} = -3.819\,\overline{a_z} - 2.546\,\overline{a_z}$

$\boxed{\overline{H_p} = -6.365\,\overline{a_z}}$ A/m

$|\overline{H_p}| = 6.365$ A/m

Topic 3.5c

Applications of Biot-Savarts Law

c. Magnetic Field Intensity on the axis of a Circular Loop

06-DEC2008/Jan 2009

State and explain Biot – savart law. Using this, find the magnetic flux density at the centre of a circular current loop of radius 'a'(m) (07 Marks)

06-DEC 2013/Jan 2014

Using Biot-Savart law, derive an expression for magnetic field intensity on the axis of a circular ring of radius 'a' carrying current 'I'. (10 Marks)

06 - June /July 2011

Obtain the expression for the magnetic flux density at any point on the axis of a circular current loop of n turns. (07 Marks)

02 - June /July 2011

State Biot-Savart law. Apply this law to determine the magnetic flux density at the center of a circular current loop. (08 Marks)

06 – Jan 2013

State and explain Biot – Savart law. Using this, find the magnetic flux density at the centre of a circular loop of radius 'a' mt. (08 Marks)

06 – May/June 2010

Derive the expression for magnetic flux density on the axis of a circular loop of radius 'a' carrying current I using Biot Savart law. (07 Marks)

Question

State Biot Savart Law apply this Law to determine the magnetic ~~flux~~ density at the Center of a Circular Current Loop (8M).

[06 - Jan 2009, 06 - Jan 2014, 06 - J|J 2011, 02 J|J 2011, 06 - Jan 2013, 06 May|June 2010].

3.5.c. Magnetic Field Intensity (H) on the axis of a Circular Loop.

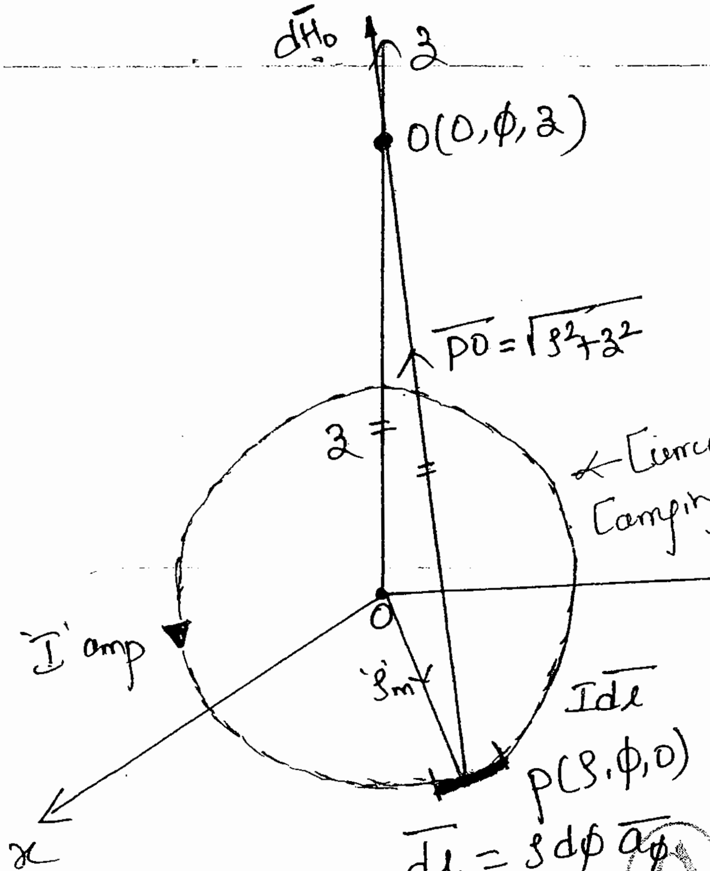


fig a: Field H on axis of a Circular Loop placed on xy-axis.

Consider a Current Carrying Circular Loop placed on xy-plane.

the Field Intensity on the axis of a Circular Loop ie @O(0,φ,z) is obtained by Considering a differential Current element $I\bar{dl}$ on Circular Loop

$\therefore I\bar{dl} = I\rho\, d\phi\, \bar{a}_\phi$.

the vector joining b/w the points P to O is

$$\overline{PO} = (0-\rho)\bar{a}_\rho + (\phi-\phi)\bar{a}_\phi + (z-0)\bar{a}_z$$

$$\overline{PO} = -\rho\bar{a}_\rho + z\bar{a}_z \; ; \quad |\overline{PO}| = \sqrt{\rho^2+z^2} \text{ m}.$$

$$d\bar{H}_0 = \frac{I\bar{dl} \times \bar{a}_{PO}}{4\pi |\overline{PO}|^2} \text{ A/m}$$

the unit vector $\bar{a}_{PO} = \dfrac{\overline{PO}}{|\overline{PO}|} = \dfrac{-\rho\bar{a}_\rho + z\bar{a}_z}{\sqrt{\rho^2+z^2}}$

$$d\bar{H}_0 = \frac{I\rho\, d\phi\, \bar{a}_\phi}{4\pi(\rho^2+z^2)} \times \left[\frac{-\rho\bar{a}_\rho + z\bar{a}_z}{\sqrt{\rho^2+z^2}}\right] \text{ A/m}$$

using Cross product of unit vectors

i.e $\overline{a_\phi} \times \overline{a_\rho} = -\overline{a_z}$ and $\overline{a_\phi} \times \overline{a_z} = \overline{a_\rho}$

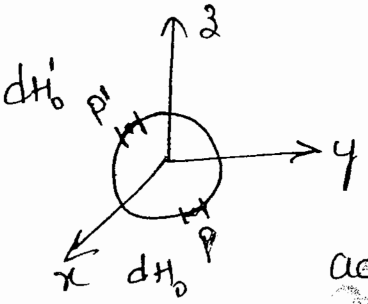
due to symmetrical current element @ exactly opposite side (fig b).

$$\therefore d\overline{H_0} = \frac{I\rho\, d\phi}{4\pi(\rho^2+z^2)^{3/2}}\left[\rho\overline{a_z} + \cancel{z\overline{a_\rho}}\right] \leftarrow (a)$$

horizontal component.

the eq^n (a) shows that $d\overline{H_0}$ has two components ($\overline{a_z}$ and $\overline{a_\rho}$).

when we considered a filament at (ρ, ϕ, z) in the above fig a. there is one more small filament at exactly diametrically opposite side point p' (shown in fig b).



The Field Intensity due to differential filament at p' also has two components when these two field Intensities (dH_0 and dH_0') added, the horizontal components get cancelled. and results only Vertical components.

Thus the result is only the vertical component.

$$\overline{H_0} = \int_{\phi=0}^{2\pi} \frac{I\rho^2}{4\pi} \frac{d\phi}{[\rho^2+z^2]^{3/2}}\overline{a_z} \quad A/m.$$

$$\overline{H_0} = \frac{I\rho^2 \, \overline{a_z}}{4\pi(\rho^2+z^2)^{3/2}} \int_{\phi=0}^{2\pi} d\phi \rightarrow 2\pi$$

$$\overline{H_0} = \frac{I\rho^2}{\cancel{4\pi}_{2}(\rho^2+z^2)^{3/2}} \times \cancel{2\pi} \, \overline{a_z}$$

$$\boxed{\overline{H_0} = \frac{I\rho^2}{2(\rho^2+z^2)^{3/2}} \overline{a_z}} \text{ A/m}$$

The magnetic Flux density $\boxed{\overline{B} = \mu_0 H = \frac{\mu I \rho^2}{2(\rho^2+z^2)^{3/2}} \overline{a_z}}$ wb/m² (or Tesla)

→ $\overline{H}$ at any point on the axis is always perpendicular to plane of the circular loop.

→ the diration of $\overline{H}$ is upward | downward is obtained by the right hand rule.

→ the above result is for any point on the axis at a distance z. if $\overline{H}$ at the center of the loop is desired ie put $z=0$, $\overline{H}$ becomes

$$\boxed{\overline{H} = \frac{I}{2\rho} \overline{a_z}} \text{ A/m} = \frac{I}{2\rho} \overline{a_z} \text{ A/m}$$

field at center of loop.

My magnetic flux density

$$\overline{B} = \frac{\mu_0 I}{2\rho} \overline{a_z} \text{ wb/m}^2.$$

problem 4.

10-DEC2011/Jan 2012

A single trun circular coil 5 cm diameter carries a current of 2.8 A. Determine the magnetic flux density $\vec{B}$ at a point on the axis 10 cm from the center. Derive the formula used.

(08 Marks)

Question.

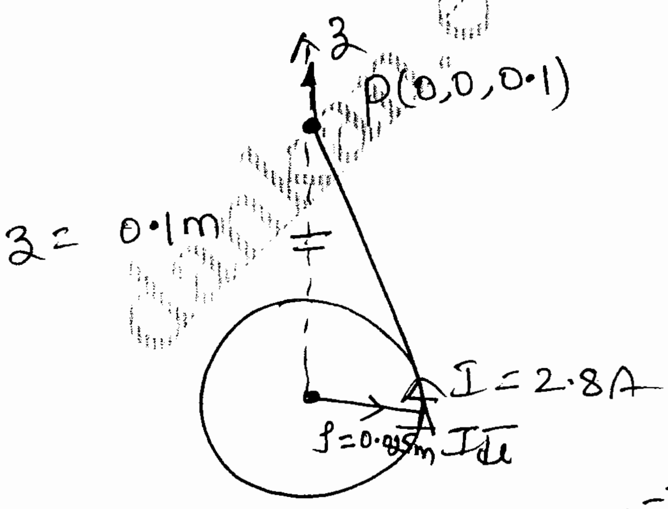
A single turn circular coil 5cm diametere carries a current of 2·8A. Determine the magnetic fluxdensity $\overline{B}$ at a point onthe axis 10cm from the center. Derive the formula used. (8m)

Solu: Step1. derive an expression Magnetic flux density ($\overline{B}$) on the axis of a circular loop

i.e $$\overline{B} = \frac{\mu_0 I \rho^2}{2(\rho^2+z^2)^{3/2}} \overline{a_z} \ wb/m^2.$$

d = 5cm = 0·05m

radius ρ = 0·025m.



$$\overline{B} = \frac{\mu_0 I \rho^2}{2(\rho^2+z^2)^{3/2}} \overline{a_z} \ wb/m^2$$

$$\overline{B} = \frac{4\pi \times 10^{-7}(2\cdot8)(0\cdot025)^2}{2\left[(0\cdot025)^2+(0\cdot1)^2\right]^{3/2}} \overline{a_z} \ wb/m^2$$

$$\overline{B} = \frac{2\cdot 1991148\times 10^{-9}}{2(0\cdot025^2+0\cdot1^2)^{1\cdot5}} \overline{a_z} \ wb/m^2$$

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$$\overline{B} = 1.00397 \times 10^{-6}\, \overline{a_z}\ wb/m^2$$

$$\boxed{\overline{B} = 1.00397\, \overline{a_z}}\ \mu wb/m^2$$

and

$$|\overline{B}| = 1.00397\ \mu wb/m^2$$

the magnetic field Intensity

$$\overline{H} = \frac{\overline{B}}{\mu_0} = \frac{1.00397 \times 10^{-6}}{4\pi \times 10^{-7}}\ A/m$$

$$\overline{H} = 0.798933\, \overline{a_z}\ A/m$$

$$\boxed{\overline{H} = 0.798933\overline{a_z}}\ A/m$$

$$\boxed{|\overline{H}| = 0.79893}\ A/m.$$

problem 5:

$\overline{H}$ due circular loop

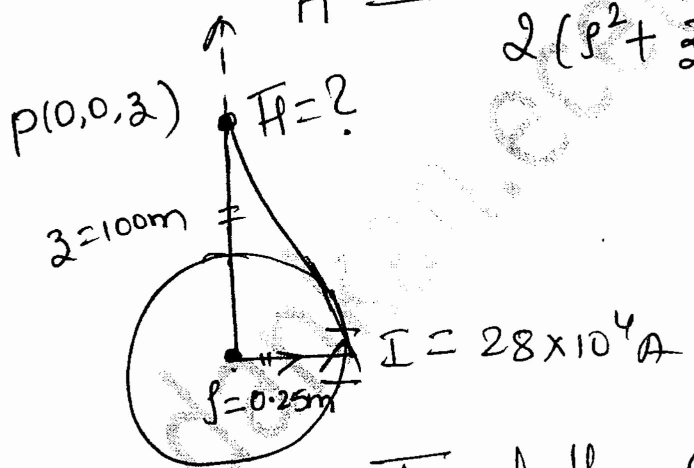
A single turn circular coil of 50 m in diameter carries a current of 28×10^4 A. Determine the magnetic field intensity $\overline{H}$ at a point on the axis of the coil and 100m from the coil. the μ_r of freespace surrounding the coil is unity.

Feb 2001 (6m)

solu:

$\overline{H}$ due to Axis of a circular current carrying loop is given by

$$\overline{H} = \frac{I\rho^2}{2(\rho^2+z^2)^{3/2}}\,\overline{a_z} \text{ A/m.} \leftarrow (a)$$



$d = 50$ m.

$\boxed{\rho = 25\text{ m}}$ = radius.

i. $\overline{H}$ at the axis of the coil. i.e $z \to 0$.

in eqn (a)

$$\overline{H} = \frac{I\rho^2}{2\rho^3}\,\overline{a_z} \text{ A/m.}$$

$$\overline{H} = \frac{I}{2\rho}\,\overline{a_z} \quad A/m.$$

$I = 28 \times 10^4$ A ; $\rho = 25$m.

$$\overline{H} = \frac{28 \times 10^4}{2(25)}\,\overline{a_z} \quad A/m$$

$$\boxed{\overline{H} = 5.6\,\overline{a_z}}\ kA/m = 5600\,\overline{a_z}\ A/m$$

ii. $\overline{H}$ at a point on the axis 100m from the coil is

$$\overline{H} = \frac{I\rho^2}{2(\rho^2+z^2)^{3/2}}\,\overline{a_z} \quad A/m.$$

$z = 100$m.

$$\overline{H} = \frac{(28 \times 10^4)(25)^2}{2(25^2+100^2)^{1.5}}\,\overline{a_z} \; A/m$$

$$\boxed{\overline{H} = 79.894088\,\overline{a_z}}\ A/m.$$

$$\boxed{|\overline{H}| = 79.894}\ A/m.$$

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695

Topic 35d

35d Magnetic Field Intensity at a point on the Axis of a solenoid

Questions

02-DEC2010

By using Biot-Savart's law, obtain the equations of magnetic field intensity and magnetic flux density at any point on the axis of a coil (06 Marks)

Dec/Jan 2017

b. Derive an expression for Magnetic flux density at any point on the axis of Solenoid. (08 Marks)

02-DEC2008/Jan 2009

Show that the magnetic field intensity at the end of a long solenoid is one half of that at the centre. (06 Marks)

06 –June/July 2014

Derive an expression for H at a point in the axis of solenoid. (06 Marks)

Derive an expression for magnetic flux density and field intensity at any point on the axis of Solenoid. (8m)

[02-Dec 2010, 10 Dec/Jan 2017, 02-Jan 2009, 06-June/July 2014].

Soln

Consider a solenoid of length 'L' meters and a point P on its axis making angles ϕ_1 and ϕ_2 with both ends as shown in the fig.

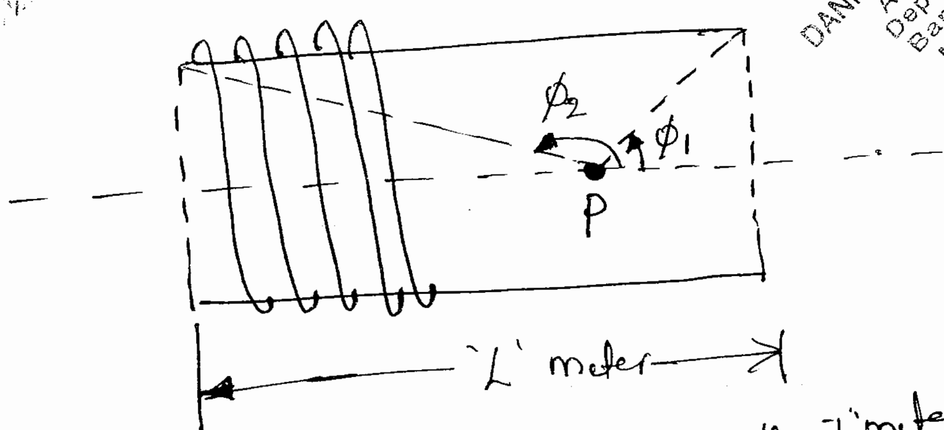

fig. Solenoid of length 'L' meter.

the incremental flux density at point p is given by

$$dB_p = \frac{\mu N I}{2} \sin\phi \, d\phi$$

from fig. ϕ varies from ϕ_1 to ϕ_2.

$\therefore$ The magnetic flux density at point p is

$$B_p = \int_{\phi_1}^{\phi_2} \frac{\mu N I}{2} \sin\phi \, d\phi$$

$$= \frac{\mu N I}{2} (-\cos\phi) \Big|_{\phi_1}^{\phi_2}$$

$$B_p = \frac{\mu N I}{2} [-\cos\phi_2 + \cos\phi_1]$$

$$\boxed{B_p = \frac{\mu N I}{2} [\cos\phi_1 - \cos\phi_2]}$$ Wb/m^2 or Tesla.

The magnetic Field intensity at a point p is

$$\boxed{H_p = \frac{B_p}{\mu} = \frac{NI}{2} [\cos\phi_1 - \cos\phi_2]}$$ A/m.

Problem 6.

06 - June /July 2013

A solenoid of 10 cm diameter and 30 cm length is wound with 150 turns and carries a current of 5 A. Find the magnetic flux density at a point on the axis at a distance of 10 cm from the midpoint of the solenoid. (08 Marks)

Question.

A Solenoid of 10cm diameter and 30cm Length is wound with 150 turns and carries a current of 5A. Find the magnetic flux density at a point on the axis at a distance of 10cm from the midpoint of the solenoid.

Solu:-

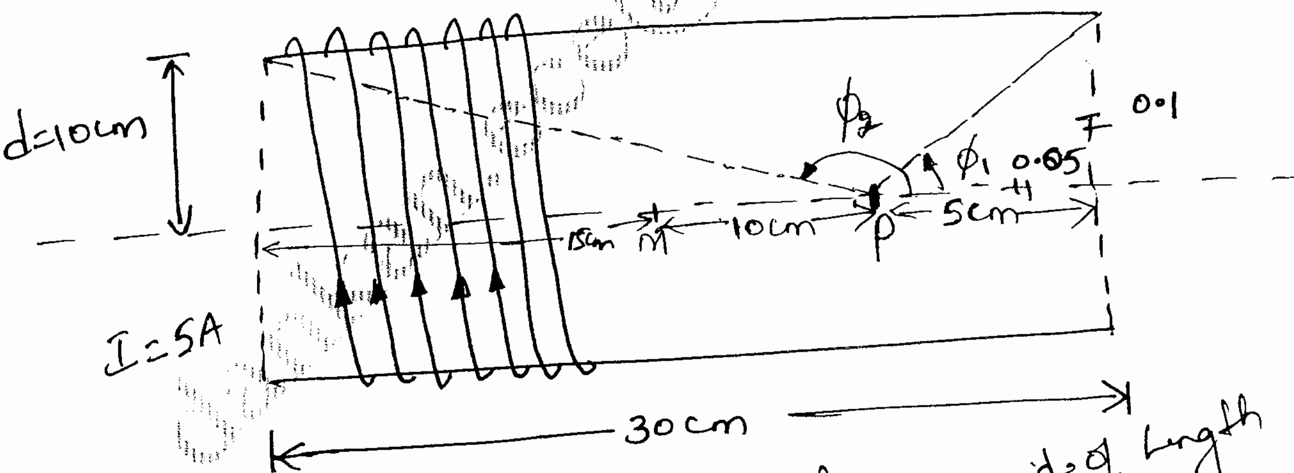

fig. Solenoid of Length 30cm.

$L = 30cm = 0.3m$.

and $d = 10cm = 0.1m$.

$I = 5A$.

and $\phi_2 = 180^\circ - \tan^{-1}\left(\frac{0.1}{0.25}\right)$

from fig. $\phi_1 = \tan^{-1}\left(\frac{0.1}{0.05}\right) \Rightarrow \tan\phi_1 = \left(\frac{0.1}{0.05}\right)$

$\Rightarrow \phi_1 = \tan^{-1}\left(\frac{0.1}{0.05}\right) = \underline{63.435^\circ}$

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the magnetic flux density at any point p along the axis is given by

$$B_p = \frac{\mu N I}{2}\left[\cos\phi_1 - \cos\phi_2\right].$$

N = number of turns per meter $= \frac{150}{l} = \frac{150}{0.3}$

$$\boxed{N = 500} \text{ turns/meter}.$$

$$\boxed{\phi_1 = 63.435^\circ}$$

and $\phi_2 = 180^\circ - \tan^{-1}\left(\frac{0.1}{0.25}\right)$

$$\phi_2 = 180^\circ - 21.8^\circ$$

$$\boxed{\phi_2 = 158.2^\circ}$$

$$\therefore B_p = \frac{4\pi\times10^{-7}\times500\times5}{2}\left[\cos(63.435^\circ) - \cos(158.2^\circ)\right]$$

$$\boxed{B_p = 2.16106\times10^{-3} = 2.16106 \text{ m wb/m}^2 \text{ (or) Tesla}}$$

the magnetic field intensity at point 'p' is

$$\boxed{H_p = \frac{B_p}{\mu_0} = 1.7196\times10^3} \text{ A/m}$$

Question: Topic 3.5e Magnetic Fluxdensity at the 3.5e Center of a Square Current Loop.

Find the magnetic flux density at the Center of a Square Current Loop of side 'a'.

Consider a Square Current Loop of side 'a' m carrying a current I, situated in free space. it is required to find the magnetic flux density at point P, the center of the square.

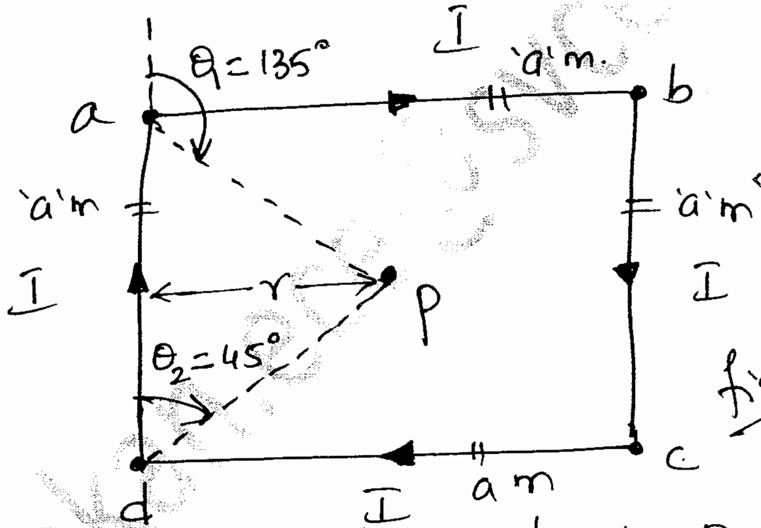


fig. Square Current Loop assumed to be in xy plane.

the flux density at point P due to current I flowing from 'd' to 'a' is given by

$$\bar{B} = \frac{\mu_0 I}{4\pi r}\left[\cos\theta_2 - \cos\theta_1\right]\bar{a}_z \ wb/m^2$$

$\theta_2 = 45^\circ$ and $\theta_1 = 135^\circ$, $r = \frac{a}{2}$ m.

$$\overline{B} = \frac{\mu_0 I}{4\pi(\frac{a}{2})}\left[\cos(45^\circ) - \cos(135^\circ)\right] \overline{a_z} \ wb/m^2$$

$$\overline{B} = \frac{\mu_0 I}{2\pi a}\left[\frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}}\right)\right] \overline{a_z}$$

$$\overline{B} = \frac{\mu_0 I}{\sqrt{2}\pi a} \overline{a_z} \quad wb/m^2.$$

Since square has 'four' sides and the current through each wire causes the magnetic flux density $\overline{B}$ pointing $\perp^r$ to paper and into it, the overall magnetic flux density $\overline{B_{net}} = 4\overline{B}$ wb/m²

$$\overline{B_{net}} = 4 \frac{\mu_0 I}{\sqrt{2}\pi a} \overline{a_z} \ wb/m^2$$

$$\boxed{\overline{B_{net}} = \frac{2\sqrt{2}\mu_0 I}{\pi a}} \overline{a_z} \ wb/m^2$$

and magnetic field intensity $\boxed{\overline{H_{net}} = \frac{\overline{B}}{\mu_0} = \frac{2\sqrt{2} I}{\pi a}} \overline{a_z}$ A/m

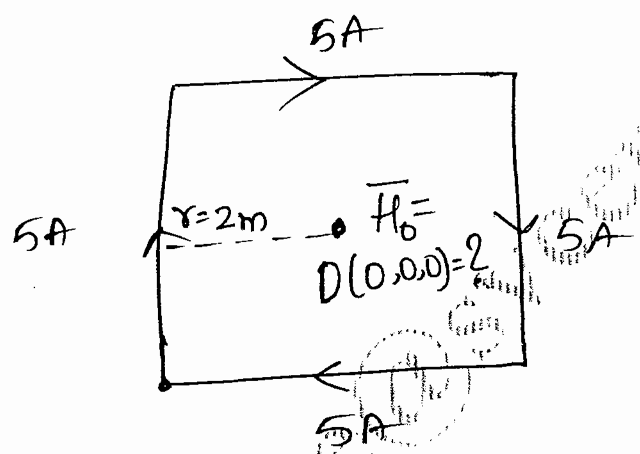
problem 7.

06 –Dec/Jan 2008

Question.

Find $\overline{H}$ at the Center of a square Current Loop of side 4 meters, if a current of 5 ampere's is passing through it. (8M).

Solu:-



$I = 5A$.

$a = 4m$

$r = \frac{a}{2} = 2m$.

Note:- Step 1: derive the general expression of $\overline{B}$ and $\overline{H}$ due to Square Current Loop.

Step 2. using the equation

$$\overline{H_{net}} = \frac{2\sqrt{2}\, I}{\pi a}\ \overline{a_z}\ A/m.$$

$$\overline{H_Q} = \frac{2\sqrt{2}(5)}{\pi(4)}\,\overline{a_z}\ A/m.$$

$$\overline{H_Q} = 1.125\,\overline{a_z}\ A/m.$$

$$\boxed{|\overline{H_0}| = 1.12539}\ A/m.$$

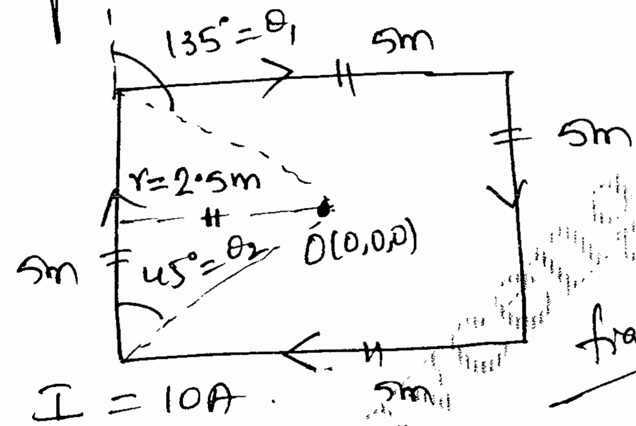
Problem 8. Find the magnetic flux density ($|\bar{B}|$) at the Centre of a square conductor of each side equal to 5m and carrying a current of 10A. take $\mu = 4\pi \times 10^{-7}$ H/m. 06-JJ 2012 (or) 06-DEC2011/Jan 2012

Find the magnetic flux density at the centre 'O' of a square of sides equal to 5m and carrying 10 amperes of current. (08 Marks)

Question.

Find the magnetic flux density at the Centre 'O' of a Square equal to 5m and carrying 10 A of current. (8m).

Sol^n:-



$I = 10A$.

fig. Square Current Loop at z=0 m plane.

Step 1. derive general Expression of $\bar{H}$ at centre of Square Current Loop.

Step 2. using above obtained expression

i.e $$\bar{H} = \frac{2\sqrt{2}\, I}{\pi a}\, \bar{a}_z \ \text{A/m}.$$

$a = 5m$ and $I = 10A$.

$$\overline{H}_{net} = \frac{2\sqrt{2}(10)}{\pi(5)} \overline{a}_z \quad A/m.$$

$$\boxed{\overline{H}_{net} = \underline{\underline{1.800632}}\, \overline{a}_z} \quad A/m.$$

$$\boxed{|\overline{H}_{net}| = 1.800632} \quad A/m.$$

problems

02 - June /July 2012

Ex. A circuit carrying direct current of 5 A forms a hexagon inscribed in a circle of radius 1 m. Calculate the magnetic flux density at the center of the current hexagon. Assume the medium is free space. (06 Marks)

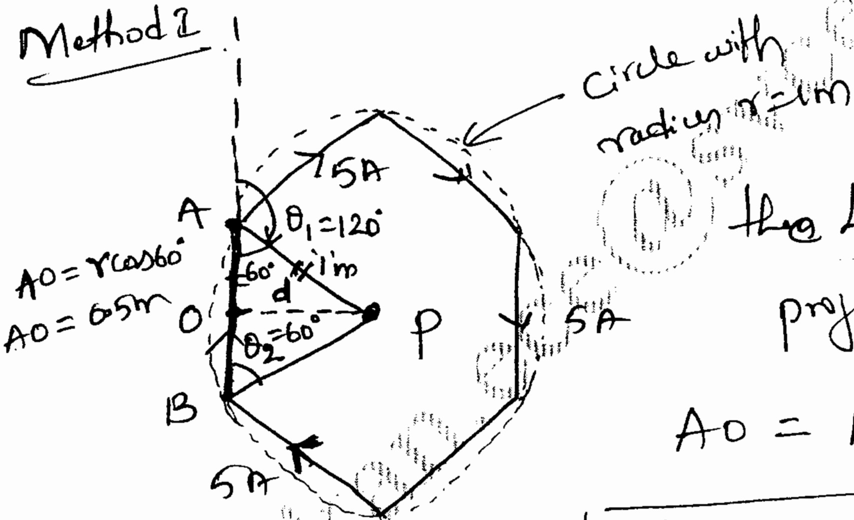
06 –June/July 2014

A circuit carrying current 5A, from rectangular hexagon inscribed in a circle of radius 1m. calculate B at the centre of hexagon. (04 Marks)

Question.

A Circuit carrying Current 5A, from rectangular hexagon inscribed in a circle of radius 1m. Calculate B at the center of hexagon.

Method I



the Length AO by using projection of AP on AO

$$AO = AP\cos\theta = r\cos\theta$$

$$\boxed{AO = 1\cos(60°) = 0.5}\ m$$

using pythagorous theorm

$$d^2 + (AO)^2 = 1^2$$

$$\Rightarrow d = \sqrt{1-(AO)^2} = \sqrt{1-0.5^2}$$

$$\boxed{d = 0.866m}$$

Magnetic flux density at a point 'p' due Current Carrying element BA is given by

$$B = \frac{\mu_0 I}{4\pi d}\left[\cos\theta_2 - \cos\theta_1\right]$$

$$B = \frac{\cancel{4\pi} \times 10^{-7} \times 5}{\cancel{4\pi}(0.866)}\left[\cos 60^\circ - \cos 120^\circ\right] \rightarrow 1$$

$$B = 5.77367 \times 10^{-7}\ wb/m^2.$$

$\therefore$ Magnetic flux density at point p due to Current in all six sides is

$$B = 6 \times 5.77367 \times 10^{-7}\ wb/m^2$$

$$\boxed{B = 3.46420 \times 10^{-6}}\ wb/m^2$$

or $B = 3.4642\ \mu wb/m^2$

Note:- for a Conductor in the form of regular polygon of n-side inscribed in a Circle of radius 'r' m

the Flux density B at the centre is

$$\boxed{B = \frac{\mu_0 n I}{2\pi r}\tan\left(\frac{\pi}{n}\right)}\ wb/m^2$$

Method II.

using above std. result.

$$B = \frac{\mu_0 n I}{2\pi r} \tan\left(\frac{\pi}{n}\right) \text{ wb/m}^2$$

given $I = 5A$, $\mu_0 = 4\pi \times 10^{-7}$ H/m.

hexagon no. of sides $\boxed{n=6}$

radius of $\odot$ $\boxed{r = 1m}$

$$B = \frac{4\cancel{\pi} \times 10^{-7} \times 6 \times 5}{2\cancel{\pi}(1)} \tan\left(\frac{\pi}{6}\right)$$

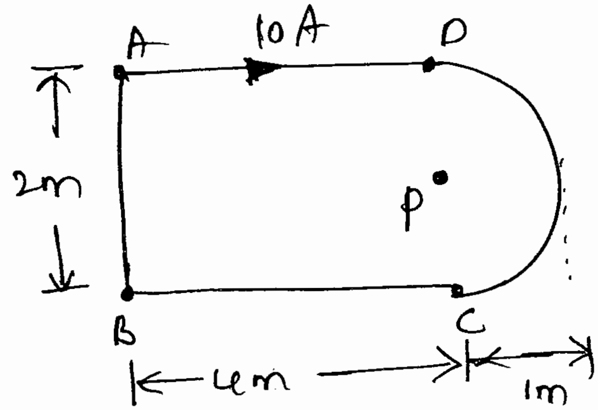
$$\boxed{B = 3.4641016 \times 10^{-6}} \text{ wb/m}^2.$$

(or)

$$\boxed{B = 3.4641 \,\mu \text{ wb/m}^2}$$

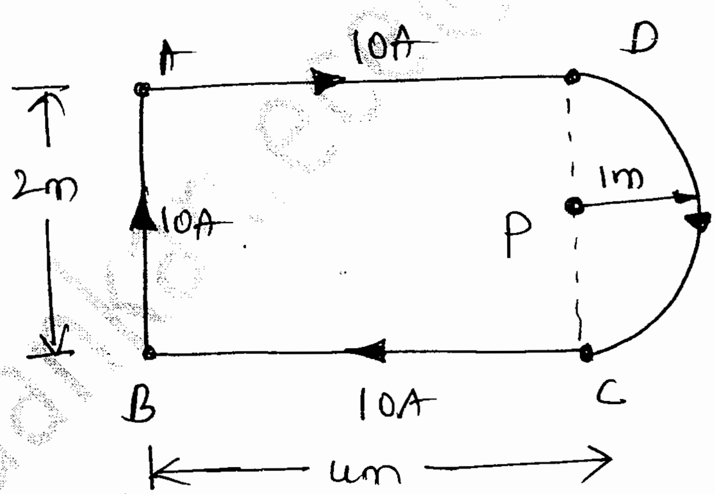
problem 10.

Find the value of the magnetic flux density at the point p for the Current Circuit shown below



Solu:- The magnetic field Intensity at point 'p' is

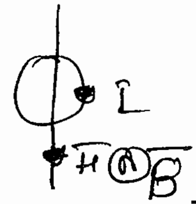
$$\overline{H_p} = \overline{H_{BA}} + \overline{H_{AD}} + \overline{H_{DC}} + \overline{H_{CB}}$$



Since Current (I) is in clockwise direction,

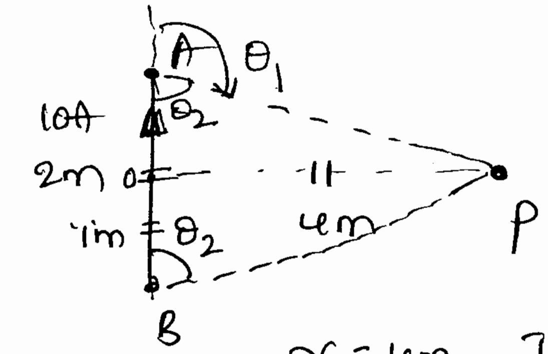
the dirn of $\overline{H}$ (a) $\overline{B} \rightarrow -\overline{a_z}$.

[using Flemming righthand rule]



(45)

→ $\overline{H_{BA}} = ?$



$r = 4m, \ I = 10A$

$$\theta_2 = \tan^{-1}\left(\frac{4}{1}\right) = \underline{75.9637^\circ}$$

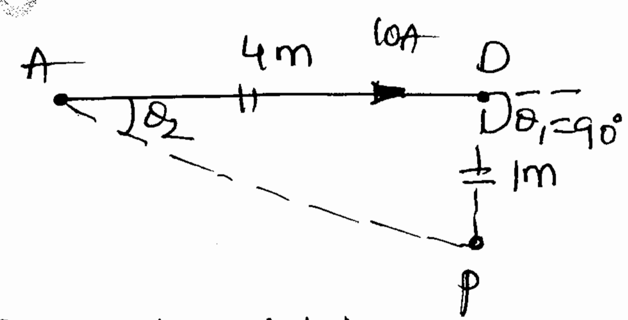
$$\theta_1 = 180^\circ - \theta_2 = \underline{104.0362^\circ}$$

$$H_{BA} = \frac{I}{4\pi r}\left[\cos\theta_2 - \cos\theta_1\right]$$

$$H_{BA} = \frac{10}{4\pi(4)}\left[\cos(75.963^\circ) - \cos(104.0362^\circ)\right]$$

$$\boxed{H_{BA} = 0.096504} \ A/m$$

→ $H_{AD} = ?$



$r = 1m.$

$\theta_1 = 90^\circ$

$$\theta_2 = \tan^{-1}\left(\frac{1}{4}\right) = 14.03624^\circ.$$

(46)

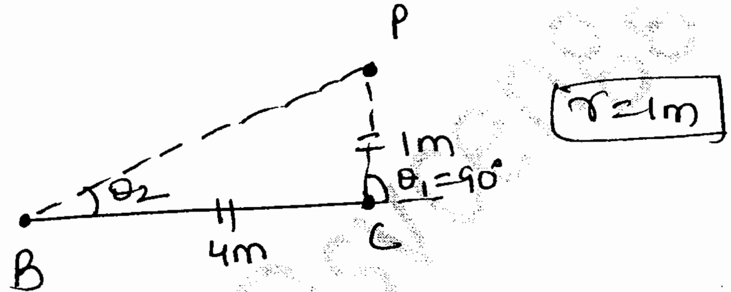
710

$$H_{AD} = \frac{I}{4\pi r}\left[\cos\theta_2 - \cos\theta_1\right]$$

$$H_{AD} = \frac{10}{4\pi(1)}\left[\cos(14.03624^\circ) - \cancel{\cos(90^\circ)}^{\,0}\right]$$

$$\boxed{H_{AD} = 0.772014}\ A/m$$

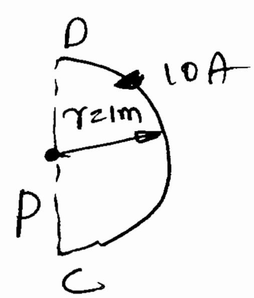
→ $H_{CB} = ?$



$$\boxed{H_{CB} = H_{AD} = 0.772014}\ A/m$$

→ $H_{DC} = ?$

$$H_{DC} = \frac{I}{4r}\ A/m$$ --- due Semi Circle.



$$H_{DC} = \frac{10}{4(1)}\ A/m$$

$$\boxed{H_{DC} = 2.5}\ A/m$$

(47)

711

Net field at point p

$$\overline{H_p} = \overline{H_{BA}} + \overline{H_{AD}} + \overline{H_{DC}} + \overline{H_{CB}}$$

$$\overline{H_p} = \left[-0.096504 + 0.772014 + 2.5 + 0.772014\right](-\overline{a_z})$$

$$\boxed{\overline{H_p} = -4.1405\,\overline{a_z}}\ A/m$$

the magnetic flux density at point 'p' is

$$\overline{B_p} = \overline{H_p}\cdot\mu_0 \quad Wb/m^2$$

$$\overline{B_p} = (-4.1405\,\overline{a_z})(4\pi\times10^{-7})$$

$$\boxed{\overline{B_p} = 5.203148\,\mu}\ Wb/m^2$$

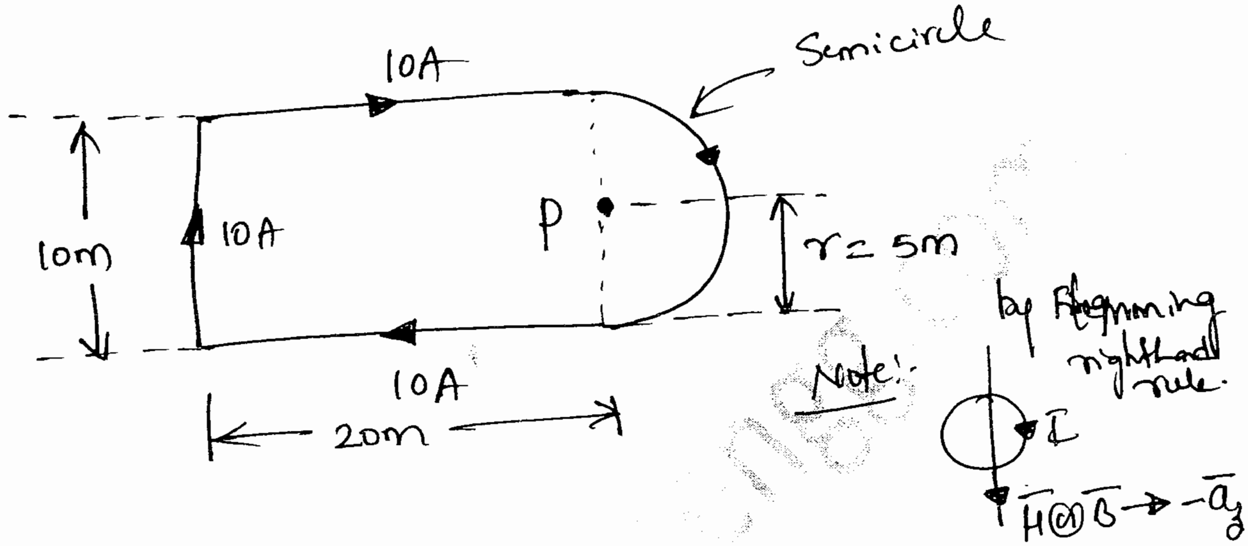
$$= 5.203148 \times 10^{-6}\ Wb/m^2$$

48

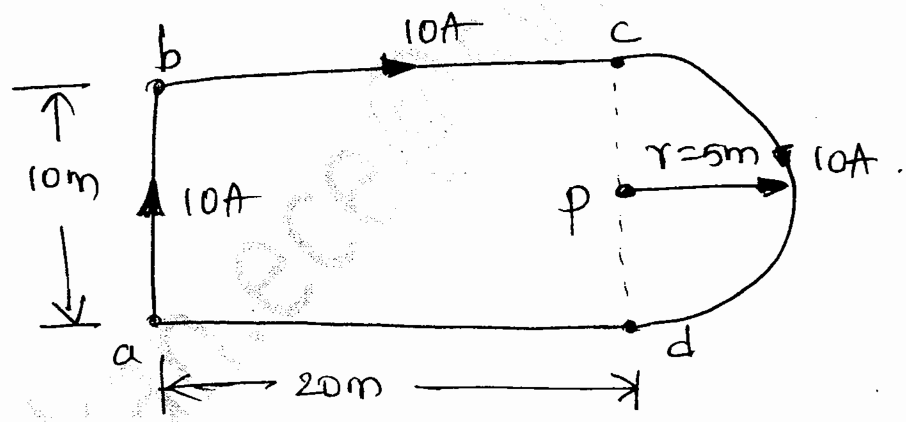
712

Question 11.

Find the magnetic field intensity at point p for the circuit shown in the fig.



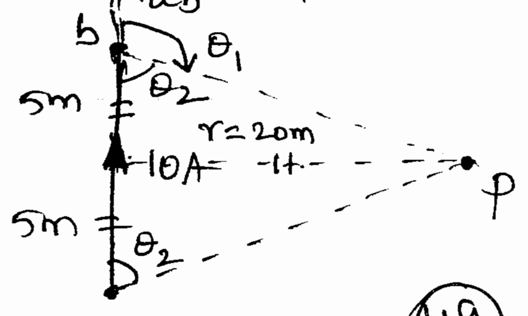
Solu:-



$$H_p = H_{\text{due to Current filament } ab} + H_{bc} + H_{cd} + H_{da} \text{ A/m}$$

$I = 10A$

Step 1. $H_{ab} = ?$



$\theta_2 = \tan^{-1}\left(\frac{20}{5}\right) = 75.963^\circ$

$\theta_1 = 180^\circ - \theta_2 = 104.036^\circ$

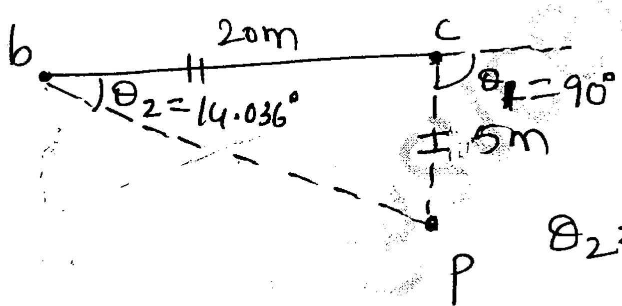
and $r = 20m$.

$$H_{ab} = \frac{I}{4\pi r}\left[\cos\theta_2 - \cos\theta_1\right] \text{ A/m}$$

$$H_{ab} = \frac{10}{4\pi(20)}\left[\cos(75.963^\circ) - \cos(104.036^\circ)\right]$$

$$\boxed{H_{ab} = 0.0193007} \text{ A/m.}$$

Step 2. $H_{bc} = ?$



$\theta_2 = \tan^{-1}\left(\frac{5}{20}\right) = 14.036^\circ$

$\theta_1 = 90^\circ$

$r = 5\text{m}.$

$$H_{bc} = \frac{I}{4\pi r}\left[\cos\theta_2 - \cos\theta_1\right] \text{ A/m.}$$

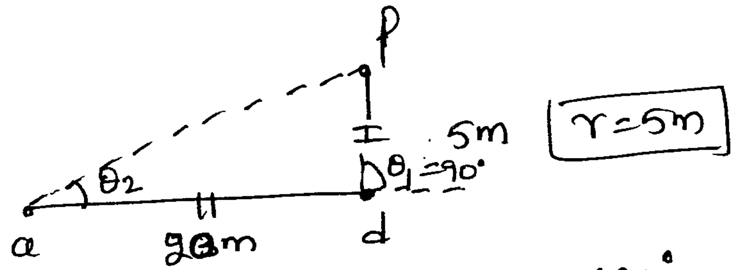
$$H_{bc} = \frac{10}{4\pi(5)}\left[\cos(14.036^\circ) - \overbrace{\cos(90^\circ)}^{0}\right]$$

$$\boxed{H_{bc} = 0.1544031} \text{ A/m}$$

$$= \underline{0.1544031} \text{ A/m}$$

50

Step 3. $H_{da} = ?$



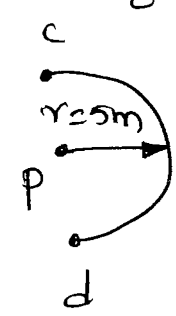
$$\theta_2 = \tan^{-1}\left(\frac{5}{20}\right) = 14.03624°$$

$$H_{da} = \frac{I}{4\pi r}\left[\cos\theta_2 - \cos\theta_1\right]$$

$$H_{da} = \frac{10}{4\pi\ (5)}\left[\cos(14.036°) - \cancel{\cos(90°)}^{\,0}\right]$$

$$\boxed{H_{da} = 0.1544031}\ A/m$$

Step 4. $H_{cd} = \frac{I}{4r}$ A/m ---- Semi circle Carrying current 'I' amperes



$$H_{cd} = \frac{10}{4(5)}\ A/m$$

$$\boxed{H_{cd} = 0.5}\ A/m$$

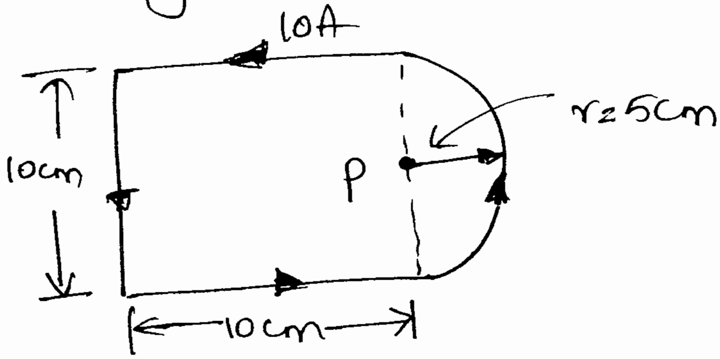
$$\bar{H}_{net} = \bar{H}_{P_{net}} = 0.0193007 + 0.154403 + 0.154403 + 0.5$$ ---- downwards direⁿ.

$$\bar{H}_{net} = 0.8281\ A/m$$ ---- acts downwards by Current is Carrying along clock wise dⁿ.

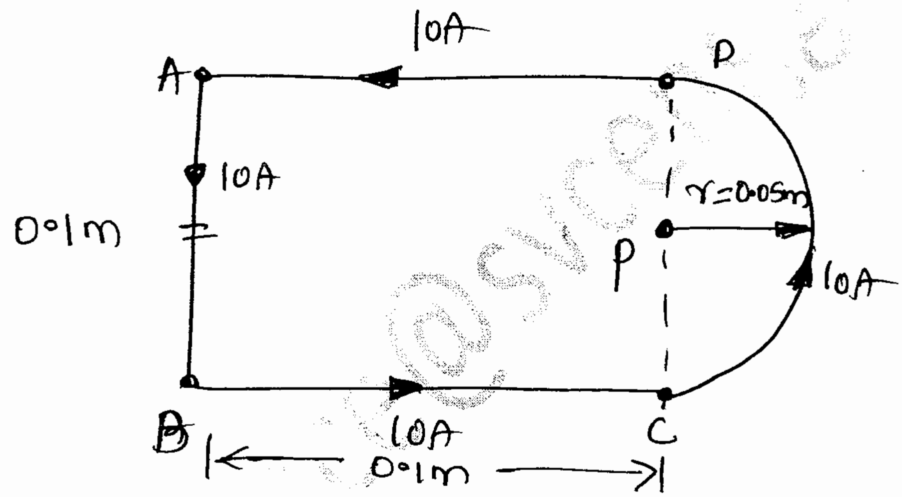
$$\boxed{\bar{H}_{net} = -0.8281\,\bar{a}_z\ A/m}$$

problem 12.

Find the magnetic field at point P in the fig. shown



Soln:

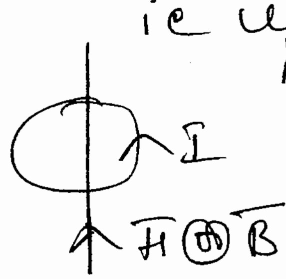


$$\overline{H}_{P_{net}} = \overline{H}_{AB} + \overline{H}_{BC} + \overline{H}_{CD} + \overline{H}_{DA} \ A/m.$$

Since the Current is in Anti-clockwise direction

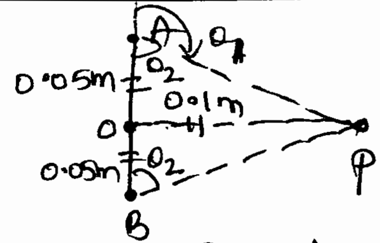
the $\overline{H}$ (or) $\overline{B}$ acts along $(+\overline{a}_z)$

i.e upward dirⁿ



(52)

$\rightarrow \overline{H}_{AB} = ?$



$\boxed{r = 0.1m}$

$$H_{AB} = \frac{I}{4\pi r}\left[\cos\theta_2 - \cos\theta_1\right]$$

$\theta_2 = \tan^{-1}\left(\frac{0.1}{0.05}\right)$

$\boxed{\theta_2 = 63.434^\circ}$

$\theta_1 = 180^\circ - \theta_2$

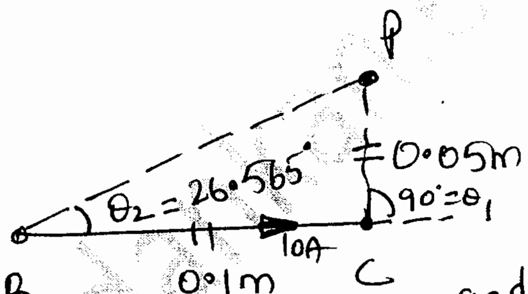
$\boxed{\theta_1 = 116.56^\circ}$

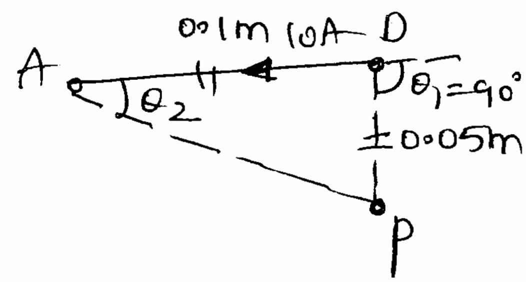
$$H_{AB} = \frac{10}{4\pi(0.1)}\left[\cos(63.434^\circ) - \cos(116.56^\circ)\right]$$

$$H_{AB} = 7.117 \ A/m$$

(or) $\overline{H}_{AB} = 7.117\,\overline{a}_z \ A/m.$

$\rightarrow H_{BC} = H_{DA} = ?$





$\theta_2 = \tan^{-1}\left(\frac{0.05}{0.1}\right)$ and $\boxed{r = 0.05m}$

$\boxed{\theta_2 = 26.565^\circ}$

$\theta_2 = 26.565^\circ,$

$\theta_1 = 90^\circ$

$r = 0.05m.$

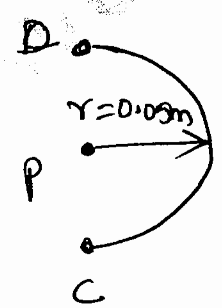
$$H_{BC} = H_{DA} = \frac{I}{4\pi r}\left[\cos\theta_2 - \cos\theta_1\right]$$

$$= \frac{10}{4\pi(0.05)}\left[\cos(26.565^\circ) - \cancel{\cos(90^\circ)}^{\,0}\right]$$

$$H_{BC} = H_{DA} = 14.235 \text{ A/m.}$$

$$\overline{H}_{BC} = \overline{H}_{DA} = \underline{\underline{14.235\,\overline{a}_z}} \text{ A/m}$$

$\rightarrow$ $H_{CD} = ?$



Semi arc circle.

$\boxed{r = 0.05}$ m

$$H_{CD} = \frac{I}{4r} \text{ A/m}$$

$$= \frac{10}{4(0.05)}$$

$$H_{CD} = 50 \text{ A/m}$$

$$\boxed{\overline{H}_{CD} = 50\,\overline{a}_z} \text{ A/m}$$

$$\overline{H}_{Pnet} = \overline{H}_{AB} + \overline{H}_{BC} + \overline{H}_{DA} + \overline{H}_{CD} = 7.117\overline{a}_z + 14.23\overline{a}_z + 14.23\overline{a}_z + 50\overline{a}_z$$

$$\boxed{\overline{H}_{Pnet} = 85.577\,\overline{a}_z} \text{ A/m}$$

problem 13

10 - June /July 2015

Calculate the magnetic field intensity at point P due to 10 A current flowing in the anticlockwise direction in the metallic block shown in Fig. Q4 (b). (06 Marks)

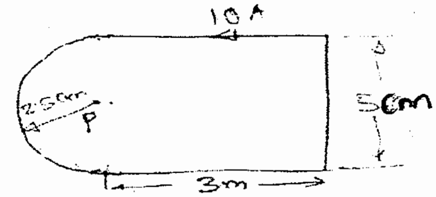
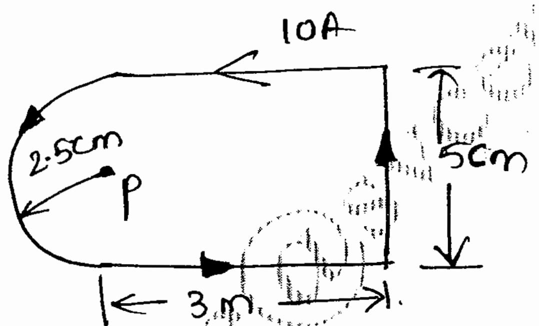


Fig. Q4 (b)

Question.

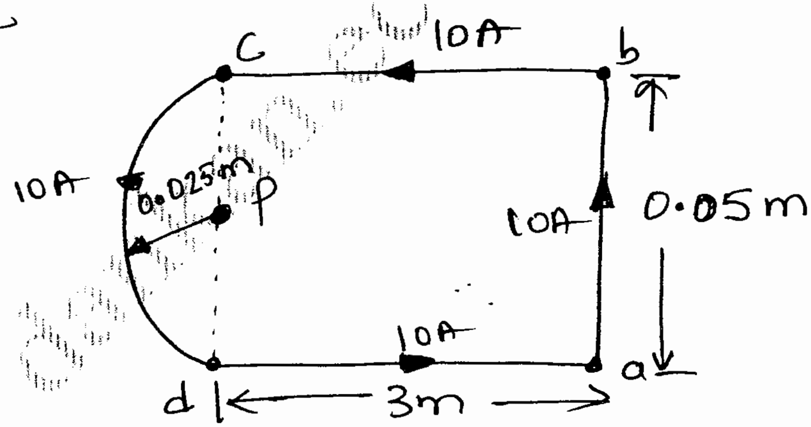
Calculate the magnetic field intensity at point P due to 10A Current flowing in the anticlockwise direction in the metallic block shown in fig. (6m)



I → Anti-clock wire direc^n

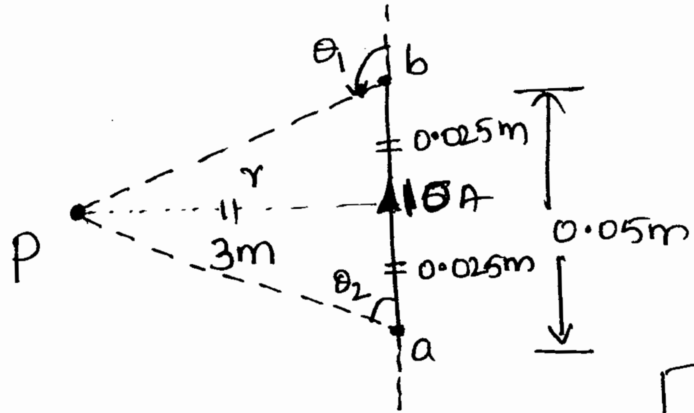
$\therefore \bar{H}$ @ $\bar{B} \rightarrow +\bar{a}_z$

Solu^n:



H at P = H due filament ab + H due to filament bc + H due to filament cd + H due to filament da.

→ B at a point p due to Current Carrying filament ab.



$$\tan\theta_2 = \frac{3}{0.025}$$

$$\boxed{\theta_2 = 89.5225^\circ}$$

$$\theta_1 = 180^\circ - \theta_2 = 180^\circ - 89.5225^\circ = 90.477^\circ$$

$$\boxed{r = 3m} \qquad \boxed{\theta_1 = 90.477^\circ}$$

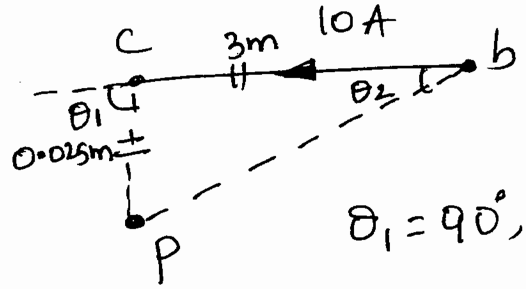
$$B_{ab} = \frac{\mu_0 I}{4\pi r}\left[\cos\theta_2 - \cos\theta_1\right] \ Wb/m^2$$

$$B_{ab} = \frac{\cancel{4\pi} \times 10^{-7}(10)}{\cancel{4\pi}(3)}\left[\cos(89.5225^\circ) - \cos(90.477^\circ)\right]$$

$$H_{ab} = 5.55299 \times 10^{-9} \ \cancel{Wb}/m^2$$

$$H_{ab} = 5.55299 \ n\cancel{Wb}/m^2$$

→ B at a point p due to Current Carrying filament bc.



$\theta_1 = 90^\circ$, and $r = 0.025m$

$r = 0.025m$

$$\tan\theta_2 = \frac{0.025}{3} \Rightarrow$$

$$\boxed{\theta_2 = 0.47745^\circ}$$

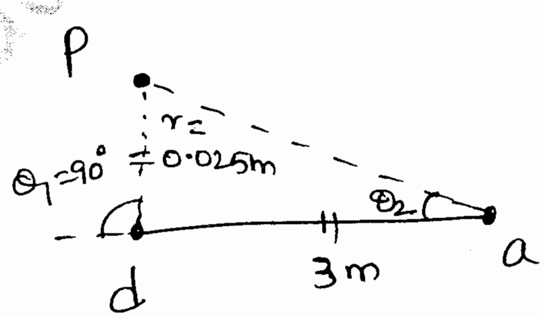
$$B_{bc} = \frac{\mu_0 I}{4\pi r}\left[\cos\theta_2 - \cos\theta_1\right] \text{ wb/m}^2$$

$$B_{bc} = \frac{\cancel{4\pi}\times 10^{-7}\times 10}{\cancel{4\pi}(0.025)}\left[\cos(0.47745^\circ) - \cancel{\cos(90^\circ)}^{\,0}\right]$$

$$B_{bc} = 39.9986\times 10^{-6} \text{ wb/m}^2$$

$$B_{bc} = 39.9986 \ \mu A/m$$

→ B at a point due to Current carrying filament da.



$\boxed{\theta_1 = 90^\circ}$ $r = 0.025m$

$$\theta_2 = \tan^{-1}\left(\frac{0.025}{3}\right)$$

$$\boxed{\theta_2 = 0.47745^\circ}$$

$$B_{da} = \frac{\mu_0 I}{4\pi r}\left[\cos\theta_2 - \cos\theta_1\right] \text{ wb/m}^2$$

$$\boxed{B_{da} = 39986\,\mu\, Wb/m^2} = 39986 \times 10^{-6}\ Wb/m^2$$

→ B due to Current Carrying element cd (i.e Semi Circle).

w.k.t $|\bar{H}|$ at the center of a Circular Current Loop is

$$|\bar{B}| = \frac{I\mu_0}{2r}\ Wb/m^2$$

$|\bar{B}|$ at the center of a Semicircular Loop with anticlockwise Current is

$$|\bar{B}| = \frac{I\mu_0}{4r}\ Wb/m^2$$

; $r = 0.025\,m$

$$B_{cd} = \frac{10 \times 4\pi \times 10^{-7}}{4(0.025)}\ Wb/m^2$$

$= 125.66\,\mu A/m.$

$$\boxed{B_{cd} = 125.66 \times 10^{-6}}\ Wb/m^2$$

the net field $|\bar{H}|$ at point P is given by

$$B_{net} = B_{ab} + B_{bc} + B_{cd} + B_{da}$$

$$B_{net} = 5.55299\,\mu + 39986\,\mu + 39986\,\mu + 125.66\,\mu$$

$$\boxed{B_{net} \simeq 205.637\,\mu Wb/m^2} = 205.63\,\bar{a}_z\ \mu Wb/m^2$$

58

@ point b $(H_{net} = \frac{B_{net}}{\mu_0} = 163.64\,A/m$ ...

problem 14

June/July 2016 EE

b. Determine magnetic flux density 'B' at 'P' for a current loop shown in Fig Q4(b). (09 Marks)

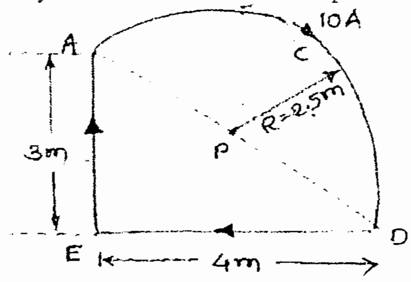
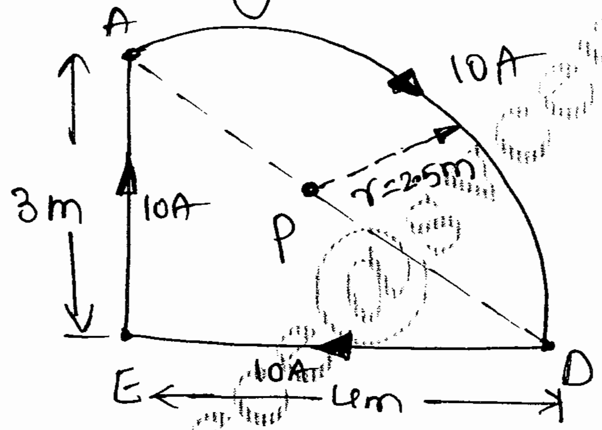


Fig. Q4(b)

problem (B)

Determine magnetic flux density at P for a current Loop shown in fig. (9m)



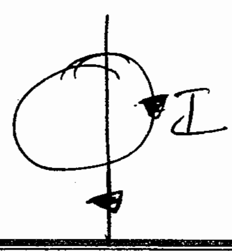
$|AD| = \sqrt{3^2+4^2} = 5m.$

Sol^n:-

$\bar{H}$ at point P

$$\bar{H}_P = \bar{H}_{AD} + \bar{H}_{DE} + \bar{H}_{EA}$$

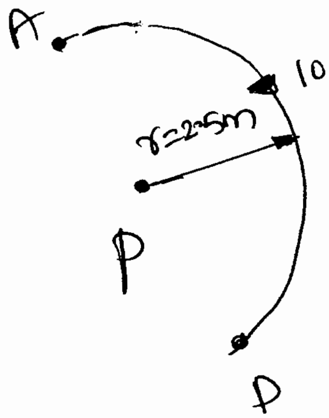
Since the Current is in clockwise direction



$\therefore \bar{H}$ & $\bar{B} \longrightarrow -\bar{a}_z$

acts downwards.

→ $\overline{H}_{AD} = ?$



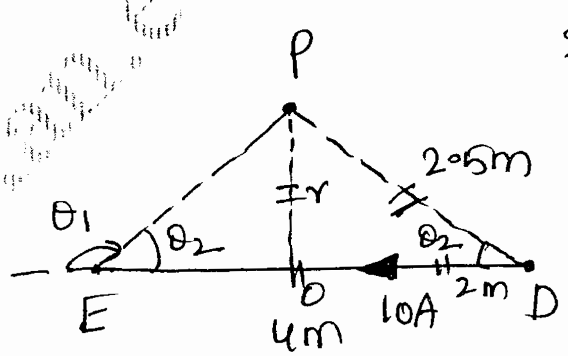
Semi arc circle.

$$\overline{H}_{AD} = \frac{I}{4r}(-\overline{a}_z)\ A/m$$

$$\overline{H}_{AD} = \frac{10}{4(2.5)}(-\overline{a}_z)\ A/m$$

$$\boxed{\overline{H}_{AD} = -\overline{a}_z}\ A/m$$

→ $\overline{H}_{DE} = ?$



$$2.5^2 = r^2 + 2^2$$

$$\Rightarrow r^2 = 2.5^2 - 2^2$$

$$\boxed{r = 1.5m}$$

$$\theta_2 = \tan^{-1}\left(\frac{r}{2}\right) = \tan^{-1}\left(\frac{1.5}{2}\right) = \underline{36.869^\circ}$$

$$\theta_1 = 180^\circ - \theta_2 = \underline{143.13^\circ}$$

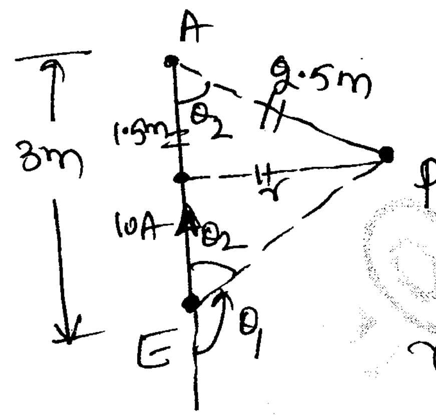
$$\overline{H}_{DE} = \frac{I}{4\pi r}\left[\cos\theta_2 - \cos\theta_1\right](-\overline{a}_z)$$

$$= \frac{10}{4\pi(1.5)}\left[\cos(36.869^\circ) - \cos(143.13^\circ)\right]$$

$$\boxed{\overline{H}_{DE} = -0.84883\,\overline{a}_z} \ A/m$$

→ $H_{EA} = ?$

$\theta_2 = 53.13^\circ$



$\tan\theta_2 = \left(\frac{r}{1.5}\right) \Rightarrow$

$$r^2 + 1.5^2 = 2.5^2$$

$$r^2 = 2.5^2 - 1.5^2$$

$$\boxed{r = 2m}$$

$$\boxed{\theta_1 = 180^\circ - \theta_2 = 126.869^\circ}$$

$$\overline{H}_{EA} = \frac{I}{4\pi r}\left[\cos\theta_2 - \cos\theta_1\right](-\overline{a}_z)$$

$$= \frac{10}{4\pi(2)}\left[\cos(53.13^\circ) - \cos(126.869^\circ)\right](-\overline{a}_z)$$

$$\boxed{\overline{H}_{EA} = -0.47746\,\overline{a}_z} \ A/m$$

$$\overline{H_p} = \overline{H_{AD}} + \overline{H_{DE}} + \overline{H_{EA}} \ A/m$$

$$\overline{H_p} = -\overline{a_z} - 0.84883\,\overline{a_z} - 0.47746\,\overline{a_z} \ A/m$$

$$\boxed{\overline{H_p} = -2.32629\,\overline{a_z}} \ A/m$$

$$|\overline{H_p}| = 2.32629 \ A/m$$

the fluxdensity $\overline{B_p}$ is given by

$$\overline{B_p} = \overline{H_p}\mu_0 = -2.32629 \times 4\pi \times 10^{-7} \ wb/m^2$$

$$\overline{B_p} = -2.9233 \times 10^{-6}\,\overline{a_z} \ wb/m^2$$

$$\boxed{\overline{B_p} = -2.9233\,\overline{a_z}\,\mu} \ wb/m^2$$

62

726

02-DEC2010

Find the magnetic field intensity and the magnetic flux density at P, as shown in the figure Q4 (c). (06 Marks)

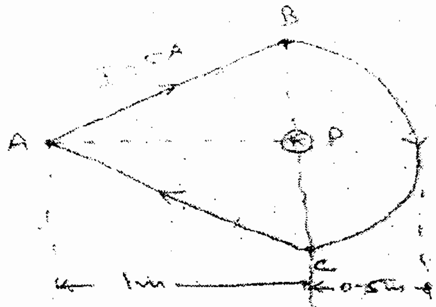


Fig. Q4 (c)

Dec/Jan 2017

c. Find the magnetic field intensity at the point P for the Fig Q5(c) shown below. (06 Marks)

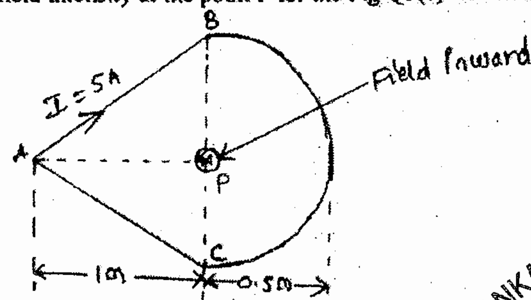
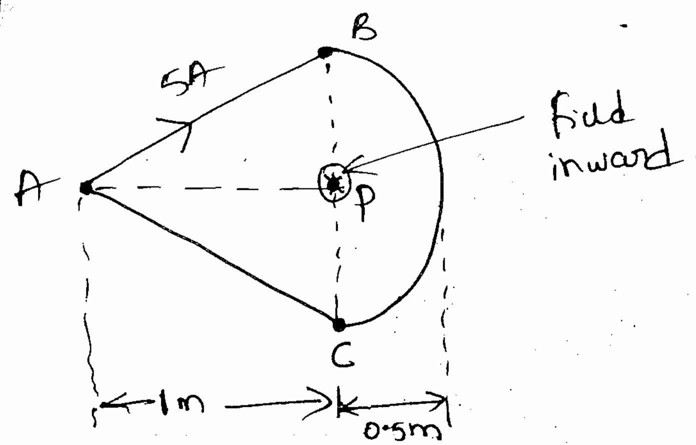


Fig Q5(c)

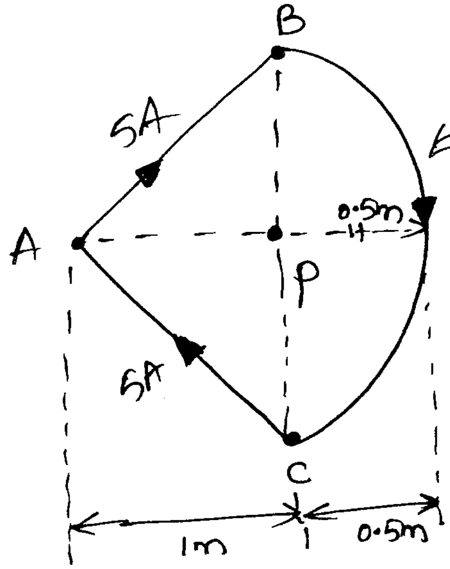
Question

Find the magnetic field intensity at the point p for the fig shown below (6m).

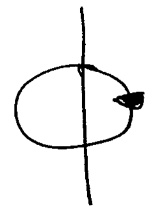


62-a

Solu:-



$I = 5A$.



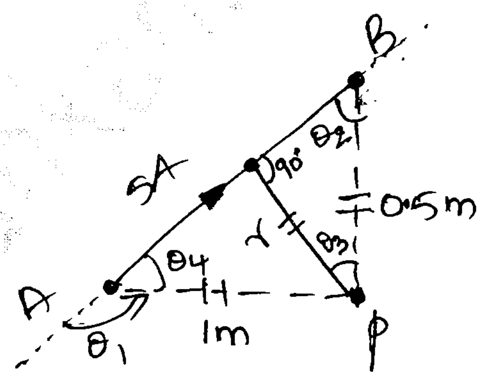
$\therefore \bar{H}$ or $\bar{B} \rightarrow (-\bar{a}_z)$

i.e acts inward dirⁿ.

the magnetic field intensity at a point 'p' is given by

$$\bar{H}_p = \bar{H}_{AB} + \bar{H}_{BC} + \bar{H}_{CA} \ \ A/m.$$

→ $\bar{H}_{AB}$:- field intensity at a point p due to current filament AB.



$AB = \sqrt{1^2 + 0.5^2} = 1.118m$

$\tan\theta_2 = \left(\frac{1}{0.5}\right)$

$$\boxed{\theta_2 = 63.434^\circ}$$

$$\theta_3 = 180 - 90^\circ - \theta_2 = 26.566^\circ$$

$$\boxed{\theta_3 = 26.566^\circ}$$

$\perp^r$ distance

$$r = 0.5\cos\theta_3 = 0.5\cos(26.566^\circ) = 0.4472m.$$

$$\boxed{r = 0.4472\,m} \qquad \theta_4 = \tan^{-1}\left(\frac{0.5}{1}\right) = 26.565^\circ$$

$$\boxed{\theta_1 = 180^\circ - \theta_4 = 153.434^\circ}$$

$$\bar{H}_{AB} = \frac{I}{4\pi r}\left[\cos\theta_2 - \cos\theta_1\right](-\bar{a}_z) \; A/m$$

$$\bar{H}_{AB} = \frac{5}{4\pi(0.4472)}\left[\cos(63.434^\circ) - \cos(153.434^\circ)\right](-\bar{a}_z)$$

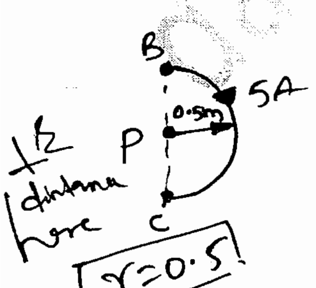
$$\boxed{\bar{H}_{AB} = 1.193704(-\bar{a}_z)} \; A/m.$$

→ $\bar{H}_{CA}$:- field intensity at a point 'p' due to the current filament CA.

$$\bar{H}_{CA} = \frac{I}{4\pi r}\left[\cos\theta_2 - \cos\theta_1\right](-\bar{a}_z) \; A/m$$

$$\Rightarrow \boxed{\bar{H}_{CA} = \bar{H}_{AB} = 1.193704(-\bar{a}_z)} \; A/m$$

→ $\bar{H}_{BC}$:- field intensity at a point 'p' due to the current filament BC (i.e. semi arc circle)



$$\bar{H}_{BC} = \frac{I}{4r}(-\bar{a}_z) \; A/m = \frac{5}{4(0.5)}(-\bar{a}_z)$$

$$\boxed{\bar{H}_{BC} = 2.5(-\bar{a}_z)} \; A/m$$

$$\bar{H}_p = \bar{H}_{AB} + \bar{H}_{BC} + \bar{H}_{CA} \; A/m \qquad (62\text{-}C)$$

$$\boxed{\bar{H}_p = 1.193704(-\bar{a}_z) + 1.193704(-\bar{a}_z) + 2.5(-\bar{a}_z) = 4.8874(-\bar{a}_z) \; A/m}$$

729

$\Rightarrow \bar{H}_p = 4.8874(-\bar{a}_z) \; A/m$

2. Ampere's circuital law

State and explain Ampere's circuital law. 10 - June /July 2014 (04 Marks)

State and explain Ampere's circuital law 06 – May/June 2010 (06 Marks)

State and prove Ampere's law. 10 - June /July 2015 (04 Marks)

June/July 2016 EE

a. State and explain Ampere's circuital law. (05 Marks)

Dec/Jan-2017

c. State and explain Amperes circuital law. (06 Marks)

Question

State and explain Ampere's Circuital Law. (6m).

[10 - June/July 2014, 06-May/June-2010, June/July 2016(EE), 10-June/July 2015, 10-Dec/Jan 2017.].

Topic 3.6

3.6. Ampere's Circuital Law :-

Statement :- The Line integral of $\overline{H}$ around a single Closed path is equal ~~to the~~ Current enclosed by that path.

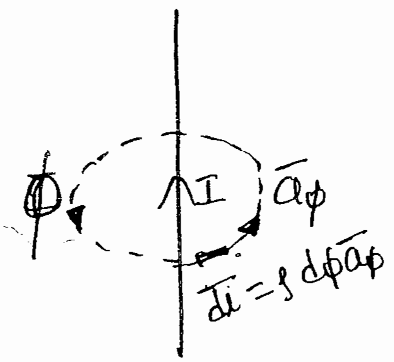


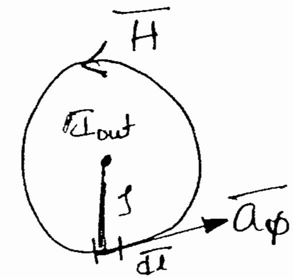
fig: Current carrying filament.

mathematically

$$\boxed{\oint_{\langle l \rangle} \overline{H} \cdot \overline{dl} = I} \leftarrow (a) \quad \text{Ampere's}$$

proof :- Consider a infinite ~~Length~~ Current Carrying filament is placed along 'z' axis. The magnetic field Intensity due to this is given by

$$\overline{H} = \frac{I}{2\pi\rho} \overline{a_\phi} \quad A/m$$



Circular path

$\overline{dl} = \rho\, d\phi\, \overline{a_\phi}$

Consider a L.H.S part of eq^n (a)

$$\oint_{\langle l \rangle} \overline{H} \cdot \overline{dl} = \int_{\phi=0}^{2\pi} \frac{I}{2\pi\rho} \overline{a_\phi} \cdot \rho\, d\phi\, \overline{a_\phi}$$

$$= \frac{I}{2\pi} \int_{\phi=0}^{2\pi} d\phi \; \overline{a_\phi} \cdot \overline{a_\phi} = \frac{I}{2\pi} \times 2\pi$$

$$= I = R.H.S$$

$$\boxed{\oint_{\langle l \rangle} \overline{H} \cdot \overline{dl} = I} \quad \text{Ampere's}$$

i.e

$$\boxed{\oint_{\langle l \rangle} \overline{H} \cdot \overline{dl} = I}$$

thus Ampere's Law is Verified

problem 15

06-DEC2010

If the magnetic field intensity in a region is H = (3y − 2)a_z + 2x a_y find the current density at the origin (06 Marks)

Question

if the magnetic field intensity in a region is $\overline{H} = (3y-2)\overline{a_z} + 2x\overline{a_y}$; find the Current density at the origin. (6M).

Solu:

given $\overline{H} = (3y-2)\overline{a_z} + 2x\overline{a_y}$ A/m.

using point form of Ampere's Circuital Law

$\overline{J} = \nabla \times \overline{H}$: A/m²

$$\nabla \times \overline{H} = \begin{vmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ \frac{\partial}{\partial x} & \partial/\partial y & \partial/\partial z \\ 0 & 2x & (3y-2) \end{vmatrix}$$

$$= \left[\frac{\partial}{\partial y}(3y-2) - \cancel{\frac{\partial(2x)}{\partial z}}^{\,0}\right]\overline{a_x} - \cancel{\frac{\partial(3y-2)}{\partial x}}^{\,0}\,\overline{a_y} + \frac{\partial(2x)}{\partial x}\,\overline{a_z}$$

$$\nabla \times \overline{H} = 3\overline{a_x} + 2\overline{a_z} \; A/m^2$$

$$\overline{J} = \nabla \times \overline{H} = 3\overline{a_x} + 2\overline{a_z} \; A/m^2.$$

Since $\overline{J}$ is a independent of spatial variable. ∴ $\overline{J}$ at origin

i) Same

$$\boxed{\overline{J_0} = \nabla \times \overline{H} = 3\overline{a_x} + 2\overline{a_z}} \; A/m^2$$

Problem 16.

06-DEC2008/Jan 2009

Magnetic field intensity in free space is $\vec{H} = 10\rho^2 \vec{a}_\phi (A/m)$ Determine

i) $\vec{J}$

ii) Integrate $\vec{J}$ over the circular surface $\rho = 1(m)$, all ϕ and $z = 0$. **(06 Marks)**

[06-Jan 2013 (6m)]

Question.

Magnetic field intensity in free space is $\overline{H} = 10\rho^2 \overline{a_\phi}$ A/m. Determine

i. $\overline{J}$

ii. Integrate $\overline{J}$ over the circular surface $\rho = 1m$, all ϕ and $z = 0m$. (6m).

Sol^n: Given $\overline{H} = 10\rho^2 \overline{a_\phi}$ A/m .... in Cylindrical C.S

using point form of Ampere's Law

$P(\rho, \phi, z)$ → $d\rho$, $\rho d\phi$, dz

$dv = \rho d\phi d\rho dz$

i.e $\overline{J} = \nabla \times \overline{H}$ A/m^2

$$\nabla \times \overline{H} = \frac{1}{\rho} \begin{vmatrix} \overline{a_\rho} & \rho\overline{a_\phi} & \overline{a_z} \\ \partial/\partial\rho & 0 & 0 \\ 0 & \rho[10\rho^2] & 0 \end{vmatrix}$$

$$= \frac{1}{\rho}\left[\frac{\partial[\rho(10\rho^2)]}{\partial\rho} - 0\right] \overline{a_z}$$

$$\nabla \times \overline{H} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(10\rho^3)\ \overline{a_z}$$

$$= \frac{10}{\cancel{\rho}} \cdot 3\rho^{\cancel{2}}\ \overline{a_z}$$

$$\nabla \times \overline{H} = 30\rho\ \overline{a_z}$$

Current density $\boxed{\overline{J} = \nabla \times \overline{H} = 30\rho\ \overline{a_z}}$ A/m².

ii) Integrate $\overline{J}$ ie $I = \oint_{\langle S \rangle} \overline{J} \cdot \overline{dS}$

given Circular Surface

$0 < \rho \leq 1m$
$0 < \phi < 2\pi$
and $z = 0m$ Surface.

$P(\rho, \phi, z)$
$d\rho \quad \rho\, d\phi \quad dz$

$$\overline{dS} = \rho\, d\rho\, d\phi\ \overline{a_z} \quad \text{-- } z = 0m \text{ surface}$$

$$I = \oint_{\langle S \rangle} \overline{J} \cdot \overline{dS} = \int_{\langle S \rangle} 30\rho\, \overline{a_z} \cdot \rho\, d\rho\, d\phi\ \overline{a_z}$$

$$= \int_{\rho=0}^{1} 30\rho^2 d\rho \int_{\phi=0}^{2\pi} d\phi\ \cancel{\overline{a_z} \cdot \overline{a_z}}^{\,1} = 10 \times 2\pi \times 1$$

$$= 20\pi \text{ Ampere's}$$

$$\boxed{I = \oint_{\langle S \rangle} \overline{J} \cdot \overline{dS} = 20\pi = 62.831} \text{ Ampere's}$$

Problem 17.

10 -Jan 2013

Given $\overline{H} = 20r^2 \overline{a\phi}$ A/m, determine the current density J also determine the total current that crosses the surface r = 1 m, $0 < \phi < 2\pi$ and z = 0 in cylindrical co-ordinate. (08 Marks)

Question.

Given $\overline{H} = 20r^2 \overline{a_\phi}$ A/m, determine the Current density $\overline{J}$ also determine the total Current that Crosses the Surface $r = 1m$, $0 < \phi < 2\pi$ and $z = 0$ in Cylindrical Co-ordinate. (8M)

Soln: Given $\overline{H} = 20r^2 \overline{a_\phi}$ A/m ... in cylindrical C.S

using point form of Ampere's Law

i.e $\overline{J} = \nabla \times \overline{H}$ A/m²

$P(r, \phi, z)$: dr, $r\,d\phi$, dz

$dv = r\,dr\,d\phi\,dz$

$$\nabla \times \overline{H} = \frac{1}{r}\begin{vmatrix} \overline{a_r} & r\overline{a_\phi} & \overline{a_z} \\ \partial/\partial r & 0 & 0 \\ 0 & r(20r^2) & 0 \end{vmatrix}$$

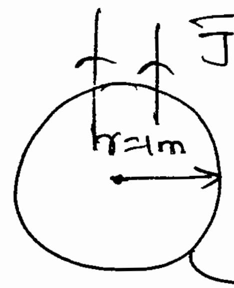
$$= \frac{1}{r}\left[\frac{\partial}{\partial r}\left[20r^3\right] - 0\right]\overline{a_z}$$

$$\nabla \times \overline{H} = \frac{1}{\cancel{r}} \cdot 20 \times 3r^{\cancel{2}} = 60r\,\overline{a_z} \text{ A/m}^2$$

Current density $\boxed{\overline{J} = \nabla \times \overline{H} = 60r\,\overline{a_z}}$ A/m^2

ii) the total current (I) crossing the surface

$$I = \oint_{\langle S \rangle} \overline{J} \cdot \overline{dS}$$



$\overline{J}$ $\overline{a_n} = +\overline{a_z}$

$r = 1m$

$0 < r < 1m$

$0 < \phi < 2\pi$

@ $z = 0m$ Surface

$$\overline{dS} = r\,dr\,d\phi\,\overline{a_z} \quad \cdots z = 0m \text{ Surface}$$

$$I = \oint_{\langle S \rangle} \overline{J} \cdot \overline{dS} = \int_{\langle S \rangle} 60r\,\overline{a_z} \cdot r\,dr\,d\phi\,\overline{a_z}$$

$$= \int_{r=0}^{1} 60r^2 dr \int_{\phi=0}^{2\pi} d\phi \; \cancel{\overline{a_z} \cdot \overline{a_z}}^{\,1}$$

$$= 20 \times 2\pi \times 1 = 40\pi \text{ Amperes}$$

$$\boxed{I = \oint_{\langle S \rangle} \overline{J} \cdot \overline{dS} = 40\pi = 125.66} \text{ Amperes}$$

problem 18.

10-DEC2011/Jan 2012

Calculate the value of vector current density at P(1.5, 90°, 0.5) if $H = \frac{2}{r}\cos 0.2\phi\, a_r$

(04 Marks)

Question.

Calculate the value of Vector Current density in cylindrical Co-ordinates at $p(1.5, 90^\circ, 0.5)$ if

$$\overline{H} = \frac{2}{\rho}\cos(0.2\phi)\,\overline{a_\rho} \quad A/m \;(\text{Wm})$$

06 - J/J 2009.

Sol^n:

Current density

$$\overline{J} = \nabla \times \overline{H} \quad A/m^2.$$

$$\overline{H} = \frac{2}{\rho}\cos(0.2\phi)\,\overline{a_\rho} \quad ..A/m \;.... \text{ in cylindrical C.S}$$

$P(\rho, \phi, z)$ → $d\rho$, $\rho\, d\phi$, dz

$$\nabla \times \overline{H} = \frac{1}{\rho}\begin{vmatrix} \overline{a_\rho} & \rho\overline{a_\phi} & \overline{a_z} \\ \partial/\partial\rho & \partial/\partial\phi & 0 \\ H_\rho & 0 & 0 \end{vmatrix}$$

$$\nabla \times \overline{H} = \frac{1}{\rho}\left[-\frac{\partial}{\partial\phi}(H_\rho)\right]\overline{a_z}$$

$$= \frac{-1}{\rho}\frac{\partial}{\partial\phi}\left[\frac{2}{\rho}\cos(0.2\phi)\right]\overline{a_z}$$

$$\nabla \times \overline{H} = \frac{-2}{\rho^2} \times -\sin(0.2\phi)\,(0.2)\,\overline{a_z}$$

$$\boxed{\overline{J} = \nabla \times \overline{H} = +\frac{0.4}{\rho^2}\sin(0.2\phi)\,\overline{a_z}} \ A/m^2$$

the current density at point $P(1.5, 90°, 0.5)$

i.e $\rho = 1.5m$, $\phi = 90°$

$$\overline{J_p} = \frac{0.4}{(1.5)^2}\sin(0.2 \times 90)\,\overline{a_z}$$

$$\boxed{\overline{J_p} = 0.05493\,\overline{a_z}} \ A/m^2$$

$$\boxed{|\overline{J_p}| = 0.054936} \ A/m^2$$

10 -Jan 2013

Calculate the value of vector current density in cylindrical co-ordinates at p(1.5, 90°, 0.5) if $\vec{H} = \frac{2}{\rho}\cos 0.2\phi \ \vec{a\phi}$. (06 Marks)

Question

Calculate the value of Vector Current density in cylindrical Co-ordinates at $p(1.5, 90°, 0.5)$ if

$$\overline{H} = \frac{2}{\rho}\cos(0.2\phi)\,\overline{a_\phi}\ A/m$$

Soln:- Given $\overline{H} = \frac{2}{\rho}\cos(0.2\phi)\,\overline{a_\phi}\ A/m.$

$H_\phi = \frac{2}{\rho}\cos(0.2\phi)\ A/m.$ - given in cylindrical C.S

$$\overline{J} = \nabla\times\overline{H} = \frac{1}{\rho}\begin{vmatrix} \overline{a_\rho} & \rho\overline{a_\phi} & \overline{a_z} \\ \partial/\partial\rho & \partial/\partial\phi & 0 \\ 0 & H_\phi & 0 \end{vmatrix}$$

$$\overline{J} = \nabla\times\overline{H} = \frac{1}{\rho}\frac{\partial H_\phi}{\partial\rho}\,\overline{a_z}$$

$$\overline{J} = \nabla\times\overline{H} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left[\frac{2}{\rho}\cos(0.2\phi)\right]\overline{a_z}\ A/m^2$$

$$= \frac{1}{\rho}\cdot\frac{-2}{\rho^2}\cdot\cos(0.2\phi)\,\overline{a_z}\ A/m^2.$$

$$\boxed{\bar{J} = \nabla \times \bar{H} = \frac{-2}{\rho^3} \cos(0.2\phi)\, \bar{a}_z} \ A/m^2$$

the current density at point $p(1.5, 90°, 0.5)$.

ie $\rho = 1.5m$ and $\phi = 90°$

$$\bar{J}_p = -\frac{2}{(1.5)^3} \cos(0.2 \times 90°)\, \bar{a}_z$$

$$\boxed{\bar{J}_p = -0.56358\, \bar{a}_z} \ A/m^2$$

$$\boxed{|\bar{J}_p| = +0.56358} \ A/m^2$$

problem 19

06 - June /July 2011

Calculate the value of the vector current density at point P(2, 3, 4) if
$\vec{H} = x^2 z \vec{a}_y - y^2 x \vec{a}_z$ (06 Marks)

Question

Calculate the value of the vector current density at point $p(2,3,4)$ if $\overline{H} = x^2 z \overline{a_y} - y^2 x \overline{a_z}$ A/m. (6m)

Solu:-

$$\overline{H} = x^2 z \overline{a_y} - y^2 x \overline{a_z} \text{ A/m} \quad \text{in cartesian C.S.}$$

Current density $\overline{J} = \nabla \times \overline{H}$ A/m^2.

$$\nabla \times \overline{H} = \begin{vmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & H_y & H_z \end{vmatrix}$$

$$\nabla \times \overline{H} = \left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right]\overline{a_x} - \left[\frac{\partial H_z}{\partial x} - 0\right]\overline{a_y} + \left[\frac{\partial H_y}{\partial x} - 0\right]\overline{a_z}$$

$$\bar{J} = \nabla \times \bar{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\bar{a_x} - \frac{\partial H_z}{\partial x}\bar{a_y} + \frac{\partial H_y}{\partial x}\bar{a_z}$$

$H_x = 0$; $H_y = x^2 z$ A/m; $H_z = -y^2 x$

$\frac{\partial H_y}{\partial z} = x^2$; $\frac{\partial H_y}{\partial x} = 2xz$;

$\frac{\partial H_z}{\partial y} = -2yx$; A/m²

$\frac{\partial H_z}{\partial x} = -y^2$; A/m²

$$\boxed{\bar{J} = \nabla \times \bar{H} = (-2yx - x^2)\,\bar{a_x} + y^2\,\bar{a_y} + 2xz\,\bar{a_z}} \text{ A/m}^2$$

Current density at point $p(2, 3, 4)$

i.e $x=2$, $y=3$, and $z=4$.

$$\bar{J_p} = \nabla \times \bar{H} = [-2(3)(2) - (2)^2]\,\bar{a_x} + (3)^2\bar{a_y} + 2(2)(4)\bar{a_z}$$

$$\boxed{\bar{J_p} = -16\,\bar{a_x} + 9\,\bar{a_y} + 16\,\bar{a_z}} \text{ A/m}^2$$

$$|\bar{J_p}| = 24.3515 \text{ A/m}^2$$

problem 20

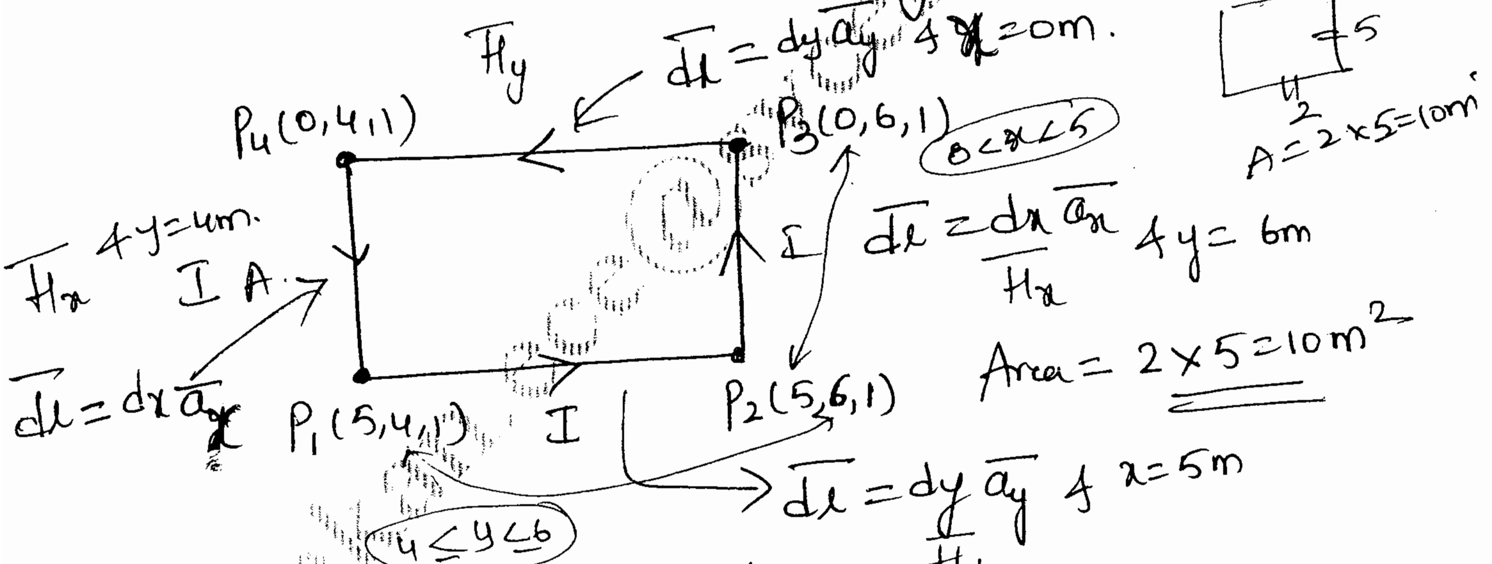
10 - June /July 2014

The magnetic field intensity is given by $H = 0.1y^3 X + 0.4xZ$ A/m. Determine current flow through the path $P_1(5, 4, 1) - P_2(5, 6, 1) - P_3(0, 6, 1) - P_4(0, 4, 1)$ and current density J.

(08 Marks)

Question.

The magnetic field intensity in given by $\overline{H} = 0.1 y^3 \overline{a_x} + 0.4\, x \overline{a_z}$ A/m. Determine Current Flow through the path $P_1(5,4,1) - P_2(5,6,1) - P_3(0,6,1) - P_4(0,4,1)$ and Current density J. (8m).



using Ampere's Circuital Law

$$I = \oint_{\langle l \rangle} \overline{H} \cdot \overline{dl}$$

$$I = \cancel{I_{P_{12}}}^{\,0} + I_{P_{23}} + \cancel{I_{P_{34}}}^{\,0} + I_{P_{41}}$$

bcz $H_y = 0$ in given field H

$$I_{P_{23}} = \int_{x=5}^{0} \overline{H_x} \cdot \overline{dl} = \int_{x=5}^{0} 0.1 y^3 \overline{a_x} \cdot dx\, \overline{a_x} \Big|_{y=6m}.$$

$$= 0.1(6)^3 [0-5]\ \cancel{\overline{a_x} \cdot \overline{a_x}}^{\,1}$$

$$\boxed{I_{P_{23}} = -108\,A.}$$

$$I_{P_{41}} = \int_{x=0}^{5} 0.1 y^3\, \overline{a_x} \cdot dx\, \overline{a_x} \Big|_{y=4m}$$

$$= 0.1(4)^3 \times 5 \times \cancel{\overline{a_x} \cdot \overline{a_x}}^{\,1}$$

$$\boxed{I_{P_{41}} = 32} \text{ Amperes}$$

$$I_{net} = I_{P_{23}} + I_{P_{41}} = -108 + 32$$

$$\boxed{I_{net} = -76} \text{ Amperes}$$

the magnitude of Current density $J = \frac{I}{Area} = \frac{-76}{10}$

$$\boxed{J = -7.6}\ A/m^2$$

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Topic 3.7 Application of Ampere's Circuital Law

3.7a. $\bar{H}$ due to infinitely Long Straight-Current carrying filament.

Questions.

06-DEC2011/Jan 2012

State and prove Ampere's circuital law. By applying it obtain expression for $\bar{H}$ due to infinitely long straight conductor (08 Marks)

02 - June /July 2011

State ampere's circuital law. Apply this law to find magnetic field, H due to an infinitely long straight conductor carrying a steady current of I, amps. (07 Marks)

Solu:- Step 1. State and prove Ampere's circuital Law.

i.e $$\boxed{\oint_{\langle L \rangle} \bar{H} \cdot \bar{dl} = I_{enclosed}}$$ Ampere's

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Applications of Ampere's Circuital Law.

3.7a) $\overline{H}$ due to infinitely Long straight Current Carrying filament:-

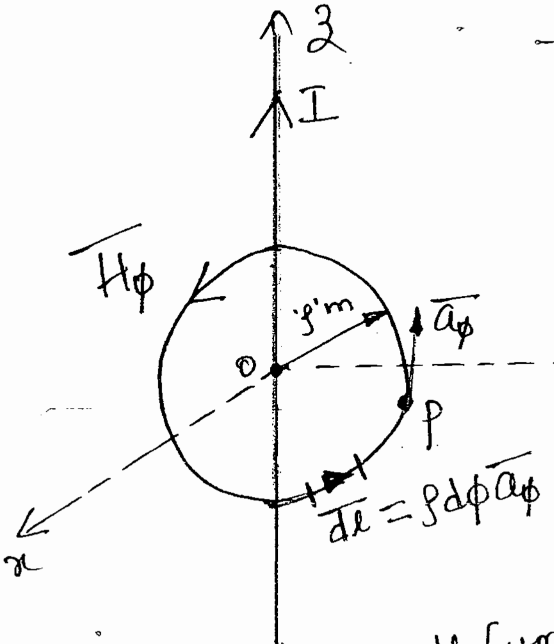


fig. infinite length Current Carrying filament.

the Magnitude of $\overline{H}$ depends on 'ρ' and the direction is always tangential to the closed path.

i.e $\overline{a_\phi}$. So $\overline{H}$ has only Component in $\overline{a_\phi}$ direc^n H_ϕ.

i.e $\overline{H} = H_\phi \overline{a_\phi}$ A/m. ← (a)

and

$$\overline{dl} = \rho d\phi \overline{a_\phi}$$

$$\overline{H} \cdot \overline{dl} = H_\phi \overline{a_\phi} \cdot \rho d\phi \overline{a_\phi}$$

$$\overline{H} \cdot \overline{dl} = H_\phi \rho d\phi \overbrace{\overline{a_\phi} \cdot \overline{a_\phi}}^{1} = \rho H_\phi d\phi$$

using Ampere's Circuital Law

$$\therefore \text{ i.e } \oint_{\langle l \rangle} \overline{H} \cdot \overline{dl} = I = \int_{\phi=0}^{2\pi} \rho H_\phi d\phi = \rho H_\phi \int_{\phi=0}^{2\pi} d\phi$$

$$I = \rho H_\phi (2\pi) \Rightarrow \boxed{H_\phi = \frac{I}{2\pi\rho}} \text{ A/m.}$$

← (b)

eq^n (b) in eq^n (a)

ie

$$\therefore \quad \overline{H} = H_\phi \, \overline{a_\phi} \quad A/m$$

$$\Rightarrow \boxed{\overline{H} = \frac{I}{2\pi\rho} \overline{a_\phi}} \; A/m$$

3.7b

3.7b Magnetic field Intensity (H) in a Co-axial Cable.

*06-DEC2010

A coaxial cable with radius of inner conductor a, inner radius of outer conductor b and outer radius c carries a current I at inner conductor and –I in the outer conductor. Determine and sketch variation of H against r for i) r < a ii) a < r < b iii) b < r < c iv) r > c (10 Marks)

10-June/July 2013

In an infinitely long coaxial cable carrying a uniformely current I in the inner conductor and -I in the outer conductor, find the magnetic field intensity is a function of radius and sketch the field intensity variation. (07 Marks)

06 - June /July 2012

In a Co-axial line, radius of inner conductor is 'a' m, inner radius of outer conductor is 'b' m and outer radius of outer conductor is 'c' m. Inner and outer conductors carry current I and -I respectively. Using Ampere circuit law, find magnetic field intensity for r < a; a < r < b, b < r < c; r > c cases. Sketch the variation of field intensity versus distance. (08 Marks)

Question.

In a co-axial line, radius of inner condutor is 'a' m, inner radius of outer condutor is 'b' m and outer radius of outer condutor is 'c' m. Inner and outer condutor carry current I and -I respectively. Using Ampere circuit Law, find magnetic field intensity for i> $r < a$; ii> $a < r < b$ iii> $b < r < c$ iv> $r > c$ cases. Sketch the variation of field Intensity versus distance. (8m).

[06 Dec 2010, 10 J/J 2013, 06-J/J 2012]

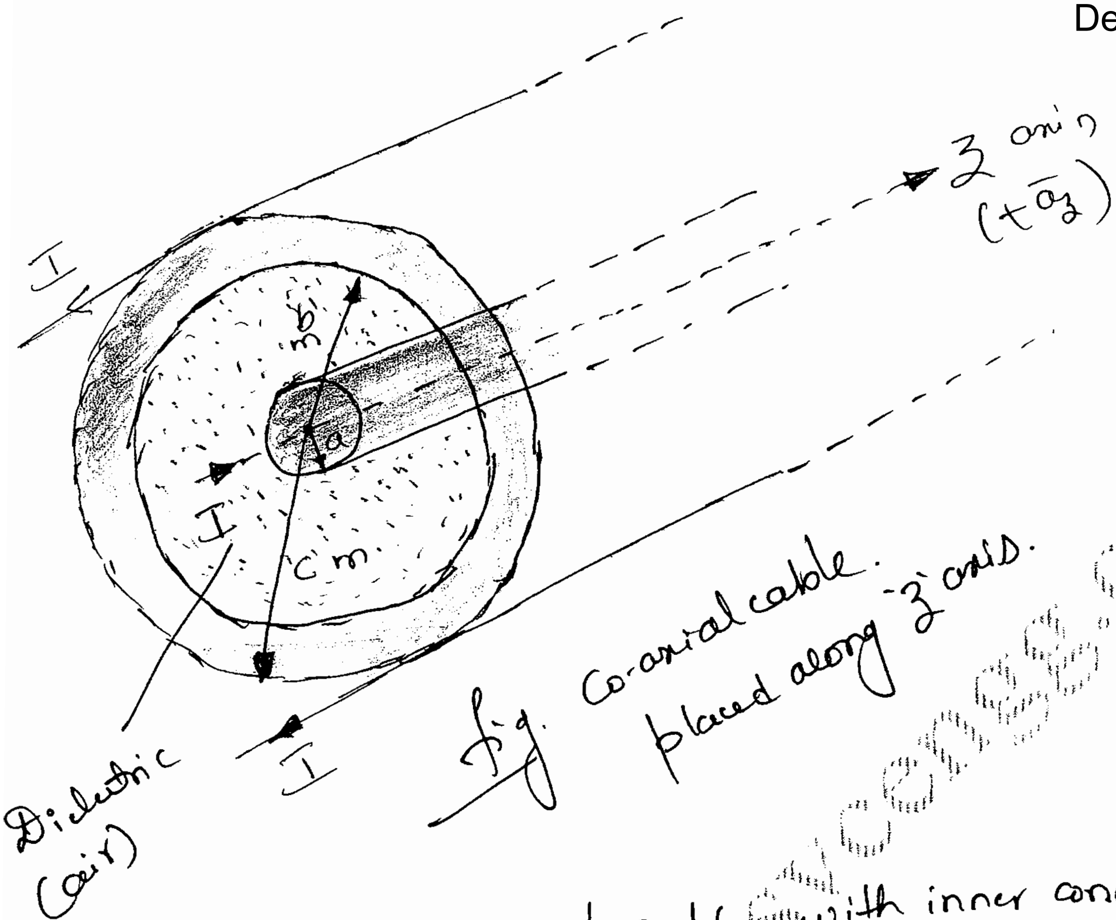


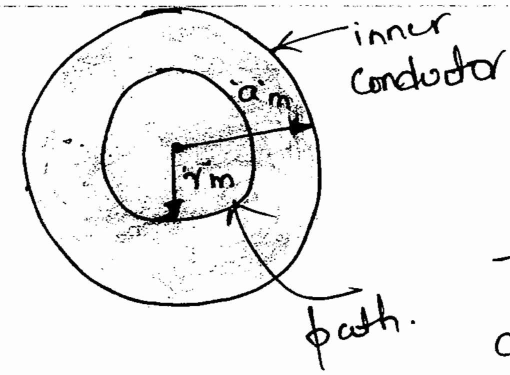
fig. Co-axial cable placed along 'z' axis.

Consider a co-axial cable with inner conductor is solid having a radius 'a' m, carrying a direct current of I ampere's.

the outer conductor is in the form of concentric cylinder whose inner radius is b and outer radius 'c' meter respectively.

The current I is uniformly distributed in the inner conductor. while $-I$ is uniformly distributed in the outer conductor.

Case i. $r < a$.



the area of cross-section enclosed is $\pi r^2\ m^2$.

the total current is 'I' through the area πa^2. hence the current enclosed by the closed path is

$$\boxed{I_p = \frac{\pi r^2}{\pi a^2} I = \frac{r^2}{a^2} I}\ \text{Ampere's.}$$

the $\overline{H}$ ~~act~~ along only $\overline{a_\phi}$ direction.

$$\therefore\ \overline{H} = H_\phi \overline{a_\phi}.$$

$$\overline{dl} = r\,d\phi\,\overline{a_\phi} \quad \ldots \text{in circular path direction.}$$

using Ampere's Circuital Law

$$\oint_{\langle l \rangle} \overline{H} \cdot \overline{dl} = I'$$

$$\oint_{\langle l \rangle} H_\phi \overline{a_\phi} \cdot r\,d\phi\,\overline{a_\phi} = \frac{r^2}{a^2} I.$$

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$$\int_{\phi=0}^{2\pi} H_\phi \, r \, d\phi = \frac{r^2}{a^2} I$$

$$H_\phi \cdot r \cdot 2\pi = \frac{r^2}{a^2} I$$

$$\therefore \boxed{H_\phi = \frac{r}{2\pi a^2} I} \text{ A/m}$$

$$\Rightarrow \boxed{\overline{H} = H_\phi \overline{a_\phi} = \frac{I r}{2\pi a^2} \overline{a_\phi}} \text{ A/m.}$$

Case i.
$r < a$.
(within conductor).

Case ii. $(a < r < b)$.

When 'r' is in b/w 'a' and 'b' ie $a < r < b$.

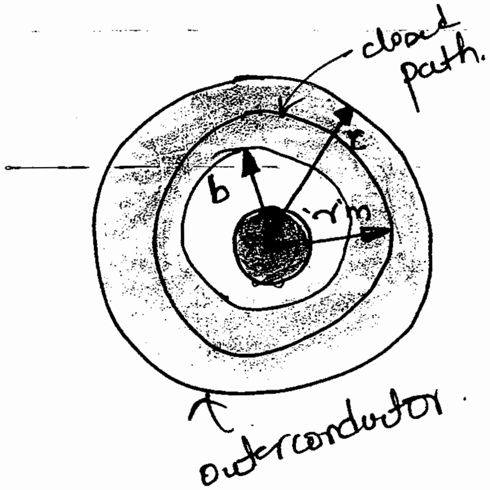
it is similar to the case of conductor carrying a direct current of I along the 'z' axis having infinite length.

$\therefore \overline{H}$ in this region is $\boxed{\overline{H} = \frac{I}{2\pi r} \overline{a_\phi}}$ A/m

... $a < r < b$

(83)

Case iii. within outer condutor i.e. $b < r < c$.



$b < r < c$.

Consider the closed path as shown in the fig. the current enclosed by the closed path is only the part of the current $-I$, in the outer conductor, the total current $-I$ is flowing through the cross section $\pi(c^2-b^2)$ while the closed path encloses the cross section $\pi(r^2-b^2)$.

Hence the current enclosed by the closed path of outer conductor is

$$I' = \frac{\pi(r^2-b^2)}{\pi(c^2-b^2)}(-I) = -\frac{(r^2-b^2)}{(c^2-b^2)} I.$$

and also the closed path encloses the inner conductor hence the current I flowing through it

$I'' = I$ = current in inner conductor enclosed total Current enclosed by the closed path is

$$I_{enc} = I' + I'' = -\frac{(r^2-b^2)}{(c^2-b^2)} I + I$$

$$I_{enc} = I\left[1 - \frac{r^2-b^2}{c^2-b^2}\right]$$

$$I_{enc} = I\left[\frac{c^2 - \cancel{b^2} - r^2 + \cancel{b^2}}{c^2-b^2}\right]$$

$$\boxed{I_{enc} = I\left[\frac{c^2-r^2}{c^2-b^2}\right]} \text{ Amperes.}$$

Using Ampere's Circuital Law

i.e
$$\oint_{\langle l \rangle} \bar{H} \cdot \bar{dl} = I_{enc}$$

$$\bar{H} = H_\phi \bar{a}_\phi \text{ and } \bar{dl} = r\,d\phi\,\bar{a}_\phi .$$

$$\oint_{\langle l \rangle} \bar{H} \cdot \bar{dl} = H_\phi (2\pi r) = I\left[\frac{c^2-r^2}{c^2-b^2}\right]$$

(87)

$$\boxed{H_\phi = \frac{I}{2\pi r}\left[\frac{c^2 - r^2}{c^2 - b^2}\right]} \ A/m$$

$$\therefore \boxed{\overline{H} = H_\phi \overline{a_\phi} = \frac{I}{2\pi r}\left[\frac{c^2 - r^2}{c^2 - b^2}\right]\overline{a_\phi}\ A/m.}$$

$$\cdots (b < r < c)$$

Case iV. Outside the cable $r > C$.

Since the total current outside the cable is zero.

i.e $I_t = -I + I = 0 \ A$.

$$\therefore \oint_{\langle l \rangle} \overline{H} \cdot \overline{dl} = 0$$

$$\Rightarrow \boxed{\overline{H} = 0} \ A/m.$$

The magnetic field doesnot exist outside the cable.

The variation of $\overline{H}$ against 'r' is shown in fig below.

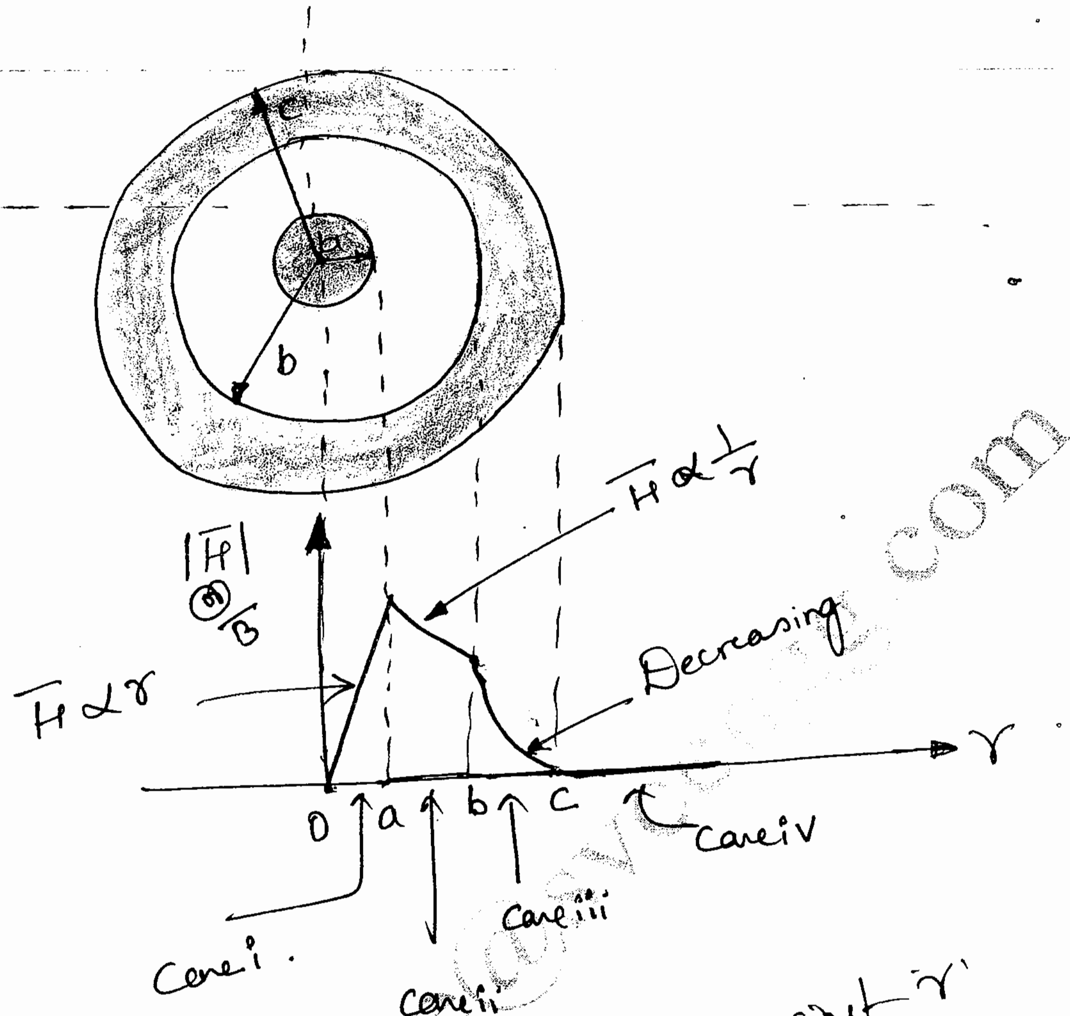


fig: Variation of $\overline{H}$ against 'r' in co-axial cable.

$$\overline{H} = \begin{cases} \dfrac{I r}{2\pi a^2}\overline{a_\phi} \; ; & r<a \\ \dfrac{I}{2\pi r}\overline{a_\phi} \; ; & a<r<b \\ \dfrac{I}{2\pi r}\left[\dfrac{c^2-r^2}{c^2-b^2}\right]\overline{a_\phi} \; ; & b<r<c \\ 0 \; ; & \text{ow.} \end{cases} \quad A/m.$$

Topic 3.7C

3.7C] $\bar{H}$ in b/w a Toroidal coil

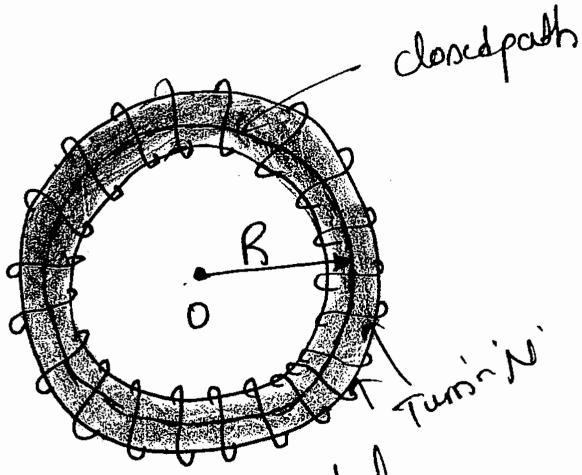


fig. Toroidal coil with Current 'I' Ampere

Consider a toroidal coil of N turns. and the Current I Flows through the coil.

using Ampere's Circuital Law

$$\oint_{\langle 1 \rangle} \bar{H} \cdot \bar{dl} = NI \text{ ; Amperes}$$

$$\oint_{\langle 1 \rangle} H \, dl \cos\theta = NI$$

field H is constant over the coil

$$H \oint_{\langle 1 \rangle} dl \cos\theta = NI$$

the closed path is circle of radius 'R' m

$$\oint_{\langle 1 \rangle} dl \cos\theta = 2\pi R \text{ ... perimeter}$$

$$\oint_{\langle \lambda \rangle} dl \cos\theta = 2\pi R$$

$$\Rightarrow \quad H(2\pi R) = NI$$

$$\boxed{H = \frac{NI}{2\pi R}} \quad A/m$$

and $\quad B = \mu H \quad wb/m^2$

$$\boxed{B = \frac{NI\mu}{2\pi R}} \quad wb/m^2$$

problem 2)

Dec/Jan 2017

a. An air cored torroid having a cross sectional area of 6cm^2 and mean radius 15cm is wound uniformly with 500 turns carrying a current of 4A. Determine the magnetic flux density and field intensity of torroid. (06 Marks)

Soln:

given $A = 6\,cm^2 = 6 \times (10^{-2})^2\, m^2$

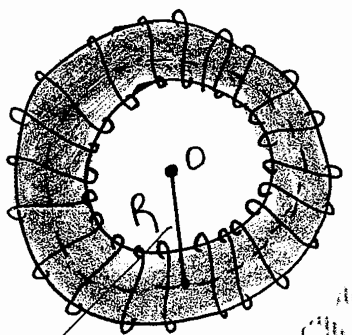
$A = 6 \times 10^{-4}\, m^2.$

$R = 15\,cm = 0.15\,m.$

$N = 500.$

$I = 4A.$

$Area = \pi r^2$



$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{6 \times 10^{-4}}{\pi}}$

$r = 0.0138\, m.$

but $R = 0.15\,m.$

$r \ll R.$

$$\therefore B = \frac{\mu_0 \mu_r N I}{2\pi R} = \frac{4\pi \times 10^{-7} \times 1 \times 500 \times 4}{2\pi (0.15)}$$

$$B = 2.666 \times 10^{-3}\ \text{wb/m}^2$$

$$\boxed{B = 2.667}\ \text{m wb/m}^2$$

$$H = \frac{B}{\mu_0 \mu_r} = \frac{2.667 \times 10^{-3}}{4\pi \times 10^{-7} \times 1}$$

$$\boxed{H = 2.122}\ \text{kA/m}$$

(or) $H = 2122.065$ A/m

Topic 3.8

- **Concept of Curl + Curl in all three co-ordinate systems+Point form of Ampere's Law**

Question

06 - June /July 2013

Explain the concept of curl with suitable derivation of curl F (06 Marks)

Explain the Concept of Curl with Suitable derivation of Curl $\bar{F}$. (6m). J/J 2013.

(or)

obtain the differential form of Ampere's work Law, in a Steady magnetic field (8m) 02 Dec 2010.

(or)

prove that ampere's Circuital Law $\nabla \times \bar{H} = \bar{J}$ A/m^2 (7m) 10-Jan 2014.

(or)

S.T $\nabla \times \bar{H} = \bar{J}$ A/m^2

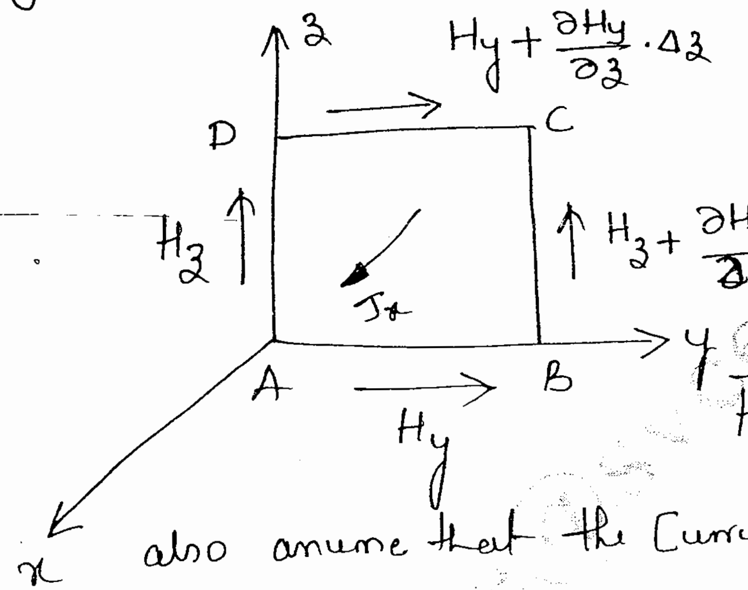
(or)

$\nabla \times \bar{B} = \mu_0 \bar{J}$ wb/m^3.

3.8 a. Application of Ampere's Law to Differential Elements of ie point form of Ampere's Law] (or) Concept of Curl.

$$\boxed{\nabla \times \bar{H} = \bar{J}} \ A/m^2$$

Let's consider ~~a differential~~ surface in Cartesian Co-ordinate system in yz plane as shown in fig.



Let's consider a region in which thin surface is having the magnetic field intensity as

$$\bar{H} = H_x \bar{a_x} + H_y \bar{a_y} + H_z \bar{a_z} \ A/m \quad \leftarrow (1)$$

also assume that the current density in a region is given by

$$\bar{J} = J_x \bar{a_x} + J_y \bar{a_y} + J_z \bar{a_z} \ A/m^2 \quad \leftarrow (2)$$

Let $\bar{H}$ along side AB equal H_y and along side AD equal H_z. if the field is not uniform, the values of $\bar{H}$ along side BC and CD are given by

$$H_z + \frac{\partial H_z}{\partial y} \Delta y \quad \text{and} \quad H_y + \frac{\partial H_y}{\partial z} \Delta z.$$

then using Ampere's Law

$$\oint_{\langle 1 \rangle} \overline{H} \cdot \overline{dl} = \int_A^B \overline{H} \cdot \overline{dl} + \int_B^C + \int_C^D + \int_D^A = J_x \Delta y \Delta z$$

$$= \Delta I \quad \longleftarrow (3)$$

i.e $\int_A^B = H_y \Delta y$

$$\left.\begin{aligned} \int_B^C &= \left(H_z + \frac{\partial H_z}{\partial y} \cdot \Delta y\right) \Delta z = H_z \Delta z + \frac{\partial H_z}{\partial y} \Delta y \Delta z \\ \int_C^D &= -\left(H_y + \frac{\partial H_y}{\partial z} \cdot \Delta z\right) \Delta y = -H_y \Delta y - \frac{\partial H_y}{\Delta z} \cdot \Delta z \Delta y \\ \int_D^A &= -H_z \Delta z \end{aligned}\right\} \longleftarrow (4)$$

using set (4) in eq^n (3)

$$\oint_{\langle 1 \rangle} \overline{H} \cdot \overline{dl} = \cancel{H_y} \Delta y + \cancel{H_z} \Delta z + \frac{\partial H_z}{\partial y} \Delta y \Delta z - \cancel{H_y} \Delta y - \frac{\partial H_y}{\partial z} \Delta z \Delta y - \cancel{H_z} \Delta z$$

$$= J_x \Delta y \Delta z = \Delta I$$

i.e $\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) \Delta \cancel{y} \Delta z = J_x \Delta \cancel{y} \Delta z$

or

$$\boxed{\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = J_x} \longleftarrow (5)$$

My Now by taking differential areas in xy plane and xz planes, we can prove that.

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = J_z \leftarrow (6) \quad \text{and}$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = J_y \leftarrow (7)$$

using eq^n (2) i.e

$$\overline{J} = J_x \overline{a_x} + J_y \overline{a_y} + J_z \overline{a_z}$$

(5), (6) and (7) in eq^n (2)

$$\overline{J} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\overline{a_x} + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\overline{a_y} + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\overline{a_z}$$

$$\Rightarrow \boxed{\overline{J} = \nabla \times \overline{H}} \quad A/m^2$$

point form of Ampere's Law.

using the relation $\overline{B} = \mu_0 \overline{H}$ wb/m^2

(or) $\overline{H} = \frac{\overline{B}}{\mu_0}$ A/m

$$\Rightarrow \overline{J} = \nabla \times \frac{\overline{B}}{\mu_0} \Rightarrow \boxed{\nabla \times \overline{B} = \mu_0 \overline{J}},\ wb/m^3$$

3.7b Curl :- When ∇ operates on vector $\overline{H}$ as a Crossproduct results in Curl $\nabla \times \overline{H}$.

- Curl : $\nabla \times \overline{H}$ A/m^2.

Curl in all three Co-ordinate System :-

a. → Cartesian Co-ordinate System :-

$$P(x, y, z) \quad x \to dx,\; y \to dy,\; z \to dz$$

$$\nabla = \frac{\partial}{\partial x}\overline{a_x} + \frac{\partial}{\partial y}\overline{a_y} + \frac{\partial}{\partial z}\overline{a_z} \quad m^{-1}$$

$$\overline{H} = H_x\overline{a_x} + H_y\overline{a_y} + H_z\overline{a_z} \quad A/m.$$

$$\nabla \times \overline{H} = \begin{vmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix} \quad A/m^2$$

XX.

$$\boxed{\nabla \times \overline{H} = \left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right]\overline{a_x} - \left[\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right]\overline{a_y} + \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right]\overline{a_z} \quad A/m^2}$$

→ Curl $\nabla \times \overline{H}$ in Cylindrical Co-ordinate System :-

$$P(\rho, \phi, z) \quad d\rho \quad \rho\, d\phi \quad dz \qquad dv = \rho\, d\rho\, d\phi\, dz$$

$$\nabla = \frac{\partial}{\partial \rho}\overline{a_\rho} + \frac{1}{\rho}\frac{\partial}{\partial \phi}\overline{a_\phi} + \frac{\partial}{\partial z}\overline{a_z} \quad m^{-1}$$

$$\overline{H} = H_\rho \overline{a_\rho} + H_\phi \overline{a_\phi} + H_z \overline{a_z} \quad A/m$$

$$\nabla \times \overline{H} = \frac{1}{\rho}\begin{vmatrix} \overline{a_\rho} & \rho\overline{a_\phi} & \overline{a_z} \\ \partial/\partial\rho & \partial/\partial\phi & \partial/\partial z \\ H_\rho & \rho H_\phi & H_z \end{vmatrix}$$

$$\nabla \times \overline{H} = \frac{1}{\rho}\left\{\left[\frac{\partial H_z}{\partial \phi} - \frac{\partial(\rho H_\phi)}{\partial z}\right]\overline{a_\rho} - \left[\frac{\partial H_z}{\partial \rho} - \frac{\partial H_\rho}{\partial z}\right]\rho\overline{a_\phi} + \left[\frac{\partial}{\partial \rho}[\rho H_\phi] - \frac{\partial H_\rho}{\partial \phi}\right]\overline{a_z}\right\}$$

$$\boxed{\nabla \times \overline{H} = \left[\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z}\right]\overline{a_\rho} - \left[\frac{\partial H_z}{\partial \rho} - \frac{\partial H_\rho}{\partial z}\right]\overline{a_\phi} + \left[\frac{1}{\rho}\frac{\partial(\rho H_\phi)}{\partial \rho} - \frac{1}{\rho}\frac{\partial H_\rho}{\partial \phi}\right]\overline{a_z}} \quad A/m^2$$

iii → Curl in Spherical Co-ordinate System.

$$p(r,\theta,\phi) \quad dr, \; r\,d\theta, \; r\sin\theta\,d\phi \qquad dv = r^2\sin\theta\,dr\,d\theta\,d\phi$$

$$\nabla = \frac{\partial}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial}{\partial\theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\bar{a}_\phi \quad m^{-1}$$

$$\bar{H} = H_r\bar{a}_r + H_\theta\bar{a}_\theta + H_\phi\bar{a}_\phi \quad A/m$$

$$\nabla\times\bar{H} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \bar{a}_r & r\bar{a}_\theta & r\sin\theta\,\bar{a}_\phi \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ H_r & rH_\theta & r\sin\theta H_\phi \end{vmatrix} \quad A/m^2$$

$$\nabla\times\bar{H} = \frac{1}{r^2\sin\theta}\left\{\left[\frac{\partial}{\partial\theta}(r\sin\theta H_\phi) - \frac{\partial}{\partial\phi}(rH_\theta)\right]\bar{a}_r - \left[\frac{\partial}{\partial r}[r\sin\theta H_\phi] - \frac{\partial H_r}{\partial\phi}\right]r\bar{a}_\theta + \left[\frac{\partial(rH_\theta)}{\partial r} - \frac{\partial H_r}{\partial\theta}\right]r\sin\theta\,\bar{a}_\phi\right\}$$

$$= \left\{\left[\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}[\sin\theta H_\phi] - \frac{1}{r\sin\theta}\frac{\partial(H_\theta)}{\partial\phi}\right]\bar{a}_r - \left[\frac{1}{r}\frac{\partial(rH_\phi)}{\partial r} - \frac{1}{r\sin\theta}\frac{\partial H_r}{\partial\phi}\right]\bar{a}_\theta + \left[\frac{1}{r}\frac{\partial(rH_\theta)}{\partial r} - \frac{1}{r}\frac{\partial H_r}{\partial\theta}\right]\bar{a}_\phi\right\}$$

$$\nabla \times \overline{H} = \left\{ \left[\frac{1}{r\sin\theta} \frac{\partial}{\partial\theta}\left[\sin\theta H_\phi\right] - \frac{1}{r\sin\theta} \frac{\partial H_\theta}{\partial\phi} \right] \overline{a_r} - \left[\frac{1}{r}\frac{\partial}{\partial r}(rH_\phi) - \frac{1}{r\sin\theta}\frac{\partial H_r}{\partial\phi} \right] \overline{a_\theta} + \left[\frac{1}{r}\frac{\partial(rH_\theta)}{\partial r} - \frac{1}{r}\frac{\partial H_r}{\partial\theta} \right] \overline{a_\phi} \right\} \ A/m^2$$

i.e

$$\boxed{\nabla \times \overline{H} = \left\{ \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta H_\phi) - \frac{\partial H_\theta}{\partial\phi} \right] \overline{a_r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial H_r}{\partial\phi} - \frac{\partial(rH_\phi)}{\partial r} \right] \overline{a_\theta} + \frac{1}{r}\left[\frac{\partial(rH_\theta)}{\partial r} - \frac{\partial H_r}{\partial\theta} \right] \overline{a_\phi} \right\}} \ A/m^2$$

Problem 22.

06-DEC2009/Jan 2010

In cylindrical coordinates, a magnetic field is given as $H = [4\rho - 2\rho^2]a_\phi$ A/m, $0 \le \rho \le 1$.

i) Find the current density or a function of ρ within cylinder.

ii) Find the total current that passes through the surface $Z = 0$ and $0 \le \rho \le 1$ m in the a_z direction. (08 Marks)

Question.

In cylindrical co-ordinates, a magnetic field is given as $\overline{H} = [4\rho - 2\rho^2]\,\overline{a_\phi}$ A/m, $0 \le \rho \le 1$ m.

i. Find the Current density (or) a function of ρ within cylinder.

ii. Find the total Current that passes through the Surface $z=0$ and $0 \le \rho \le 1$ m. in the $\overline{a_z}$ direction.

Soln:

given $\overline{H} = (4\rho - 2\rho^2)\,\overline{a_\phi}$ A/m

$\overline{H} = H_\phi\,\overline{a_\phi}$ --- in cylindrical C.S

i. $J = ?$ using point form of Ampere's Law

$$\overline{J} = \nabla \times \overline{H} \quad A/m^2.$$

$$\nabla \times \overline{H} = \frac{1}{\rho}\begin{vmatrix} \overline{a_\rho} & \rho\,\overline{a_\phi} & \overline{a_z} \\ \partial/\partial\rho & 0 & 0 \\ 0 & \rho H_\phi & 0 \end{vmatrix} \quad A/m^2$$

$$\nabla \times \overline{H} = \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}(\rho H_\phi) - 0\right]\overline{a_z}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho(4\rho - 2\rho^2)\right]\overline{a_z}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[4\rho^2 - 2\rho^3\right]\overline{a_z}$$

$$= \frac{1}{\rho}\left[4(2\rho) - 6\rho^2\right]\overline{a_z}$$

$$\nabla \times \overline{H} = \frac{1}{\rho}\left[8\rho - 6\rho^2\right]\overline{a_z}$$

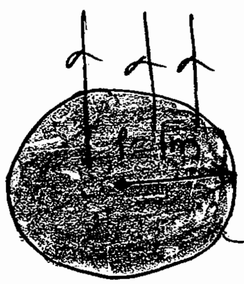
$$\boxed{\nabla \times \overline{H} = (8 - 6\rho)\overline{a_z}} \; A/m^2$$

$$\boxed{\overline{J} = J_z \overline{a_z}} \; A/m^2.$$

∴ the Current density in terms of 'ρ' is

$$\boxed{\overline{J} = \nabla \times \overline{H} = (8 - 6\rho)\overline{a_z}} \; A/m^2$$

ii) the total current passes through the surface $z = 0\,m$ and $0 \le \rho \le 1\,m$ in $\overline{a_z}$ direcⁿ is



$\overline{a_n} = +\overline{a_z}$ $P(\rho, \phi, z)$ $d\rho$ $\rho d\phi$ dz

$\overline{ds} = \rho\, d\rho\, d\phi\, \overline{a_z}$

$0 < \rho < 1$
$0 < \phi < 2\pi$

$\rightarrow z = 0\,m$; constant surface.

$$I = \oint_{\langle S \rangle} \overline{J} \cdot \overline{ds}$$

$$I = \oint_{\langle S \rangle} (8 - 6\rho)\overline{a_z} \cdot \rho\, d\rho\, d\phi\, \overline{a_z}$$

$$I = \int_{\rho=0}^{1} (8 - 6\rho)\rho\, d\rho \int_{\phi=0}^{2\pi} d\phi \qquad \cancel{\overline{a_z} \cdot \overline{a_z}}^{\,1}$$

$$I = 2 \times 2\pi \times 1$$

$$I = 4\pi \text{ Amperes}$$

$$\boxed{I = 4\pi = 12.5663} \text{ Amperes}$$

problem 23

06-DEC 2013/Jan 2014

Given $J = 10^3 \sin\theta \, a_r$ A/m^2 in spherical coordinate system. Find the current crossing the spherical shell of $r = 0.02$ m, where r = radius of shell. (04 Marks)

Question

Given $\overline{J} = 10^3 \sin\theta \, \overline{a_r}$ A/m^2 in Spherical co-ordinate System. Find the Current crossing the spherical shell of $r = 0.02m$, where r = radius of shell. (4M).

Soln:

$$\overline{J} = 10^3 \sin\theta \, \overline{a_r} \; A/m^2$$

— in spherical C.S

$$p(r, \theta, \phi)$$

$dr \quad r\,d\theta \quad r\sin\theta \, d\phi$.

$$\overline{ds} = r^2 \sin\theta \, d\theta \, d\phi \; \overline{a_r} \quad \text{.. } r = 0.02m \text{ Surface.}$$

$0 < \theta < 2\pi$

$0 < \phi < 2\pi$

$$I = \oint_{\langle S \rangle} \overline{J} \cdot \overline{ds}$$

$$I = \oint_{\langle S \rangle} 10^3 \sin\theta \, \overline{a_r} \cdot r^2 \sin\theta \, d\theta \, d\phi \; \overline{a_r} \Big|_{r = 0.02m} \; ; \text{Amperes}$$

$$I = 10^3 (0.02)^2 \int_{\theta=0}^{\pi} \sin^2\theta \, d\theta \int_{\phi=0}^{2\pi} d\phi \ \cancel{\overline{a_r} \cdot \overline{a_r}}^{\,1}$$

$$I = 10^3 (0.02)^2 (1.57079)(2\pi)(1)$$

$$\boxed{I = 3.9478} \text{ Ampers}$$

the Current Crossing the spherical shell of radius $r = 0.02\,m$ is $\boxed{I = 3.9478}$ A

problem 24.

10 - June /July 2012

In the region $0 < r < 0.5$m, in cylindrical co-ordinates, the current density is $\bar{J} = 4.5e^{-2r}\bar{a}_z$ A/m² and $\bar{J} = 0$ elsewhere. Use amperes circuital law to find $\bar{H}$ (05 Marks)

Question.

In the region $0 < r < 0.5m$, in cylindrical co-ordinates, the current density is $\bar{J} = 4.5e^{-2r}\,\bar{a}_z$ A/m² and $\bar{J} = 0$ elsewhere. Use Amperes Circuital Law to find $\bar{H}$. (5M)

Solu:- method I:- using ampere's Circuital Law

$$\oint_{\langle l \rangle} \bar{H}\cdot\bar{dl} = I \quad \text{Ampere's}$$

$$I = \oint_{\langle s \rangle} \bar{J}\cdot\bar{ds}.$$

$p(r, \phi, z)$ Cylindrical C.S

$dr \quad rd\phi \quad dz$

$\bar{ds} = r\,dr\,d\phi\,\bar{a}_z \cdots z = k$ Surface

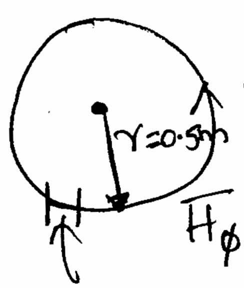
$$I = \oint_{\langle s \rangle} 4.5e^{-2r}\bar{a}_z \cdot r\,dr\,d\phi\,\bar{a}_z \; : \; \text{Ampere's}$$

$0 < r < 0.5m \, ; \quad 0 < \phi < 2\pi.$

$$I = \int_{r=0}^{0.5} 4.5 r e^{-2r} dr \int_{\theta=0}^{2\pi} d\phi \; \cancel{\bar{a}_z \cdot \bar{a}_z}^{\,1}$$

$$I = 0.29727 \times 2\pi \times 1$$

$$\boxed{I = 1.867802} \text{ Amperes}$$



$I = 1.867802A$

$$\bar{H}_\phi = H_\phi \bar{a}_\phi : A/m$$

$$\bar{dl} = r d\phi \bar{a}_\phi \qquad \oint_{\langle l \rangle} \bar{H}_\phi \cdot \bar{dl} = I$$

$$\oint_{\langle l \rangle} H_\phi \bar{a}_\phi \cdot r d\phi \bar{a}_\phi = I$$

$$H_\phi r \int_0^{2\pi} d\phi \; \cancel{\bar{a}_\phi \cdot \bar{a}_\phi}^{\,1} = I$$

$$H_\phi = \frac{I}{2\pi r} \; A/m$$

$$H_\phi = \frac{1.867802}{(2\pi)\, r}$$

$$H_\phi = \frac{0.297269}{r} \quad A/m.$$

the magnetic field intensity $\overline{H}$ is given by $\overline{H} = H_\phi \overline{a_\phi}$ A/m.

$$\boxed{\overline{H} = \frac{0.297269}{r}\overline{a_\phi}\ A/m} \quad \text{for } 0 < r < 0.5m$$

at $r = 0.5m$.

$$\boxed{\overline{H} = 0.594539\,\overline{a_\phi}} \ A/m.$$

Method II, using point form of Ampere's Circuital Law.

i.e $\overline{J} = \nabla \times \overline{H}$ A/m².

and $\overline{H} = H_\phi \overline{a_\phi}$ A/m.

and $H_\phi = f^n(r)$ alone.

$$\nabla \times \overline{H} = \frac{1}{r} \begin{vmatrix} \overline{a_r} & r\overline{a_\phi} & \overline{a_z} \\ \partial/\partial r & 0 & 0 \\ 0 & rH_\phi & 0 \end{vmatrix}$$

$$\overline{J} = \nabla \times \overline{H} = \frac{1}{r} \frac{\partial (H_\phi r)}{\partial r} \overline{a_z}.$$

given $\overline{J} = 4.5 e^{-2r} \overline{a_z}$

$$4.5 e^{-2r} \overline{a_z} = \frac{1}{r} \frac{\partial (H_\phi r)}{\partial r} \overline{a_z}$$

Equating 'z' components on bothside

$$4.5 e^{-2r} = \frac{1}{r} \frac{\partial (H_\phi \cdot r)}{\partial r}$$

$$\frac{\partial (H_\phi \cdot r)}{\partial r} = 4.5 r e^{-2r} \qquad 0 < r < 0.5$$

$$r \cdot H_\phi = \int_{r=0}^{0.5} 4.5 r e^{-2r} dr$$

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$$H_\phi = \frac{1}{r}\int_{r=0}^{0.5} 4.5\, r e^{-2r}\, dr \quad \text{------ we calc:}$$

$$H_\phi = \frac{4.5}{r}\left[r \cdot \frac{e^{-2r}}{-2}\Big|_{r=0}^{0.5} + \int_{r=0}^{0.5} \frac{e^{-2r}}{-2}\cdot 1 \right]$$

$$H_\phi = \frac{1}{r}\left[0.29727\right]$$

$$\boxed{H_\phi = \frac{0.29727}{r}} \text{ A/m.}$$

the magnetic field intensity b/w the region $0 < r < 0.5$ is

$$\overline{H} = H_\phi \, \overline{a_\phi} \text{ A/m}$$

i.e $\boxed{\overline{H} = \frac{0.29727}{r}\overline{a_\phi}}$ A/m

for $0 < r < 0.5$.

at $r = 0.5$m

$$\boxed{\overline{H} = 0.59453\,\overline{a_\phi}} \text{ A/m.}$$

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4. **Stokes' theorem**

06-DEC2009/Jan 2010

State and prove the Stoke's theorem. (06 Marks)

06 - June /July 2011

State and prove stokes theorem. (07 Marks)

06 –June/July 2014

Show that $\oint H \cdot dl = \int \nabla \times H \cdot dS = I$ with definition of the same. (10 Marks)

Dec/Jan 2016 (4M)

State and explain the following ii) Stokes theorem.

06-DEC2010

State and prove the Stoke's theorem. (04 Marks)

Question's.

State and prove the stoke's theorem. (6m)

(or)

Show that $\oint_{\langle l \rangle} \bar{H} \cdot \bar{dl} = \oint_{\langle S \rangle} (\nabla \times \bar{H}) \cdot \bar{dS} = I$, with definition of the same.

[06 - Dec 2009/Jan 2010, 06 - June/July 2011, 06 June/July -2014, 10-Dec/Jan 2016, 06-Dec 2010].

3.9. Stoke's theorem:-

Statement:- "Integration of any vector around a closed path is always equal to integration of the curl of that vector through out the surface enclosed by that path." ie

$$\boxed{\oint_{\langle l \rangle} \overline{H} \cdot \overline{dl} = \oint_{\langle S \rangle} (\nabla \times \overline{H}) \cdot \overline{dS}}$$ Ampere

proof: from Ampere's Circuital Law

$$\oint_{\langle l \rangle} \overline{H} \cdot \overline{dl} = I \text{ ampere's} \quad \leftarrow (1)$$

from the concept of current density

$$I = \oint_{\langle S \rangle} \overline{J} \cdot \overline{dS} \quad \text{Ampere's} \quad \leftarrow (2)$$

equating eq^n (1) and (2),

$$\oint_{\langle l \rangle} \overline{H} \cdot \overline{dl} = I = \oint_{\langle S \rangle} \overline{J} \cdot \overline{dS} \quad \leftarrow (3)$$

using point form of Ampere's Law

ie $\nabla \times \overline{H} = \overline{J} \ A/m^2 \quad \leftarrow (4)$

eq^n (4) in (3)

$$\oint_{\langle l \rangle} \overline{H} \cdot \overline{dl} = I = \oint_{\langle S \rangle} \overline{J} \cdot \overline{dS} = \oint_{\langle S \rangle} (\nabla \times \overline{H}) \cdot \overline{dS}$$

$$\Rightarrow \boxed{\oint_{\langle l \rangle} \overline{H} \cdot \overline{dl} = \oint_{\langle S \rangle} (\nabla \times \overline{H}) \cdot \overline{dS}} \quad \text{Ampere's}$$

→ Stokes theorem.

Note:- In general for any Vector $\overline{A}$

i) Divergence theorem

$$\oint_{\langle S \rangle} \overline{A} \cdot \overline{dS} = \int_{\langle Vol \rangle} (\nabla \cdot \overline{A})\, dv$$

obs:- Surface to volume integral

ii) Stokes theorem:-

$$\oint_{\langle l \rangle} \overline{A} \cdot \overline{dl} = \oint_{\langle S \rangle} (\nabla \times \overline{A}) \cdot \overline{dS}$$

obs:- line to Surface integral.

problem 25.

10-DEC2011/Jan 2012

Evaluate both sides of the Stoke's theorem for the field $\overline{H} = 6xy\,\hat{a}_x - 3y^2\,\hat{a}_y$ A/m and the rectangular path around the region $2 \le x \le 5$, $-1 \le y \le 1$, $z = 0$. (10 Marks)

(or)

Question.

Verify the stoke's theorem for the field $\overline{H} = 6xy\,\overline{a}_x - 3y^2\,\overline{a}_y$ A/m and the rectangular path around the region, $2 \le x \le 5$, $-1 \le y \le 1$, $z=0$. Let the positive direction of $\overline{dS}$ be $\overline{a}_z$. (8m)

[15-June/July 2017 (CBCS) (8m)] [06-Jan 2009].

Solu: Stoke's theorem

$$\oint_{\langle l \rangle} \overline{H}\cdot\overline{dl} = \oint_{\langle S \rangle} (\nabla \times \overline{H})\cdot\overline{dS}.$$

R.H.S

$$\oint_{\langle S \rangle} (\nabla \times \overline{H})\cdot\overline{dS} = ?$$

$$\overline{H} = 6xy\,\overline{a_x} - 3y^2\,\overline{a_y} \quad A/m$$

$$2 \le x \le 5, \quad -1 \le y \le +1, \quad z = 0.$$

$$\nabla \times \overline{H} = \begin{vmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & 0 \end{vmatrix}$$

$$= -\frac{\partial H_y}{\partial z}\overline{a_x} - \left[-\frac{\partial H_x}{\partial z}\right]\overline{a_y} + \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right]\overline{a_z}$$

$$\nabla \times \overline{H} = -\cancel{\frac{\partial H_y}{\partial z}}^{0}\,\overline{a_x} + \cancel{\frac{\partial H_x}{\partial z}}^{0}\,\overline{a_y} + \left(\cancel{\frac{\partial H_y}{\partial x}}^{0} - \frac{\partial H_x}{\partial y}\right)\overline{a_z}$$

$$H_x = 6xy \; A/m \quad ; \quad H_y = -3y^2$$

$$\frac{\partial H_y}{\partial z} = 0. \quad \frac{\partial H_y}{\partial x} = 0.$$

$$\frac{\partial H_x}{\partial z} = 0.$$

$$\frac{\partial H_x}{\partial y} = 6x \; A/m^2$$

$$\boxed{\nabla \times \overline{H} = -6x\,\overline{a_z}} \; A/m^2$$

and $\overline{ds} = dx\,dy\,(+\overline{a_z})$

$\hookrightarrow$ z=0 plane
given +ve z direct"

$$\oint_{\langle S\rangle} (\nabla \times \bar{H}) \cdot \bar{ds} = \oint_{\langle S\rangle} (-6x\,\bar{a}_z) \cdot dx\,dy\,\bar{a}_z$$

$$= -6 \int_{x=2}^{5} x\,dx \int_{y=-1}^{1} dy$$

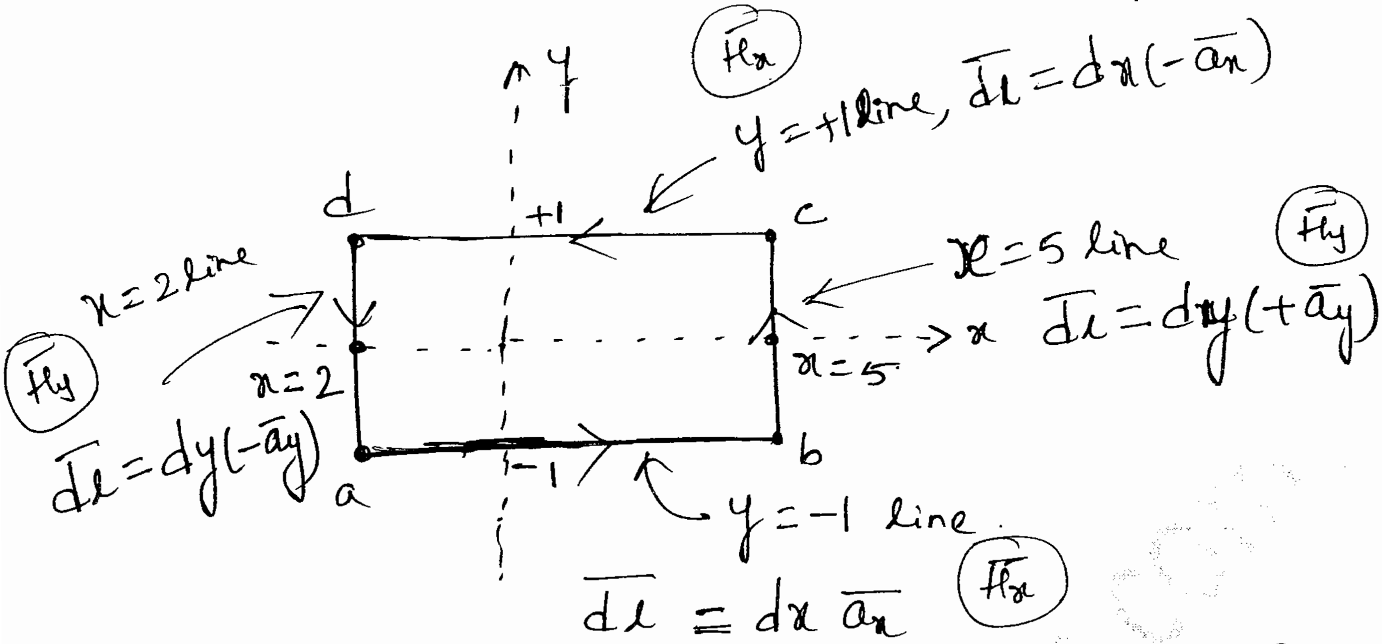
$$= -6 \left.\frac{x^2}{2}\right|_2^5 \times (+2)$$

$$= -6 \cdot \frac{[5^2 - 2^2]}{\cancel{2}} (\cancel{2}) = -6[25-4]$$

$$\oint_{\langle S\rangle} (\nabla \times \bar{H}) \cdot \bar{ds} = -126 \text{ Ampere} \longleftarrow (a)$$

L.H.S $\oint_{\langle l\rangle} \bar{H} \cdot \bar{dl} = ?$

$$\boxed{\begin{array}{c} 2 \le x \le 5 \\ -1 \le y \le 1 \end{array}}$$ rectangle dimension.



$$\oint_{\langle 1 \rangle} \overline{H} \cdot \overline{dl} = \int_a^b \overline{H} \cdot \overline{dl} + \int_b^c \overline{H} \cdot \overline{dl} + \int_c^d \overline{H} \cdot \overline{dl} + \int_d^a \overline{H} \cdot \overline{dl} \; ; A$$

$$= \int_{x=2}^{5} \overline{H_x} \cdot \overline{dl} + \int_{y=-1}^{1} \overline{H_y} \cdot \overline{dl} + \int_{x=5}^{2} \overline{H_x} \cdot \overline{dl} + \int_{y=+1}^{-1} \overline{H_y} \cdot \overline{dl} \; ; A$$

$$\oint_{\langle 1 \rangle} \overline{H} \cdot \overline{dl} = \int_{x=2}^{5} 6xy\,dx \Big|_{y=-1} + \int_{y=-1}^{1} -3y^2\,dy + \int_{x=5}^{2} 6xy\,(-dx) \Big|_{y=+1} + \int_{y=1}^{-1} (-3y^2)(-dy).$$

$$= -6(10.5) - 2 - 63 + 2 = -126 \text{ Ampere's}$$

i.e $\oint_{\langle 1 \rangle} \overline{H} \cdot \overline{dl} = -126$ Ampere's ——— (b)

Since eq^n (a) = eq^n (b)

$\therefore$ Stokes theorem is Verified.

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problem 26

10-Dec/Jan 2015

Verify Stoke's theorem for the field $\bar{H} = 2r\cos\theta\, \bar{a}_r + r\bar{a}_\theta$ for the path shown $r = 0$ to 1, $\theta = 0$ to $90°$. **(08 Marks)**

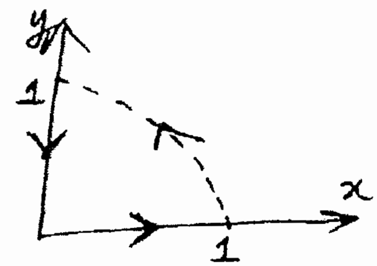
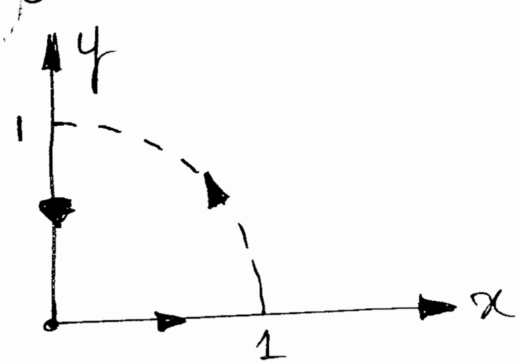


Fig. Q4(b)

Question

Verify stoke's theorem for the field $\overline{H} = 2r\cos\theta\, \overline{a}_r + r\, \overline{a}_\theta$ for the path shown $r = 0$ to 1. and $\theta = 0$ to $90°$. (8m)

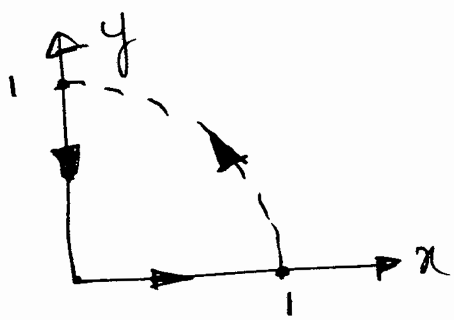


Soln: Given $\overline{H}$ in Spherical Co-ordinate System.

$P(r, \theta, \phi)$

$dr \quad rd\theta \quad r\sin\theta\, d\phi$

$$\overline{H} = 2r\cos\theta\,\overline{a_r} + r\,\overline{a_\theta}\ \text{A/m}$$



... $\phi = k$ surface.

R.H.S

$0 < r \le 1\,\text{m}$ and $0 < \theta < 90^\circ$

$$\oint_{\langle S \rangle} (\nabla \times \overline{H}) \cdot \overline{ds}$$

$P(r, \theta, \phi)$

$dr \quad r\,d\theta \quad r\sin\theta\,d\phi$

$$\overline{ds} = r\,dr\,d\theta\,\overline{a_\phi}.$$

$$\nabla \times \overline{H} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \overline{a_r} & r\overline{a_\theta} & r\sin\theta\,\overline{a_\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & 0 \\ H_r & rH_\theta & 0 \end{vmatrix}$$

$$= \frac{1}{r^{\cancel{2}}\cancel{\sin\theta}}\left[\frac{\partial}{\partial r}(rH_\theta) - \frac{\partial H_r}{\partial \theta}\right] \cancel{r}\cancel{\sin\theta}\,\overline{a_\phi}$$

$H_r = 2r\cos\theta$ A/m and $H_\theta = r$.

$$= \frac{1}{r}\left[\frac{\partial}{\partial r}(r^2) - \frac{\partial}{\partial \theta}(2r\cos\theta)\right]\overline{a_\phi}$$

$$= \frac{1}{r}\left[2r - 2r(-\sin\theta)\right]\overline{a_\phi}$$

$$= \frac{1}{r}\left[2r + 2r\sin\theta\right]\overline{a_\phi}$$

$$\boxed{\nabla \times \overline{H} = 2[1+\sin\theta]\overline{a_\phi}} \text{ A/m}^2$$

$$\oint_{\langle S\rangle}(\nabla\times\overline{H})\cdot\overline{ds} = \oint_{\langle S\rangle} 2(1+\sin\theta)\overline{a_\phi}\cdot r\,dr\,d\theta\,\overline{a_\phi}$$

$$= \int_{r=0}^{1} r\,dr \int_{\theta=0}^{90^\circ} 2(1+\sin\theta)\,d\theta$$

$$= 0.5 \times 5.1416 = 2.5708$$

$$\boxed{\oint_{\langle S\rangle}(\nabla\times\overline{H})\cdot\overline{ds} = 2.5708 \text{ Ampere's}}$$ (a)

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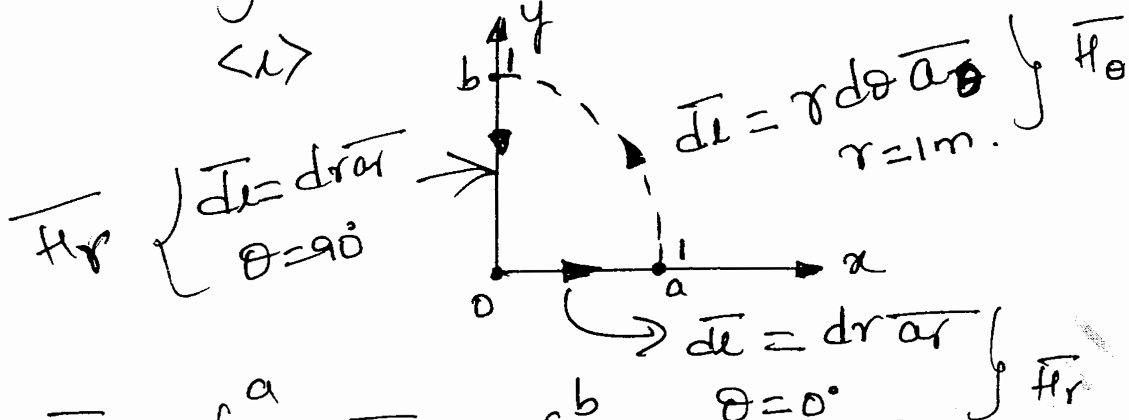
L.H.S

$$\oint_{\langle l \rangle} \overline{H} \cdot \overline{dl} = ? \qquad \overline{H} = 2r\cos\theta\, \overline{a_r} + r\,\overline{a_\theta} \text{ A/m.}$$



$$\oint_{\langle l \rangle} \overline{H} \cdot \overline{dl} = \int_0^a \overline{H} \cdot \overline{dl} + \int_a^b \overline{H} \cdot \overline{dl} + \int_b^0 \overline{H} \cdot \overline{dl} \text{ : Ampere's}$$

$$\int_0^a \overline{H} \cdot \overline{dl} = \int_{r=0}^{1} 2r\cos\theta\, \overline{a_r} \cdot dr\,\overline{a_r} \Big|_{\theta=0°}$$

$$= \int_{r=0}^{1} 2r\,dr = 1 \text{. Ampere's}$$

$$\int_a^b \overline{H} \cdot \overline{dl} = \int_{\theta=0°}^{\pi/2} r\,\overline{a_\theta} \cdot r\,d\theta\,\overline{a_\theta} = r^2 \int_0^{\pi/2} d\theta \Big|_{r=1m} = (1)^2 \times \pi/2$$

$$= 0.5\pi \text{. Ampere's.}$$

$$\int_b^0 \overline{H} \cdot \overline{dl} = \int_{r=1}^{0} 2r\cos\theta\, \overline{a_r} \cdot dr\,\overline{a_r} \Big|_{\theta=90°} = 0 \text{ Ampere's}$$

[$\cos(90°) = 0$]

$$\boxed{\oint_{\langle l \rangle} \overline{H} \cdot \overline{dl} = 1 + \pi/2 + 0 = 2.570796} \text{ Ampere's} \quad \text{(b)}$$

Since equation (a) = eq^n (b) ∴ Stokes theorem is Verified.

Problem 27. Given the magnetic field $\overline{H} = 2r^2(z+1)\sin\phi\,\overline{a_\phi}$. Verify Stokes theorem for the portion of a cylindrical surface defined by $r=2m$, $\frac{\pi}{4} < \phi < \pi/2$, $1 < z < 1.5$ and for its perimeter. (8m)

(or) 10-J/J 2012 10 - June /July 2015

Verify stokes theorem for a field having $\dot{H} = 2\rho^2(\tau+1)\sin\phi\, a_\phi$ for the portion of a cylindrical surface defined by $\rho = 2$, $\frac{\pi}{4} \le \phi \le \frac{\pi}{2}$, $1 \le \tau \le 1.5$ and for its perimeter.

(10 Marks)

Question (or)

Verify stokes theorem for a field having $\overline{H} = 2\rho^2(z+1)\sin\phi\,\overline{a_\phi}$ for the portion of a cylindrical surface defined by $\rho = 2m$, $\frac{\pi}{4} \le \phi \le \pi/2$, and $1 \le z \le 1.5m$ and for its perimeter. (10m)

Soln:- stokes theorem

$$\oint_{\langle l \rangle} \overline{H} \cdot \overline{dl} = \oint_{\langle S \rangle} (\nabla \times \overline{H}) \cdot \overline{ds}$$

R.H.S

$$\oint_{\langle S \rangle} (\nabla \times \overline{H}) \cdot \overline{ds} = ?$$

Given field $\overline{H} = 2\rho^2(z+1)\sin\phi\,\overline{a_\phi}$ is in cylindrical co-ordinate system.

$$\bar{H} = 2\rho^2(z+1)\sin\phi\,\bar{a}_\phi \ \text{A/m}.$$

$$\bar{H} = H_\phi \bar{a}_\phi \ \text{A/m}.$$

$P(\rho, \phi, z)$ $\rightarrow$ $d\rho$, $\rho d\phi$, dz

$$\nabla \times \bar{H} = \frac{1}{\rho}\begin{vmatrix} \bar{a}_\rho & \rho\bar{a}_\phi & \bar{a}_z \\ \partial/\partial\rho & \partial/\partial\phi & \partial/\partial z \\ 0 & \rho H_\phi & 0 \end{vmatrix}$$

$$= \frac{1}{\rho}\left[-\frac{\partial(\rho H_\phi)}{\partial z}\bar{a}_\rho + \left[0 + \frac{\partial(\rho H_\phi)}{\partial \rho}\right]\bar{a}_z\right]$$

$$= \frac{1}{\cancel{\rho}}\cdot -\cancel{\rho}\frac{\partial H_\phi}{\partial z}\bar{a}_\rho + \frac{1}{\cancel{\rho}}\cdot\rho\frac{\partial H_\phi}{\partial \rho}\bar{a}_z$$

$$\nabla \times \bar{H} = -\frac{\partial H_\phi}{\partial z}\bar{a}_\rho + \frac{\partial H_\phi}{\partial \rho}\bar{a}_z$$

$$\nabla \times \bar{H} = -\frac{\partial}{\partial z}\left[2\rho^2(z+1)\sin\phi\right]\bar{a}_\rho + \frac{\partial}{\partial \rho}\left[2\rho^2(z+1)\sin\phi\right]\bar{a}_z$$

$$\boxed{\nabla \times \bar{H} = -2\rho^2\sin\phi\,\bar{a}_\rho + 4\rho(z+1)\sin\phi\,\bar{a}_z} \ \text{A/m}.$$

$$P(\rho, \phi, z) \rightarrow d\rho \quad \rho\, d\phi \quad dz$$

$$\overline{dS} = \rho\, d\phi\, dz\, \overline{a}_\rho \quad \ldots\ldots \quad \rho = 2 \text{ Surface}.$$

$$\oint_{\langle S\rangle} (\nabla \times \overline{H}) \cdot \overline{dS} = \oint_{\langle S\rangle} \left[-2\rho^2 \sin\phi\, \overline{a}_\rho + 4\rho(z+1)\sin\phi\, \overline{a}_z\right] \cdot \rho\, d\phi\, dz\, \overline{a}_\rho$$

$$\overline{a}_z \cdot \overline{a}_\rho = 0$$
$$\overline{a}_\phi \cdot \overline{a}_\rho = 0.$$

$$= \oint_{\langle S\rangle} \left[-2\rho^2 \sin\phi\, \overline{a}_\rho \cdot \rho\, d\phi\, dz\, \overline{a}_\rho\right] \Bigg|_{\rho = 2m.}$$

$$= \oint_{\langle S\rangle} -2\rho^3 \sin\phi\, d\phi\, dz\, \cancel{\overline{a}_\rho \cdot \overline{a}_\rho}^{\,1} \Bigg|_{\rho = 2m. \text{ Surface.}}$$

$$= -2(2)^3 \times \int_{\phi = \pi/4}^{\pi/2} \sin\phi\, d\phi \times \int_{z=1}^{1.5} dz$$

$$\oint_{\langle S\rangle} (\nabla \times \overline{H}) \cdot \overline{dS} = -16 \times 0.7071 \times 0.5 = -5.6568\, A$$

RHS.

$$\boxed{\oint_{\langle S\rangle} (\nabla \times \overline{H}) \cdot \overline{dS} = -5.6568 \text{ Amperes}}$$

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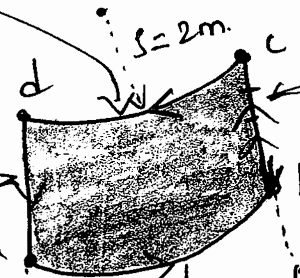
L. H 1.5 i.e $\oint_{\langle 1 \rangle} \overline{H} \cdot \overline{dl} = ?$

$$\overline{H} = 2\rho^2 (z+1) \sin\phi \, \overline{a_\phi} \ \text{A/m}.$$

$\rho = 2\text{m}$. $\pi/4 \le \phi \le \pi/2$; $1 \le z \le 1.5$.

$\rightarrow$ 'ρ' = k Surface.

$(\overline{H_\phi})$ $\overline{dl} = \rho d\phi \, \overline{a_\phi}$ (radial path) $(z = 1.5\text{m}, \rho = 2\text{m})$



$\rho = 2\text{m}$

height $\overline{dl} = dz \, \overline{a_z} \Big|_{\rho = 2\text{m} \ \& \ \phi = \pi/2}$ $(\overline{H_z})$

$z = 1.5$ d c b a $z = 1$

$\overline{dl} = dz \, \overline{a_z}$ $(\overline{H_z})$ $\rho = 2\text{m}$ & $\phi = \pi/4$.

$\phi = \pi/4$ $\phi = \pi/2$.

$\overline{dl} = \rho d\phi \, \overline{a_\phi}$ and $z = 1$, $\rho = 2\text{m}$ (H_ϕ) (given only H_ϕ)

bcz given only H_ϕ component

Circular path

$$\oint_{\langle 1 \rangle} \overline{H} \cdot \overline{dl} = \int_a^b \overline{H_\phi} \cdot \overline{dl} + \cancel{\int_b^c \overline{H_z} \cdot \overline{dl}}^{\,0} + \int_c^d \overline{H_\phi} \cdot \overline{dl} + \cancel{\int_d^a \overline{H_z} \cdot \overline{dl}}^{\,0}$$

$$\int_a^b \overline{H_\phi} \cdot \overline{dl} = \int_{\phi=\pi/4}^{\pi/2} 2\rho^2 (z+1) \sin\phi \, \overline{a_\phi} \cdot \rho \, d\phi \, \overline{a_\phi} \Bigg|_{z = 1\text{m} \ \& \ \rho = 2\text{m}.}$$

$$= 2(2)^3 (1+1) \int_{\phi=\pi/4}^{\pi/2} \sin\phi \, d\phi \ \cancel{\overline{a_\phi} \cdot \overline{a_\phi}}^{\,1}$$

$$= 16 \times 2 \times 0.7071 = 22.6275 \text{ Amperes}$$

$$\int_c^d \overline{H_\phi} \cdot \overline{dl} = \left. \int_{\phi=\pi/2}^{\pi/4} 2\rho^2(z+1)\sin\phi\, \overline{a_\phi} \cdot \rho\, d\phi\, \overline{a_\phi} \right|_{\substack{\rho=2m \\ z=1.5m}}$$

$$= 2(2)^3(1.5+1)\int_{\phi=\pi/2}^{\pi/4} \sin\phi\, d\phi\ \cancel{\overline{a_\phi} \cdot \overline{a_\phi}}^{\,1}$$

$$\int_c^d \overline{H_\phi} \cdot \overline{dl} = 2^4 \times 2.5 \times -0.7071$$

$$= -28.2844 \text{ Ampere's}$$

$$\therefore \oint_{\langle 1 \rangle} \overline{H} \cdot \overline{dl} = 22.6275 + 0 - 28.2844 + 0$$

$$= -5.6568 \text{ Ampere's}$$

L.H.S

$$\boxed{\oint_{\langle 1 \rangle} \overline{H} \cdot \overline{dl} = -5.6568 \text{ Ampere's}} \longleftarrow (b)$$

Since eq^n (a) = eq^n (b) $\therefore$ Stokes theorem is Verified.

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Topic 3.10

Magnetic flux and magnetic flux density

Question.

Define Magnetic Flux (ϕ), Magnetic Field Intensity ($\vec{H}$), and magnetic flux density and mention the Expression.

10-Dec-Jan 2012 (6M)

Magnetic Flux (ϕ) * ϕ – Scalar in nature.

∴ the Magnetic Flux (ϕ) crossing any surface is found by

$$\boxed{\phi = \oint_{S} \overline{B} \cdot \overline{dS}} \text{ wb} \quad \text{or} \quad \phi = \int_{S} \overline{B} \cdot \overline{dS}$$

open surface.

where ϕ – magnetic Flux (wb)

$\overline{B}$ – magnetic Flux density wb/m2

Magnetic Flux density ($\overline{B}$) :–

* The total Magnetic Lines of force or Magnetic Flux crossing a unit area in a plane normal to the direction of Flux is called Magnetic Flux density ($\overline{B}$).

* ie. $\boxed{\overline{B} = \dfrac{d\phi}{dS}}$ wb/m^2 or $\overline{B} = \dfrac{d\phi}{dS}\,\overline{a_n}$ wb/m^2 or Tesla(T).

* $\overline{B}$ is Vector in nature and Measured in wb/m^2 or Tesla.

Magnetic Field Intensity ($\overline{H}$) :–

* The quantitative Measure of strongness or weakness of the Magnetic field is given by Magnetic field Intensity ($\overline{H}$).

* $\overline{H}$ is Vector in nature and Measured in A/m or N/wb.

Relation b/w $\overline{B}$ and $\overline{H}$:–

$$\boxed{\overline{B} = \mu \overline{H}} \text{ wb/m}^2$$

where μ – permeability of the medium.

$\mu = \mu_0 \mu_r$ H/m

μ_0 → absolute permeability ($4\pi \times 10^{-7}$ H/m)

μ_r → relative permeability (H/m)

and $\boxed{\mu_0 = 4\pi \times 10^{-7}}$ H/m

$$\boxed{\bar{B} = \mu\bar{H} = \mu_0\mu_r\bar{H}} \quad Wb/m^2$$

for all non-magnetic Material $\mu_r = 1$ while for magnetic material $\mu_r > 1$.

Note:- Direction of $\bar{B}$ and $\bar{H}$ are same.

Topic 3.11

Scalar and Vector Magnetic Potentials

3.11
XYX
V.Imp

02-DEC2010
Explain . i) Scalar magnetic potential ii) Vector magnetic potential (04 Marks)

02-DEC2008/Jan 2009
Arrive at an expression for vector magnetic potential. (06 Marks)

10-June/July 2013
Discuss the scalar and vector magnetic potentials. (05 Marks)

02 - June /July 2011
Differentiate between scalar magnetic potential and vector magnetic potentials. (06 Marks)

02 - June /July 2012
Distinguish between scalar and vector magnetic potential. Derive an expression for the vector magnetic potential. (08 Marks)

06- June /July 2009
Explain scalar and vector Magnetic Potential (08 Marks)

02 - June /July 2010
Explain similarities and differences between electric potential and vector magnetic potential using their definitions. (08 Marks)

10-Dec/Jan 2015
Explain scalar and vector magnetic potential. (04 Marks)

10 - June /July 2014
Explain scalar and vector magnetic potential (08 Marks)

06 – Jan 2013
Explain scalar and vector magnetic potentials. (06 Marks)

June/July 2016

c. Clearly distinguish between scalar magnetic potential and vector magnetic potential. (06 Marks)

Dec/Jan 2017

a. Explain the concept of scalar and vector magnetic potential. (08 Marks)

Dec/Jan 2017 CBCS

b. Explain the concepts of scalar and vector magnetic potential. (08 Marks)

Question:

Explain the concepts of scalar and vector magnetic potential. (8M).

[02-Dec 2010, 02-Jan 2009, 10-June/July 2013, 02-J/J 2011, 02-J/J 2012, 06 J/J 2009, 02-J/J 2010, 10-Jan 2015, J/J 2016, Dec/Jan 2017, 10-J/J 2014, 06-Jan 2013, 15-Dec/Jan 2017 (CBCS)].

3.11 ~~Explain the concepts of scalar and vector magnetic potential.~~ (08 Marks)

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Soln:- The Electric Scalar potential of Electrostatics is given by V.

from the concept of potential gradient the Electric field intensity $\bar{E}$ is related to the Scalar potential V is given by $\bar{E} = -\nabla V$ V/m ← (1)

lly in magnetic field there are two types of potentials defined

i. Scalar magnetic potential (V_m).

ii. Vector magnetic potential ($\bar{A}$).

i. Scalar magnetic potential (V_m):-

Consider the Vector identities:

$\nabla \times \nabla V = 0$, where V — Scalar ← (2)

$\nabla \cdot (\nabla \times \bar{A}) = 0$, where $\bar{A}$ - vector. ← (3)

if V_m is said to be the Scalar magnetic potential

then $\nabla \times \nabla V_m = 0$ ← (4)

~~the magnetic scalar potential V_m, related to $\bar{H}$ is~~

given by $\boxed{\bar{H} = -\nabla V_m}$ A/m. ← (3)

$\Rightarrow \nabla V_m = -\bar{H}$

using eqn (4)

$\nabla \times (-\bar{H}) = 0.$

$\Rightarrow \boxed{\nabla \times \bar{H} = 0}$ ← (6)

but from point form of Ampere's circuital Law

$\boxed{\nabla \times \bar{H} = \bar{J}}$ ← (7)

by comparing eqn (6) and (7)

$\Rightarrow \boxed{\nabla \times \bar{H} = 0}$ valid only if $\bar{J} = 0$.

and $\bar{J} = 0$ only when $\sigma = 0$ i.e

freespace (a) Source-free region.

∴ Scalar magnetic potential V_m can be defined for Source free region where $\bar{J} = 0$, i.e Current density is zero.

$\Rightarrow \boxed{\bar{H} = -\nabla V_m}$ valid only when $\bar{J} = 0$ A/m².

$\Rightarrow \boxed{V_m = -\oint \bar{H} \cdot d\bar{l}}$

ii. Vector magnetic potential ($\bar{A}$)

Vector magnetic potential $\bar{A}$, measured in Wb/m.

using eq^n (3)

$$\boxed{\nabla\cdot(\nabla\times\bar{A})=0} \longleftarrow (8)$$

i.e divergence of a curl of a vector is zero.

the relationship b/w magnetic flux density $\bar{B}$ and the vector Magnetic potential is given by

$$\bar{B}=\nabla\times\bar{A} \longleftarrow (9)$$

using point form of Ampere's Circuital Law

$$\nabla\times\bar{H}=\bar{J} \quad A/m^2 \longleftarrow (10)$$

$$\bar{B}=\mu_0\bar{H} \quad \ldots Wb/m^2 \text{ (freespace)}$$

$$\nabla\times\left(\frac{\bar{B}}{\mu_0}\right)=\bar{J} \longleftarrow (11)$$

$$\nabla\times\bar{B}=\mu_0\bar{J} \longleftarrow (12)$$

from eq^n (9)

$$\nabla\times\bar{B}=\nabla\times\nabla\times\bar{A} \longleftarrow (13)$$

eq^n (12) in (13)

$$\nabla\times\nabla\times\bar{A}=\mu_0\bar{J} \longleftarrow (14)$$

~~using vector identity~~ $\nabla(\nabla\cdot\bar{A}) - \nabla^2\bar{A} = \nabla\times\nabla\times\bar{A}$

in eq^4 (14) $= \nabla\times\nabla\times\bar{A}$.

$$\nabla(\nabla\cdot\bar{A}) - \nabla^2\bar{A} = \mu_0\bar{J}$$

$$\Rightarrow \boxed{\bar{J} = \frac{1}{\mu_0}\left[\nabla(\nabla\cdot\bar{A}) - \nabla^2\bar{A}\right]}$$

The vector magnetic potential $\bar{A}$ in Measured a distance 'R' from due to differential Current element is given by

$$\bar{A} = \int_{\langle l \rangle} \frac{\mu_0 \bar{I}\, d\bar{l}}{4\pi R} \quad \text{... for line current}$$

$$\bar{A} = \int_{\langle S \rangle} \frac{\mu_0 \bar{K}\, ds}{4\pi R} \quad \text{... for Surface current}$$

where $\bar{K}$ – Surface current density (A/m^2)

$$\bar{A} = \int_{\langle V \rangle} \frac{\mu_0 \bar{J}\, dv}{4\pi R} \quad \text{... for volume current.}$$

problem 28.

prove that $\nabla\cdot\bar{B} = 0$ from the concept of vector magnetic potential A.

Soln:- $\bar{B} = \nabla\times\bar{A}$

taking Divergence operation on both sides, we get

$$\nabla\cdot\bar{B} = \nabla\cdot(\nabla\times\bar{A})$$

RHS.
$$\nabla\times\bar{A} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\bar{a}_x - \left[\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right]\bar{a}_y + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\bar{a}_z$$

$$\therefore \nabla\cdot(\nabla\times\bar{A}) = \frac{\partial}{\partial x}\left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right] - \frac{\partial}{\partial y}\left[\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right] + \frac{\partial}{\partial z}\left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]$$

$$= \cancel{\frac{\partial^2 A_z}{\partial x\partial y}} - \cancel{\frac{\partial^2 A_y}{\partial x\partial z}} - \cancel{\frac{\partial^2 A_z}{\partial x\partial y}} + \cancel{\frac{\partial^2 A_x}{\partial y\partial z}} + \cancel{\frac{\partial^2 A_y}{\partial x\partial z}} - \cancel{\frac{\partial^2 A_x}{\partial y\partial z}} = 0$$

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$$\therefore \boxed{\nabla\cdot\bar{B} = 0}$$

$$\therefore \boxed{\nabla \cdot \overline{B} = 0}$$

problem 29.

At a point $p(x, y, z)$, the components of A_x, A_y, and A_z of vector magnetic potential A are given by $A_x = 4x + 3y + 2z$, $A_y = 5x + 6y + 3z$ and $A_z = 2x + 3y + 5z$. Determine the magnitude and direction of B at p. what is the nature of this field? (6m) [15-June/July 2017 (4M) CBCS] [J/J 2001]

Soln: $$\overline{B} = \nabla \times \overline{A} = \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\overline{a_x} + \left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right]\overline{a_y} + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\overline{a_z}.$$

$A_x = 4x + 3y + 2z$

$\frac{\partial A_x}{\partial y} = 3$ and $\frac{\partial A_x}{\partial z} = 2$.

$A_y = 5x + 6y + 3z$

$\frac{\partial A_y}{\partial x} = 5$ and $\frac{\partial A_y}{\partial z} = 3$.

$A_z = 2x + 3y + 5z$

$\frac{\partial A_z}{\partial x} = 2$ and $\frac{\partial A_z}{\partial y} = 3$.

$$\therefore \overline{B} = (3-3)\overline{a_x} + (2-2)\overline{a_y} + (5-3)\overline{a_z}$$

$$\boxed{\overline{B} = 2\overline{a_z}} \ wb/m^2$$

$\therefore |\overline{B}| = 2 \ wb/m^2$ and is directed along z-direction. The nature of this field is uniform.

$\overline{A} = 100\rho^{1.5}\overline{a_z}$ wb/m.

problem 30.

06-DEC2009/Jan 2010

If the vector magnetic potential at a point in a space is given as $A = 100\,\rho^{1.5}a_z$ wb/mt, find the following: i) H ii) J and show that $\oint H.dl = I$ for the circular path with $\rho = 1$. **(06 Marks)**

$\oint H \cdot dl = I$ 06 - J/J 2010 (9m)

Question

Vector magnetic potential in free space is given by $\overline{A} = 100\rho^{1.5}\overline{a_z}$ wb/m. Find the magnetic field intensity and Current density and hence prove Ampere's Circuital Law for $\rho = 1$m. (9M).

soln: Given vector magnetic potential in free space: $\mu = \mu_0$ H/m

$\overline{A} = 100\rho^{1.5}\overline{a_z}$ ~~A~~ wb/m ... in cylindrical C.S

w.k.t $\overline{B} = \nabla \times \overline{A}$ wb/m^2.

$$\nabla \times \overline{A} = \frac{1}{\rho}\begin{vmatrix} \overline{a_\rho} & \rho\overline{a_\phi} & \overline{a_z} \\ \partial/\partial\rho & 0 & 0 \\ 0 & 0 & 100\rho^{1.5} \end{vmatrix}$$

$$\nabla \times \overline{A} = \frac{-1}{\cancel{\rho}}\left[\frac{\partial}{\partial\rho}(100\rho^{1.5}) - 0\right]\cancel{\rho}\overline{a_\phi} : \text{Wb/m}^2$$

$$\nabla \times \overline{A} = -100 \times 1.5 \, \rho^{0.5} \, \overline{a_\phi} \ wb/m^2$$
$$= -150 \rho^{0.5} \overline{a_\phi} \ wb/m^2$$

$$\boxed{\overline{B} = \nabla \times \overline{A} = \frac{-150}{\rho^{0.5}} \overline{a_\phi}} \ wb/m^2$$

i. Magnetic field intensity $\overline{H}$

$$\boxed{\overline{H} = \frac{\overline{B}}{\mu_0} = \frac{-150}{\mu_0 (\rho^{0.5})} \overline{a_\phi}} \ A/m = \frac{-150 \rho^{0.5}}{\mu_0} \overline{a_\phi}$$

$\mu = 4\pi \times 10^{-7}$ H/m.

(or)

$$\boxed{\overline{H} = -\frac{119.3662}{\rho^{0.5}} \overline{a_\phi}} \ MH/m$$

(or) $$\boxed{\overline{H} = -\frac{119.366 \times 10^6}{\rho^{0.5}} \overline{a_\phi}} \ H/m.$$

ii) Current density $\overline{J} = \nabla \times \overline{H}$ A/m^2.

$$\nabla \times \overline{H} = \frac{1}{\rho} \begin{vmatrix} \overline{a_\rho} & \rho \overline{a_\phi} & \overline{a_z} \\ \frac{\partial}{\partial \rho} & 0 & 0 \\ 0 & \rho H_\phi & 0 \end{vmatrix} \ A/m^2$$

$$\overline{J} = \nabla \times \overline{H} = \frac{1}{\rho}\left[\frac{\partial(\rho H_\phi)}{\partial \rho} - 0\right]\overline{a_z}$$

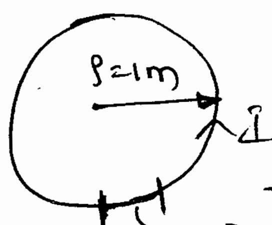
$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho \cdot \left(\frac{-150\rho^{0.5}}{\mu_0}\right)\right]\overline{a_z}$$

$$\overline{J} = \frac{-1}{\rho}\frac{\partial}{\partial \rho}\left[\rho^{1.5}\right]\cdot\frac{150}{\mu_0}\overline{a_z}$$

$$\overline{J} = \frac{-1}{\rho}\left(1.5\rho^{0.5}\right)\left(\frac{150}{\mu_0}\right)\overline{a_z}$$

$$\boxed{\overline{J} = \frac{-225}{\mu_0}\rho^{-0.5}\,\overline{a_z}} \text{ A/m}^2 \longleftarrow (a)$$

iii) To show $\oint_L \overline{H}\cdot\overline{dl} = I$ Ampere's at $\rho = 1$m surface.



$P(\rho, \phi, z)$

$\overline{dl} = \rho\, d\phi\, \overline{a_\phi}$.

(Circular path). ($\rho = 1$m ; along the path 'ρ' value is '1') from eqn ⓐ R.H.S

$$I = \oint_{\langle S \rangle} \overline{J} \cdot \overline{ds} \quad \text{for}$$

$z = 0$ in surface
$0 \le \rho \le 1$
$0 < \phi < 2\pi$

$$\overline{ds} = \rho \, d\rho \, d\phi \, \overline{a_z}$$

$$I = \oint_{\langle S \rangle} \frac{-225}{\mu_0} \rho^{-0.5} \overline{a_z} \cdot \rho \, d\rho \, d\phi \, \overline{a_z}$$

$$I = \frac{-225}{\mu_0} \int_{\rho=0}^{1} \rho^{0.5} d\rho \int_{\phi=0}^{2\pi} d\phi \, \overline{a_z} \cdot \overline{a_z}$$

$$I = \frac{-225}{\mu_0} \times 0.6667 \times 2\pi \times 1$$

$$I = \frac{-942.52}{\mu_0} = -7.50037 \times 10^{-6} A$$

$$\boxed{I = \frac{-942.52}{\mu_0} = -7.5 \, \mu \text{Ampere}}$$

(6)

(141)

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L.H.S

$$\oint_{\langle 1\rangle} \overline{H}\cdot\overline{dl} = \int_{\langle 1\rangle} \overline{H_\phi}\cdot\overline{dl}$$

$$= \int_{\langle 1\rangle} \left. \frac{-150\rho^{0.5}}{\mu_0}\,\overline{a_\phi}\cdot\rho\,d\phi\,\overline{a_\phi}. \right|_{\rho=1m}$$

$$= \frac{-150}{\mu_0}(\rho)^{1.5} \int_{\phi=0}^{2\pi} d\phi \times \left.\cancel{\overline{a_\phi}\cdot\overline{a_\phi}}^{\,1}\right|_{\rho=1m}$$

$$= -\frac{150}{\mu_0}\cancel{(1)^{1.5}}^{\,1} \times 2\pi \times 1$$

$$= \frac{-150}{\mu_0}\times 2\pi.$$

$$\boxed{I = -\frac{300\pi}{\mu_0} \simeq \frac{942.52}{\mu_0} = -7.5\mu A}$$ ← (C)

Since eq^n (b) = eq^n (c) ie $\oint_{\langle 1\rangle} \overline{H}\cdot\overline{dl} = I$; A.

∴ Ampere's Law is Verified along Circular path with $\rho = 1m$.

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problem 3)

Dec/Jan 2017

c. Given the vector magnetic potential
$\vec{A} = x^2 \vec{ax} + 2yz \vec{ay} + (-x^2) \vec{az}$
Find magnetic flux density. (04 Marks)

Soln:- Question.

Given the vector magnetic potential
$\bar{A} = x^2 \bar{a}_x + 2yz \bar{a}_y - x^2 \bar{a}_z$ wb/m. Find magnetic flux density. (6m). (or) Aug-2004.

Soln:-
$$\bar{A} = x^2 \bar{a}_x + 2yz \bar{a}_y - x^2 \bar{a}_z \text{ wb/m.}$$
$$\bar{A} = A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_z$$
$$\bar{B} = \nabla \times \bar{A} \text{ wb/m}^2$$
$$\nabla \times \bar{A} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$$
$$= \left[\cancelto{0}{\frac{\partial A_z}{\partial y}} - \cancelto{2y}{\frac{\partial A_y}{\partial z}}\right]\bar{a}_x - \left[\cancelto{-2x}{\frac{\partial A_z}{\partial x}} - \cancelto{0}{\frac{\partial A_x}{\partial z}}\right]\bar{a}_y$$
$$+ \left[\cancelto{0}{\frac{\partial A_y}{\partial x}} - \cancelto{0}{\frac{\partial A_x}{\partial y}}\right]\bar{a}_z \text{ : wb/m}^2$$

$A_x = x^2$; $A_y = 2yz$, $A_z = -x^2$.

$$\nabla \times \bar{A} = -2y\,\bar{a}_x + 2x\,\bar{a}_y \quad \text{wb/m}^2.$$

the magnetic flux density

$$\boxed{\bar{B} = \nabla \times \bar{A} = -2y\,\bar{a}_x + 2x\,\bar{a}_y} \ \text{wb/m}^2.$$

problem 32.

06 –Dec/Jan 2008

Given $\vec{A} = (y\cos ax)\vec{a}_x + (y + e^x)\vec{a}_z$, find $\nabla \times \vec{A}$ at the origin. (06 Marks)

Question

Given $\overline{A} = (y\cos ax)\,\overline{a_x} + (y + e^x)\,\overline{a_z}$ wb/m

find $\nabla \times \overline{A}$ at the origin.

Solu:- $\overline{A} = y\cos(ax)\,\overline{a_x} + (y + e^x)\,\overline{a_z}$ wb/m.

$$\nabla \times \overline{A} = \begin{vmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ \partial/\partial x & \partial/\partial y & 0 \\ A_x & 0 & A_z \end{vmatrix}$$

$$\nabla \times \overline{A} = \left[\frac{\partial A_z}{\partial y}\,\overline{a_x} - \frac{\partial A_z}{\partial x}\,\overline{a_y} - \frac{\partial A_x}{\partial y}\,\overline{a_z} \right]$$

$$\frac{\partial A_z}{\partial y} = 1 \,; \quad \frac{\partial A_z}{\partial x} = e^x \,; \quad \frac{\partial A_x}{\partial y} = \cos(ax)$$

$$\nabla \times \overline{A} = \overline{a_x} - e^x\,\overline{a_y} - \cos(ax)\,\overline{a_z} \quad \text{wb/m}^2$$

$\nabla \times \overline{A}$ at Origin $O(0,0,0)$ is

$x=0,\ y=0$ and $z=0$

$$\boxed{\bar{B}_0 = \nabla \times \bar{A} = \bar{a}_x - \bar{a}_y - \bar{a}_z} \ \text{wb/m}^2$$

Magnitude of $\nabla \times \bar{A}_0$ at origin

$$\boxed{|\bar{B}_0| = |\nabla \times \bar{A}| = \sqrt{1+1+1} = \sqrt{3}} \ \text{wb/m}^2$$

Module-3 (Part B)

Summary:

I. List of Symbols. unit.

→ magnetic flux (ϕ) — wb.

→ magnetic field intensity ($\bar{H}$) — A/m.

→ Current Element $I\,\bar{dl}$ — A-m.

→ Current (I) — Ampere.

→ magnetic flux density ($\bar{B}$) — wb/m² (or) Tesla.

→ Current density ($\bar{J}$) — A/m².

→ permeability (μ) — H/m.

$\mu = \mu_0 \mu_r$ H/m.

$\mu_0 = 4\pi \times 10^{-7}$ H/m.

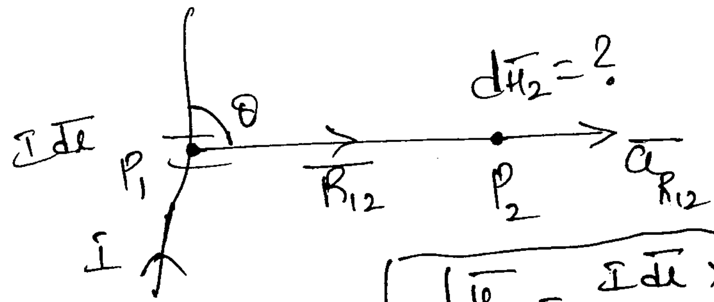
→ Vector magnetic potential ($\bar{A}$) — wb/m.

→ Surface Current density ($\bar{K}$) — A/m².

(11). List of formulae.

Note:- $\boxed{\overline{B} = \mu \overline{H}}$ wb/m^2.
freespace $\mu = \mu_0$ H/m $= 4\pi \times 10^{-7}$ H/m.

1. Biot-Savart-Law:-



$$\boxed{d\overline{H_2} = \frac{I\overline{dl} \times \overline{a_{R_{12}}}}{4\pi R_{12}^2}} \text{ A/m}$$

(or)

$$\boxed{d\overline{H_2} = \frac{I\overline{dl} \times \overline{R_{12}}}{4\pi R_{12}^3}} \text{ A/m.}$$

2. Magnetic field intensity ($\overline{H}$) due to infinite Long Straight Current carrying filament.

$$\boxed{\overline{H_P} = \frac{I}{2\pi\rho}\overline{a_\phi}} \text{ A/m (or) At/Wb}$$

where ρ – $\perp^r$ distance from point 'p' to infinite Length Current carrying filament.

3. $\overline{H}$ due to finite Length Current carrying filament.

$$\boxed{\overline{H} = \frac{I}{4\pi\rho}\left[\sin\alpha_2 - \sin\alpha_1\right]\overline{a_\phi}} \text{ A/m.}$$

4. $\overline{H}$ due on the axis of a Circular Current Loop.

$$\boxed{\overline{H} = \frac{I\rho^2}{2(\rho^2+z^2)^{3/2}}\overline{a_z}} \text{ A/m}$$

Special case

$\overline{H}$ at center of the Loop i.e $z=0$ is given by

$$\boxed{\overline{H} = \frac{I}{2\rho}\overline{a_z}} \text{ A/m}$$

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5. $\overline{H}$ ... at a point on the axis of a finite Length Solenoid.

$$\boxed{H = \frac{NI}{2}[\cos\phi_1 - \cos\phi_2]} \text{ A/m}$$

6. $\overline{H}$ at the center of a square Current Loop.

$$\boxed{\overline{H} = \frac{2\sqrt{2}I}{\pi a}\,\overline{a_z}} \text{ A/m.}$$

7. Ampere's Circuital Law.

The Line integral of $\overline{H}$ around a single closed path is equal to the Current enclosed by that path.

mathematically $\boxed{\oint_{\langle l\rangle} \overline{H}\cdot\overline{dl} = I_{enc}}$ Ampere's

8. Magnetic field intensity ($\overline{H}$) of a Co-axial Cable.

$$\overline{H} = \begin{cases} \dfrac{Ir}{2\pi a^2}\,\overline{a_\phi} & ; \; r<a \\ \dfrac{I}{2\pi r}\,\overline{a_\phi} & ; \; a<r<b \\ \dfrac{I}{2\pi r}\left[\dfrac{c^2-r^2}{c^2-b^2}\right]\overline{a_\phi} & ; \; b<r<c \\ 0 & ; \; \text{ow.} \end{cases} \text{ A/m}$$

9. $\overline{H}$ in b/w a Toroidal coil.

$$\boxed{H = \frac{NI}{2\pi R}} \text{ A/m}$$

10. For a Conductor in the form of regular polygon of n-side inscribed in a Circle of radius 'r' m the Flux density B at the centre is

$$\boxed{B = \frac{\mu_0 n I}{2\pi r} \tan\left(\frac{\pi}{n}\right)} \ wb/m^2$$

11. point form of Ampere's Law

$$\boxed{\nabla \times \bar{H} = \bar{J}} \ A/m^2$$

12. Stoke's theorem.

$$\boxed{\oint_{\langle l \rangle} \bar{H} \cdot \bar{dl} = \oint_{\langle S \rangle} (\nabla \times \bar{H}) \cdot \bar{ds}} \ \text{Ampere's}$$

13. Current $I = \oint_{\langle l \rangle} \bar{H} \cdot \bar{dl} = \oint_{\langle S \rangle} \bar{J} \cdot \bar{ds}$ Ampere's

14. Magnetic Flux (ϕ).

$$\boxed{\phi = \oint_{\langle S \rangle} \bar{B} \cdot \bar{ds}} \ wb.$$

15. magnetic flux density ($\bar{B}$)

$$\boxed{\bar{B} = \frac{d\phi}{ds}} \ wb/m^2; \quad \bar{B} = \frac{d\phi}{ds} \bar{a}_n \ wb/m^2. \text{ (or) Tesla.}$$

16. $\boxed{\bar{B} = \mu_0 \bar{H}}$ wb/m^2

17. Vector magnetic potential ($\bar{A}$); $\boxed{\bar{B} = \nabla \times \bar{A}}$; wb/m^2

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Module -4

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Part-A : Magnetic Forces
Force on a moving charge, differential current elements, Force between differential current elements.
Part-B: Magnetic Materials
Magnetization and permeability, Magnetic boundary conditions, Magnetic circuit, Potential Energy and forces on magnetic materials.

Part-A : Magnetic Forces

Force on a moving charge, differential current elements, Force between differential current elements.

<u>Topics:</u>
4.1 Force on Moving charge or Lorentz force equation
Solved Problems
4.2 Force on a differential current element
4.3 Force between differential current elements
  a. Magnetic Force between two current elements
  b. Force between two parallel conductors

Summary
- List of Symbols
- List of Formulae

Part-B : Magnetic Materials

Magnetization and permeability, Magnetic boundary conditions, Magnetic circuit, Potential Energy and forces on magnetic materials.

<u>Topics:</u>
4.4 Concept of Magnetization and Permeability
4.5 Magnetic Boundary conditions
4.6 Magnetic Circuits
  a. Reluctance of a Magnetic circuits
  b. Comparison between electric and magnetic circuits
4.7 Reluctance in a series magnetic circuits
4.8 Potential Energy and Forces on Magnetic Materials.

Summary
- List of Symbols
- List of Formulae

Topic 4.1

Force on Moving charge or Lorentz force equation

Derive an expression for magnetic force on
i) Moving point charge and (5marks) 06-DEC2011/Jan 2012. 010-Dec/Jan 2015

10-Jan 2013

Derive lorentz force equation. (05 Marks)

02 - June /July 2010

Derive the Lorentz force equation for the force exerted on a moving charged particle charge Q, with velocity $\overline{v}$, in a magnetic field $\overline{B}$ and electric field $\overline{E}$ (06 Marks)

10 - June /July 2015

What is Lorentz force equation? (02 Marks)

10 - June /July 2014

Derive Lorentz's force equation. (05 Marks)

06 - June /July 2013

State and prove the Lorentz force equation. (08 Marks)

06 –Dec/Jan 2008

[illegible] (06 Marks)

06 –June/July 2014

Obtain the expression for magnetic force on moving point charge (5marks)

June/July 2016 EE

a. Derive Lorentz force equation for a moving change placed in a combined electric and magnetic field. (06 Marks)

Questions

Derive Lorentz force equation (5m)

(or)

obtain the expression for magnetic force on moving point charge (5m).

(or)

State and prove Lorentz force Equation and mention the applications of its solution. (6m).

[06 Jan 2012, 10-Jan 2015, 02-J|J 2010, 10-J|J 2015, 06 J|J 2014, 10 J|J 2014, 06 J|J 2013, 06-Jan 2008, J|J 2016 (EE)]

Solⁿ:- Force on moving point charge (or) Lorentz force Equation:-

A positive charge 'Q' moving with velocity $\bar{v}$ in a uniform magnetic field of flux density $\bar{B}$, experiences a force $\bar{F}$ given by

$$\bar{F}_1 = Q(\bar{v} \times \bar{B}) \text{ Newton} \quad \leftarrow (1)$$

The magnitude of the force is given by

$$|\bar{F}_1| = QvB \sin\theta \text{ Newton} \quad \leftarrow (2)$$

where 'θ' is the angle between the velocity vector $\bar{v}$ and $\bar{B}$. the direction of force is perpendicular to the plane containing $\bar{v}$ and $\bar{B}$. and points in the direction along which a right handed screw would move if rotated from $\bar{v}$ to $\bar{B}$.

$\bar{F}_1 = Q(\bar{v} \times \bar{B})$; N

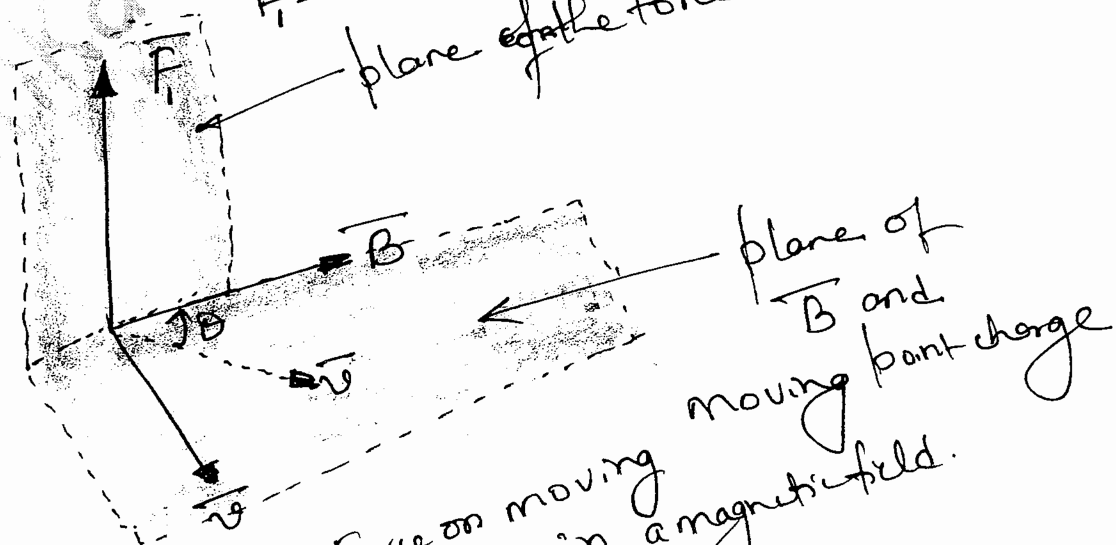

fig. Force on moving charge in a magnetic field.

if the charge 'Q' is subjected to only the influence of an electric field of strength $\overline{E}$ then the force experienced by it will be

$$\overline{F}_2 = Q\overline{E} \longleftarrow (3)$$

if it is subjected to the combined influence of a magnetic field of flux density $\overline{B}$ and an electric field of strength $\overline{E}$, then the resultant $\overline{F}$ will be sum of two forces $\overline{F}_1$ and $\overline{F}_2$.

$$\therefore \boxed{\overline{F} = Q(\overline{E} + \overline{v} \times \overline{B})} \text{ : Newton} \longleftarrow (4)$$

Eq (4) is called the Lorentz force equation.

Applications of Lorentz force equation.

Lorentz force equation and its solution is required in determining electron orbits in the magnetron, proton paths in the cyclotron, plasma characteristics in a magneto-hydro dynamics (MHD) generator, (or) in general charged particle motion in combined electric and magnetic fields.

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06 - June /July 2011

The point charge Q=18 nc has a velocity of 5×10^6 m/s in the direction $\vec{a}_v = 0.6\vec{a}_x + 0.75\vec{a}_y + 0.3\vec{a}_z$. Calculate the force exerted on the charge Q by the field $\vec{B} = -3\vec{a}_x + 4\vec{a}_y + 4\vec{a}_z$ mT. **(05 Marks)**

10 - June /July 2012

A point charge Q = 18 nc has a velocity of 5×10^6 m/s in the direction $\vec{a}_v = 0.6\vec{a}_x + 0.75\vec{a}_y + 0.3\vec{a}_z$. Calculate the magnitude of the force exerted on the charge by the field :

i) $\vec{E} = -3\vec{a}_x + 4\vec{a}_y + 6\vec{a}_z$ kV/m

ii) $\vec{B} = -3\vec{a}_x + 4\vec{a}_y + 6\vec{a}_z$ mT

iii) $\vec{B}$ and $\vec{E}$ acting together. **(08 Marks)**

06 – May/June 2010

A point charge Q = 18 nC has a velocity of 5×10^6 m/s in the direction $\vec{a} = 0.6\vec{a}_x + 0.75\vec{a}_y + 0.3\vec{a}_z$. Calculate the magnitude of the force exerted on the charge by the field $\vec{B} = -3\vec{a}_x + 4\vec{a}_y + 6\vec{a}_z$ mT. **(06 Marks)**

Question 1.

A point charge of $Q = 18$nc has a velocity of 5×10^6 m/sec in the direction $\bar{a}_v = 0.6\bar{a}_x + 0.75\bar{a}_y + 0.3\bar{a}_z$. Calculate the Magnitude of the force exerted on the charge by the field:

i. $\bar{E} = -3\bar{a}_x + 4\bar{a}_y + 6\bar{a}_z$ kV/m.

ii. $\bar{B} = -3\bar{a}_x + 4\bar{a}_y + 6\bar{a}_z$ mTesla.

iii. $\bar{B}$ and $\bar{E}$ acting together. (8 m)

06 – June/July 2011, 10 – June/July 2012 and 06 – may/June 2010. J/J 2016(EE)

Soln:- Given

$Q = 18\,nC$, $v = 5\times10^{6}$ m/sec.

$\bar{a}_v = 0.6\,\bar{a}_x + 0.75\,\bar{a}_y + 0.3\,\bar{a}_z$

$\bar{B} = -3\,\bar{a}_x + 4\,\bar{a}_y + 6\,\bar{a}_z$ mTesla.

i.e. $\bar{F}_m = Q\,\bar{v}\times\bar{B}$; Newton

$\bar{v} = v\,\bar{a}_v$ m/sec.

$\therefore\ \bar{F}_m = Q\cdot v\ \bar{a}_v\times\bar{B}$

$$\bar{F}_m = 18\times10^{-9}\times5\times10^{6}\begin{vmatrix}\bar{a}_x & \bar{a}_y & \bar{a}_z\\ 0.6 & 0.75 & 0.3\\ -3 & 4 & 6\end{vmatrix}\times10^{-3}\ ;\ N$$

$$\bar{F}_m = 90\times10^{-6}\left[3.3\,\bar{a}_x - 4.5\,\bar{a}_y + 4.65\,\bar{a}_z\right]$$

$$\boxed{\bar{F}_m = 297\,\bar{a}_x - 405\,\bar{a}_y + 418.5\,\bar{a}_z}\ \mu N.$$

Magnitude of force $\bar{F}_m$ is

$$|\bar{F}_m| = \sqrt{297^2 + 405^2 + 418.5^2} = 653.7402\,\mu N$$

$$\boxed{|\bar{F}_m| = 653.7402\,\mu N}$$

i. $\overline{F_E} = Q\overline{E}$: N

$$\overline{F_E} = 18\times10^{-9}\left[-3\overline{a_x}+4\overline{a_y}+6\overline{a_z}\right]\times10^{3}$$

$$\boxed{\overline{F_E} = -54\overline{a_x}+72\overline{a_y}+108\overline{a_z}} : \mu N$$

$$\boxed{|\overline{F_E}| = 140.5844} \ \mu N$$

iii. $\overline{B}$ and $\overline{E}$ acting together.

$$\overline{F}_{total} = \overline{F_E}+\overline{F_m} : N$$

$$= 297\overline{a_x}-405\overline{a_y}+418.5\overline{a_z}-54\overline{a_x}+72\overline{a_y}+108\overline{a_z} : \mu N$$

$$\boxed{\overline{F}_{total} = 243\overline{a_x}-333\overline{a_y}+526.5\overline{a_z}} : \mu N$$

$$|\overline{F}_{total}| = \sqrt{243^2+333^2+526.5^2} : \mu N$$

$$\boxed{|\overline{F}_{total}| = 668.6854} : \mu N$$

problem 2.

A positive point charge $Q = 20nC$ is moving with a velocity of 12×10^6 m/sec in a direction specified by the unit vector $\bar{a}_v = -0.48\,\bar{a}_x - 0.6\,\bar{a}_y + 0.64\,\bar{a}_z$.

Find

i. the magnitude of the vector force exerted on the moving particle by the magnetic field

$$\bar{B} = 2\bar{a}_x - 3\bar{a}_y + 5\bar{a}_z \ ; mT.$$

ii. by the electric field $\bar{E} = 2\bar{a}_x - 3\bar{a}_y + 5\bar{a}_z$ kv/m.

iii. Both $\bar{B}$ and $\bar{E}$ acting together.

Solu:-

i. $\bar{F}_m = Q\,\bar{v} \times \bar{B}$

$$\bar{F}_m = Q\,v\,\bar{a}_v \times \bar{B}$$

$$\bar{F}_m = (20 \times 10^{-9})(12 \times 10^6) \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ -0.48 & -0.6 & 0.64 \\ 2 & -3 & 5 \end{vmatrix} \times 10^{-3}.$$

$$\bar{F}_m = 240 \times 10^{-6}\left[-1.08\,\bar{a}_x + 3.68\,\bar{a}_y + 2.64\,\bar{a}_z\right]$$

$$\boxed{\bar{F}_m = -259.2\,\bar{a}_x + 883.2\,\bar{a}_y + 633.6\,\bar{a}_z} \ ; \mu N$$

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Magnitude of force $\overline{F}_m$ is

$$\boxed{|\overline{F}_m| = 1117.44\,\mu N}$$

ii. $\overline{F}_E = Q\overline{E}$

$$\overline{F}_E = 20\times10^{-9}\times10^{3}\left[2\overline{a}_x - 3\overline{a}_y + 5\overline{a}_z\right] N.$$

$$\boxed{\overline{F}_E = 40\overline{a}_x - 60\overline{a}_y + 100\overline{a}_z}\ ;\ \mu N$$

$$\boxed{|\overline{F}_E| = 123.2882\,\mu N}$$

iii. $\overline{F}_{total} = \overline{F}_E + \overline{F}_m$; N.

$$= 40\overline{a}_x - 60\overline{a}_y + 100\overline{a}_z - 259.2\overline{a}_x + 883.2\overline{a}_y + 633.6\overline{a}_z\ ;\ \mu N$$

$$\boxed{\overline{F}_{total} = -219.2\overline{a}_x + 823.2\overline{a}_y + 733.6\,\overline{a}_z}\ ;\ \mu N$$

$$\boxed{|F_{total}| = 1124.22\,\mu N}$$

Note:- $|z_1 + z_2| \le |z_1| + |z_2|$
$|z_1 - z_2| \ge |z_1| - |z_2|$ } properties of Modulus.

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Problem 3.

A point charge of $Q = -1.2C$ has velocity $\bar{v} = 5\bar{a}_x + 2\bar{a}_y - 3\bar{a}_z$ m/sec. Find the magnitude of the force exerted on the charge if,

i. $\bar{E} = -18\bar{a}_x + 5\bar{a}_y - 10\bar{a}_z$ v/m.

ii. $\bar{B} = -4\bar{a}_x + 4\bar{a}_y + 3\bar{a}_z$ Tesla.

iv. both are present Simultaneously.

[15-June/July 2017 (8M) CBCS]

Solu:- i) $\bar{F}_E = Q\bar{E}$

$$\bar{F}_E = -1.2\left[-18\bar{a}_x + 5\bar{a}_y - 10\bar{a}_z\right] \text{ N}$$

$$\boxed{\bar{F}_E = +21.6\bar{a}_x - 6\bar{a}_y + 12\bar{a}_z} \text{ N}$$

$$|\bar{F}_E| = \sqrt{21.6^2 + 6^2 + 12^2} = 25.4275 \text{ Newton}$$

$$\boxed{|\bar{F}_E| = 25.4275} \text{ Newton}$$

ii) $\bar{F}_m = Q\,\bar{v} \times \bar{B}$ Newton

$$\bar{F}_m = -1.2 \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 5 & 2 & -3 \\ -4 & 4 & 3 \end{vmatrix} \text{ Newton}$$

$$\boxed{\overline{F_m} = -21.6\,\overline{a_x} + 3.6\,\overline{a_y} - 33.6\,\overline{a_z}} \text{ ; Newton.}$$

$$|\overline{F_m}| = \sqrt{21.6^2 + 3.6^2 + 33.6^2} \text{ ; N.}$$

$$\boxed{|\overline{F_m}| = 40.1058} \text{ Newton}$$

iii. Both $\overline{E}$ and $\overline{B}$ acting Simultaneously

$$\overline{F_{total}} = \overline{F_E} + \overline{F_m}$$

$$= 21.6\,\overline{a_x} - 6\,\overline{a_y} + 12\,\overline{a_z} - 21.6\,\overline{a_x} + 3.6\,\overline{a_y} - 33.6\,\overline{a_z}$$

$$\boxed{\overline{F_{total}} = -2.4\,\overline{a_y} - 21.6\,\overline{a_z}} \text{ ; Newton}$$

$$|\overline{F_{total}}| = \sqrt{2.4^2 + 21.6^2}$$

$$\boxed{|\overline{F_{total}}| = 21.739} \text{ Newton}$$

Topic 4.2

Force on a differential current element

02-DEC2010

Derive an equation for the force acting as a current element. (06 Marks)

Derive an expression for magnetic force on : 010-Dec/Jan 2015

ii) Differential current element. 06-DEC2011/Jan 2012 (5marks)

06 –June/July 2014

Obtain the expression for magnetic force on differential current element.

06-DEC 2013/Jan 2014

Derive an expression for the force on a differential current carrying element. (06 Marks)

10-June/July 2013

Discuss the force on a differential current element and also obtain the expression for force. (08 Marks)

02 - June /July 2011

Obtain an expression for force on differential current element placed in a magnetic field. (07 Marks)

Dec/Jan 2016

a. Derive expression for force on a differential current element (06 Marks)

Dec/Jan 2017 CBCS scheme

a. Find the expression for force on differential current element moving in a steady magnetic field. Deduce the result to a straight conductor in a uniform magnetic field. (08 Marks)

Questions

Obtain the expression for magnetic force on differential Current Element.

(or)

Derive an expression for the force on a differential Current Carrying element.

(or)

Find the expression for force on differential Current Element moving in a steady magnetic field. Deduce the result to a straight Conductor in a uniform magnetic field.

Topic U-2. Force on a differential Current Element

8 a. Find the expression for force on differential current element moving in a steady magnetic field. Deduce the result to a straight conductor in a uniform magnetic field. (08 Marks)

[15-June/July 2017 (4M) CBCS] 15-Dec/Jan 2017 (CBCS) Scheme.

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soln:- The Force on a charge particle moving through steady magnetic field can be written as the differential force exerted on a differential Element of charge

$$d\bar{F} = dQ\,(\bar{v} \times \bar{B}) \quad \text{Newton} \longleftarrow (1)$$

the Current density interms of volume charge density is given by

$$\bar{J} = \rho_v \bar{v} \quad A/m^2 \longleftarrow (2)$$

and $dQ = \rho_v\, dv \longleftarrow (3)$

using eqn (3) in eqn (1)

$$\therefore\; d\bar{F} = \rho_v\, dv\; \bar{v} \times \bar{B}$$

$$d\bar{F} = \rho_v \bar{v} \times \bar{B}\; dv$$

$$d\bar{F} = (\bar{J} \times \bar{B})\, dv \longleftarrow (4)$$

the differential Current element interms of Surface Current (A/m) is given by

$$I\,\bar{dL} = \bar{K}\, dS = \bar{J}\, dv \longleftarrow (5)$$

Lorentz force eqn can be applied to Surface Current density

i.e use eqn (5) in eqn (4)

$$d\bar{F} = \bar{K} \times \bar{B} \, ds \longleftarrow (6)$$

Ily for a differential current filament

$$d\bar{F} = I \, \overline{dl} \times \bar{B} \longleftarrow (7)$$

the net force

$$\bar{F} = \int_{\langle vol \rangle} \bar{J} \times \bar{B} \, dv \longleftarrow (8)$$

$$\bar{F} = \int_{\langle s \rangle} \bar{K} \times \bar{B} \, ds \longleftarrow (9)$$

and

$$\bar{F} = \oint_{\langle l \rangle} I \, \overline{dl} \times \bar{B} = -I \oint_{\langle l \rangle} \bar{B} \times \overline{dl} \longleftarrow (10)$$

if consider the conductor to be straight and in a uniform magnetic field

$$\boxed{\oint_{\langle l \rangle} \overline{dl} = \bar{L}}$$

$$\Rightarrow \bar{F} = I \bar{L} \times \bar{B} = I L B \sin\theta \, \bar{a}_n \, ; N$$

$$\boxed{|\bar{F}| = F = BIL \sin\theta}$$ Newton

where 'θ' is the angle b/w the vectors representing the direction of the current flow and the direction of the magnetic flux density.

Problem 4

~~a. A conductor 4m long lies along the y axis with a current of 10A in the~~ $\overline{ay}$ direction. Find the force on the conductor if the field in the region is $\overline{B} = 0.005\overline{ax}$ Tesla. (04 Marks)

15-Dec/Jan 2017 (CBCS) Scheme.

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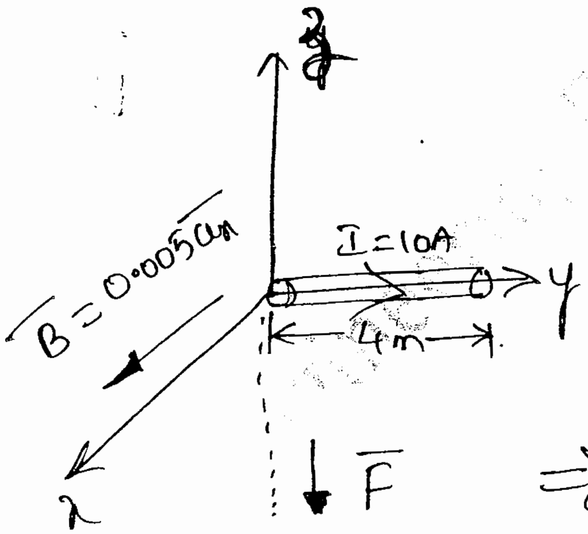
[15-June/July 2017(4m) CBCS]

Soln:- given $I = 10A$.

$\overline{B} = 0.005\overline{a_x}\,T$; $\overline{dl} = dl\,\overline{a_y} = 4\overline{a_y}$

Force $(\overline{F})$ experienced by a Current Carrying conductor in presence of magnetic field $\overline{B}$ is given by

$$\overline{F} = I\,\overline{dl} \times \overline{B} \quad \text{Newton}$$



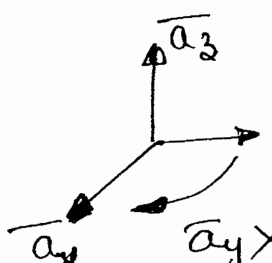
Since the conductor is placed along 'y' dirn

$\therefore \overline{dl} = dl\,\overline{a_y}$

$\overline{dl} = 4\overline{a_y}$

$$\Rightarrow \overline{F} = 10\,(4\overline{a_y} \times 0.005\overline{a_x})$$

$$\overline{F} = [10 \times 4 \times 0.005]\,(\overline{a_y} \times \overline{a_x}) \rightarrow (-\overline{a_z})$$



$\overline{a_y} \times \overline{a_x} = -\overline{a_z}$

$$\Rightarrow \overline{F} = 0.2(-\overline{a_z})$$

$$\boxed{\overline{F} = -0.2\,\overline{a_z}} \text{ Newton.}$$

$$\boxed{|\overline{F}| = 0.2} \text{ Newton}$$

problem 5

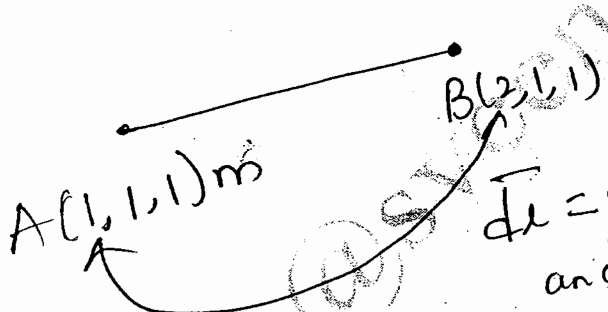
02 - June /July 2010

The field $B = -2\hat{a}_x + 3\hat{a}_y + 4\hat{a}_z$ mT is present in free space. Find the vector force exerting a straight wire, carrying 12A in the a_{AB} direction given A(1, 1, 1)m and B(2, 1, 1)m (06 Marks)

Question:

The field $\bar{B} = -2\bar{a}_x + 3\bar{a}_y + 4\bar{a}_z$ mTesla in present in free space. Find the vector force exerting a straight wire, carrying 12A in the $\bar{a}_{AB}$ direction given A(1,1,1)m and B(2,1,1)m. (6 m).

soln:



$\bar{dl} = dx\,\bar{a}_x$.

and $dx = 2 - 1 = 1$ m.

Force exerting on straight wire

$$\bar{F} = I\bar{dl} \times \bar{B} \text{ : Newton}$$

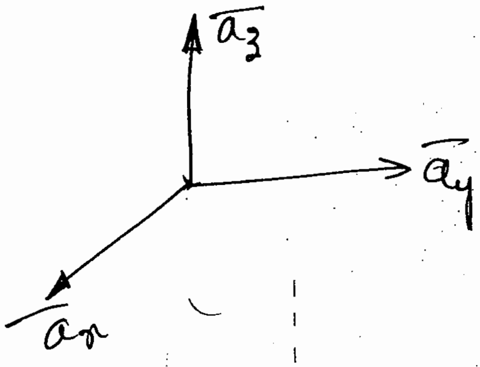
$I = 12A$.

$$I\bar{dl} = 12\,dx\,\bar{a}_x$$

$$I\bar{dl} = 12\bar{a}_x \text{ A-m}$$

$$\bar{F} = 12\bar{a}_x \times (-2\bar{a}_x + 3\bar{a}_y + 4\bar{a}_z)\cdot(1 \times 10^{-3}) \text{ : N}$$

$$\bar{F} = 12\bar{a}_x \times (-2\bar{a}_x + 3\bar{a}_y + 4\bar{a}_z)(1\times10^{-3})$$



$$\bar{a}_x \times \bar{a}_x = 0.$$

$$\bar{a}_x \times \bar{a}_y = -\bar{a}_z$$

$$\bar{a}_x \times \bar{a}_z = -\bar{a}_y$$

$$\bar{F} = 12\times1\times10^{-3}\left[0 + 3(-\bar{a}_z) + 4(-\bar{a}_y)\right]$$

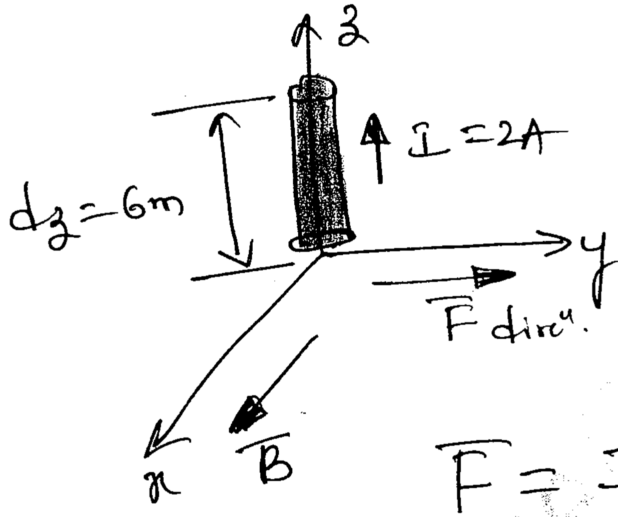
$$\bar{F} = 12\times10^{-3}\left[-4\bar{a}_y - 3\bar{a}_z\right]$$

$$\boxed{\bar{F} = -48\bar{a}_y - 36\bar{a}_z \ ; \ mN}$$

problem 6.

A conductor 6m Long lies along 'z' axis with a Current of 2A in $\bar{a}_z$ direction. Find the force exerted by conductor if $\bar{B} = 0.08\,\bar{a}_x$ Tesla.

Solu:-



$$\bar{F} = I\,\bar{dl} \times \bar{B} \; ; N$$

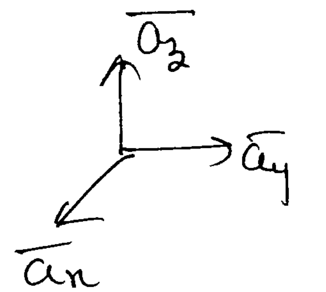
Since the conductor is placed along 'z' direction

$$\therefore \bar{dl} = dz \cdot \bar{a}_z = 6\,\bar{a}_z$$

$$\bar{F} = I\,\bar{dl} \times \bar{B}$$

$$\bar{F} = 2\,(6\,\bar{a}_z) \times 0.08\,\bar{a}_x$$

$$\bar{F} = 2 \times 6 \times 0.08\,[\bar{a}_z \times \bar{a}_x]$$



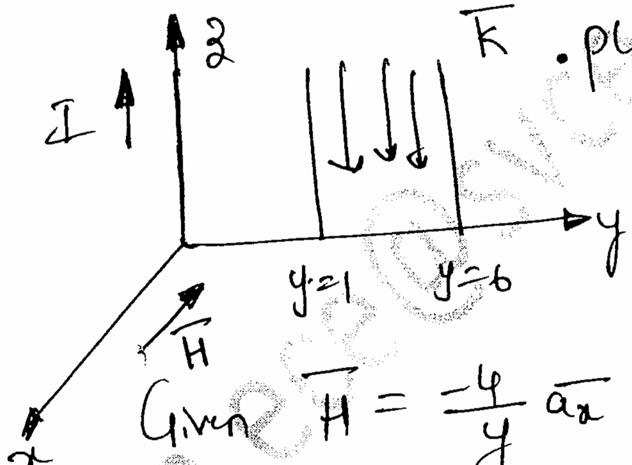
$$\bar{a}_z \times \bar{a}_x = +\bar{a}_y$$

$$\boxed{\bar{F} = 0.96\,\bar{a}_y} \; ; \text{Newton}$$

Problem 7.

A perfectly conducting current element on the z-axis produces the magnetic field $\bar{H} = -\frac{4}{y}\bar{a}_x$ A/m at P(0, y, z) in free space. Find the force exerted on a current carrying strip lying between y=1 and y=6m in the yz plane carrying a current density $\bar{K} = -\pi \bar{a}_z$ A/m.

Soln:



$ds = dz\,dy$.

$\bar{K} = -\pi \bar{a}_z$ A/m.

Given $\bar{H} = \frac{-4}{y}\bar{a}_x$; $\bar{B} = \mu_0 \bar{H}$ wb/m²

$$d\bar{F} = \bar{K} \times \bar{B}\, ds$$

$$d\bar{F} = -\pi \bar{a}_z \times \left(\frac{-4\mu_0}{y}\bar{a}_x\right) dy\, dz$$

$$d\bar{F} = 4\pi\mu_0 \frac{dy}{y} \cdot dz\ \bar{a}_y \qquad \bar{a}_z \times \bar{a}_x = \bar{a}_y$$

$$\bar{F} = 4\pi\mu_0 \int_{y=1}^{6} \frac{1}{y} dy \int_{z=0}^{1} dz\ \bar{a}_y$$

$$\boxed{\bar{F} = 4\pi\mu_0 \ln(6)\ \bar{a}_y} \text{ N/m}$$

problem 8

02-DEC2008/Jan 2009

Find the force per meter length between two long parallel wires separated by 10 cm in air and carrying a current of 10 A in the same direction. Derive any formula used. (05 Marks)

10-Jan 2013

Find the force per meter length between two long parallel wires separated by 10 cm in air and carrying a current of 10A in the same direction. **(05 Marks)**

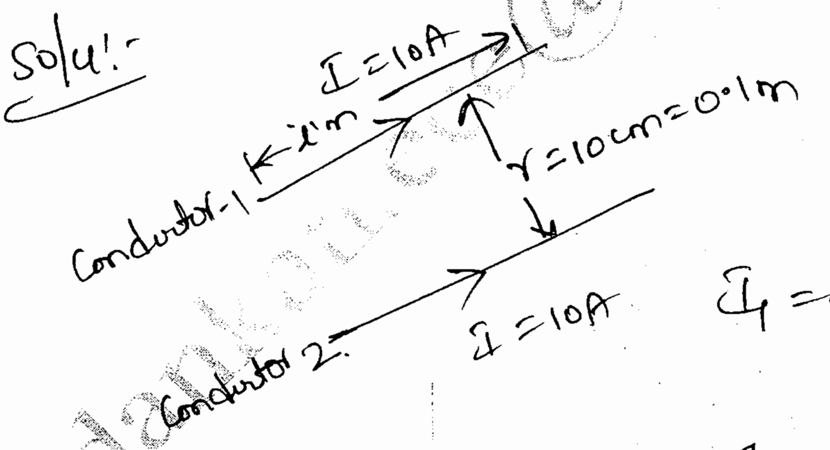
06 - June /July 2013

Find the force per meter length between two long parallel wires separated by 10 cm in air and carrying a current of 10 A in the same direction. (04 Marks)

Question

Find the force permeter Length between two Long parallel wires separated by 10cm in air and Carrying a Current of 10A in the Same direction. (4M).

Solu:-



$I_1 = I_2 = 10A$.

$$\mu = \mu_0 = 4\pi \times 10^{-7} \ H/m.$$

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r} \ N/m.$$

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$$\frac{F}{l} = \frac{4\pi \times 10^{-7} \times 10 \times 10}{2\pi (0.1)}$$

$$F/l = 0.2 \times 10^{-3} \text{ N/m}$$

$$\boxed{F/l = 0.2 \text{ m N/m}}$$

Since the Current is in the same direction, the nature of force is attractive.

problem.

10 - June /July 2015

A square loop carrying 2 mA current is placed in the field of an infinite element carrying current of 15 A as shown in Fig. Q5 (b). Find the force exerted on the loop. (08 Marks)

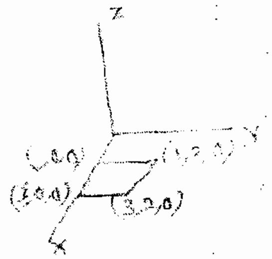


Fig. Q5 (b)

06 – May/June 2010

A sq. loop carrying 2 mA current is placed in the field of an infinite filament carrying current of 15 Amp as shown, fig. Q5 (c). Find the force exerted on the sq.loop (08 Marks)

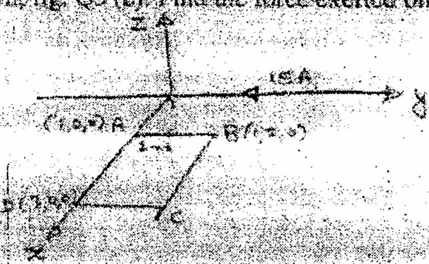


Fig. Q5 (c)

Question

A square Loop carrying 2mA Current is placed in the field of an infinite filament carrying Current of 15A as shown in fig.

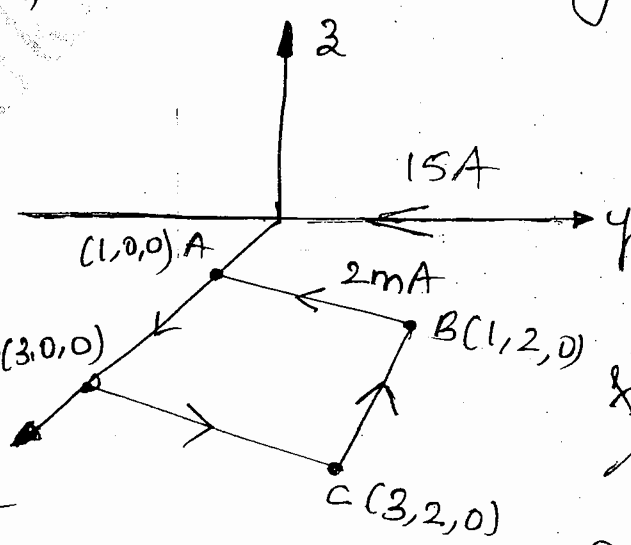
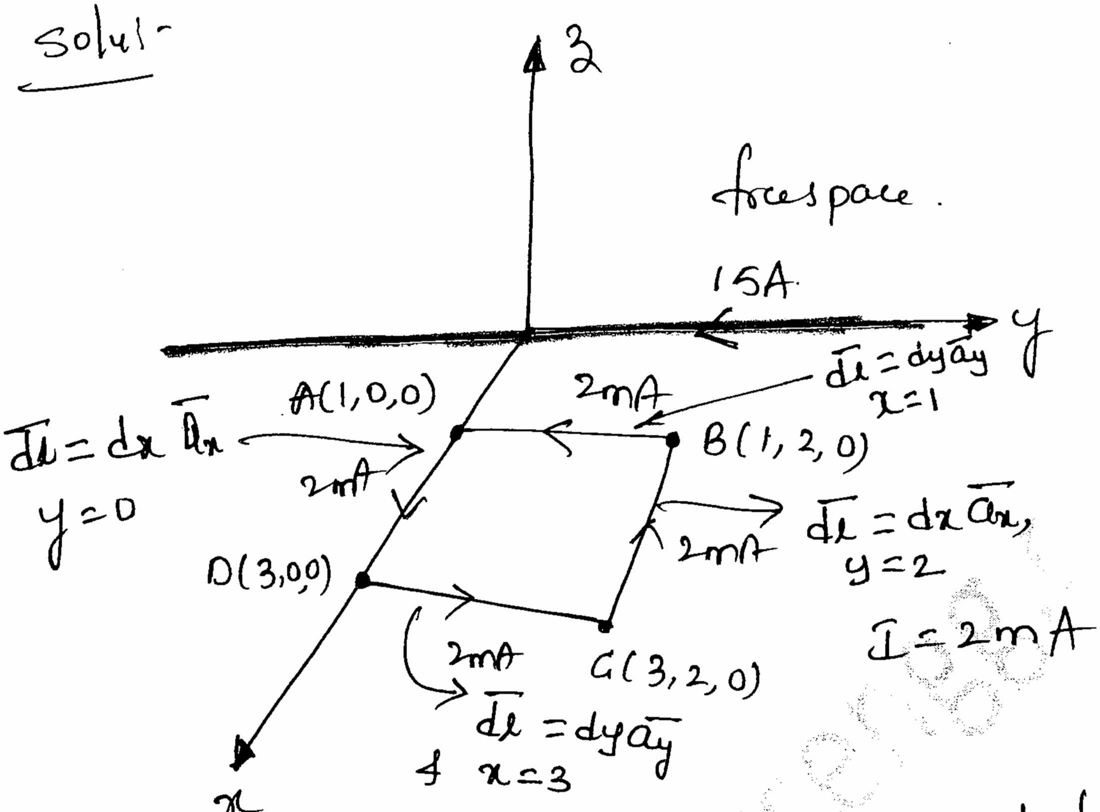


fig. A square loop of wire in xy plane carrying 2mA is subjected to a non-uniform $\bar{B}$ field.

Solu :-



The Field produced in the plane of the Loop due to infinite Length Current carrying filament placed along 'y' axis is

$$\overline{H} = \frac{I}{2\pi x}\,\overline{a_z} \quad A/m.$$

$$\therefore \overline{B} = \mu_0 \overline{H} = \cancel{4\pi}^{2} \times 10^{-7} \cdot \frac{15}{\cancel{2\pi} x} = \frac{3 \times 10^{-6}}{x}\,\overline{a_z}$$

$$\boxed{\overline{B} = \frac{3}{x}\,\overline{a_z}} \ \mu\,wb/m^2$$

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the Force exerted on a Square Loop is given by

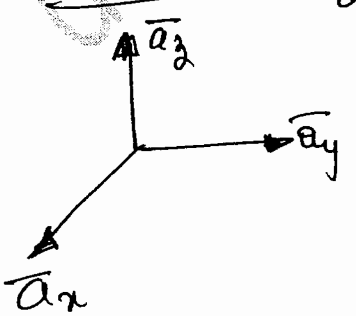
$$\overline{F} = -I \oint \overline{B} \times \overline{dl} \quad ; \text{Newton}$$

$$\overline{F} = -2\times10^{-3}(3\times10^{-6})\left[\int_{x=1}^{3} \frac{\overline{a_z}}{x} \times dx\,\overline{a_x} + \int_{y=0}^{2} \frac{\overline{a_z}}{3} \times dy\,\overline{a_y} + \int_{x=3}^{1} \frac{\overline{a_z}}{x} \times dx\,\overline{a_x} + \int_{y=2}^{0} \frac{\overline{a_z}}{1} \times dy\,\overline{a_y}\right]$$

$$\overline{F} = -6\times10^{-9}\left[\ln x\Big|_1^3 \overline{a_y} + \frac{1}{3} y\Big|_0^2 (-\overline{a_x}) + \ln x\Big|_3^1 (\overline{a_y}) + y\Big|_2^0 (-\overline{a_x})\right].$$

Note: $\overline{a_z} \times \overline{a_y} = -\overline{a_x}$,

$\overline{a_z} \times \overline{a_x} = +\overline{a_y}$



$$\overline{F} = -6\times10^{-9}\left[\ln(3)\,\overline{a_y} - \frac{2}{3}\,\overline{a_x} + \ln\left(\frac{1}{3}\right)\overline{a_y} + 2\overline{a_x}\right]$$

$$\overline{F} = -8\,\overline{a_x} \times 10^{-9} \text{ Newton}$$

$$\boxed{\overline{F} = -8\,\overline{a_x}\ \eta \text{ Newton}}$$

$$|F| = 8\ \eta N.$$

∴ the net force on the Loop is in the $-ve$ x direction. i.e $-\overline{a_x}$.

Topic 4.3

Force between differential current elements

a. Magnetic Force between two current elements
b. Force between two parallel conductors

4.3a. Magnetic force between two differential Current Element.

06-DEC2010

Derive an equation for the force between the two differential current elements. (06 Marks)

06-DEC2008/Jan 2009

Obtain the expression of magnetic force between two current elements and hence f current loops. **(06 Mark**

06-DEC2009/Jan 2010

With usual notations, derive the equation for magnetic force between two differential current elements. **(06 Marks)**

10-DEC 2013/Jan 2014

Deduce the expression for force between the differential current elements. (10 Marks)

10 - June /July 2012

Obtain the expression of magnetic force between differential current elements. (05 Marks)

06 – Jan 2013

Derive the equation for magnetic force between two differential current elements. (06 Marks)

Dec/Jan 2017 CBCS scheme

a. Derive an equation for the magnetic force between two differential current elements. **(06 Marks)**

Questions

Derive an equation for the force between the two differential Current Elements

(or)

Obtain the expression for force between the differential Current elements.

(or)

Derive the equation for magnetic force between two differential Current Elements

15- Dec/Jan 2017 (CBCS) Scheme.

Topic 4.3a

Derive an equation for the magnetic force between two differential current elements. (06 Marks)

Sol:-

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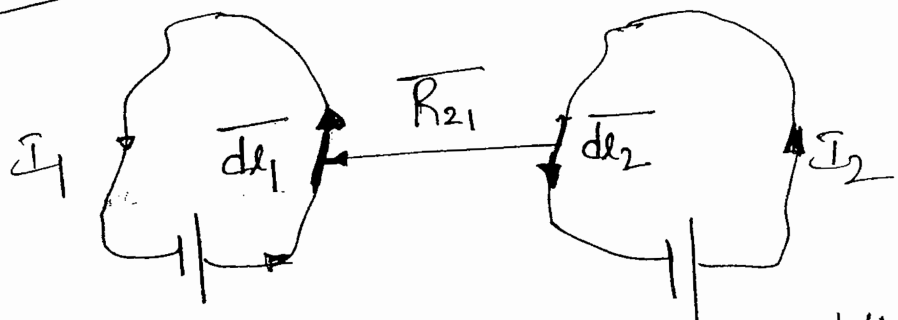

fig. magnetic force b/w two differential Current elements.

Consider two current loops with currents I_1 and I_2. The loops are divided into small vector line segments $\overline{dl}$ and the current element $I\overline{dl}$. And the current elements are respectively $I_1\overline{dl_1}$ and $I_2\overline{dl_2}$.

According to Biot-Savart Law, both the current elements produces magnetic fields.

The magnetic field produced by $I_2\overline{dl_2}$ at $I_1\overline{dl_1}$ is

$$d\overline{B}_2 = \frac{\mu_0 I_2 \overline{dl_2} \times \overline{a}_{R_{21}}}{4\pi R_{21}^2} \longleftarrow (1)$$

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Hence, the force on Current Element $I_1 \overline{dl_1}$ due to the field $d\overline{B_2}$ produced by the current element $I_1 \overline{dl_1}$ is

$$d\overline{F_1} = I_1 \overline{dl_1} \times d\overline{B_2} \longleftarrow (2)$$

using eq^n (1) in eq^n (2)

$$d\overline{F_1} = \frac{\mu_0 I_1 \overline{dl_1} \times [I_2 \overline{dl_2} \times \overline{a}_{R_{21}}]}{4\pi R_{21}^2} \quad \text{Newton's} \longleftarrow (3)$$

the above equation in similar to Coulomb's Law and can be determined experimentally.

the total force $\overline{F_1}$ on Current Loop 1 due to Current Loop 2 is

$$\overline{F_1} = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{\langle l_1 \rangle} \oint_{\langle l_2 \rangle} \frac{\overline{dl_1} \times (\overline{dl_2} \times \overline{a}_{R_{21}})}{R_{21}^2} \quad \text{Newtons} \longleftarrow (4)$$

∴ Illy the force $\overline{F_2}$ on Loop 2 due to magnetic field $\overline{B_1}$ produced by Loop 1 is

$$\overline{F_2} = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{\langle l_2 \rangle} \oint_{\langle l_1 \rangle} \frac{\overline{dl_2} \times (\overline{dl_1} \times \bar{a}_{R_{12}})}{R_{12}^2} \text{ Newton's} \quad \leftarrow (5)$$

from eqⁿ (4) and eqⁿ (5)

$$\boxed{\overline{F_2} = -\overline{F_1}} \text{ Newton's}$$

The above condition indicates that both forces F_1 and $\overline{F_2}$ obey Newton's third Law i.e for Every action there is equal and opposite reaction.

Topic 4.3b.

4.3b Force between two parallel Conductors

Consider a two long parallel conductors of length 'l' m placed in free space, having a distance of separation 'r' m between them.

assume that the conductors carries current in opposite direction as shown in fig.

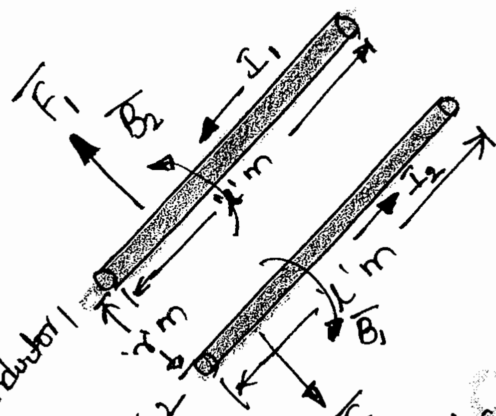


fig. Two parallel conductors carrying current in opposite direction.

The Force $\overline{F_1}$ on a length 'l' of conductor-1 due to magnetic field produced by conductor-2 is

$$\overline{F_1} = I_1 \overline{l} \times \overline{B_2} \text{ ; Newton}$$

$$\overline{F_1} = I_1 l B_2 \sin\theta \, \overline{a_n} \text{ ; Newton}$$

$$|\overline{F_1}| = I_1 l B_2 \sin\theta \text{ ; N}$$

Since $\theta = 90°$ (from fig)

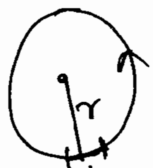
$$\boxed{|\overline{F_1}| = I_1 l B_2}$$

Note:- from fig. $I_1 \overline{l}$ and $\overline{B_2}$ are at right angles to each other $\therefore \theta = 90°$.

To find B_2, using Amperes Circuital Law

i.e $\oint \overline{H_2} \cdot \overline{dl} = I_2$ Ampere's

$$H_2 \cdot (2\pi r) = I_2$$



$\overline{H_2} = H_\phi \overline{a_\phi}$

$\overline{dl} = r\,d\phi\, \overline{a_\phi}$

$$\oint H_2 \overline{a_\phi} \cdot r\,d\phi\, \overline{a_\phi} = H_2 \int_{\phi=0}^{2\pi} r\,d\phi\, \cancel{\overline{a_\phi} \cdot \overline{a_\phi}}^{\,1} \Rightarrow H_2 (2\pi r).$$

$$\boxed{H_2 = \frac{I_2}{2\pi r}} \text{ A/m.}$$

$$\therefore \boxed{B_2 = \mu_0 H_2 = \frac{\mu_0 I_2}{2\pi r}} \text{ wb/m}^2$$

$\therefore$ the magnitude of force F_1 is

$$\boxed{|\overline{F_1}| = \frac{\mu_0 I_1 I_2 l}{2\pi r}} \text{ Newton's}$$

lly the magnitude of force acting on a length 'l' of conductor-2 due to the magnetic field produced by conductor-1 is

$$\boxed{|\overline{F_2}| = \frac{\mu_0 I_1 I_2 l}{2\pi r} = I_2 l B_1} \text{ Newton.}$$

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i.e $$\boxed{B_1 = \frac{\mu_0 I_1}{2\pi r}} \quad \text{wb/m}^2$$

<u>obs</u>. from the expression $I_1 \bar{l} \times \bar{B}_2$ and $I_2 \bar{l} \times \bar{B}_1$, the force is repulsive if the currents in the conductors are in opposite directions and attractive if they are in same direction.

this is My opposite to the case of electrostatic field i.e like charges repel and unlike charges attract Each other.

Problem 1.0.

A current element $I_1\overline{dl_1} = -3\overline{a}_y$ Am at $P_1(5,2,1)$ and $I_2\overline{dl_2} = -4\overline{a}_z$ A·m at $P_2(1,8,5)$. Find the differential force on $\overline{dl_2}$.

Soln:

$$d\overline{F_2} = I_2\overline{dl_2} \times d\overline{B_1}$$

$$d\overline{F_2} = I_2\overline{dl_2} \times \left\{ \frac{\mu_0 I_1\overline{dl_1} \times \overline{a}_{R_{12}}}{4\pi R_{12}^2} \right\}$$

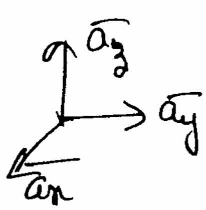
$$d\overline{F_2} = \frac{\mu_0}{4\pi R_{12}^2} \; I_2\overline{dl_2} \times (I_1\overline{dl_1} \times \overline{a}_{R_{12}})$$

$P_1(5,2,1)$ $P_2(1,8,5)$

$$\overline{R_{12}} = -4\overline{a}_x + 6\overline{a}_y + 4\overline{a}_z.$$

$$d\overline{F_2} = \frac{\mu_0}{4\pi R_{12}^3} \; I_2\overline{dl_2} \times (I_1\overline{dl_1} \times \overline{R_{12}})$$

$$d\overline{F_2} = \frac{\cancel{4\pi} \times 10^{-7}}{\cancel{4\pi}} \; \frac{(-4\overline{a}_z) \times [(-3\overline{a}_y) \times (-4\overline{a}_x + 6\overline{a}_y + 4\overline{a}_z)]}{[16+36+16]^{3/2}}$$



$\overline{a}_y \times \overline{a}_x = -\overline{a}_z$; $\overline{a}_y \times \overline{a}_y = 0$; $\overline{a}_y \times \overline{a}_z = +\overline{a}_x$

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$$= 10^{-7} \frac{(-4\bar{a}_z) \times [12(-\bar{a}_z) - 12(+\bar{a}_x)]}{(68)^{1.5}}$$

$\bar{a}_z \times \bar{a}_x = +\bar{a}_y$
$\bar{a}_z \times \bar{a}_z = 0$

$$= \frac{10^{-7}[+48\bar{a}_y]}{(68)^{1.5}}$$

$d\bar{F}_2 = 8.56008\,y\,\bar{a}_y$ Newton.

(a)

$$\boxed{d\bar{F}_2 = 8.56008\,\bar{a}_y} \; y\,N$$

Problem 11

010-Dec/Jan 2015

A current element $I_1 dl_1 = 10^{-4}\,\hat{a}_z$ (Am) is located at $P_1(2, 0, 0)$ and another current element $I_2 dl_2 = 10^{-6}\,[\hat{a}x - 2\hat{a}y + 3\hat{a}z]$ (Am) is located at $P_2(-2, 0, 0)$. Both are in free space :

i) Find force exerted on $I_2 dl_2$ by $I_1 dl_1$.

ii) Find force exerted on $I_1 dl_1$ by $I_2 dl_2$. **(10 Marks)**

Question.

A Current Element $I_1\overline{dl_1} = 10^{-4}\,\overline{a_z}$ A-m is Located at $P_1(2,0,0)$ another Current Element $I_2\overline{dl_2} = 10^{-6}[\overline{a_x} - 2\overline{a_y} + 3\overline{a_z}]$ A-m is Located at $P_2(-2,0,0)$ and both are in free space.

i> Find the Force Exerted on $I_2\overline{dl_2}$ by $I_1\overline{dl_1}$.

ii> Find the Force Exerted on $I_1\overline{dl_1}$ by $I_2\overline{dl_2}$.

Solu:-

$P_1\,(2,0,0)$ —— $P_2(-2,0,0)$

i. Force exerted on $I_2\,\overline{dl_2}$ by $I_1\,\overline{dl_1}$

$$d\overline{F_2} = I_2\,\overline{dl_2} \times d\overline{B_1}$$

$$= I_2\,\overline{dl_2} \times \left\{ \frac{\mu_0 I_1 \overline{dl_1} \times \overline{a}_{R_{12}}}{4\pi R_{12}^2} \right\}$$

$$d\overline{F_2} = \frac{\mu_0}{4\pi R_{12}^3}\left[I_2\,\overline{dl_2} \times (I_1\,\overline{dl_1} \times \overline{R_{12}}) \right]$$

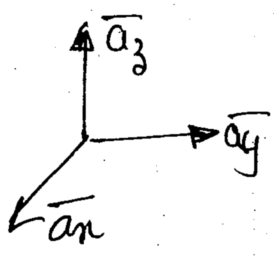
$$\overline{R_{12}} = (-2-2)\,\overline{a_x} = -4\,\overline{a_x}$$

$$|\overline{R_{12}}| = R_{12} = 4\ \text{m}.$$

$$d\overline{F_2} = \frac{\cancel{4\pi} \times 10^{-7}}{\cancel{4\pi}(4)^3}\left[10^{-6}(\overline{a_x} - 2\overline{a_y} + 3\overline{a_z}) \times (10^{-4}\,\overline{a_z} \times -4\,\overline{a_x})\right]$$

$$d\overline{F_2} = \frac{10^{-7}}{64}\left[10^{-6} \times 10^{-4} \times -4\,(\overline{a_x} - 2\overline{a_y} + 3\overline{a_z}) \times (\overline{a_z} \times \overline{a_x})\right]$$

$$= \frac{-4 \times 10^{-17}}{64}\left[(\overline{a_x} - 2\overline{a_y} + 3\overline{a_z}) \times \overline{a_y}\right]$$



$$\overline{a_z} \times \overline{a_x} = +\overline{a_y}$$
$$\overline{a_x} \times \overline{a_y} = +\overline{a_z}$$
$$\overline{a_y} \times \overline{a_y} = 0$$
$$\overline{a_z} \times \overline{a_y} = -\overline{a_x}$$

$$d\bar{F}_2 = \frac{-4\times10^{-17}}{64}\left[\bar{a}_z - 2(0) + 3(-\bar{a}_x)\right]$$

$$d\bar{F}_2 = \frac{-10^{-17}}{16}\left[\bar{a}_z - 3\bar{a}_x\right]$$

$$\boxed{d\bar{F}_2 = \left[1.875\,\bar{a}_x - 0.625\,\bar{a}_z\right]\times10^{-18}}\ \text{Newton.}$$

ii. Force Exerted on $I_1\bar{dl}_1$ by $I_2\bar{dl}_2$.

$$d\bar{F}_1 = I_1\bar{dl}_1 \times d\bar{B}_2$$

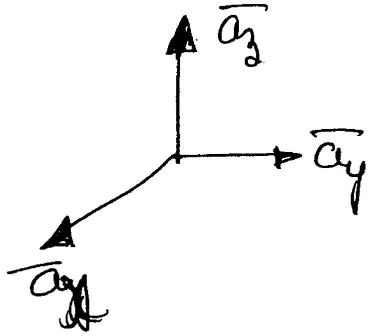
$$= I_1\bar{dl}_1 \times \left\{\frac{\mu_0 I_2\bar{dl}_2\times\bar{a}_{R_{21}}}{4\pi R_{21}^2}\right\}$$

$$d\bar{F}_1 = \frac{\mu_0}{4\pi R_{21}^3}\left\{I_1\bar{dl}_1 \times \left(I_2\bar{dl}_2\times\bar{R}_{21}\right)\right\}.$$

$$\bar{R}_{21} = 4\bar{a}_x\,; \qquad |\bar{R}_{21}| = R_{21} = 4\,m.$$

$$d\bar{F}_1 = \frac{4\pi\times10^{-7}}{4\pi\times4^3}\left[10^{-4}\bar{a}_z \times \left[10^{-6}(\bar{a}_x - 2\bar{a}_y + 3\bar{a}_z)\times4\bar{a}_x\right]\right.$$

$$d\bar{F}_1 = \frac{\cancel{4\pi}\times 10^{-7}\times 10^{-4}\times 10^{-6}\times \cancel{4}^{1}}{\cancel{4\pi}\times \cancel{64}_{16}}\left[\bar{a}_z \times \left[(\bar{a}_x - 2\bar{a}_y + 3\bar{a}_z)\times \bar{a}_x\right]\right]$$



$\bar{a}_x \times \bar{a}_x = 0$

$\bar{a}_y \times \bar{a}_x = -\bar{a}_z$

$\bar{a}_z \times \bar{a}_x = +\bar{a}_y$

$\bar{a}_z \times \bar{a}_z = 0$ and

$\bar{a}_z \times \bar{a}_y = -\bar{a}_x$

$$d\bar{F}_1 = \frac{10^{-17}}{16}\left[\bar{a}_z \times (+2\bar{a}_z + 3\bar{a}_y)\right]$$

$$d\bar{F}_1 = \frac{10^{-17}}{16}\left[2(0) + 3(-\bar{a}_x)\right]$$

$$d\bar{F}_1 = -0.1875\times 10^{-17}\,\bar{a}_x$$

$$\boxed{d\bar{F}_1 = -1.875\times 10^{-18}\,\bar{a}_x}$$ Newton.

problem 12

06-DEC2011/Jan 2012

Two differential current elements, $I_1\Delta\bar{L}_1 = 10^{-5}\bar{a}_z$ A.m. at $P_1(1, 0, 0)$ and $I_2\Delta\bar{L}_2 = 10^{-5}(0.6\bar{a}_x - 2\bar{a}_y + 3\bar{a}_z)$ A.m. at $P_2(-1, 0, 0)$ are located in free space. Find vector force exerted on $I_2\Delta\bar{L}_2$ by $I_1\Delta\bar{L}_1$. (10 Marks)

Dec/Jan 2016

A current element $I_1 \Delta L_1 = 10^{-5}\ \vec{az}$ A.m is located at $P_1(1, 0, 0)$ while second element $I_2 \Delta L_2 = 10^{-5} (0.6\ \vec{ax}\ 2\ \vec{ay} + 3\ \vec{az})$ A.m is at P_2 (-1, 0, 0) both in free space find the vector force exerted on $I_2 \Delta L_2$ by $I_1 \Delta L_1$ (08 Marks)

Question.

A Current element $I_1\overline{\Delta L_1} = 10^{-5}\,\bar{a}_z$ A.m is Located at $P_1(1,0,0)$ while Second element $I_2\overline{\Delta l_2} = 10^{-5}(0.6\ \bar{a}_x - 2\bar{a}_y + 3\bar{a}_z)$ A.m is at $P_2(-1,0,0)$ both in free space find the vector force Exerted on $I_2\overline{\Delta l_2}$ by $I_1\overline{\Delta l_1}$.

Solu:- The force exerted on $I_2\overline{dl_2}$ due to $I_1\overline{dl_1}$ is

$$d\bar{F}_2 = I_2\overline{dl_2} \times d\bar{B}_1$$

$$d\bar{B}_1 = \mu_0 d\bar{H}_1$$

$$d\bar{H}_1 = \frac{I_1\overline{dl_1} \times \bar{a}_{R_{12}}}{4\pi R_{12}^2}$$

$$d\bar{F}_2 = I_2 \, \overline{dl_2} \times \left\{ \frac{\mu_0 I_1 \overline{dl_1} \times \bar{a}_{R_{12}}}{4\pi R_{12}^2} \right\}$$

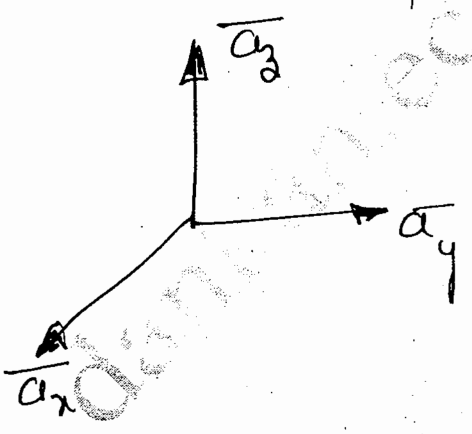
$$\overline{R_{12}} = (-1-1)\,\bar{a}_x = -2\bar{a}_x$$

$$|\overline{R_{12}}| = 2$$

$$d\bar{F}_2 = \frac{\mu_0}{4\pi R_{12}^3} \left\{ I_2 \overline{dl_2} \times (I_1 \overline{dl_1} \times \overline{R_{12}}) \right\}$$

$$d\bar{F}_2 = \frac{\mu_0}{4\pi (2)^3} \left\{ 10^{-5}(0.6\,\bar{a}_x - 2\bar{a}_y + 3\bar{a}_z) \times (10^{-5}\bar{a}_z \times -2\bar{a}_x) \right\}$$

$$d\bar{F}_2 = \frac{\cancel{4\pi} \times 10^{-7} \times 10^{-5} \times 10^{-5} \times -\cancel{2}^{\,1}}{\cancel{4\pi} \times \cancel{8}\,4} \left\{ (0.6\,\bar{a}_x - 2\bar{a}_y + 3\bar{a}_z) \times (\underbrace{\bar{a}_z \times \bar{a}_x}_{+\bar{a}_y}) \right\}$$



$$\bar{a}_z \times \bar{a}_x = \bar{a}_y$$
$$\bar{a}_x \times \bar{a}_y = +\bar{a}_z$$
$$\bar{a}_y \times \bar{a}_y = 0$$
$$\bar{a}_z \times \bar{a}_y = -\bar{a}_x$$

$$d\bar{F}_2 = -\frac{10^{-17}}{4} \left\{ 0.6\,\bar{a}_z - 2(0) + 3(-\bar{a}_x) \right\}$$

$$d\bar{F}_2 = -\frac{10^{-17}}{4} \left[-3\bar{a}_x + 0.6\,\bar{a}_z \right] \text{ Newton}$$

$$\boxed{d\bar{F}_2 = [7.5\,\bar{a}_x - 1.5\,\bar{a}_z] \times 10^{-18}} \text{ Newton.}$$

Module3 (part B)

Topic 4.4 Concept of Magnetization and permeability

Definitions:-

Magnetic pole strength:-

Magnetic pole strength of a pole is said to be unity if it experiences a force of 1 Newton when placed at a distance of 1 meter from a similar one, in air (or) vacuum.

Magnetic Moment (m):-

Magnetic moment (m) of a magnet is the product of magnetic pole strength and the distance between the two poles.

Magnetization ($\bar{M}$). 06-June/July 2009 (2M)

The magnetic moment / unit volume of a magnet is called magnetization.

$$\boxed{M = \frac{m}{v}}$$

where v - is the volume of the magnetic material.

Note:- Magnetization (M) = no. of atoms × dipole moment.

$$\boxed{M = nm}$$

Magnetic Susceptibility (χ) :- (2m) 06-June/July 2009.

The ratio of magnetization (M) to the strength of the field (H) is called the magnetic Susceptibility (χ) of the material.

$$\boxed{\chi = \frac{M}{H}}$$

Magnetic Field :- Magnetic field is the region where a magnetic pole experiences a force.

Magnetic field Intensity (H)

Field intensity (H) at any point in a magnetic field is equal to, and directed along the force experienced by a unit north pole at that point.

Permeability (μ) (2m) 06-June/July 2009.

The permeability of vaccum (or) free space is taken as the Standard reference with respect to which permeabilities of other materials are expressed.

The permeability of vaccum or freespace is denoted by μ_0. and $\mu_0 = 4\pi \times 10^{-7}$ H/m.

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$$\boxed{\mu = \mu_0 \mu_r} \text{ H/m.}$$

μ_0 - absolute permeability ; $\mu_0 = 4\pi \times 10^{-7}$ H/m

μ_r - relative permeability. $\boxed{\mu_r = 1}$ for air medium.

Relation between $\bar{B}$ and $\bar{H}$:-

$$\bar{B} = \mu \bar{H} \text{ wb/m}^2.$$

$$\bar{B} = \mu_0 \mu_r \bar{H} \text{ wb/m}^2.$$

for air (or) vaccum $\mu_r = 1$.

$$\therefore \boxed{\bar{B} = \mu_0 \bar{H}} \text{ wb/m}^2.$$

Relation between B, M and H :-

The magnetic flux density due to the field 'H' is given by $B = \mu_0 H$ wb/m² ← ①

the magnetic flux density due to the extra flux in the medium is given by the product $\mu_0 M$, where M is the magnitization of the specimen.

ie $B = \mu_0 M$ wb/m² ← ②

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Thus the resultant flux density B in the material medium is given by

$$B = \mu_0 H + \mu_0 M$$

$$\boxed{B = \mu_0 (H+M)} \text{ wb/m}^2 \leftarrow (3)$$

if μ_r is the relative permeability of the medium, then $B = \mu_0 \mu_r H \leftarrow (4)$

from eqn (3) and (4)

$$\cancel{\mu_0} \mu_r H = \cancel{\mu_0} (H+M)$$

$$\boxed{M = (\mu_r - 1) H} \leftarrow (5)$$

the magnitization M interms of Susceptibility (χ) is $\boxed{M = \chi H} \leftarrow (6)$

using eqn (6) in eqn (5)

$$\chi \cancel{H} = (\mu_r - 1) \cancel{H}$$

$$\Rightarrow \boxed{\chi = \mu_r - 1} \text{ (or) } \boxed{\mu_r = 1 + \chi = \frac{\mu}{\mu_0}}$$

$$\Rightarrow \boxed{\mu_r = 1 + \chi = \frac{\mu}{\mu_0}}$$

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problem 13

02 - June /July 2011

Find the magnetic field intensity inside a magnetic material, for the following conditions:
a. M = 100 A/m and $\mu = 1.5 \times 10^{-5}$ H/m
b. B = 200 μT, χm, (magnetic susceptibility) = 15.

(05 Marks)

Question.

Find the magnetic field intensity inside a magnetic material for the following conditions.

a. $M = 100$ A/m and $\mu = 1.5 \times 10^{-5}$ H/m.

b. $B = 200\,\mu T$, χ_m (magnetic susceptibility) = 15.

c. there are 8×10^{28} atoms/m^3, Each atom has a dipole moment of 2.5×10^{-27} A-m^2 and $\mu_r = 30$.

Soln:

a. Given $M = 100$ A/m

$\mu = 1.5 \times 10^{-15}$ H/m

$\mu = \mu_0 \mu_r$ H/m.

$$\mu_r = \frac{\mu}{\mu_0} = \frac{1.5 \times 10^{-5}}{4\pi \times 10^{-7}} = 11.94$$

$$\boxed{\mu_r = 11.94}$$

$$\mu_r = 1 + \chi_m \Rightarrow \boxed{\chi_m = \mu_r - 1}$$

$$\chi_m = 11.94 - 1 = 10.94$$

$$\boxed{\chi_m = 10.94}$$

$$M = \chi_m H$$

$$H = \frac{M}{\chi_m} = \frac{100}{10.94}$$

$$\boxed{H = 9.14}\ A/m$$

b) Given $B = 200\mu T$ and $\chi_m = 15$

$\mu_r = 1 + \chi_m = 1 + 15 = 16$; we $\boxed{B = \mu_0 \mu_r H}$ Wb/m²

$$H = \frac{B}{\mu_0 \mu_r} = \frac{200 \times 10^{-6}}{4\pi \times 10^{-7} \times 16}$$

$$\boxed{H = 9.95}\ A/m$$

c) Given no of atom $N = 8 \times 10^{28}$ atoms/m³

$m = 2.5 \times 10^{-27}$ A-m² and $\mu_r = 30$

$$M = N \cdot m = 8 \times 10^{28} \times 2.5 \times 10^{-27}$$

$$\boxed{M = 200}\ A/m$$

$$M = \chi_m H = (\mu_r - 1) H$$

$$200 = (30 - 1) H$$

$$\Rightarrow \boxed{H = \frac{200}{29} = 6.89}\ A/m$$

problem 14

06-DEC2008/Jan 2009

Find Magnetization in magnetic material, where (08 Marks)
i) $\mu = 1.8 \times 10^{-5}$ H/m and H = 120 A/m; ii) $\mu_r = 22$, there are 8.3×10^{28} atoms/m^3 and each atoms has a dipole moment of 4.5×10^{-27} A – m^2; iii) B = 300 μT and $X_m = 15$

06- June /July 2009

Find the magnetization in a magnetic material where:

i) $\mu = 1.8 \times 10^{-5}$ (H/m) and H = 120 (A/m).

ii) $\mu_r = 22$, there are 8.3×10^{28} atoms/m^3 and each atom has a dipole moment of 4.5×10^{-27} (A/m^2) and

iii) B = 300 (μT) and $\chi_m = 15$.

(06 Marks)

02 - June /July 2010

Find the magnetization in a magnetic material, where $\mu = 1.8 \times 10^{-5}$ H/m, H = 120 A/m

(03 Marks)

Dec/Jan 2017 CBCS scheme

b. Find the magnetization in a material where : i) $\mu = 1.8 \times 10^{-5}$ H/m and H = 120 A/m ii) $\mu_r = 22$. There are 8.3×10^{28} atom/m^3 and each atom has a dipole moment of 4.5×10^{-27} A/m^2. iii) B = 300 μT and $X_{m} = 15$. (06 Marks)

Question

Find the magnetization in a material where:

i) $\mu = 1.8 \times 10^{-5}$ H/m and H = 120 A/m.

ii) $\mu_r = 22$. there are 8.3×10^{28} atoms/m^3 and Each Atom has a dipole moment of 4.5×10^{-27} A/m^2 and

iii. B = 300 μT and $\chi_m = 15$.

Soln

i) Given $\mu = 1.8 \times 10^{-5}$ H/m. and H = 120 A/m.

$$M = (\mu_r - 1) H$$

$$M = \left(\frac{\mu}{\mu_0} - 1\right) H$$

$$M = \left(\frac{1.8 \times 10^{-5}}{4\pi \times 10^{-7}} - 1\right)(120) = 1598.87$$

$$\boxed{M \simeq 1599 \text{ A/m}}$$

$\boxed{M \simeq 1599}$ A/m

ii. $\mu_r = 22$, $n = 8.3 \times 10^{28}$ atoms/m³.

$m = 4.5 \times 10^{-27}$ Am²

$M = nm$

$M = (8.3 \times 10^{28})(4.5 \times 10^{-27})$

$\boxed{M = 373.5}$ A/m

iii. $B = 300 \times 10^{-6}$ Tesla and $\chi_m = 15$.

$B = \mu_0 \mu_r H$ Wb/m²

$\chi_m = \frac{M}{H}$ or $H = \frac{M}{\chi_m}$

$B = \frac{\mu_0 \mu_r M}{\chi_m}$

$M = \frac{B \chi_m}{\mu_0 \mu_r}$

$\chi_m = (\mu_r - 1)$ or $\mu_r = (\chi_m + 1)$

$$M = \frac{B \chi_m}{\mu_0 (\chi_m + 1)} = \frac{(300 \times 10^{-6})(15)}{4\pi \times 10^{-7}(15+1)}$$

$M = 223.811$ A/m

$\boxed{M \simeq 224}$ A/m

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problem 15

10-June/July 2013

Given a ferrite material which we shall specify to be operating in a linear mode with $B = 0.05$ T, let us assume $\mu_r = 50$, and calculate values for χ_m, M and H. (06 Marks)

Question

A ferrite material is operating in Linear mode with $B = 0.05$ T. assume $\mu_r = 50$. Calculate magnetic Susceptibility (χ_m), magnetization (M) and magnetic field intensity (H).

4m - 10 J/J 2014

6m - 06-Jan2013

Soln :-

i. Susceptibility (χ_m)

$$\chi_m = \mu_r - 1$$

$$\chi_m = 50 - 1$$

$$\boxed{\chi_m = 49}$$

ii. Magnetic field Intensity (H)

$$B = \mu H = \mu_0 \mu_r H$$

$$H = \frac{B}{\mu_0 \mu_r} = \frac{0.05}{4\pi \times 10^{-7} \times 50}$$

$$\boxed{H = 796} \text{ A/m}$$

iii. Magnetization

$$M = \chi_m H$$

$$M = 49\,(796)$$

$$\boxed{M = 39004}\ \text{A/m}$$

problem 6

06 - June /July 2012

Find the magnetic field intensity within a magnetic material for the following cases with $\mu_o = 4\pi * 10^{-7}$ H/m.

i) Magnetization M = 180 A/m, permeability $\mu_r = 1.8 * 10^{-5}$ H/m

ii) Magnetic flux density |B| = 450 * 10^{-6} Tesla and (Chi m) $\chi_m = 15$. **(06 Marks)**

Question

Find the magnetic field intensity within a magnetic material for the following cases with $\mu_o = 4\pi \times 10^{-7}$ H/m.

i. magnetization $M = 180$ A/m, $\mu = 1.8 \times 10^{-5}$ H/m

ii. Magnetic flux density $B = 450 \times 10^{-6}$ Tesla and $\chi_m = 15$.

soln: i. Given $M = 180$ A/m

$\mu = 1.8 \times 10^{-5}$ H/m.

$$\mu_r = \frac{\mu}{\mu_0} = \frac{1.8 \times 10^{-5}}{4\pi \times 10^{-7}} = 14.323$$

$$\boxed{\mu_r = 14.3239}$$

$$\chi_m = \mu_r - 1 = 14.3239 - 1$$

$$\boxed{\chi_m = 13.3239}$$

$$M = \chi_m H$$

$$\Rightarrow H = \frac{M}{\chi_m} = \frac{180}{13.3239} = 13.509 \text{ A/m}$$

$$\boxed{H = 13.509} \text{ A/m}$$

ii. Given $B = 450 \times 10^{-6}$ Tesla and

$\chi_m = 15$.

$$\mu_r = 1 + \chi_m = 15 + 1 = 16.$$

$$H = \frac{B}{\mu_0 \mu_r} \quad A/m$$

$$H = \frac{450 \times 10^{-6}}{(4\pi \times 10^{-7})(16)}$$

$$\boxed{H = 22.381} \; A/m$$

problem 17

10 - June /July 2012

If $\bar{B} = 0.05x\,\bar{a}_y$ T in a material for which $\chi_m = 2.5$,

find : i) μ_r ; ii) μ ; iii) $\bar{H}$; iv) $\bar{M}$; v) $\bar{J}$ and vi) $\bar{J}_b$. (07 Marks)

Question

if $\overline{B} = 0.05x\,\overline{a_y}$ Tesla in a material for which $\chi_m = 2.5$ find

a) μ_r b) μ c) $\overline{H}$ d) $\overline{m}$ e) $\overline{J}$

and f) $\overline{J_d}$. [15-June/July-2017 (8M) CBCS]

Soln: Given $\chi_m = 2.5$ and $B = 0.05x$ wb/m²

a) $\mu_r = 1 + \chi_m$

$$\boxed{\mu_r = 1 + 2.5 = 3.5}$$

b) $\mu = \mu_0 \mu_r = 4\pi \times 10^{-7} \times 3.5$

$$\boxed{\mu = 4.398 \times 10^{-6}} \text{ H/m}$$

c) $\overline{B} = \mu\overline{H} = \mu_0\mu_r\overline{H}$

$$\overline{H} = \frac{\overline{B}}{\mu_0\mu_r} = \frac{0.05x\,\overline{a_y}}{4.398 \times 10^{-6}}$$

$$\boxed{\overline{H} = 11.368 \times 10^3 x\,\overline{a_y}} \text{ A/m.}$$

d) Magnetization

$$\overline{M} = \chi_m \overline{H}$$

$$\overline{M} = 2.5\left[11.368 \times 10^3 x\right] \overline{a_y}$$

$$\boxed{\overline{M} = 28.42 \times 10^3 x \, \overline{a_y}} \; A/m$$

e) the total Current density

$$\overline{J} = \nabla \times \overline{H} = \begin{vmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 11.36 \times 10^3 x & 0 \end{vmatrix}$$

$$= \frac{\partial}{\partial x}\left[11.36 \times 10^3 x\right] \overline{a_z}$$

$$\boxed{\overline{J} = 11.36 \times 10^3 \, \overline{a_z}} \; A/m^2$$

f). the bound Current density

$$\overline{J_b} = \nabla \times \overline{m} = \begin{vmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 28.42 \times 10^3 x & 0 \end{vmatrix}$$

$$\overline{J_b} = \frac{\partial}{\partial x}\left[28.42 \times 10^3 x\right] \overline{a_z}$$

$$\boxed{\overline{J_b} = 28.42 \times 10^3 \, \overline{a_z}} \; A/m^2$$

problem 18.

Find the Magnetic field intensity within a magnetic material where

a. $M = 150\,A/m$ and $\mu = 1.5 \times 10^{-5}\,H/m$.

b. $B = 300\,\mu T$ and $\chi_m = 15$.

c. there are 8.2×10^{28} atoms/m^3 Each atom has a dipole moment of $5 \times 10^{-27}\,A\text{-}m^2$ and $\mu_r = 30$.

Soln: a. $\mu_r = \frac{\mu}{\mu_0} = \frac{1.5 \times 10^{-5}}{4\pi \times 10^{-7}} = 11.936$

$$\boxed{\mu_r = 11.936}$$

$$M = \chi_m H = (\mu_r - 1) H$$

$$H = \frac{M}{(\mu_r - 1)} = \frac{150}{(11.936-1)} = 13.7154\,A/m$$

$$\boxed{H = 13.7154}\,A/m$$

b. the magnetic flux density is given by

$$B = \mu H = \mu_0 \mu_r H \quad wb/m^2$$

$$\mu_r = 1 + \chi_m$$

$$B = \mu_0 (1 + \chi_m) H \quad wb/m^2$$

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$$H = \frac{B}{\mu_0(1+\chi_m)} = \frac{300\times10^{-6}}{4\pi\times10^{-7}(1+15)}$$

$$\boxed{H = 14.92}\ A/m$$

c) $n = 8.2\times10^{28}$ atoms/m^3

$m = 5\times10^{-27}$ $A\text{-}m^2$

$$M = n\cdot m = (8.2\times10^{28})(5\times10^{-27})$$

$$\boxed{M = 410}\ A/m$$

$$H = \frac{M}{(\mu_r-1)} = \frac{410}{(30-1)} = 14.137\ A/m.$$

$$\boxed{H = 14.137}\ A/m$$

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Problem 19.

Through a suitable experiment on a magnetic material, the magnetic flux density $\bar{B}$ is found to be 1.2T when H = 300 A/m when H is increased to 1500 A/m, the B field increased to 1.5T. what is the percentage change in the magnetization vector.

Soln:

$$B = \mu H \ \text{wb/m}^2$$

$$B = \mu_0 \mu_r H \ \text{Wb/m}^2$$

$$\mu_{r_1} = \frac{B_1}{\mu_0 H_1} = \frac{1.2}{4\pi \times 10^{-7}(300)} = 3183.1$$

$$\mu_{r_2} = \frac{B_2}{\mu_0 H_2} = \frac{1.5}{4\pi \times 10^{-7}(1500)} = 795.8$$

$$M = \chi_m H = (\mu_r - 1) H$$

$$M_1 = 3183.1 \times 300 = 954.6 \text{ KA/m}$$

$$M_2 = 795.8 \times 1500 = 1.19 \times 10^6 \text{ A/m}$$

$$\% \text{ change} = \frac{M_2 - M_1}{M_1} \times 100 = \frac{1.19\times10^6 - 954.6\times10^3}{954.6\times10^3} \times 100$$

$$\boxed{\% \text{ change} = 24.66\ \%}$$

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Topic 4.5

Magnetic Boundary conditions

06-DEC2010

Derive the magnetic boundary conditions at the interface between the two different magnetic materials. Discuss the conditions. (08 Marks)

10-DEC2011/Jan 2012

Obtain boundary conditions at the interface between two magnetic materials. (06 Marks)

06 - June /July 2012

Derive the boundary conditions for magnetic flux density (B), magnetic field intensity (H) at the interface between two different magnetic materials (08 Marks)

06- June /July 2009

Obtain the boundary conditions at interface between two magnetic materials. (06 Marks)

10 - June /July 2014

Derive the magnetic boundary conditions at the interface between two different magnetic materials. (06 Marks)

06 - June /July 2013

Consider two different media placed adjacently in a region where there is a magnetic field. Explain with suitable mathematical steps the magnetic boundary conditions. (08 Marks)

10-Dec/Jan 2010

b. Derive the boundary conditions at the interface between two different magnetic materials. (06 Marks)

Soln:

Magnetic boundary conditions as the conditions that $\overline{H}$ (or) $\overline{B}$ field must satisfy at the boundary between two different media.

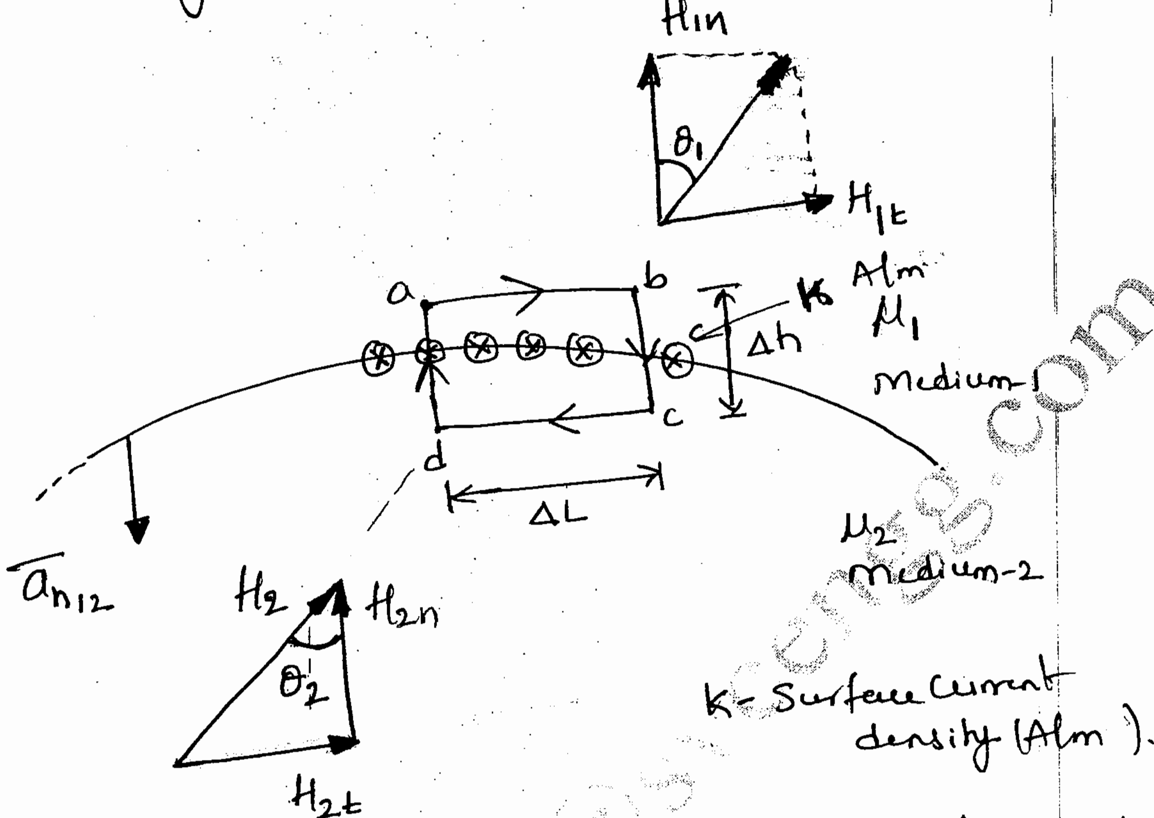
Gauss's Law for magnetic fields

$$\oint_{\langle S\rangle} \overline{B}\cdot\overline{dS} = 0 \quad \text{(1)}$$

and Ampere's Circuital Law

$$\oint_{\langle l\rangle} \overline{H}\cdot\overline{dl} = I \quad \text{(2)}$$

a] Tangential Components :-



Consider boundary between two magnetic media-1 and Media-2 characterized respectively μ_1 and μ_2 as shown in fig.

applying Ampere's circuital Law to closed path abcda assume Δh is vanishingly small

i.e $\Delta h \to 0$.

$$H_{1t} \cdot \Delta L - H_{2t} \cdot \Delta L = K \Delta L$$

where K - Surface current density (A/m).

$$\therefore \quad H_{1t}\Delta L - H_{2t}\Delta L = K\,\Delta L$$

$$(H_{1t} - H_{2t})\cancel{\Delta L} = K\cancel{\Delta L}$$

$$H_{1t} - H_{2t} = K \longleftarrow ③$$

$$\frac{B_{1t}}{\mu_1} - \frac{B_{2t}}{\mu_2} = K$$

from eq^n ③ in general form

$$\boxed{(\bar{H}_1 - \bar{H}_2) \times \bar{a}_{n_{12}} = \bar{K}} \ \text{A/m.}$$

where $a_{n_{12}}$ is a unit vector normal to the interface and is directed from medium-1 to medium-2.

if the boundary is free of current (or) the media are not conductors (for k is free current density)

$K = 0$.

eq^n ③ becomes

$$\boxed{H_{1t} = H_{2t}} \quad \text{(or)} \quad \boxed{\frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}}$$

thus the tangential components of $\bar{H}$ is continuous while that of $\bar{B}$ is discontinuous at the boundary

b) Normal Components:-

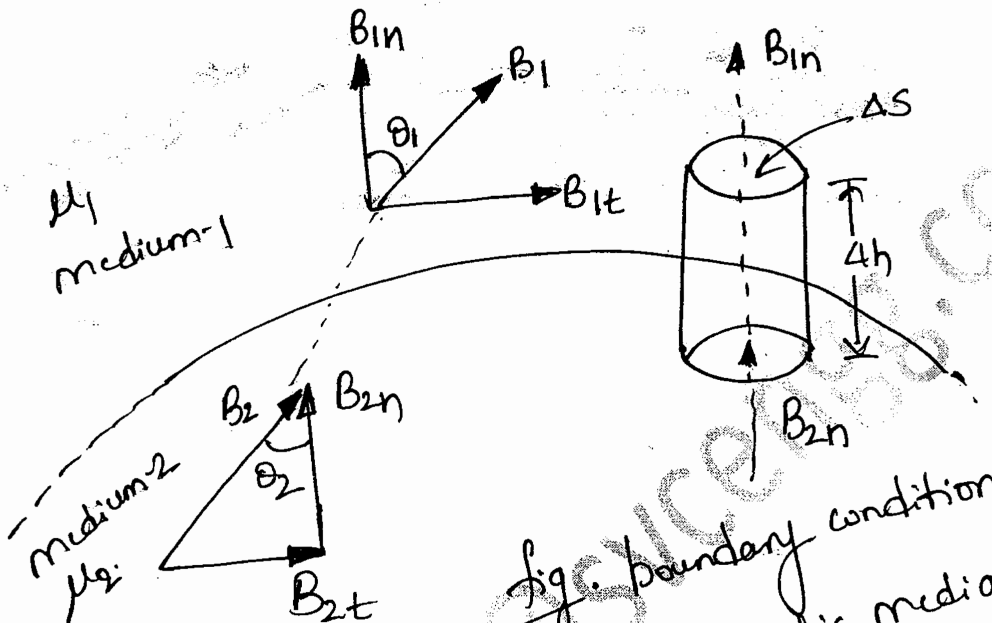


fig. boundary conditions between two magnetic media $\bar{B}$ and $\bar{H}$.

applying Gauss's Law to pill box.

ie $\oint_{\langle S \rangle} \bar{B} \cdot \bar{ds} = 0$

$B_{1n} \Delta S - B_2 \Delta S = 0$

$\Rightarrow \boxed{B_{1n} = B_{2n}}$ (or) $\boxed{\mu_1 H_{1n} = \mu_2 H_{2n}}$ ← (4)

Since $B = \mu H$ wb/m² equ(4) shows that normal component of $\bar{B}$ is continuous at the boundary. it also

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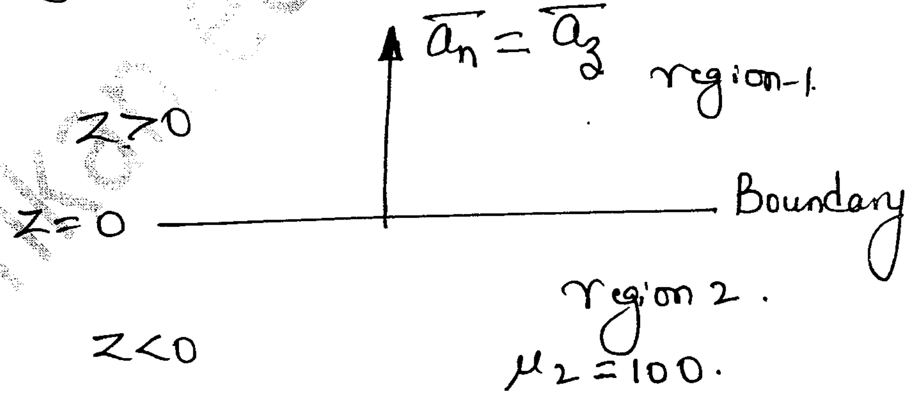
shows that normal component of $\overline{H}$ is discontinuous. i.e at the boundary $\overline{H}$ undergoes some changes at the interface.

problem 20.

The $z=0$ plane makes the boundary between two magnetic materials. region-1 characterized by $z>0$ and region-2 by $z<0$. the magnetic flux density in region-1 is $\overline{B_1} = 1.5\overline{a_x} + 0.8\overline{a_y} + 0.6\overline{a_z}$ mT. Find the magnetic flux density in region 2. assume relative permeability of region-1 as free space and relative permeability of region-2 as 100.

Soln:-



Given $\overline{B_1} = 1.5\overline{a_x} + 0.8\overline{a_y} + 0.6\overline{a_z}$ mT.

w.k.t $B_{n_2} = B_{n_1}$ @ boundary.

$$\Rightarrow \boxed{B_{z_2} = B_{z_1} = 0.6 \text{ mT}}$$

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also at boundary $\overline{a_n} \times (\overline{H_1} - \overline{H_2}) = \overline{K}$

since $\overline{K} = 0$ and $\overline{a_n} = \overline{a_z}$

$$\overline{a_z} \times (\overline{H_1} - \overline{H_2}) = 0$$

$$\overline{a_z} \times \left(\frac{\overline{B_1}}{\mu_0} - \frac{\overline{B_2}}{\mu_0 (100)} \right) = 0$$

$$\overline{a_z} \times (100\overline{B_1} - \overline{B_2}) = 0$$

$$\overline{a_z} \times \left\{ [100(1.5) - B_{x_2}]\overline{a_x} + [100(0.8) - B_{y_2}]\overline{a_y} + [100(0.6) - B_{z_2}]\overline{a_z} \right\} = 0$$

$$(150 - B_{x_2})\overline{a_y} - (80 - B_{y_2})\overline{a_x} = 0.$$

$\Rightarrow 150 - B_{x_2} = 0 \Rightarrow \boxed{B_{x_2} = 150}$ mT

and $80 - B_{y_2} = 0 \Rightarrow \boxed{B_{y_2} = 80}$ mT.

and $B_{z_2} = B_{z_1} = 0.6$ mT.

$$\overline{B_2} = B_{x_2}\overline{a_x} + B_{y_2}\overline{a_y} + B_{z_2}\overline{a_z} \quad \text{wb/m}^2$$

$$\boxed{\overline{B_2} = 150\overline{a_x} + 80\overline{a_y} + 0.6\overline{a_z}} \text{ mT}$$

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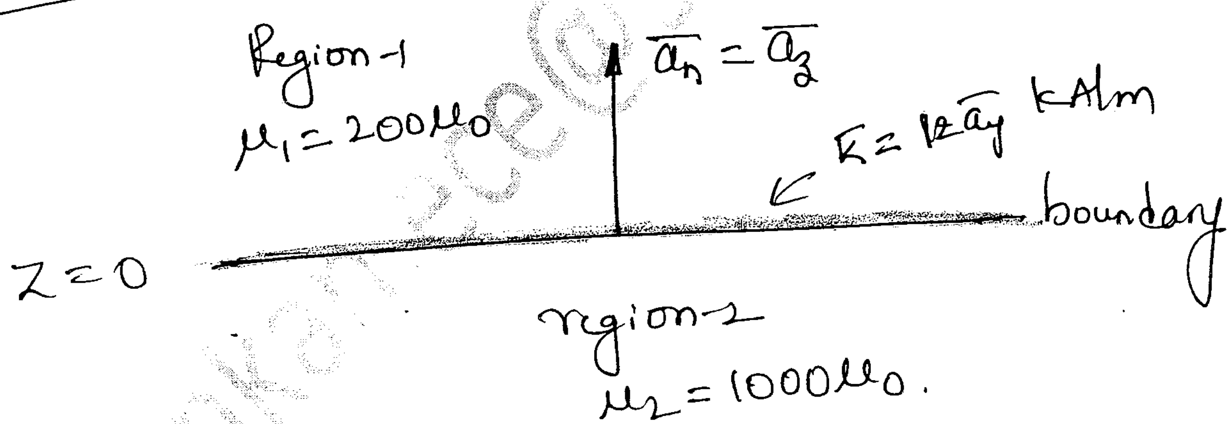
Problem 21

The $z=0$ plane makes the boundary between two magnetic materials. Region-1 is defined by $z>0$ and the magnetic field intensity in this region is $40\bar{a}_x + 50\bar{a}_y + 12\bar{a}_z$ kA/m. Region 2 is defined by $z<0$ and has a relative permeability of 1000. If the relative permeability of medium-1 is 200. Find the magnetic field intensity in medium-2. A current sheet of $12\bar{a}_y$ kA/m is present at the boundary.

Soln.



$$\bar{H}_1 = 40\bar{a}_x + 50\bar{a}_y + 12\bar{a}_z \text{ kA/m.}$$

at the Boundary $\quad B_{n_2} = B_{n_1}$

$$\boxed{B_{z_2} = B_{z_1}}$$

$$\mu_2 H_{z_2} = \mu_1 H_{z_1}$$

$$H_{z_2} = \frac{\mu_1}{\mu_2} H_{z_1}.$$

$$= \frac{200}{1000}(12 \times 10^3)$$

$$\boxed{H_{z_2} = 2.4\ kA/m.}$$

also at boundary

$$\bar{a}_n \times (\bar{H}_1 - \bar{H}_2) = \bar{K}$$

$$\bar{a}_z \times (\bar{H}_1 - \bar{H}_2) = 12\bar{a}_y$$

$$\bar{a}_z \times [(H_{x_1} - H_{x_2})\bar{a}_x + (H_{y_1} - H_{y_2})\bar{a}_y + (H_{z_1} - H_{z_2})\bar{a}_z] = 12\bar{a}_y$$

$$(H_{x_1} - H_{x_2})\bar{a}_y - (H_{y_1} - H_{y_2})\bar{a}_x = 12\bar{a}_y$$

$$H_{x_1} - H_{x_2} = 12 \Rightarrow \boxed{H_{x_2} = 40 - 12 = 28\ kA/m}$$

$$H_{y_1} - H_{y_2} = 0 \Rightarrow \boxed{H_{y_1} = 50}\ kA/m$$

$$\therefore \bar{H}_2 = H_{x_2}\bar{a}_x + H_{y_2}\bar{a}_y + H_{z_2}\bar{a}_z\ A/m.$$

$$\boxed{\bar{H}_2 = 28\bar{a}_x + 50\bar{a}_y + 2.4\bar{a}_z}\ kA/m.$$

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problem 22.

b. ~~For region 1, $\mu_1 = 4\mu H/m$ and for region 2, μ_2~~ $= 6\mu$ H/m. The regions are separated by $z = 0$ plane. The surface current density at the boundary is $\overline{K} = 100\overline{a}x$ A/m. Find $\overline{B}_2$ if $\overline{B}_1 = 2\hat{a}x - 3\hat{a}y + \hat{a}z$ militesla for $z > 0$. 15-Dec/Jan 2017 (CBCS) (08 Marks)

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Solu:-

Medium-1 $\mu_1 = 6\mu H/m$

$z > 0$ $\quad \overline{B}_1 = 2\overline{a}_x - 3\overline{a}_y + \overline{a}_z$ mT

$\overline{a}_N = \overline{a}_z$

$z = 0$ plane (Boundary)

Medium-2 $\quad \overline{a}_{N_{12}} = -\overline{a}_z \quad \overline{K} = 100\,\overline{a}_x$ A/m.

$(z<0)$ $\quad \mu_2 = 6\mu H/m$.

$\overline{B}_2 = B_{x_2}\overline{a}_x + B_{y_2}\overline{a}_y + B_{z_2}\overline{a}_z$ mTeda.

$B_{x_1} = 2$mT ; $B_{y_1} = -3$mT and $B_{z_1} = 1$mT.

w.k.t from the concept of magnetic Boundary conditions the normal components of Magnetic Flux densities of both the medium are equal.

i.e $\boxed{B_{n_1} = B_{n_2}}$ @ the boundary.

Given Boundary is $z=0$ plane

∴ the unit normal vector $\boxed{\overline{a}_n = \overline{a}_z}$

Since $\boxed{B_{n_1} = B_{n_2}}$ and $\boxed{\bar{a}_n = a_z}$

$$\Rightarrow \boxed{B_{z_1} = B_{z_2} = 1\text{m Tesla}}$$

To find B_{x_2} and B_{y_2}

using $(\bar{H}_1 - \bar{H}_2) \times \bar{a}_{n_{12}} = \bar{K}$ ← (1)

where $\bar{a}_{n_{12}}$ - unit vector normal to the interface and is directed from medium 1 to medium 2.

$$\bar{H}_1 = \frac{\bar{B}_1}{\mu_1} = \frac{B_{x_1}\bar{a}_x + B_{y_1}\bar{a}_y + B_{z_1}\bar{a}_z}{\mu_1} \text{ A/m}$$

$$\bar{H}_2 = \frac{\bar{B}_2}{\mu_2} = \frac{B_{x_2}\bar{a}_x + B_{y_2}\bar{a}_y + B_{z_2}\bar{a}_z}{\mu_2} \text{ A/m.}$$

and $\boxed{\bar{a}_{n_{12}} = -\bar{a}_z}$; $\bar{K} = 100\,\bar{a}_x$.

using eq^n (1)

$$\left[\left(\frac{B_{x_1}}{\mu_1} - \frac{B_{x_2}}{\mu_2}\right)\bar{a}_x + \left(\frac{B_{y_1}}{\mu_1} - \frac{B_{y_2}}{\mu_2}\right)\bar{a}_y + \left(\frac{B_{z_1}}{\mu_1} - \frac{B_{z_2}}{\mu_2}\right)\bar{a}_z\right](-\bar{a}_z) = 100\,\bar{a}_x \quad \leftarrow (2)$$

using Cross product of unit vector's

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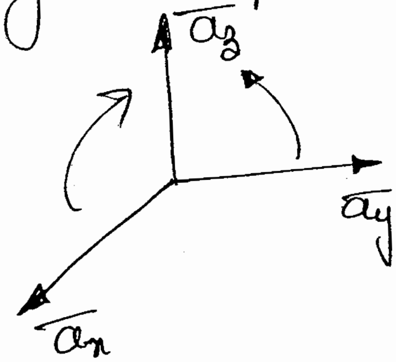

$$\bar{a}_x \times \bar{a}_z = -\bar{a}_y$$
$$\bar{a}_y \times \bar{a}_z = +\bar{a}_x$$
$$\bar{a}_z \times \bar{a}_z = 0.$$

$$\left(\frac{B_{x_1}}{\mu_1} - \frac{B_{x_2}}{\mu_2}\right)\bar{a}_y + \left(\frac{B_{y_1}}{\mu_1} - \frac{B_{y_2}}{\mu_2}\right)(-\bar{a}_x) + 0 = 100\,\bar{a}_x$$

$$\left[\frac{B_{x_1}}{\mu_1} - \frac{B_{x_2}}{\mu_2}\right]\bar{a}_y + \left(\frac{B_{y_2}}{\mu_2} - \frac{B_{y_1}}{\mu_1}\right)\bar{a}_x = 100\,\bar{a}_x + 0\,\bar{a}_y$$

Equating Components along 'x' and 'y' dire^n.

y - component

$$\Rightarrow \frac{B_{x_1}}{\mu_1} - \frac{B_{x_2}}{\mu_2} = 0$$

$$\Rightarrow B_{x_2} = \frac{\mu_2}{\mu_1} B_{x_1} = \frac{\cancel{6}^{\,3}\cancel{\mu}}{\cancel{4}_{\,2}\cancel{\mu}} \times \cancel{2}\,m = 3\,m\,Tesla$$

$$\boxed{B_{x_2} = 3\,m\,Tesla}$$

x - component

$$\frac{B_{y_2}}{\mu_2} - \frac{B_{y_1}}{\mu_1} = 100$$

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$$\frac{B_{y_2}}{\mu_2} = 100 + \frac{B_{y_1}}{\mu_1}$$

$$B_{y_2} = 100\mu_2 + \frac{\mu_2}{\mu_1} B_{y_1}$$

$$B_{y_2} = 100(6\mu) + \frac{6\cancel{\mu}}{4\cancel{\mu}} \times (-3m)$$

$$B_{y_2} = -3.9 \times 10^{-3} \text{ Tesla}$$

$$\boxed{B_{y_2} = -3.9 \text{ m Tesla}}$$

∴ the magnetic flux density $\overline{B_2}$ for $z<0$ is given by

$$\overline{B_2} = B_{x_2}\overline{a_x} + B_{y_2}\overline{a_y} + B_{z_2}\overline{a_z} \text{ Tesla}$$

$$\boxed{\overline{B_2} = 3\overline{a_x} - 3.9\overline{a_y} + \overline{a_z}} \text{ m Tesla}$$

Topic 4.6

Magnetic Circuits

a. Reluctance of a Magnetic circuits

b. Comparison between electric and magnetic circuits

Question: write a note on magnetic circuits [15-Jun/July 2017 (4M) CBCS]

"A magnetic circuit is a closed path of magnetic lines of force through one (or) more materials".

Reluctance: The property by which the flux passage is affected is referred to as reluctance.

The resistance of electric circuit can be expressed in terms of conductivity σ as

$$R = \frac{l}{\sigma S} \quad \Omega$$

where l – length in (m)

S – Area of crosssection (m^2)

σ – conductivity of the material.

In case of magnetic circuits, we can define reluctance in very much like way as

$$\boxed{R = \frac{l}{\mu S}} \quad H^{-1}$$

where μ – permeability of the isotropic Linear homogenous material.

The Reciprocal of reluctance is called permeance denoted by P. and measured in Henry.

$$\boxed{P = \frac{\mu S}{l}} \ H$$

4.6 a] Relutance of a magnetic Circuit

Consider a magnetic Circuit formed with a Loop of a magnetic material in which flux is setup by using a coil of N turns in which Current I is flowing.

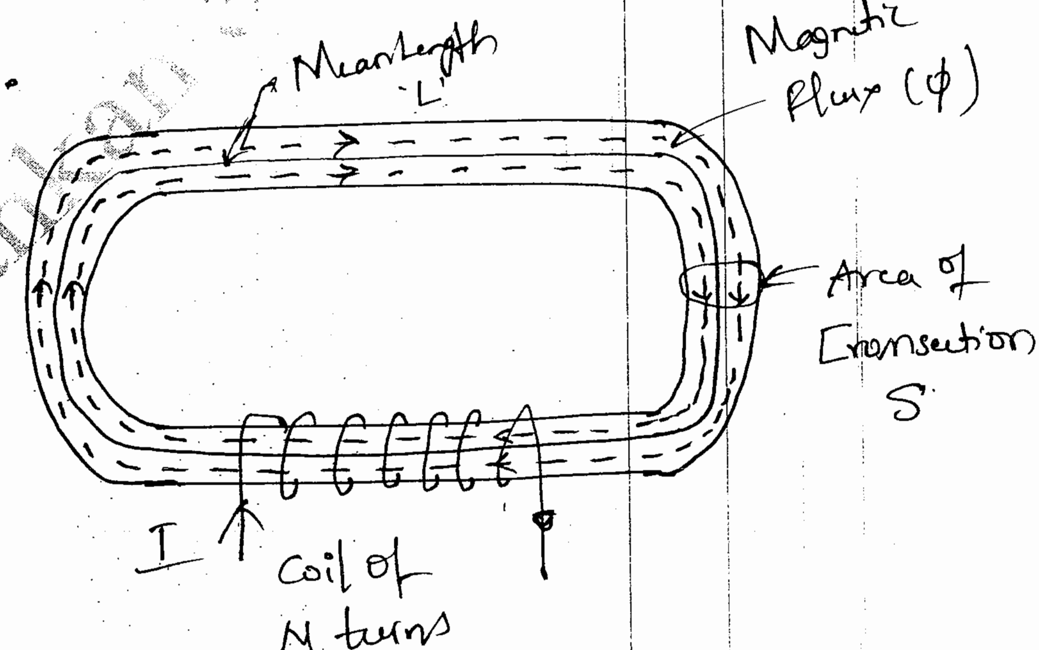

fig. Magnetic circuit.

let 'S' be the Area of Cross section and 'L' be the mean length of material.

then $B = \frac{\phi}{S}$ wb/m² (or) $\boxed{\phi = B \cdot S}$ wb.

$B = \mu H$ and $H = \frac{V_m}{l} = \frac{m.m.f}{l}$ A/m.

$B = \mu \left(\frac{V_m}{l}\right)$; wb/m²

$\phi = \mu \frac{V_m}{l} S \leftarrow (1)$

and $\phi = \frac{V_m}{R} \leftarrow (2)$

where R - reluctance

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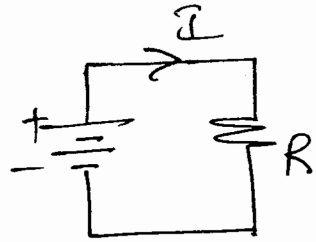
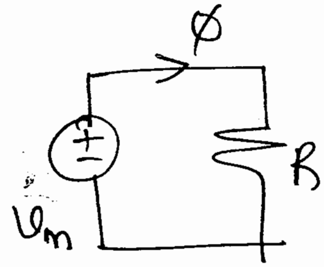
equating eqⁿ (1) and eqⁿ (2)

$$\frac{V_m}{R} = \mu \frac{V_m}{l} S$$

$$\Rightarrow \boxed{R = \frac{l}{\mu S}} \; H^{-1} \text{ (or) } 1/\text{Henry}.$$

Fringing Effect :- if there is an air gap between the path of the magnetic Flux it spreads, and bulges out this effect called fringing effect.

4.b Comparision b/w Electric and Magnetic Circuits

Parameter	Equation in Electrical case	Analogous eq^n in Magnetic case.
i. field Equation	$\bar{E} = -\nabla V$ V/m	$\bar{H} = -\nabla V_m$ A/m.
ii. potential difference b/w point A & B	$V_{AB} = \int_A^B \bar{E} \cdot \overline{dl}$ volts	$V_{mAB} = \int_A^B \bar{H} \cdot \overline{dl}$.
iii. ohm's Law	$\bar{J} = \sigma \bar{E}$ A/m² (or) $V = RI$	$\bar{B} = \mu \bar{H}$ wb/m² (or) $V_m = S\phi$.
iv. Current / flux density	$I = \int_S \bar{J} \cdot \overline{dS}$	$\phi = \int_S \bar{B} \cdot \overline{dS}$
v. emf / mmf	$V = \oint \bar{E} \cdot \overline{dl} = E \cdot L$	$V_m = \oint \bar{H} \cdot \overline{dl} = HL$
vi. Resistance / Reluctance	$R = \frac{l}{\sigma S}$ Ω	$R = \frac{l}{\mu S}$: H^{-1}
vii. Closed path integral in the field	$\oint \bar{E} \cdot \overline{dl} = 0$	$\oint \bar{H} \cdot \overline{dl} = NI$.
viii. Electric / magnetic Circuit		

Topic 4.7 Reluctance in a Series Magnetic Ckts.

Reluctance in a series magnetic circuits

Obtain the expression for reluctance in a series magnetic circuit. 06 - June /July 2011 (05 Marks)

Obtain the expression for reluctance in a series magnetic circuit. 10 - June /July 2014 (05 Marks)

Question. obtain the expression for reluctance in a Series magnetic circuit (5M).

Soln:

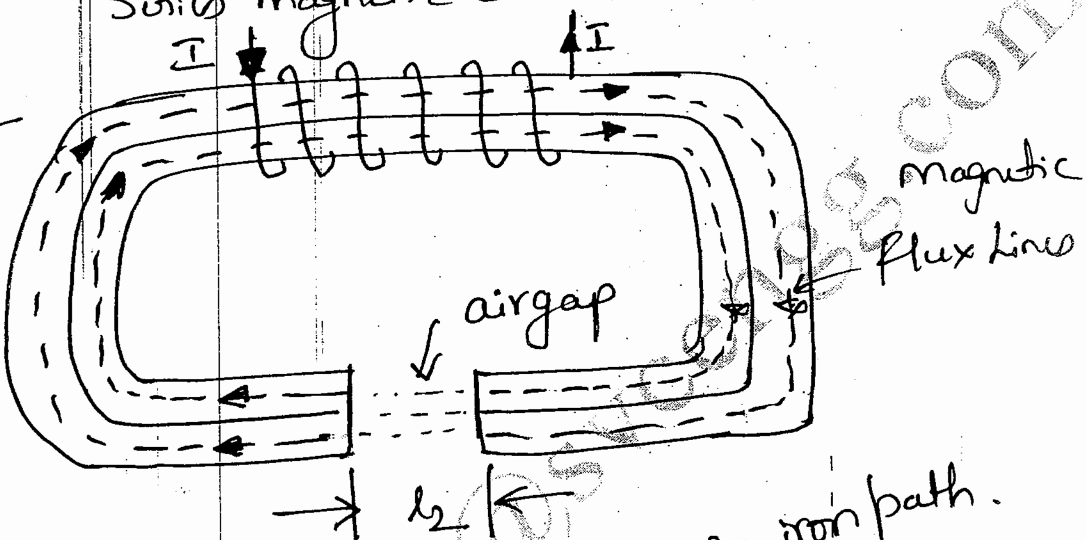


fig. airgap in iron path.

A Series magnetic Circuit is the one in which the Magnetic flux remains the same in all parts of the Circuit.

A typical Series magnetic circuit can be considered to be consisting of an iron part of Length l, with area of cross-section 'S' and an air gap over a Length l_2.

let R_1 and R_2 be the reluctance of the iron path and the airgap respectively.

if V_m is the m.m.f required to set up a flux ϕ, then

$$V_{m_1} = \phi_1 R_1 \longleftarrow (a) \text{ mmf across iron path}$$

$$V_{m_2} = \phi_2 R_2 \longleftarrow (b) \text{ mmf across airgap}$$

$$V_m = V_{m_1} + V_{m_2} \quad \ldots \text{ in a Series ckt}$$

$$V_m = \phi_1 R_1 + \phi_2 R_2$$

in a Series Circuit $\phi_1 = \phi_2 = \phi$.

$$\therefore V_m = \phi (R_1 + R_2) \longleftarrow (c)$$

if R is the equivalent reluctance for entire circuit, then

$$V_m = \phi R \longleftarrow (d)$$

Comparing (c) and (d)

$$\therefore \boxed{R = R_1 + R_2} \quad H^{-1}$$

$$\Rightarrow \boxed{R = \frac{l_1}{\mu_1 S} + \frac{l_2}{\mu_2 S}} \quad H^{-1}$$

whe μ_1 and μ_2 are the permeability of the iron and air media.

problem 23.

Calculate the reluctance of a magnetic circuit of mean length 0.5m of area of cross section 0.3 cm^2. the relative permeability of the medium is 100. also calculate the flux if the coil used to set up the magnetic flux has 1000 turns with a current of 0.2A.

Soln :- given $l = 0.5m.$

$S = 0.3 \times 10^{-4}\ m^2$

$\mu_r = 100$; $N = 1000$, $I = 0.2A$.

$R = ?$ $\phi = ?$

Reluctance $R = \frac{l}{\mu S} = \frac{l}{\mu_0 \mu_r S}$

$$R = \frac{0.5}{4\pi \times 10^{-7} \times 100 \times 0.3 \times 10^{-4}}$$

$$\boxed{R = 1.326 \times 10^{8}\ A/wb \text{ (or) } H^{-1}}$$

$$\phi = \frac{V_m}{R} = \frac{NI}{R} = \frac{100 \times 0.2}{1.326 \times 10^{8}}$$

$$\boxed{\phi = 1.5 \times 10^{-6}}\ wb$$

Topic 4.8

. Potential Energy and Forces on Magnetic Materials.

Derive the equation for energy density in a magnetic field.

02-DEC2008/Jan 2009 (05 Marks)

Question. Derive magnetic Energy density in a magnetic field.

Ea. Magnetic Energy S.

The Energy stored in Inductor is given by

$$W_m = \frac{1}{2} L I^2 \text{ joules} \quad \text{(1)}$$

the general expression for Energy in Electro-static is given by

$$W_E = \frac{1}{2} \int_{vol} (D \cdot E)\, dv \text{. Joules}$$

Hly in magnetostatics

$$W_m = \frac{1}{2} \int_{vol} (\bar{B} \cdot \bar{H})\, dv \text{ joules.}$$

but $\bar{B} = \mu H$ (or) $\bar{H} = \bar{B}/\mu$.

$$\boxed{W_H = \frac{1}{2} \int_{vol} \mu H^2\, dv}$$ and

$$\boxed{W_B = \frac{1}{2} \int_{vol} \frac{B^2}{\mu}\, dv} \text{ Joules.}$$

the magnetic Energy density is nothing but the magnetic energy stored per unit volume. measured in J/m³.

i.e $e = \frac{W_H}{v} = \frac{1}{2}\mu H^2$ joule/m³

(or) $e = \frac{1}{2} B^2/\mu$ J/m³

$$\boxed{e_m = \frac{1}{2}\mu H^2 = \frac{1}{2}\frac{B^2}{\mu}}$$ Joules/m³

b) Force on Magnetic Materials:- [15-June/July 2017 (4M) CBCS]

Question: write a note on forces on magnetic materials.

Energy = Force × distance = work done.

$$dw_H = F\,dl = \frac{1}{2}\frac{B^2}{\mu}\cdot S\cdot dl$$

$dv = S\cdot dl$
$= Sdl$

$$\boxed{F = \frac{B^2 S}{2\mu}}$$ Newtons

the tractive pressure is the ratio of force on a magnetic surface per area measured in N/m²

$$\boxed{F/S = \frac{1}{2\mu_0}B^2 = \frac{1}{2}BH = \frac{1}{2}\mu H^2}$$ N/m²

Module-4 (Summary)

1. Lorentz force equation $\boxed{\bar{F} = Q(\bar{v} \times \bar{B})}$; N.

$\boxed{F = QvB\sin\theta}$ N.

2. Force experienced by the point charge in presence of both Electric and magnetic field. $\bar{v}$ - velocity vector (m/sec)

$\boxed{\bar{F} = Q(\bar{E} + \bar{v} \times \bar{B})}$; Newton.

3. Force on a differential Current element.

$\boxed{d\bar{F} = I\,\bar{dl} \times \bar{B}}$ N.

(or) $\bar{F} = I\bar{L} \times \bar{B} = ILB\sin\theta\,\bar{a}_n$; N.

4. magnetic Force b/w two differential Current Element.

$d\bar{F}_1 = I_1\bar{dl}_1 \times d\bar{B}_2$; N.

$$\boxed{d\bar{F}_1 = \frac{\mu_0 I_1 \bar{dl}_1 \times (I_2\,\bar{dl}_2 \times \bar{a}_{R_{21}})}{4\pi R_{21}^2}}\ ;\ \text{Newton}.$$

5. Force between two parallel conductors

$$\boxed{F = \frac{\mu_0 I_1 I_2 l}{2\pi r}}\ \text{N}.$$

6. Magnetization and permeability

$\boxed{M = \frac{m}{\text{Volume}}}$ (or) $\boxed{M = n \cdot m}$

(78)

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Magnetic Susceptibility $(\chi) = \frac{M}{H}$.

$$\boxed{\chi = \frac{M}{H}}$$

permeability $\boxed{\mu = \mu_0 \mu_r}$ H/m

$\boxed{B = \mu_0 \mu_r H}$ wb/m^2

7. Relation between B, M and H

$\boxed{B = \mu_0 H}$ wb/m^2 and $\boxed{B = \mu_0 M}$ wb/m^2

$\boxed{B_{net} = \mu_0 (H + M)}$ wb/m^2

$\boxed{M = (\mu_r - 1) H}$ A/m.

$\boxed{M = \chi H}$ A/m.

$\boxed{\chi = \mu_r - 1}$ or $\boxed{\mu_r = \chi + 1}$

$$\boxed{\mu_r = 1 + \chi = \frac{\mu}{\mu_0}}$$

8. Total Current density $\boxed{\bar{J} = \nabla \times \bar{H}}$ A/m^2.

9. Bound Current density $\boxed{\bar{J_b} = \nabla \times \bar{m}}$ A/m^2

10. % of change in Magnetization $= \frac{M_2 - M_1}{M_1} \times 100\%$.

11. Magnetic Boundary conditions.

$$\oint_{\langle S\rangle} \bar{B} \cdot \overline{dS} = 0 \quad \text{and} \quad \oint_{\langle l\rangle} \bar{H} \cdot \overline{dl} = I.$$

$$\boxed{(\bar{H}_1 - \bar{H}_2) \times \bar{a}_{n_{12}} = \bar{K}} \text{ A/m}$$

- tangential components of magnetic field intensity $\bar{H}$ are equal

$$\boxed{H_{1t} = H_{2t}} \text{ A/m} \quad \text{(or)} \quad \boxed{\frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}}$$

- Normal components of Magnetic flux density $(\bar{B})$ are equal.

$$\boxed{B_{1n} = B_{2n}} \text{ wb/m}^2 \quad \text{(or)} \quad \boxed{\mu_1 H_{1n} = \mu_2 H_{2n}}$$

12. Reluctance $\boxed{R = \dfrac{l}{\mu S}}$. H^{-1}.

13. Permeance $\boxed{P = \dfrac{\mu S}{l} = R^{-1}}$; H. $\boxed{\phi = \dfrac{V_m}{R}} = \dfrac{NI}{R}$; wb

$$\boxed{B = \frac{\phi}{S}} \text{ wb/m}^2 \quad \text{(or)} \quad \boxed{\phi = B \cdot S} \text{ wb.}$$

14. Reluctance of a series magnetic circuit

$$\boxed{R = R_1 + R_2} \; H^{-1} \quad \text{(or)} \quad \boxed{R = \frac{l_1}{\mu_1 S} + \frac{l_2}{\mu_2 S}} \; H^{-1}$$

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15. Magnetic Energy

$$\boxed{W_m = \frac{1}{2} L I^2 = \frac{1}{2} \int_{vol} (\bar{B} \cdot \bar{H}) \, dv} \text{ Joules}$$

$$\boxed{W_H = \frac{1}{2} \int_{vol} \mu H^2 \, dv = \frac{1}{2} \int_{vol} \frac{B^2}{\mu} \, dv} \text{ Joules}$$

16. magnetic Energy density

$$\boxed{e_m = \frac{1}{2} \mu H^2 = \frac{1}{2} \frac{B^2}{\mu}} \text{ Joules/m}^3$$

17. Force on a magnetic materials

$$\boxed{F = \frac{B^2 S}{2\mu}} \text{ Newton}$$

18. $$\boxed{\frac{F}{S} = \frac{1}{2} \mu_0 B^2 = \frac{1}{2} B H = \frac{1}{2} \mu H^2} \text{ N/m}^2.$$

Module -5(Part-A)

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Part-A : Time-varying fields and Maxwell's equations
Faraday's law, displacement current, Maxwell's equations in point form, Maxwell's equations in integral form.

Topics:

5.1 a. Faraday's law
b. Lenz's law
c. Maxwell's Equation from Faraday's Law
d. Transformer and Motional EMF
Solved Problems

5.2 Inconsistency of Amperes Law (Modified Ampere's Law) +
a. Concept of Conduction and displacement current and Current densities
b. Loss tangent and its importance
c. Continuity current equation from Maxwell's Equation
d. Conduction and Displacement current in capacitor

Solved Problems

5.3 Maxwell's equations in point form Maxwell's equations in integral form
a. Maxwell's Equations for static fields
b. Maxwell's Equations for Time-varying fields
c. Maxwell's Equations in free space medium
d. Maxwell's Equations in Good conducting medium
e. Maxwell's Equations in Good dielectrics or Low loss dielectric medium
Solved Problems

Summary
- List of Symbols
- List of Formulae

Module-5 . part (A).

Topic 5.1 . a. Faraday's Law ~~to design a law~~

b. Lenz's Law.

c. Maxwell's Equation from Faraday's law

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \cdot V/m^2$$

d. Transformer and motional E.m.f.

Questions

Using the Faraday's law, deduce the Maxwell's equation, to relate time varying electric and magnetic field (8m).

(or)

prove that $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$. (6m)

(or)

State Faraday's Law and obtain point and integral forms of Faraday's Law of EMI . (5m).

(or)

For a closed stationary path in space linked with a changing magnetic field prove that $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$. (8m)

(or)

Starting from the concept of Faraday's Law of electromagnetic induction derive the maxwell's equation

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \cdot \quad (6M)$$

(or)

Using Faraday's Law derive an expression for emf induced in a stationary conductor placed in a time varying magnetic field. (4M)

Explain Faraday's law and Lenz's Law. (6M)

[06-Dec 2010, 02-Dec 2010, 02-Jan 2009, 06-Jan 2012, 06-Jan 2014, 10-Jan 2014, 06-June/July-2011, 06-June/July 2012, 02-June/July 2012, 02-June/July 201 06 Jan 2013, 06-June/July 2014].

06 - June /July 2012

8 State and explain Farday's law of electromagnetic induction. Hence obtain Maxwell's equation in differential form. (04 Marks)

02 - June /July 2012

9 Obtain Faraday's law of electromagnetic induction in integral form and hence arrive at the differential form of Faraday's law. (08 Marks)

02 - June /July 2010

10 Derive $\nabla\times\bar{E}=-\frac{\partial\bar{B}}{\partial t}$

06 – Jan 2013

11 Derive the Maxwell's equation in point form as derived from Faraday's law. (06 Marks)

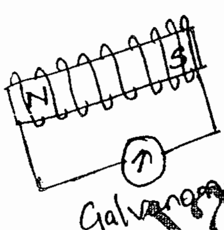
06 –June/July 2014

12 Explain Faraday's law and Lenz's law. (06 M)

Soln: Topic 5.1a Faraday's Law.

Faraday's Law:-

Faraday's Law can be stated as "the magnitude of the induced emf in a circuit is equal to the rate of change of the magnetic flux through it and its direction opposes the flux change.



i.e $\boxed{e=-\frac{d\phi}{dt}}$ volt's ← (1)

where ϕ – Flux Linkage with the circuit or coil.

if coil has N turns then emf induced across the coil is

$$\boxed{e=-N\frac{d\phi}{dt}} \text{ volt's} \leftarrow (2)$$

Note:- the interpretation of -ve sign is given by Lenz Law.

5.1b. Lenz's Law.

- Lenz Law:- The induced emf is in such a direction as to oppose the charge causing it.

(i.e the -ve sign indicates ~~that the~~ the direction of induced emf is such that to produce a current which will produce a magnetic field which will oppose the original field).

5.1c Maxwell's Equation from Faraday's Law.

The induced emf is a scalar quantity measured in volts, and is given by

Note:- Start from Faraday's Law

$$e = \oint_{\langle L \rangle} \bar{E} \cdot \bar{dl} \longleftarrow (3)$$

from Faraday Law,

$$e = -\frac{d\phi}{dt} \text{ volt's} \longleftarrow (3a)$$

from defn of Magnetic flux density $(\bar{B})$

$$\bar{B} = \frac{d\phi}{dS} \text{ wb/m}^2$$

The total Magnetic flux (ϕ) passing through specified area is given by

$$\phi = \int_{\langle S \rangle} \bar{B} \cdot \bar{dS} \quad \text{wb's} \longleftarrow (4)$$

where $\bar{B}$ - Magnetic Fluxdensity (wb/m² (or) Tesla).

From Faraday's Law

$$e = -\frac{d\phi}{dt} = \frac{-d}{dt}\left[\int_{\langle S\rangle} \overline{B}\cdot\overline{ds}\right] \longleftarrow (4a)$$

equating eqn (3) and eqn (4a)

ie $$e = \oint_{\langle l\rangle} \overline{E}\cdot\overline{dl} = -\frac{d\phi}{dt} = \frac{-d}{dt}\left[\int_{\langle S\rangle} \overline{B}\cdot\overline{ds}\right] \text{ volts}$$

$$\oint_{\langle l\rangle} \overline{E}\cdot\overline{dl} = -\frac{d}{dt}\left[\int_{\langle S\rangle} \overline{B}\cdot\overline{ds}\right] \text{ volts}$$

$$\boxed{\oint_{\langle l\rangle} \overline{E}\cdot\overline{dl} = -\int_{\langle S\rangle} \frac{\partial \overline{B}}{\partial t}\cdot\overline{ds}} \text{ volts} \longleftarrow (5)$$

eqn (5) called Integral form of Maxwell's eqn derived from Faraday's Law

using Stoke's theorem i.e

$$\oint_{\langle l\rangle} \overline{A}\cdot\overline{dl} = \int_{\langle S\rangle} (\nabla \times \overline{A})\cdot\overline{ds}$$

$\Rightarrow$ L.H.S of eqn (5)

$$\oint_{\langle l \rangle} \overline{E} \cdot \overline{dl} = \int_{\langle S \rangle} (\nabla \times \overline{E}) \cdot \overline{dS} \leftarrow (6)$$

$\therefore$ eqn (5) becomes

$$\oint_{\langle l \rangle} \overline{E} \cdot \overline{dl} = \int_{\langle S \rangle} \left(-\frac{\partial \overline{B}}{\partial t}\right) \cdot \overline{dS} = \int_{\langle S \rangle} (\nabla \times \overline{E}) \cdot \overline{dS}$$

$$\Rightarrow \boxed{\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}} \leftarrow (7) \quad V/m^2$$

eqn (7) is called as point form of Maxwell's eqn derived from Faraday's Law.

i.e $\boxed{\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}}$ V/m²

eqn (7) indicates that time-varying magnetic fields is responsible for rotational electric field.

Note:- 1. if $\overline{B}$ is not a function of time 't' (i.e not varying with time) then $\frac{\partial \overline{B}}{\partial t} = 0$,

eqn (7) becomes $\boxed{\nabla \times \overline{E} = 0}$ (6)

$\Rightarrow \boxed{\nabla \times \bar{E} = 0}$; $\Rightarrow$ which is same as electrostatic result ie $\underset{\langle 1 \rangle}{\oint} \bar{E} \cdot \bar{dl} = 0$.

2. if $\boxed{\nabla \times \bar{E} = 0}$ then $\bar{E}$ is said to be arised from a static distribution of charges.

3. if $\boxed{\nabla \times \bar{E} \neq 0}$ then $\bar{E}$ is not arised from static distribution of charges.

Now let us consider the force experienced by the charge 'Q' in presence of magnetic field is given by [from concept of motional emf]

i.e from Lorentz force eqn

$$\bar{F}_m = Q(\bar{v} \times \bar{B}) \quad \text{Newton's} \leftarrow (8)$$

the motional electric field intensity $\bar{E}_m = \frac{\bar{F}_m}{Q} = \bar{v} \times \bar{B}$ V/m $\leftarrow (9)$

$\therefore$ Induced emf is given by

$$e_m = \underset{\langle 1 \rangle}{\oint} \bar{E}_m \cdot \bar{dl} = \underset{\langle 1 \rangle}{\oint} (\bar{v} \times \bar{B}) \cdot \bar{dl} \leftarrow (10)$$

eqn (10) represents total emf induced when a conductor is moved in a Uniform constant magnetic field.

In case, the Magnetic flux density is also varying with time then the induced emf is the combination of transformer and motional emf.

given by

$$\boxed{e_{total} = e_{transformer} + e_{motional}} \text{ volts}$$

iv)

$$\boxed{e = \underbrace{\oint_{\langle l \rangle} \bar{E}\cdot\overline{dl}}_{\text{total emf}} = \underbrace{-\int_{\langle s \rangle} \frac{\partial \bar{B}}{\partial t}\cdot\overline{ds}}_{\text{transformer emf }(e_t)} + \underbrace{\oint_{\langle l \rangle} (\bar{v}\times\bar{B})\cdot\overline{dl}}_{\text{motional emf }(e_m)}} \text{ volts}$$

Topic 5.1d. Transformer and motional Emf.

Questions.

06-DEC2008/Jan 2009

13 Explain transformer and motional induced emfs. (06 Marks)

(or)

02 - June /July 2011

14 State Faraday's law. Apply Faraday's law to i) Stationary conductor and changing field and ii) Stationary field and moving conductor and derive necessary expressions. (09 Marks) field and (9m)

(or)

EE- June /July 2016

15 a. Explain Faraday's laws applied to : i) stationary path, changing field and ii) steady field, moving circuit. (06 Marks)

Soln:- Transformer emf (or) Stationary Conductor & changing field.

it is defined as the emf induced in a stationary circuit due to change of Magnetic Flux density across the circuit with time called as transformer induced e.m.f.

i.e. $e = -\dfrac{d\phi}{dt} = -\int_{\langle S \rangle} \dfrac{\partial \bar{B}}{\partial t} \cdot d\bar{S}$ volts

$\therefore$ $$\boxed{e_{traf} = \oint_{\langle l \rangle} \bar{E} \cdot \bar{dl} = -\int_{\langle S \rangle} \frac{\partial \bar{B}}{\partial t} \cdot d\bar{S}}$$ volts

transformer emf.

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ii> Motional e.m.f / Stationary field and Moving Conductor :-

it is defined as the emf induced b/w the two ends of a conductor due to its motion in a steady Magnetic field.

→ from Lorentz force eqn

$$\overline{F}_m = Q(\overline{v} \times \overline{B}) \text{ Newtons}$$

$$\overline{E}_m = \frac{\overline{F}_m}{Q} = (\overline{v} \times \overline{B}) \text{ V/m.}$$

the induced motional emf

$$e_m = \oint_{\langle l \rangle} \overline{E}_m \cdot \overline{dl} = \oint_{\langle l \rangle} (\overline{v} \times \overline{B}) \cdot \overline{dl}$$

$$\Rightarrow \boxed{e_m = \oint_{\langle l \rangle} \overline{E}_m \cdot \overline{dl} = \oint_{\langle l \rangle} (\overline{v} \times \overline{B}) \cdot \overline{dl}} \text{ volts}$$

Motional emf.

Problem 1

$\bar{E} = E_m \sin(\omega t - \beta z)\bar{a}_y$ $\bar{D}, \bar{B}, \bar{H}$

06-DEC2010

16 Given $\bar{E} = E_m \sin(\omega t - \beta z)\hat{a}_y$ in free space. Find $\bar{D}$, $\bar{B}$ and $\bar{H}$. (06 Marks)

(or)

10 - June /July 2012

17 Given $\bar{E}$= EmSin(wt – βz)$\hat{a}_y$ in force space, find $\bar{D}$, $\bar{B}$ and $\bar{H}$. Sketch $\bar{E}$ and $\bar{H}$ at t = 0. , at $t=0$. (10 Marks)

$\bar{E}$ $\bar{H}$

06 - June /July 2013

18 ~~Given $\bar{E} = E_m \sin(\omega t - \beta z)\hat{a}_y$ in free space. Calculate $\bar{D}$, $\bar{B}$ and $\bar{H}$.~~ (08 Marks)

Mention years.

10-Dec/Jan 2016

19 b. ~~$\vec{E}$= Em sin (wt – Bz) ay in free space find $\vec{D}$, $\vec{B}$, $\vec{H}$~~ (05 Marks)

Soln:- given

$$\bar{E} = E_m \sin(\omega t - \beta z)\bar{a}_y = E_y a_y \quad V/m$$

in free space $\mu_r = 1$ and $\epsilon_r = 1$

$\Rightarrow$ $\boxed{\mu = \mu_0}$ H/m and $\boxed{\epsilon = \epsilon_0}$ F/m

i) Electric Flux density $(\bar{D})$ C/m²

$$\bar{D} = \epsilon_0 \bar{E} \quad C/m^2$$

$$\therefore \boxed{\bar{D} = \epsilon_0 E_m \sin(\omega t - \beta z)\bar{a}_y} \quad C/m^2$$

ii) To Find $\bar{B} = ?$ from Faraday's Law using point form of Maxwell's eqn

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \longleftarrow (1)$$

Since $\bar{E}$ is fn of only 'z' and has only 'E_y' component.

and $E_y = E_m \sin(\omega t - \beta z)$ V/m (II)

$$\nabla \times \bar{E} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 0 & 0 & \partial/\partial z \\ 0 & E_y & 0 \end{vmatrix}$$

$$= \left[0 - \frac{\partial E_y}{\partial z}\right] \bar{a}_x = \frac{-\partial E_y}{\partial z} \bar{a}_x \quad V/m^2$$

$$\frac{-\partial E_y}{\partial z} \bar{a}_x = -\frac{\partial}{\partial z}\left[E_m \sin(\omega t - \beta z)\right] \cdot \bar{a}_x$$

$$= -E_m \left[\cos(\omega t - \beta z)\right] \times -\beta \, \bar{a}_x$$

$$-\frac{\partial E_y}{\partial z} \bar{a}_x = + E_m \beta \cos[\omega t - \beta z] \, \bar{a}_x \quad V/m^2$$

∴ using eq^n (1) i.e

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \quad V/m^2$$

$$E_m \beta \cos[\omega t - \beta z] \, \bar{a}_x = -\frac{\partial \bar{B}}{\partial t}$$

(or)

$$\bar{B} = \int \frac{\partial \bar{B}}{\partial t} \cdot dt = -\int E_m \beta \cos(\omega t - \beta z) \, \bar{a}_x \, dt$$

$$\bar{B} = -E_m \beta \sin(\omega t - \beta z) \times \frac{1}{\omega} \cdot \bar{a}_x$$

$$\therefore \boxed{\bar{B} = -\frac{E_m \beta}{\omega} \sin(\omega t - \beta z) \, \bar{a}_x} \quad wb/m^2 \text{ (or) Tesla}$$

iii> Relationship b/w $\overline{B}$ and $\overline{H}$

$$\overline{B} = \mu_0 \overline{H} \quad wb/m^2$$

(or) $$\overline{H} = \frac{\overline{B}}{\mu_0} \quad A/m$$

**

$$\therefore \boxed{\overline{H} = -\frac{E_m \beta}{\omega \mu_0} \sin(\omega t - \beta z)\, \overline{a_x}} \quad A/m \text{ (or) } N/wb.$$

at time $t = 0$

$$\overline{E} = E_m \sin(\omega t - \beta z)\, \overline{a_y}$$

$$\Rightarrow \overline{E} = -E_m \sin(\beta z)\, \overline{a_y} \; V/m$$

(or)

$$\overline{E} = E_m \sin(\beta z)\,(-\overline{a_y}) \; V/m \leftarrow (a)$$

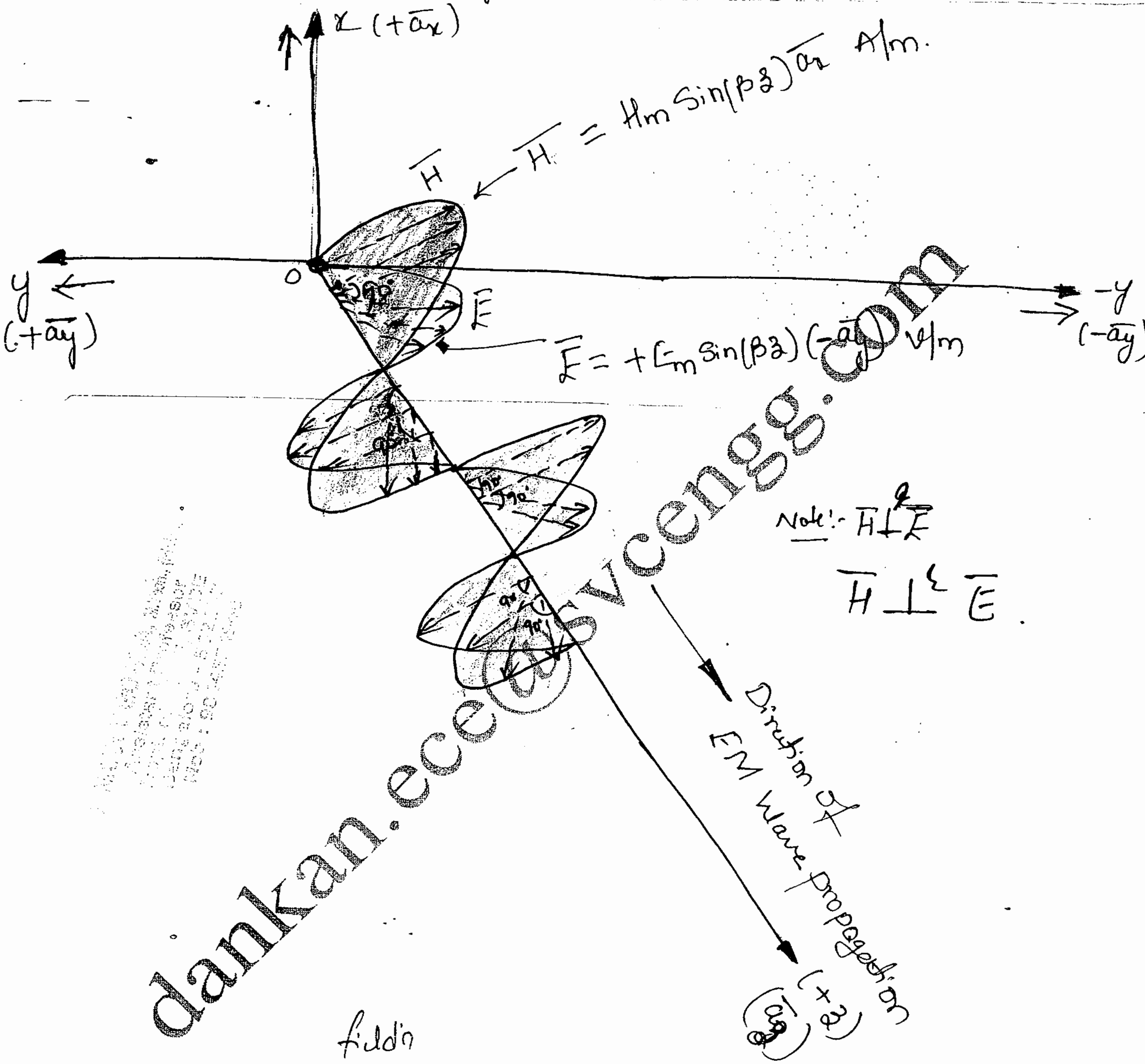
and

$$\overline{H} = \frac{-\beta E_m}{\omega \mu_0} \sin(-\beta z)\, \overline{a_x} = +\frac{\beta E_m}{\omega \mu_0} \sin(\beta z)\, \overline{a_x} \; A/m$$

(or) $$\overline{H} = H_m \sin(\beta z)\, \overline{a_x} \; A/m \leftarrow (b)$$

where $H_m = \dfrac{\beta E_m}{\omega \mu_0}$ A/m.

$\overline{E} = E_m \sin(\beta z)(-\overline{a_y})$ V/m and $\overline{H} = H_m \sin(\beta z)\,\overline{a_x}$ A/m.



Note:- $\overline{H} \perp^r \overline{E}$

$\overline{H} \perp^r \overline{E}$.

Note:- field's $\overline{E}$ and $\overline{H}$ are $\perp^r$ to each other.

Problem 2

If the Electric Field Intensity in free space is given in the rectangular Co-ordinates as $E = E_m Sin(\alpha x) Sin(\omega t - \beta z) a_y\ V/m$. Find the magnetic field Intensity **H** using Faraday's Law

Solu:- given $\bar{E} = E_m \sin(\alpha x)\sin(\omega t - \beta z)\,\bar{a}_y$ V/m.

using Faraday's Law, and In free space $\boxed{\mu = \mu_0}$ H/m, $\boxed{\epsilon = \epsilon_0}$ F/m

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \quad V/m^2$$

$$\bar{E} = E_y \bar{a}_y \ V/m \ ; \ \therefore E_y = E_m \sin(\alpha x)\sin(\omega t - \beta z)\ V/m$$

and $E_y = f^y(x, z)$

$$\therefore \nabla \times \bar{E} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \partial/\partial x & 0 & \partial/\partial z \\ 0 & E_y & 0 \end{vmatrix} = -\frac{\partial \bar{B}}{\partial t} = -\mu_0 \frac{\partial \bar{H}}{\partial t} \quad V/m^2$$

$$\frac{\partial \bar{H}}{\partial t} = -\frac{1}{\mu_0}\left[-\frac{\partial E_y}{\partial z}\bar{a}_x + \frac{\partial E_y}{\partial x}\bar{a}_z\right]$$

$$\frac{\partial \bar{H}}{\partial t} = \frac{1}{\mu_0}\frac{\partial}{\partial z}\left[E_m \sin(\alpha x)\sin(\omega t - \beta z)\right]\bar{a}_x$$

$$-\frac{1}{\mu_0}\frac{\partial}{\partial x}\left[E_m \sin(\alpha x)\sin(\omega t - \beta z)\right]\bar{a}_z$$

$$\frac{\partial \bar{H}}{\partial t} = -\frac{E_m \beta}{\mu_0} \sin(\alpha x)\cos(\omega t - \beta z)\,\bar{a}_x$$

$$- \frac{E_m \alpha}{\mu_0} \cos(\alpha x)\sin(\omega t - \beta z)\,\bar{a}_z \quad A/m\text{-sec}$$

$$\therefore \quad \bar{H} = \int \frac{\partial \bar{H}}{\partial t}\, dt \quad A/m$$

$$\bar{H} = \frac{-E_m \beta}{\mu_0 \omega} \sin(\alpha x)\sin(\omega t - \beta z)\,\bar{a}_x$$

$$+ \frac{E_m \alpha}{\mu_0 \omega} \cos(\alpha x)\cos(\omega t - \beta z)\,\bar{a}_z \quad A/m.$$

$$\therefore \quad \boxed{\bar{H} = \frac{-E_m \beta}{\omega \mu_0} \sin(\alpha x)\sin(\omega t - \beta z)\,\bar{a}_x + \frac{E_m \alpha}{\omega \mu_0} \cos(\alpha x)\cos(\omega t - \beta z)\,\bar{a}_z} \quad A/m.$$

problem 3

$e = -\frac{\omega B a^2}{2}$ volt's

06-DEC2008/Jan 2009

Show that an emf induced in a Faraday's disc generator is $e = -\frac{WBa^2}{2}$ (Volts), where 'W' is the angular velocity in rad/sec, B is the magnetic flux density in Tesla and 'a' is the radius of the disc in metre. (06 Marks)

'ω' rad/sec, B 'a'

Sol^n:-

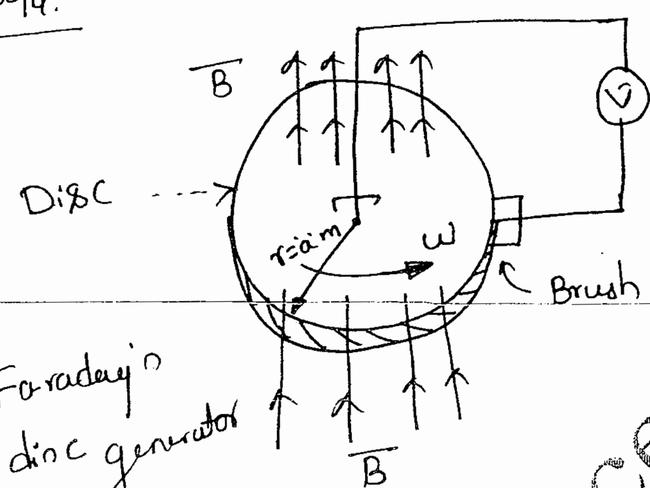


fig. Faraday's disc generator

from fig. the direction of magnetic flux density $\overline{B}$ is

$$\overline{B} = B_z \overline{a_z} \ \text{wb/m}^2$$

$r = a\,m$ - radius of the disc.

the Linear velocity of the disc $\overline{v} = \omega r \overline{a_\phi}$ m/sec.

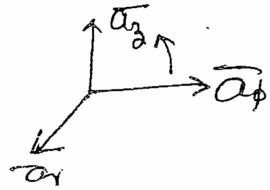
where ω - angular velocity (rad/sec);

the Electric field $\overline{E_m} = \overline{v} \times \overline{B}$ v/m

$$= \omega r \overline{a_\phi} \times B \overline{a_z}$$

$$\boxed{\overline{E_m} = \omega r B \overline{a_r}} \ \text{v/m}$$

$\overline{a_\phi} \times \overline{a_z} = +\overline{a_r}$



$\overline{dl} = dr\,\overline{a_r}$ ... along radial path

the e.m.f induced $e = -\oint_{\langle l \rangle} \overline{E_m} \cdot \overline{dl}$ volt's

from potential concept

$e = -\int_{\langle l \rangle} \overline{E} \cdot \overline{dl}$ volt's

$$e = -\int_{r=0}^{a} \omega r B\,\overline{a_r} \cdot dr\,\overline{a_r}$$

$$= -\omega B \int_{r=0}^{a} r\,dr \quad \overline{a_r} \cdot \overline{a_r} = 1$$

$$e = -\omega B \left. \frac{r^2}{2} \right|_0^a$$

$$e = \frac{-\omega B}{2}\left[a^2 - 0\right]$$

$$\boxed{e = -\frac{\omega a^2 B}{2}}$$ volt's

$\therefore$ Induced e.m.f in a Faraday's disc is given by

$$\boxed{e = \frac{-\omega B a^2}{2}}$$ volt's

Note:- if we assume the direction of Magnetic field acting downwards ie $\overline{B} = B_z(-\overline{a_z}) = -B_z\overline{a_z}$ wb/m²

$\Longrightarrow$ $\boxed{e = +\frac{\omega B a^2}{2}}$ volt's

the induced emf will be in +ve sign.

problem 4 $\overline{E} = 2x^3 \overline{a_x} + 4x^4 \overline{a_y}$ V/m.

06-DEC2009/Jan 2010

22 With usual notations, derive the Maxwell's equation in point form as derived from Faraday's law. Hence show that electric field $E = 2x^3 a_x + 4x^4 a_y$ v/m can not arise from a static distribution of charges. (08 Marks) static

Soln:- Maxwell's eqn in point form from Faraday's Law ~~refer~~ ~~question No-1~~. i.e $\boxed{\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}}$ V/m²

problem: soln

given $\overline{E} = 2x^3 \overline{a_x} + 4x^4 \overline{a_y}$ V/m $= E_x \overline{a_x} + E_y \overline{a_y}$ V/m

and

$E_x = 2x^3$ V/m

$E_y = 4x^4$ V/m.

If $\overline{E}$ is said to be not arise from a static distribution of charges then $\boxed{\nabla \times \overline{E} \neq 0}$ $\nabla \times E \neq 0$

Check

$$\nabla \times \overline{E} = \begin{vmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ \partial/\partial x & 0 & 0 \\ E_x & E_y & 0 \end{vmatrix}$$

$$= \begin{vmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ \partial/\partial x & 0 & 0 \\ 2x^3 & 4x^4 & 0 \end{vmatrix}$$

$$= \frac{\partial(4x^4)}{\partial x} \overline{a_z} = 4 \times 4 x^3 \overline{a_z} = 16 x^3 \overline{a_z}$$

$$\boxed{\nabla \times \overline{E} = 16x^3\, \overline{a_z}} \; V/m^2$$

Since $\nabla \times \overline{E} = 16x^3\, \overline{a_z} \neq 0$ $\therefore$ the given field $\overline{E}$ is not arised from static distribution of charges.

problem 5 $\bar{B} = 0.5\cos(377t)[3\bar{a}_y + 4\bar{a}_z]$ Tesla.

10-DEC2011/Jan 2012

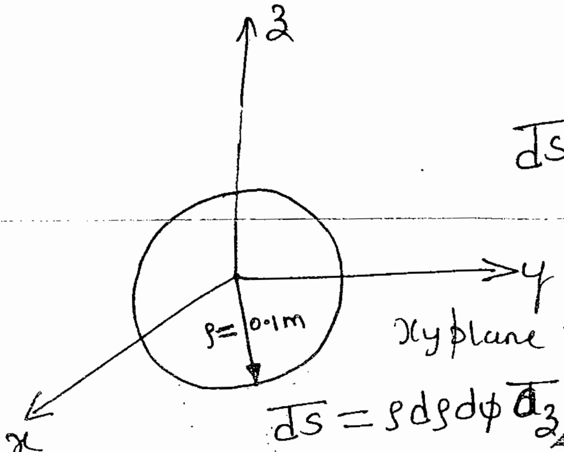
A circular loop of 10 cm radius is located in xy plane with magnetic field $\vec{B} = 0.5\cos(377t)[3\hat{a}_y + 4\hat{a}_z]$ T. Calculate the voltage induced by the loop. (06 Marks)

Solu:- given $\bar{B} = 0.5\cos(377t)[3\bar{a}_y + 4\bar{a}_z]$ Tesla.

radius $\rho = 10$ cm $= 0.1$m

$P(\rho, \phi, z)$ — $d\rho$, $\rho d\phi$, dz

$\overline{ds} = \rho\, d\rho\, d\phi\, \bar{a}_z$



xy plane $\Rightarrow$ z = 0 plane

$\overline{ds} = \rho\, d\rho\, d\phi\, \bar{a}_z$

the total flux (ϕ) leaving the surface is

$$\boxed{\phi = \int_{\langle S\rangle} \bar{B}\cdot\overline{ds}} \text{ wb}$$

$\bar{a}_y \cdot \bar{a}_z = 0$

$\bar{a}_z \cdot \bar{a}_z = 1$

$$\phi = \int_{\langle S\rangle} [0.5\cos(377t)][3\bar{a}_y + 4\bar{a}_z] \cdot \rho\, d\rho\, d\phi\, \bar{a}_z$$

$$= \int_{\langle S\rangle} [0.5\cos(377t)](4\bar{a}_z)\cdot \rho\, d\rho\, d\phi\, \bar{a}_z$$

$$= 4 \times 0.5\cos(377t) \underbrace{\int_{\rho=0}^{0.1} \rho\, d\rho}_{5\times 10^{-3}} \underbrace{\int_{\phi=0}^{2\pi} d\phi}_{2\pi} \underbrace{\bar{a}_z \cdot \bar{a}_z}_{1}$$

$$= 2\cos(377t) \times (5\times10^{-3}) \times (2\pi) \times 1$$

$$\boxed{\phi = 20\pi \times 10^{-3} \cos(377t)} \text{ wb}$$

$$= 6.28318 \cos(377t) \text{ mwb}$$

the induced voltage/emf in the loop is given by

$$v = e = -\frac{d\phi}{dt} \text{ volts}$$

$$= -\frac{d}{dt}\left[20\pi \times 10^{-3} \cos(377t)\right]$$

$$= -20\pi \times 10^{-3} \times -\sin(377t) \times 377$$

$$= +7540\pi \times 10^{-3} \sin(377t)$$

$$\boxed{v = 23.6876 \sin(377t)} \text{ volts}$$

Notes:- if Circular Loop has 'N' number of turns the induced emf $v = 23.6876 N \sin(377t)$ volts

i.e $v = -N\frac{d\phi}{dt}$ volts

eg: if $N = 10$ turns $\boxed{v = 230.6876 \sin(377t)}$ volts problem6

40cm xy plane

10 - June /July 2015

20Ω

A circular conducting loop of radius 40 cm lies in xy plane and has resistance of 20Ω. If the magnetic flux density in the region is given as, → if the

$\hat{B} = 0.2\cos 500t\hat{X} + 0.75\sin 400t\hat{Y} + 1.2\cos 314t\hat{Z}T$.

Determine effective value of induced current in the loop. (06 Marks)

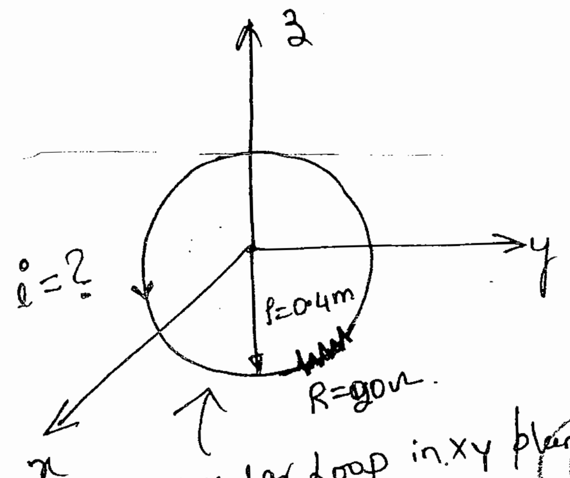
RMS

Soln:

$$\overline{B} = 0.2\cos(500t)\,\overline{a_x} + 0.75\sin(400t)\,\overline{a_y} + 1.2\cos(314t)\,\overline{a_z}\,T.$$

$1.2\cos(314t)\overline{a_z}$

$R = 20\Omega$. $\rho = 0.4\,m = 40\,cm$.



$P(\rho, \phi, z)$ → $d\rho$, $\rho d\phi$, dz

the total flux (ϕ) leaving the surface is

$$\boxed{\phi = \int_{\langle S\rangle} \overline{B}\cdot\overline{ds}}\ wb$$

→ z=0 plane.

$$\boxed{\overline{ds} = \rho\, d\rho\, d\phi\, \overline{a_z}}$$

$$\phi = \int_{\langle S\rangle} \left[0.2\cos(500t)\,\overline{a_x} + 0.75\sin(400t)\,\overline{a_y} + 1.2\cos(314t)\,\overline{a_z}\right]\cdot\left[\rho\, d\rho\, d\phi\, \overline{a_z}\right]$$

$\overline{a_x}\cdot\overline{a_z} = 0$

$\overline{a_y}\cdot\overline{a_z} = 0$

$\overline{a_z}\cdot\overline{a_z} = 1$

$$\phi = \int_{\langle S\rangle} 1.2\cos(314t)\,\overline{a_z}\cdot\rho\, d\rho\, d\phi\, \overline{a_z}$$

$$= 1.2\cos(314t)\underbrace{\int_{\rho=0}^{0.4} \rho\, d\rho}_{0.08}\ \underbrace{\int_{\phi=0}^{2\pi} d\phi}_{2\pi}\ \underbrace{\overline{a_z}\cdot\overline{a_z}}_{1}$$

$$= 1\cdot2 \cos(314t) \times 0\cdot08 \times 2\pi \times 1$$

$$\boxed{\phi = 0\cdot192\pi \cos(314t)} \text{ wb}$$

$\therefore$ the induced voltage $e = -\frac{d\phi}{dt}$ volts

$$e = -0\cdot192\pi \times -\sin(314t) \times 314$$

$$\boxed{e = 189\cdot4 \sin(314t)} \text{ volts}$$

$\therefore$ the Current in the Loop

$$i = \frac{e}{R} = \frac{189\cdot4 \sin(314t)}{20} \text{ Amperes}$$

$$\boxed{i = 9\cdot47 \sin(314t)} \text{ Amperes}$$

problem 7

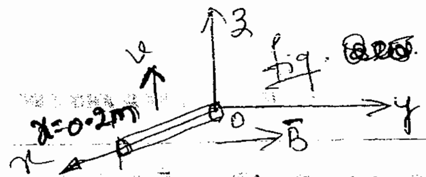


fig. problem 7

10 - June /July 2012

Find the induced voltage in the conductor if $\bar{B} = 0.04\hat{a}_y$ T and $\bar{v} = 2.5\sin 10^3 t\,\hat{a}_z$ m/s, find induced emf, if $\bar{B}$ is changed to 0.04 $\hat{a}_x$ T. $\bar{B} = 0.04\,\bar{a}_y$ $0.04\,\bar{a}_x$ T (or) — $\bar{v} = 2.5\sin 10^3 t\,\bar{a}_z$ m/sec (05 Marks)

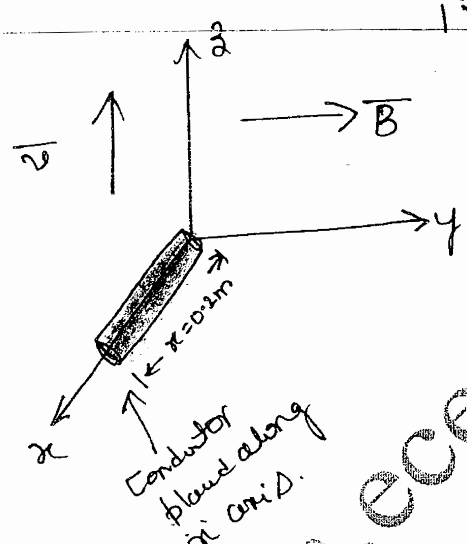
EE- June /July 2016

c. A straight conductor of length 0.2m, lies on x-axis with one end at origin. The conductor is subjected to a magnetic flux density $B = 0.04\bar{a}_y$ Tesla and the velocity $\bar{v} = 2.5 \sin 10^3 t\,a_z$ m/sec. Determine motional emf induced in the conductor. (06 Marks)

sol^n:- i) given $\bar{B} = 0.04\,\bar{a}_y$ Tesla ; $|\bar{B}| = 0.04$.

and $\bar{v} = 2.5\sin(10^3 t)\,\bar{a}_z$ m/sec. — velocity vector

$|\bar{v}| = 2.5\sin(10^3 t)$



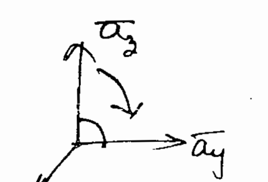
the motional emf induced in a conductor is given by

$$\boxed{e_m = \oint_{\langle l\rangle} \bar{E}_m \cdot \bar{dl} = \int_{\langle l\rangle} (\bar{v}\times\bar{B})\cdot\bar{dl} \text{ volts}}$$

$$\bar{E}_m = \bar{v}\times\bar{B} = |\bar{v}||\bar{B}|\sin\theta\,\bar{a}_n$$

$$\bar{E}_m = 2.5\sin(10^3 t)\times 0.04\times \sin(90^\circ)(-\bar{a}_x)$$

$$\boxed{\bar{E}_m = -0.1\sin(10^3 t)\,\bar{a}_x} \text{ v/m}$$



clockwise dir^n

$\bar{a}_z\times\bar{a}_y = -\bar{a}_x$

and $\theta = 90^\circ$

$\therefore\ \bar{a}_n = -\bar{a}_x$

the induced emf $e_m = \int_{<l>} E_m \cdot \overline{dl}$ volts

Since the Conductor is placed along x axis.

and Length $x = 0.2m$

$\therefore 0 < x < 0.2m$

$\overline{dl} = dx\,\overline{a_x}$.

$$\therefore \quad e_m = \int_{x=0}^{0.2} -0.1 \sin(10^3 t)\, \overline{a_x} \cdot dx\, \overline{a_x}$$

$$= -0.1 \sin(10^3 t) \int_{x=0}^{0.2} dx \; \overbrace{\overline{a_x} \cdot \overline{a_x}}^{1} \text{ volts}$$

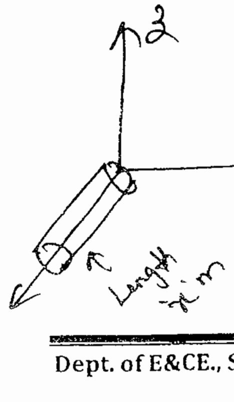
$$e_m = -0.1 \sin(10^3 t)\,(0.2)$$

$$\therefore \quad e_m = -0.02 \sin(10^3 t) \text{ volts}$$

i.e $\boxed{e_m = -0.02 \sin(10^3 t)}$ volts

Problem No. 7

Note:- In question No. 25 Length of the Conductor is not mention $\therefore$ we can assume it Length to be x m. ; $\overline{dl} = dx\,\overline{a_x}$



$\Rightarrow$ induced motional emf

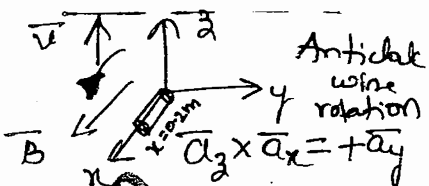
$$e_m = \int_{x=0}^{x} -0.1 \sin(10^3 t)\, \overline{a_x} \cdot dx\, \overline{a_x}$$

$$e_m = -0.1 \sin(10^3 t) \int_{x=0}^{x} dx \times \overbrace{\overline{a_x} \cdot \overline{a_x}}^{1}$$

$\boxed{e_m = -0.1\,x \sin(10^3 t)}$ volts

ii) If $\overline{B} = 0.04\,\overline{a_x}$ Tesla then the induced motional emf is $e_m = \int_{<l>} \overline{E_m} \cdot \overline{dl} = \int_{<l>} (\overline{v} \times \overline{B}) \cdot \overline{dl}$ Volts

$\overline{v} = 2.5 \sin(10^3 t)\,\overline{a_z}$ m/sec.



$\overline{a_z} \times \overline{a_x} = +\overline{a_y}$

$\theta = 90^\circ$

$\therefore \overline{a_n} = +\overline{a_y}$

$$\overline{E_m} = \overline{v} \times \overline{B} = |\overline{v}||\overline{B}| \sin\theta\,\overline{a_n}$$

$$\overline{E_m} = 2.5 \sin(10^3 t) \times 0.04 \,\cancelto{1}{\sin(90^\circ)}\,(+\overline{a_y})$$

$$\boxed{\overline{E_m} = 0.1 \sin(10^3 t)\,\overline{a_y}} \text{ volts}$$

Since the Conductor placed along 'x' axis $\overline{dl} = dx\,\overline{a_x}$ f $0 < x < 0.2m$

$$e_m = \int_{<l>} \overline{E_m} \cdot \overline{dl} \qquad \overline{a_y} \cdot \overline{a_x} = 0$$

$$e_m = \int_{x=0}^{0.2} 0.1 \sin(10^3 t)\,\overline{a_y} \cdot dx\,\overline{a_x}$$

$$e_m = 0.1 \sin(10^3 t) \int_{x=0}^{0.2} dx \;\cancelto{0}{\overline{a_y} \cdot \overline{a_x}}$$

$= 0.1 \sin(10^3 t) \times 0.2 \times 0$

$= 0$

$\therefore \boxed{e_m = 0}$ volts

Obs: In this case motional induced emf (e_m) is zero bcz the Conductor Cuts No field Lines. ∴ the induced voltage must be zero. (or) $\overline{B}$ and $\overline{dl}$ ie magnetic field & Conductor plane are parallel to each other ∴ No field lines Cut the Conductor. ∴ No induced emf.

problem 8 J/J 2013

A copper disc 40 cm diameter is rotated at 3000 rpm on a horizontal axis perpendicular to and through the centre of disc axis, lying in magnetic meridian. Two brushes make contact with the disc at diametrically opposite points on the edge. If horizontal component of earth's field is 0.02 mT, find the induced e.m.f between brushes. (04 Marks)

Sol'n:- given $\overline{B} = 0.02\,\overline{a_r}$ mTesla.

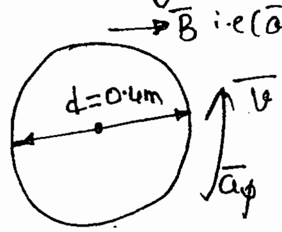
$\overline{E_m} = \overline{v} \times \overline{B}$ volts/m.

where $\overline{v}$ = linear velocity

total rotation per minute = 3000 rpm

$\omega = 2\pi f = \frac{2\pi}{T}$ /sec

$\omega = 2\pi$ rad/sec



$\therefore r = 0.2m$

$\overline{v} = 2\pi r \overline{a_\phi}$ m/sec. $= \omega r \overline{a_\phi} \rightarrow \omega r \overline{a_\phi}$

1 rev/sec i.e one revolution/second.

given $\overline{B} = B_r \overline{a_r}$ horizontal component.

for 1 minute $\longrightarrow$ 3000 revolution (given)

1 minute = 60 seconds

1 second $\longrightarrow \frac{3000}{60} = 50$ revolutions/sec.

for r = 50 revolutions/sec

$\overline{v} = 100\pi r \overline{a_\phi}$ m/sec

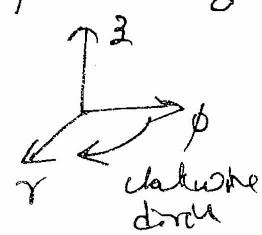
radius $r = d/2 = \frac{40}{2}$ cm = 20cm = 0.2m.

$\overline{v} = 100\pi (0.2) \overline{a_\phi} = 20\pi \overline{a_\phi}$ m/sec

the field $\overline{E_m} = \overline{v} \times \overline{B} = 20\pi \overline{a_\phi} \times 0.02 \overline{a_r}$

$= 0.4\pi (-\overline{a_z})$

$\overline{a_\phi} \times \overline{a_r} = -\overline{a_z}$



and $\overline{dl} = dz\,\overline{a_z}$

the induced emf (e_m).

$$e_m = \oint_{<L>} \bar{E}_m \cdot \overline{dl} = \int_{z=0}^{z} 0.4\pi(-\bar{a}_z) \cdot dz\,\bar{a}_z$$

$$e = -0.4\pi \int_{z=0}^{z} dz \quad \bar{a}_z \cdot \bar{a}_z$$

$$\therefore \boxed{e = -0.4\pi z} \text{ volts}$$

Problem   B.U 2000.

Calculate the voltage across $1 ohm$ and 2 ohm resistors shown in fig. the loop is located in the XY plane and $\varphi = 0.1t\ wb.$

$\phi = 0.1t\ wb$ given $\phi = 0.1t$ wb.

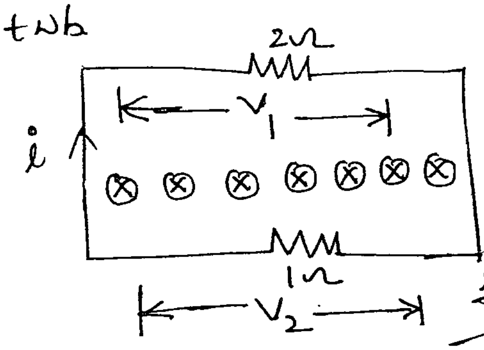


fig. problem

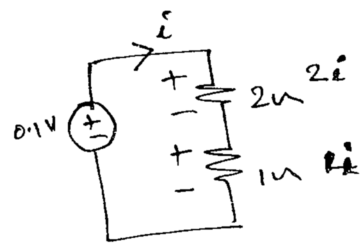
Soln: the induced emf 'e' is given by

$$e = -\frac{d\phi}{dt} = -0.1\frac{d(t)}{dt} = -0.1\ \text{volts}$$

$$\boxed{e = -0.1}\ \text{volts}$$

the Magnitude of induced emf $|e| = 0.1$ volts.

applying KVL to the loop



$$0.1 = 2i + 1i$$

$$0.1 = 3i \Rightarrow i = \frac{0.1}{3}$$

$$\boxed{i = 0.033}\ \text{Amperes.}$$

The voltage across the 2Ω resistor

$$V_{2\Omega} = 2 \times i = 2 \times 0.0333$$

$$\boxed{V_{2\Omega} = 0.0666}\ \text{volts}$$

Ily: the voltage across 1Ω resistor

$$V_{1\Omega} = 1 \times i = 0.0333 \text{ volts}$$

$$\boxed{V_{1\Omega} = 0.0333} \text{ volts}$$

$\Rightarrow$ KVL $\quad e = V_{2\Omega} + V_{1\Omega}$

$$\boxed{0.1 \simeq 0.0666 + 0.033}$$ ✓ Verified.

problem 10

$$\overline{B} = \begin{cases} 4\sin\omega t\,\overline{a_z} ; & \rho \le \rho_0 \\ 0 ; & \rho > 0 \end{cases}$$

06-DEC2009/Jan 2010

The time varying magnetic field in free space is given as $B = \begin{cases} 4\sin\omega t\, a_z & \rho \le \rho_0 \\ 0 & \rho > 0 \end{cases}$

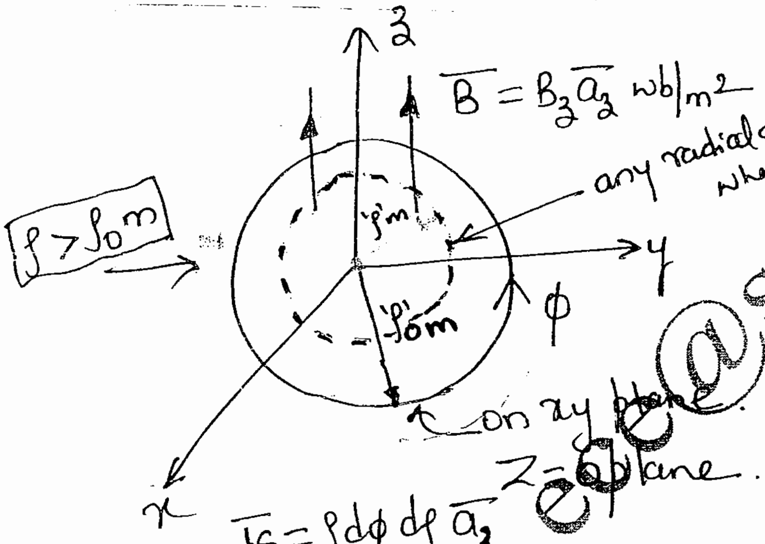
Determine E using Faraday's law. Verify the same using Maxwell's equations. (08 Marks)

$\overline{E}$ same

Solu:- given

$$\overline{B} = \begin{cases} 4\sin(\omega t)\,\overline{a_z} & ; \rho \le \rho_0 \\ 0 & ; \rho > \rho_0 \end{cases}$$

Note:- Should be correction



The total Flux Crossing the Surface

$$\boxed{\phi = \int_{\langle S \rangle} \overline{B} \cdot \overline{dS}} \text{ wb}$$

$\overline{dS} = \rho\, d\phi\, d\rho\, \overline{a_z}$

$$\phi = \int_{\langle S \rangle} 4\sin(\omega t)\,\overline{a_z} \cdot \rho\, d\rho\, d\phi\, \overline{a_z}$$

i) if $\rho \le \rho_0$

$$= 4\sin(\omega t) \int_{\rho=0}^{\rho} \rho\, d\rho \int_{\phi=0}^{2\pi} d\phi \;\; \overline{a_z} \cdot \overline{a_z}^{\,1}$$

$$= 4\sin(\omega t) \cdot \frac{\rho^2}{2}\Big|_0^{\rho} \times 2\pi \times 1$$

$$\phi = 4\sin(\omega t) \times \frac{\rho^2}{\cancel{2}} \times \cancel{2}\pi = 4\pi\rho^2 \sin(\omega t) \text{ wb}.$$

$$\therefore \boxed{\phi = 4\pi\rho^2 \sin(\omega t)} \text{ wb for } \rho \le \rho_0 \text{ m}.$$

ii) if $\rho > \rho_0 m \Rightarrow 0 < \rho < \rho_0$

$$\Rightarrow \phi = \int_{\rho=0}^{\rho_0} \int_{\phi=0}^{2\pi} 4\sin(\omega t)\, \rho\, d\rho\, d\phi$$

$$\phi = 4\sin(\omega t) \int_{\rho=0}^{\rho_0} \rho\, d\rho \int_{\phi=0}^{2\pi} d\phi$$

$$\phi = 4\sin(\omega t) \cdot \frac{\rho^2}{2}\Big|_0^{\rho_0} \times 2\pi$$

$$\phi = 4\sin(\omega t) \cdot \frac{\rho_0^2}{\cancel{2}} \times \cancel{2}\pi$$

$$\therefore \boxed{\phi = 4\pi\rho_0^2 \sin(\omega t)} \text{ wb for } \rho > \rho_0 m$$

$$\therefore \phi = \int_{\langle S\rangle} \overline{B}\cdot\overline{dS} = \begin{cases} 4\pi\rho^2 \sin(\omega t) & ; \rho \le \rho_0\, m \\ 4\pi\rho_0^2 \sin(\omega t) & ; \rho > \rho_0\, m \end{cases}$$

the induced emf

$$e = \frac{-d\phi}{dt}\text{ wb} = \begin{cases} -\frac{d}{dt}\left[4\pi\rho^2 \sin(\omega t)\right] & ; \rho \le \rho_0\, m \\ -\frac{d}{dt}\left[4\pi\rho_0^2 \sin(\omega t)\right] & ; \rho > \rho_0\, m \end{cases}$$

$$\therefore e = \begin{cases} -4\pi\rho^2 \cos(\omega t) \times \omega & ; \rho \le \rho_0 \\ -4\pi\rho_0^2 \cos(\omega t) \times \omega & ; \rho > \rho_0 \end{cases}$$

$$e = \begin{cases} -4\pi\rho^2\omega \cos(\omega t)\ \text{volts} & ; \rho \le \rho_0 \\ -4\pi\rho_0^2\omega \cos(\omega t)\ \text{volts} & ; \rho > \rho_0 \end{cases} \longleftarrow (1)$$

and $e = \oint_{\langle l\rangle} \overline{E_m}\cdot\overline{dl} = \int_{\langle l\rangle} E_\phi \overline{a_\phi} \cdot \rho\, d\phi\, \overline{a_\phi} = E_\phi \rho \int_{\phi=0}^{2\pi} d\phi\ \overline{a_\phi}\cdot\overline{a_\phi}$ ^{1}

along circular path $\overline{dl} = \rho\, d\phi\, \overline{a_\phi}$ m

$$e = \oint_{\langle l\rangle} \overline{E_m}\cdot\overline{dl} = 2\pi\rho E_\phi\ \text{volts} \longleftarrow (2)$$

equating eq^n ① and eq^n ②

i.e

$$\Rightarrow \cancel{2\pi}\rho E_\phi = -\cancel{4\pi}\rho^2 \omega \cos(\omega t) \quad ; \rho \le \rho_0$$

$$\Rightarrow \boxed{E_\phi = -2\omega\rho \cos(\omega t)} \quad ; \rho \le \rho_0 \quad V/m$$

and $2\pi\rho E_\phi = -4\pi\rho_0^2 \omega \cos(\omega t) \; ; \rho > \rho_0$

$$\Rightarrow \boxed{E_\phi = -2\frac{\rho_0^2}{\rho} \omega \cos(\omega t)} \quad ; \rho > \rho_0 \quad V/m$$

$$\therefore \boxed{E_\phi = \begin{cases} -2\omega\rho\cos(\omega t) \; ; \rho \le \rho_0 & V/m \\ -2\frac{\rho_0^2}{\rho}\omega\cos(\omega t) \; ; \rho > \rho_0 & V/m \end{cases}}$$

ii. P-4b :- Verification using Maxwell's eq^n

i.e $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \quad V/m^2$

given

$$\bar{B} = \begin{cases} 4\sin(\omega t)\bar{a}_z \; ; \rho \le \rho_0 m. \\ 0 \; ; \rho > \rho_0 m \end{cases}$$

$$\frac{\partial \bar{B}}{\partial t} = \begin{cases} 4\cos(\omega t)(\omega)\bar{a}_z \; ; \rho \le \rho_0 m \\ 0 \; ; \rho > \rho_0 m \end{cases}$$

35

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} = \begin{cases} -4\omega\cos(\omega t)\,\bar{a_z} & ; \ \rho \le \rho_0 \\ 0 & ; \ \rho > \rho_0 \, m. \end{cases} \quad \text{(a)}$$

the field $\bar{E}$ must be

$$\bar{E} = E_\phi \bar{a_\phi} \ \text{V/m} \ ; \qquad P(\rho, \phi, z) \rightarrow d\rho, \ \rho d\phi, \ dz$$

$$dv = \rho \, d\rho \, d\phi \, dz \ m^3$$

$$\therefore \ \nabla \times \bar{E} = \frac{1}{\rho}\begin{vmatrix} \bar{a_\rho} & \rho\bar{a_\phi} & \bar{a_z} \\ \partial/\partial\rho & 0 & 0 \\ 0 & \rho E_\phi & 0 \end{vmatrix}$$

$$\nabla \times \bar{E} = \frac{1}{\rho}\frac{\partial(\rho E_\phi)}{\partial \rho}\bar{a_z} \quad \text{(b)}$$

equating equation (a) and eq^n (b)

$$\text{ie} \quad \nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

Case i $\rho \le \rho_0$

$$\frac{1}{\rho}\frac{\partial(\rho E_\phi)}{\partial \rho}\bar{a_z} = -4\omega\cos(\omega t)\,\bar{a_z}$$

$$\Rightarrow \frac{1}{\rho}\frac{\partial(\rho E_\phi)}{\partial \rho} = -4\omega\cos(\omega t)$$

$$\frac{\partial(\rho E_\phi)}{\partial \rho} = -4\omega\cos(\omega t)\,\rho$$

Integrating w.r.t 'ρ' on both side

$$\rho E_\phi = -\cancel{4}^{2}\omega \cos(\omega t) \times \frac{\rho^{\cancel{2}}}{\cancel{2}}$$

$$\Rightarrow \boxed{E_\phi = -2\omega\rho\cos(\omega t)} ; \quad \rho \leq \rho_0 m. \quad \leftarrow (c)$$

Case ii. $\rho > \rho_0 m$

$$\Rightarrow \frac{1}{\rho} \cdot \frac{\partial(\rho E_\phi)}{\partial \rho} \bar{a_z} = 0 \cdot \bar{a_z}$$

$$\Rightarrow \frac{\partial(\rho E_\phi)}{\partial \rho} = 0$$

Integrating w.r.t ρ

$$\rho E_\phi = C_1 \Rightarrow \boxed{E_\phi = C_1/\rho} \leftarrow (d)$$

using Boundary Condition i.e

at $\rho = \rho_0 m$; both field's are equal

i.e both eq^n (c) = eq^n (d)

$$-2\omega\rho\cos(\omega t) = C_1/\rho \quad @ \rho = \rho_0 m$$

$$\Rightarrow -2\omega\rho_0\cos(\omega t) = C_1/\rho_0$$

$$\Rightarrow \boxed{C_1 = -2\omega\rho_0^2\cos(\omega t)} \leftarrow (e)$$

using eq^n (e) in eq^n (d)

$$E_\phi = \frac{-2\omega\rho_0^2}{\rho}\cos(\omega t) \;;\; \rho > \rho_0 \text{ m} \quad \leftarrow (f)$$

Finally By combining eq^n (c) & (f)

$$\Rightarrow E_\phi = \begin{cases} -2\omega\rho\cos(\omega t) \text{ V/m} \;;\; \rho \le \rho_0 \text{ m} \\ \dfrac{-2\rho_0^2\omega}{\rho}\cos(\omega t) \text{ V/m} \;;\; \rho > \rho_0 \text{ m} \end{cases} \leftarrow (Y)$$

it is observed that eq^n (X) = eq^n (Y)

∴ Both the method's gives same Ans.

∴ using Maxwell's eq^n Ans. is verified.

Topic 5.2 . Inconsistency of Amperes Law (Modified Amperes Law).

2. ~~Inconsistency of Amperes Law + displacement current + Loss tangent~~

Questions.

02-DEC2008/Jan 2009

What is the inconsistency of Ampere's law with equation of continuity? Derive the modified form of Ampere's law by Maxwell. (06 Marks)

(or)

10-DEC2011/Jan 2012

What is displacement current and equation of continuity? Derive Maxwell's equation for Ampere's circuit law. (06 Marks)

06 - June /July 2012

Define displacement current density. (02 Marks)

06-DEC 2013/Jan 2014

Derive Maxwell's equation from Ampere's law. (06 Marks)

06 - June /July 2011

Modify the Ampere's circuital law to suit the time varying condition. (06 Marks)

02 - June /July 2010

(or)

Derive $\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$. → $\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$.

06 – May/June 2010

Maxwell's.

What do you mean by displacement current and equation of continuity? Derive Maxwell's equation from Ampere's circuital law. (08 Marks)

06-DEC2009/Jan 2010

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Solⁿ:-

The point form of Ampere's Law States that

$$\nabla \times \bar{H} = \bar{J} \quad A/m^2 \leftarrow (1).$$

taking divergence on bothside

$$\nabla \cdot (\nabla \times \bar{H}) = \nabla \cdot \bar{J} \quad A/m^3$$

Note:- According to the vector identity, divergence of a Curl of any vector is zero.

i.e $\quad \nabla\cdot(\nabla\times\bar{A}) = 0$

$\Rightarrow$ Illy $\quad \nabla\cdot(\nabla\times\bar{H}) = 0.$

$\therefore$ eq^n (1) becomes

$$\nabla\cdot(\overset{0}{\cancel{\nabla\times\bar{H}}}) = \nabla\cdot\bar{J} \ A/m^3$$

$$\Rightarrow \quad \nabla\cdot\bar{J} = 0 \ A/m^3 \longleftarrow (2)$$

the above result ie $\nabla\cdot\bar{J} = 0$ is not Consistent with the Continuity Current equation.

i.e $\quad \nabla\cdot\bar{J} = -\frac{\partial \rho_v}{\partial t} \ A/m^3.$

$\therefore$ it is observed that Ampere's Law is inconsistent and Some modification is required in it.

Let Suppose if we add an unknown Vector $\bar{G}$ to eq^n (1)

i.e $\quad \nabla\times\bar{H} = \bar{J} + \bar{G} \longleftarrow (3)$

Now, taking divergence on bothside

$$\nabla\cdot(\cancel{\nabla\times H}) = \nabla\cdot\bar{J} + \nabla\cdot\bar{G} \Rightarrow 0$$

$$\Rightarrow \nabla\cdot\bar{J} + \nabla\cdot\bar{G} = 0$$

$$\nabla\cdot\bar{J} = -\nabla\cdot\bar{G}$$

using $\nabla\cdot\bar{J} = -\frac{\partial\rho_v}{\partial t}$ A/m3 in the above eqn

Continuity Current eqn

$$\Rightarrow -\frac{\partial\rho_v}{\partial t} = -\nabla\cdot\bar{G}$$

$$\Rightarrow \nabla\cdot\bar{G} = \frac{\partial\rho_v}{\partial t} = \dot{\rho}_v \quad \leftarrow (4)$$

w.k.t from point form of Gauss's Law

i.e $\nabla\cdot\bar{D} = \rho_v$

taking differentiation on bothside w.r.t 't'

$$\left[\nabla\cdot\frac{\partial\bar{D}}{\partial t}\right] = \frac{\partial\rho_v}{\partial t}$$

$$(or) \quad \boxed{\nabla\cdot\dot{\bar{D}} = \dot{\rho}_v} \leftarrow (5)$$

using eqn (5) in eqn (4)

i.e $\nabla \cdot \bar{G} = \frac{\partial \rho_v}{\partial t} = \nabla \cdot \frac{\partial \bar{D}}{\partial t}$

By Comparing both sides of above eqn

we can write the unknow vector

$$\boxed{\bar{G} = \frac{\partial \bar{D}}{\partial t} = \dot{\bar{D}}} \quad A/m^2 \text{ or } C/m^2\text{-sec.}$$

∴ Ampere's Law is Modified to

$$\boxed{\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}} \quad A/m^2$$

$$\boxed{\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}} \quad A/m^2 \leftarrow (6)$$

point form of Modified Ampere's Law

$\bar{J}$: A/m^2 $\frac{\partial \bar{D}}{\partial t}$: $C/m^2\text{-sec}$ or A/m^2

both $\bar{J}$ and $\frac{\partial \bar{D}}{\partial t}$ has same units i.e A/m^2 called Current density.

The term $\frac{\partial \bar{D}}{\partial t}$ or $\dot{\bar{D}}$ called Current density and is also called as displacement Current density. while $\bar{J}$ is

Called as Conduction Current density.

Thus the defn of Current density can be formed as

$$\boxed{\overline{J}_{total} = \overline{J}_{conduction} + \overline{J}_{displacement} = \overline{J}_C + \overline{J}_D} \; A/m^2 \quad \text{--- (7)}$$

By Integration of eqn (6) over a Surface

i.e $\int_{\langle S \rangle} (\nabla \times \overline{H}) \cdot \overline{dS} = \int_{\langle S \rangle} \left(\overline{J} + \frac{\partial \overline{D}}{\partial t}\right) \cdot \overline{dS} = \int_{\langle S \rangle} (\overline{J}_C + \overline{J}_D) \cdot \overline{dS}$

using stokes theorem & Ampere's Circuital Law

i.e $\boxed{\oint_{\langle l \rangle} \overline{H} \cdot \overline{dl} = I}$ Ampere's Law

$$I = \oint_{\langle l \rangle} \overline{H} \cdot \overline{dl} = \int_{\langle S \rangle} (\nabla \times \overline{H}) \cdot \overline{dS} \; A$$

the above eqn becomes

$$I = \oint_{\langle l \rangle} \overline{H} \cdot \overline{dl} = \underbrace{\int_{\langle S \rangle} \overline{J}_C \cdot \overline{dS}}_{\text{Conduction Current}} + \underbrace{\int_{\langle S \rangle} \overline{J}_D \cdot \overline{dS}}_{\text{displacement Current.}}$$

the total Current

$$\boxed{|I|_{total} = \sqrt{I_C^2 + I_D^2}} \; \text{Ampere's}$$

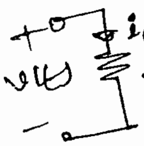
(or) $|i|_{total} = \sqrt{i_C^2 + i_D^2}$ Ampere's

bcz $\overline{J}_D \perp^{r} \overline{J}_D$ its a vector Sum

total Current — Conduction Current — displacement Current

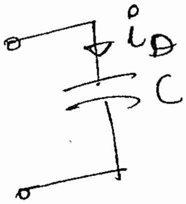
Conduction Current density ($\bar{J}_C$) :- [15-June/July 2017 (2M) CBCS]

the term $\bar{J}_C = \sigma \bar{E}$ A/m² is called Conduction Current density. and Conduction Current $i_C = |J_C| A$ Ampere



i.e $\boxed{i_C = |\bar{J}_C| \cdot A = \int_{\langle S \rangle} \bar{J}_C \cdot \overline{dS}}$ Ampere where A - Area of Conduction

[15-J/J 2017 (2M) CBCS] Displacement Current density ($\bar{J}_D$) :- the term $\bar{J}_D$ is called displacement Current density.



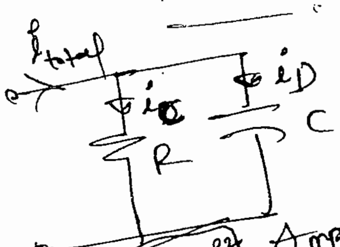
i.e $\boxed{\bar{J}_D = \dot{\bar{D}} = \frac{\partial \bar{D}}{\partial t}}$ ~~A/m²~~ $= \epsilon \frac{\partial \bar{E}}{\partial t}$ A/m²

and Displacement Current $\boxed{i_D = |\bar{J}_D| \cdot \text{Area}}$ Ampere

$\boxed{i_D = \frac{|\bar{J}_D|}{\text{Area}} = \int_{\langle S \rangle} \bar{J}_D \cdot \overline{dS}}$ Ampere $i_D = |\bar{J}_D| \cdot A$; Ampere.

Total Current density :- $\bar{J}_{total} = \bar{J}_C + \bar{J}_D$ A/m²

$|J_{total}| = \sqrt{J_C^2 + J_D^2}$



$$\bar{J}_{total} = \sigma \bar{E} + \frac{\partial \bar{D}}{\partial t} \text{ A/m}^2$$

$\|^{ly}$ $\boxed{i_{total} = \sqrt{i_C^2 + i_D^2}}$

$\boxed{i_{total} = \sqrt{i_C^2 + i_D^2}}$ Ampere

$$\bar{J}_{total} = \sigma \bar{E} + \epsilon \frac{\partial \bar{E}}{\partial t} \text{ A/m}^2$$

$\boxed{i_{total} = \sqrt{i_C^2 + i_D^2}}$ Ampere

vector addⁿ by $i_C \perp^r i_D$ (or) $J_C \perp^r J_D$.

Note :- Time domain TD $\quad$ FD (frequency domain)

$$f(t) \xleftrightarrow{F.T} F(\omega)$$

$$\partial/\partial t [f(t)] \xleftrightarrow{F.T} j\omega F(\omega)$$

$$\therefore \partial/\partial t \xleftrightarrow{F.T} j\omega$$

$$\therefore \bar{J}_{total} = \sigma \bar{E} + j\omega\epsilon \bar{E} \text{ A/m}^2$$

(or) $\boxed{\bar{J}_{total} = (\sigma + j\omega\epsilon) \bar{E}}$ A/m²

Topic 5.2a Concept of Conduction and displacement densities

from point form Ampere's Law $\nabla \times \overline{H} = \overline{J}_c$ A/m^2 ← (1)

the modified Ampere's Law

i.e $\nabla \times \overline{H} = \overline{J}_c + \frac{\partial \overline{D}}{\partial t}$ A/m^2 ← (2)

This is Maxwell's equation (based on Ampere's Circuital Law) for a time-varying field.

the term $\overline{J}_d = \frac{\partial \overline{D}}{\partial t}$ A/m^2 is known as displacement Current density. and $\overline{J}_c$ is the condition Current density.

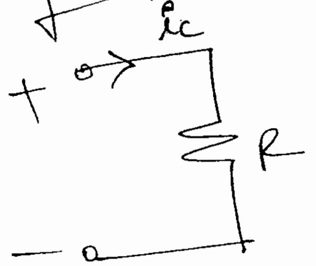
i.e $\overline{J}_c = \sigma \overline{E}$ A/m^2 --- also called point form of ohm's Law.

$\overline{J}_c$ → A/m^2, σ → ℧/m, E → V/m

℧ = A/V ⇒ $\boxed{V = IR}$ volt's.

⇒ $\boxed{℧ = \text{A/V}}$ the Conduction Current density is due to motion of free electron's.

$\overline{J}_c = \sigma \overline{E}$ A/m^2 ⇒ $|\overline{J}_c| = \frac{i_c}{A}$ A/m^2



the Conduction Current

$\boxed{i_c = |\overline{J}_c| \times A}$ Ampere's

The insertion of $\overline{J}_D$ into eqn (1) was one of the major contributions of Maxwell.

Without the term $\overline{J_D}$, the propogation of electromagnetic waves (Eg. radio (or) TV waves) would be impossible.

at Low frequencies, $\overline{J_D}$ is usually neglected compared with $\overline{J_C}$.

* However, at radio frequencies, the two terms (ie $\overline{J_D}$ & $\overline{J_C}$) are comparable.

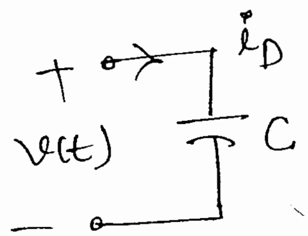
* Displacement Current is a result of time-varying electric field. A typical example of such current is the current through a Capacitor when an alternating voltage source is applied to its plates.

$$\overline{J_D} = \frac{\partial \overline{D}}{\partial t} \ A/m^2 \quad \text{and} \quad \boxed{i_D = |\overline{J_D}|\, A} \ \text{Amperes}$$

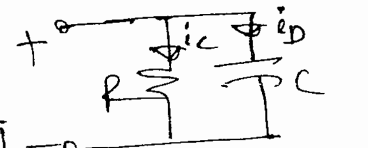
(or) $\boxed{|\overline{J_D}| = i_D / A} \ A/m^2$

Mag of $\overline{J_D}$

$$J_D = \frac{\partial D}{\partial t} \ A/m^2$$



$$\Rightarrow J_D = \frac{\partial(\epsilon E)}{\partial t}$$



$$J_D = \epsilon \frac{\partial E}{\partial t}$$

$\overline{J}_{total} = \overline{J_C} + \overline{J_D} \ A/m^2$

and $E = \frac{v(t)}{d}$ v/m ..for Sinusoidal field

$\boxed{i_{total} = \sqrt{i_C^2 + i_D^2}}$ A

vector sum b/w $\overline{J_C} \perp^r \overline{J_D}$

$$\Rightarrow \boxed{J_D = \frac{\epsilon}{d} \frac{\partial v(t)}{dt}} \ A/m^2$$

the Displacement Current $i_D = J_D \times A = \underbrace{\frac{\epsilon A}{d}}_{C} \frac{\partial v(t)}{dt}$

$$\Rightarrow \boxed{i_D = C\frac{dv(t)}{dt}} \ \text{Amperes}$$

∴ Displacement Current is nothing but Current in a Capacitor.

problem 11

$\to 50\sin 10^3 t$ volts $\quad \swarrow 5\,cm^2$

06 –Dec/Jan 2008

44 A parallel plate capacitor with plate area 5 cm² and plate separation of 3 mm has a voltage of 50 sin 10³ t volts applied to its plates. Calculate the displacement current assuming $\epsilon = 2\,\epsilon_0$. $\leftarrow \epsilon = 2\epsilon_0\, F/m$.

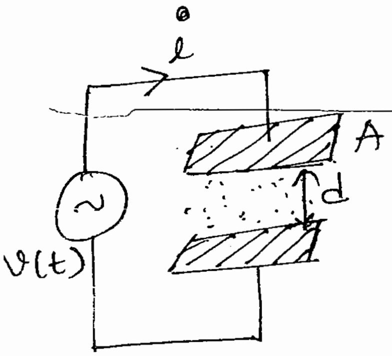
(08 Marks)

solu:- given Area $= 5\,cm^2 = 5\times(10^{-2})^2\, m^2$

$= 5\times10^{-4}\, m^2$.

$d = 3mm = 3\times10^{-3}\,m$. $v(t) = 50\sin(10^3 t)$ volt's

$\epsilon = 2\epsilon_0 \quad i_d = ?$



the displacement current i_d is given by

$\epsilon = 2\epsilon_0$

$$\boxed{i_D = |\bar{J}_D|\cdot A} \text{ Ampere's}$$

In capacitor $i_C = i_D = \dfrac{C\,dv(t)}{dt}$

$= \dfrac{\epsilon A}{d}\dfrac{dv(t)}{dt}$ Amp

$$\bar{J}_D = \frac{\partial \bar{D}}{\partial t} = \epsilon\frac{\partial \bar{E}}{\partial t}\ A/m^2 \quad \& \quad |\bar{E}| = \frac{v(t)}{d}\ v/m$$

$$|\bar{J}_D| = \frac{\epsilon}{d}\frac{dv(t)}{dt}\ A/m^2$$

$$\therefore |\bar{J}_D| = \frac{\epsilon}{d}\frac{d}{dt}\left[50\sin(10^3t)\right]$$

$$|\bar{J}_D| = \frac{\epsilon}{d}\times 50\cos(10^3t)\times10^3\ A/m^2$$

$$i_D = |\bar{J}_D|\times \text{Area of Cross Section} = \frac{50\epsilon\times10^3}{d}\cos(10^3t)\times \text{Area}$$

$$= \frac{50\times2\times8.854\times10^{-12}\times10^3}{3\times10^{-3}}\cos(10^3t)\times5\times10^{-4}$$

$$\boxed{i_D = 0.14756\cos(10^3t)\ \mu\text{ Ampere's}}$$

(or)

$$\boxed{i_D = 147.566\cos(10^3t)\ nA}$$

(47) 424-A

Topic 5.b Loss tangent (or) dissipation factor of dielectric material :-

10-DEC2011/Jan 2012

Question

What is loss tangent? Explain its practical importance. (06 Marks)

Sol^n :- w.k.t the Conduction Current density

$$\bar{J_c} = \sigma \bar{E} \quad A/m^2$$

$$|\bar{J_c}| = \sigma |\bar{E}| \xleftarrow{A/m^2} (1)$$

and displacement Current density $\bar{J_D}$

$$\bar{J_D} = \frac{\partial \bar{D}}{\partial t} = \epsilon \frac{\partial \bar{E}}{\partial t} = j\omega\epsilon \bar{E} \quad A/m^2$$

$$|\bar{J_D}| = \omega \epsilon |\bar{E}| \quad A/m^2 \quad (2)$$

Note:-

$j = e^{j\pi/2}$

$|j| = |e^{j\pi/2}| = 1$

$e^{j\pi/2} = \cos\pi/2 + j\sin\pi/2$

$|j| = \sqrt{0+1^2} = \sqrt{1} = 1$

Loss tangent :- the ratio of Magnitude of Conduction Current density to the displacement Current density is nothing but Loss tangent.

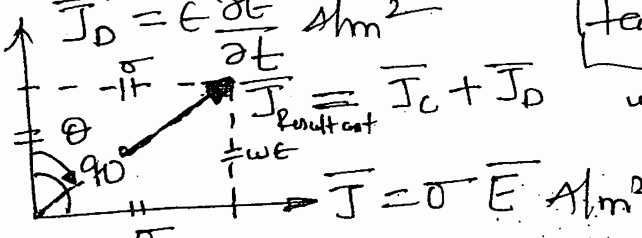
i.e $$\left|\frac{\bar{J_c}}{\bar{J_D}}\right| = \frac{\sigma |\cancel{\bar{E}}|}{\omega\epsilon|\cancel{\bar{E}}|} = \frac{\sigma}{\omega\epsilon}$$

i.e $$\boxed{\frac{|\bar{J_c}|}{|\bar{J_D}|} = \frac{\sigma}{\omega\epsilon}}$$ ← Loss tangent (or) dissipation factor of the dielectric materials.

thus the ratio of the Magnitude of the Conduction Current density to the displacement Current density depends on the properties of the medium σ, ϵ and frequency (ω)

Note :- the Conduction Current density $\overline{J_c} = \sigma \overline{E}$ and displacement Current density $\overline{J_D} = \epsilon \frac{\partial E}{\partial t}$ A/m² are perpendicular ($\perp^r$) to each other.

$\overline{J_D} = \epsilon \frac{\partial E}{\partial t}$ A/m²



i.e $\boxed{\overline{J_{total}} = \overline{J_c} + \overline{J_D}}$ A/m²

$i_{total} = \sqrt{i_c^2 + i_D^2}$ Amperes

$\boxed{\tan\theta = \frac{opp}{adj} = \frac{\sigma}{\omega\epsilon}}$

where θ - Loss angle

Vector sum. $|\overline{J_t}| = \sqrt{J_c^2 + J_D^2}$ A/m

$\boxed{i_t = \sqrt{i_c^2 + i_D^2}}$ Amperes

Significance of Loss tangent $\left(\frac{\sigma}{\omega\epsilon}\right)$:-

i. for a Conductors the value of Conductivity (σ) is very Large

∴ The Condion Current in very Large Compared to displacement Current.

i.e if $\frac{\sigma}{\omega\epsilon} >> 1 \Rightarrow \boxed{\frac{\sigma}{\omega\epsilon} >> 1}$ ← indicates that given medium in Conducting

Eg :- Silver $\sigma = 6.17 \times 10^7$ ℧/m

Copper $\sigma = 5.8 \times 10^7$ ℧/m.

Gold $\sigma = 4.1 \times 10^7$ ℧/m

Aluminum $\sigma = 3.82 \times 10^7$ ℧/m.

i.e $\frac{|\overline{J_c}|}{|\overline{J_D}|} >> 1 \Rightarrow |\overline{J_c}| >> |\overline{J_D}|$

(or) $\sigma >> \omega\epsilon$

ii. In case of dielectric Medium 'σ' in very Small.

∴ the displacement Current is greater than Compare to the Condution Current.

i.e $\frac{|\overline{J_c}|}{|\overline{J_D}|} = \frac{\sigma}{\omega\epsilon} << 1 \Rightarrow$ $|\overline{J_c}| << |\overline{J_D}|$ (or) $\boxed{\sigma << \omega\epsilon}$

Eg :- Teflon, Lime stone, Soil, etc.

iii. if $\left(\frac{\sigma}{\omega\epsilon}\right) \longrightarrow 0$ i.e $\left|\frac{\overline{J_c}}{\overline{J_D}}\right| \rightarrow 0$ then the given Medium in Called perfect dielectric Medium.

$\boxed{\frac{\sigma}{\omega\epsilon} \simeq 0}$ Eg :- Diamond, Insulator, quartz, marble etc.

Topic 5.2C Continuity Current Equation from Maxwell's equation.

Questions With usual notations, derive the differential form of continuity equation from the Maxwell's equations. (04 Marks)

(or) 10-DEC 2015/Jan 2016

d. Derive continuity equation from Maxwell's equation. (05 Marks)

Sol:- i.e $\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t}$ A/m³ using Maxwell's eqn.
Continuity eqn

from generalized Ampere's Circuital Law (or) Modified Ampere's Law

$$\boxed{\nabla \times \bar{H} = \bar{J}_c + \frac{\partial \bar{D}}{\partial t}} \text{ A/m}^2 \quad \text{(1)}$$

taking divergence on both side

$$\nabla \cdot (\nabla \times \bar{H}) = \nabla \cdot \bar{J} + \nabla \cdot \left(\frac{\partial \bar{D}}{\partial t}\right)$$

0 (using vector identity) i.e $\nabla \cdot (\nabla \times \bar{A}) = 0$ any vector $\bar{A}$.

$$\Rightarrow 0 = \nabla \cdot \bar{J} + \nabla \cdot \left(\frac{\partial \bar{D}}{\partial t}\right)$$

$$\nabla \cdot \bar{J} = -\nabla \cdot \left(\frac{\partial \bar{D}}{\partial t}\right)$$

$$\nabla \cdot \bar{J} = -\frac{\partial}{\partial t}(\nabla \cdot \bar{D}) \text{ A/m}^3$$

using point form of Gauss's Law $\nabla \cdot \bar{D} = \rho_v$ C/m³ i.e Maxwell's first eqn (electrostatic)

the above eqn becomes

$$\Rightarrow \boxed{\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t}} \text{ A/m}^3$$

(50) Differential form of Continuity eqn from the Maxwell's eqn.

Topic 5.2d Conduction and Displacement Current in a Capacitor.

Questions

02-DEC2010

For a time varying field, having a capacitor, show that, the conduction current is equal to the displacement current. (04 Marks) (4m)

(or)

02-DEC2008/Jan 2009

Justify that for the case of a parallel plate capacitor the displacement current is equivalent to conduction current. Comment on the ratio of magnitudes of conduction current density to displacement current density. (06 Marks)

(or)

10-DEC 2013/Jan 2014

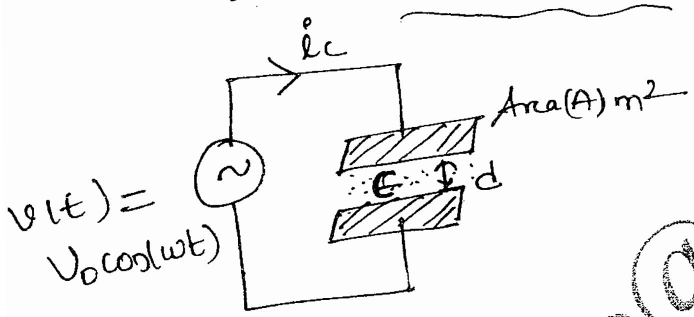
Show that, in a capacitor the conduction current density is equal to displacement current density for the applied voltage of $v(t) = v_0 \cos wt$. (10 Marks)

(or)

10-DEC2011/Jan 2012

Q. Explain Concept of displacement Current in Capacitor ... (04 Marks) 06-Jul 2012 → 2012.

Sol: a. Conduction Current in Capacitor 'C'



Consider a parallel plate Capacitor connected across an A.C Source of voltage

$$v(t) = V_0 \cos(wt) \text{ volts} \leftarrow (1)$$

Let, the Area of each of the plate be 'A' m² and their distance of separation 'd' m.

The Capacitance b/w the parallel plates is given by

$$C = \frac{\epsilon A}{d} \text{ Farad's} \leftarrow (2)$$

the Conduction Current

$$I_c = C\frac{dv(t)}{dt} \text{ Ampere's} \leftarrow (3)$$

using eqn (1) in eqn (3)

$$I_c = \frac{\epsilon A}{d} \frac{d}{dt}\left[V_0 \cos(\omega t)\right]$$

$$I_c = \frac{\epsilon A V_0}{d}\left[-\sin(\omega t) \times \omega\right]$$

$$\therefore \boxed{I_c = -\frac{\epsilon A V_0 \omega}{d} \sin(\omega t)} \text{ Amperes} \quad (4)$$

~~Mag of Conduction Current density $|\bar{J}_c|$~~

Ily

$$\boxed{|\bar{J}_c| = \frac{I_c}{A} = \frac{-\epsilon V_0 \omega}{d} \sin(\omega t)} \text{ Ampere}/m^2 \quad (4a)$$

b. Displacement Current:- (I_D)

w.k.t

$$\bar{D} = \epsilon \bar{E} \quad C/m^2$$

and the applied voltage

$$v(t) = V_0 \cos(\omega t) \quad \text{volt's}$$

$$|\bar{E}| = \frac{v(t)}{d} = \frac{V_0}{d}\cos(\omega t) \quad v/m$$

$$\therefore |\bar{D}| = \epsilon|\bar{E}| = \frac{\epsilon V_0}{d}\cos(\omega t) \quad v/m.$$

the displacement Current density is given by

$$\bar{J}_D = \frac{\partial \bar{D}}{\partial t} \quad A/m^2 \quad (a) \qquad |\bar{J}_D| = \left|\frac{\partial \bar{D}}{\partial t}\right| \quad A/m^2$$

$$|\bar{J}_D| = \frac{\partial |\bar{D}|}{\partial t} = \frac{\partial}{\partial t}\left[\frac{\epsilon V_0}{d}\cos(\omega t)\right]$$

$$= \frac{\epsilon V_0}{d} \times -\sin(\omega t) \times \omega$$

$$\therefore \quad |\bar{J}_D| = -\frac{\epsilon V_0 \omega}{d}\sin(\omega t) \quad A/m^2$$

and the displacement Current i_D

$$i_D = |\bar{J}_D| \times \text{Area of Cross section}(A)$$

$$\boxed{i_D = |\bar{J}_D|\, A} \quad \text{Ampere's}$$

$$\therefore \boxed{i_D = \frac{-\epsilon V_0 \omega A}{d}\sin(\omega t)} \quad A \quad \leftarrow (5)$$

and Mag of displacement Current density $|\bar{J}_D|$

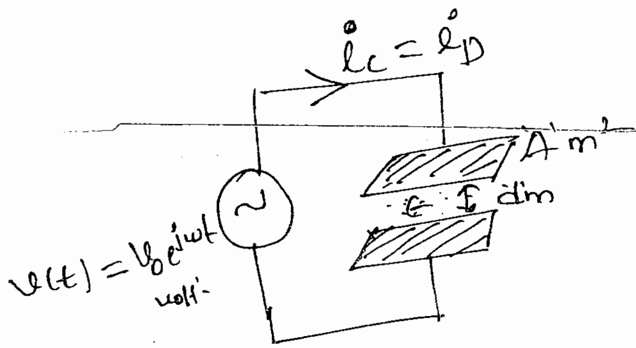
$$\boxed{|\bar{J}_D| = i_D/A = -\frac{\epsilon V_0 \omega}{d}\sin(\omega t)} \leftarrow (5a)$$

From eq^n (4) and (5) | it is observed that In a Capacitor
Ily eq^n (4a) and eq^n (5a) | the Conduction and displacement Current and Current densities are equal.

Key Note:- if we consider the applied voltage across the parallel plate capacitor is to be complex exponential signal i.e

$\boxed{v(t) = V_0 e^{j\omega t}}$ volt's then

the conduction and displacement current & current density are given below



$$\boxed{i_C = i_D = \frac{j\omega\epsilon A}{d} \cdot V_0 e^{j\omega t}} \text{ Ampere's.}$$

and

$|\bar{J}_C| = i/A$ A/m^2

$$\boxed{|\bar{J}_C| = |\bar{J}_D| = \frac{j\omega\epsilon V_0}{d} e^{j\omega t}} \ A/m^2$$

or

$$\boxed{|\bar{J}_C| = |\bar{J}_D| = \frac{\omega\epsilon V_0}{d}} \ A/m^2$$

bcz $|j| = 1$ & $|e^{j\omega t}| = 1$

Key Note:-

Loss tangent $\left(\frac{\sigma}{\omega\varepsilon}\right)$

I. Good Conductor

$\left(\frac{\sigma}{\omega\varepsilon} >> 1\right)$

Eg:- Silver $\sigma = 6.17 \times 10^7$ ℧/m

Copper, $\sigma = 5.8 \times 10^7$ ℧/m

Gold, $\sigma = 4.10 \times 10^7$ ℧/m

Aluminium $\sigma = 3.82 \times 10^7$ ℧/m

Tungsten $\sigma = 1.82 \times 10^7$ ℧/m

Zinc $\sigma = 1.67 \times 10^7$ ℧/m

Brass $\sigma = 1.5 \times 10^7$ ℧/m

Nickel $\sigma = 1.45 \times 10^7$

Iron $\sigma = 1.03 \times 10^7$

etc.

II. Good dielectrics (or) Low Loss dielectrics

$\left(\frac{\sigma}{\omega\varepsilon} << 1\right)$

Note practically

Eg:- $\boxed{\frac{\sigma}{\omega\varepsilon} << 0.01}$

Teflon,

Limestone $\sigma = 10^{-2}$ ℧/m

Freshwater $\sigma = 10^{-3}$ ℧/m.

etc.

III. $\frac{\sigma}{\omega\varepsilon} \simeq 0$

i.e. $\frac{\sigma}{\omega\varepsilon} \to 0$

first

Perfect Dielectrics (Loss Less dielectrics)

Eg:- all Insulators

i.e

Soil $\sigma = 10^{-5}$ ℧/m

Granite $\sigma = 10^{-6}$ ℧/m

Marble $\sigma = 10^{-8}$ ℧/m

Bakelite $\sigma = 10^{-9}$ ℧/m

porcelain $\sigma = 10^{-10}$ ℧/m

Diamond $\sigma = 2 \times 10^{-13}$ ℧/m

polystyrene $\sigma = 10^{-16}$ ℧/m

Quartz $\sigma = 10^{-17}$ ℧/m.

$$\overline{J_D} = -10^6 \times -\sin(377t + 1.2566\times10^6 z) \times 1.2566\times10^6 \,\overline{a_x}\, A/m^2$$

$$\overline{J_D} = +1.2566\times10^{12} \sin(377t + 1.2566\times10^6 z)\,\overline{a_x}\, A/m^2$$

the magnitude (or Amplitude) of $\overline{J_D}$

i.e $\boxed{|\overline{J_D}| = 1.2566\times10^{12}\, A/m^2}$

Note:- $|\sin(\omega t+\theta)|_{max} = 1$

ie Maximum value of Sin | cos f^n is one.

the vector $\overline{J_D}$ is given by

$$\boxed{\begin{array}{l}\overline{J_D} = 1.2566\times10^{12} \sin(377t + 1.2566\times10^6 z)\,\overline{a_x}\, A/m^2 \\ \text{or} \\ \text{amplitude of } \overline{J_D} \quad |\overline{J_D}| = 1.2566\times10^{12}\, A/m^2\end{array}}$$

Obs:- In free space $|\overline{J_D}| >> \cancel{|\overline{J_C}|} \to 0$ bcz freespace.

problem 14 $J_c = 0.02\sin(10^9 t)\ A/m^2$.

In a given lossy dielectric medium, conduction current density $J_c = 0.02\sin(10^9 t)$ A/m^2. Find the displacement current density if $\sigma = 10^3\ s/m$ and $\epsilon_r = 6.5$ J/J-2012

(6M)

Sol'n:- for a Lossy dielectric Medium $\boxed{\frac{\sigma}{\omega\epsilon} >> 1}$

given

$\bar{J}_c = 0.02\sin(10^9 t)\ A/m^2$; $|J_c| = 0.02\ A/m^2$

given $\sigma = 10^3\ S/m$ and $\epsilon_r = 6.5\ F/m$

$\omega = 10^9\ r/sec$

$\frac{\sigma}{\omega\epsilon} = \frac{10^3}{10^9 \times 6.5 \times 8.854 \times 10^{-12}}$

$\frac{\sigma}{\omega\epsilon} = 17375.89 >>> 1$

$\Rightarrow \boxed{\sigma >> \omega\epsilon}$

$$\frac{|\bar{J}_c|}{|\bar{J}_D|} = \frac{\sigma}{\omega\epsilon}$$

$$|\bar{J}_D| = \frac{\omega\epsilon|\bar{J}_c|}{\sigma} = \frac{10^9 \times 6.5 \times 8.854 \times 10^{-12} \times 0.02}{10^3}$$

$$\boxed{|\bar{J}_D| = 1.151 \times 10^{-6}}\ A/m^2$$

(or) $1.151\ \mu A/m^2$.

w.k.t $\bar{J}_c$ and $\bar{J}_D$ are $\perp^r$ to each other (or) right angles to each other

$$\therefore \boxed{\bar{J}_D = 1.151\cos(10^9 t)}\ \mu A/m^2$$

b/c given $\bar{J}_c$ is in Sin $\therefore$ $\bar{J}_D$ must be in Cos b/c $\bar{J}_c \perp^r \bar{J}_D$

Obs:- In Lossy dielectric Medium

$|\bar{J}_c| >> |\bar{J}_D|$

$\therefore$ the medium is Conducting Medium.

problem 15

- Show that for a Sinusoidal varying field the conduction current and the displacement current are always displaced by 90^o *in phase.*

Soln:- i.e S.T $\overline{J_c} \perp^{r} \overline{J_D}$ (or) $i_c \perp^{r} i_D$

let us Consider a Sinusoidal varying field

$$E = E_m \cos(\omega t) \text{ volt's}/m \leftarrow (1)$$

the Conduction Current $|\overline{J_c}| = \sigma |E| \; A/m^2$

$$\boxed{i_c = |\overline{J_c}| \cdot A = J_c \cdot A} \text{ Ampere's}$$

i.e $i_c = \sigma E_m \cos(\omega t) A$ Ampere's

(or)

$$\boxed{i_c = \sigma A E_m \cos(\omega t)} \text{ Ampere's} \leftarrow (2)$$

$$J_c = \frac{i_c}{A} = \sigma E_m \cos \omega t \; A/m^2 \leftarrow (2a)$$

the displacement Current

$$i_d = J_D \cdot A = \frac{\partial D}{\partial t} A$$

$$i_d = \epsilon A \frac{\partial E}{\partial t} = A \epsilon \frac{\partial}{\partial t} [E_m \cos(\omega t)]$$

$$i_d = -A E_m \omega \epsilon \sin(\omega t) \text{ Ampere's}$$

(or)

$$\boxed{i_d = A E_m \omega \epsilon \cos(\omega t + \pi/2)} \leftarrow (3) \text{ Ampere's}$$

$$J_D = \frac{i_d}{A} = E_m \omega \epsilon \cos(\omega t + \pi/2) \; A/m^2 \leftarrow (3a)$$

by observing eqn (2) & (3) lly (2a) & (3a) i_c and i_D are displaced in phase by $90°$.

$\Rightarrow$ and also $|\overline{J_c}|$ & $|\overline{J_D}|$ are also $\perp^{r}$ to Each other. $\overline{J_c} \perp^{r} \overline{J_D}$. (60)

problem 16

Show that for a Sinusoidal varying field the conduction and the displacement current densities are always displaced by 90^o *in phase*.

soln: from ~~(eqn)~~ soln (contd) problem 15

$\Rightarrow$ from eqn (2a)

i.e $|\bar{J}_C| = \sigma E_m \cos(\omega t)$ A/m^2

and eqn (3a)

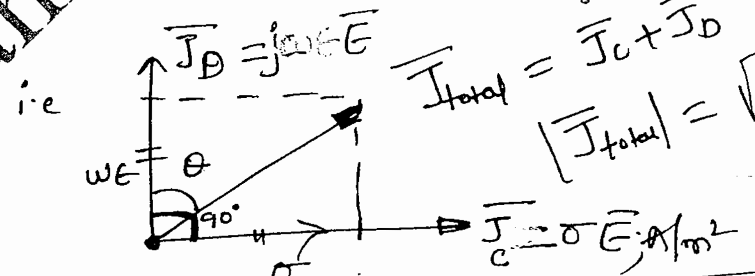
$$|\bar{J}_D| = E_m \omega \epsilon \cos(\omega t + \pi/2) \text{ A/m}^2$$

it is clear that both conduction and displacement current densities are always displaced by 90° in phase.

$$\bar{J}_C \perp^r \bar{J}_D$$

(or) $|\bar{J}_C| \perp^r |\bar{J}_D|$.

i.e



$\bar{J}_{total} = \bar{J}_C + \bar{J}_D$

$|\bar{J}_{total}| = \sqrt{J_C^2 + J_D^2}$ A/m^2

$$\frac{|i_{total}|}{A} = \sqrt{\left(\frac{i_C}{A}\right)^2 + \left(\frac{i_D}{A}\right)^2}$$

$$\Rightarrow \frac{i_{total}}{\cancel{A}} = \frac{\sqrt{i_C^2 + i_D^2}}{\cancel{A}}$$

(61) $\Rightarrow \boxed{i_{total} = \sqrt{i_C^2 + i_D^2}}$ Ampere's.

problem 17 $\left|\frac{J_d}{J_c}\right| = \frac{\omega\epsilon}{\sigma}$; $\sigma = 10^5$ S/m. 50Hz 50MHz

Show that $\left|\frac{J_d}{J_c}\right| = \frac{\omega\epsilon}{\sigma}$; and calculate its value for aluminum at frequency of 50Hz and 50MHz given the Conductivity $\sigma = 10^5 \frac{S}{m}$. Jan-2009 (6M)

Solu:- $|\bar{J_c}| = \sigma|\bar{E}|$ A/m² and

$|\bar{J_D}| = \omega\epsilon|\bar{E}|$ A/m².

$$\Rightarrow \left|\frac{\bar{J_D}}{\bar{J_c}}\right| = \frac{\omega\epsilon\cancel{|\bar{E}|}}{\sigma\cancel{|\bar{E}|}} = \frac{\omega\epsilon}{\sigma} \Rightarrow \boxed{\left|\frac{\bar{J_D}}{\bar{J_c}}\right| = \frac{\omega\epsilon}{\sigma}}$$

a. @ $f = 50$Hz, $\sigma = 10^5$ S/m

$$\left|\frac{\bar{J_D}}{\bar{J_c}}\right| = \frac{2\pi f\epsilon_0\epsilon_r}{\sigma} = \frac{2\pi\times 50\times 8.854\times 10^{-12}\times 1}{10^5}$$

$$\boxed{\left|\frac{\bar{J_D}}{\bar{J_c}}\right| = \frac{\omega\epsilon}{\sigma} = 2.7815\times 10^{-14}}$$

$$\Rightarrow \frac{\sigma}{\omega\epsilon} = 3.595\times 10^{13}$$

b. @ $f = 50$MHz

$$\left|\frac{\bar{J_D}}{\bar{J_c}}\right| = \frac{2\pi f\epsilon_0\epsilon_r}{\sigma} = \frac{2\pi\times 50\times 10^6\times 8.854\times 10^{-12}\times 1}{10^5}$$

$$\boxed{\left|\frac{\bar{J_D}}{\bar{J_c}}\right| = 2.7815\times 10^{-8}}$$

$$\Rightarrow \frac{\sigma}{\omega\epsilon} = 35.95096\times 10^6$$

obs:- both the frequencies the medium to be ~~good~~ conductor / good Conducting medium bcz $\left(\frac{\sigma}{\omega\epsilon} >> 1\right)$.

$$\Rightarrow |\bar{J_c}| >> |\bar{J_D}| \text{ (or) } |i_c| >> |i_D|$$

Problem 18

18._Wet marshy soil is characterized by $\sigma = 10^{-2} s/m$, ε_r=15 and $\mu_r = 1$ at frequencies 60Hz, 1M Hz, 100M Hz, and 10G Hz indicate whether the soil may be considered, a dielectric or neither. (10M)—June2012.

Solu:- W.k.t the distinguishing conditions are

1. for good conductor $\frac{\sigma}{\omega\varepsilon} >> 1$.
2. For a perfect dielectrics $\frac{\sigma}{\omega\varepsilon} \to 0$.
3. for a good dielectrics $\frac{\sigma}{\omega\varepsilon} << 1$.

i) ~~at frequency~~ $f = 60\,Hz$.

$\sigma = 10^{-2}\, S/m$, $\varepsilon_r = 15\, F/m$, $\mu_r = 1\, H/m$.

$$\frac{\sigma}{2\pi f \varepsilon_0 \varepsilon_r} = \frac{10^{-2}}{2\pi \times 60 \times 8.854 \times 10^{-12} \times 15}$$

$$\frac{\sigma}{\omega\varepsilon} = 1.9972 \times 10^{5} \simeq 2 \times 10^{5} >> 1$$

$$\Rightarrow \boxed{\frac{\sigma}{\omega\varepsilon} >> 1}$$

at $f = 60\,Hz$
∴ the given medium in Considered to be good Conductor.

ii) at frequency $f = 1\,MHz$.

$$\frac{\sigma}{\omega\varepsilon} = \frac{\sigma}{2\pi f \varepsilon_0 \varepsilon_r} = \left(\frac{\sigma}{2\pi \varepsilon_0 \varepsilon_r}\right) \times \frac{1}{f}$$

$$= 12 \times \cancel{10^{6}} \times \left(\frac{1}{1 \times \cancel{10^{6}}}\right)$$

$$= 12$$

$$\frac{\sigma}{\omega\epsilon} = 12 > 1$$

$$\Rightarrow \boxed{\frac{\sigma}{\omega\epsilon} > 1}$$

$\therefore$ at $f = 1\,MHz$ the given medium is considered to be Good conductor.

iii) at $f = 100\,MHz$.

$$\frac{\sigma}{\omega\epsilon} = \left(\frac{\sigma}{2\pi\epsilon_0\epsilon_r}\right)\frac{1}{f} = 12\times10^6 \times \frac{1}{100M}$$

$$\frac{\sigma}{\omega\epsilon} = 0.12 < 1$$

$$\Rightarrow \boxed{\frac{\sigma}{\omega\epsilon} < 1}$$ $\therefore$ at $f = 100\,MHz$ the given medium is considered to be Good dielectric.

iv) at $f = 10\,GHz$.

$$\frac{\sigma}{\omega\epsilon} = 12\times10^6 \times \frac{1}{10G} = 1.2\times10^{-3} <<< 1$$

i.e $\frac{\sigma}{\omega\epsilon} \simeq 0 \Rightarrow \left(\frac{\sigma}{\omega\epsilon}\right) \to 0$

$\therefore$ at $f = 10\,GHz$ the given medium is considered to be perfect dielectric.

problem 19

Show that the ratio of amplitude of conduction current density and displacement current density is $\frac{\sigma}{\omega \epsilon}$ for an applied field $E = E_0 \cos(\omega t)\, v/m$ assume $\epsilon = \epsilon_0$

Sol^n:- given $E = E_0 \cos(\omega t)$ v/m
and $\epsilon = \epsilon_0$ F/m.

the conduction current density $\overline{J_c} = \sigma \overline{E}$ A/m²

$|\overline{J_c}| = \sigma |\overline{E}|$ A/m² ; $J_c = \sigma E$ ~~A/m²~~

$\therefore J_c = \sigma E_0 \cos(\omega t)$ A/m² ← (1)

the displacement current density $\overline{J_D}$ is given by

$$\overline{J_D} = \frac{\partial D}{\partial t} \text{ A/m}^2$$

$$\overline{J_D} = \epsilon \frac{\partial E}{\partial t} \text{ A/m}^2$$

$$J_D = \left[\epsilon \frac{\partial E}{\partial t}\right] \text{ A/m}^2$$

$$J_D = \left[\epsilon \cdot \frac{\partial}{\partial t}\left[E_0 \cos(\omega t)\right]\right] \text{ A/m}^2$$

$$= \left[\epsilon E_0 \left[-\sin(\omega t)\right]\right] \times \omega \text{ A/m}^2$$

$$= -\epsilon E_0 \sin(\omega t) \times \omega \text{ A/m}^2$$

$$J_D = \epsilon E_0 \omega \cos(\omega t + \pi/2) \text{ A/m}^2 \leftarrow (2)$$

the ratio of amplitude of conduction current density & J_D is

$$\left|\frac{J_c}{J_D}\right| = \left|\frac{\sigma \cancel{E_0} \cos(\omega t)}{\omega \epsilon \cancel{E_0} \cos(\omega t + \pi/2)}\right| = \frac{\sigma}{\omega \epsilon}$$

$$\boxed{\left|\frac{J_c}{J_D}\right| = \frac{\sigma}{\omega \epsilon}}$$

Note:- $|\cos(\omega t)| = 1$ and $|\cos(\omega t + \pi/2)| = 1$

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problem 20: $\overline{H} = H_m e^{j(\omega t+\beta z)} \overline{a_x}$ A/m.

10-Jan 2013

Mention year.

~~Given $H = H_m e^{j(\omega t+\beta z)} \overline{ax}$ A/m in free space find E~~ (05 Marks

10 - June /July 2016

b. Given $\overline{H} = H_m e^{j(\omega t + \beta z)} \hat{a}_x$ A/m in free space. Find $\overline{E}$. (06 Marks)

Sol'n: given medium in free space

$\sigma = 0$ and $\overline{J_c} = \sigma \overline{E} = 0$ A/m²

using Modified Ampere's Law (ie Maxwell's eqn)

$$\nabla \times \overline{H} = \underbrace{\cancel{\overline{J_c}}}_{\approx 0} + \frac{\partial \overline{D}}{\partial t} \text{ A/m}^2$$

$$\boxed{\nabla \times \overline{H} = \frac{\partial \overline{D}}{\partial t}} \text{ A/m}^2 \quad (1)$$

H_x

$$\overline{H} = H_m e^{j(\omega t + \beta z)} \overline{a_x} \text{ A/m.}$$

$\overline{H} = f^n(z)$ and has only H_x component.

$$\nabla \times \overline{H} = \begin{vmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ 0 & 0 & \partial/\partial z \\ H_x & 0 & 0 \end{vmatrix} = -\frac{\partial H_x}{\partial z}(-\overline{a_y})$$

$$\nabla \times \overline{H} = +\frac{\partial H_x}{\partial z} \overline{a_y} \text{ A/m}^2 \quad (2)$$

eq^n (2) in eq^n (1)

i.e $$\frac{\partial \vec{D}}{\partial t} = \frac{\partial H_x}{\partial z}\,\overline{a_y} \quad A/m^2$$

$$\epsilon_0 \frac{\partial \vec{E}}{\partial t} = \frac{\partial}{\partial z}\left[H_m e^{j(\omega t + \beta z)}\right]\overline{a_y} \quad A/m^2$$

$$\frac{\partial \vec{E}}{\partial t} = \frac{H_m}{\epsilon_0} \cdot e^{j(\omega t + \beta z)} \times \beta\, \overline{a_y}$$

$$\frac{\partial \vec{E}}{\partial t} = \frac{H_m \beta}{\epsilon_0} e^{j(\omega t + \beta z)}\,\overline{a_y} \quad V/m\text{-sec}$$

$$\overline{E} = \int \frac{\partial \vec{E}}{\partial t}\, dt$$

$$= \frac{H_m \beta}{\epsilon_0} \frac{e^{j(\omega t + \beta z)}}{\omega}\,\overline{a_y}$$

$$\therefore \boxed{\overline{E} = \frac{\beta H_m}{\omega \epsilon_0} e^{j(\omega t + \beta z)}\,\overline{a_y}} \quad V/m$$

or $$\boxed{\overline{E} = E_m e^{j(\omega t + \beta z)}\,\overline{a_y}} \quad V/m$$

where $E_m = \dfrac{\beta H_m}{\omega \epsilon_0}$ V/m.

problem 21) $\overline{J_d} = 20\cos[1.5\times10^8 t - \beta x]\,\overline{a_y}\ \mu A/m^2$ $\epsilon_r = 5$ $\sigma = 0$

5.2 In a certain dielectric media the relative permittivity $\epsilon_r = 5$, conductivity $\sigma = 0$, the displacement current density $\overline{J}_d = 20\cos(1.5\times10^8 t - bx)\hat{a}_y\ \mu A/m^2$. Determine the electric flux density and electric field intensity. (06 Marks)

'β' phase constant rad/m.

soln: given $\epsilon_r = 5$ F/m ; $\sigma = 0$ $\mho$/m.

$$\overline{J_D} = 20\cos(1.5\times10^8 t - \beta x)\,\overline{a_y}\ \mu A/m^2$$

$$\beta = \frac{2\pi}{\lambda}\ \text{rad/m} \ \ldots \text{phase constant.}$$

$\overline{D} = ?$ and $\overline{E} = ?$

$$\overline{J_D} = \frac{\partial \overline{D}}{\partial t}$$

$$\overline{D} = \int \overline{J_D}\,dt \ \ C/m^2$$

$$\therefore \overline{D} = \frac{20\sin(1.5\times10^8 t - \beta x)\,\overline{a_y}}{1.5\times10^8}$$

$$\therefore \overline{D} = \frac{20}{1.5}\times10^{-8}\sin[1.5\times10^8 t - \beta x]\,\overline{a_y}\ C/m^2$$

$$\therefore \overline{D} = 13.333\times10^{-8}\sin[1.5\times10^8 t - \beta x]\,\overline{a_y}\ C/m^2$$

$$\boxed{\overline{D} = 133.33\sin[1.5\times10^8 t - \beta x]\,\overline{a_y}\ ;\ nC/m^2}$$

the Electric Field Intensity $\bar{E}$ is given by

$$\bar{D} = \epsilon \bar{E} \; C/m^2$$

$$\Rightarrow \bar{E} = \frac{\bar{D}}{\epsilon_0 \epsilon_r} \bar{E} \; V/m$$

$$\bar{E} = \frac{133.33 \times 10^{-9}}{8.854 \times 10^{-12} \times 5} \sin[1.5 \times 10^8 t - \beta x] \bar{a}_y \; V/m$$

$$\bar{E} = 3011.746 \sin[1.5 \times 10^8 t - \beta x] \bar{a}_y \; V/m$$

$\bar{a}_y$

or

$$\boxed{\bar{E} = 3.011746 \sin[1.5 \times 10^8 t - \beta x] \bar{a}_y} \; kV/m$$

Derive point form of Ampere's Law i.e $\nabla \times \bar{H} = \bar{J}$ A/m²

(or) $\nabla \times \bar{B} = \mu_0 \bar{J}$ wb/m³ 02 - June /July 2012

(or)

Using Ampere's circuital law, derive Maxwell's curl equation

$\nabla \times \vec{B} = \mu_0 \cdot \vec{J}$ (or) $\nabla \times \bar{H} = \bar{J}$ A/m² (06 Marks)

Soln: w.k.t from Ampere's Circuital Law i.e the Line integral of $\bar{H}$ around a Single closed path is equal to the Current enclosed.

mathematically $\boxed{I = \oint_{<l>} \bar{H} \cdot \bar{dl}}$ Ampere's

using Stokes theorem i.e $\oint_{<l>} \bar{H} \cdot \bar{dl} = \int_{<s>} (\nabla \times \bar{H}) \cdot \bar{ds}$

from Concept of Current density

$I = \int_{<s>} \bar{J} \cdot \bar{ds}$ Ampere's

i.e $\boxed{\oint_{<l>} \bar{H} \cdot \bar{dl} = \int_{<s>} (\nabla \times \bar{H}) \cdot \bar{ds} = I = \int_{<s>} \bar{J} \cdot \bar{ds}}$ A

$$\int_{<s>} (\nabla \times \bar{H}) \cdot \bar{ds} = \int_{<s>} \bar{J} \cdot \bar{ds}$$

$\Rightarrow \boxed{\nabla \times \bar{H} = \bar{J}}$ A/m² point form of ampere's Law

using relationship b/w B and $\bar{H}$ i.e

$\overline{B} = \mu_0 \overline{H}$ wb/m^2

(a) $\overline{H} = \overline{B}/\mu_0$ A/m.

the above eqn becomes

$\nabla \times (\overline{B}/\mu_0) = \overline{J}$ A/m^2

(a) $\boxed{\nabla \times \overline{B} = \mu_0 \overline{J}}$ wb/m^3

problem 23

06 – Jan 2013

Determine the frequency at which conduction current density and displacement current density are equal in a medium with $\sigma = 2 \times 10^{-4}$ S/mt and $\epsilon_r = 81$. (06 Marks)

$\sigma = 2\times10^{-4}$ S/m. $\epsilon_r = 81$

(or)

10 – June/July 2016

Find the frequency at which conduction current density and displacement current density are equal in a medium with $\sigma = 2\times10^{-4}$ ℧/m and $\epsilon_r = 81$. (04 Marks)

Soln: given $|\bar{J}_C| = |\bar{J}_D|$ A/m^2 $\Rightarrow f = ?$

i.e @what frequency (f), the Conduction current density and displacement current density are equal.

$\sigma = 2\times10^{-4}$ S/m and $\epsilon_r = 81$ F/m.

W.K.T

$$\left|\frac{\bar{J}_C}{\bar{J}_D}\right| = \frac{\sigma}{\omega\epsilon}$$

if $|\bar{J}_C| = |\bar{J}_D|$

then $\left|\frac{\bar{J}_C}{\bar{J}_D}\right| = 1$

$$\therefore \frac{\sigma}{\omega\epsilon} = 1$$

$\omega = 2\pi f$ rad/sec

$$\sigma = \omega\epsilon$$

$$\sigma = 2\pi f \epsilon_0 \epsilon_r$$

$$f = \frac{\sigma}{2\pi \epsilon_0 \epsilon_r} \text{ Hz}$$

$$f = \frac{2\times10^{-4}}{2\pi \times 8.854\times10^{-12}\times 81}$$

$$\boxed{f = 44.3839 \text{ kHz}}$$

i.e the frequency at which $|\bar{J}_C| = |\bar{J}_D|$ is $\boxed{f = 44.3839}$ kHz problem 24 $\sigma = 10^{-8}$ S/m. $\epsilon_r = 4$.

The dry earth has a conductivity $\sigma = 10^{-8}\ S/m$ and a relative permittivity $\epsilon_r = 4$. Find the frequency above which the conduction current dominates the displacement current.

Solu: given $\sigma = 10^{-8}$ S/m. $\epsilon_r = 4$.

find $f = ?$ at which $\left|\frac{i_c}{i_D}\right| \gg 1$

i.e $|i_c| \gg |i_D|$

$\Rightarrow |\bar{J}_c| \gg |\bar{J}_D|$

(or) $\left|\frac{J_c}{J_D}\right| \gg 1$

$\Rightarrow \frac{\sigma}{\omega \epsilon} \gg 1$

$\frac{\sigma}{2\pi f \epsilon_0 \epsilon_r} > 1$

(or)

$\frac{\sigma}{2\pi \epsilon_0 \epsilon_r} > f$

$\frac{10^{-8} \times 18 \times 10^{9}}{(4)} > f$

$\boxed{45 Hz > f}$

(or) $f < 45 Hz$.

$\frac{\sigma}{\omega \epsilon} = \frac{\sigma}{2\pi f \epsilon_0 \epsilon_r}$

$= \frac{\sigma}{8\pi \epsilon_0 f}$.

Eg. if $f = 44 Hz$

$\left(\frac{\sigma}{\omega \epsilon}\right) > 1 \Rightarrow i_c > i_D$

if $f = 46 Hz$.

$\left(\frac{\sigma}{\omega \epsilon}\right) < 1 \Rightarrow i_c < i_D$

i.e $\rightarrow$ if $0 < f < 45 Hz \Rightarrow$ Conduction Current dominates the displacement Current. ($i_c > i_D$)

if $f > 45 Hz \Rightarrow$ Displacement Current dominates the Conduction Current. ($i_c < i_D$)

problem 25

$i = 5.5\sin(4\times10^{10}t)\,\mu A$

1.5mm

06 –June/July 2014

A circular cross section conductor of radius 1.5mm carries a current $i = 5.5\sin(4\times10^{10}t)\,\mu A$. Find amplitude of the displacement current density if $\sigma = 35\,\mho/m$, $\epsilon_r = 10$. (06 Marks)

Sol^n:- given $r = 1.5mm$, $\sigma = 35\,\mho/m$, $\epsilon_r = 10$.

$i_c = 5.5\sin(4\times10^{10}t)\,\mu A$. $\omega = 4\times10^{10}$ rad/sec.

$\sigma = 35\,\mho/m$ and $\epsilon_r = 10$.

the ratio of $\left|\frac{\bar{J}_c}{\bar{J}_d}\right| = \frac{\sigma}{\omega\epsilon} = \frac{\sigma}{\omega\epsilon_0\epsilon_r}$

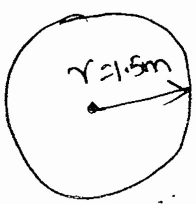
$$\frac{\sigma}{\omega\epsilon} = \frac{35}{4\times10^{10}\times10\times8.854\times10^{-12}}$$

$$\boxed{\frac{\sigma}{\omega\epsilon} = 9.8825}$$

given $i_c = 5.5\sin(4\times10^{10}t)\,\mu A$

$|i_c| = 5.5\,\mu A$

the amplitude of the displacement current density is given by



$$\left|\frac{\bar{J}_c}{\bar{J}_D}\right| = 9.8825$$

$$|\bar{J}_D| = \frac{|\bar{J}_c|}{9.8825} = \left(\frac{|i_c|}{A}\right)\times\frac{1}{9.8825}$$

$$|\bar{J}_D| = \frac{5.5\times10^{-6}}{\pi(1.5m)^2}\times\frac{1}{9.8825}$$

Area $= \pi r^2$
$= \pi(1.5m)^2$

$$\boxed{|\bar{J}_D| = 0.078733}\ A/m^2$$

(or) $|\bar{I}_D| = |\bar{J}_D|\times A = 5.5653\times10^{-7}\,A$

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(or) $\boxed{|i_D| = 0.55653\,\mu A}$

$$\boxed{i_D = 0.55653 \cos(4\times10^{10} t)}\ \mu A$$

the amplitude of Conduction Current density

$$\left|\frac{\bar{J}_c}{\bar{J}_D}\right| = 9.8825$$

$$|\bar{J}_c| = 9.8825\,|\bar{J}_D|$$

$$= 9.8825 \times 0.078733$$

$$\boxed{|\bar{J}_c| = 0.77807}\ A/m^2$$

Obs: Since $\frac{\sigma}{\omega\epsilon} = 9.8825$

i.e $\frac{\sigma}{\omega\epsilon} > 1$ $\therefore$ the given medium is Conducting Medium.

Topic 5.3

3. Maxwell's equations in point form Maxwell's equations in integral form

06-DEC2010

Derive the Maxwell's equations in the point form of the Gauss's law for time varying fields. **(06 Marks)**

02-DEC2010

Write Maxwell's equation in,

i) Steady magnetic field.
ii) Time varying field. **(08 Marks)**

06-DEC2008/Jan 2009

Write the Maxwell's equations in point form for static fields and in integral form for time varying fields. **(08 Marks)**

02-DEC2008/Jan 2009

List Maxwell's equations in integral forms for i) Static fields ii) Time-varying fields.(08 Marks)

06-DEC2011/Jan 2012

Write the Maxwell's equations in point form. **(04 Marks)**

10-Jan 2013

Explain Maxwell's equations for time varying fields. **(10 Marks)**

06-DEC 2013/Jan 2014

List Maxwell's equation in differential form and integral form. **(08 Marks)**

10-June/July 2013

List the Maxwell's equations in point and integral forms for time varying field. **(06 Marks)**

06 - June /July 2011

List the Maxwell's equations derived from Faraday's law, and Ampere's circuital law both in differential and integral form for i) Steady fields and ii) Time – varying fields.

10 - June /July 2012

Write an explanatory note on : Maxwell's equations in point and integral forms applicable to time varying fields. **(05 Marks)**

06- June /July 2009

List Maxwell's equations in point form and integral form. **(08 Marks)**

010-Dec/Jan 2015

mention year

List Maxwell's equations in point form and integral form. 12 **(08 Marks)**

10 - June /July 2015

10 - June /July 2015

State Maxwell's equations for a good conductor and for perfect dielectrics. (08 Marks)

10 - June /July 2014

List Maxwell's equations in differential and integral forms. (08 Marks)

06 – May/June 2010

~~Write Maxwell's equations in point form and integral form.~~ (06 Marks)

06 – June/July 2014 mention year.

Write Maxwell's equations in differential form and integral form. (08 Marks)

010-Dec/Jan 2016

Derive Maxwell's equations for time varying fields. (08 Marks)

EE- 10 J/J 2016

List Maxwell's equations for both : i) steady and ii) Time varying fields in differential and integral form, also mention the relevant laws they demonstrate. (08 Marks)

[15- June/July 2017 (6M) CBCS]

Maxwell's equations classified into

Static fields

- I. Workdone concept
- II. Gauss's Law (Electro-statics)
- III. Gauss's Law (Magneto statics)
- IV. Ampere's Circuital Law
- V. Continuity eq^n

Time-Varying fields (General set)

- I. Faraday's Law
- II. Modified Ampere's Law
- III. Gauss's Law (electro statics)
- IV. Gauss's Law (Magneto-statics)
- V. Continuity equation

Maxwell's eq^n for Time-Varying fields (General set)

Specific cases

1 Maxwell's eq^n for free Space

$\epsilon_r = 1$ & $\mu_r = 1$

$\boxed{\epsilon = \epsilon_0}$ $\boxed{\mu = \mu_0}$ $\left(\frac{\sigma}{\omega\epsilon}\right) \to 0$

$\rho_v \to 0$ and $\sigma = 0$

$\therefore \boxed{\bar{J}_c = \sigma\bar{E} = 0}$

2 Maxwell's eq^n for Good conductors

$\left(\frac{\sigma}{\omega\epsilon}\right) >> 1$

$\Rightarrow \boxed{\bar{J}_c >> \frac{\partial \bar{D}}{\partial t}}$ $\therefore$

neglect the term $\frac{\partial \bar{D}}{\partial t}$.

Since $\frac{\partial \bar{D}}{\partial t}$ is Very-Very less

$\rho_v = \nabla \cdot \bar{D} \to 0$ $\therefore \boxed{\rho_v = 0}$

3 Perfect dielectrics

$\left(\frac{\sigma}{\omega\epsilon}\right) << 1$

ie $\bar{J}_c << \frac{\partial \bar{D}}{\partial t}$ $\therefore$

neglect the term $\bar{J}_c$

$\therefore \boxed{\bar{J}_c = 0}$

and $\boxed{\rho_v = 0}$ bcz No free charges in dielectric Medium.

5.3 a Maxwell's equations for Static field :-

The fields taken into consideration are static Electric field due to charges at rest and the static magnetic field due steady currents.

The equations governing these fields may be summarized as follows

Work done Concept :-

i) the workdone required to move a point charge of unit over a closed path is equal to zero.

+ve Coulomb's

i.e $$\boxed{\oint_{\langle l \rangle} \bar{E} \cdot \bar{dl} = 0} \leftarrow ① \ldots \text{Integral form.}$$

using Stokes' theorem

$$\oint_{\langle l \rangle} \bar{E} \cdot \bar{dl} = \int_{\langle S \rangle} (\nabla \times \bar{E}) \cdot \bar{dS} = 0$$

the above result is true only when

but $\bar{dS} \neq 0$

$$\nabla \times \bar{E} = 0$$

i.e $$\boxed{\nabla \times \bar{E} = 0} \leftarrow ② \ldots \text{Differential/point form.}$$

ii. By Gauss's Law [Electrostatics]

The total Flux Comingout of any Closed Surface is equal to the net charge enclosed by that surface.

i.e $\oint_{\langle S \rangle} \overline{D} \cdot \overline{dS} = Q_{enclosed} = \int_{\langle vol \rangle} \rho_v \, dv$ Coulomb's

$$\therefore \boxed{\oint_{\langle S \rangle} \overline{D} \cdot \overline{dS} = \int_{\langle vol \rangle} \rho_v \, dv} \leftarrow \text{(3)}$$

C. --- integral form.

using Divergence theorem

$$\oint_{\langle S \rangle} \overline{D} \cdot \overline{dS} = \int_{\langle vol \rangle} (\nabla \cdot \overline{D}) \, dv = \int_{\langle vol \rangle} \rho_v \, dv \text{ Coulomb's}$$

$$\boxed{\nabla \cdot \overline{D} = \rho_v} \; C/m^3 \leftarrow \text{(4)}$$

Differential/ --- point form.

i.e the total electric Flux Crossing per unit volume is nothing but the volume charge density enclosed by that volume.

iii) Gauss's Law (Magneto Statics)

W.k.t Magnetic flux Lines form a closed path. i.e the total outgoing flux is equal to the incoming magnetic flux.

[Since non Existence of isolated South (or) isolated north poles in the magnetic fields].

$$\boxed{\oint_{S} \bar{B} \cdot \bar{dS} = 0} \text{ wb} \leftarrow (5) \text{ --- Integral form.}$$

using divergence theorem

$$\oint_{S} \bar{B} \cdot \bar{dS} = \int_{vol} (\nabla \cdot \bar{B})\, dv = 0$$

$$\Rightarrow \boxed{\nabla \cdot \bar{B} = 0} \text{ wb/m}^3 \leftarrow (6) \text{ Differential / --- point form.}$$

IV. Ampere's Circuital Law :-

The Line integral of $\bar{H}$ around a closed path is equal to the Current enclosed.

$$\text{i.e} \quad \oint_{L} \bar{H} \cdot \bar{dl} = I = \int_{S} \bar{J} \cdot \bar{dS}$$

$$\Rightarrow \boxed{\oint_{S} \bar{H} \cdot \bar{dl} = \int_{S} \bar{J} \cdot \bar{dS}} \text{ Ampere's} \leftarrow (7) \text{ --- Integral form.}$$

using stokes theorem i.e

$$\oint_{\langle l\rangle} \overline{H}\cdot\overline{dl} = \int_{\langle S\rangle} (\nabla\times\overline{H})\cdot\overline{dS} = \int_{\langle S\rangle} \overline{J}\cdot\overline{dS}$$

$$\Rightarrow \boxed{\nabla\times\overline{H} = \overline{J}} \text{ A/m}^2 \leftarrow (8)$$

--- Differential / point form.

V. ~~Continuity~~ Current eqn:-

From the Concept of Current density ($\overline{J}$)

ie $\overline{J} = \frac{dI}{dS}$ A/m²

(or) $I = \oint_{\langle S\rangle} \overline{J}\cdot\overline{dS} = \frac{dQ}{dt}$; Since for a static field's $Q = K$ (constant)

$\therefore \frac{dQ}{dt} = 0$.

$$\therefore \boxed{\oint_{\langle S\rangle} \overline{J}\cdot\overline{dS} = 0} \text{ Ampere's} \leftarrow (9)$$

--- Integral form.

using divergence theorem

$$\oint_{\langle S\rangle} \overline{J}\cdot\overline{dS} = \int_{\langle Vol\rangle} (\nabla\cdot\overline{J})\, dv = 0$$

$$\Rightarrow \boxed{\nabla\cdot\overline{J} = 0} \text{ A/m}^3 \leftarrow (10)$$

Differential / --- point form.

The above equations from (1) to (10) are called as Maxwell's equation for static field. i.e for the field whose time variation is equal to zero.

The eqn (1), 3, 5, 7, and 9 are called Maxwell's eqns in integral form. while eqn (2), 4, 6, 8 and 10 are called Maxwell's eqn's in point form for static fields.

Table No - 1 Summary :- Maxwell's eqn's for static field's

15 Jun/July 2017 (6M) CBCS.

Sl No.	Integral form	Point form (or) Differential form	Remark
1.	$\oint_{\langle l \rangle} \bar{E}\cdot \bar{dl} = 0$; Volts	$\nabla \times \bar{E} = 0$; V/m²	No work done is required to move a unit positive charge over a closed path.
2.	$\oint_{\langle S \rangle} \bar{D}\cdot \bar{ds} = \int_{\langle vol \rangle} \rho_v dv$; C	$\nabla \cdot \bar{D} = \rho_v$; C/m³	Gauss's Law (Electrostatics)
3.	$\oint_{\langle S \rangle} \bar{B}\cdot \bar{dS} = 0$; Wb	$\nabla \cdot \bar{B} = 0$; Wb/m³	Gauss's Law (Magnetostatics)
4.	$\oint_{\langle l \rangle} \bar{H}\cdot \bar{dl} = \int_{\langle S \rangle} \bar{J}\cdot \bar{dS}$; A	$\nabla \times \bar{H} = \bar{J}$; A/m²	Ampere's Circuital Law
5.	$\oint_{\langle S \rangle} \bar{J}\cdot \bar{dS} = 0$; A	$\nabla \cdot \bar{J} = 0$; A/m³	Continuity eqn.

In addition to these equations two more concepts are introduced i.e

i) the electric field is produced by changing magnetic field. [i.e Time-varying Magnetic field] --- Faraday's Law

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \; V/m^2$$

ii) The magnetic Field is produced by changing Electric field. [i.e Time-varying Electric field]. --- Modified Ampere's Law

$$\text{i.e } \nabla \times \bar{H} = \bar{J}_c + \frac{\partial \bar{D}}{\partial t} \; A/m^2$$

The First Concept was introduced by Michael Faraday and the Second by Maxwell's they form the basic equations of electromagnetic theory.

5.6 Maxwell's Equations for Time-varying Field's

1. Modified Ampere's Law
2. Faraday's Law
3. Gauss's Law (Electrostatics)
4. Gauss's Law Magnetostatics
5. Continuity equation.

1 Maxwell's equation from Modified Ampere's Law :-

The total Current $I = \oint_{\langle l \rangle} \bar{H} \cdot \overline{dl} = \int_{\langle s \rangle} (\bar{J}_c + \bar{J}_D) \cdot \overline{ds}$ Amperes

where $\bar{J}_c = \sigma \bar{E} \; A/m^2$; Conduction Current density.

$\bar{J}_D = \frac{\partial \bar{D}}{\partial t} \; A/m^2$; displacement Current density.

$$\therefore \boxed{\oint_{\langle l \rangle} \overline{H} \cdot \overline{dl} = \int_{\langle S \rangle} \left[\overline{J_c} + \frac{\partial \overline{D}}{\partial t} \right] \cdot \overline{ds}} \leftarrow (1)$$

Ampere's ---- Integral form.

using stoke's theorem $\oint_{\langle l \rangle} \overline{H} \cdot \overline{dl} = \int_{\langle S \rangle} (\nabla \times \overline{H}) \cdot \overline{ds}$ Ampere's

$$\int_{\langle S \rangle} (\nabla \times \overline{H}) \cdot \overline{ds} = \int_{\langle S \rangle} \left(\overline{J_c} + \frac{\partial \overline{D}}{\partial t} \right) \cdot \overline{ds}$$

$$\boxed{\nabla \times \overline{H} = \overline{J_c} + \frac{\partial \overline{D}}{\partial t}} \quad A/m^2 \leftarrow (2)$$

Differential / ---- point form.

Word Statement :- The Magnetomotive force around a closed path is equal to the Conduction Current plus the time derivative of electric flux density through any Surface bounded by the path.

[2] Maxwell's equation from Faraday's Law :-

In a closed path or Loop the Electric potential (emf) is developed due to time-varying Magnetic field in the Vicinity of that Closed path.

ie $e = -\frac{d\phi}{dt} = -\frac{d}{dt}\left[\int_{\langle S \rangle} \overline{B} \cdot \overline{ds} \right] = -\int_{\langle S \rangle} \frac{\partial \overline{B}}{\partial t} \cdot \overline{ds}$ volt's

$$e = \oint_{\langle l \rangle} \overline{E} \cdot \overline{dl} = \int_{\langle S \rangle} \left(-\frac{\partial \overline{B}}{\partial t} \right) \cdot \overline{dS}$$

$$\boxed{\oint_{\langle l\rangle} \bar{E}\cdot \bar{dl} = \int_{\langle S\rangle} -\frac{\partial \bar{B}}{\partial t}\cdot \bar{dS}} \leftarrow \text{volt's} \ (3)$$

----- Integral form.

using Stokes theorem ie $\oint_{\langle l\rangle} \bar{E}\cdot\bar{dl} = \int_{\langle S\rangle} (\nabla\times\bar{E})\cdot\bar{dS}$

$$\int_{\langle S\rangle} (\nabla\times\bar{E})\cdot\bar{dS} = \int_{\langle S\rangle}\left(-\frac{\partial\bar{B}}{\partial t}\right)\cdot\bar{dS}$$

$$\boxed{\nabla\times\bar{E} = -\frac{\partial\bar{B}}{\partial t}} \ \text{V/m}^2 \ \text{or Wb/m}^2\text{-sec} \leftarrow (4)$$

---- Differential / point form.

Word statement :- The electromotive force around a closed path is equal to the time derivative of the magnetic flux density ($\bar{B}$) through any surface bounded by the path.

3 Maxwell's equation from Gauss's Law (Electrostatic)

Total Electric flux crossing the closed surface is equal to the total charge enclosed by the closed surface.

i.e $\oint_{\langle S\rangle} \bar{D}\cdot\bar{dS} = Q_{enclosed} = \int_{\langle vol\rangle} \rho_v \, dv$ Coulomb's

ie $$\boxed{\oint_{\langle S\rangle} \bar{D}\cdot\bar{dS} = \int_{\langle vol\rangle} \rho_v \, dv} \ C \leftarrow (5)$$

---- Integral form.

using Divergence theorem i.e $\oint_{\langle S\rangle} \bar{D}\cdot\bar{ds} = \int_{\langle vol\rangle} (\nabla\cdot\bar{D})\, dv$ C

$$\oint_{\langle S\rangle} \overline{D} \cdot \overline{dS} = \int_{\langle vol\rangle} (\nabla \cdot \overline{D})\, dv = \int_{\langle vol\rangle} \rho_v\, dv \quad \text{Coulomb's}$$

$$\boxed{\nabla \cdot \overline{D} = \rho_v} \ C/m^3 \longleftarrow (6) \quad \text{Differential/ point form.}$$

Word Statement :- The total Electric Flux density through the Surface enclosing a volume is equal to the charge within the volume.

[4] Maxwell's Equation from Gauss's Law [Magnetostatics]

In Case of Magnetic field the total outgoing flux is equal to incoming magnetic flux.

i.e $$\boxed{\oint_{\langle S\rangle} \overline{B} \cdot \overline{dS} = 0} \longleftarrow (7) \quad \text{--- Integral form.}$$

using divergence theorem

$$\oint_{\langle S\rangle} \overline{B} \cdot \overline{dS} = \int_{\langle vol\rangle} (\nabla \cdot \overline{B})\, dv = 0$$

true only when $dv \neq 0$

but $$\boxed{\nabla \cdot \overline{B} = 0} \longleftarrow (8) \quad \text{--- Differential/ point form.}$$

Word Statement :- The net Magnetic flux Emerging through any closed Surface is zero.

[5] Maxwell's equation from Continuity eq^n:-

W.k.t Continuity Current eq^n from Law of Conservation of charge

i.e $$I = \oint_{\langle S \rangle} \overline{J} \cdot \overline{ds} = -\frac{dQ}{dt} = -\int_{\langle vol \rangle} \frac{\partial \rho_v}{\partial t} dv$$

$$\boxed{\oint_{\langle S \rangle} \overline{J} \cdot \overline{ds} = \int_{\langle vol \rangle} \left(-\frac{\partial \rho_v}{\partial t}\right) dv} \leftarrow (9)$$

--- Integral form

using divergence theorem

i.e $$\oint_{\langle S \rangle} \overline{J} \cdot \overline{ds} = \int_{\langle vol \rangle} (\nabla \cdot \overline{J}) dv = \int_{\langle vol \rangle} \left(-\frac{\partial \rho_v}{\partial t}\right) \cdot dv$$

$$\Rightarrow \boxed{(\nabla \cdot \overline{J}) = -\frac{\partial \rho_v}{\partial t}} \, A/m^3 \leftarrow (10)$$

--- Differential/ point form.

Word Statement:- Current diverging from a small volume per unit volume is equal to the rate of decrease of charge per unit volume at every point.

Table-2 :- Summary → Maxwell's eq^n for Time-varying field :- (General Set).

Sl. No.	Integral form	Differential / point form	Remark
1.	$\oint_{\langle l \rangle} \overline{H}\cdot\overline{dl} = \int_{\langle S \rangle}\left(\overline{J}_c + \frac{\partial \overline{D}}{\partial t}\right)\cdot\overline{dS}$; Amps	$\nabla \times \overline{H} = \overline{J}_c + \frac{\partial \overline{D}}{\partial t}$; A/m²	Modified Ampere's Law
2.	$\oint_{\langle l \rangle} \overline{E}\cdot\overline{dl} = -\int_{\langle S \rangle}\frac{\partial \overline{B}}{\partial t}\cdot\overline{dS}$; volts	$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$; v/m²	Faraday's Law.
3.	$\oint_{\langle S \rangle} \overline{D}\cdot\overline{dS} = \int_{\langle vol \rangle} \rho_v\, dv$; C	$\nabla\cdot\overline{D} = \rho_v$; C/m³	Gauss's Law (Electro-statics)
4.	$\oint_{\langle S \rangle} \overline{B}\cdot\overline{dS} = 0$; Wb	$\nabla\cdot\overline{B} = 0$; Wb/m³	Gauss's Law (Magneto Statics).
5.	$\oint_{\langle S \rangle} \overline{J}\cdot\overline{dS} = \int_{\langle vol \rangle}\left(-\frac{\partial \rho_v}{\partial t}\right) dv$; Amps	$\nabla\cdot\overline{J} = -\frac{\partial \rho_v}{\partial t}$; A/m³	Continuity eq^n.

Table 3 :- Maxwell's eq^n for free Space

Note:- In free space $\mu_r = 1$ and $\epsilon_r = 1$

$\therefore \boxed{\mu = \mu_0}$ H/m and $\boxed{\epsilon = \epsilon_0}$ F/m

Conductivity $\boxed{\sigma = 0}$ ℧/m $\therefore \bar{J}_c = \sigma \bar{E} = 0$ A/m²

and $\boxed{\rho_v = 0}$ C/m³

Note:- use these conditions in General set

Sl.No	Integral form	Differential / point form	Remark.
1.	$\oint_{\langle l \rangle} \bar{H} \cdot \bar{dl} = \int_{\langle S \rangle} \frac{\partial \bar{D}}{\partial t} \cdot \bar{dS}$; A	$\nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t}$; A/m²	Modified Ampere's Law
2.	$\oint_{\langle l \rangle} \bar{E} \cdot \bar{dl} = -\int_{\langle vol \rangle} \frac{\partial \bar{B}}{\partial t} \cdot \bar{dS}$; volt	$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$; V/m²	Faraday Law
3.	$\oint_{\langle S \rangle} \bar{D} \cdot \bar{dS} = 0$; C	$\nabla \cdot \bar{D} = 0$ C/m³	Gauss's Law (electrostatics)
4.	$\oint_{\langle S \rangle} \bar{B} \cdot \bar{dS} = 0$; wb	$\nabla \cdot \bar{B} = 0$ wb/m³	Gauss's Law (Magnetostatics).
5.	$\oint_{\langle S \rangle} \bar{J} \cdot \bar{dS} = 0$; Ampere	$\nabla \cdot \bar{J} = 0$ A/m³	Continuity eq^n.

Key Note:- If given Medium to be perfect dielectrics, Consider it to be on free space case only. Dk Air medium/free Space is the best example for perfect dielectric.

Topic 5.3d

Table 4:- Maxwell's eq^n for Good Conductors:-

In Good Conductor's $\left(\frac{\sigma}{\omega\epsilon}\right) >> 1 \Rightarrow \sigma >> \omega\epsilon$.

$\Rightarrow \overline{J_C} >> \frac{\partial D}{\partial t}$ ∴ neglect the term $J_D \left(\frac{\partial D}{\partial t}\right)$

Since $\frac{\partial D}{\partial t}$ is very less ∴ $\rho_v = \nabla \cdot \overline{D} \to 0$ ∴ $\boxed{\rho_v = 0}$ C/m^3

Cond^n:- $J_D \to 0$ and $\rho_v \to 0$; are then condition's in General set.

Sl. No.	Integral form	Differential / Point form	Remark
01.	$\oint_{\langle l \rangle} \overline{H} \cdot \overline{dl} = \int_{\langle S \rangle} \overline{J_c} \cdot \overline{dS}$; Ampere's	$\nabla \times \overline{H} = \overline{J_c}$; A/m^2	Modified Ampere's Law
2.	$\oint_{\langle l \rangle} \overline{E} \cdot \overline{dl} = -\int_{\langle S \rangle} \frac{\partial \overline{B}}{\partial t} \cdot \overline{dS}$; volts	$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$; V/m^2	Faraday's Law
3.	$\oint_{\langle S \rangle} \overline{D} \cdot \overline{dS} = 0$; C	$\nabla \cdot \overline{D} = 0$; C/m^3	Gauss's Law (Electrostatics)
4.	$\oint_{\langle S \rangle} \overline{B} \cdot \overline{dS} = 0$; Wb's	$\nabla \cdot \overline{B} = 0$; Wb/m^3	Gauss's Law (Magnetostatics)
5.	$\oint_{\langle S \rangle} \overline{J} \cdot \overline{dS} = 0$; Ampere	$\nabla \cdot \overline{J} = 0$; A/m^3	Continuity eq^n.

Tables :- Maxwell's equation for Good dielectric (or) Lowloss dielectric

~~for Low Loss dielectrics $\left(\frac{\sigma}{\omega\epsilon}\right) << 1 \Rightarrow \sigma << \omega\epsilon$~~

$\therefore \quad \overline{J_c} << \frac{\partial \overline{D}}{\partial t} \quad \therefore$ neglected the term $\overline{J_c}$.

i.e $\boxed{\overline{J_c} \to 0}$

and $\boxed{\rho_v = 0}$ be, NO free charges in dielectrics.

Note:- use $\boxed{\overline{J_c} \to 0}$ and $\boxed{\rho_v \to 0}$ in the general set.

Sl. NO.	Integral form	Point form	Remark
1.	$\oint_{\langle l \rangle} \overline{H} \cdot \overline{dl} = \int_{\langle S \rangle} \frac{\partial \overline{D}}{\partial t} \cdot \overline{dS}$; A	$\nabla \times \overline{H} = \frac{\partial \overline{D}}{\partial t}$; A/m²	Modified Ampere's Law.
2.	$\oint_{\langle l \rangle} \overline{E} \cdot \overline{dl} = -\int_{\langle S \rangle} \frac{\partial \overline{B}}{\partial t} \cdot \overline{dS}$; volt	$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$; V/m²	Faraday's Law
3.	$\oint_{\langle S \rangle} \overline{D} \cdot \overline{dS} = 0$; C	$\nabla \cdot \overline{D} = 0$; C/m³	Gauss's Law (electrostatics)
4.	$\oint_{\langle S \rangle} \overline{B} \cdot \overline{dS} = 0$; wb	$\nabla \cdot \overline{B} = 0$; wb/m³	Gauss's Law (magnetostatics)
5.	$\oint_{\langle S \rangle} \overline{J} \cdot \overline{dS} = 0$; A	$\nabla \cdot \overline{J} = 0$; A/m³	Continuity eqn.

Key Note :-

1. In Free space Medium

$\boxed{\sigma = 0}$, $\boxed{\epsilon = \epsilon_0}$ and $\boxed{\mu = \mu_0}$

2. Lossless dielectrics (or) perfect dielectrics $\boxed{\frac{\sigma}{\omega\epsilon} \to 0}$

i.e $\boxed{\sigma \simeq 0}$ $\boxed{\epsilon = \epsilon_r \epsilon_0}$, $\boxed{\mu = \mu_r \mu_0}$

3) Low Loss Dielectrics (or) Good dielectrics $\boxed{\frac{\sigma}{\omega\epsilon} << 1}$

$\boxed{\sigma \neq 0}$, $\boxed{\epsilon = \epsilon_r \epsilon_0}$, $\boxed{\mu = \mu_r \mu_0}$

4. Good Conductors / Lossy Medium $\left(\frac{\sigma}{\omega\epsilon}\right) >> 1$.

$\boxed{\sigma \simeq \infty}$, $\boxed{\epsilon = \epsilon_0}$ $\boxed{\mu = \mu_r \mu_0}$

9b. List Maxwell's equations for steady and time varying fields in 15- June/July 2017 (CBCS)
i) Point form ii) Integral from. (06 Marks)

Soln:-
Maxwell's Equation's for Static or Steady field.

Sl.NO.	Integral form	point form	Remark
01.	$\oint_{\langle l \rangle} E \cdot \overline{dl} = 0$; volts	$\nabla \times E = 0$; V/m	No work done is required to move a unit positive charge over a closed path.
02.	$\oint_{\langle S \rangle} \overline{D} \cdot \overline{ds} = \int_{\langle vol \rangle} \rho_v \, dv$; C	$\nabla \cdot D = \rho_v$; C/m^2	Gauss's Law (Electrostatics)
03.	$\oint_{\langle S \rangle} \overline{B} \cdot \overline{dS} = 0$; wb	$\nabla \cdot B = 0$; wb/m^2	Gauss's Law (Magneto statics).
04.	$\oint_{\langle l \rangle} \overline{H} \cdot \overline{dl} = \int_{\langle S \rangle} \overline{J} \cdot \overline{ds}$; A	$\nabla \times \overline{H} = \overline{J}$; A/m^2	Ampere's Circuital Law.
05.	$\oint_{\langle S \rangle} \overline{J} \cdot \overline{dS} = 0$; A	$\nabla \cdot \overline{J} = 0$; A/m^3	Continuity Current equation.

Maxwell's equations for Time-varying field.

Sl.No.	Integral form	Differential (or) Point form	Remark
01.	$\oint_{\langle l \rangle} \overline{E}\cdot\overline{dl} = -\int_{\langle S \rangle} \frac{\partial \overline{B}}{\partial t}\cdot\overline{ds}$; volts	$\nabla\times\overline{E} = -\frac{\partial \overline{B}}{\partial t}$; V/m²	Faraday's Law
02.	$\oint_{\langle l \rangle} \overline{H}\cdot\overline{dl} = \int_{\langle S \rangle} \left(\overline{J}_c + \frac{\partial \overline{D}}{\partial t}\right)\cdot\overline{ds}$; Ampere	$\nabla\times\overline{H} = \overline{J}_c + \frac{\partial \overline{D}}{\partial t}$; A/m²	Modified Ampere's Law
03.	$\oint_{\langle S \rangle} \overline{D}\cdot\overline{ds} = \int_{\langle vol \rangle} \rho_v\cdot dv$; C	$\nabla\cdot\overline{D} = \rho_v$ C/m³	Gauss's Law (Electrostatics)
04.	$\oint_{\langle S \rangle} \overline{B}\cdot\overline{ds} = 0$; wb	$\nabla\cdot\overline{B} = 0$; wb/m²	Gauss's Law (Magnetostatics)
05.	$\oint_{\langle S \rangle} \overline{J}\cdot\overline{ds} = \int_{\langle vol \rangle} -\frac{\partial \rho_v}{\partial t}\cdot dv$	$\nabla\cdot\overline{J} = -\frac{\partial \rho_v}{\partial t}$; A/m³	Continuity Current eqⁿ.

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Problem 26 $\bar{E} = [kx - 100t]\,\bar{a}_y$ V/m.
$\bar{H} = [x + 20t]\,\bar{a}_z$ A/m.

06-DEC2011/Jan 2012

Determine the value of K such that following pairs of fields satisfies Maxwell's equation in the region where $\sigma = 0$ and $\rho_v = 0$.

a) $\bar{E} = [Kx - 100t]\,\bar{a}_y$ V/m $\bar{H} = [x + 20t]\,\bar{a}_z$ A/m
$\mu = 0.25$ H/m, $\varepsilon = 0.01$ F/m

(08 Marks)

b) $\bar{D} = 5x\,\bar{a}_x - 2y\,\bar{a}_y + kz\,\bar{a}_z$ $\mu C/m^2$
$\bar{B} = 2\bar{a}_y$ mT and $\mu = \mu_0$; $\epsilon = \epsilon_0$ F/m.

Sol^n:- a) Given $\bar{E}$ and $\bar{H}$ are time-varying field's
$\therefore$ the Maxwell's eq^n is

$$\nabla \times \bar{E} = -\frac{\partial B}{\partial t} \; V/m^2 \quad \text{and} \quad \bar{B} = \mu_0 \bar{H} \; Wb/m^2.$$

$$\begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \partial/\partial x & 0 & 0 \\ 0 & E_y & 0 \end{vmatrix} = -\mu_0 \frac{\partial H}{\partial t} \; ; V/m^2.$$

given
$\bar{E} = [kx - 100t]\,\bar{a}_y$ V/m.
$E_y = (kx - 100t)$ V/m $\bar{E} \Rightarrow f^n(x, t)$ only.
$\bar{H} = [x + 20t]\,\bar{a}_z$ A/m
$\bar{H} \Rightarrow f^n(x, t)$ and $H_y = (x + 20t)$ A/m.

$$\Rightarrow \frac{\partial E_y}{\partial x}\,\bar{a}_z = -\mu_0 \frac{\partial H}{\partial t}$$

$$\frac{\partial}{\partial x}[kx - 100t]\,\bar{a}_z = -\mu_0 \frac{\partial}{\partial t}[x + 20t]\,\bar{a}_z$$

$k \bar{a_z} = -\mu_0 (20) \bar{a_z}$

$\Rightarrow$ Equating the 'z' components on bothside

$k = -20\mu_0 = -20(0.25)$

$\Rightarrow \boxed{k = -5} \; V/m^2$

Unit of 'k'

$E_y = (kx - 100t) \, V/m$

$E_y \to V/m$, $kx \to V/m$, $100t \to V/m$

$kx \Rightarrow V/m$

Unit of 'k' in 'm'

$\boxed{k \to V/m^2}$

$\therefore$ for $k = -5 \, V/m^2$ the pair of fields $\bar{E}$ and $\bar{H}$ are satisfies the maxwells equation.

i.e $\boxed{\nabla \times \bar{E} = -\frac{\partial B}{\partial t}}$ V/m^2

b) given $\bar{D} = 5x \bar{a_x} - 2y \bar{a_y} + kz \bar{a_z} \; \mu C/m^2$

$\bar{B} = 2\bar{a_y}$ mT and $\mu = \mu_0$, $\epsilon = \epsilon_0$.

Since the given fields $\bar{D}$ and $\bar{B}$ are not time-varying fields

$\therefore$ using Maxwell's eq^ns $\nabla \cdot \bar{D} = \rho_v \; C/m^3$

and $\nabla \cdot \bar{B} = 0 \; wb/m^3$

given $\rho_v = 0$

$\Rightarrow \nabla \cdot \bar{D} = 0 \; C/m^3$

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = 0$$

$D_x = 5x\ \mu C/m^2$; $D_y = -2y\ \mu C/m^2$; $D_z = kz\ \mu C/m^2$

$$\therefore \nabla \cdot \bar{D} = \frac{\partial}{\partial x}(5x)\mu + \frac{\partial}{\partial y}(-2y)\mu + \frac{\partial}{\partial z}(kz)\mu = 0$$

$$[5 - 2 + k]\mu = 0$$

$$\Rightarrow \boxed{k = -3}\ C/m^3$$

$\therefore$ the value of $\boxed{k = -3}\ C/m^3$ such that ~~the given field~~ $\bar{D}$, satisfies the maxwell's eq^n $\nabla \cdot D = 0\ C/m^3$.

Illy. given $\bar{B} = 2\bar{a}_y$ mT

$B_y = 2\ mWb/m^2$ Maxwell's eq^n $\nabla \cdot B = 0$

$$\nabla \cdot \bar{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}$$

$$= 0 + \cancel{\frac{\partial}{\partial y}(2)}^{0} + 0$$

$$\therefore \boxed{\nabla \cdot \bar{B} = 0}$$

$\therefore$ given $\bar{B} = 2\bar{a}_y$ mT satisfies the Maxwell's equation $\boxed{\nabla \cdot B = 0}$ wb/m^2 (or) Tesla.

Problem 27 [15- June/July 2017 (6M) CBCS]

10-DEC2011/Jan 2012

27 Determine whether or not the following pairs of fields satisfy Maxwell's equation.

$\vec{E} = E_m \sin x \sin t \, \hat{a}_y$ v/m $\rightarrow \bar{E} = E_m \sin x \sin t \, \bar{a}_y$ v/m

$\vec{H} = \frac{E_m}{\mu} \cos x \cos t \, \hat{a}_z$ $\Rightarrow$ A/m; $\bar{H} = \frac{E_m}{\mu} \cos x \cos t \, \bar{a}_z$ A/m. (06 Marks)

06.- June /July 2013

(or)

Do the fields $\vec{E} = E_m \cdot \sin x \cdot \sin t \hat{a}_y$ and $\overline{H} = \frac{E_m}{\mu_0} \cdot \cos x \cdot \cos t \hat{a}_z$ ($\bar{a}_z$) satisfy the Maxwell's equations? (08 Marks)

Soln:- the given field eqn are Time-varying field's

i.e $\bar{E} = E_m \sin x \sin t \, \bar{a}_y$ v/m.

$\bar{E} \Rightarrow f^n(x,t)$ and $E_y = E_m \sin x \sin t$ V/m.

and $\bar{H} = \frac{E_m}{\mu} \cos x \cos t \, \bar{a}_z$ A/m

$\bar{H} \Rightarrow f^n(x,t)$ and $H_z = \frac{E_m}{\mu} \cos x \cos t$ A/m

Since the fields $\bar{E}$ and $\bar{H}$ are Time-varying $\therefore$ the Maxwell's eqn's related to time-varying fields are

1. Faraday's Law i.e $\nabla \times \bar{E} = -\frac{\partial B}{\partial t}$ v/m^2 ← (1)

2. Modified Ampere's Law

$\nabla \times \bar{H} = \bar{J}_c + \frac{\partial D}{\partial t}$; A/m^2 ← (2)

Case i. By Considering eqn (1)

$\nabla \times \bar{E} = -\frac{\partial B}{\partial t}$ v/m^2 and $\bar{B} = \mu_0 \bar{H}$ wb/m^2

$$\therefore \nabla \times \bar{E} = -\mu_0 \frac{\partial \bar{H}}{\partial t} \; v/m^2$$

$$\begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \partial/\partial x & 0 & 0 \\ 0 & E_y & 0 \end{vmatrix} = -\mu_0 \frac{\partial}{\partial t}\left[\frac{E_m}{\mu_0} \cos x \cos t\right] \bar{a}_z$$

$$\frac{\partial E_y}{\partial x} \bar{a}_z = -\cancel{\mu_0} \cdot \frac{E_m}{\cancel{\mu_0}} \cos(x) \left[-\sin t\right] \bar{a}_z$$

$$\frac{\partial}{\partial x}\left[E_m \sin x \sin t\right] \bar{a}_z = + E_m \cos x \sin t \; \bar{a}_z$$

$$\Rightarrow E_m \cos x \sin t \; \bar{a}_z = E_m \cos x \sin(t) \; \bar{a}_z$$

$\Rightarrow$ Equating the 'z' components

$$\boxed{E_m \cos x \sin(t) = E_m \cos x \sin t}$$

thus the given fields $\bar{E}$ and $\bar{H}$ satisfies the Maxwell's equation $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$; v/m^2.

Case ii: By considering eqn ② ie $\nabla \times \bar{H} = \bar{J}_c + \frac{\partial \bar{D}}{\partial t}$ ← ⓐ

given $\bar{E} = E_m \sin x \sin t \; \bar{a}_y$ v/m

and $\bar{H} = \frac{E_m}{\mu_0} \cos x \cos t \; \bar{a}_z$ A/m

Since given $\mu = \mu_0$ H/m $\therefore$ the given Medium is to be considered as free space.

In free space $\boxed{\sigma = 0}$ S/m

$\therefore \boxed{\overline{J_c} = \sigma \overline{E} = 0}$ A/m²

$\therefore$ eqⁿ ⓐ becomes

$$\nabla \times \overline{H} = \frac{\partial \overline{D}}{\partial t} \text{ A/m}^2 \text{ and } \overline{D} = \epsilon_0 \overline{E} \text{ C/m}^2$$

$$\nabla \times \overline{H} = \frac{\partial \overline{D}}{\partial t} = \epsilon_0 \frac{\partial \overline{E}}{\partial t} \text{ ; A/m}^2$$

$$\nabla \times \overline{H} = \epsilon_0 \frac{\partial \overline{E}}{\partial t} \text{ ; A/m}^2$$

$$\begin{vmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ \partial/\partial x & 0 & 0 \\ 0 & 0 & H_z \end{vmatrix} = \epsilon_0 \frac{\partial \overline{E}}{\partial t}$$

$$-\frac{\partial H_z}{\partial x} \overline{a_y} = \epsilon_0 \frac{\partial \overline{E}}{\partial t}$$

$$-\frac{\partial}{\partial x}\left[\frac{E_m}{\mu_0} \cos x \cos t\right] \overline{a_y} = \epsilon_0 \frac{\partial}{\partial t}\left[E_m \sin x \sin t\right] \overline{a_y}$$

$$-\frac{E_m}{\mu_0} \cos t \times -\sin x \; \overline{a_y} = \epsilon_0 E_m \sin x \cos t \; \overline{a_y}$$

Equating the y-components on bothside

$$+\frac{E_m}{\mu_0}\cancel{\cos t}\cancel{\sin x} = \epsilon_0 \cancel{E_m}\cancel{\sin x}\cancel{\cos t}$$

$$\Rightarrow \boxed{\frac{1}{\mu_0} \neq \epsilon_0}$$ by $\epsilon_0 = 8.854\times10^{-12}$ F/m & $\mu_0 = 4\pi\times10^{-7}$ H/m

$$\Rightarrow \boxed{\epsilon_0 \neq 1/\mu_0}$$

$\therefore$ the given field's $\overline{E}$ and $\overline{H}$ "doesnot satisfies" the Maxwell's equation $\nabla\times\overline{H} = \frac{\partial D}{\partial t}$ A/m^2.

i.e $\nabla\times\overline{H} \neq \frac{\partial D}{\partial t}$ ~~A/m~~ for the given field's $\overline{E}$ & $\overline{H}$.

problem 28 $\mu = 10^{-5}$ H/m $\epsilon = 4\times10^{-9}$ F/m $\rho_v = 0$ K

10-June/July 2013 (6m)
06 J/J -2009
02 J/J 2010 (7m)
10-Dec/Jan 2013 (6m)
15-Dec/Jan 2017 (8m) (CBCS scheme)

79 Let $\mu = 10^{-5}$ H/m, $\epsilon = 4\times10^{-9}$ F/m, $\sigma = 0$ and $\rho_v = 0$. Find K so that each of the following pair of fields satisfies Maxwell's equations:

i) $D = (6a_x - 2ya_y + 2za_z\,nC/m^2)$

$H = (Kxa_x + 10ya_y - 25za_z)$ A/m

ii) $E = (20y - Kt)a_x$ V/m

$H = (y + 2\times10^6 t)\,a_z$ A/m

(06 Marks)

$$\bar{D} = [6\bar{a}_x - 2y\bar{a}_y + 2z\bar{a}_z]\ nC/m^2$$

$$\bar{H} = (kx\bar{a}_x + 10y\bar{a}_y - 25z\bar{a}_z)\ A/m.$$

(06 Marks)

80 For the given medium $\epsilon = 4\times10^{-9}$ F/m and $\sigma = 0$. Find 'k' so that the following pair of fields satisfy Maxwell's equations.

$\vec{E} = (20y - kt)\vec{a}_x$ V/m

$\vec{H} = (y + 2\times10^6 t)\vec{a}_z$ A/m

(07 Marks)

$$\bar{E} = (20y - kt)\,\bar{a}_x\ V/m$$

$$\bar{H} = (y + 2\times10^6 t)\,\bar{a}_z\ A/m.$$

06- June /July 2009

Let $\mu = 10^{-5}$ H/m, $\epsilon = 4\times10^{-9}$ F/m, $\sigma = 0$ and $\rho_v = 0$. Find K so that each of the following pair of fields satisfies Maxwell's equation.

[illegible]

(06 Marks)

02 - June /July 2010

If $\mu = 10^{-5}$ H/m, $\epsilon = 4*10^{-9}$ F/m, $\sigma = 0$ and $\rho_v = 0$, find k (including units) to satisfy $\bar{D} = 6\hat{a}_x - 2y\hat{a}_y + 2z\hat{a}_z$ nc/m² and $\bar{H} = kx\hat{a}_x + 10y\hat{a}_y - 25z\hat{a}_z$ A/m, satisfying Maxwell equation.

(06 Marks)

soln: Recalling the Maxwell's eqn's

$$\nabla \times \bar{H} = \bar{J}_c + \frac{\partial \bar{D}}{\partial t} \leftarrow (1)\ A/m^2$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \leftarrow (2)\ V/m^2$$

$$\nabla \cdot \bar{D} = \rho_v\ C/m^3 \leftarrow (3)$$

$$\nabla \cdot \bar{B} = 0\ Wb/m^3 \leftarrow (4)$$

given data

$\mu = 10^{-5}$ H/m ; $\epsilon = 4\times10^{-9}$ F/m, $\sigma = 0$ S/m and

$\rho_v = 0$ C/m³

Case i :- (i) Set 1. $\overline{D} = 6\overline{a}_x - 2y\overline{a}_y + 2z\overline{a}_z$ nC/m²

and $\overline{H} = kx\,\overline{a}_x + 10y\,\overline{a}_y - 25z\,\overline{a}_z$ A/m

Since the given field's are static ∴ using ③ and eqⁿ ④. bcz $\overline{D}$ & $\overline{H}$ are independent of time 't'.

using eqⁿ ③. $\nabla\cdot\overline{D} = \rho_v$

given $\rho_v = 0$ C/m³

$$\Rightarrow \nabla\cdot\overline{D} = 0 \text{ C/m}^3$$

$D_x = 6$ nC/m², $D_y = -2y$ nC/m², $D_z = 2z$ nC/m²

$$\nabla\cdot\overline{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$\nabla\cdot\overline{D} = \frac{\partial}{\partial x}(6) + \frac{\partial}{\partial y}(-2y) + \frac{\partial}{\partial z}(2z)$$

$$\nabla\cdot\overline{D} = 0 - \cancel{2} + \cancel{2} = 0$$

$$\Rightarrow \boxed{\nabla\cdot\overline{D} = 0} \text{ C/m}^3$$

∴ given field $\overline{D}$ Satisfies the Maxwell's eqⁿ

$$\nabla\cdot\overline{D} = 0.$$

using eq^n (4) i.e $\nabla \cdot \overline{B} = 0$

given $\overline{H} = kx\,\overline{a_x} + 10y\,\overline{a_y} - 25z\,\overline{a_z}$ A/m.

$\nabla \cdot \overline{B} = 0 \Rightarrow \nabla \cdot (\mu \overline{H}) = 0$

$\overline{B} = \mu \overline{H}$ wb/m^2

$\mu \neq 0$

$\therefore \Rightarrow \nabla \cdot \overline{H} = 0$

$$\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = 0$$

$$\frac{\partial}{\partial x}[kx] + \frac{\partial}{\partial y}(10y) + \frac{\partial}{\partial z}(-25z) = 0$$

$k + 10 - 25 = 0$

$\Rightarrow \boxed{k = 15}$ A/m^2.

unit of 'k'

obs $H_x = kx$; A/m

$kx \rightarrow$ A/m

x m $\therefore \boxed{k \rightarrow \text{A/m}^2}$

$\therefore$ the value of $k = 15$ A/m^2 such that $\overline{H}$ satisfies the Maxwell's eq^n $\nabla \cdot \overline{B} = 0$ wb/m^3 i.e $\nabla \cdot \overline{H} = 0$ A/m^2

Conci Set 2 eqⁿ given $\bar{E} = (20y - kt)\bar{a}_x$ V/m.

$\bar{E} \Rightarrow f^n(y,t)$ and $E_x = (20y - kt)$ V/m.

and $\bar{H} = (y + 2\times10^6 t)\bar{a}_z$ A/m

$\bar{H} \Rightarrow f^n(y,t)$ and $H_z = (y + 2\times10^6 t)$ A/m.

Since the given field's are time-varying field's and given $\sigma = 0$ ∴ using eqⁿ (1)

i.e $\nabla\times\bar{H} = \cancel{\bar{J}_c}^{0} + \frac{\partial \bar{D}}{\partial t}$; given $\sigma = 0$ S/m.

$$\therefore \Rightarrow \nabla\times\bar{H} = \frac{\partial \bar{D}}{\partial t} = \epsilon\frac{\partial \bar{E}}{\partial t}$$

$$\nabla\times\bar{H} = \epsilon\frac{\partial \bar{E}}{\partial t}$$

$$\begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 0 & \partial/\partial y & 0 \\ 0 & 0 & H_z \end{vmatrix} = \epsilon\frac{\partial \bar{E}}{\partial t}$$

$$\frac{\partial H_z}{\partial y}\bar{a}_x = \epsilon\frac{\partial \bar{E}}{\partial t}$$

$$\frac{\partial}{\partial y}\left[y + 2\times10^6 t\right]\bar{a}_x = \epsilon\frac{\partial}{\partial t}\left[20y - kt\right]\bar{a}_x$$

$$\left[1 + 0\right]\bar{a}_x = \epsilon\left[0 - k\right]\bar{a}_x$$

Equating the component'n of 'x'

$1 = -K\epsilon$

$\Rightarrow K = -1/\epsilon \, (F/m)^{-1}$ or $V/m\text{-sec}$

given $\epsilon = 4\times10^{-9}$ F/m.

$\therefore \boxed{K = -250\times10^{6}}$ m/F or V/m-sec

$\therefore$ the value of $K = -\frac{1}{\epsilon}\,(F/m)^{-1}$ or -250×10^{6} m/F or V/m-sec

Such that $\bar{H}$ and $\bar{E}$, satisfies the Maxwell's eqn $\nabla\times\bar{H} = \frac{\partial \bar{D}}{\partial t}$ A/m²

Note:- unit of 'K'

given

$\bar{E} = (20y - kt)\,\bar{a}_x$ V/m

$20y \rightarrow$ V/m, $kt \rightarrow$ V/m

$kt \rightarrow$ V/m

$t \rightarrow$ sec

$k \rightarrow$ V/m-sec.

$\therefore \boxed{K = -\frac{1}{\epsilon}\ \text{m/F or } K = -250\times10^{6}\ \text{m/F or V/m-sec}}$

problem 29 $\epsilon = 9.5\epsilon_0$, $\mu = 10\mu_0$ 06 - June /July 2012 $\sigma = 0$

Determine whether following pair of fields satisfy Maxwell's equations in the region $\sigma = 0$; $\epsilon = 9.5\epsilon_0$ and $\mu = 10\mu_0$

(i) $\bar{E} = 3y\,\hat{a}_y$ and $\bar{H} = 4x\,\hat{a}_z$. $\rightarrow \bar{H} = 4x\,\bar{a}_x$ A/m (12 Marks)

$\bar{E} = 3y\,\bar{a}_y$ v/m

Sol^n:- given $\bar{E} = 3y\,\bar{a}_y$ v/m and $\bar{H} = 4x\,\bar{a}_x$ A/m.

$\sigma = 0$ S/m ; $\epsilon = 9.5\epsilon_0$ F/m and $\mu = 10\mu_0$ H/m.

Since the given field's are not time-varying field's ∴ the Maxwell's eq^n that relates to static/steady field's are

$$\nabla\cdot\bar{D} = \rho_v \text{ C/m}^3 \leftarrow (1) \quad \text{and} \quad \nabla\cdot\bar{B} = 0 \text{ wb/m}^3 \leftarrow (2)$$

Since ρ_v is not given assume $\rho_v = 0$ C/m³

∴ the Maxwell's eq^n becomes $\nabla\cdot\bar{D} = 0$ C/m³

$$\nabla\cdot(\epsilon\bar{E}) = 0$$

$$\epsilon \neq 0$$

$$\therefore \nabla\cdot\bar{E} = 0.$$

given $E_y = 3y$ v/m.

$$\nabla\cdot\bar{E} = \frac{\partial E_x}{\partial x}\Big|^{0} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\Big|^{0}$$

$$\nabla\cdot\bar{E} = \frac{\partial}{\partial y}(3y) = 3 \neq 0$$

$$\therefore \boxed{\nabla\cdot\bar{E} \neq 0} \text{ V/m}^2$$

∴ the given field $\overline{E} = 3y\,\overline{a_y}$ V/m doesnot satisfies the Maxwell's eqⁿ $\nabla\cdot\overline{D} = 0$ C/m³.

Ily. using eqⁿ ie $\nabla\cdot\overline{B} = 0$

using $\overline{B} = \mu\overline{H}$ wb/m²

$$\therefore \nabla\cdot(\mu\overline{H}) = 0$$

$$\mu \neq 0$$

$$\therefore \boxed{\nabla\cdot\overline{H} = 0} \ \text{A/m}^2$$

$$\frac{\partial H_x}{\partial x} + \cancel{\frac{\partial H_y}{\partial y}}^{0} + \cancel{\frac{\partial H_z}{\partial z}}^{0} = \nabla\cdot\overline{H} \ ; \ \text{A/m}^2$$

given $\overline{H} = 4x\overline{a_x} = H_x\overline{a_x}$ A/m

$$\therefore \nabla\cdot\overline{H} = \frac{\partial}{\partial x}(4x)$$

$$\boxed{\nabla\cdot\overline{H} = 4 \neq 0} \ \text{A/m}^2$$

∴ the given field $\overline{H} = 4x\overline{a_x}$: A/m doesnot satisfies the Maxwell's eqⁿ $\nabla\cdot\overline{B} = 0$ or $\nabla\cdot\overline{H} = 0$.

ie $\Rightarrow \nabla\cdot\overline{H} \neq 0$ Ily $\nabla\cdot\overline{B} \neq 0$.

problem 30

$\epsilon = 2\times10^{-6}$ F/m $\quad \mu = 1.25\times10^{-5}$ H/m $\quad \sigma = 0$

010-Dec/Jan 2015

A homogeneous material has $\varepsilon = 2\times10^{-6}$ F/m and $\mu = 1.25\times10^{-5}$ H/m and $\sigma = 0$. Electric field intensity $\vec{E} = 400\cos(10^9 t - kz)\hat{a}_x$ V/m. If all the fields vary sinusoidally, find $\vec{D}$, $\vec{B}$, $\vec{H}$ and k using Maxwell's equations. (12 Marks)

$\rightarrow \bar{D}, \bar{B}, \bar{H}$ & K.

Soln:- given $\epsilon = 2\times10^{-6}$ F/m and $\mu = 1.25\times10^{-5}$ H/m and $\sigma = 0$.

$\rightarrow \bar{E} = 400\cos(10^9 t - Kz)\,\bar{a}_x$ v/m.

$\bar{E} \Rightarrow f^n(z,t)$ and $E_x = 400\cos(10^9 t - Kz)$ v/m.

i) $\bar{D} = \epsilon \bar{E}$ C/m²

given $\epsilon = 2\times10^{-6}$ F/m

$\therefore \bar{D} = 2\times10^{-6}\left[400\cos(10^9 t - Kz)\right]\bar{a}_x$ C/m²

$$\Rightarrow \boxed{\bar{D} = 800\cos\left[10^9 t - Kz\right]\bar{a}_x} \ \mu C/m^2 \leftarrow (a)$$

ii) To find $\bar{B}$, using $\nabla\times\bar{E} = -\dfrac{\partial\bar{B}}{\partial t}$

$$\begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 0 & 0 & \partial/\partial z \\ E_x & 0 & 0 \end{vmatrix} = -\frac{\partial\bar{B}}{\partial t}$$

$$-\frac{\partial E_x}{\partial z}(-\bar{a}_y) = -\frac{\partial\bar{B}}{\partial t}$$

$$\Rightarrow \frac{\partial\bar{B}}{\partial t} = -\frac{\partial E_x}{\partial z}\bar{a}_y$$

$$\frac{\partial \overline{B}}{\partial t} = -\frac{\partial}{\partial z}\left[400\cos(10^9 t - kz)\right]\overline{a_y}$$

$$= +400\sin(10^9 t - kz) \times -k\,\overline{a_y}$$

$$\frac{\partial \overline{B}}{\partial t} = -400k\sin(10^9 t - kz)\,\overline{a_y}$$

$$\overline{B} = \int \frac{\partial \overline{B}}{\partial t}\cdot dt = +400k\,\frac{\cos(10^9 t - kz)\overline{a_y}}{10^9}$$

$$\therefore \boxed{\overline{B} = 400k\cos(10^9 t - kz)\overline{a_y}}\ \text{n}\,\text{wb/m}^2 \text{ (or) } \text{n Tesla.}$$

iii) $\overline{H} = \dfrac{\overline{B}}{\mu}$ A/m.

given $\mu = 1.25 \times 10^{-5}$ H/m

$$\therefore \overline{H} = \frac{400k}{1.25\times10^{-5}}\cos[10^9 t - kz]\,\overline{a_y} \times 10^{-9}$$

$$\therefore \boxed{\overline{H} = 32k\cos[10^9 t - kz]\overline{a_y}}\ \text{mA/m}$$

To find value of 'k' ; using Maxwell's eqn

$$\nabla \times \overline{H} = \overline{J_c} + \frac{\partial \overline{D}}{\partial t} \text{ A/m}^2$$

given $\sigma = 0 \quad \therefore \overline{J_c} = \sigma \overline{E}^{\,0} = 0.$

$$\therefore \quad \nabla \times \overline{H} = \frac{\partial \overline{D}}{\partial t}$$

$$\begin{vmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ 0 & 0 & \partial/\partial z \\ 0 & H_y & 0 \end{vmatrix} = \frac{\partial \overline{D}}{\partial t}$$

$$-\frac{\partial H_y}{\partial z} \overline{a_x} = \frac{\partial \overline{D}}{\partial t}$$

$$\frac{\partial \overline{D}}{\partial t} = -\frac{\partial}{\partial z}\left[32k \cos[10^9 t - kz]\right] \times 10^{-3} \overline{a_x}$$

$$= -32k \times -\sin(10^9 t - kz) \times -k \times 10^{-3} \overline{a_x}$$

$$\frac{\partial \overline{D}}{\partial t} = -32k^2 \times 10^{-3} \sin[10^9 t - kz] \overline{a_x}$$

$$\overline{D} = \int \frac{\partial \overline{D}}{\partial t} \cdot \partial t$$

$$\overline{D} = \frac{-32k^2 \times 10^{-3} \times -\cos(10^9 t - kz)}{10^9} \overline{a_x}$$

$$\overline{D} = +32k^2 \times 10^{-12} \cos[10^9 t - kz] \overline{a_x} \leftarrow (b)$$

Equating magnitudes of eq^n (a) and (b)
[i.e component of 'x']

$$10^{-6} \times 800 \cos[\cancel{10^9 t} - kz] = 32k^2 \times 10^{-12} \cos[\cancel{10^9 t} - kz]$$

$$\Rightarrow k^2 = \frac{800 \times 10^{-6}}{32 \times 10^{-12}}$$

$$k^2 = 25 \times 10^6$$

$$k = \pm\sqrt{25 \times 10^6}$$

$$\boxed{k = \pm 5000} \text{ rad/m}$$

note:- Unit of 'k'

$\bar{E} = 400 \cos(10^9 t - kz)\bar{a}_x$ V/m

$\bar{E} = E_0 \cos(\omega t \pm \phi)\bar{a}_x$ V/m

General form.

phase angle in 'rad'

$kz \to$ rad

'm'

$\therefore \boxed{k \to \text{rad/m}}$ in

$\therefore$ using 'k' value in obtained $\bar{D}$, $\bar{H}$ and $\bar{B}$

i.e $\to \bar{D} = 800 \cos[10^9 t - kz]$ μC/m²

put $k = \pm 5000$ rad/m

$$\therefore \boxed{\bar{D} = 800 \cos[10^9 t \mp 5000z]} \ \mu C/m^2$$

$\to \bar{B} = 400k \cos[10^9 t - kz]\,\bar{a}_y$ nwb/m²

$\bar{B} = 400(\pm 5000) \cos[10^9 t \mp 5000z]\,\bar{a}_y$ nwb/m²

$$\therefore \boxed{\bar{B} = \pm 2 \cos[10^9 t \mp 5000z]\,\bar{a}_y} \ mwb/m^2$$

and

$$\overline{H} = 32k\cos[10^9t - kz]\,\overline{a_y} \;;\; mA/m.$$

$$\overline{H} = 32(\pm 5000)\cos[10^9t \mp 5000z]\,\overline{a_y} \;;\; mA/m.$$

$$\boxed{\overline{H} = \pm 160\cos(10^9t \mp 5000z)\,\overline{a_y}} \;;\; A/m$$

problem 31)

What values of A and β are required for two fields $\bar{E} = 120\pi Cos[10^6\pi t - \beta x]a_y$ v/m and $\bar{H} = ACos[10^6\pi t - \beta x]a_z$ A/m. Satisfies Maxwell's equation in a medium where $\epsilon_r = \mu_r = 4$ and $\sigma = 0$.

soln:- given $\bar{E} = 120\pi \cos[10^6\pi t - \beta x]\bar{a}_y$ v/m.

$\bar{H} = A\cos[10^6\pi t - \beta x]\bar{a}_z$ A/m.

$\epsilon_r = \mu_r = 4$ and $\sigma = 0$.

using Maxwell's eqn

$$\nabla \times \bar{H} = \cancel{\bar{J}_c}^{\,0} + \frac{\partial \bar{D}}{\partial t}$$

$$\nabla \times \bar{H} = \epsilon \frac{\partial \bar{E}}{\partial t} \leftarrow (1) \quad A/m^2$$

and $$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \quad v/m^2$$

$$\Rightarrow \nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t} \quad v/m^2 \leftarrow (2).$$

using eqn (1)

$$\begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \partial/\partial x & 0 & 0 \\ 0 & 0 & H_z \end{vmatrix} = \epsilon \frac{\partial \bar{E}}{\partial t}$$

$$-\frac{\partial H_z}{\partial x} \cdot \bar{a}_y = \epsilon \frac{\partial \bar{E}}{\partial t}$$

$$-\frac{\partial}{\partial x}\left[A\cos(10^6\pi t - \beta x)\right]\bar{a}_y = -\epsilon \times 120\pi \times \sin[10^6\pi t - \beta x]\bar{a}_y \times 10^6\pi$$

$$\Rightarrow A\beta \sin(10^6\pi t - \beta x)\,\overline{a_y} = -120\pi^2 \epsilon \times 10^6 \sin(10^6\pi t - \beta x)\,\overline{a_y}$$

Equating 'y' components on bothside

$$A\beta \cancel{\sin(10^6\pi t - \beta x)} = -120\pi^2 \epsilon \times 10^6 \cancel{\sin(10^6\pi t - \beta x)}$$

$$\Rightarrow A\beta = -120\pi^2 \epsilon \times 10^6 \longleftarrow (a).$$

Ily using eqn (2) $\nabla \times \overline{E} = -\mu \dfrac{\partial \overline{H}}{\partial t}$ v/m²

$$\begin{vmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ \partial/\partial x & 0 & 0 \\ 0 & E_y & 0 \end{vmatrix} = -\mu \frac{\partial \overline{H}}{\partial t}$$

$$\frac{\partial E_y}{\partial x}\overline{a_z} = -\mu \frac{\partial \overline{H}}{\partial t}$$

$$120\pi\beta \sin(10^6\pi t - \beta x)\overline{a_z} = -A\mu \sin(10^6\pi t - \beta x)\overline{a_z} \times 10^6\pi$$

Equating 'z' components on bothside.

$$\Rightarrow 120\cancel{\pi}\beta = -A\mu \times 10^6 \cancel{\pi}$$

$$\beta = \frac{-\mu A \times 10^6}{120} \longleftarrow (b)$$

using eqn (b) in eqn (a)

$$A\left[\frac{-\mu A \times 10^6}{120}\right] = -120\pi^2 \epsilon \times 10^6 \Rightarrow A^2 = \frac{(120)^2\pi^2 \epsilon}{\mu}$$

$$A^2 = 1.00136 \Rightarrow \boxed{A = \pm 1.00068}$$

if $\boxed{A = +1.00068} \longrightarrow \boxed{B = -0.0425}$

(a) if $\boxed{A = -1.00068} \longrightarrow \boxed{B = +0.0425}$

problem 32

A certain material has conductivity $\sigma = 0$ and relative permeability $\mu_r = 1$. Make use of Maxwell's equations to find the following.

I. $H(z,t)$ and II. ϵ_r

assume $\overline{E} = 800 Sin(10^6 t - 0.1z)\overline{a}_y v/m$ inside the material.

[Schaum's outline]

soln:-

given

$$\overline{E} = 800 \sin(10^6 t - 0.1z)\,\overline{a_y} \text{ v/m} \quad \leftarrow (a)$$

$\sigma = 0$ v/m ; $\mu_r = 1$.

using Maxwell's eqn derived from Faraday's law

i.e $\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$ v/m²

$$\begin{vmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ 0 & 0 & \partial/\partial z \\ 0 & E_y & 0 \end{vmatrix} = -\frac{\partial \overline{B}}{\partial t}$$

$$-\frac{\partial E_y}{\partial z}\overline{a_x} = -\frac{\partial \overline{B}}{\partial t} \Rightarrow \frac{\partial \overline{B}}{\partial t} = \frac{\partial E_y}{\partial z}\overline{a_x}$$

$$\Rightarrow \frac{\partial \overline{B}}{\partial t} = \frac{\partial}{\partial z}\left[800 \sin(10^6 t - 0.1z)\right]\overline{a_x}$$

$$\frac{\partial \overline{B}}{\partial t} = 800 \cos[10^6 t - 0.1z] \times -0.1\,\overline{a_x}$$

$$\frac{\partial \overline{B}}{\partial t} = -80 \cos[10^6 t - 0.1z]\,\overline{a_x}$$

$$\overline{B} = \int \frac{\partial \overline{B}}{\partial t} \cdot \partial t = \frac{-80 \times \sin(10^6 t - 0.1z)}{10^6}\,\overline{a_x}$$

$$\overline{B} = -80 \times 10^{-6} \sin(10^6 t - 0.1z)\,\overline{a_x}$$

$$\overline{B} = \mu_0 \mu_r \overline{H} \quad wb/m^2$$

$$\therefore \overline{H} = \frac{\overline{B}(z,t)}{\mu_0 \mu_r} = \frac{-80\times10^{-6} \sin(10^6 t - 0.1z)}{4\pi\times10^{-7}} \overline{a_x}$$

$$\boxed{\overline{H}(z,t) = -63.66 \sin(10^6 t - 0.1z)\,\overline{a_x}} \; A/m$$

using Maxwell's eqn derived from Ampere's Circuital Law

ie $\nabla\times\overline{H} = \cancelto{0}{\overline{J_c}} + \frac{\partial \overline{D}}{\partial t} \Rightarrow \boxed{\nabla\times\overline{H} = \frac{\partial \overline{D}}{\partial t}} \; A/m^2$

$$\begin{vmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ 0 & 0 & \partial/\partial z \\ H_x & 0 & 0 \end{vmatrix} = \frac{\partial \overline{D}}{\partial t}$$

$$\frac{\partial H_x}{\partial z}\overline{a_y} = \frac{\partial \overline{D}}{\partial t} \Rightarrow \frac{\partial \overline{D}}{\partial t} = \frac{\partial}{\partial z}\left[-63.66 \sin(10^6 t - 0.1z)\right]\overline{a_y}$$

$$\frac{\partial \overline{D}}{\partial t} = -63.66 \cos(10^6 t - 0.1z) \times -0.1\,\overline{a_y}$$

$$\frac{\partial \overline{D}}{\partial t} = +6.366 \cos(10^6 t - 0.1z)\overline{a_y}$$

$$\overline{D} = \int \frac{\partial \overline{D}}{\partial t}\,\partial t = \frac{6.366 \sin(10^6 t - 0.1z)}{10^6}\overline{a_y}$$

$$\overline{D} = 6.366\times10^{-6} \sin(10^6 t - 0.1z)\overline{a_y} \quad C/m^2$$

$$\overline{D} = \epsilon \overline{E} = \epsilon_0 \epsilon_r \overline{E} \Rightarrow \overline{E} = \frac{\overline{D}}{\epsilon_0 \epsilon_r} \; v/m$$

ie $\overline{E} = \frac{6.366\times10^{-6}}{\epsilon_0 \epsilon_r} \sin[10^6 t - 0.1z]\,\overline{a_y} \; v/m \leftarrow (b)$

Comparing eqn (a) and eqn (b) $\Rightarrow \frac{6.366\times10^{-6}}{8.854\times10^{-12}\epsilon_r} = 800$

$$\Rightarrow \boxed{\epsilon_r = 898.7} \; F/m.$$

problem 3

Show that an emf induced in a Faradays disc generator is $e = -\frac{\omega B a^2}{2}$ volts where 'ω' is the angular velocity in rad/sec, B is the magnetic Flux density in Tesla and 'a' is the radius of the disc in meter.

problem 6

A circular conducting Loop of radius 40cm Lies in xy plane and has resistance of 20Ω. If the magnetic flux density in the region is given as

$$\bar{B} = 0.2\cos(500t)\,\bar{a}_x + 0.75\sin(400t)\,\bar{a}_y + 1.2\cos(314t)\,\bar{a}_z \text{ Tesla.}$$

Determine Effective value of induced current in the Loop

Problem 7. A Straight conductor of Length 0.2m, Lies on x-axis with one end at origin. The conductor is Subjected to a magnetic flux density $\bar{B} = 0.04\,\bar{a}_y$ Tesla and the velocity $\bar{v} = 2.5\sin 10^3 t\;\bar{a}_z$ m/sec. Determine motional Emf induced in the conductor. (6m).

Problem 8.

A copper disc 40cm diameter is rotated at 3000 rpm on a horizontal axis perpendicular to and through the centre of disc axis, lying in magnetic meridian. Two brushes make contact with disc at diametrically opposite points on the edge. If horizontal components of earth's field is 0.02mT, find the induced emf between brushes.

problem 33

S.) A Conductor Carries Steady Current of I amperes. The components of Current density vector $\bar{J}$ are $J_x = 2ax$ and $J_y = 2ay$. Find the third component J_z. Derive any relation employed. June - 2006 (10M).

Note:- Module-5A Question.

Soln:- using Continuity eqn

$$\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t} \; A/m^3.$$

if Conductor Carries Steady Current then

$$\rho_v = \text{Constant} \Rightarrow \frac{\partial \rho_v}{\partial t} = 0 \; C/m^3\text{-sec}.$$

$$\Rightarrow \nabla \cdot \bar{J} = 0$$

$$\frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} = 0$$

$$\frac{\partial}{\partial x}(2ax) + \frac{\partial}{\partial y}(2ay) + \frac{\partial J_z}{\partial z} = 0$$

$$2a + 2a + \frac{\partial J_z}{\partial z} = 0$$

$$\frac{\partial J_z}{\partial z} = -4a.$$

Integrating wrt z

$$\boxed{J_z = -4az + K} \; A/m^2$$

where K - integral Constant.

Module-5 (part A)

Summary.

a) List of Symbols.

1. angular frequency (ω) $= 2\pi f$ rad/sec.

2. frequency (f) $\longrightarrow$ Hz or cycles/sec.

3. Time period (T) $= \frac{1}{f} \longrightarrow$ seconds (Sec).

4. Speed of Light $v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ m/sec.

$$\boxed{v = 3\times10^8} \text{ m/sec}$$

5. Faraday's Law

$$\boxed{e = -\frac{d\phi}{dt}} \text{ volts}$$

$$\phi = NLI ; \text{ wb}$$

$$\boxed{e = -NL\frac{dI}{dt}} \text{ volts}$$

6. $\boxed{\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}}$ V/m² .. Maxwell's eqn from Faraday's Law.

7. Modified Ampere's Circuital Law

$$\boxed{\nabla \times \bar{H} = \bar{J}_c + \frac{\partial \bar{D}}{\partial t}} \quad A/m^2.$$

8. $\boxed{\bar{J}_c = \sigma \bar{E}}$ A/m^2. - - - point form of ohm's Law
(or) Conducting current density

9. displacement Current density

$$\boxed{\bar{J}_D = \frac{\partial \bar{D}}{\partial t} = j\omega\epsilon\bar{E}} \quad A/m^2$$

10. Loss tangent $\left|\frac{\bar{J}_c}{\bar{J}_d}\right| = \left(\frac{\sigma}{\omega\epsilon}\right)$ - - dimension Less.

11. $\nabla \cdot \bar{B} = 0$ Wb/m^3 - - - point form of Gauss's Law (magnetostatics).

b. List of Formulaes:-

i. Faraday's Law:- The Magnitude of the induced emf in a Circuit is equal to the rate of change of the magnetic flux through it and its direction opposes the Flux change.

$$\boxed{e = -\frac{d\phi}{dt}} \text{ volts}$$

coil with 'N' turns

$$\boxed{e = -N\frac{d\phi}{dt}} \text{ volts.}$$

2. Lenz Law:- the induced emf is in Such a direction as to oppose the change causing it.

3. Maxwell's equation from Faraday's Law

integral form.

$$\boxed{\oint_{\langle l \rangle} \overline{E}\cdot\overline{dl} = \int_{\langle S \rangle} (\nabla \times \overline{E})\cdot\overline{ds} = -\int_{\langle S \rangle} \frac{\partial \overline{B}}{\partial t}\cdot\overline{ds}} \text{ volts}$$

$$\boxed{\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}} \text{ V/m}^2 \text{ ---- point form}$$

4. Total (or) net Emf.

$$\boxed{\oint_{\langle l \rangle} \overline{E}\cdot\overline{dl} = -\underbrace{\int_{\langle S \rangle} \frac{\partial \overline{B}}{\partial t}\cdot\overline{ds}}_{\text{Transformer emf}} + \underbrace{\oint_{\langle l \rangle} (\overline{v} \times \overline{B})\cdot\overline{dl}}_{\text{motional Emf.}}}$$

5. if given $\bar{E}$ field is said to be arise from static distribution of charges only when $\boxed{\nabla \times \bar{E} = 0}$ V/m².

if $\boxed{\nabla \times \bar{E} \neq 0}$ V/m² then $\bar{E}$ donot arise from static distribution of charges.

6. Ampere's Circuital Law $\boxed{\nabla \times \bar{H} = \bar{J}}$ A/m².

7. Modified Ampere's Circuital Law.

$$\boxed{\oint_{\langle l \rangle} \bar{H} \cdot \bar{dl} = \int_{\langle s \rangle} \bar{J} \cdot \bar{ds} + \int_{\langle s \rangle} \frac{\partial \bar{D}}{\partial t} \cdot \bar{ds}}$$

integral form. Ampere's

$$\underbrace{I_t}_{\text{total current}} = \underbrace{I_c}_{\text{Conduction current}} + \underbrace{I_D}_{\text{displacement current}}$$

Ampere's

$$\boxed{\nabla \times \bar{H} = \bar{J}_c + \frac{\partial \bar{D}}{\partial t}} \text{ A/m}^2$$

$$\bar{J}_t = \bar{J}_c + \bar{J}_D$$

total Current density = Conduction Current density + displacement Current density

8. Conduction Current density ($\bar{J}_c$).

$$\boxed{\bar{J}_c = \sigma \bar{E}} \text{ A/m}^2$$ --- point form of ohm's Law.

9. Displacement Current density ($\bar{J}_D$)

$$\boxed{\bar{J}_D = \frac{\partial \bar{D}}{\partial t} = j\omega\epsilon\bar{E}} \text{ A/m}^2$$

Note:- $\boxed{\bar{J}_c \perp \bar{J}_D}$ always.

10. Loss tangent.

$$\boxed{\left|\frac{\bar{J}_c}{\bar{J}_D}\right| = \frac{\sigma}{\omega\epsilon}}$$ ---- Loss tangent.

if $\left(\frac{\sigma}{\omega\epsilon}\right) >> 1$; medium is Good conductor.

$\left(\frac{\sigma}{\omega\epsilon}\right) << 1$; Medium is Good dielectric.

$\left(\frac{\sigma}{\omega\epsilon}\right) \rightarrow 0$; medium is perfect dielectric.

11. Maxwell's equations in point and integral form.

a. Maxwell's equations for steady (or) static fields.

Sl.No	Integral form	Point form	Remark
01.	$\oint_{\langle l \rangle} \bar{E}\cdot\bar{dl} = 0 ; V$	$\nabla\times\bar{E} = 0 ; V/m^2$	Work done.
02.	$\oint_{\langle S \rangle} \bar{D}\cdot\bar{dS} = \int_{\langle vol \rangle} \rho_v \, dv ; C$	$\nabla\cdot\bar{D} = \rho_v ; C/m^3$	Gauss's Law (Electrostatics).
03.	$\oint_{\langle l \rangle} \bar{H}\cdot\bar{dl} = \int_{\langle S \rangle} \bar{J}\cdot\bar{dS} ; A$	$\boxed{\nabla\times\bar{H} = \bar{J}} ; A/m^2$	Ampere's Law
04.	$\oint_{\langle S \rangle} \bar{B}\cdot\bar{dS} = 0 ; wb$	$\nabla\cdot\bar{B} = 0 ; wb/m^3$	Gauss's Law (magnetostatics)
05.	$\oint_{\langle S \rangle} \bar{J}\cdot\bar{dS} = 0 ; A$	$\nabla\cdot\bar{J} = 0 ; A/m^3$	Continuity Current eq^n.

d. Maxwell's Equations in perfect Dielectric Medium. (Lossless medium)

$\left(\frac{\sigma}{\omega\epsilon}\right) \rightarrow 0 \quad \therefore \bar{J}_C = 0$ and No free charge Exist in dielectric Medium

$\therefore \rho_v = 0.$

$\epsilon = \epsilon_r \epsilon_0$ F/m and $\mu = \mu_0 \mu_r$ H/m

Sl.No.	Integral form	point form	Remark.
01.	$\oint_{\langle l \rangle} \bar{E} \cdot \bar{dl} = -\int_{\langle S \rangle} \left(\frac{\partial \bar{B}}{\partial t}\right) \cdot \bar{ds}$	$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} = -\mu_0 \mu_r \frac{\partial \bar{H}}{\partial t}$	Faraday's Law.
02.	$\oint_{\langle l \rangle} \bar{H} \cdot \bar{dl} = \int_{\langle S \rangle} \frac{\partial \bar{D}}{\partial t} \cdot \bar{ds}$	$\nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t} = \epsilon_0 \epsilon_r \frac{\partial \bar{E}}{\partial t}$	Modified ampere's Law.
03.	$\oint_{\langle S \rangle} \bar{D} \cdot \bar{ds} = 0$	$\nabla \cdot \bar{D} = 0$	Gauss's Law (Electrostatic)
04.	$\oint_{\langle S \rangle} \bar{B} \cdot \bar{ds} = 0$	$\nabla \cdot \bar{B} = 0$	Gauss's Law (magnetostatics)
05.	$\oint_{\langle S \rangle} \bar{J} \cdot \bar{ds} = 0$	$\nabla \cdot \bar{J} = 0$	Continuity current equation.

e. Maxwell's Equation In Good Conducting Medium. (Lossy-medium)

$\left(\frac{\sigma}{\omega\epsilon}\right) >> 1 \Rightarrow \overline{J_C} >> \overline{J_D}$

$\therefore$ neglect $\overline{J_D}$. Since $\frac{\partial \overline{D}}{\partial t}$ is Small.

Since $\frac{\partial \overline{D}}{\partial t}$ is very less $\therefore \rho_v = \nabla \cdot \overline{D} \rightarrow 0$

$\therefore \rho_v = 0 \; C/m^3$.

Sl.No.	Integral form	Point form	Remark
01.	$\oint_{\langle l \rangle} \overline{E} \cdot \overline{dl} = -\int_{\langle S \rangle} \frac{\partial \overline{B}}{\partial t} \cdot \overline{dS}$	$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$	Faraday's Law
02	$\oint_{\langle l \rangle} \overline{H} \cdot \overline{dl} = \int_{\langle S \rangle} \overline{J} \cdot \overline{dS}$	$\nabla \times \overline{H} = \overline{J}$	Modified ampere's Law.
03.	$\oint_{\langle S \rangle} \overline{D} \cdot \overline{dS} = 0$	$\nabla \cdot \overline{D} = 0$	Gauss's Law (Electrostatics)
04.	$\oint_{\langle S \rangle} \overline{B} \cdot \overline{dS} = 0$	$\nabla \cdot \overline{B} = 0$	Gauss's Law (magnetostatics)
05.	$\oint_{\langle S \rangle} \overline{J} \cdot \overline{dS} = 0$	$\nabla \cdot \overline{J} = 0$	Continuity of current equation.

f. Maxwell's Equation ~~for~~ Good dielectric Medium (or) Lossless Medium.

In Lossless (or) Good dielectric medium $\left(\frac{\sigma}{\omega\epsilon}\right) << 1$.

$\Rightarrow \sigma << \omega\epsilon \quad \therefore \quad \bar{J}_c << \frac{\partial \bar{D}}{\partial t}$

$\therefore$ neglect the term $\bar{J}_c$. ie $\bar{J}_c \rightarrow 0$.

and $\boxed{\rho_v = 0}$ bcz no free charges in dielectrics.

S/No.	Integral form	Point form	Remark.
01.	$\oint_{\langle l \rangle} \bar{E}\cdot\bar{dl} = -\int_{\langle S \rangle} \frac{\partial \bar{B}}{\partial t}\cdot\bar{ds}$	$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} ; V/m^2$	Faraday's Law.
02.	$\oint_{\langle l \rangle} \bar{H}\cdot\bar{dl} = \int_{\langle S \rangle} \frac{\partial \bar{D}}{\partial t}\cdot\bar{ds} ; A ;$	$\nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t} A/m^2$	~~Modified~~ Modified ampere's Law
03.	$\oint_{\langle S \rangle} \bar{D}\cdot\bar{dS} = 0$	$\nabla\cdot\bar{D} = 0 ; C/m^3$	Gauss Law (electrostatics)
04.	$\oint_{\langle S \rangle} \bar{B}\cdot\bar{ds} = 0$	$\nabla\cdot\bar{B} = 0 ; Wb/m^3$	Gauss's Law (magnetostatics)
05.	$\oint_{\langle S \rangle} \bar{J}\cdot\bar{ds} = 0 ; A$	$\nabla\cdot\bar{J} = 0 ; A/m^3$.	Continuity Current equation.

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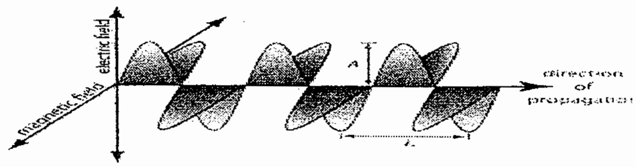
Module -5(Part-B): Electro-Magnetic Waves



Part-B : Uniform Plane Wave

Wave propagation in free space and good conductors. Poynting's theorem and wave power, Skin Effect.

Topics:

5.4 Introduction to Electro-Magnetic Waves.
- Recap of Maxwell's Equations in Free Space.
- Concept of Wave Motion.
- Concept of Wave Equation.

5.5 Wave Propagation in Free Space/General Wave Equation in Free space + Solution of Wave Equation in Free Space.

5.6 Definition of Plane Waves and Uniform Plane Waves.

5.7 Wave Propagation in Good Conductors/ Wave Equation in Good Conducting medium + Solution of Wave Equation in Good Conducting Medium.

5.8 Wave Propagation in Good Dielectrics and Perfect dielectrics / Wave Equation in Good Dielectrics and Perfect dielectric medium + its Solution.

5.9 Transverse nature of Electro-Magnetic Waves.

5.10 Relationship b/w $|\mathbf{E}|$ and $|\mathbf{H}|$.

5.11 Characteristics of Medium/ General Definitions of:
- Propagation Constant (γ)
- Attenuation Constant (α)
- Phase Constant (β)
- Wave Velocity (v)
- Wave Length (λ)
- Intrinsic Impedance (η)

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5.12 Wave Equation in Phasor form.

5.13 Expressions for $\alpha, \beta, \gamma, \lambda$, v, and η in
- General case
- Free Space
- Perfect Dielectrics
- Good Conductors and
- Good Dielectrics

5.14 Concept of Skin effect and Skin depth for Good Conductors.

5.15 Poynting's theorem and wave power.
- State and Prove Poynting's theorem
- Expression for wave power/ average Power density in Lossless and Lossy medium

Miscellaneous Topics:

5.16 Polarization of Uniform Plane waves.

5.17 Brewster angle in Wave Propagation.

Summary
- List of Symbols
- List of Formulae

Topics:

1. *Introduction to Electro-Magnetic Waves.*
   - *Recap of Maxwell's Equations in Free Space.*
   - *Concept of Wave Motion.*
   - *Concept of Wave Equation.*

Topic 5.4

2. *Definition of Plane Waves and Uniform Plane Waves.*
3. *Wave Propagation in Free Space/General Wave Equation in Free space + Solution of Wave Equation in Free Space.*

Introduction :-

An Electro-magnetic wave propogation can be explained by using Maxwell's equations. The Existence of EM waves was stated by Prof. Heinrich Hertz. actually Maxwell himself predicted the Existence of EM waves earlier. Hertz was the first Scientist who generated and detected radio waves Succesfully.

EM waves are function of Space and time.

Typical examples of EM waves are radio waves, TV Signals etc.

1.

02-DEC2010

Derive the wave equation of E & H. (08 Marks)

02-DEC2008/Jan 2009

Derive the wave equations for $\vec{E}$ and $\vec{H}$ in a general medium. (07 Marks)

06-DEC2011/Jan 2012

Derive general wave equations in terms of $\overline{D}$ and $\overline{B}$ in uniform medium using Maxwell's equations. (08 Marks)

10-DEC2011/Jan 2012

With usual notations, obtain the general wave equations for electric and magnetic fields. (06 Marks)

10-Jan 2013

Starting from Maxwell's equations obtain the general wave equations in electric and magnetic field. (10 Marks)

10-DEC 2013/Jan 2014

Using Maxwell's equation derive an expression for uniform plane wave in free space. (08 Marks)

10-June/July 2013

Starting from Maxwell's equations, obtain the wave equations in free space. (07 Marks)

06 - June /July 2011

Starting from Maxwell's equations obtain the general wave equations in electric and magnetic fields. (10 Marks)

10 - June /July 2012

Starting from Maxwell's equation, derive the wave equation for a uniform plane wave travelling in free space. (08 Marks)

02 - June /July 2012

Using Maxwell's equations, show that the free space wave equation in E may be written as

$$\nabla^2\overline{E} - \mu \in \frac{\partial \overline{E}}{\partial t^2} = 0$$ (06 Marks)

010-Dec/Jan 2015

Starting form Maxwell's equations derive wave equation in E and H for a uniform plane wave travelling in free space. (10 Marks)

06 – May/June 2010

What is meant by 'uniform plane wave'? Derive the expression for UPW in free space. (07 Marks)

06 –June/July 2014

Derive an equation for wave propagation in free-space. (10 Marks)

06 –Dec/Jan 2008

What is uniform plane wave? Explain its propagation in free space with necessary equations. (08 Marks)

Method -I :-

In free space [Source free region, where $\rho_v = 0$; $\sigma = 0$ and $\bar{J_c} = 0$] - the Maxwell's equation for free space which are $\epsilon = \epsilon_0$ F/m and $\mu = \mu_0$ H/m.

$$\nabla \times \bar{H} = \bar{J_c} + \frac{\partial \bar{D}}{\partial t}; \; A/m^2 \implies \nabla \times \bar{H} = \epsilon_0 \frac{\partial \bar{E}}{\partial t} \rightarrow (1)$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}; \; V/m^2 \implies \nabla \times \bar{E} = -\mu_0 \frac{\partial \bar{H}}{\partial t} \rightarrow (2)$$

$$\nabla \cdot \bar{D} = \rho_v : C/m^3 \implies \nabla \cdot \bar{D} = 0$$

w.k.t $\bar{D} = \epsilon_0 \bar{E}$ C/m²

$$\left. \begin{array}{l} \text{and } \epsilon_0 \nabla \cdot \bar{E} = 0 \\ \text{(or) } \nabla \cdot \bar{E} = 0 \end{array} \right\} \rightarrow (3)$$

$$\nabla \cdot \bar{B} = 0 \implies \mu_0 (\nabla \cdot \bar{H}) = 0 \implies \nabla \cdot \bar{H} = 0 \rightarrow (4)$$

and $\bar{B} = \mu_0 \bar{H}$ wb/m²

Concept of Wave motion :-

eqn(1) states that if the Electric field $\bar{E}$ changes with time at some point this change produce a rotating curling $\bar{H}$ varying spatially in magnetic field at that point; a direction normal to its orientation.

eqn(2) at time-varying $\bar{H}$ generates a rotating $\bar{E}$ [curl $\bar{E}$] and this $\bar{E}$ varies spatially in a direction normal to its orientation.

Concept of Wave Equation:-

assume an EM wave travelling in free space. Consider that an Electric field is in x-direction; while a Magnetic field is in y-direction. both the field will not vary with x and y but with z only. they will also change with time as the wave propogating in free space.

Consider a Maxwell's eqn expressed in $\bar{E}$ and $\bar{H}$ as

$$\nabla \times \bar{H} = \bar{J_c} + \frac{\partial \bar{D}}{\partial t} : A/m^2$$

in free space $\sigma = 0 \ \mho/m \ \therefore \ \bar{J_c} \to 0$.

$$\nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t} \leftarrow (1)$$

let $\bar{D} = D_x \bar{a_x} + D_y \bar{a_y} + D_z \bar{a_z} \ C/m^2$

and
$$\nabla \times \bar{H} = \begin{vmatrix} \bar{a_x} & \bar{a_y} & \bar{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix}$$

$$= \left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right]\bar{a_x} + \left[\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right]\bar{a_y} + \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right]\bar{a_z}$$

$\therefore$ eq^n (1) can be written as

$$\left[\cancel{\frac{\partial H_z}{\partial y}}^{\to 0} - \frac{\partial H_y}{\partial z}\right]\bar{a_x} + \left[\cancel{\frac{\partial H_x}{\partial z}}^{\to 0} - \cancel{\frac{\partial H_z}{\partial x}}^{0}\right]\bar{a_y} + \left[\cancel{\frac{\partial H_y}{\partial x}}^{0} - \cancel{\frac{\partial H_x}{\partial y}}^{0}\right]\bar{a_z}$$

$$= \frac{\partial}{\partial t}\left[D_x\bar{a_x} + D_y\bar{a_y} + D_z\bar{a_z}\right] \longleftarrow (2)$$

as wave $\bar{H}$ is travelling in y-direction $\therefore H_x = H_z = 0$.

and $H_y = f^n(z,t)$ only $\therefore \frac{\partial H_y}{\partial x} = 0;\ \frac{\partial H_y}{\partial y} = 0$.

$\therefore$ eq^n (2) becomes

$$-\frac{\partial H_y}{\partial z}\bar{a_x} = \frac{\partial}{\partial t}\left[D_x\bar{a_x} + D_y\bar{a_y} + D_z\bar{a_z}\right]$$

equating 'x' component on both side

$$\Rightarrow -\frac{\partial H_y}{\partial z} = \frac{\partial D_x}{\partial t} \quad \text{and} \quad \bar{D} = \epsilon\bar{E}\ C/m^2$$

(or) $D_x = \epsilon E_x\ C/m^2$

$$\therefore \boxed{\frac{\partial H_y}{\partial z} = -\epsilon\frac{\partial E_x}{\partial t}} \longleftarrow (a)$$

Now consider a Maxwell's eq^n derived from Faraday's Law

i.e $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$

$$\left[\frac{\partial E_y}{\partial z}^{\nearrow 0} - \frac{\partial E_z}{\partial y}^{\nearrow 0}\right]\bar{a}_x + \left[\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right]\bar{a}_y + \left[\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right]\bar{a}_x$$

$$= -\frac{\partial}{\partial t}\left[B_x\bar{a}_x + B_y\bar{a}_y + B_z\bar{a}_z\right] \longleftarrow (3)$$

as $\bar{E}$ is in x-direction $\therefore E_y = E_z = 0$.

and $E_x = f^n(z,t)$ only $\therefore \frac{\partial E_x}{\partial x} = \frac{\partial E_x}{\partial y} = 0$.

$\therefore$ the eq^n (3) becomes

$$\frac{\partial E_x}{\partial z}\bar{a}_y = -\frac{\partial}{\partial t}\left[B_x\bar{a}_x + B_y\bar{a}_y + B_z\bar{a}_z\right]$$

Comparing y-components on bothside

$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}$$

using $\bar{B} = \mu\bar{H}$ wb/m^2

and $B_y = \mu H_y$ wb/m^2

$$\frac{\partial E_x}{\partial z} = -\mu\frac{\partial H_y}{\partial t} \quad \text{(or)} \quad \boxed{\frac{\partial H_y}{\partial t} = -\frac{1}{\mu}\frac{\partial E_x}{\partial z}} \longleftarrow (b)$$

Differentiating eq^n (a) w.r.t 't'

$$\frac{\partial}{\partial t}\left[\frac{\partial H_y}{\partial z}\right] = -\epsilon\frac{\partial^2 E_x}{\partial t^2} \longleftarrow (c)$$

Differentiating eq^n (b) w.r.t 'z'

$$\frac{\partial}{\partial z}\left[\frac{\partial H_y}{\partial t}\right] = -\frac{1}{\mu}\frac{\partial^2 E_x}{\partial z^2} \leftarrow (d)$$

from eq^n (c) and (d) L.H.S are same by interchanging the order of differentiation.

∴ Equating R.H.S of eq^n (c) and eq^n (d)

$$\epsilon\frac{\partial^2 E_x}{\partial t^2} = \frac{1}{\mu}\frac{\partial^2 E_x}{\partial z^2} \leftarrow (e)$$

$$\therefore \boxed{\frac{\partial^2 E_x}{\partial t^2} = \frac{1}{\mu\epsilon}\frac{\partial^2 E_x}{\partial z^2}} \leftarrow (f)$$

The Classical wave eq^n is represented by $\nabla^2 F = \frac{1}{v^2}\frac{\partial^2 F}{\partial t^2} \leftarrow (g)$

the above eq^n (g) represents a wave travelling with a velocity 'v' m/sec

Comparing eq^n (g) with (f), it is clear that $\boxed{v = \frac{1}{\sqrt{\mu\epsilon}}}$ m/sec $\leftarrow (h)$

With this as reference Maxwell's predicted that, the empty space supports the propogation of Electromagnetic wave at speed

$$v = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 3\times10^8 \text{ m/sec} \quad \ldots \text{ in free space.}$$

Hence we can write the eq^n (f)

$$\boxed{\frac{\partial^2 E_x}{\partial t^2} = v^2\frac{\partial^2 E_x}{\partial z^2}} \leftarrow (i)$$

the eq^n (i) Called as Wave eq^n in freespace.

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The solution of this wave equation is given by

$$\boxed{E_x = E_m^+ \cos(\omega t - \beta z) + E_m^- \cos(\omega t + \beta z)} \text{ V/m} \quad \leftarrow (j)$$

Solu consists of one component of field travelling in positive z-direction having amplitude E_m^+; while other component having amplitude of E_m^- travelling in negative z-direction.

the wave velocity $\boxed{v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{\omega}{\beta}}$ m/sec.

i.e $\beta = \frac{2\pi}{\lambda}$ rad/m $= \frac{2\pi f}{\lambda f} = \frac{\omega}{v}$

$\therefore \boxed{\beta = \frac{\omega}{v}}$ rad/m or $\boxed{v = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}}}$ m/sec

we can obtain ||ly type of eqn for magnetic field H by considering eqn (b)

i.e $\frac{\partial H_y}{\partial t} = -\frac{1}{\mu}\frac{\partial E_x}{\partial z}$

$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu}\frac{\partial}{\partial z}\left[E_m^+ \cos(\omega t - \beta z) + E_m^- \cos(\omega t + \beta z)\right]$$

$$= -\frac{1}{\mu}\left[E_m^+ \beta \sin(\omega t - \beta z) - E_m^- \beta \sin(\omega t + \beta z)\right]$$

By Integrating w.r.t time 't'

$$H_y = -\frac{1}{\mu}\left[\frac{-E_m^+\beta}{\omega}\cos(\omega t - \beta z) + \frac{E_m^-\beta}{\omega}\cos(\omega t + \beta z)\right]$$

$$\therefore \boxed{H_y = \frac{E_m^+\beta}{\omega\mu}\cos(\omega t - \beta z) - \frac{E_m^-\beta}{\omega\mu}\cos(\omega t + \beta z)} \text{ A/m.}$$

$$\therefore \boxed{H_y = H_m^+ \cos(\omega t - \beta z) - H_m^- \cos(\omega t + \beta z)} \; A/m \longleftarrow (k)$$

this eq^n ||ly to eq^n (3) representing two components of a Magnetic field one is in forward direction while other is in backward dire^n.

the wave eq^n ie from eq^n (i)

$$\frac{\partial^2 E_x}{\partial t^2} = v^2 \frac{\partial^2 E_x}{\partial z^2} \longleftarrow (i)$$

Since $\bar{E} = E_x \bar{a}_x$; v/m acts only in 'x' dire^n $\therefore E_y = E_z = 0$.

and $E_x = f^n(z,t)$ only.

$$\therefore \frac{\partial^2 E_x}{\partial z^2} = \nabla^2 \bar{E} ; \longleftarrow (l)$$

using eq^n (l) in (i)

$$\frac{\partial^2 E_x}{\partial t^2} = v^2 \nabla^2 \cdot \bar{E}$$

$$\Rightarrow \nabla^2 \cdot \bar{E} = \frac{1}{v^2} \frac{\partial^2 E_x}{\partial t^2} \text{ and } v = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\therefore \nabla^2 \cdot \bar{E} = \frac{1}{(1/\sqrt{\mu\epsilon})^2} \frac{\partial^2 E_x}{\partial t^2}$$

$$\therefore \nabla^2 \cdot \bar{E} = \mu\epsilon \frac{\partial^2 E_x}{\partial t^2} \quad (d)$$

$\bar{E} = E_x \bar{a}_x$ v/m
$E_x = f^n(z,t)$
$\bar{H} = H_y \bar{a}_y$ A/m
$H_y = f^n(z,t)$

In general

$$\boxed{\nabla^2 \cdot \bar{E} - \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2} = 0} \longleftarrow (\alpha)$$

||ly we can write for Magnetic field in free space

i.e.

$$\boxed{\nabla^2 \cdot \bar{H} - \mu\epsilon \frac{\partial^2 \bar{H}}{\partial t^2} = 0} \longleftarrow (\beta) \quad (9)$$

Topics 5.5

V.imp. Method-II :- Wave Equation in free Space

Soln:- Let us Consider the two Time-Varying Maxwell's eqns

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} ; A/m^2 \text{ and } \nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} ; V/m^2$$

but $\bar{D} = \epsilon \bar{E}$ C/m² and $\bar{B} = \mu \bar{H}$ wb/m²

in free Space $\mu = \mu_0$ H/m and $\epsilon = \epsilon_0$ F/m. ;

$$\nabla \times \bar{H} = \bar{J} + \epsilon_0 \frac{\partial \bar{E}}{\partial t} \leftarrow (1)$$

$$\nabla \times \bar{E} = -\mu_0 \frac{\partial \bar{H}}{\partial t} \leftarrow (2)$$

from eqn (2) i.e $\nabla \times \bar{E} = -\mu_0 \frac{\partial \bar{H}}{\partial t}$

Curl on both side to above eqn

$$\nabla \times \nabla \times \bar{E} = -\mu_0 \frac{\partial}{\partial t}(\nabla \times \bar{H}) \leftarrow (3)$$

using Vector identity i.e $\nabla \times \nabla \times \bar{A} = \nabla(\nabla \cdot \bar{A}) - \nabla^2 A$

we can write

$$\nabla \times \nabla \times \bar{E} = \nabla(\nabla \cdot \bar{E}) - \nabla^2 \bar{E}$$

using poisson's eqn i.e $\nabla \cdot \bar{D} = \rho_v$ C/m³

$\nabla \cdot \bar{E} = \rho_v / \epsilon_0$ V/m²

$$\nabla \times \nabla \times \bar{E} = \nabla(\rho_v/\epsilon_0) - \nabla^2 \bar{E} \leftarrow (4)$$

$\therefore$ eqn (3) becomes

$$\nabla(\rho_v/\epsilon_0) - \nabla^2 \bar{E} = -\mu_0 \frac{\partial}{\partial t}(\nabla \times \bar{H})$$

using eqn (1) in R.H.S

$$\nabla(\rho_v/\epsilon_0) - \nabla^2 \bar{E} = -\mu_0 \frac{\partial}{\partial t}\left[\bar{J} + \epsilon_0 \frac{\partial \bar{E}}{\partial t}\right] \leftarrow (4a)$$

$$\boxed{\nabla^2 \bar{E} - \mu_0 \epsilon_0 \frac{\partial^2 \bar{E}}{\partial t^2} = \mu_0 \frac{\partial \bar{J}}{\partial t} + \nabla(\rho_v/\epsilon_0)} \leftarrow (5)$$

the L.H.S of above eqn is in the characteristic form of a wave eqn. the soln of such an equation represents a propogating wave. The R.H.S represents the sources which are responsible for the wave field i.e the charges and current.

∴ Hence eqn (5) represents the wave equation in $\bar{E}$ for a medium with constant 'μ' and ϵ; i.e for a homogeneous and isotropic medium.

Now taking curl on bothside for eqn (1).

$$\nabla \times \nabla \times \bar{H} = \nabla \times \bar{J} + \epsilon_0 \frac{\partial}{\partial t}(\nabla \times \bar{E})$$

using eqn (2) in R.H.S

$$\nabla(\nabla \cdot \bar{H}) - \nabla^2 \bar{H} = \nabla \times \bar{J} - \mu_0 \epsilon_0 \frac{\partial^2 \bar{H}}{\partial t^2}$$

asper Maxwell's eqn $\nabla \cdot \bar{B} = 0$ and $\nabla \cdot \bar{H} = 0$

$\mu_0 \neq 0$.

$$\therefore \boxed{\nabla^2 \bar{H} - \mu_0 \epsilon_0 \frac{\partial^2 \bar{H}}{\partial t^2} = -(\nabla \times \bar{J})} \leftarrow (6)$$

The above equation represent the wave eqn in $\bar{H}$ for a medium with constant μ_0 and ϵ_0.

for a Source free region and free Space

ie $\sigma = 0$ ∴ $\bar{J} = 0$ and $\rho_v = 0$..

∴ eq^n ⑤ and eq^n ⑥ becomes

$$\boxed{\nabla^2 \bar{E} - \mu_0 \epsilon_0 \frac{\partial^2 \bar{E}}{\partial t^2} = 0} \quad \leftarrow (7)$$

and

$$\boxed{\nabla^2 \bar{H} - \mu_0 \epsilon_0 \frac{\partial^2 \bar{H}}{\partial t^2} = 0} \quad \leftarrow (8)$$

the Classical Wave equation is represented by

$$\nabla^2 \bar{F} - \frac{1}{v^2} \frac{\partial^2 \bar{F}}{\partial t^2} = 0 \quad \leftarrow (9).$$

where $\boxed{v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{\omega}{\beta}}$ m/sec ; wave velocity

ie in free space

$\boxed{v = 3 \times 10^8 \text{ m/sec}}$

Note:- Students are advised to write Method-II in Examination.

2.

06-DEC2010

Obtain the solution of wave equation for uniform plane wave in free space. (10 Marks)

06-DEC 2013/Jan 2014

Obtain the solution of wave equation for uniform plane wave in free space. (08 Marks)

10 - June /July 2014

Obtain solution of the wave equation for a uniform plane wave (UPW) in free space. (06 Marks)

Soln:- w.k.t the wave eqn in free space

$$\nabla^2 \bar{E} - \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2} = 0 \leftarrow (1)$$

let soln $\bar{E} = E_0 e^{j\omega t}$ v/m $\leftarrow (2)$

where $\bar{E}$ - instantaneous field at time 't'.

E_0 - amplitude of $\bar{E}$. $j = \sqrt{-1}$; $\omega = 2\pi f$ rad/sec (angular frequency).

$$\frac{\partial^2 \bar{E}}{\partial t^2} = j^2 \omega^2 \left[E_0 e^{j\omega t} \right] = -\omega^2 \bar{E} \leftarrow (3)$$

using eqn (3) we have wave eqn for Lossless Medium as

$$\boxed{\nabla^2 \bar{E} + \omega^2 \mu\epsilon \bar{E} = 0} \leftarrow (4)$$ this eqn called Helmholtz eqn.

and the soln Consists of

$$\boxed{\bar{E} = E_m^+ \cos(\omega t - \beta z) + E_m^- \cos(\omega t + \beta z)} \text{ v/m}$$

wave travelling along +ve z direction. wave travelling -z direc'n

lly. $\nabla^2 \bar{H} - \mu\epsilon \frac{\partial^2 \bar{H}}{\partial t^2} = 0$; the soln

$$\boxed{\bar{H} = H_m^+ \cos(\omega t - \beta z) - H_m^- \cos(\omega t + \beta z)} \text{ A/m}$$

Note:- for more details refer Page NO - 512A and 512B

Topic 5.6

3. Define Plane Waves and Uniform Plane Waves

Plane Waves :- Plane waves are waves that poms Variation only in the direction of wave propogation and their Characteristics remain constant across planes normal to the direction of propogation.

Uniform Plane Waves (UPW) :-

In the Case of Electromagnetic wave propogating along x-axis, they are referred to as "Uniform plane waves" if the Electric field and Magnetic fields are independent of y and z but function of x and t only. Further for such a wave, it is important to note that there will be no field component along the direction of wave propogation, this is Called transverse nature of Electromagnetic wave. (TEM - wave).

Topic 5.7

Topic:

4. *Wave Propagation in Good Conductors/ Wave Equation in Good Conducting medium + Solution of Wave Equation in Good Conducting Medium.*

4.

06-DEC2008/Jan 2009

Discuss the uniform plane wave propagation in a good conducting medium. (06 Marks)

06-DEC 2013/Jan 2014

Derive an expression for uniform plane waves in good conductor. (06 Marks)

06 - June /July 2012

With suitable assumption work out the solution of wave equation for uniform plane wave propogating in a good conductor. (10 Marks)

06- June /July 2009

Discuss the behaviour of good conductor when uniform ϕ line wove propagates through it. (10 Marks)

10 - June /July 2014

Discuss uniform plane wave propagation in a good conducting media. (06 Marks)

06 – Jan 2013

Discuss the uniform plane wave propagation in a good conducting medium. (06 Marks)

Obtain the Solution of Wave Equation in Conducting Medium.

Soln: In a Conducting medium (or) Good Conductor's

$\sigma \neq 0$.

w.k.t from Maxwell's eqn's

$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$; A/m² and $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$; V/m²

and $\bar{D} = \epsilon \bar{E}$ C/m², $\bar{B} = \mu \bar{H}$ wb/m²;

$\mu = \mu_0 \mu_r$ H/m and $\epsilon = \epsilon_0 \epsilon_r$ F/m.

$$\nabla \times \bar{H} = \bar{J} + \epsilon \frac{\partial \bar{E}}{\partial t} \leftarrow (1)$$

and $$\nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t} \leftarrow (2)$$

from eqn (2) i.e $\nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t}$

Curl on both side

$$\nabla\times\nabla\times\bar{E} = -\mu\frac{\partial}{\partial t}(\nabla\times\bar{H}) \longleftarrow (3)$$

using vector identity i.e $\nabla\times\nabla\times\bar{A} = \nabla(\nabla\cdot\bar{A}) - \nabla^2\bar{A}$

$$\nabla\times\nabla\times\bar{E} = \nabla(\nabla\cdot\bar{E}) - \nabla^2\bar{E}$$

w.k.t $\nabla\cdot\bar{D} = \rho_v\, C/m^3$ and $\nabla\cdot\bar{E} = \rho_v/\epsilon\ V/m^2$

$$\nabla\times\nabla\times\bar{E} = \nabla(\rho_v/\epsilon) - \nabla^2\bar{E} \longleftarrow (4)$$

eq^n (3) becomes

$$\nabla(\rho_v/\epsilon) - \nabla^2\bar{E} = -\mu\frac{\partial}{\partial t}\left[\bar{J} + \epsilon\frac{\partial\bar{E}}{\partial t}\right]$$

$$\nabla^2\bar{E} = \nabla(\rho_v/\epsilon) + \mu\sigma\frac{\partial\bar{E}}{\partial t} + \mu\epsilon\frac{\partial^2\bar{E}}{\partial t^2}$$

most of the region are source free $\therefore \rho_v = 0\ C/m^3$.

$\therefore$ the above eq^n becomes

$$\boxed{\nabla^2\bar{E} = \mu\sigma\frac{\partial\bar{E}}{\partial t} + \mu\epsilon\frac{\partial^2\bar{E}}{\partial t^2}} \longleftarrow (a)$$

lly taking Curl on both side to eq^n (1)

$$\nabla\times\nabla\times\bar{H} = \nabla\times\bar{J} + \epsilon\frac{\partial}{\partial t}(\nabla\times\bar{E})$$

using eq^n (2) and vector identity

$$\nabla(\cancel{\nabla}\cdot\bar{H}) - \nabla^2\bar{H} = \nabla\times\bar{J} + \epsilon\frac{\partial}{\partial t}\left[-\mu\frac{\partial\bar{H}}{\partial t}\right]$$

0 using Maxwell's eqn $\nabla\cdot\bar{B}=0$ $\therefore \nabla\cdot(\mu\bar{H})=0$

i.e $\mu\neq 0 \Rightarrow \boxed{\nabla\cdot\bar{H}=0}$

and $\bar{J}=\sigma\bar{E}$ A/m²; $\nabla\times\bar{E} = -\mu\frac{\partial\bar{H}}{\partial t}$ V/m²

$$\Rightarrow -\nabla^2\bar{H} = \sigma(\nabla\times\bar{E}) - \mu\epsilon\frac{\partial^2\bar{H}}{\partial t^2}$$

$$-\nabla^2\bar{H} = \sigma\left[-\frac{\mu\partial\bar{H}}{\partial t}\right] - \mu\epsilon\frac{\partial^2\bar{H}}{\partial t^2}$$

$$\boxed{\nabla^2\bar{H} = \mu\sigma\frac{\partial\bar{H}}{\partial t} + \mu\epsilon\frac{\partial^2\bar{H}}{\partial t^2}} \longleftarrow (b)$$

The equation (a) and eqn (b) are called Electric and Magnetic field's of wave equations in conducting Medium

(or) Good conductors.

Imp: eqn (a) and eqn (b) in general we can write for all field vectors $\bar{E}$, $\bar{D}$, $\bar{H}$ and $\bar{B}$

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$$\boxed{\nabla^2\begin{bmatrix}\bar{E}\\ \bar{D}\\ \bar{H}\\ \bar{B}\end{bmatrix} = \mu\sigma\frac{\partial}{\partial t}\begin{bmatrix}\bar{E}\\ \bar{D}\\ \bar{H}\\ \bar{B}\end{bmatrix} + \mu\epsilon\frac{\partial^2}{\partial t^2}\begin{bmatrix}\bar{E}\\ \bar{D}\\ \bar{H}\\ \bar{B}\end{bmatrix}}$$

The presence of the first order term in the Second order differential eqn indicates that the fields decay as the propogate through the Medium. [$\therefore$ Conducting medium ($\sigma\neq 0$) called Lossy medium].

Solution of Wave equation in Conducting Medium :-

The Wave equation in Conducting Medium

$$\nabla^2 \bar{E} = \mu\sigma \frac{\partial \bar{E}}{\partial t} + \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2} \longleftarrow (a)$$

and

$$\nabla^2 \bar{H} = \mu\sigma \frac{\partial \bar{H}}{\partial t} + \mu\epsilon \frac{\partial^2 \bar{H}}{\partial t^2} \longleftarrow (b)$$

assume EM wave in propogating in 'z' direction

$\therefore$ $\bar{E} = E_x \bar{a}_x$ V/m and $E_x = f^n(z,t)$ only

$\bar{H} = H_y \bar{a}_y$ A/m and $H_y = f^n(z,t)$ only

the solution of eq^n (a) is

$$\boxed{E_x(z,t) = E_m^+ e^{-\alpha z}\cos(\omega t - \beta z) + E_m^- e^{+\alpha z}\cos(\omega t + \beta z)} \text{ V/m}$$

and $H_m = \dfrac{E_m}{\eta}$ A/m and $\boxed{\eta = |\eta|\angle\theta_\eta}$ Ω

$= |\eta| e^{-j\theta_\eta}$ Ω.

$\therefore$ the soln of eq^n (b) is

$$H_y(z,t) = \frac{E_m^+}{|\eta|} e^{-\alpha z}\cos(\omega t - \beta z - \theta_\eta) - \frac{E_m^-}{|\eta|} e^{+\alpha z}\cos(\omega t + \beta z - \theta_\eta)$$

$$\boxed{H_y(z,t) = H_m^+ e^{-\alpha z}\cos(\omega t - \beta z - \theta_\eta) - H_m^- e^{\alpha z}\cos(\omega t + \beta z - \theta_\eta)} \text{ A/m.}$$

Note1. if $\bar{E} \rightarrow \bar{a}_x$ then $\bar{E} = E_x \bar{a}_x$ v/m
$\bar{H} \rightarrow \bar{a}_y$ then $\bar{H} = H_y \bar{a}_y$ A/m
then direction of EM wave propagates in 'z' dir^n.
and $E_x, H_y \Rightarrow f^n(z,t)$ only.

Note2. if $\bar{E} \rightarrow \bar{a}_y$ then $\bar{E} = E_y \bar{a}_y$ v/m.
$\bar{H} \rightarrow \bar{a}_z$ then $\bar{H} = H_z \bar{a}_z$ A/m.
then direction of EM wave propagation is in 'x' dir^n
and $E_y, H_z \Rightarrow f^n(x,t)$ only

Note3. if $\bar{E} \rightarrow \bar{a}_z$; $\bar{E} = E_z \bar{a}_z$ v/m.
$\bar{H} \rightarrow \bar{a}_x$, $\bar{H} = H_x \bar{a}_x$ A/m
then direction of EM wave propagation is in 'y' dir^n
and $E_z, H_x \Rightarrow f^n(y,t)$ only.

Note4. Non-Existance of field components along the direction of wave propagation.
Eg ie if EM wave is propagating along 'z' direction then $\boxed{E_z = H_z = 0}$; ie field components along 'z' direction doesnot Exists.

Topic 5.8

Topic:

5. ***Wave Propagation in Good Dielectrics and Perfect dielectrics / Wave Equation in Good Dielectrics and Perfect dielectric medium + its Solution.***

5. 02-DEC2010

Discuss the wave propagation in a good dielectric (absorption medium). (12 Marks)

02 - June /July 2011

Starting from Maxwell's equations derive wave equation for a uniform plane wave travelling in dielectric media. (08 Marks)

Sol:- A dielectric Medium is a one in which the Conduction Current is almost zero in comparision to the displacement Current and Such a Medium Called as dielectric Medium.

Dielectric Medium

- perfect dielectric's: $\frac{\sigma}{\omega \epsilon} \to 0$
- Good dielectrics: $\left(\frac{\sigma}{\omega \epsilon}\right) << 1$.

Wave eq^n:- w.k.t from Maxwell's eq^{n}s

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} \ A/m^2 \quad \text{and} \quad \nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \ v/m^2$$

but $\bar{D} = \epsilon \bar{E}$ c/m^2 and $\bar{B} = \mu \bar{H}$ wb/m^2

$$\nabla \times \bar{H} = \bar{J} + \epsilon \frac{\partial \bar{E}}{\partial t} \leftarrow (1); \quad \nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t} \leftarrow (2)$$

from eq^n (2) i.e $\nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t}$ v/m^2

Curl on both side

$$\nabla \times \nabla \times \bar{E} = -\mu \frac{\partial}{\partial t} (\nabla \times \bar{H}) \leftarrow (3)$$

using vector identity $\nabla \times \nabla \times \bar{E} = \nabla(\nabla \cdot \bar{E}) - \nabla^2 \bar{E}$

$$\nabla\cdot\bar{D} = \rho_v \ C/m^3 \Rightarrow \nabla\cdot\bar{E} = \rho_v/\epsilon \ V/m^2$$

$$\nabla\times\nabla\times\bar{E} = \nabla(\rho_v/\epsilon) - \nabla^2\bar{E} \quad \leftarrow (4)$$

eqn (3) becomes

$$\nabla(\rho_v/\epsilon) - \nabla^2\bar{E} = -\mu\frac{\partial}{\partial t}\left[\bar{J} + \epsilon\frac{\partial \bar{E}}{\partial t}\right]$$

$$\nabla^2 E = \nabla(\rho_v/\epsilon) + \mu\sigma\frac{\partial \bar{E}}{\partial t} + \mu\epsilon\frac{\partial^2 E}{\partial t^2}$$

most of the region are source free $\therefore \rho_v = 0 \ C/m^3$

the above eqn becomes

$$\boxed{\nabla^2\bar{E} = \mu\sigma\frac{\partial \bar{E}}{\partial t} + \mu\epsilon\frac{\partial^2 \bar{E}}{\partial t^2}}$$

for a perfect dielectrics $\frac{\sigma}{\omega\epsilon} \to 0 \ \therefore \sigma \simeq 0 \ \mho/m$.

using these conditions the above eqn becomes

$$\nabla^2\bar{E} = \mu\epsilon\frac{\partial^2 \bar{E}}{\partial t^2}$$

$$\boxed{\nabla^2\bar{E} - \mu\epsilon\frac{\partial^2 \bar{E}}{\partial t^2} = 0}$$

$\leftarrow$ Wave eqn of $\bar{E}$ field for perfect dielectric Medium (Lossless Medium) (ie $\frac{\sigma}{\omega\epsilon} \to 0$).

Note!:- for a Good dielectric Medium $\frac{\sigma}{\omega\epsilon} << 1$; $\therefore$ the Wave ~~eqn~~ eqn for Good dielectric Medium is

(2)

$$\boxed{\nabla^2\bar{E} = \mu\sigma\frac{\partial \bar{E}}{\partial t} + \mu\epsilon\frac{\partial^2 \bar{E}}{\partial t^2}}$$

$\leftarrow$ Wave eqn $\bar{E}$ field Good dielectric Medium (Low Loss dielectrics) $\frac{\sigma}{\omega\epsilon} << 1$.

lly the Magnetic field Wave eqn, take Curl on bothside to eqn (1).

$$\nabla \times \nabla \times \overline{H} = \nabla \times \overline{J} + \epsilon \frac{\partial}{\partial t}(\nabla \times E)$$

using eqn (2) and vector identity

$$\nabla(\cancel{\nabla} \cdot \overline{H}) - \nabla^2 \overline{H} = \nabla \times \overline{J} + \epsilon \frac{\partial}{\partial t}\left[-\mu \frac{\partial \overline{H}}{\partial t}\right]$$

0 using Maxwell's eqn $\nabla \cdot B = 0 \Rightarrow \nabla \cdot \overline{H} = 0 \; A/m$

and $\overline{J} = \sigma \overline{E} \; A/m^2$

$$-\nabla^2 \overline{H} = \sigma(\nabla \times E) - \mu\epsilon \frac{\partial^2 \overline{H}}{\partial t^2}$$

$$-\nabla^2 \overline{H} = \sigma\left[-\mu \frac{\partial \overline{H}}{\partial t}\right] - \mu\epsilon \frac{\partial^2 \overline{H}}{\partial t^2}$$

$$\boxed{\nabla^2 \overline{H} = \sigma\mu \frac{\partial \overline{H}}{\partial t} + \mu\epsilon \frac{\partial^2 \overline{H}}{\partial t^2}}$$

for a perfect dielectric Medium $\frac{\sigma}{\omega\epsilon} \to 0$; $\sigma \simeq 0 \; \mho/m$.

$\therefore$ the above equation becomes

$$\nabla^2 \overline{H} = \mu\epsilon \frac{\partial^2 \overline{H}}{\partial t^2}$$

$$\boxed{\nabla^2 \overline{H} - \mu\epsilon \frac{\partial^2 \overline{H}}{\partial t^2} = 0}$$

Wave eqn of $\overline{H}$ field for perfect dielectric Medium. $\left(\frac{\sigma}{\omega\epsilon} \to 0\right)$

ie [Lossless dielectrics (or) Medium]

Note:- for a Good dielectric Medium $\frac{\sigma}{\omega\epsilon} << 1$

$$\therefore \boxed{\nabla^2 \bar{H} = \mu\sigma \frac{\partial \bar{H}}{\partial t} + \mu\epsilon \frac{\partial^2 \bar{H}}{\partial t^2}}$$ ← Wave eqⁿ of $\bar{H}$ field for Good dielectrics (Low Loss dielectrics) Medium i.e $\frac{\sigma}{\omega\epsilon} << 1$.

Note:- Dielectric Medium Classified into

I. [Perfect dielectrics / Loss Less dielectrics]

Condⁿ. $\left(\frac{\sigma}{\omega\epsilon}\right) \to 0$; $\sigma \simeq 0$

$\mu = \mu_0\mu_r$ H/m & $\epsilon = \epsilon_0\epsilon_r$ F/m.

Ex:- all Insulators

diamond $\sigma = 2\times10^{-13}$ ℧/m.

polystyrene, Quartz, marble, Bakelite etc.

II. (Good dielectrics / Low Loss dielectrics)

Condⁿ. $\frac{\sigma}{\omega\epsilon} << 1$.

Eg:- Teflon, Limestone, Freshwater, Soil. etc.

Topic 5.9

Topic:

6. *Transverse Nature of EM waves (TEM)/ Non Existence of field components along the direction of wave propagation for a Uniform Plane Waves.*

6.

Show that the uniform plane wave is transverse in nature. 02-DEC2010 (04 Marks)

What are uniform plane waves? Show that a UPW is transverse in nature. 02-DEC2008/Jan 2009 (07 Marks)

Prove that traveling electromagnetic waves are transverse in nature. 02 - June /July 2012 (06 Marks)

Prove that a Uniform plane wave travelling along x-direction has no x-component of **E** or **H**.

Soln:- Defn of UPW :- refer page NO-517.

assume that EM wave is propogating along x-direction then corresponding wave equ is given by

$$\frac{\partial^2 E}{\partial x^2} = \mu\epsilon \frac{\partial^2 E}{\partial t^2}$$

the three scalar equation's along x, y and z directions are given by

$$\frac{\partial^2 E_x}{\partial x^2} = \mu\epsilon \frac{\partial^2 E_x}{\partial t^2} \longleftarrow (1)$$

$$\frac{\partial^2 E_y}{\partial x^2} = \mu\epsilon \frac{\partial^2 E_y}{\partial t^2} \longleftarrow (2)$$

$$\frac{\partial^2 E_z}{\partial x^2} = \mu\epsilon \frac{\partial^2 E_z}{\partial t^2} \longleftarrow (3)$$

In a region where there is NO charge density (free space)

$\rho_v = 0 \quad \therefore \nabla\cdot\bar{D} = \rho_v \; C/m^3$

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$\Rightarrow \nabla\cdot\bar{D} = 0 \; C/m^3$

$$\bar{D} = \epsilon \bar{E} \quad C/m^2$$

$$\therefore \epsilon \nabla \cdot \bar{E} = 0$$

$$\epsilon \neq 0 \Rightarrow \nabla \cdot \bar{E} = 0$$

$$\text{i.e } \frac{\partial E_x}{\partial x} + \cancel{\frac{\partial E_y}{\partial y}}^{\,0} + \cancel{\frac{\partial E_z}{\partial z}}^{\,0} = 0$$

Since the Wave is travelling along 'x' direction then E is independent of 'y' and 'z'. ∴ the Last two terms are equal to zero.

$$\Rightarrow \frac{\partial E_x}{\partial x} = 0 \quad \text{(4)}$$

This means that there is no Variation of E_x along x-direction.

from eqn (1) $$\frac{\partial^2 E_x}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial E_x}{\partial x}\right) = 0 = \mu \epsilon \frac{\partial^2 E_x}{\partial t^2} \quad \text{(5)}$$

$$\epsilon \neq 0 \text{ and } \mu \neq 0$$

$$\Rightarrow \frac{\partial^2 E_x}{\partial t^2} = 0 \quad \text{(5)}$$

it is Seen from eqn (5) that the second derivative of E_x w.r.t 't' must be zero. This requires that E_x must either i) to be zero.

ii) (or) Constant in time

iii) increasing Uniformly with time.

A Field Satisfying (ii) and (iii) above Can never be a Wave hence E_x must be zero.

Hence a UPW is transverse and hence components of E and H only in a direction perpendicular to the direction of propogation.

Note:- Wave is nothing but a periodic oscillations.

Topic 5.10.

Topic:

7. Relationship b/w $|E|$ and $|H|$.

7. 10-DEC2011/Jan 2012

For an electromagnetic wave propagating in free space prove that $\frac{|\bar{E}|}{|\bar{H}|} = \eta$. (08 Marks)

06 - June /July 2013

With suitable mathematical steps, prove the relation between $\overline{E}$ and $\overline{H}$ for a travelling uniform plane wave. (10 Marks)

Solu: We know that, the general wave equation in free space; assuming wave propogating along 'z' direction ---- Electric field.

$$\nabla^2 \bar{E} - \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2} = 0 \quad \leftarrow (1)$$

the solution of the above equ is given by

$$E_x = E_m^+ \cos(\omega t - \beta z) + E_m^- \cos(\omega t + \beta z) \ V/m \quad \leftarrow (2)$$

lly. the Magnetic field.

the wave equ $\nabla^2 \bar{H} - \mu\epsilon \frac{\partial^2 \bar{H}}{\partial t^2} = 0 \leftarrow (3)$

the solu:-

$$H_y = H_m^+ \cos(\omega t - \beta z) - H_m^- \cos(\omega t + \beta z) \ A/m$$

$$H_y = \frac{\beta E_m^+}{\omega\mu} \cos(\omega t - \beta z) - \frac{\beta E_m^-}{\omega\mu} \cos(\omega t + \beta z) \ A/m$$

$\therefore$ the ratio's of $\dfrac{|E|}{|H|} = \dfrac{|E^+|}{|H^+|} = \dfrac{|E^-|}{|H^-|} = \dfrac{E_m}{\left(\frac{\beta E_m}{\omega \mu}\right)}$

$$\left|\frac{E}{H}\right| = \frac{\omega \mu}{\beta} = \left(\frac{\omega}{\beta}\right)\cdot \mu \; ; \; \Omega \longleftarrow (4)$$

Note:- $|E| \rightarrow V/m$ & $|H| \rightarrow A/m$ $\left|\frac{E}{H}\right| \rightarrow V/A$ or Ω

the wave velocity $v = \dfrac{\omega}{\beta}$ m/sec $= \dfrac{1}{\sqrt{\mu\epsilon}}$ m/sec. $\longleftarrow (5)$

using eq^n (5) in eq^n (4)

$$\left|\frac{\bar{E}}{\bar{H}}\right| = \frac{1}{\sqrt{\mu\epsilon}} \times \mu = \frac{1}{\sqrt{\mu\epsilon}} (\sqrt{\mu})^2$$

iiy. $\boxed{\left|\frac{\bar{E}}{\bar{H}}\right| = \sqrt{\frac{\mu}{\epsilon}} = \eta}$ Ω.

where η intrinsic impedance.

for a free space $\mu = \mu_0$ H/m and $\epsilon = \epsilon_0$ F/m

$$\frac{|E|}{|H|} = \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{8.854 \times 10^{-12}}} = 120\pi \text{ or } 377\,\Omega$$

28. $\boxed{\eta = 120\pi \text{ or } 377}$ Ω

Characteristic (or) Intrinsic Impedance (η):- The ratio of amplitude / magnitude of $\bar{E}$ to $\bar{H}$ of the waves in either direction is called intrinsic impedance of the material in which wave is travelling and is denoted by 'η'.

i.e $$\boxed{\eta = \left|\frac{\bar{E}}{\bar{H}}\right| = \frac{E_m}{H_m} = \frac{\omega\mu}{\beta} = \sqrt{\frac{\mu}{\epsilon}}}\ \Omega$$

Note:- in free space $\mu = \mu_0$ and $\epsilon = \epsilon_0$ F/m

$$\boxed{\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi \text{ or } 377}\ \Omega$$

7a. A UPW with an Electric field Intensity equal to 1v/m is travelling in free space. Find the magnitude associated Magnetic field.

Soln: given $|E_m| = 1\ V/m$

$|H_m| = ?$

w.k.t In free space

$$\eta = \frac{|E_m|}{|H_m|} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\ \Omega$$

$$\Rightarrow |H_m| = \frac{1}{377}\ A/m$$

$$\Rightarrow |H_m| = 2.6525 \times 10^{-3}\ A/m.$$

$$\boxed{H_m = 2.6525 \times 10^{-3}}\ A/m$$

7b. Show that Electric and Magnetic energy densities in a travelling Plane wave are Equal.

solu:- w.k.t the UPW travelling in a free space (or) perfect dielectric medium

$$\eta = \left|\frac{\bar{E}}{\bar{H}}\right| = \sqrt{\frac{\mu}{\epsilon}} \ \Omega$$

Note:- Energy density = $\frac{\text{Energy stored}}{\text{volume}}$

$\Rightarrow J/m^3$

i.e

$$\Rightarrow \frac{E}{H} = \sqrt{\frac{\mu}{\epsilon}}$$

$$\left(\frac{E}{H}\right)^2 = \frac{\mu}{\epsilon}$$

$$\frac{E^2}{H^2} = \frac{\mu}{\epsilon}$$

$$\Rightarrow \epsilon E^2 = \mu H^2$$

$\times$ by '$\frac{1}{2}$' on bothside

$$\boxed{\frac{1}{2}\epsilon E^2 = \frac{1}{2}\mu H^2}$$

Energy density in an electric field $(\frac{1}{2}\epsilon E^2)$ Joules/m^3.

Energy density in an Magnetic field $(\frac{1}{2}\mu H^2)$ J/m^3

Topic 5.11

Topic:

8. 8. *Characteristics of Medium/ General Definitions of:*
   - *Propagation Constant (γ)*
   - *Attenuation Constant (α)*
   - *Phase Constant (β)*
   - *Wave Velocity (v)*
   - *Wave Length (λ)*
   - *Intrinsic Impedance (η)*

Define phase velocity, wavelength and propagation constant. **10 - June /July 2015 (06 Marks)**

i) attenuation Constant (α):-

In general when any wave propogates in the medium, it get's attenuated. i.e the amplitude of the signal reduces. this is represented by "attenuation constant(α)". and is Measured in neper per meter (Np/m).

$$\boxed{1\,N_p = 8.686\,dB}$$

ii) phase Constant (β):- When a wave propogates, phase change also takesplace. Such a phase change is expressed by a phase constant (β). and is Measured in (rad/m).

$$\boxed{\beta = \frac{2\pi}{\lambda} = \frac{\omega}{v}} \text{ rad/m}$$

iii) Propogation Constant (γ) :-

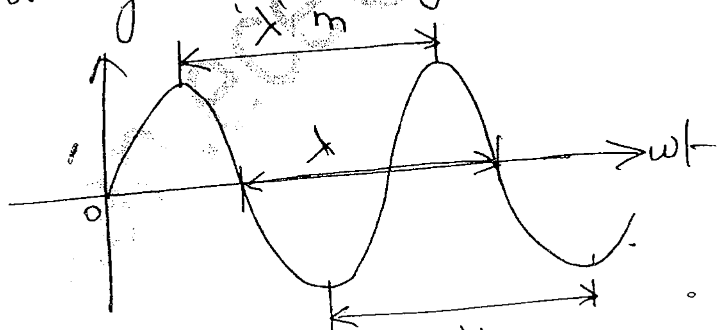
attenuation constant (α) and phase constant (β) together Constitutes a propogation constant of Medium for uniform plane wave and it is represented by γ.

it is expressed per unit Length

$$\boxed{\gamma = \alpha + j\beta} \; m^{-1}$$

iv) WaveLength (λ)

WaveLength (λ) is the distance b/w any two points with the Same phase, Such as b/w Crests (or) troughs (or) Corresponding Zero Crossings as shown in fig.



the WaveLength of a Sinusoidal wave is the spatial period of the wave - the distance over which the wave's shape repeats.

$$\therefore \boxed{\text{Wave Length } (\lambda) = \frac{2\pi}{\beta}} \; m$$

v) Characteristics (or) Intrinsic Impedance (η)

The ratio of amplitudes of $\bar{E}$ to $\bar{H}$ of the waves in either direction is called intrinsic impedance of the material in which wave is travelling, and is denoted by (η).

$$\boxed{\eta = \left|\frac{\bar{E}}{\bar{H}}\right| = \frac{E_m}{H_m} = \frac{\omega\mu}{\beta} = v\mu = \sqrt{\frac{\mu}{\epsilon}}}\ \Omega$$

In free space $\mu = \mu_0$ H/m and $\epsilon = \epsilon_0$ F/m

$$\therefore \boxed{\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi \text{ (or) } 377}\ \Omega$$

In general

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0\mu_r}{\epsilon_0\epsilon_r}} = \sqrt{\frac{\mu_0}{\epsilon_0}}\sqrt{\frac{\mu_r}{\epsilon_r}}$$

$$\therefore \boxed{\eta = 377\sqrt{\frac{\mu_r}{\epsilon_r}}}\ \Omega.$$

vi) Phase velocity (or) Wave Velocity (v_p) :-

The phase velocity (v) of a plane wave is the velocity with which the phase of the wave propogates. For a wave travelling in +ve 'z' direction the E field is given by $E = E_m^+ \cos(\omega t - \beta z)$ V/m the phase = Constant(k)

$$(\omega t - \beta z) = k$$

the phase velocity $v_p = \frac{dz}{dt}$ m/sec

$$\omega(1) - \beta \frac{dz}{dt} = 0 \Rightarrow \omega = \beta \frac{dz}{dt}$$

(or) $\boxed{v_p = \frac{dz}{dt} = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}}}$ m/sec

Note:- Wave velocity in free space $v_p = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3\times10^8$ m/sec

$\boxed{v_p = 3\times10^8}$ m/sec

(or) In general $v_p = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}}$ m/sec

$\boxed{v_p = 3\times10^8 \frac{1}{\sqrt{\mu_r \epsilon_r}}}$ m/sec

Relation b/w Wavelength (λ) and phase constant (β)

if 'λ' is the Length of one complete cycle of Sinusoidal wave

then $\beta = \frac{2\pi}{\lambda}$ rad/m (or) $\boxed{\lambda = \frac{2\pi}{\beta}}$ m

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{\lambda f} = \frac{\omega}{v} \Rightarrow \boxed{\beta = \frac{\omega}{v}} \text{ rad/m}$$

(or) $$\beta = \frac{\omega}{1/\sqrt{\mu\epsilon}} = \omega\sqrt{\mu\epsilon}$$

$$\Rightarrow \boxed{\beta = \frac{\omega}{v} = \omega\sqrt{\mu\epsilon}} \text{ rad/m}$$

(or) phase velocity $\boxed{v = f\lambda = \frac{1}{\sqrt{\mu\epsilon}} = \frac{\omega}{\beta}}$ m/sec

Topic: 5.1 Topic:

9. *Wave Equation in Phasor form.*

9.Derive Wave Equation in Phasor form.

w.k.t Maxwell's eqn from Faraday's Law

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \;;\; V/m^2$$

$$\bar{B} = \mu \bar{H} \quad wb/m^2$$

$$\therefore \nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t} \quad \leftarrow (1)$$

Note:- Vector identity

$$\boxed{\nabla \times \nabla \times \bar{A} = \nabla \cdot (\nabla A) - \nabla^2 \bar{A}}$$

Maxwell's eqn from Modified Ampere's Law

$$\nabla \times \bar{H} = \bar{J}_c + \frac{\partial \bar{D}}{\partial t} \;;\; A/m^2 \quad \leftarrow (2)$$

taking Curl on bothside to eqn (1)

$$\nabla \times \nabla \times \bar{E} = -\mu \frac{\partial}{\partial t}[\nabla \times \bar{H}]$$

using vector identity and eqn (2)

$$\nabla(\cancel{\nabla \cdot E}) - \nabla^2 \bar{E} = -\mu \frac{\partial}{\partial t}\left[\bar{J}_c + \frac{\partial \bar{D}}{\partial t}\right]$$

for source-free region $\boxed{\nabla \cdot \bar{E} = 0}$

$\therefore$ above equation becomes

$$-\nabla^2 \bar{E} = -\mu \frac{\partial}{\partial t}\left[\bar{J}_c + \frac{\partial \bar{D}}{\partial t}\right]$$

using $\frac{\partial}{\partial t} \rightarrow j\omega$ and $\bar{J}_c = \sigma E \; A/m^2$; $\bar{D} = \epsilon \bar{E} \; c/m^2$

$$\nabla^2 \bar{E} = \mu j\omega\left[\sigma \bar{E} + j\omega \epsilon \bar{E}\right]$$

$$\boxed{\nabla^2 \bar{E} = j\omega\mu[\sigma + j\omega\epsilon]\,\bar{E}}$$

$$\boxed{\nabla^2 \bar{E} = \gamma^2 \bar{E}}$$

3b

Ily if we take Curl on bothside for eq^n (2)

$$\nabla \times \nabla \times \bar{H} = \nabla \times \bar{J} + \epsilon \frac{\partial}{\partial t}[\nabla \times \bar{E}]$$

using vector identity and eq^n (1)

$$\nabla(\cancel{\nabla \cdot \bar{H}}) - \nabla^2 \bar{H} = \nabla \times \bar{J} + \epsilon \frac{\partial}{\partial t}\left[-\mu \frac{\partial \bar{H}}{\partial t}\right]$$

For a source free region $\nabla \cdot \bar{H} = 0$ and $\bar{J} = \sigma \bar{E}$ A/m^2.

$$-\nabla^2 \cdot \bar{H} = \epsilon \frac{\partial}{\partial t}\left[-\mu \frac{\partial \bar{H}}{\partial t}\right] + \sigma[\nabla \times \bar{E}]$$

$$\nabla \times \bar{E} = -j\omega\mu \bar{H} = -\frac{\partial B}{\partial t}$$

$$\Rightarrow -\nabla^2 \bar{H} = \epsilon j\omega[-\mu j\omega \bar{H}] + \sigma[-j\omega\mu \bar{H}]$$

$$-\nabla^2 \bar{H} = -[\sigma + j\omega\epsilon]\mu j\omega \bar{H}$$

$$\boxed{\nabla^2 \bar{H} = j\omega\mu[\sigma + j\omega\epsilon]\bar{H}}$$

$$\boxed{\nabla^2 \bar{H} = \gamma^2 \bar{H}}$$

$$\Rightarrow \nabla^2 \bar{E} = \gamma^2 \bar{E} \leftarrow (a) \quad \text{and} \quad \nabla^2 \bar{H} = \gamma^2 \bar{E} \leftarrow (b)$$

eq^n (a) and eq^n (b) are called phasor form of Wave equation.

where $\boxed{\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}}$ m^{-1} is the propogation Constant Can be expressed interms of α and β.

a General Expressions

Topic 5.13

Topic:

10. Expressions for $\alpha, \beta, \gamma, \lambda, v$, and η in

- *Free Space*
- *Perfect Dielectrics (loss less dielectrics)*
- *~~Good Conductors~~ (Lossy dielectrics/medium) and*
- *Good Dielectrics (Low Loss dielectrics/medium)*

Specific Cases.

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10. *Expressions for $\alpha, \beta, \gamma, \lambda, v$, and η in*
    - *Free Space*

a. General Expressions for $\alpha, \beta, \gamma, \eta, v_p$ and λ -

I. Expression for attenuation constant (α), phase constant (β) and propogation constant (γ)

w.k.t from phasor form representation of wave equation for Electric field is given by

$$\nabla^2 \bar{E} = j\omega\mu[\sigma + j\omega\epsilon]\bar{E}$$

$$\nabla^2 \bar{E} = \gamma^2 \bar{E}$$

where $\gamma^2 = j\omega\mu[\sigma + j\omega\epsilon]$

and $\gamma = \alpha + j\beta \quad m^{-1}$

Square on both side

$$\gamma^2 = (\alpha + j\beta)^2 = j\omega\mu[\sigma + j\omega\epsilon]$$

$$\alpha^2 - \beta^2 + j2\alpha\beta = -\omega^2\epsilon\mu + j\sigma\omega\mu$$

Comparing real and Imaginary parts

$$\alpha^2 - \beta^2 = -\omega^2\epsilon\mu \quad \text{(i)}$$

and $2\alpha\beta = \sigma\omega\mu \quad \text{(2)}$

for simplicity put $\omega^2 \epsilon \mu = a^2 \leftarrow (3)$ and $\sigma \omega \mu = b^2 \leftarrow (4)$

$$\alpha^2 - \beta^2 = -a^2 \leftarrow (5)$$

$$2\alpha\beta = b^2 \leftarrow (6)$$

Square and adding eq^n (5) and eq^n (6)

$$(\alpha^2 - \beta^2)^2 + 4\alpha^2\beta^2 = a^4 + b^4$$

using $(x+y)^2 = (x-y)^2 + 4xy$

$$\therefore \quad (\alpha^2 - \beta^2)^2 + 4\alpha^2\beta^2 = a^4 + b^4$$

$$(\alpha^2 + \beta^2)^2 = a^4 + b^4$$

$$\alpha^2 + \beta^2 = \sqrt{a^4 + b^4} \leftarrow (7)$$

$\Rightarrow$ eq^n (5) + eq^n (7)

$$2\alpha^2 = \sqrt{a^4 + b^4} - a^2$$

$$2\alpha^2 = \sqrt{a^4\left[1 + \frac{b^4}{a^4}\right]} - a^2$$

$$2\alpha^2 = a^2\sqrt{1 + \frac{b^4}{a^4}} - a^2$$

$$\alpha^2 = \frac{a^2}{2}\left[\sqrt{1 + \frac{b^4}{a^4}} - 1\right] \leftarrow (8)$$

Square and divide eq^n (3) / eq^n (4)

$$\frac{a^4}{b^4} = \frac{(\omega^2 \epsilon \mu)^2}{(\sigma \omega \mu)^2} \Rightarrow \frac{b^4}{a^4} = \frac{\sigma^2 \cancel{\omega^2} \cancel{\mu^2}}{(\omega^2)^{\cancel{2}} \cancel{\mu^2} \epsilon^2}$$

$$\frac{b^4}{a^4} = \frac{\sigma^2}{\omega^2 \epsilon^2} \leftarrow (9)$$

using eq^n (9) in eq^n (8) and $a^2 = \omega^2 \epsilon \mu$

$$\alpha^2 = \frac{\omega^2 \epsilon \mu}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}} - 1\right]$$

xx.

$$\therefore \boxed{\alpha = \omega\sqrt{\mu\epsilon}\left\{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]\right\}^{1/2}}$$

$\Rightarrow$ eq^n (7) − eq^n (5)

$$\cancel{\alpha^2} + \beta^2 - \cancel{\alpha^2} + \beta^2 = \sqrt{a^4+b^4} + a^2$$

$$2\beta^2 = \sqrt{a^4+b^4} + a^2$$

$$\beta^2 = \frac{a^2}{2}\left[\sqrt{1+\frac{b^4}{a^4}} + 1\right]$$

using $\frac{b^4}{a^4} = \frac{\sigma^2}{\omega^2\epsilon^2}$; and $a^2 = \omega^2 \epsilon \mu$

$$\beta^2 = \frac{\omega^2\epsilon\mu}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]$$

xxx

$$\boxed{\beta = \omega\sqrt{\mu\epsilon}\left\{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]\right\}^{1/2}} \text{ rad/m}$$

Note:- 1. attenuation Constant (α)

$$\alpha = \omega\sqrt{\mu\epsilon}\left\{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]\right\}^{1/2} \text{ Np/m}$$

2. phase Constant (β)

$$\beta = \omega\sqrt{\mu\epsilon}\left\{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]\right\}^{1/2} \text{ rad/m}$$

3. propagation Constant (γ)

$$\boxed{\gamma = (\alpha + j\beta)} \text{ m}^{-1}$$

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2). Intrinsic Impedance (or) Characteristic impedance (η)

w.k.t from Faraday's Law $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$; v/m²

$$\begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix} = -\mu \frac{\partial}{\partial t}\left[H_x \bar{a}_x + H_y \bar{a}_y + H_z \bar{a}_z\right]$$

assume that EM wave is propogating along 'z' direction

then $\bar{E} = E_x \bar{a}_x$ v/m and $\bar{H} = H_y \bar{a}_y$ A/m

$$E_x, H_y \Rightarrow f^n(z,t) \text{ only and } \begin{matrix} E_y = E_z = 0 \\ H_x = H_z = 0. \end{matrix}$$

$$\left[\cancelto{0}{\frac{\partial E_y}{\partial z}} - \cancelto{0}{\frac{\partial E_z}{\partial y}}\right]\bar{a}_x + \left[\frac{\partial E_x}{\partial z} - \cancelto{0}{\frac{\partial E_z}{\partial x}}\right]\bar{a}_y + \left[\cancelto{0}{\frac{\partial E_x}{\partial y}} - \cancelto{0}{\frac{\partial E_y}{\partial x}}\right]\bar{a}_z$$

$$= -\mu\left[\frac{\partial}{\partial t}\left[\cancelto{0}{H_x}\bar{a}_x + H_y \bar{a}_y + \cancelto{0}{H_z}\bar{a}_z\right]\right]$$

$$\frac{\partial E_x}{\partial z}\bar{a}_y = -\mu \frac{\partial H_y}{\partial t}\bar{a}_y$$

Comparing y-components on bothside

$$\boxed{\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t}} \leftarrow (1)$$

the representation of UPW travelling in +z' direction is given by $\boxed{E_x = E_m e^{-\gamma z}}$ v/m

where γ - propogation Constant.

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$$\frac{\partial E_x}{\partial z} = E_m e^{-\gamma z}(-\gamma) = -\gamma E_m e^{-\gamma z} \rightarrow E_x$$

$$\frac{\partial E_x}{\partial z} = -\gamma E_x \longleftarrow (2)$$

using eq^n (2) in eq^n (1)

$$-\gamma E_x = -\mu \frac{\partial H_y}{\partial t}$$

$$\Rightarrow \quad \gamma E_x = \mu \frac{\partial H_y}{\partial t} = \mu (j\omega) H_y$$

using $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$ and $\frac{\partial}{\partial t} \rightarrow j\omega$

$$\eta = \frac{E_x}{H_y} = \frac{j\omega\mu}{\gamma}$$

$$\eta = \frac{E_x}{H_y} = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\epsilon)}} = \frac{\sqrt{(j\omega\mu)^2}}{\sqrt{j\omega\mu(\sigma + j\omega\epsilon)}}$$

$$\eta = \sqrt{\frac{(j\omega\mu)^{\cancel{2}}}{\cancel{(j\omega\mu)}(\sigma + j\omega\epsilon)}} = \sqrt{\frac{j\omega\mu}{(\sigma + j\omega\epsilon)}}$$

Intrinsic impedance

$$\Rightarrow \quad \boxed{\eta = \frac{E_x}{H_y} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}}$$

3) Wave Velocity (or) phase velocity (v_p) :-

- the phase velocity v of a plane wave is the velocity with which the phase of the wave propagates. for a wave travelling in +ve 'z' direction the E field is $E = E_m^+ \cos(\omega t - \beta z)$ v/m.

the phase = constant (k)

$$\omega t - \beta z = k$$

the phase velocity $v_p = \frac{dz}{dt}$ m/sec.

$$\omega(1) - \beta \frac{dz}{dt} = 0 \Rightarrow \omega = \beta \frac{dz}{dt}$$

$$\boxed{v_p = \frac{dz}{dt} = \frac{\omega}{\beta}} \text{ m/sec}$$

or

$$\boxed{v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu \epsilon}}} \text{ m/sec}$$

4. Wave Length (λ) :-

$$\boxed{\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu \epsilon}}} \text{ m}$$

Summary:-

1. attenuation constant (α):-

$$\boxed{\alpha = \omega\sqrt{\mu\epsilon}\left\{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]\right\}^{1/2}} \text{ Np/m.}$$

2. phase constant (β):-

$$\boxed{\beta = \omega\sqrt{\mu\epsilon}\left\{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]\right\}^{1/2}} \text{ rad/m.}$$

3. propogation constant (γ):-

$$\boxed{\begin{aligned}\gamma &= (\alpha + j\beta)\, m^{-1} \\ &= \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}\ ; m^{-1}\end{aligned}}$$

4. Intrinsic Impedance 'η':-

$$\boxed{\eta = \sqrt{\frac{j\omega\mu}{(\sigma + j\omega\epsilon)}}}\ \Omega.$$

5. phase velocity (v_p):-

$$\boxed{v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}}} \text{ m/sec.}$$

6. Wave Length (λ):-

$$\boxed{\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f}} \text{ m.}$$

a. $\alpha, \beta, \gamma, \eta, v_p$ and λ in free Space :-

Note In free Space $\sigma = 0$ ℧/m ; $\epsilon = \epsilon_0$ F/m

$\mu = \mu_0$ H/m . Note:- Use the above condn's in general expressions and Simplify.

i) attenuation Constant (α) :-

xx $\boxed{\alpha = 0}$ Np/m.

ii) phase Constant (β) :-

xx $\boxed{\beta = \omega\sqrt{\mu\epsilon}}$ rad/m.

iii) propogation Constant (γ) :-

$\gamma = \alpha + j\beta = 0 + j\omega\sqrt{\mu\epsilon}$; m^{-1}

xx $\therefore \boxed{\gamma = j\omega\sqrt{\mu\epsilon} = \omega\sqrt{\mu\epsilon}\,\angle 90^\circ}$; m^{-1}.

iv) Intrinsic Impedance (η)

xx $\boxed{\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\,\Omega \text{ (or) } 120\pi}$ Ω

v) Wave velocity (or) phase velocity (v_p) :-

xx $\boxed{v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 3\times10^8}$ m/sec

vi) Wave Length (λ) xx $\boxed{\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f}}$ m

b. $\alpha, \beta, \gamma, \eta, V_p$ and λ for Perfect Dielectric (or) Lossless dielectric Medium :- $\frac{\sigma}{\omega\epsilon} \to 0$; $\boxed{\sigma \simeq 0}$

and $\mu = \mu_0\mu_r$ H/m and $\epsilon = \epsilon_0\epsilon_r$ F/m. Note:- use the above condition general expression and Simplify

i) attenuation Constant (α) :-

w.k.t the general expression of 'α'

$$\alpha = \omega\sqrt{\mu\epsilon}\left[\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]\right]^{1/2} ; Np/m$$

using cond^n $\frac{\sigma}{\omega\epsilon} \to 0$

$$\alpha = \omega\sqrt{\mu\epsilon}\left[\frac{1}{2}(\cancel{1}^{0}-1)\right]^{1/2} = 0 ; Np/m$$

$$\therefore \boxed{\alpha = 0} \; Np/m$$

ii) phase Constant (β)

$$\beta = \omega\sqrt{\mu\epsilon}\left\{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]\right\}^{1/2} ; rad/m$$

using cond^n $\frac{\sigma}{\omega\epsilon} \to 0$

$$\beta = \omega\sqrt{\mu\epsilon}\left\{\frac{1}{2}[1+1]\right\}^{1/2} = \omega\sqrt{\mu\epsilon} \; rad/m$$

$$\therefore \boxed{\beta = \omega\sqrt{\mu\epsilon}} \; rad/m.$$

iii) propogation Constant (γ) :-

$$\gamma = \alpha + j\beta = 0 + j\beta = 0 + j\omega\sqrt{\mu\epsilon}$$

$$\therefore \boxed{\gamma = j\omega\sqrt{\mu\epsilon} = \omega\sqrt{\mu\epsilon}\,\angle 90^\circ} \; m^{-1}$$

(4b)

Note:- $Z = a + jb = |Z|\angle Z$

$|Z| = \sqrt{a^2+b^2}$; $\angle Z = \tan^{-1}(b/a)$

iv) Intrinsic Impedance (η) :-

w.k.t the general expression of $\eta = \sqrt{\frac{j\omega\mu}{(\sigma + j\omega\epsilon)}}$ Ω

using $\frac{\sigma}{\omega\epsilon} \to 0$

$$\eta = \sqrt{\frac{j\omega\mu}{\omega\epsilon\left[\cancel{\frac{\sigma}{\omega\epsilon}} + j\right]}} = \sqrt{\frac{j\omega\mu}{j\omega\epsilon}} = \sqrt{\frac{\mu}{\epsilon}}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0\mu_r}{\epsilon_0\epsilon_r}} = \sqrt{\frac{\mu_0}{\epsilon_r}}\sqrt{\frac{\mu_r}{\epsilon_r}} = 377\sqrt{\frac{\mu_r}{\epsilon_r}}$$

$$\therefore \boxed{\eta = \sqrt{\frac{\mu}{\epsilon}} = 377\sqrt{\frac{\mu_r}{\epsilon_r}}} \ \Omega$$

v) phase velocity (or) Wave velocity (v_p) :-

$$\boxed{V_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}}} \ m/sec$$

(or)

$$v_p = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}} = \frac{3\times10^8}{\sqrt{\mu_r\epsilon_r}} \ m/sec$$

$$\boxed{v_p = \frac{1}{\sqrt{\mu\epsilon}} = \frac{3\times10^8}{\sqrt{\mu_r\epsilon_r}}} \ m/sec.$$

vi) Wave Length (λ) :-

$$\boxed{\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f}} \ m$$

11. Expressions for $\alpha, \beta, \gamma, \lambda, v,$ and η in

- *Perfect Dielectrics (loss less dielectrics)*

Summary :- for Perfect dielectrics (or) Lossless dielectrics

$$\frac{\sigma}{\omega\epsilon} \to 0 \; ; \; \boxed{\sigma \simeq 0} \; \text{℧/m}$$

and $\mu = \mu_0 \mu_r$ H/m ; $\epsilon = \epsilon_0 \epsilon_r$ F/m.

i) attenuation Constant (α) :-

so. $\boxed{\alpha = 0}$ Np/m

ii) phase constant (β) :-

$$\boxed{\beta = \omega\sqrt{\mu\epsilon}} \text{ rad/m.}$$

iii) Propogation Constant (γ) :-

$$\gamma = \alpha + j\beta = 0 + j\omega\sqrt{\mu\epsilon}$$

$$\boxed{\gamma = j\omega\sqrt{\mu\epsilon} = \omega\sqrt{\mu\epsilon}\,\angle 90^\circ} \; m^{-1}.$$

iv) Intrinsic impedance (η) :-

$$\boxed{\eta = \sqrt{\frac{\mu}{\epsilon}} = 377\sqrt{\frac{\mu_r}{\epsilon_r}}} \; \Omega.$$

v) wave velocity (or) phase velocity (v_p) :-

$$\boxed{v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{3\times10^8}{\sqrt{\mu_r\epsilon_r}}} \; \text{m/sec.}$$

vi) Wavelength (λ) :- $\boxed{\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f}}$ m.

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12. Expressions for $\alpha, \beta, \gamma, \lambda, v$, and η in
- *Good Dielectrics (Low Loss dielectrics/medium)* (or) Lossy dielectrics.

10 - June /July 2015

Derive the expression for α, β, γ and V for low loss dielectric. (06 Marks)

$\frac{\sigma}{\omega\epsilon} << 1 \Rightarrow \boxed{\sigma \neq 0}$ $\epsilon = \epsilon_0 \epsilon_r$ F/m

and $\mu = \mu_0 \mu_r$ H/m.

i. α, β and γ :-

w.k.t the propogation constant of a general medium

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} \ m^{-1} = (\alpha + j\beta) \ m^{-1}$$

$$\gamma = \sqrt{j\omega\mu(j\omega\epsilon)\left[\frac{\sigma}{j\omega\epsilon} + 1\right]} = \sqrt{j^2\omega^2\mu\epsilon\left(\frac{\sigma}{j\omega\epsilon} + 1\right)}$$

$$\gamma = j\omega\sqrt{\mu\epsilon}\left[1 - j\frac{\sigma}{\omega\epsilon}\right]^{1/2} m^{-1}$$

note :- $\frac{1}{j} = -j$

using Bionomial theorem

$$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots$$

for $|x| < 1$.

if $n = 1/2$

$$(1+x)^{1/2} = 1 + \frac{x}{2} - \frac{x^2}{8}.$$

$$\boxed{x = -\frac{j\sigma}{\omega\epsilon}}$$

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$$\therefore \gamma = j\omega\sqrt{\mu\epsilon}\left\{1 + \frac{1}{2}\left(-j\frac{\sigma}{\omega\epsilon}\right) - \frac{1}{8}\left(-j\frac{\sigma}{\omega\epsilon}\right)^2\right\}$$

$$\gamma = j\omega\sqrt{\mu\epsilon}\left\{1 - j\frac{\sigma}{2\omega\epsilon} + \frac{1}{8}\left(\frac{\sigma}{\omega\epsilon}\right)^2\right\}$$

$$= j\omega\sqrt{\mu\epsilon} + \cancel{\omega}\sqrt{\mu\epsilon}\frac{\sigma}{2\cancel{\omega}\epsilon} + j\omega\sqrt{\mu\epsilon} \times \frac{1}{8}\left(\frac{\sigma}{\omega\epsilon}\right)^2$$

$$\boxed{\gamma = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} + j\omega\sqrt{\mu\epsilon}\left\{1 + \frac{1}{8}\left(\frac{\sigma}{\omega\epsilon}\right)^2\right\}\ m^{-1}}$$

$$\gamma = (\alpha + j\beta)\ m^{-1}$$

Comparing real and Imaginary parts

$$\boxed{\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}\ \text{Np/m}} \text{ and}$$

$$\boxed{\beta = \omega\sqrt{\mu\epsilon}\left\{1 + \frac{1}{8}\left(\frac{\sigma}{\omega\epsilon}\right)^2\right\}}\ \text{rad/m.}$$

iv) Intrinsic impedance (η)

w.k.t the general expression of $\eta = \sqrt{\frac{j\omega\mu}{(\sigma + j\omega\epsilon)}}\ \Omega$

$$\eta = \sqrt{\frac{\cancel{j\omega}\mu}{\cancel{j\omega}\epsilon\left[\frac{\sigma}{j\omega\epsilon} + 1\right]}} = \sqrt{\frac{\mu}{\epsilon}\left[\frac{\sigma}{j\omega\epsilon} + 1\right]^{-1}}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}\left[1 - j\frac{\sigma}{\omega\epsilon}\right]^{-1}}$$

$\frac{1}{j} = -j$ (or) $j^2 = -1$.

if $\left|\frac{\sigma}{\omega\epsilon}\right| << 1$; using Bionomial theorem

$$\eta = \sqrt{\frac{\mu}{\epsilon}}\left[1 - j\frac{\sigma}{\omega\epsilon}\right]^{-1/2}$$

Note:- $(1-x)^{-1/2} = 1 + x/2$ if $|x| << 1$.

$$\boxed{\eta = \sqrt{\frac{\mu}{\epsilon}}\left[1 + j\frac{\sigma}{2\omega\epsilon}\right]}\ \Omega.$$

V. Phase velocity (or) Wave velocity (v_p) m/sec

$$v_p = \frac{\omega}{\beta} = \frac{\cancel{\omega}}{\cancel{\omega}\sqrt{\mu\epsilon}\left[1 + \frac{\sigma^2}{8\omega^2\epsilon^2}\right]} \text{ m/sec.}$$

$$v_p = \frac{1}{\sqrt{\mu\epsilon}}\left[1 + \frac{\sigma^2}{8\omega^2\epsilon^2}\right]^{-1} \text{ m/sec.}$$

using Bionomial theorem $(1+x)^{-1} = (1-x)$; $|x| << 1$.

$$\therefore \boxed{v_p = \frac{1}{\sqrt{\mu\epsilon}}\left[1 - \frac{\sigma^2}{8\omega^2\epsilon^2}\right]} \text{ m/sec.}$$

VI. Wavelength (λ):- $\boxed{\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f}}$ m

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Summary :- $\alpha, \beta, \gamma, \eta, v_p$ and λ in Good dielectric (or) Lossy dielectric Medium.

Cond^n :- $\boxed{\frac{\sigma}{\omega\epsilon} << 1}$ $\boxed{\sigma \neq 0}$ v/m

$\boxed{\mu = \mu_0 \mu_r}$ H/m ; $\boxed{\epsilon = \epsilon_0 \epsilon_r}$ F/m

i) attenuation constant (α) :-

$$\boxed{\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}} \text{ Np/m}$$

ii) phase constant (β) :-

$$\boxed{\beta = \omega\sqrt{\mu\epsilon}\left[1 + \frac{\sigma^2}{8\omega^2\epsilon^2}\right]} \text{ rad/m}$$

iii) propogation Constant (γ) :-

$$\boxed{\gamma = \alpha + j\beta = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} + j\omega\sqrt{\mu\epsilon}\left[1 + \frac{\sigma^2}{8\omega^2\epsilon^2}\right]} ; m^{-1}$$

iv) Intrinsic impedance (η)

$$\boxed{\eta = \sqrt{\frac{\mu}{\epsilon}}\left[1 + j\frac{\sigma}{2\omega\epsilon}\right]} \ \Omega.$$

v) Wave (or) phase velocity :-

$$\boxed{v_p = \frac{1}{\sqrt{\mu\epsilon}}\left[1 - \frac{\sigma^2}{8\omega^2\epsilon^2}\right]} \text{ m/sec}$$

vi) Wave Length (λ) :- $\boxed{\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f}}$ m

13. *Expressions for $\alpha, \beta, \gamma, \lambda, v$, and η in*
- *Good Conductors(Lossy medium)* / Lossy medium.

10-DEC 2013/Jan 2014

Derive an expression for propagation constant, intrinsic impedance and phase velocity in good conducting media if the uniform plane wave is propagating. (06 Marks)

06 – May/June 2010

Deduce the expressions for α and β for a wave traveling in lossy medium. (07 Marks)

06-DEC2009/Jan 2010

With usual notations, derive the expression for intrinsic impendence for lossy media. (06 Marks)

Soln:- In Good Conducting Medium $\frac{\sigma}{\omega\epsilon} >> 1$

$\therefore \sigma \neq 0 \quad \epsilon = \epsilon_0 \epsilon_r$ F/m ; $\mu = \mu_0 \mu_r$ H/m.

$\boxed{\sigma \simeq \infty}$

i) α, β and γ :-

w.k.t from phasor form of wave representation

$$\nabla^2 \bar{E} = \gamma^2 \bar{E}$$

where $\gamma^2 = [j\omega\mu(\sigma + j\omega\epsilon)]$

γ - propogation Constant (m^{-1})

$$\gamma = \sqrt{j\omega\mu[\sigma + j\omega\epsilon]} = \alpha + j\beta$$

$$\gamma = \sqrt{j\omega\mu\sigma\left(1 + j\frac{\omega\epsilon}{\sigma}\right)}$$

w.k.t for a good Conductor $\left(\frac{\sigma}{\omega\epsilon}\right) >> 1$

(or) $\left(\frac{\omega\epsilon}{\sigma}\right) << 1$

$\therefore$ neglect the term $\left(\frac{\omega\epsilon}{\sigma}\right)$.

53 i.e $\frac{\omega\epsilon}{\sigma} \to 0$

$$\gamma = \sqrt{j\omega\mu\sigma}\ (1)$$

$$\gamma = \sqrt{\omega\mu\sigma} \times \sqrt{j}$$

$$\boxed{\gamma = \sqrt{\omega\mu\sigma}\ \angle 45^\circ}\ m^{-1}$$

Note:-

$$j = e^{j\pi/2}$$

$$\sqrt{j} = (e^{j\pi/2})^{1/2} = e^{j\pi/4}$$

$$\sqrt{j} = 1\angle \pi/4$$

(or) $= 1\angle 45^\circ$

$$\gamma = \sqrt{\omega\mu\sigma} \times 1\angle 45^\circ$$

$$\gamma = \sqrt{\omega\mu\sigma}\left[1e^{j\pi/4}\right]$$

$$\gamma = \sqrt{\omega\mu\sigma}\left[\cos(45^\circ) + j\sin(45^\circ)\right]$$

$$\gamma = \sqrt{\omega\mu\sigma}\left[\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right]$$

$$\gamma = \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}}\ ;\ m^{-1}$$

$$\gamma = (\alpha + j\beta)\ ;\ m^{-1}$$

By Comparing real and imaginary part's

$$\boxed{\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}}\ Np/m \quad \text{and} \quad \boxed{\beta = \sqrt{\frac{\omega\mu\sigma}{2}}}\ rad/m$$

Note: $\boxed{\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}}$

Note:- for a lossy medium (or) Good conductor's

$$\boxed{\alpha = \beta}$$

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iv) Intrinsic Impedance (η)

W.k.t the general expression of 'η' is given by

$$\eta = \sqrt{\frac{j\omega\mu}{(\sigma + j\omega\epsilon)}} \ \Omega$$

and for a good conductors $\left(\frac{\sigma}{\omega\epsilon}\right) >> 1$.

i.e $\sigma >> \omega\epsilon$

$$\therefore \boxed{\sigma + j\omega\epsilon \simeq \sigma}$$

$$\Rightarrow \eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}} \times \sqrt{j}$$

Note: $\sqrt{j} = e^{j\pi/4} = 1\angle 45^\circ$

xx.

$$\therefore \boxed{\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ} \ \Omega .$$

v) Phase Velocity (or) Wave velocity (v_p)

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\frac{\omega\mu\sigma}{2}}} = \sqrt{\frac{2\omega}{\mu\sigma}} \ m/sec.$$

xx

$$\therefore \boxed{v_p = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}}} \ m/sec$$

vi) Wavelength (λ) : $\boxed{\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f}}$ m

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Summary :- $\alpha, \beta, \gamma, \eta, v_p$ and λ in Good Conducting (or) Long Medium :-

i) attenuation Constant (α) :-

$$\boxed{\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}} \text{ Np/m.}$$

ii) phase Constant (β) :-

$$\boxed{\beta = \sqrt{\frac{\omega\mu\sigma}{2}}} \text{ rad/m.}$$

Note :- $\boxed{\alpha = \beta}$ in Good conducting / Long medium.

iii) propagation Constant (γ)

$$\boxed{\gamma = \alpha + j\beta = \sqrt{\omega\mu\sigma} \angle 45^\circ} \text{ m}^{-1}$$

iv) Intrinsic Impedance (η) :-

$$\boxed{\eta = \sqrt{\frac{\omega\epsilon}{\sigma}} \angle 45^\circ} \ \Omega.$$

v) phase velocity (or) Wave velocity (v_p)

$$\boxed{v_p = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}}} \text{ m/sec}$$

vi) Wave Length (λ)

$$\boxed{\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f}} \text{ m}$$

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Tables:- Summary:- α, β, γ, η, v_p and λ in all the Cases.

Sl.No / Condn	parameter	Free Space $\sigma = 0$; $\mu = \mu_0$ H/m; $\epsilon = \epsilon_0$ F/m	Perfect dielectrics (or) Lossless dielectrics | medium $\sigma \simeq 0$; $\frac{\sigma}{\omega\epsilon} \to 0$; $\mu = \mu_0\mu_r$ H/m; $\epsilon = \epsilon_0\epsilon_r$ F/m	Good dielectrics (or) Low Loss dielectrics $\left(\frac{\sigma}{\omega\epsilon}\right) << 1$; $\mu = \mu_0\mu_r$ H/m; $\epsilon = \epsilon_0\epsilon_r$ F/m; $\boxed{\sigma \neq 0}$	Good Conducting | Lossy medium. $\frac{\sigma}{\omega\epsilon} >> 1$; $\sigma \simeq \infty$; $\mu = \mu_0\mu_r$ H/m; $\epsilon = \epsilon_0\epsilon_r$ F/m
1>	attenuation Constant (α) Np/m	$\boxed{\alpha = 0}$ Np/m	$\boxed{\alpha = 0}$ Np/m	$\boxed{\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}}$ Np/m	$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$; Np/m
2>	phase Constant (β) → rad/m	$\boxed{\beta = \omega\sqrt{\mu_0\epsilon_0}}$ rad/m.	$\boxed{\beta = \omega\sqrt{\mu\epsilon}}$ rad/m; $\mu = \mu_0\mu_r$ H/m; $\epsilon = \epsilon_0\epsilon_r$ F/m	$\beta = \omega\sqrt{\mu\epsilon}\left[1 + \frac{\sigma^2}{8\omega^2\epsilon^2}\right]$; rad/m	$\beta = \sqrt{\frac{\omega\mu\sigma}{2}}$; rad/m; Note:- $\boxed{\alpha = \beta}$ ***
3>	propagation Constant (γ) $\gamma = (\alpha + j\beta)\ m^{-1}$	$\gamma = j\omega\sqrt{\mu_0\epsilon_0}$; $\boxed{\gamma = \omega\sqrt{\mu_0\epsilon_0}\ \angle 90^\circ}$; m^{-1}	$\gamma = j\omega\sqrt{\mu\epsilon}$; $\boxed{\gamma = \omega\sqrt{\mu\epsilon}\ \angle 90^\circ}$; m^{-1}	$\gamma = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} + j\omega\sqrt{\mu\epsilon}\left[1 + \frac{\sigma^2}{8\omega^2\epsilon^2}\right]$; m^{-1}; $\gamma = \alpha + j\beta$;	$\gamma = \sqrt{\omega\mu\sigma}\ \angle 45^\circ$; m^{-1}
4>	Intrinsic Impedance (η) → ohms (Ω)	$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\ \Omega$ (or) $120\pi\ \Omega$	$\eta = 377\sqrt{\frac{\mu_r}{\epsilon_r}}$; Ω; $\eta = \sqrt{\frac{\mu}{\epsilon}}$; Ω	$\eta = \sqrt{\frac{\mu}{\epsilon}}\left[1 + j\frac{\sigma}{2\omega\epsilon}\right]$; Ω	$\eta = \sqrt{\frac{\omega\mu}{\sigma}}\ \angle 45^\circ$; Ω
5>	phase Velocity (or) Wave Velocity (v_p) → m/sec.	$v_p = \frac{1}{\sqrt{\mu_0\epsilon_0}}$ m/sec $= 3\times10^8$ m/sec	$v_p = \frac{1}{\sqrt{\mu\epsilon}}$ m/sec; $v_p = \frac{3\times10^8}{\sqrt{\mu_r\epsilon_r}}$ m/sec	$v_p = \frac{1}{\sqrt{\mu\epsilon}}\left[1 - \frac{\sigma^2}{8\omega^2\epsilon^2}\right]$; m/sec	$v_p = \sqrt{\frac{2\omega}{\mu\sigma}}$; m/sec
6>	Wavelength (λ) → meters (m)	$\boxed{\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f}}$	$\boxed{\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f}}$; m	$\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f}$; m	$\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f}$; m

Topic : 5.14

13. Concept of Skin effect and Skin depth for Good Conductors.

14. 02-DEC2008/Jan 2009

What do you mean by depth of penetration?

06-DEC2011/Jan 2012

Define : ii) Skin effect. (04 Marks)

10-June/July 2013

Derive an expression for depth of penetration. (07 Marks)

02 - June /July 2011

Describe and derive an expression for the depth of penetration. (04 Marks)

Skin Effect :- When an Electromagnetic wave enters into a Conducting Medium, its amplitude decreases exponentially and becomes practically zero after penetrating a small distance. as a result, the current induced by the wave exists only near the surface of the conductor. This effect is called "Skin effect".

The "Skindepth" (or) "depth of penetration" is defined as the depth of a conductor at which the amplitude of an incident wave drops down to $1/e$ (or) 37.1. of its original value.

if 'x' is the distance travelled in the medium and E_0 is the amplitude then the field E is given by

$$E = E_0 e^{-\alpha x} \longleftarrow (1)$$

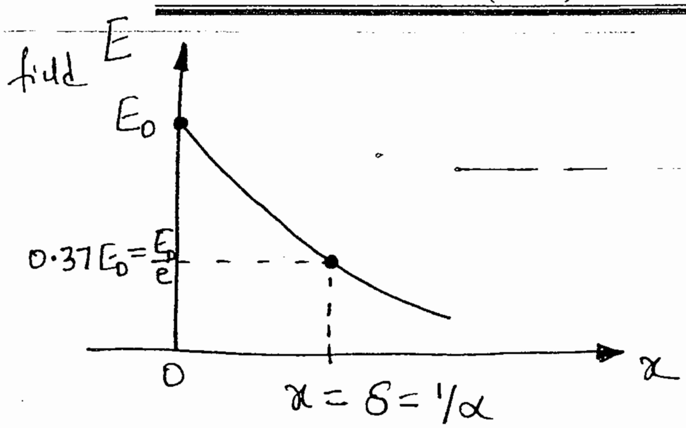


fig. decaying of amplitude in Conducting Medium

Where E_0 is the amplitude either at the time of incidence (or) at some point in the medium where 'x' is taken as zero.

α – attenuation Constant (Np/m).

and w.k.t $\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$ Np/m for good Conducting Medium.

at the depth of penetration of $x = \delta$ m. Let the value of $x = \frac{1}{\alpha}$ (i.e $x = \delta = 1/\alpha$) at which time

$$\Rightarrow E = E_0 e^{-\alpha x} \text{ v/m} \quad e^{-\alpha x} = 1/e = 0.3678 \simeq 0.37$$

$$\therefore E = \frac{E_0}{e} = 0.3678 E_0 \simeq 0.37 E_0 \longleftarrow (2)$$

Where 'δ' is called the depth of penetration (or) Skindepth Measured in meters (m).

w.k.t In Conducting medium

$$\boxed{\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} = \beta}$$

$$\therefore \text{ Skin depth } \delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}} = \frac{1}{\beta};$$

XXX $\boxed{\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{1}{\pi f \mu \sigma}} = \frac{1}{\beta}}$; meters.

Case study 1 :- for copper $\sigma_{copper} = 5.8 \times 10^7$ v/m and $\omega = 2\pi f$ rad/s

by considering free space

$$\boxed{\delta = \frac{0.066}{\sqrt{f}}}\ m$$

@ $f = 50\ Hz \Rightarrow \delta = 1\ cm$;

@ $f = 1\ kHz \Rightarrow \delta = 2\ mm$;

@ $f = 1\ MHz \Rightarrow \delta = 0.066\ mm$;

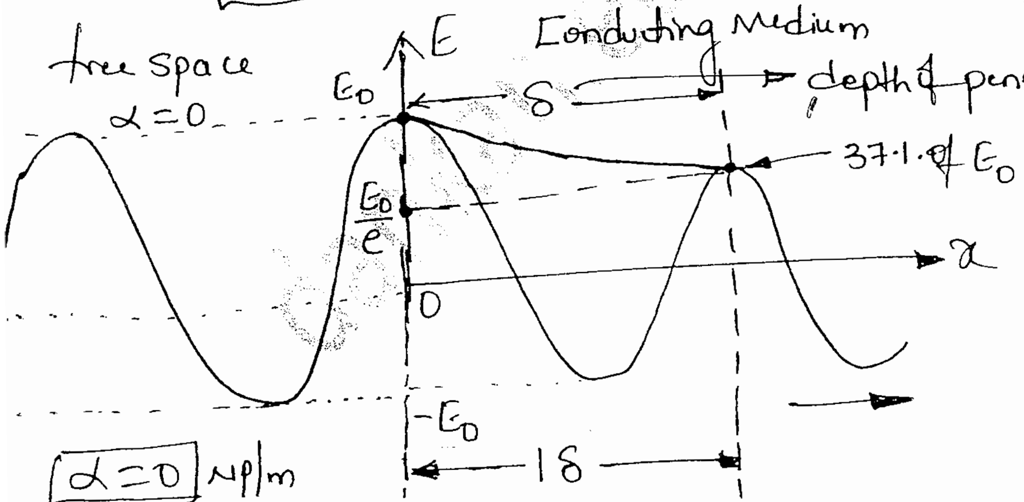
obs:- as $f \uparrow \Rightarrow \delta \downarrow$

as frequency of operation increases the depth of penetration decreases.

Case study 2 :- for Silver $\sigma_{Silver} = 6.17 \times 10^7$ S/m;

$$\boxed{\delta = \frac{1}{\alpha} = \sqrt{\frac{1}{\pi f \mu \sigma}} = \frac{0.064}{\sqrt{f}}}\ ; \text{meters}$$

$E = E_0 e^{-\alpha x}$.



Note 1 :- Skin depth is a Measure of the depth to which an EM wave Can penetrate the Medium.

x = 0 m. boundary

In Conducting medium

$$\boxed{\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}}\ Np/m$$

Note 2 :- the concept of skin effect and skindepth (or) depth of penetration is defined only for Good conductors (or) wave propagation in Conducting Medium.

(60)

Express $\alpha, \beta, \gamma, \eta, v_p$ and λ in terms of Skindepth 'δ' :-

1. In a Conducting Medium $\alpha = \beta = \sqrt{\frac{\omega \mu \sigma}{2}}$

$$\therefore \boxed{\delta = \frac{1}{\alpha} = \frac{1}{\beta} = \sqrt{\frac{2}{\omega \mu \sigma}} = \sqrt{\frac{1}{\pi f \mu \sigma}}} \; ; \text{meters}$$

$$\Rightarrow \boxed{\delta = \alpha^{-1} = \beta^{-1}} \; ; \text{m.}$$

2. phase (or) Wave velocity $v_p = \frac{\omega}{\beta} = \omega(\delta)$

$$\therefore \boxed{v_p = \omega \delta} \; \text{m/sec} \quad \text{(or)} \Rightarrow \boxed{\delta = \frac{v_p}{\omega}} \; ; \text{meters}$$

3. $\gamma = \alpha + j\beta \; ; \text{m}^{-1}$

Sin $\alpha = \beta \quad \Rightarrow \gamma = \alpha(1+j1) = \alpha\sqrt{2}\,\angle 45^\circ$

$$\therefore \boxed{\gamma = \sqrt{2}\,\delta^{-1}\,\angle 45^\circ} \; \text{m}^{-1}$$

4) $\eta = \sqrt{\frac{\omega \mu}{\sigma}}\,\angle 45^\circ \; \Omega$

using $\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega \mu \sigma}}$; meters

$$\frac{1}{\delta} = \sqrt{\frac{\omega \mu \sigma}{2}} = \sqrt{\frac{\omega \mu}{\sigma}}\sqrt{\frac{\sigma^2}{2}} = \sqrt{\frac{\omega \mu}{\sigma}} \cdot \frac{\sigma}{\sqrt{2}}$$

$$\therefore \Rightarrow \sqrt{\frac{\omega \mu}{\sigma}} = \frac{\sqrt{2}}{\sigma}\left(\frac{1}{\delta}\right)$$

$$\therefore \boxed{\eta = \frac{\sqrt{2}}{\sigma \delta}\,\angle 45^\circ} \; \Omega$$

5. the wavelength $\lambda = 2\pi/\beta$; m

$\delta = 1/\beta$; m

$$\boxed{\lambda = 2\pi\delta} \; ; \text{meters}$$

$$\Rightarrow \boxed{\eta = \frac{\sqrt{2}}{\delta \sigma}\,\angle 45^\circ} \; \Omega$$

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e. The depth of penetration in a conducting medium is 0.1m and the frequency of the electromagnetic wave is 1 MHz. Find the conductivity of the conducting medium. (03 Marks)

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15-Dec/Jan 2017
[CBCS-scheme]

Soln: Given

$\delta = 0.1m$.

$f = 1MHz$.

Skindepth.

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{\cancel{2}}{\cancel{2}\pi f\mu\sigma}}$$

$$\delta = \sqrt{\frac{1}{\pi f\mu\sigma}} \quad \text{meters}$$

$$\delta^2 = \frac{1}{\pi f\mu\sigma}$$

$$\sigma = \frac{1}{\pi f\mu\delta^2} \quad \mho/m.$$

$$\sigma = \frac{1}{\pi \times 1\times 10^6 \times 4\pi\times 10^{-7}\times (0.1)^2}$$

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$$\boxed{\sigma = 25.3302} \quad \mho/m \text{ (or) } S/m.$$

$$= 25.3302\ \mho/m \text{ (or) } mho/meter$$

15. 10-June/July 2013

Find the depth of penetration at a frequency of 1.6 MHz in aluminium, where $\sigma = 38.2$ Mz/m and $\mu_r = 1$. Also find γ, λ and V_P. (06 Marks)

06 –Dec/Jan 2008

Find the skin depth δ at a frequency of 1.6 MHz in aluminium, where $\sigma = 38.2$ MS/m and $\mu_r = 1$. Also find γ, λ and V_p. (06 Marks)

soln:- given $f = 1.6\,MHz$ and $\sigma = 38.2 \times 10^6$ v/m.

$\mu_r = 1. \Rightarrow \mu = \mu_0 = 4\pi \times 10^{-7}$ H/m.

→ the depth of penetration

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{1}{\pi f \mu \sigma}}\ ;m$$

$$\delta = \sqrt{\frac{1}{\pi \times 1.6 \times 10^6 \times 4\pi \times 10^{-7} \times 38.2 \times 10^6}} = 6.44 \times 10^{-5} m$$

$$\boxed{\delta = 64.4\,\mu m} = 64.4 \times 10^{-6}\,m$$

→ the propogation constant 'γ' in conducting medium is given by $\gamma = \sqrt{2}\,\delta^{-1} \angle 45^\circ\ ; m^{-1}$

$$\gamma = \sqrt{2}\,(64.4\,\mu)^{-1} \angle 45^\circ ; m^{-1}$$

$$\boxed{\gamma = 2.20 \times 10^4 \angle 45^\circ}\ ; m^{-1}$$

→ Wavelength $(\lambda) = \frac{2\pi}{\beta} = 2\pi\delta$; m

$$\lambda = 2\pi(64.4\,\mu) \Rightarrow \boxed{\lambda = 0.4053 \times 10^{-3}} \text{ meters}$$

(or) $\lambda = 405.3\,\mu$; meters

→ Wave velocity (or) phase velocity (V_p) :-

$$V_p = \frac{\omega}{\beta} = \omega \cdot \delta = 2\pi \times 1.6 \times 10^6 \times 64.4\,\mu$$

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$$\Rightarrow \boxed{V_p = 647.419} \text{ m/sec}$$

16. 06 - June /July 2012

Determine the depth of penetration for copper at 3MHz frequency. The conductivity for copper is $58 * 10^7$ s/m and permeability (μ) is $1.26 * 10^{-6}$ H/m (1.26 μ H/m). (05 Marks)

Sol^n: given $f = 3MHz$. $\sigma = 58\times10^7$ ℧/m.

$\mu = 1.26\mu H/m$.

the depth of penetration (or) skin depth

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{1}{\pi f\mu\sigma}} \text{ ; meters}$$

$$\delta = \sqrt{\frac{1}{\pi\times3\times10^6\times1.26\times10^{-6}\times58\times10^7}} = 1.204939\times10^{-5} \text{ ; meters}$$

$$\boxed{\delta = 12.0493\mu m} \Rightarrow 12.0493\times10^{-6} \text{ meters}$$

Note:-

	Material	Conductivity (σ)
1.	Silver	$\sigma = 6.17\times10^7$ S/m.
2.	Copper	$\sigma = 5.8\times10^7$ S/m.
3.	Gold	$\sigma = 4.10\times10^7$ S/m
4.	Aluminium	$\sigma = 3.82\times10^7$ S/m.

Note2:- In the above problem if we consider $\sigma_{copper} = 5.8\times10^7$ S/m then $\boxed{\delta = 38.1035\mu m}$

$\Rightarrow \delta = 38.1035\times10^{-6}$ meters

02-DEC2010

17. Find the depth of penetration, when a 20 MHz signal is propagating the free space and penetrating into a conductor of conductivity $\sigma = 5\times10^7$ ℧/m. (02 Marks)

Soln:- $f = 20MHz = 20\times10^6$ Hz.

In free space $\mu = \mu_0 = 4\pi\times10^{-7}$ H/m.

$\sigma = 5\times10^7$ ℧/m.

$$\delta = \sqrt{\frac{2}{\mu\omega\sigma}} = \sqrt{\frac{1}{\pi f \mu \sigma}} \text{ meters}$$

$$\delta = \sqrt{\frac{1}{\pi\times20\times10^6\times4\pi\times10^{-7}\times5\times10^7}}$$

$$\delta = 1.59154\times10^{-5} \text{ meters}$$

or $\boxed{\delta = 15.9154\ \mu}$ meters

$= 15.9154\times10^{-6}$ meters

18 a. For silver the conductivity is $\sigma = 3.0\times10^4$ s/m. At what frequency will the depth of penetration be 1 mm? **06 - June /July 2013** **(04 Marks)**

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18 b. For silver the conductivity is $\sigma = 3.0 \times 10^6$ s/m . at what frequency the depth of penetration be 1mm.

Solu:- given $\sigma_{silver} = 3.0\times10^4$ s/m.

$\delta = 1mm.$ $f = ?$

$$\delta = \sqrt{\frac{1}{\pi f \mu \sigma}} \Rightarrow \delta^2 = \frac{1}{\pi f \mu \sigma}$$

$$f = \frac{1}{\delta^2 \pi \mu \sigma}$$

$$f = \frac{1}{(1m)^2 \times \pi \times 4\pi\times10^{-7} \times 3.0\times10^4}$$

$$\boxed{f = 8.4434\ M}Hz = \underline{\underline{8.4434\times10^6}}\ Hz$$

Note:- if $\sigma = 3.0\times10^6$ s/m then

$$\boxed{f = 84.4343\ k}Hz = \underline{\underline{84.4343\times10^{+3}}}Hz$$

10 - June /July 2012

19. Wet marshy soil is characterized by $\sigma = 10^{-2}$ s/m, $\epsilon_r = 15$ and $\mu_r = 1$. At frequencies 60Hz and 10 GHz indicate whether soil be considered a conductor or a dielectric. (04 Marks)

Soln:- given $\sigma = 10^{-2}$ v/m. $\epsilon_r = 15$ and $\mu_r = 1$.

i) $f = 60 Hz$ ii) $f = 10 GHz$.

W.k.t → for a good Conductor's $\left(\frac{\sigma}{\omega\epsilon}\right) >> 1$.

→ for a perfect dielectrics $\left(\frac{\sigma}{\omega\epsilon}\right) \to 0$

→ for a good dielectric $\left(\frac{\sigma}{\omega\epsilon}\right) << 1$.

Case i. $f = 60 Hz$.

$$\frac{\sigma}{\omega\epsilon} = \frac{\sigma}{2\pi f \epsilon_0 \epsilon_r} = \frac{10^{-2}}{2\pi(60)(8.854\times10^{-12})(15)}$$

$$\frac{\sigma}{\omega\epsilon} = (12\times10^{6})\left(\frac{1}{60}\right)$$

$$\Rightarrow \frac{\sigma}{\omega\epsilon} = 2\times10^{5} >> 1.$$

∴ Marshy soil at $f = 60 Hz$ act as a "Good conductor".

Case ii. $f = 10 GHz$; $\frac{\sigma}{\omega\epsilon} = 12\times10^{6}\left(\frac{1}{10\times10^{9}}\right) = 1.2\times10^{-3}$

$$\Rightarrow \frac{\sigma}{\omega\epsilon} = 1.2\times10^{-3} << 1$$

∴ Marshy soil at $f = 10 GHz$ act as a "Dielectric".

20. 10 - June /July 2015

A material is characterized by $\varepsilon_r = 2.5$, $\mu_r = 1$ and $\sigma = 4\times10^{-5}$ ℧/m at f = 1 MHz. Determine the value of the loss tangent, attenuation constant and phase constant. (09 Marks)

Soln:- given $\epsilon_r = 2.5$, $\mu_r = 1$.

$\sigma = 4\times10^{-5}$ ℧/m at $f = 1\,MHz = 1\times10^{6}\,Hz$.

Note: In the given problem medium is not specified

$\therefore$ find, Lossttangent $= \dfrac{\sigma}{\omega\epsilon} = \dfrac{\sigma}{2\pi f \epsilon_0 \epsilon_r}$

$$= \frac{4\times10^{-5}}{2\pi\times1\times10^{6}\times8.854\times10^{-12}\times2.5} = 0.287$$

$\Rightarrow \dfrac{\sigma}{\omega\epsilon} = 0.287 < 1$ $\boxed{\dfrac{\sigma}{\omega\epsilon} = 0.287}$; Losstangent

Since $\left(\dfrac{\sigma}{\omega\epsilon}\right) < 1$ $\therefore$ the medium is considered to be "Good dielectric"

Since the Medium is Good dielectric.

→ attenuation constant (α)

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}\ Np/m = \frac{\sigma}{2}\sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}}$$

$$\alpha = \frac{4\times10^{-5}}{2}\sqrt{\frac{4\pi\times10^{-7}\times1}{8.854\times10^{-12}\times2.5}} = 0.00476\ Np/m$$

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$\boxed{\alpha = 4.7653\times10^{-3}}$ Np/m.

or

$\alpha = 4.7653$ m Np/m

→ phase constant (β)

$$\beta = \omega\sqrt{\mu\epsilon}\left[1 + \frac{\sigma^2}{8\omega^2\epsilon^2}\right] \text{ rad/m}$$

$$\beta = 2\pi f\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}\left[1 + \frac{\sigma^2}{8(2\pi f)^2(\epsilon_0\epsilon_r)^2}\right]$$

$$\beta = 2\pi \times 1\times10^6 \times \sqrt{4\pi\times10^{-7}\times 1\times 8.854\times10^{-12}\times 2.5} \times \left[1 + \frac{(4\times10^{-5})^2}{8\times(2\pi\times10^6)^2(2.5\times 8.854\times10^{-12})^2}\right]$$

$$\beta = 0.033137[1 + 0.010339]$$

$$\boxed{\beta = 0.033479} \text{ rad/m}$$

Summary :- Lontangent $\boxed{\frac{\sigma}{\omega\epsilon} = 0.287}$

attenuation Constant $\boxed{\alpha = 0.00476}$ Np/m

phase Constant $\boxed{\beta = 0.033479}$ rad/m.

21. 02-DEC2010

A 10 MHz signal with Ex = 100 mV/m is ~~propagating in a~~ nature of medium with $\epsilon_r = 1.5$ and $\mu_r = 3.5$. Find, i) Velocity ii) Phase constant iii) Wavelength iv) Intrinsic impedance and v) Hz. (04 Marks)

Solu:- given $f = 10\,MHz$. $E_x = 100\,mV/m$. $\epsilon_r = 1.5\,F/m$. $\mu_r = 3.5\,H/m$.

Since σ value not given ∴ according to the given data Medium is assumed to be perfect dielectrics.

i) Wave (or) phase velocity (v_p):-

$$v_p = \frac{1}{\sqrt{\mu\epsilon}}\ m/sec$$

$$v_p = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}} = \frac{3\times10^8}{\sqrt{\mu_r \epsilon_r}} = \frac{3\times10^8}{\sqrt{3.5\times1.5}}$$

$$\boxed{v_p = 1.3093\times10^8}\ m/sec$$

ii) phase constant (β) :- $\beta = \frac{\omega}{v_p}$ rad/m

$$\beta = \frac{2\pi f}{v_p} = \frac{2\pi \times 10\times10^6}{(1.3093\times10^8)} = 0.47988\ rad/m$$

$$\boxed{\beta = 0.47988}\ rad/m$$

iii) Wave Length (λ) :-

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.47988} \text{ meters}$$

$$\boxed{\lambda = 13.093} \text{ m}$$

iv) Intrinsic impedance (η)

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = 377\sqrt{\frac{\mu_r}{\epsilon_r}} = 377\sqrt{\frac{3.5}{1.5}}$$

$$\boxed{\eta = 575.877} \ \Omega$$

v) the Value of H_z : given $E_x = 100$ m V/m.

w.k.t $\frac{|E_x|}{|H_z|} = \eta$

$$H_z = \frac{E_x}{\eta} = \frac{100 \times 10^{-3}}{575.877}$$

$$H_z = 0.17364 \times 10^{-3} \text{ A/m}$$

or $\boxed{H_z = 0.17364}$ mA/m

06-DEC2008/Jan 2009

22. The magnetic field intensity of uniform plane wave in air is 20 (A/m) in $\vec{a_y}$ direction. The wave is propagating in the $\vec{a_z}$ direction at an angular frequency of 2×10^9(rad/sec)
Find: i) Phase shift constant; ii) Wavelength;
iii) Frequency and iv) Amplitude of electric field intensity. (06 Marks

Soln:- $H_y = 20\bar{a}_y$ A/m

EM wave propogation $\Rightarrow \bar{a}_z$ (z-dirn).

$\omega = 2\times10^9$ rad/sec.

i) phase shift constant (β):-

$$\beta = \frac{2\pi}{\lambda} = \frac{\omega}{v_p} \text{ rad/m}$$

assume given medium to be freespace

$\therefore v_p = 3\times10^8$ m/sec.

$$\Rightarrow \beta = \frac{2\times10^9}{3\times10^8} = 6.667 \text{ rad/m}$$

$\therefore \boxed{\beta = 6.667}$ rad/m

ii) Wavelength (λ):- $\beta = \frac{2\pi}{\lambda}$ rad/m

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{6.667} = 0.942 \text{ m}$$

$\Rightarrow \boxed{\lambda = 0.942}$ m

iii) frequency (f) :-

given $\omega = 2\pi f = 2\times10^9$

$$f = \frac{2\times10^9}{2\pi} = 0.318\times10^9\ Hz$$

$$\therefore \Rightarrow \boxed{f = 0.318\ G}\ Hz$$

iv) Amplitude of Electric field Intensity (E_x)

$H_y = 20\ A/m\ ;\ \bar{a_y}$

W.K.T $\eta = \frac{E_x}{H_y}\ \Omega$; assume Medium to be freespace

$$E_x = \eta H_y = 377(20)$$

$$\boxed{\bar{E_x} = 7540\,\bar{a_x}}\ V/m$$

or $\bar{E_x} = 7.54\,\bar{a_x}\ kV/m$

$$|E_x| = 7.54\ kV/m$$

23. 02-DEC2008/Jan 2009

For damp soil at a frequency of 1 MHz given that $\varepsilon_r = 12$, $\mu_r = 1$ and conductivity (a) = 20m ℧/m. Determine i) Attenuation constant ii) Phase constant iii) Propagation constant iv) Wavelength v) ~~Phase velocity~~ vi) Intrinsic impedance. (06 Marks)

σ

Soln:- given $f = 1 MHz$. $\epsilon_r = 12\ F/m$, $\mu_r = 1\ H/m$.

$\sigma = 20\ m℧/m$.

the losstangent value $\frac{\sigma}{\omega\epsilon} = \frac{20\times10^{-3}}{2\pi(10^6)\times12\times8.854\times10^{-12}}$

$\frac{\sigma}{\omega\epsilon} = 29.959 >> 1$

$\Rightarrow$ Since $\left(\frac{\sigma}{\omega\epsilon}\right) >> 1$.

$\therefore$ the given medium is Considered to be Good Conductor (or) Conducting medium.

In a Good Conducting Medium

i) attenuation Constant (α):-

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\frac{2\pi f \mu_0 \mu_r \sigma}{2}}$$

$$\alpha = \sqrt{\pi f \mu_0 \mu_r \sigma}$$

$$\alpha = \sqrt{\pi\times10^6\times4\pi\times10^{-7}\times1\times20\times10^{-3}}$$

$$\boxed{\alpha = 0.28099}\ Np/m.$$

ii) Phase Constant (β) :-

$$\boxed{\beta = \alpha = \sqrt{\frac{\omega\mu\sigma}{2}} = 0.28099} \text{ rad/m}$$

iii) propagation Constant (γ)

$$\gamma = \alpha + j\beta = \alpha\sqrt{2}\,\angle 45^\circ$$

Since $\alpha = \beta$

$$\gamma = \sqrt{2} \times 0.28099\,\angle 45^\circ$$

$$\boxed{\gamma = 0.3897\,\angle 45^\circ}\ m^{-1}$$

(or) $\gamma = (0.2809 + j0.2809)\ m^{-1}$.

iv) Wave velocity (or) phase velocity (v_p) :-

$$v_p = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \frac{2\pi \times 10^6}{0.28099} = 22.368 \times 10^6 \text{ m/sec}$$

$$\boxed{v_p = 22.368 \times 10^6} \text{ m/sec}$$

v) Intrinsic impedance (η) :-

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}}\,\angle 45^\circ = \sqrt{\frac{2\pi f \mu_0 \mu_r}{\sigma}}\,\angle 45^\circ$$

$$\eta = \sqrt{\frac{2\pi \times 10^6 \times 4\pi \times 10^{-7} \times 1}{20 \times 10^{-3}}}\,\angle 45^\circ = 19.869\,\angle 45^\circ\ \Omega$$

$$\boxed{\eta = 19.869\,\angle 45^\circ}\ \Omega.$$

vi) Wavelength (λ) :- $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.2809} = 22.36$ m

$$\Rightarrow \boxed{\lambda = 22.3680} \text{ m}$$

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XD. Special kind

24. 06-DEC2009/Jan 2010

The electric field intensity of 300 MHz uniform plane wave in free space is given by $E = (20 + j50)(a_x + 2a_y)e^{-j\beta z}$ V/m. Find

i) ω, λ, u and β ii) E at t = 1 ns z = 10 cm iii) What is $|H|_{max}$? (10 Marks)

Sol^n:- given $\bar{E} = (20 + j50)(\bar{a}_x + 2\bar{a}_y)e^{-j\beta z}$ v/m

$f = 300$ MHz and medium - freespace.

i) a) $\omega = 2\pi f = 2\pi \times 300 \times 10^6 = 6\pi \times 10^8$ rad/sec

b) $\lambda = \frac{v_p}{f} = \frac{3\times10^8}{300\times10^6} = 1\,m \Rightarrow \boxed{\lambda = 1m}$.

c) $\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{1} \Rightarrow \boxed{\beta = 2\pi}$ rad/m

d) $v_p = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3\times10^8$ m/sec $\Rightarrow \boxed{v_p = 3\times10^8}$ m/sec.

ii) $E(z,t) = Re\{E e^{j\omega t}\}$; given t = 1 nsec and z = 10 cm.

$$= Re\{(\underset{a}{20} + j\underset{b}{50})(\bar{a}_x + 2\bar{a}_y)[\underset{c}{\cos(\omega t - \beta z)} + j\underset{d}{\sin(\omega t - \beta z)}]\}$$

Note:- $(a + jb)(c + jd) = (ac - bd) + j(bd - bc)$

$$= Re\{-11-\} = (\bar{a}_x + 2\bar{a}_y)[20\cos(\omega t - \beta z) - 50\sin(\omega t - \beta z)]$$

$$= [20\cos(\omega t - \beta z) - 50\sin(\omega t - \beta z)]\bar{a}_x + [40\cos(\omega t - \beta z) - 100\sin(\omega t - \beta z)]\bar{a}_y$$

the value of $\omega t - \beta z = ?$

(Fb) $\omega t - \beta z = 6\pi \times 10^8 \times 1\times10^{-9} - 2\pi \times 10 \times 10^{-2} = 0.4\pi$ rad

$$E(z,t) = \left[20\cos(0.4\pi) - 50\sin(0.4\pi)\right]\bar{a}_x + \left[40\cos(0.4\pi) - 100\sin(0.4\pi)\right]\bar{a}_y \text{ V/m.}$$

$$\cos(0.4\pi) = 0.309$$
$$\sin(0.4\pi) = 0.951$$

$$E(z,t) = -41.372\,\bar{a}_x - 82.74\,\bar{a}_y \text{ V/m.}$$

$$\Rightarrow \boxed{E(z,t) = -41.372\,\bar{a}_x - 82.74\,\bar{a}_y} \text{ V/m}$$

iii) $$|H|_{max} = \frac{|E|_{max}}{\eta}$$

$$|E|_{max} = \sqrt{E E^*} \text{ V/m.}$$

and use $\eta = 377\,\Omega$ (bcz given medium is freespace)

given $E = (20+j50)\,(\bar{a}_x + 2\bar{a}_y)\,e^{-j\beta z}$ V/m.

$$E^* = (20-j50)\,(\bar{a}_x + 2\bar{a}_y)\,e^{+j\beta z} \text{ V/m}$$

$$EE^* = (20+j50)\,(20-j50)\,(\bar{a}_x + 2\bar{a}_y)(\bar{a}_x + 2\bar{a}_y)\,\cancel{e^{-j\beta z}}\,\cancel{e^{+j\beta z}}$$

Note:- $(a+jb)(a-jb) = a^2 + b^2$

$$EE^* = [20^2 + 50^2] \times \sqrt{1+4} \times \sqrt{1+4}$$

$$EE^* = 14500$$

$$\therefore \quad |E_{max}| = \sqrt{EE^*} = \sqrt{14500} = 120.415 \ V/m$$

$$\Rightarrow \quad |H_{max}| = \frac{|E_{max}|}{\eta} = \frac{120.415}{377} = 0.319405 \ A/m$$

$$\boxed{|H_{max}| = 0.319405} \ A/m$$

Summary :-

i) a. $\omega = 6\pi \times 10^8$ rad/sec

b. $\lambda = 1m$

c. $v_p = 3 \times 10^8$ m/sec.

d. $\beta = 2\pi$ rad/m.

ii) $\boxed{E(z,t) = -41.372\bar{a}_x - 82.74\bar{a}_y}$ V/m

iii) $\boxed{|H_{max}| = 0.319405}$ A/m

25. 06-DEC2011/Jan 2012

A 300 MHz uniform plane wave propagates through (lossless med.) fresh water for which $\sigma = 0$, $\mu r = 1$ and $\varepsilon r = 78$. Calculate : i) α, ii) β, iii) λ, iv) η. (08 Marks)

10-Jan 2013

A 300 MHz uniform plane wave propagates through fresh water for which $\sigma = 0$, $\mu_r = 1$, $\epsilon_r = 78$, calculate :

i) Attenuation constant
ii) Phase constant
iii) Wave length
iv) Intrinsic impedance. (05 Marks)

Solu:- given $f = 300 \times 10^6$ Hz. $\sigma = 0$, $\mu_r = 1$ and $\epsilon_r = 78$.

$\left(\frac{\sigma}{\omega\epsilon}\right) \rightarrow 0$ ∴ the given medium in considered to be a "perfect dielectric" (or) "Lossless Medium"

i) attenuation constant (α):-

Since given $\boxed{\sigma = 0}$

∴ $\boxed{\alpha = 0}$ Np/m : for Lossless Medium.

ii) phase constant (β):-

$\beta = \omega\sqrt{\mu\epsilon} = \omega\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}$ rad/m

$\beta = 2\pi \times 300 \times 10^6 \sqrt{4\pi \times 10^{-7} \times 1 \times 8.854 \times 10^{-12} \times 78}$

$\boxed{\beta = 55.5294}$ rad/m

iii) Wave Length (λ) :-

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{55.5294} = 0.11315 \text{ meters}$$

$$\Rightarrow \boxed{\lambda = 0.11315} \text{ m}$$

iv) Intrinsic impedance (η) :-

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = 377\sqrt{\frac{\mu_r}{\epsilon_r}} = 377\sqrt{\frac{1}{78}}$$

$$\boxed{\eta = 42.686} \ \Omega$$

v) phase velocity (or) Wave velocity (V_p)

$$V_p = \frac{1}{\sqrt{\mu\epsilon}} = \frac{3\times10^8}{\sqrt{\mu_r\epsilon_r}} = \frac{3\times10^8}{\sqrt{78}}$$

$$\boxed{V_p = 0.33968\times10^8} \text{ m/sec}$$

26. 10-DEC2011/Jan 2012

Calculate intrinsic impedance η, propagation constant γ and wave velocity v for a ~~conducting~~ medium in which $\sigma = 58$ MS/m, $\mu_r = 1$, $\epsilon_r = 1$ at frequency of 100 MHz.

(06 Marks)

06-DEC 2013/Jan 2014

Calculate intrinsic impedance η. $\sigma = 58$ Ms/m, $\mu_r = 1$, $\epsilon_r = 1$ at frequency of 100 MHz.

(06 Marks)

Sol^n: given $\sigma = 58 \times 10^6$ ℧/m.

$\mu_r = 1$ and $\epsilon_r = 1$ at $f = 100 \times 10^6$ Hz.

Lontangent $\frac{\sigma}{\omega\epsilon} = \frac{58 \times 10^6}{2\pi \times 100 \times 10^6 \times 8.854 \times 10^{-12}}$

$\frac{\sigma}{\omega\epsilon} = 1.0425 \times 10^{10} >> 1$

$\Rightarrow \left(\frac{\sigma}{\omega\epsilon}\right) >> 1$ ∴ the given medium is considered to be Good Conductor.

i) Intrinsic impedance (η)

$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ = \sqrt{\frac{2\pi f \mu_0 \mu_r}{\sigma}} \angle 45^\circ$

$\eta = \sqrt{\frac{2\pi \times 100 \times 10^6 \times 4\pi \times 10^{-7} \times 1}{58 \times 10^6}} \angle 45^\circ$

$\boxed{\eta = 3.6896 \times 10^{-3} \angle 45^\circ}$ Ω

(or) $\boxed{\eta = 0.0036896 \angle 45^\circ}$ Ω

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ii) propagation Constant (γ) :-

$$\gamma = \sqrt{\omega\mu\sigma} \angle 45^\circ \ m^{-1}$$

$$= \sqrt{2\pi f \mu_0 \mu_r \sigma} \angle 45^\circ$$

$$\gamma = \sqrt{2\pi \times 100\times10^6 \times 4\pi\times10^{-7} \times 58\times10^6} \angle 45^\circ$$

$$\boxed{\gamma = 213.997\times10^3 \angle 45^\circ} \ m^{-1}$$

iii) Wave velocity (v_p) :-

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\frac{\omega\mu\sigma}{2}}} = \sqrt{\frac{2\omega^2}{\omega\mu\sigma}} = \sqrt{\frac{2\omega}{\mu\sigma}}$$

$$v_p = \sqrt{\frac{2\times2\pi\times100\times10^6}{4\pi\times10^{-7}\times58\times10^6}} = 4.15227 \ km/sec$$

$$v_p = 4.1522\times10^3 \ m/sec$$

$$\boxed{v_p = 4.1522 \ km/sec}$$

iv) attenuation Constant (α) and phase Constant (β)

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\frac{2\pi f \mu_0\mu_r\sigma}{2}}$$

(82)

$$\alpha = \sqrt{\pi f \mu_0\mu_r\sigma} \ Np/m$$

$$\alpha = \sqrt{\pi \times 100 \times 10^{6} \times 4\pi \times 10^{-7} \times 58 \times 10^{6}}$$

$$\boxed{\alpha = 151.319 \times 10^{3}} \text{ Np/m}$$

Since the Medium is Good conductor

$$\Rightarrow \quad \alpha = \beta = 151.319 \times 10^{3}$$

∴ phase constant (β)

$$\boxed{\beta = 151.319 \times 10^{3}} \text{ rad/m}$$

the value of Skin depth (or) depth of penetration

$$\delta = \frac{1}{\alpha} = \alpha^{-1} = (151.319 \times 10^{3})^{-1}$$

$$\delta = 6.608 \times 10^{-6} \text{ meter}$$

$$\boxed{\delta = 6.608} \ \mu\text{m}$$

27. 10-DEC 2013/Jan 2014

The $\bar{H}$ field in free space is given by $\bar{H}(x,t) = 10\cos(10^8 t - \beta x)\hat{a}y$ A/mt. Find β, λ and $E(x, t)$ at $P(0.1, 0.2, 0.3)$ and t = 1ns. **(06 Marks)**

soln: given

$$\bar{H}(x,t) = 10\cos(10^8 t - \beta x)\,\bar{a}_y \text{ A/m.}$$

a) By comparing with std field

$$\bar{H}(x,t) = H_m^+ \cos(\omega t - \beta x)\,\bar{a}_y \text{ A/m.}$$

$$\Rightarrow H_m^+ = 10 \text{ A/m}; \quad \omega = 10^8 \text{ rad/sec}$$

i) $\beta = \frac{2\pi}{\lambda}$ rad/m.

given medium in freespace $v_p = \frac{\omega}{\beta}$ m/sec

$$\Rightarrow \beta = \frac{\omega}{v_p} = \frac{10^8}{3\times10^8} = 1/3$$

$$\boxed{\beta = 0.3333} \text{ rad/m}$$

ii) $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{(1/3)} = 6\pi$; meter

$$\boxed{\lambda = 6\pi} \text{ meter} = 18.8495 \text{ meter}$$

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iii) $E(x,t) = E_m^+ \cos(\omega t - \beta x)\,\overline{a_z}$ V/m.

Since given EM wave propagates along 'x' direction ∴ $\overline{H} \rightarrow \overline{a_y}$ ($\overline{E_z} \rightarrow \overline{a_z}$)

i.e $\overline{E} \rightarrow \overline{a_z}$ V/m.

$\eta = \frac{|E|}{|H|}\ \Omega$

$\Rightarrow E_m^+ = \eta H_m^+ = 377(10) = 3770$ V/m

$\boxed{E_m^+ = 3.77}$ kV/m

∴ $E(x,t) = E_m^+ \cos(\omega t - \beta x)\,\overline{a_z}$ V/m

and $\omega = 10^8$ rad/sec

$\beta = 1/3$ rad/m

$E(x,t) = 3770 \cos(10^8 t - \frac{1}{3}x)\,\overline{a_z}$ V/m.

$E(x,t)$ at $P(0.1, 0.2, 0.3)$ and $t = 1$ nsec.

$x = 0.1$ m ; $t = 1\times10^{-9}$ sec

$E(x,t) = 3770 \cos\left[10^8 \times 10^{-9} - \frac{0.1}{3}\right]\overline{a_z}$ V/m

$E(x,t) = 3769.99\,\overline{a_z}$ V/m

85 $\Rightarrow \boxed{|E(x,t)| = 3769.99}$ V/m

06 - June /July 2011
(10 Marks)

28. A uniform plane wave with 10 MHz frequency ~~has~~ average pointing vector 1 w/m^2. If the medium is perfect dielectric with $\mu_r = 2$ and $\epsilon_r = 3$, $\mu_0 = 4\pi \times 10^{-7}$ H/m, $\epsilon_0 = 8.854 \times 10^{-12}$ F/m;
Find :
i) Velocity
ii) Wavelength
iii) Intrinsic impedance
iv) rms value of electric field.

(10 Marks)

Solu:- given $f = 10 MHz$.

$\sigma \simeq 0$; $\frac{\sigma}{\omega \epsilon} \to 0$; $\epsilon_r = 3$ F/m and

$\mu_r = 2$ H/m.

$\therefore$ Medium is Considered to be perfect dielectrics.

$\mu_0 = 4\pi \times 10^{-7}$ H/m ; $\epsilon = \epsilon_0 \epsilon_r$ F/m

$\epsilon_0 = 8.854 \times 10^{-12}$ F/m.

i) wave velocity (v_p):-

$$v_p = \frac{1}{\sqrt{\mu \epsilon}} = \frac{3\times 10^8}{\sqrt{\mu_r \epsilon_r}} = \frac{3\times 10^8}{\sqrt{2\times 3}}$$

$$v_p = 122.474 \times 10^6 \text{ m/sec}$$

$$\boxed{v_p = 1.22474 \times 10^8} \text{ m/sec}$$

ii) Wave Length (λ)

$$v_p = f\lambda \text{ m/sec}$$

$$\lambda = \frac{v_p}{f} = \frac{122.474\times10^6}{10\times10^6} = 12.2474\text{ m}$$

$$\boxed{\lambda = 12.2474} \text{ meter}$$

iii) Intrinsic impedance (η)

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = 377\sqrt{\frac{\mu_r}{\epsilon_r}} = 377\sqrt{\frac{2}{3}}$$

$$\boxed{\eta = 307.8192}\ \Omega$$

iv) using poynting theorem:-

$$P_{avg} = \frac{1}{2}\frac{E_m^2}{\eta} = \frac{E_m^2}{2\eta} \text{ w/m}^2$$

given $P_{avg} = 1\text{ w/m}^2$

$$E_m = \sqrt{2\eta(P_{avg})} = \sqrt{2\times307.8192\times1}$$

$$\boxed{E_m = 24.812} \text{ v/m}$$

r.m.s value of E

$$E_{rms} = \frac{E_m}{\sqrt{2}} = \frac{24.812}{\sqrt{2}} = 17.544 \text{ V/m}$$

$$\boxed{E_{rms} = 17.544} \text{ V/m}$$

My if $E_m = 24.812$ V/m

$$\left|\frac{E_m}{H_m}\right| = \eta \ \Omega$$

$$\Rightarrow H_m = \frac{E_m}{\eta} = \frac{24.812}{307.8192}$$

$$\boxed{H_m = 0.080605} \text{ A/m}$$

$$= 80.6057 \text{ mA/m}$$

$$H_{rms} = \frac{H_m}{\sqrt{2}} = 56.9968 \text{ mA/m}$$

29. 02 - June /July 2011

A uniform plane wave propagating in a perfect dielectric medium has
$E = 500 \cos[10^7 t - \beta z]\, a_x$ ~~Vm and~~
$H = 1.1 \cos[10^5 t - \beta z]\, a_y$ A/m. If the wave is travelling with a velocity u = 0.5 C (m/s), find ϵ_r and μ_r, where $c = 3 \times 10^8$ m/s. (04 Marks)

and also find iii) β iv) λ v) η.

Solu:- given $\bar{E} = 500 \cos[10^7 t - \beta z]\, \bar{a}_x$ v/m ← (1)

$\bar{H} = 1.1 \cos[10^5 t - \beta z]\, \bar{a}_y$ A/m.

$v_p = 0.5\, C = 0.5 \times 3 \times 10^8 = 1.5 \times 10^8$ m/sec.

$\epsilon_r = ?$ $\mu_r = ?$

Note:- given $\bar{E}$ and $\bar{H}$ to be of same angular frequency. i.e $\boxed{\omega_E = \omega_H}$ rad/sec.

∴ Consider $\omega = 10^7$ rad/sec.

$\Rightarrow$ $\bar{H} = 1.1 \cos[10^7 t - \beta z]\, \bar{a}_y$ A/m, ← (2)

$\omega = 10^7$ rad/sec, $E_m = 500$ v/m ; $H_m = 1.1$ A/m.

$$\eta = \frac{|E|}{|H|} = \frac{E_m}{H_m} = 377\sqrt{\frac{\mu_r}{\epsilon_r}} = \frac{500}{1.1}$$

$$\Rightarrow \sqrt{\frac{\mu_r}{\epsilon_r}} = 1.20569 \quad \leftarrow (a)$$

given $v_p = 1.5 \times 10^8 = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu_0 \epsilon_0 \mu_r \epsilon_r}}$

$$v_p = \frac{3 \times 10^8}{\sqrt{\mu_r \epsilon_r}} \quad \leftarrow (b)$$

Cntd in
Note:- refer page No. 593(a)

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Note:- Solu contd Problem NO.29.

from eqⁿ (a) $\frac{\mu_r}{\epsilon_r} = 1.453688$.

$\Rightarrow \mu_r = 1.453688\,\epsilon_r \leftarrow$ (c)

from eqⁿ (b) $\mu_r \epsilon_r = 4$.

using eqⁿ (c) $(1.453688)(\epsilon_r)(\epsilon_r) = 4$

$\epsilon_r^2 = \frac{4}{1.45368} \Rightarrow \epsilon_r = \pm 1.6588$ F/m

ϵ_r to be +ve $\therefore$ $\boxed{\epsilon_r = 1.6588}$ F/m

and using eqⁿ (c) $\mu_r = 1.453688(1.6588)$

$\boxed{\mu_r = 2.41137}$ H/m

iii) phase constant $\beta = \frac{\omega}{v_p} = \frac{10^7}{1.5\times10^8}$ rad/m

$\boxed{\beta = 0.0667}$ rad/m $\Rightarrow \beta = 0.0667$ rad/m

iv) WaveLength (λ); $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.0667}$ m

$\boxed{\lambda = 94.2477}$ m

v) $\eta = \sqrt{\frac{\mu}{\epsilon}} = 377\sqrt{\frac{\mu_r}{\epsilon_r}} = 377\sqrt{\frac{2.41137}{1.6588}} = 454.02\,\Omega$

$\boxed{\eta = 454.02}\,\Omega$

(or) $\eta = \frac{E_m^+}{H_m^+} = \frac{500}{1.1} = 454.02\,\Omega$

30.

06 - June /July 2012

A 10GHz plane wave travelling in free space has an amplitude of 15V/m. Find:

i) Velocity of propogation.
ii) Wave length.
iii) Characteristic impedance.
iv) Amplitude of $\overline{H}$.
v) Propogation constant (β). (05 Marks)

A 10 GHz plane wave in free space has electric field intensity 15 V/m. Find :

i) Velocity of propagation
ii) Wavelength
iii) Characteristic impedance of the medium
iv) Amplitude of magnetic field intensity
v) Propagation constant β. (10 Marks)

---

solu:- given $f = 10\,GHz = 10\times10^9\,Hz$.

$|E| = 15\,V/m$.

medium:- free space. $\epsilon = \epsilon_0$ and $\mu = \mu_0$ H/m

i) $v_p = \dfrac{1}{\sqrt{\mu_0\epsilon_0}} = 3\times10^8$ m/sec.

$\boxed{v_p = 3\times10^8}$ m/sec

ii) wave Length (λ):-

$\lambda = \dfrac{v_p}{f} = \dfrac{3\times10^8}{10\times10^9} = 0.03\,m$

$\boxed{\lambda = 0.03}$ m

iii) Characteristic impedance (η):-

$$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377 \text{ or } 120\pi \ \Omega$$

$$\boxed{\eta = 377 \text{ or } 120\pi} \ \Omega$$

iv) $|\bar{H}| = \frac{|E|}{\eta} = \frac{15}{377} = 0.03978 \text{ A/m}$

$$\boxed{|\bar{H}| = 0.039787} \text{ A/m}$$

v) propogation constant:- (γ)

$$\gamma = \alpha + j\beta \ m^{-1}$$

In freespace $\boxed{\alpha = 0}$ Np/m

and $\beta = \omega\sqrt{\mu_0 \epsilon_0} = \frac{\omega}{v_p} = \frac{2\pi \times 10 \times 10^9}{3 \times 10^8}$

$$\boxed{\beta = 209.439} \text{ rad/m}$$

$$\gamma = \alpha + j\beta = 0 + j\beta = j\beta = \beta \angle 90^\circ$$

$$\therefore \boxed{\gamma = 209.439 \angle 90^\circ} \ m^{-1}$$

31. 10 - June /July 2012

A 800 MHz plane wave travelling has an average Poynting vector of 8 mW/m^2. If the medium is losses with $\mu_r = 1.5$ and $\epsilon_r = 6$. Find :

i) ~~Velocity of wave~~ → its "Lon Len" Medium. | perfect dielectric Medium

ii) Wavelength

iii) Impedance of the medium

iv) r.m.s. electric field E and

v) r.m.s. magnetic field H.

(08 Marks)

Soln:- given $\mu_r = 1.5\ H/m$; $\epsilon_r = 6\ F/m$.

$P_{avg} = 8\ mW/m^2$. $f = 800\ MHz$.

i) Velocity of wave $v_p = \frac{1}{\sqrt{\mu\epsilon}}$ m/sec

$$v_p = \frac{3\times10^8}{\sqrt{\mu_r\epsilon_r}} = \frac{3\times10^8}{\sqrt{1.5\times6}} = 1\times10^8\ m/sec$$

$$\boxed{v_p = 1\times10^8}\ m/sec$$

ii) Wavelength (λ)

$$\lambda = \frac{v_p}{f} = \frac{1\times10^8}{800\times10^6} = 0.125\ m$$

$$\boxed{\lambda = 0.125}\ meter$$

iii) Impedance of the medium (η)

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = 377\sqrt{\frac{\mu_r}{\epsilon_r}} = 377\sqrt{\frac{1.5}{6}}$$

$$\boxed{\eta = 188.5}\ \Omega$$

iv) r.m.s Value of Electric field E using poynting theorem ---- given $P_{avg} = 8\,mW/m^2$

$$P_{avg} = \frac{E_m^2}{2\eta}\ W/m^2$$

$$E_m = \sqrt{2 P_{avg} \eta} = \sqrt{2 \times 8 \times 10^{-3} \times 188.5}$$

$$\boxed{E_m = 1.7366}\ V/m$$

$$H_m = \frac{E_m}{\eta} = \frac{1.7366}{188.5} = 9.2127 \times 10^{-3}\ A/m$$

$$E_{rms} = \frac{E_m}{\sqrt{2}} = \frac{1.7366}{\sqrt{2}} = 1.228\ V/m$$

$$\Rightarrow \boxed{E_{rms} = 1.228}\ V/m$$

$H_m = 9.2127 \times 10^{-3}\ A/m$; $\quad H_{rms} = \frac{H_m}{\sqrt{2}} = \frac{9.212 \times 10^{-3}}{\sqrt{2}}$

$$\boxed{H_{rms} = 6.5143 \times 10^{-3}}\ A/m$$

or $\quad H_{rms} = \frac{E_{rms}}{\eta} = \frac{1.228}{188.5} = 6.514 \times 10^{-3}\ A/m$

$$\Rightarrow H_{rms} = 6.514 \times 10^{-3}\ A/m$$

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32. 10 - June /July 2015

A uniform plane wave traveling in +z direction in air has $H = 20\hat{a}_y$ A/m the frequency of the signal is $\frac{1}{\pi} \times 10^9$ Hz. Find λ, T and E. (06 Marks)

Soln:- given $\bar{H}_y = 20\bar{a}_y$ A/m. $H_y = 20$ A/m.

$f = \frac{1}{\pi} \times 10^9$ Hz.

given medium in air ⇒ free space.

⇒ $\mu = \mu_0$ H/m ; $\epsilon = \epsilon_0$ F/m.

i) $v_p = f\lambda$ m/sec.

$v_p = 3\times10^8$ m/sec ... in free space.

$$\lambda = \frac{v_p}{f} = \frac{3\times10^8}{\left(\frac{1}{\pi}\times10^9\right)} = \frac{3\pi\times10^8}{10^9} = \underline{0.94247}$$

$\boxed{\lambda = 0.94247}$ meters

ii) $T = \frac{1}{f} = \frac{1}{\left(\frac{1}{\pi}\times10^9\right)} = \pi\times10^{-9}$ Sec

$= 3.14159\times10^{-9}$ sec $= \underline{3.14154 / \text{sec}}$

$T = 3.14159\times10^{-9}$

$T = \pi\ \eta$ sec or $\boxed{T = 3.14159\ \eta \text{sec}}$

iii) $|\bar{E}| = \eta |\bar{H}|$ or $E_x = \eta H_y$ V/m

and $\eta = 377\ \Omega$.

$E_x = 377\,(20) = 7540$ V/m

$\boxed{E_x = 7.540}$ kV/m

$|E_x| = 7540$ V/m $\qquad \bar{E}_x = 7540\,\bar{a}_x$ V/m

$\boxed{|\bar{E}_x| = 7.54}$ kV/m

10 - June /July 2015

33. For a uniform plane wave, $E_y = 10.4e^{(-j\beta x + 2\pi \times 10^9 t)}$ V/m. Find
i) The direction of propagation.
ii) Phase constant β
iii) Expression for H. (05 Marks)

Soln:- given $E_y = 10.4\, e^{(-j\beta x + 2\pi \times 10^9 t)}$ v/m. ← wrong format.

Comparing with Std. form.

$E_y = E_m e^{j(\omega t - \beta x)}$ v/m.

given E_y should be of ~~the~~ form

$\therefore E_y = 10.4\, e^{+j(-\beta x + 2\pi \times 10^9 t)} = 10.4\, e^{-j\beta x + j2\pi\times10^9 t}$ v/m.

$\omega = 2\pi \times 10^9$ rad/sec ; $E_m = 10.4$ v/m.

i) given $E_y = 10.4\, e^{j(2\pi \times 10^9 t - \beta x)}$ v/m ← indicates that

$E_y \Rightarrow f(x,t) \therefore$ dirc^n of EM wave propogation.

$\Rightarrow$ the Direction of EM wave propagation is $+x$ dirc^n ($\bar{a}_x$).

Note:- $E_y(x,t) = E_m^+ e^{j(\omega t - \beta x)} + E_m^- e^{j(\omega t + \beta x)}$ v/m.

Real part
$Re\{E_y(x,t)\} = \underbrace{E_m^+ \cos(\omega t - \beta x)}_{\text{Forward travlling wave } (+x \text{ dirc}^n)} + \underbrace{E_m^- \cos(\omega t + \beta x)}_{\text{Reverse travlling wave } (-x \text{ dirc}^n)}$

ii) Phase constant (β):- assume that given medium to be free space.

$$\beta = \frac{\omega}{v_p} = \frac{2\pi \times 10^9}{3 \times 10^8} = 20.94 \text{ rad/m}.$$

$$\boxed{\beta = 20.94} \text{ rad/m}$$

iii) Phase velocity (v_p):-

$$v_p = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/sec}.$$

$$\boxed{v_p = 3 \times 10^8} \text{ m/sec}$$

iv) Expression for ($\bar{H}$):-

w.k.t $\eta = \frac{|\bar{E}|}{|\bar{H}|} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377 \Omega$

$|\bar{E}| = 10.41$ v/m.

$$\Rightarrow |\bar{H}| = \frac{|\bar{E}|}{\eta} = \frac{10.4}{377} = 0.027586 \text{ A/m}.$$

$$\Rightarrow \boxed{|\bar{H}| = 0.027586} \text{ A/m}$$

$\therefore$ Expression for $\bar{H}(x,t)$

Since $\bar{E} \xrightarrow{\text{in}} \bar{a_y}$; EM $\Rightarrow$ +x' dirⁿ $\Rightarrow \bar{H} \xrightarrow{\text{to be}} \bar{a_z}$

$$\bar{H}_z(x,t) = H_m e^{j(\omega t - \beta x)} \text{ A/m}$$

$$\boxed{\bar{H}_z(x,t) = 0.02756\, e^{j(2\pi \times 10^9 t - \beta x)}\, \bar{a_z}} \text{ A/m}.$$

Note:- In original problem (a) if $E_y = 10.4\, e^{-j\beta x + j2\pi \times 10^9 t}\, \bar{a_y}\, \mu$ v/m.

then $\Rightarrow \bar{H}_z(x,t) = 0.02756\, e^{j(2\pi \times 10^9 t - \beta x)}\, \bar{a_z}\, \mu$A/m.

34a.

06 – Jan 2013

A 9375 MHz uniform plane wave is propagating in polystyrene ($\mu_r = 1$, $\epsilon_r = 2.56$). If the amplitude of electric field intensity is 20V/mt and the material is assumed to be lossless. ~~Find~~ ~~i)~~ Phase constant ii) Wavelength iii) Velocity of propagation iv) Intrinsic impedance v) Magnetic field intensity. (10 Marks)

Soln: given: $f = 9375\,MHz = 9375 \times 10^6\,Hz$.

$\mu_r = 1\,H/m$, $\epsilon_r = 2.56\,F/m$; $|\bar{E}| = 20\,V/m$.

given medium is Lossless ⟹ ie perfect dielectric Medium.

i) phase constant (β)

$$\beta = \frac{\omega}{v_p} = \omega\sqrt{\mu\epsilon}\ \text{rad/m}$$

$$\beta = 2\pi f\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}\ \text{rad/m}$$

$$\beta = 2\pi \times 9375 \times 10^6\sqrt{4\pi \times 10^{-7} \times (1) \times 8.854 \times 10^{-12} \times 2.56}$$

$$\boxed{\beta = 314.373}\ \text{rad/m}$$

iii) velocity of propagation (v_p)

$$v_p = \frac{1}{\sqrt{\mu\epsilon}} = \frac{3\times10^8}{\sqrt{\mu_r\epsilon_r}} = \frac{3\times10^8}{\sqrt{2.56}}$$

$$v_p = \frac{3\times10^8}{\sqrt{2.56}} = 187.5\times10^6\ \text{m/sec}$$

$$v_p = 187.5\times10^6\ \text{m/sec}$$

$$\Rightarrow \boxed{v_p = 1.875\times10^8}\ \text{m/sec}$$

iii) Wave Length (λ) :-

$$v_p = f\lambda \text{ m/sec}$$

$$\lambda = \frac{v_p}{f} = \frac{1.87\times10^8}{9375\times10^6}$$

$$\boxed{\lambda = 0.02} \text{ meters}$$

iv) Intrinsic impedance (η) :-

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0}}\cdot\sqrt{\frac{\mu_r}{\epsilon_r}} = 377\sqrt{\frac{\mu_r}{\epsilon_r}}$$

$$\eta = 377\sqrt{\frac{1}{2.56}} = 235.625\ \Omega$$

$\therefore$ $$\boxed{\eta = 235.625}\ \Omega$$

v) given $|E| = 20$ V/m.

$|H| = ?$ using $\eta = \frac{|E|}{|H|}$

$$\Rightarrow |H| = \frac{|E|}{\eta} = \frac{20}{235.625} = 0.08488 \text{ A/m}$$

$\therefore$ $$\boxed{|H| = 84.88} \text{ mA/m}$$

vi) Propogation Constant (γ)

$$\gamma = (\alpha + j\beta)\ m^{-1}$$

for a Lossless (or) perfect dielectric Medium

$$\boxed{\alpha = 0}\ Np/m$$

$$\therefore \gamma = 0 + j\beta = j\beta = \beta\angle 90^\circ\ m^{-1}$$

$$\boxed{\gamma = 314.37\angle 90^\circ}\ m^{-1}$$

34b.

A 9.375 GHz uniform plane wave is propagating in polyethylene ($\epsilon_r = 2.26$). If the amplitude of the $\vec{E}$ is 500 V/m and the material is assumed to be lossless, find
i) Phase constant ii) Wavelength iii) Velocity of propagation
iv) Intrinsic impedance v) Magnetic field intensity (06 Marks)

Solu:- given $f = 9.375\,GHz = 9.375 \times 10^6\,Hz$.

$\boxed{\epsilon_r = 2.26}$ F/m ; $|E| = 500$ V/m.; assume $\boxed{\mu_r = 1}$ H/m

given medium is Lossless or perfect dielectric.

i) phase constant (β):-

$$\beta = \frac{2\pi}{\lambda} = \omega\sqrt{\mu\epsilon} = \frac{\omega}{v_p} \text{ rad/m}$$

$$\beta = 2\pi f \sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}$$

$$\beta = 2\pi \times 9.375 \times 10^9 \sqrt{4\pi \times 10^{-7} \times 1 \times 8.854 \times 10^{-12} \times 2.26}$$

$$\boxed{\beta = 295.379} \text{ rad/m}$$

ii) WaveLength (λ)

$$\lambda = \frac{v_p}{f} = \frac{2\pi}{\beta} \text{ m}$$

$$\lambda = \frac{2\pi}{295.379} = 0.02127 \text{ m}$$

$$\boxed{\lambda = 0.02127} \text{ meter}$$

iii) Velocity of propagation (v_p):-

$$v_p = \frac{1}{\sqrt{\mu \epsilon}} = \frac{3\times10^8}{\sqrt{\mu_r \epsilon_r}} = \frac{3\times10^8}{\sqrt{1\times2.26}}$$

$$\therefore \boxed{v_p = 1.9955\times10^8} \text{ m/sec.}$$

iv) Intrinsic Impedance (η)

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = 377\sqrt{\frac{\mu_r}{\epsilon_r}} = 377\sqrt{\frac{1}{2.26}} = \frac{377}{\sqrt{2.26}}$$

$$\boxed{\eta = 250.776} \ \Omega$$

v) Magnetic field Intensity ($|H|$)

$$|H| = \frac{|E|}{\eta} = \frac{500}{250.776}$$

$$\boxed{|H| = 1.9938} \text{ A/m}$$

Summary :-

$\boxed{\beta = 295.379}$ rad/m

$\boxed{\lambda = 0.02127}$ meters

$\boxed{v_p = 1.9955\times10^8}$ m/sec

$\boxed{\eta = 250.776}$ Ω

$\boxed{|H| = 1.9938}$ A/m

35. 06 –June/July 2014

A plane wave traveling in positive x-direction in a lossless unbounded medium having permeability 4.5 times that of free space and a permittivity twice that of the wave.

i) Find phase velocity of the wave.

ii) If E has only y-component with a amplitude 20 V/m, find the amplitude and direction of H. (04 Marks)

Soln:- given wave is travelling in x-direction

and $\mu = 4.5\mu_0$ H/m.

$\epsilon = 2\epsilon_0$ F/m.

and E has only y-component i.e $E_y = 20$ V/m.

Given medium is Lossless (or) perfect dielectric medium.

i) phase velocity of the wave

$$v_p = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{(4.5\mu_0)(2\epsilon_0)}} = \frac{1}{\sqrt{9}\sqrt{\mu_0\epsilon_0}}$$

$$v_p = \frac{\cancel{3}\times 10^8}{\cancel{3}} = 1\times 10^8 \text{ m/sec}$$

$$\boxed{v_p = 1\times 10^8} \text{ m/sec}$$

ii) Direction of (H):-

A plane wave travelling in x-direction has components only in y-z plane in which its electric and Magnetic vectors are normal to each other.

Since 'E' is directed along y-direction as per data

'H' must be along 'z' direction. so the concerned vectors are E_y and H_z.

$H_z = ?$ given $E_y = 20\ V/m$

using $\frac{|E|}{|H|} = \eta = \frac{E_y}{H_z} = \sqrt{\frac{\mu}{\epsilon}}$

$$\frac{E_y}{H_z} = 377\sqrt{\frac{\mu_r}{\epsilon_r}} = 377\sqrt{\frac{4.5}{2}} = 565.10\ \Omega$$

$$\Rightarrow \frac{E_y}{H_z} = 565.10\ \Omega \Rightarrow H_z = \frac{E_y}{565.10}$$

$$\Rightarrow H_z = \frac{20}{565.10} = 0.03539\ A/m.$$

$\therefore \boxed{H_z = 0.03539}\ A/m$

36. If the electric field vector in free space is $E = 800Cos(10^8 t - \beta y)a_z$ v/m. Find the following $i. \beta$ $ii. \lambda$ $iii. H$ at the point P(1,1.5,0.4) at t=8nsec.

soln:- $\bar{E} = 800 \cos(10^8 t - \beta y)\bar{a}_z$ v/m.

$\omega = 10^8$ rad/sec ; freespace $v_p = 3\times10^8$ m/sec.

i) $\beta = \frac{\omega}{v_p} = \frac{10^8}{3\times10^8} = 1/3 = 0.333$ rad/m.

$\boxed{\beta = 0.333}$ rad/m

ii) $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{(1/3)} = 6\pi = 18.86$ meter

$\boxed{\lambda = 18.86}$ meter

iii) $|\bar{H}| = H_x = \frac{|E_z|}{\eta} = \frac{800}{377} = 2.12$ A/m.

$\therefore \boxed{\bar{H}(y,t) = \bar{H} = 2.12\cos(10^8 t - \beta y)\bar{a}_x}$ A/m.

$\omega = 10^8$ rad/sec ; $\beta = 1/3$ rad/m ; $t = 8$nsec.

and $y = 1.5$

$\bar{H} = 2.12 \cos[10^8 \times 8\times10^{-9} - 1/3 \times 1.5]\ \bar{a}_x$

$\boxed{\bar{H} = 2.02\,\bar{a}_x}$ A/m

37. A UPW $E_y = 10Sin(2\pi \times 10^8 t - \beta x) is\ travelling\ in$ x-direction in free space. Find the β, v_p, H_z component. assume $\overline{E}_z = \overline{H}_y = 0$.

solu:- given $E_y = 10 \sin(2\pi \times 10^8 t - \beta x)$ v/m.

and Medium is free space.

$E_m = 10$ v/m. $\Rightarrow |E_y| = 10$ v/m.

i) $\beta = \frac{\omega}{v_p} = \frac{2\pi \times 10^8}{3 \times 10^8} = 2.09$ rad/m.

$\boxed{\beta = 2.09}$ rad/m

ii) $v_p = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8$ m/sec.

$\boxed{v_p = 3 \times 10^8}$ m/sec

iii) $H_z = \frac{E_y}{\eta} = \frac{10}{377} = 0.0265$ A/m

$\therefore \overline{H}_z(x,t) = 0.0265 \sin[2\pi \times 10^8 t - \beta x]\ \overline{a}_z$ A/m.

or $\boxed{H_z = 0.0265 \sin(2\pi \times 10^8 t - \beta x)}$ A/m

38. The electric field of UPW is given by
$E = 40Sin(30\pi \times 10^6 t - 2\pi z)\boldsymbol{a}_x + 40Cos(30\pi \times 10^6 t - 2\pi z)\boldsymbol{a}_y . v/m$ Find
i.f in Hz ii. λ iii. direction of wave propagation iv. the associated Magnetic Field **H**

Soln: given

$$\bar{E} = 40\sin(30\pi \times 10^6 t - 2\pi z)\,\bar{a}_x + 40\cos(30\pi \times 10^6 t - 2\pi z)\,\bar{a}_y \ : \ v/m$$

assume given medium in free space.

i) $$f = \frac{\omega}{2\pi} = \frac{\cancel{30}^{15}\cancel{\pi} \times 10^6}{\cancel{2\pi}} = 15 \times 10^6 \ Hz$$

$$\boxed{f = 15} \ MHz$$

ii) $$\lambda = \frac{v_p}{f} = \frac{3 \times 10^8}{15 \times 10^6} = 20 \ m.$$

$$\boxed{\lambda = 20} \ \text{meter}$$

iii) Since in given $\bar{E} \Rightarrow (\omega t - \beta z)$

z ← dirn of EM wave propogation.

$\therefore$ Direction of Wave propagation in $+z$ dirn

or $\bar{a}_z$.

iv) $$|H| = \frac{|E|}{\eta} = \frac{40}{377} = 0.106 \ A/m$$

$$\therefore \boxed{\bar{H}(z,t) = 0.106\sin(30\pi \times 10^6 t - 2\pi z)\,\bar{a}_x + 0.106\cos(30\pi \times 10^6 t - 2\pi z)\,\bar{a}_y} \ : \ A/m.$$

39. A 10G Hz UPW travelling in free space in 'x' direction has E_z=1v/m. Find Magnetic field associated and Propagation Constant.

Feb. 2004 (6M)

Soln: $f = 10\,GHz$, $E_z = 1v/m$.

In free space $v_p = 3\times10^8$ m/sec.

$\eta = 377\ \Omega$.

i) $H_y = \frac{E_z}{\eta} = \frac{1}{377} = 2.65\times10^{-3}$ A/m.

$\boxed{H_y = 2.65\times10^{-3}}$ A/m

ii) In free space $\boxed{\alpha = 0}$ Np/m and $\beta = \omega\sqrt{\mu_0\epsilon_0}$ rad/m

$\beta = \frac{\omega}{v_p} = \frac{2\pi\times10\times10^9}{3\times10^8} = 209.4$ rad/m

$\boxed{\beta = 209.4}$ rad/m $= 209.439$ rad/m

$\gamma = \alpha + j\beta = 0 + j\beta = j\beta = \beta\angle 90^\circ\ m^{-1}$

$= 209.439\angle 90^\circ\ m^{-1}$

$\boxed{\gamma = 209.439\angle 90^\circ}\ m^{-1}$

40. Determine $\alpha, \lambda, \beta, \gamma, \eta, v_p$ $for\ damped\ soil\ at\ frequency\ of\ 1M\ Hz\ given\ that\ \varepsilon_r = 12, \mu_r = 1\ and\ \sigma = 20 \times 10^{-3}\ s/m.$

Soln:- given $f = 1\,MHz$. $\varepsilon_r = 12\ F/m$; $\mu_r = 1$

and $\sigma = 20 \times 10^{-3}\ s/m$. $\Rightarrow \frac{\sigma}{\omega\varepsilon} = 29.959 >> 1$

∴ Medium in Good Conducting.

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}\ m^{-1} = \sqrt{j\omega\mu\sigma - \omega^2\mu\epsilon}\ ;\ m^{-1}$$

$$\gamma = \sqrt{j2\pi \times 10^6 \times 4\pi \times 10^{-7} \times 20 \times 10^{-3} - (2\pi \times 10^6)^2 \times 4\pi \times 10^{-7} \times 12 \times 8.854 \times 10^{-12}}$$

$$\gamma = \sqrt{j0.1579 - 5.27 \times 10^{-3}}$$

the term $\Rightarrow 5.27 \times 10^{-3}$ is very small ∴ neglect.

$$\gamma = \sqrt{j0.1579} = \underline{\underline{0.397 \angle 45^\circ}}\ m^{-1}$$

$$\gamma = 0.281 + j0.281 \Rightarrow (\alpha + j\beta)\ m^{-1}$$

$\Rightarrow \boxed{\alpha = 0.281}$ Np/m and $\boxed{\beta = 0.281}$ rad/m

$$v_p = \frac{\omega}{\beta} = \frac{2\pi \times 10^6}{0.281} = 2.236 \times 10^7\ m/sec$$

$$\boxed{v_p = 2.236 \times 10^7}\ m/sec$$

$$\lambda = \frac{v_p}{f} = \frac{2.236 \times 10^7}{10^6} = 22.36\ meters$$

$\Rightarrow \boxed{\lambda = 22.36}$ meters

In Good Conductor

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ = \underline{\underline{19.87 \angle 45^\circ}}\ \Omega$$

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S39a) A Conductor Carries Steady Current of I amperes. The components of Current density vector $\bar{J}$ are $J_x = 2ax$ and $J_y = 2ay$. Find the third component J_z. Derive any relation employed. June - 2006 (10M).

Note:- Module-5A Question.

Soln:- using Continuity eqn

$$\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t} \; A/m^3.$$

if Conductor carries Steady Current then

$$\rho_v = \text{Constant} \Rightarrow \frac{\partial \rho_v}{\partial t} = 0 \; C/m^3\text{-sec}.$$

$$\Rightarrow \nabla \cdot \bar{J} = 0$$

$$\frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} = 0$$

$$\frac{\partial}{\partial x}(2ax) + \frac{\partial}{\partial y}(2ay) + \frac{\partial J_z}{\partial z} = 0$$

$$2a + 2a + \frac{\partial J_z}{\partial z} = 0$$

$$\frac{\partial J_z}{\partial z} = -4a.$$

Integrating w.r.t z

$$\boxed{J_z = -4az + K} \; A/m^2$$

where K - integral constant.

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$E = 10 V/m.$

b. A plane wave of 16 GHz frequency and ~~E = 10 v/m~~ propagates through the body of salt water having constants $\varepsilon = 100$, $\mu_r = 1$ and $\sigma = 100$ S/m. Determine attenuation constant, phase shift, phase velocity and intrinsic impedance of the medium and depth of penetration. (08 Marks)

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15-Dec/Jan 2017 (CBCS-Scheme)

Soln:-

Given

$f = 16$ GHz. $E = 10 V/m.$

$\varepsilon = 100$, $\mu_r = 1$ and $\sigma = 100$ S/m.

Loss tangent $\frac{\sigma}{\omega\varepsilon} = \frac{100}{2\pi \times 10^9 \times 16 \times 8.854 \times 10^{-12} \times 100}$

$\frac{\sigma}{2\pi f \varepsilon_0 \varepsilon_r} =$

$$\frac{\sigma}{\omega\varepsilon} = 1.12346 >> 1$$

$$\Rightarrow \left(\frac{\sigma}{\omega\varepsilon}\right) >> 1$$

i.e $\sigma >> \omega\varepsilon$

The given medium is considered to be the Good Conducting Medium.

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i. ~~Attenuation Constant~~ (α)

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} \text{ Np/m} = \sqrt{\frac{2\pi\times16\times10^{9}\times4\pi\times10^{-7}\times100}{2}}$$

$$\boxed{\alpha = 2513.27} \text{ Np/m}$$

ii. phase constant (β)

In conducting medium

$$\boxed{\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = 2513.27} \text{ rad/m}$$

iii. phase velocity (v_p)

$$v_p = \sqrt{\frac{2\omega}{\mu\sigma}} = \frac{\omega}{\beta} \quad \text{m/sec.}$$

$$v_p = \sqrt{\frac{2\times2\pi\times16\times10^{9}}{4\pi\times10^{-7}\times100}} = 40\times10^{6} \text{ m/sec}$$

$$\boxed{v_p = 0.4\times10^{8}} \text{ m/sec} = 0.4\times10^{8} \text{ m/sec.}$$

iv. Intrinsic impedance of the medium

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ \ \Omega$$

$$\eta = \sqrt{\frac{2\pi f \mu}{\sigma}} \angle 45^\circ$$

$$\eta = \sqrt{\frac{2\pi \times 16 \times 10^{9} \times 4\pi \times 10^{-7}}{100}} \angle 45^\circ$$

$$\boxed{\eta = 35.543 \angle 45^\circ \ \Omega}$$

v. Skin depth (or) depth of pentration

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

$$\delta = \alpha^{-1} = (2513.27)^{-1}$$

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$$\boxed{\delta = 397.888 \, \mu m}$$ (or)

$$\boxed{\delta = 0.3978 \, mm}$$

15-June/July 2017
CBCS

OR

a. What is Forward travelling wave and Backward travelling wave in free space? **(02 Marks)**

Consider a wave equation for field E in freespace is given by

$$\boxed{\frac{\partial^2 E_x}{\partial t^2} = v^2 \frac{\partial^2 E_x}{\partial z^2}}$$

assume that the field $\bar{E}$ points along x-direction and wave propogates along z direction.

the solution of this wave is given by

$$\boxed{E_x = E_m^+ \cos(\omega t - \beta z) + E_m^- \cos(\omega t + \beta z)} \text{ V/m}$$

Solution consists of one component of field travelling in positive z-direction having E_m^+ i.e forward travelling wave; while other component having amplitude E_m^- travelling in negative z-direction called backward travelling wave.

15-June/July 2017
CBCS

A uniform plane wave in free space is given by $E_s = 200 \angle 30^\circ \cdot e^{-j250z}\, \hat{a}_x$ V/m.
Find β, w, f, λ, η, $|\vec{H}|$ **(06 Marks)**

Soln:- given field $\bar{E}_s = 200 \angle 30^\circ e^{-j250z} \bar{a}_x$ v/m. ← ⓐ

Note:- $200 \angle 30^\circ = 200 e^{j\pi/6}$

General Expression of field E points along 'x' direcn assum Em wave is propagating along 'z' direcn is

$$\bar{E}_s = E_m e^{j(wt-\beta z)} \bar{a}_x \text{ v/m} \leftarrow ⓑ$$

Comparing equation ⓐ and eqn ⓑ

$\boxed{E_m = 200}$ v/m

i. $\boxed{\beta = 250}$ rad/m

ii. $\beta = \frac{2\pi}{\lambda}$ rad/m

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{250} = 0.02513 \text{ meters}$$

$\boxed{\lambda = 25.132 \text{ mm}}$

given wave is travelling in free space

$\mu = \mu_0$ H/m and $\epsilon = \epsilon_0$ F/m.

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3\times10^8 \text{ m/sec.}$$

$$\boxed{v = 3\times10^8} \text{ m/sec}$$

iii. $v = f\lambda$ m/sec

$$f = \frac{v}{\lambda} = \frac{3\times10^8}{25.132\times10^{-3}}$$

$$\boxed{f = 11.9366 \text{ GHz}}$$

iv. angular frequency (ω)

$$\omega = 2\pi f \text{ rad/sec}$$

$$\omega = 2\pi(11.9366\times10^9)$$

$$\boxed{\omega = 7.5\times10^{10}} \text{ rad/sec}$$

$= 75$ G rad/sec

114-C

vi. Intrinsic impedance (η)

$$\boxed{\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\ \Omega}$$ or

or $120\pi\ \Omega$.

vii. $|\bar{H}| = ?$

$$\eta = \frac{|\bar{E}_m|}{|\bar{H}_m|} = \frac{E_m}{H_m}$$

$$\Rightarrow H_m = \frac{E_m}{\eta} = \frac{200}{120\pi}$$

$$\boxed{H_m = 0.53051}\ A/m$$

Topic: 5.15

14. Poynting's theorem and wave power.

41.	State and explain the Poynting's theorem.	06-DEC2010 (04 Marks)
	State and prove the pointing vector theorem.	02-DEC2010 (08 Marks)
	State and explain Polynting theorem.	06-DEC2009/Jan 2010 (04 Marks)
	Define : i) Poynting's theorem	06-DEC2011/Jan 2012 (04 Marks)
	State and explain Poynting's theorem.	10-DEC2011/Jan 2012 (06 Marks)
	State and explain Poynting theorem.	10-Jan 2013 (05 Marks)
	Write a short note on Poynting theorem.	10-DEC 2013/Jan 2014 (10 Marks)
	State and prove pointing theorem.	02 - June /July 2011 (08 Marks)
	Prove and explain the Poynting theorem using Maxwell's equations.	02 - June /July 2012 (08 Marks)
	State and prove Poynting's theorem.	06- June /July 2009 (10 Marks)
	State and explain Poynting theorem.	010-Dec/Jan 2015 (10 Marks)
	Poynting vector.	10 - June /July 2015 (12 Marks)
	State and prove Poynting theorem.	10 - June /July 2014 (08 Marks)
	State and prove Poynting theorem.	06 - June /July 2013 (06 Marks)
	State and explain Poynting theorem.	06 – Jan 2013 (04 Marks)
	Short note on -10Mark Poynting's theorem.	06 – May/June 2010

State and explain poynting theorem 15 - Dec/Jan 2017 (CBCS-Scheme)

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[15 - June/July 2017 (8M) CBCS Scheme]

06 –June/July 2014

State and prove Poyntings theorem. (06 Marks

15- J/J 2017 (CBCS) (8M)

Note:- $\bar{H} \cdot \frac{\partial \bar{H}}{\partial t} = \frac{1}{2} \frac{\partial H^2}{\partial t}$ ← (1)

Illy $\bar{E} \cdot \frac{\partial \bar{E}}{\partial t} = \frac{1}{2} \frac{\partial E^2}{\partial t}$ ← (2)

proof :- Consider the eq^n (1)

$$\bar{H} \cdot \frac{\partial \bar{H}}{\partial t} = \frac{1}{2} \frac{\partial H^2}{\partial t}$$

let the vector $\bar{H} = H_x \bar{a}_x + H_y \bar{a}_y + H_z \bar{a}_z$ A/m.

$$\bar{H} \cdot \bar{H} = H_x^2 + H_y^2 + H_z^2 = H^2.$$

$$|\bar{H}| = \sqrt{H_x^2 + H_y^2 + H_z^2} \text{ A/m.}$$

i.e $\bar{H} \cdot \bar{H} = H^2$

$$\frac{\partial}{\partial t}[\bar{H} \cdot \bar{H}] = \bar{H} \cdot \frac{\partial \bar{H}}{\partial t} + \bar{H} \cdot \frac{\partial \bar{H}}{\partial t} = 2\bar{H} \cdot \frac{\partial \bar{H}}{\partial t}$$

$$\Rightarrow \frac{\partial}{\partial t}(H^2) = 2\bar{H} \cdot \frac{\partial \bar{H}}{\partial t}$$

$$\Rightarrow \boxed{\bar{H} \cdot \frac{\partial \bar{H}}{\partial t} = \frac{1}{2} \frac{\partial H^2}{\partial t}}$$

(or) Consider L.H.S of eq^n (1)

$$\bar{H} \cdot \frac{\partial \bar{H}}{\partial t} = [H_x \bar{a}_x + H_y \bar{a}_y + H_z \bar{a}_z] \cdot \frac{\partial}{\partial t}[H_x \bar{a}_x + H_y \bar{a}_y + H_z \bar{a}_z]$$

$$\bar{H} \cdot \frac{\partial \bar{H}}{\partial t} = H_x \frac{\partial H_x}{\partial t} + H_y \frac{\partial H_y}{\partial t} + H_z \frac{\partial H_z}{\partial t}$$

$\div$ & x^k by '2'

$$= \frac{1}{2}\left[2H_x\frac{\partial H_x}{\partial t} + 2H_y\frac{\partial H_y}{\partial t} + 2H_z\frac{\partial H_z}{\partial t}\right]$$

$$= \frac{1}{2}\left[\frac{\partial H_x^2}{\partial t} + \frac{\partial H_y^2}{\partial t} + \frac{\partial H_z^2}{\partial t}\right] = \frac{1}{2}\frac{\partial H^2}{\partial t} = R.H.S$$

i.e $\boxed{\bar{H}\cdot\frac{\partial \bar{H}}{\partial t} = \frac{1}{2}\frac{\partial H^2}{\partial t}}$ ||ly $\boxed{\bar{E}\cdot\frac{\partial \bar{E}}{\partial t} = \frac{1}{2}\frac{\partial E^2}{\partial t}}$

Poynting theorem :-

Statement :- "it states that net power flowing out of a given volume v is equal to the time rate of decrease in the energy stored within volume v minus the ohmic losses."

proof :- using the Maxwell's eq^n's for Time-varying fields

$$\nabla\times\bar{E} = -\mu\frac{\partial \bar{H}}{\partial t} \quad \longleftarrow (1)$$

$$\nabla\times\bar{H} = \sigma\bar{E} + \epsilon\frac{\partial \bar{E}}{\partial t} \quad \longleftarrow (2)$$

take dot product on bothside of eq^n (2) with $\bar{E}$.

$$\bar{E}\cdot(\nabla\times\bar{H}) = \sigma E^2 + \epsilon\bar{E}\cdot\frac{\partial \bar{E}}{\partial t} \quad \longleftarrow (3)$$

Note:- $\bar{E}\cdot\bar{E} = E^2$

using vector identity

i.e

$$\nabla(\bar{A}\times\bar{B}) = \bar{B}\cdot(\nabla\times\bar{A}) - \bar{A}\cdot(\nabla\times\bar{B})$$

$$\therefore \nabla(\bar{E}\times\bar{H}) = \bar{H}\cdot(\nabla\times\bar{E}) - \bar{E}\cdot(\nabla\times\bar{H})$$

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$$\Rightarrow \bar{E}\cdot(\nabla\times\bar{H}) = \bar{H}\cdot(\nabla\times\bar{E}) - \nabla\cdot(\bar{E}\times\bar{H}) \longleftarrow (3a)$$

using eqn (3a) in eqn (3) and use $\bar{E}\cdot\frac{\partial \bar{E}}{\partial t} = \frac{1}{2}\frac{\partial E^2}{\partial t}$

$$\bar{H}\cdot(\nabla\times\bar{E}) - \nabla\cdot(\bar{E}\times\bar{H}) = \sigma E^2 + \frac{\epsilon}{2}\frac{\partial E^2}{\partial t} \longleftarrow (4)$$

take dot on bothside of eqn (1) with $\bar{H}$

$$\bar{H}\cdot(\nabla\times\bar{E}) = \bar{H}\cdot\left(-\mu\frac{\partial\bar{H}}{\partial t}\right)$$

$$\Rightarrow \bar{H}\cdot(\nabla\times\bar{E}) = -\mu\,\bar{H}\cdot\frac{\partial\bar{H}}{\partial t}$$

$$\bar{H}\cdot(\nabla\times\bar{E}) = -\frac{\mu}{2}\frac{\partial H^2}{\partial t} \longleftarrow (5)$$

using eqn (5) in (4).

$$-\frac{\mu}{2}\frac{\partial H^2}{\partial t} - \nabla\cdot(\bar{E}\times\bar{H}) = \sigma E^2 + \frac{\epsilon}{2}\frac{\partial E^2}{\partial t}$$

Re-arranging the terms and taking volume integral on both-side.

$$\nabla\cdot(\bar{E}\times\bar{H}) = -\sigma E^2 - \frac{1}{2}\frac{\partial}{\partial t}\left[\mu H^2 + \epsilon E^2\right]$$

$$\Rightarrow \boxed{\nabla\cdot\bar{P} = -\sigma E^2 - \frac{1}{2}\frac{\partial}{\partial t}\left[\mu H^2 + \epsilon E^2\right]} \longleftarrow$$ poynting theorem in point form.

$$\int_{vol}\nabla\cdot(\bar{E}\times\bar{H})\,dv = -\frac{\partial}{\partial t}\int_{vol}\left[\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2\right]dv - \int_{vol}\sigma E^2\,dv \;\; ; \text{watt's}$$

poynting theorem in integral form.

$$\boxed{\int_{vol} \nabla\cdot(\bar{E}\times\bar{H})\,dv = -\frac{\partial}{\partial t}\int_{vol}\left[\frac{1}{2}\epsilon E^2+\frac{1}{2}\mu H^2\right]dv - \int_{vol}\sigma E^2\,dv} \text{ watt's}$$

{total power Leaving the volume.} = {rate of decrease in energy stored in electric and magnetic field's} − {ohmic power dissipated}.

using divergence theorem

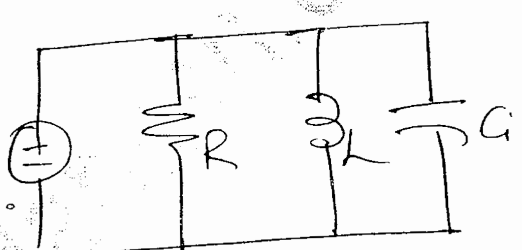
$$\int_{vol} \nabla\cdot(\bar{E}\times\bar{H})\,dv = \oint_{S}(\bar{E}\times\bar{H})\cdot\overline{ds}$$

integral form.

$$\boxed{\oint_{S}(\bar{E}\times\bar{H})\cdot\overline{ds} = -\frac{\partial}{\partial t}\int_{vol}\left[\frac{1}{2}\epsilon E^2+\frac{1}{2}\mu H^2\right]dv - \int_{vol}[\sigma E^2]\,dv} \text{ watt's}$$

Note:- 1)

Source V



$$\boxed{P_{delivered} = P_R + P_L + P_C} \text{ watt's}$$

2) the ohmic power dissipated $P_{ohmic} = \int_{vol}(\sigma E^2)\cdot dv$ watt's.

w.k.t $J = \sigma E$ A/m²

$E \cdot J \Rightarrow$ W/m³

$\uparrow$ V/m $\uparrow$ A/m²

$E\cdot J \rightarrow$ W/m³

$$P_{ohmic} = \int_{vol}(E\cdot J)\,dv \quad \text{Watt's}$$

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Alternative Statement of Poynting theorem :-

- Poynting theorem states that vector product of Electric field Intensity $\bar{E}$ and Magnetic field Intensity $\bar{H}$ at any point is a measure of the rate of energy flow per unit area at that point.

i.e $\bar{P} = \bar{E} \times \bar{H}$ Watt/m²

$\bar{E}$ in V/m, $\bar{H}$ in A/m $= V\text{-}A/m^2 = \underline{\underline{W/m^2}}$

$\therefore \bar{P}$ measured in W/m^2.

the direction of Energy flow is perpendicular to $\bar{E}$ and $\bar{H}$ in the direction of the vector $\bar{E} \times \bar{H}$.

→ Thus, the vector product $\bar{E} \times \bar{H}$ represents the rate of energy flow per unit area.

The product $\bar{E} \times \bar{H}$ itself is another vector denoted by $\bar{P}$, directed perpendicular to the plane containing the $\bar{E}$ and $\bar{H}$ vectors, in the sense of a right hand cork screw rule.

$\therefore \bar{P}$ is called poynting vector, named after the mathematician J. H poynting.

Poynting vector $\boxed{\bar{P} = \bar{E} \times \bar{H}}$ W/m²

Power density (P_{avg}):-

$$\overline{P} = \overline{E} \times \overline{H} ; \ W/m^2$$

Power density $\overline{P} = E_m H_m \sin\theta \, \overline{a_n}$

by $\overline{E} \perp^r \overline{H} \ \therefore \theta = 90°$ and $\sin(90°) = 1$.

$$H_m = \frac{E_m}{\eta} \ A/m.$$

$$\overline{P} = E_m \frac{E_m}{\eta} \overline{a_n} = \frac{E_m^2}{\eta} \overline{a_n}$$

Power density $\boxed{|\overline{P}| = \frac{E_m^2}{\eta}}$ W/m²

$$\Rightarrow \boxed{P = \frac{E_m^2}{\eta}} \ W/m^2$$

xii. Wave power (or) Average power density :-

the average power density of a EM wave is given by

$$\boxed{\overline{P_{avg}} = \frac{1}{2} Re\{\overline{E} \times \overline{H}^*\} = \frac{1}{2} Re\{\overline{E}^* \times \overline{H}\}} \ W/m^2 \quad \text{(a)}$$

eqn (a) is valid if the field's are complex valued.

if field's $\overline{E}$ and $\overline{H}$ are real valued then

$$\boxed{\overline{P_{avg}} = \frac{1}{2}[\overline{E} \times \overline{H}]} \ W/m^2 \qquad (121)$$

Expression for average power density :- → Lossless Medium
→ Lossy medium.

I case a. P_{avg} in Lossless Medium :-

Consider Lossless medium ; assume $\bar{E} \rightarrow \bar{a_x}$

$$\bar{E} = E_m \cos(\omega t - \beta z)\,\bar{a_x} \Rightarrow \therefore \bar{E} = E_m \bar{a_x} \text{ V/m}.$$

$$|\bar{E}| = |E_m \cos(\omega t - \beta z)| = E_m \text{ V/m}.$$

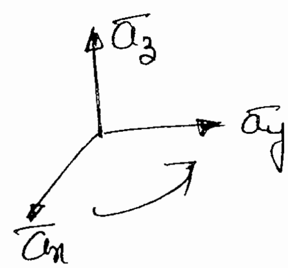
Illy $\bar{H} = H_m \cos(\omega t - \beta z)\,\bar{a_y}$ A/m. ; $\bar{H} \rightarrow \bar{a_y}$

$$|\bar{H}| = |H_m \cos(\omega t - \beta z)| = H_m \Rightarrow \therefore \bar{H} = H_m \bar{a_y} \text{ A/m}.$$

the direction of wave propagation is along 'z' direction.

$$\bar{P}_{avg} = \frac{1}{2}\bar{E} \times \bar{H} = \frac{1}{2} E_m \bar{a_x} \times H_m \bar{a_y}$$

$$\bar{P}_{avg} = \frac{1}{2} E_m H_m\, \bar{a_x} \times \bar{a_y}$$



$\bar{a_x} \times \bar{a_y} = +\bar{a_z}$

$$\bar{P}_{avg} = \frac{1}{2} E_m H_m \bar{a_z}$$

$$= \frac{1}{2} E_m \cdot \frac{E_m}{\eta} \bar{a_z} = \frac{E_m^2}{2\eta} \bar{a_z} \quad \text{W/m}^2$$

$$\boxed{\bar{P}_{avg} = \frac{E_m^2}{2\eta} \bar{a_z}} \quad \text{W/m}^2 \Rightarrow \bar{P}_{avg} = \frac{E_m^2}{2\eta} \bar{a_n} \quad \text{W/m}^2$$

(or) In general

$$\bar{P}_{avg} = \frac{E_m^2}{2\eta} \bar{a_n} ; \text{ W/m}^2$$

Where $\bar{a_n}$ - unit normal vector which is $\perp$ to both $\bar{E}$ and $\bar{H}$.

$$\boxed{|\bar{P}_{avg}| = P_{avg} = \frac{E_m^2}{2\eta}} \; w/m^2$$

The average power passing through an area A m^2 in xy-plane

$$\boxed{[Power]_{avg} = \frac{E_m^2}{2\eta} \times Area} \quad Watts = \frac{E_m^2 A}{2\eta}; watt$$

i.e $\boxed{Power = Powerdensity \times Area}$

Case b. Average powerdensity in Lossy medium / In Conducting medium :-

In Lossy Medium the fields E and H can be expressed as assume $\bar{E} \rightarrow \bar{a}_x$ direc^n

$$\therefore \bar{E} = E_m e^{-\alpha z} \cos(\omega t - \beta z)\, \bar{a}_x \; v/m.$$

$\|$ly $\bar{H} \rightarrow \bar{a}_y$ direc^n

$$\bar{H} = H_m e^{-\alpha z} \cos(\omega t - \beta z)\, \bar{a}_y \; A/m.$$

$\Rightarrow$ the direction of EM wave propagation is in 'z' direc^n.

$$|\bar{E}| = |E_m e^{-\alpha z} \cos(\omega t - \beta z)| = E_m e^{-\alpha z} \; v/m.$$

$$|\bar{H}| = |H_m e^{-\alpha z} \cos(\omega t - \beta z)| = H_m e^{-\alpha z} \; A/m.$$

$$\overline{P}_{avg} = \frac{1}{2} Re[\overline{E} \times \overline{H}^*] = \frac{1}{2} Re[\overline{E} \times \overline{H}^*] \ \text{W/m}^2$$

$$\overline{P}_{avg} = \frac{1}{2} \times |E|\,|H| \sin\theta\ \overline{a}_n$$

$\overline{E} \perp \overline{H}$

$\theta = 90^\circ$

$\therefore \sin(90) = 1.$

$\overline{a}_x \times \overline{a}_y = +\overline{a}_z$

$\therefore \overline{a}_n = +\overline{a}_z$

$H_m = \frac{E_m}{\eta}$ A/m.

$\boxed{\eta = |\eta| \angle \theta_\eta}\ \Omega$

$= |\eta| e^{+j\theta_\eta}\ \Omega$

$$\overline{P}_{avg} = \frac{1}{2} E_m e^{-\alpha z} H_m e^{-\alpha z}\ \overline{a}_z$$

$$\overline{P}_{avg} = \frac{E_m}{2} \cdot \frac{E_m}{\eta} e^{-2\alpha z}\ \overline{a}_z;\ \text{W/m}^2$$

$$\overline{P}_{avg} = \frac{E_m^2}{2\eta} e^{-2\alpha z}\ \overline{a}_z;\ \text{W/m}^2$$

W.k.t $\eta = |\eta| \angle \theta_\eta = |\eta| e^{j\theta_\eta}\ \Omega$

$$\overline{P}_{avg} = \frac{E_m^2}{2|\eta| e^{j\theta_\eta}} e^{-2\alpha z}\ \overline{a}_z;\ \text{W/m}^2$$

$$\overline{P}_{avg} = \frac{1}{2} \frac{E_m^2}{|\eta|} e^{-2\alpha z} e^{-j\theta_\eta}\ \overline{a}_z$$

$$Re\{e^{-j\theta_\eta}\} = \cos(\theta_\eta)$$

$$\therefore \boxed{\overline{P}_{avg} = \frac{E_m^2}{2|\eta|} e^{-2\alpha z} \cos(\theta_\eta)\ \overline{a}_z}\ \text{W/m}^2$$

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$$\Rightarrow \boxed{\overline{P}_{avg} = \frac{E_m^2}{2|\eta|} e^{-2\alpha z} \cos(\theta_\eta)\ \overline{a}_z}\ \text{Watt/m}^2$$

$\Leftarrow$ valid in Lossy Medium.

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Note:- The average power passing through an Area A, in any plane is given by $\boxed{[P_{avg}] = \frac{E_m^2}{2|\eta|} e^{-2\alpha z} \cos(\theta_\eta) \times \text{Area}(A)}$ Watt's

watt's $\quad$ $W/m^2 \times m^2$ = watt's

42. 06 – May/June 2010

A 100 V/m wave of frequency 300 MHz is traveling through a lossy medium having $\epsilon_r = 9$, $\mu_r = 1$ and $\sigma = 10$ S/m. Find the power dissipated over a distance of 1 μm with surface area of 2 m². (06 Marks)

Solu:- given $|E| = 100\,V/m$; $f = 300\,MHz$.

$\epsilon_r = 9$, $\mu_r = 1$ and $\sigma = 10\,\mho/m$.

Given medium is Lossy ⇒ i.e Good conductor (or) Conducting medium.

$P_{avg} = ?$ $\quad P_{dissipated} = ?$ $\quad A = 2m^2$.

distance $d = z = 1\,\mu m$ [assume EM wave propagating along 'z' direction].

Intrinsic Impedance of the medium :-

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ\ \Omega$$

$$\eta = \sqrt{\frac{2\pi f \mu_0 \mu_r}{\sigma}} \angle 45^\circ = \sqrt{\frac{2\pi \times 300 \times 10^6 \times 4\pi \times 10^{-7}}{10}} \angle 45^\circ$$

$$\boxed{\eta = 15.390 \angle 45^\circ}\ \Omega.$$

Propagation constant (γ) :-

$$\gamma = (\alpha + j\beta)\ m^{-1}$$

for a Good Conductor $\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$

$$\boxed{\gamma = \sqrt{2}\,\alpha \angle 45^\circ} \; m^{-1}$$

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\frac{2\pi f \mu_0 \mu_r \sigma}{2}} = \sqrt{\frac{\pi \times 300\times10^6 \times 4\pi\times10^{-7} \times 1 \times 10}{1}}$$

$$\boxed{\alpha = 108.827} \; Np/m.$$

$$\therefore \gamma = \sqrt{2}\,\alpha \angle 45^\circ \Rightarrow \boxed{\alpha = 153.905 \angle 45^\circ} \; m^{-1}$$

the power dissipated in the medium is the difference b/w the power entering the medium and power Leaving the medium, into total Area.

$$\left[\text{Power}\right]_{\text{dissipated}} = \left\{ \left[\text{Power}\right]_{\substack{avg \\ @z=0m}} - \left[\text{Power}\right]_{\substack{avg \\ @z=1\mu m}} \right\} \times \text{Area.}$$

$$= \left[\frac{E_m^2}{2|\eta|} e^{-2\alpha z} \cos(\theta_\eta) \Big|_{\substack{z=0m \\ \theta_\eta=45^\circ}} - \frac{E_m^2}{2|\eta|} e^{-2\alpha z} \cos(\theta_\eta) \right]_{\substack{z=1\mu m \\ \theta_\eta=45^\circ}} \times A$$

$$\left[\text{Power}\right]_{\text{dissipated}} = \frac{E_m^2}{2|\eta|} \cos(\theta_\eta)\, A \left[1 - e^{-2\alpha(1\times10^{-6})}\right]$$

$$= \frac{(100)^2}{2(15.39)} \cos(45^\circ) \times 2 \left[1 - e^{-2(108.827 \times 1\times10^{-6})}\right]$$

$$\boxed{\left[\text{Power}\right]_{\text{dissipated}} = 99.992\times10^{-3}} \; \text{Watts} = 99.99 \text{ m Watts}$$

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43. 06-DEC2010

For a wave traveling in air, the electric field is given by $\bar{E} = 6\cos(\omega t - \beta t)\hat{a}_z$ at f = 10 MHz. Calculate the average Poynting vector. (06 Marks)

should not be 't' ∴ take 'x'

Sol^n:-

given $\bar{E} = 6\cos(\omega t - \beta x)\bar{a}_z$

$f = 10\,MHz$; given airmedium ⇒ Lossless medium.

the average poynting vector

$\bar{P}_{avg} = \frac{1}{2} Re\{E \times H^*\}$: W/m²

$\bar{P}_{avg}$ in Lossless medium is given by

$\bar{P}_{avg} = \frac{E_m^2}{2\eta}\bar{a}_n$: W/m²

$\bar{E} \rightarrow \bar{a}_z$

$\bar{H} \rightarrow \bar{a}_y$ by we considered wave propagation direc^n to be 'x'.

$\bar{a}_z \times \bar{a}_y = -\bar{a}_x$.

$E_m = 6$ V/m ; $\eta = 120\pi$ Ω.

$\bar{P}_{avg} = \frac{6^2}{2(120\pi)}(-\bar{a}_x)$ W/m²

$\bar{P}_{avg} = -0.04776\,\bar{a}_x$: W/m²

$\boxed{|\bar{P}_{avg}| = 0.04776}$ W/m²

44. 02-DEC2010

The electric field intensity at 10 km in free space from a radio station was found to be 2.2 mv/m. Calculate : i) The power density ii) The total power radiated from the station. Assume the radiation to be spherically symmetric. (04 Marks)

Soln: given $E_m = 2.2\ mV/m$; air medium.

distance (or) radius $r = 10\ km$.

$\therefore$ Area of sphere with radius 'r'

$$A = 4\pi r^2\ m^2$$

$$= 4\pi (10k)^2$$

$$\boxed{A = 1.256637 \times 10^9}\ m^2$$

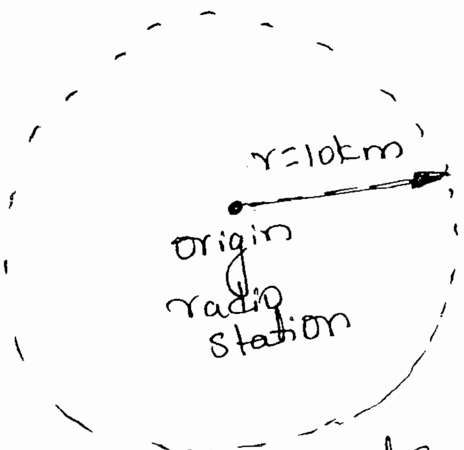

fig. spherical Symmetry with radius 10km

Area of sphere $= 4\pi r^2\ m^2$

i) the power density $|\bar{P}| = |\bar{E} \times \bar{H}|\ W/m^2$

$$= E_m \cdot H_m\ W/m^2$$

$$= \frac{E_m^2}{\eta}\ W/m^2$$

and $\eta = 120\pi\ \Omega$; in free space

$$|\bar{P}| = \frac{(2.2 \times 10^{-3})^2}{120\pi}\ W/m^2 = 12.838 \times 10^{-9}\ W/m^2$$

Power density $|\bar{P}| = 12.83849\ nW/m^2$

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$$\boxed{|\bar{P}| = 12.8384 \times 10^{-9}}\ W/m^2$$

ii) total power radiated from the station

$$P_{radiated} = \text{Powerdensity} \times \text{Area}$$

$$= 12.8384 \times 10^{-9} \times 4\pi r^2$$

$$P_{radiated} = 12.8384 \times 10^{-9} \times 4\pi (10\times10^3)^2$$

$$\boxed{P_{radiated} = 16.133} \text{ Watts}$$

Average powerdensity :-

iii) $P_{avg} = \frac{E_m^2}{2\eta} = \frac{\text{Powerdensity}}{2} = \frac{12.8384\times10^{-9}}{2}$ w/m²

$$\therefore \boxed{P_{avg} = 6.4\times10^{-9}} \text{ W/m}^2$$

iv) total average power radiated from the station

$$P_{avg(radiated)} = P_{avg} \times \text{Area} = P_{avg} \times 4\pi r^2$$

$$= 6.4\times10^{-9} (4\pi)(10\times10^3)^2$$

$$\therefore \boxed{P_{avg(radiated)} = 8.0424} \text{ Watts}$$

Note:- 1. Powerdensity $|P| = E_m^2/\eta$; w/m²

(129) 2. average Powerdensity $|P_{avg}| = \frac{E_m^2}{2\eta}$; w/m²

45. In free Space E(z,t)=50 Cos(wt-βz) a_x v/m. Find the average Power crossing a circular area of radius 2.5m in the plane Z=constant.

Solu: given $\bar{E}(z,t) = 50\cos(wt-\beta z)\,\bar{a_x}$ v/m.

EM wave ⇒ 'z' direction and Medium is free space.

Circular radius $r = 2.5m$; $E_m = 50 V/m$ and $\eta = 377\,\Omega$

Area of Circle $A = \pi r^2\ m^2$.

$\therefore \bar{P}_{avg} = \frac{E_m^2}{2\eta}\,\bar{a_n}\ W/m^2$ in freespace / Lossless medium.

Since $E \rightarrow \bar{a_x}$ and EM wave ⇒ $\bar{a_z}$

$\therefore H$ must be $\rightarrow \bar{a_y}$ | $\bar{a_n} = \bar{a_x} \times \bar{a_y}$; $= \bar{a_z}$.

$$\bar{P}_{avg} = \frac{(50)^2}{2\times 377}\,\bar{a_z}\ W/m^2$$

$$\boxed{\bar{P}_{avg} = 3.3156\,\bar{a_z}}\ W/m^2$$

$|\bar{P}_{avg}|$ = average power density $= 3.3156\ W/m^2$

The average power crossing over a circular area

$= \text{average power density} \times \text{Area}$

$= P_{avg} \times \pi r^2$

$= 3.3156 \times \pi (2.5)^2$

$\boxed{P_{avg.\,crossing} = 65.10}$ Watts

46. 06-DEC2008/Jan 2009

A circular wire having a conductivity σ and radius 'a' carrying a direct current I(Amperes). Using Poynting's theorem, ~~determine the net power~~ entering the wire of length l(m).

(08 Marks)

Sol^n:- Method I :- wkt from poynting's theorem the ohmic power dissipation

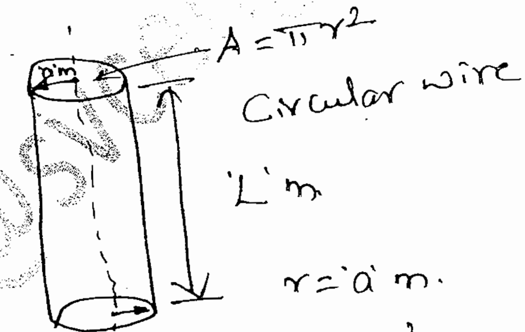
$$P_{ohmic} = \int_{\langle vol \rangle} (\bar{E} \cdot \bar{J})\, dv = \int_{\langle vol \rangle} \sigma E^2\, dv$$

σ ,

I

$R = \frac{\rho l}{A}\ \Omega$.

$P_{dissipation} = ?$



$\therefore \underline{A = \pi a^2}\ m^2$

L - Length of the circular wire.

volume of cylinder $v = \pi r^2 L\ m^3 = \underline{\pi a^2 L}\ m^3$

and $\bar{E} = \frac{\text{potential}}{\text{distance}} = \frac{V}{L}$ v/m.

Resistivity $\rho = \frac{RA}{l}\ \Omega.m$

$\boxed{\frac{1}{\rho} = \sigma}$ $\boxed{\sigma = \frac{1}{\rho} = \frac{l}{RA}}$ $\mho$/m.

132 $|\bar{J}| = \frac{I}{A}$ A/m²

$$P_{ohmic} = \int_{\langle vol \rangle} (E\,J)\,dv = \frac{V}{L} \times \frac{I}{A} \int_{\langle vol \rangle} dv = \frac{VI}{\cancel{LA}} \times \cancel{\pi a^2} L$$

volume of cylinder ; Area $A = \pi a^2 ; m^2$

$$\boxed{P_{ohmic\ dissipated} = V \cdot I = I^2 R} \text{ watt's}$$

Method-II :- using $P_{ohmic} = \int_{\langle vol \rangle} \sigma E^2 dv$ watt's

$$E = \frac{V}{L} \ v/m.$$

$$P_{ohmic} = \sigma E^2 \int_{\langle vol \rangle} dv = \sigma E^2 \times \text{volume of the circular wire}$$

$$P_{ohmic} = \sigma E^2 \times \pi r^2 L = \sigma \left(\frac{V}{L}\right)^2 \times \pi r^2 L$$

using $\sigma = \frac{L}{R \cdot A}$ v/m and $A = \pi r^2 ; m^2$

$$= \frac{\cancel{L}}{R(\cancel{A})} \times \frac{V^2}{\cancel{L^2}} \times \cancel{\pi r^2 L} \Rightarrow \frac{V^2}{R} \text{ watt's}$$

$$P_{ohmic} = \frac{V^2}{R} = \frac{(IR)^2}{R} = I^2 R \text{ ; watt's}$$

$$\Rightarrow \boxed{P_{ohmic} = VI = \frac{V^2}{R} = I^2 R} \text{ watt's.}$$

Miscellaneous Topics:

Topic 5.16 *Polarization of Uniform Plane waves.*
5.17 *Brewster angle in Wave Propagation.*

02 - June /July 2011

47. Discuss in brief the various polarizations of uniform plane waves. (08 Marks)

* Polarization of a wave refers to the time-varying behaviour of the electric field strength vector at some fixed point in space.

* Consider a plane wave propagating in 'z' direction the relative orientation b/w the planar components E_x and E_y vectors defines the polarization of the wave.

* Types :-
  a) Linear polarization.
  b) Circular polarization.
  c) Elliptical polarization.

* Linear polarization :- The planar components are in-phase with either equal (or) unequal amplitudes.

* Circular polarization :- The planar component are out of phase by 90° with equal amplitude.

* Elliptical polarization :- The planar components are out of phase by 90° with unequal amplitude.

Topic 5.17

48. 06 - June /July 2012

Write a note on Brewster angle in wave propogation. (04 Marks)

if Light (wave) strikes an interface so that their is a 90° angle b/w the reflected and refracted ray's, the reflected Light will be Linearly polarized. The direction of polarization is parallel to the plane of the interface.

The special angle of incidence that produces a 90° angle b/w the reflected and refracted ray is Called the Brewster angle (θ_B).

using Snell's Law

$$\boxed{\tan(\theta_B) = \frac{n_2}{n_1}}$$

where n_1, n_2 are refractive index of different medium.

Module-5 (Part B)

Summary:

a. List of symbols.

1. attenuation constant (α) $\rightarrow$ Np/m.
2. phase constant (β) $\rightarrow$ rad/m.
3. propagation constant (γ) $= \alpha + j\beta$; m^{-1}
4. wavelength (λ) $\rightarrow$ meters (m).
5. Intrinsic impedance (η) $\rightarrow$ ohm.
6. phase velocity (v_p) $\rightarrow$ m/sec.
7. skin depth (δ) $\rightarrow$ meters (m).
8. poynting vector $\overline{P} = \overline{E} \times \overline{H}$ $\rightarrow$ W/m^2.
9. Average power density (P_{avg}) $\rightarrow$ W/m^2.
10. total power (P_{total}) $\rightarrow$ Watt.

$$\boxed{P_{total} = P_{avg} \times Area.}$$ watt's

$W/\cancel{m^2} \times \cancel{m^2}$ = watt's.

6. List of Formulae:-

1. Wave equation (General form)

E-field $$\nabla^2 \overline{E} - \mu \epsilon \frac{\partial^2 \overline{E}}{\partial t^2} = \mu \frac{\partial \overline{J}}{\partial t} + \nabla(\rho_v/\epsilon)$$

and

H-field. $$\nabla^2 \overline{H} - \mu \epsilon \frac{\partial^2 \overline{H}}{\partial t^2} = -(\nabla \times \overline{J})$$

2. Wave equation in free space $[\sigma = 0, \rho_v = 0, \overline{J} = 0, \mu = \mu_0, \epsilon = \epsilon_0]$.

$$\nabla^2 \overline{E} - \mu_0 \epsilon_0 \frac{\partial^2 \overline{E}}{\partial t^2} = 0$$

and

$$\nabla^2 \overline{H} - \mu_0 \epsilon_0 \frac{\partial^2 \overline{H}}{\partial t^2} = 0$$

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/sec.}$$

3. Solution of Wave eqn.

assume $\overline{E} \rightarrow x$ dirn, $\overline{H} \rightarrow y$ dirn and EM wave $\rightarrow z$ dirn.

$$\boxed{\overline{E} = E_m^+ \cos(\omega t - \beta z) + E_m^- \cos(\omega t + \beta z)} \text{ v/m.}$$

$$\boxed{\overline{H} = H_m^+ \cos(\omega t - \beta z) - H_m^- \cos(\omega t + \beta z)} \text{ A/m.}$$

4. Wave Equation in phasorform.

$$\nabla^2 \bar{E} = \gamma^2 \bar{E} \quad \text{and} \quad \nabla^2 \bar{H} = \gamma^2 \bar{H}$$

where $\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$

(or) $\boxed{\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}}\ m^{-1}$

5. Wave Equation in Good conducting Medium.

$$\nabla^2 \bar{E} = \mu\sigma \frac{\partial \bar{E}}{\partial t} + \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2} \text{ and.}$$

$$\nabla^2 \bar{H} = \sigma\mu \frac{\partial \bar{H}}{\partial t} + \mu\epsilon \frac{\partial^2 \bar{H}}{\partial t^2}.$$

Soln:- assume EM wave propagating in 'z' direction with attenuation constant 'α' Np/m.

$$E_x(z,t) = E_m^+ e^{-\alpha z} \cos(\omega t - \beta z) + E_m^- e^{+\alpha z} \cos(\omega t + \beta z)$$

$$H_y(z,t) = H_m^+ e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta) - H_m^- e^{+\alpha z} \cos(\omega t + \beta z - \theta_\eta)$$

6) Wave equation in perfect Dielectric Medium.

In perfect dielectric Medium

$\mu = \mu_0 \mu_r$ H/m

$\epsilon = \epsilon_0 \epsilon_r$ F/m.

$$\boxed{\nabla^2 \bar{E} = \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2}} \quad \text{and} \quad \boxed{\nabla^2 \bar{H} = \mu\epsilon \frac{\partial^2 \bar{H}}{\partial t^2}}$$

Soln:- assume EM wave is propagating along 'z' direction

$$\bar{E} = E_m^+ \cos(\omega t - \beta z) + E_m^- \cos(\omega t + \beta z) \text{ V/m}$$

$$\bar{H} = H_m^+ \cos(\omega t - \beta z) - H_m^- \cos(\omega t + \beta z) \text{ A/m.}$$

7. Relationship b/w E, H and η.

$$\boxed{\frac{|E|}{|H|} = \eta}$$ ... intrinsic impedance of the Medium.

8. Uniform plane Wave (UPW)

In case of Electromagnetic wave propagating along x-axis, they are referred to as "Uniform plane wave" if the electric and magnetic fields are independent of y and z but function of 'x' and 't' only. Further for such a wave, it is important to note that there will be no field component

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along the direction of propagation this is called Transverse nature of electromagnetic Wave (TEM-wave).

9. Wave propagation in Good conductors (Skin effect)

Skin depth (or) depth of penetration is a measure of the depth to which an EM wave can penetrate the medium.

$$\boxed{\delta = \sqrt{\frac{1}{\pi f \mu \sigma}} = \frac{1}{\alpha}} \text{ meter.}$$

Note:- $f \uparrow \Rightarrow \delta \downarrow$

i. $\delta = \frac{1}{\alpha} = \frac{1}{\beta} = \sqrt{\frac{1}{\pi f \mu \sigma}}$; meters

ii. $v_p = \frac{\omega}{\beta} = \omega\delta$ m/sec $\Rightarrow \boxed{v_p = \omega\delta}$ m/sec.

iii. $\gamma = \alpha + j\beta$; $m^{-1} = \sqrt{2}\,\alpha \angle 45^\circ = \sqrt{2}\,\delta^{-1} \angle 45^\circ$

$$\boxed{\gamma = \sqrt{2}\,\delta^{-1} \angle 45^\circ}$$

iv. $\boxed{\eta = \frac{\sqrt{2}}{\delta\sigma} \angle 45^\circ}$ Ω.

$\therefore$ $\boxed{v_p = \omega\delta}$ m/sec. and $\boxed{\lambda = 2\pi\delta}$ meters

Note:- $\sigma_{Silver} = 6.17 \times 10^7$ S/m.

$\sigma_{Copper} = 5.80 \times 10^7$ S/m.

$\sigma_{alumin} = 3.82 \times 10^7$ S/m.

$\sigma_{gold} = 4.10 \times 10^7$ S/m.

10. Poynting theorem and Wave power

Statement:- it states that net power flowing out of a given volume (v) is equal to the time rate of decrease in the Energy stored within volume v minus the ohmic Losses.

watt.

$$\left.\begin{array}{l} \oint_{S} (\bar{E} \times \bar{H}) \cdot \overline{dS} \\ \text{(or)} \\ \int_{vol} \nabla \cdot (\bar{E} \times \bar{H})\, dv \end{array}\right\} = -\frac{\partial}{\partial t}\left[\int_{vol}\left(\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2\right) dv\right] - \int_{vol} \sigma E^2\, dv$$

{total power Leaving the volume} = {rate of decrease in Energy stored in Electric and magnetic field} − {Ohmic power dissipation}

Note:- $$\boxed{P_{\substack{\text{ohmic}\\ \text{dissipation}}} = \int_{\langle vol \rangle} (\bar{E} \cdot \bar{J})\, dv = \int_{\langle vol \rangle} \sigma E^2\, dv}$$ Watts.

11. Wave Power:-

→ Average power density in Loss Less Medium ($\alpha = 0$)

$$\boxed{\bar{P}_{avg} = \frac{E_m^2}{2\eta}\, \bar{a}_n} \quad W/m^2$$

(or) $$|P_{avg}| = \frac{E_m^2}{2\eta} \; W/m^2.$$

→ The average power passing through an area 'A' is given by

$$\boxed{[Power] = \frac{E_m^2}{2\eta} \times Area\,(A)} \quad Watts$$

$$= P_{avg} \times Area \;; \text{watts}$$

→ The average power density in Lossy medium (Conducting medium $\alpha \neq 0$).

assume EM wave is propagating in 'z' direction

$$\boxed{P_{avg} = \frac{E_m^2}{2|\eta|}\, e^{-2\alpha z} \cos(\theta_\eta)\, \bar{a}_z} \quad W/m^2$$

total power passing through Area (A) = $P_{avg} \times Area$; Watts

Sl.No.	parameter	General Medium (or) practical dielectrics (or) Lossy dielectrics	Good conductors (or) Lossy medium $\sigma \simeq \infty$; $\left(\frac{\sigma}{\omega\epsilon}\right) >> 1$, $\epsilon = \epsilon_0$; $\mu = \mu_0\mu_r$	perfect dielectrics [Lossless dielectrics] $\sigma \to 0$; $\left(\frac{\sigma}{\omega\epsilon}\right) \to 0$, $\mu = \mu_0\mu_r$ and $\epsilon = \epsilon_0\epsilon_r$	Good dielectrics [Lowloss dielectrics] $\sigma \neq 0$, $\left(\frac{\sigma}{\omega\epsilon}\right) << 1$, $\mu = \mu_0\mu_r$, $\epsilon = \epsilon_0\epsilon_r$	Free space $\sigma = 0$, $\epsilon = \epsilon_0$, $\mu = \mu_0$
01.	attenuation Constant (α) Np/m.	$\alpha = \omega\sqrt{\mu\epsilon}\left\{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]\right\}^{1/2}$ Np/m	$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$ Np/m	$\alpha = 0$	$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$; Np/m	$\boxed{\alpha = 0}$ Np/m
02.	phase constant (β) $\to$ rad/m.	$\beta = \omega\sqrt{\mu\epsilon}\left\{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]\right\}^{1/2}$ rad/m	$\beta = \sqrt{\frac{\omega\mu\sigma}{2}}$; rad/m Note:- $\boxed{\alpha = \beta}$	$\beta = \omega\sqrt{\mu\epsilon}$	$\beta = \omega\sqrt{\mu\epsilon}\left[1+\frac{\sigma^2}{8\omega^2\epsilon^2}\right]$ rad/m	$\boxed{\beta = \omega\sqrt{\mu_0\epsilon_0}}$ rad/m.
03.	propagation constant (γ) $\gamma = (\alpha + j\beta)\ m^{-1}$	$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}\ m^{-1}$ $= (\alpha + j\beta)\ m^{-1}$	$\gamma = \alpha + j\beta$ $= \sqrt{\omega\mu\sigma}\ \angle 45^\circ$; m^{-1}	$\gamma = \omega\sqrt{\mu\epsilon}\ \angle 90^\circ$; m^{-1}	$\gamma = (\alpha + j\beta)$; m^{-1}	$\boxed{\gamma = \omega\sqrt{\mu_0\epsilon_0}\ \angle 90^\circ}$ m^{-1}
04.	Intrinsic impedance (η) $\to \Omega$.	$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}\ \Omega$	$\eta = \sqrt{\frac{\omega\mu}{\sigma}}\ \angle 45^\circ$; Ω	$\eta = \sqrt{\frac{\mu}{\epsilon}}\ \Omega$ $= 377\sqrt{\frac{\mu_r}{\epsilon_r}}\ \Omega$	$\eta = \sqrt{\frac{\mu}{\epsilon}}\left[1 + j\frac{\sigma}{2\omega\epsilon}\right]$ Ω	$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\ \Omega$ $= 120\pi$ $= 120\pi$
5.	phase velocity (v_p) $\to$ m/sec	$v_p = \frac{\omega}{\beta}$ m/sec	$v_p = \sqrt{\frac{2\omega}{\mu\sigma}}$; m/sec	$v_p = \frac{1}{\sqrt{\mu\epsilon}}$ m/sec $= \frac{3\times10^8}{\sqrt{\mu_r\epsilon_r}}$ m/sec	$v_p = \frac{1}{\sqrt{\mu\epsilon}}\left[1 - \frac{\sigma^2}{8\omega^2\epsilon^2}\right]$ m/sec	$v_p = \frac{1}{\sqrt{\mu_0\epsilon_0}}$ $= 3\times10^8$ m/sec