Geodetic Surveying and Theory of Errors

Geodetic Surveying: Principle and Classification of triangulation system, Selection of baseline and stations, Orders of triangulation, Triangulation figures, Reduction to Centre, Selection and marking of stations

Theory of Errors: Introduction, types of errors, definitions, laws of accidental errors, laws of weights, theory of least squares, rules for giving weights and distribution of errors to the field observations, determination of the most probable values of quantities.
Points to be discussed

• Difference between Geodetic and Plane surveying
• What is Triangulation.
• Objectives of Triangulation.
• Principle of Triangulation.
• Triangulation figures or System.
• Classification of Triangulation.
• Inter visibility of triangulation stations (examples)
• Selection of triangulation stations.
• Measurements of Horizontal angle
SURVEYING TECHNIQUES

Horizontal Positioning
- Astronomical methods
- Triangulation
- Trilateration
- Traverse
- Satellite techniques

Vertical Positioning
- Differential levelling
- Trigonometric levelling
- Barometric levelling
- Satellite techniques
Triangulation

- Method of determining distance based on the principles of geometry
- A distant object is sighted from two well-separated locations
- The **distance** between the two locations and the **angle** between the line joining them and the line to the distant object
TRIANGULATION

• Utilizes geometric figures composed of triangles.
• Horizontal angles and a limited no. of sides (base lines) are measured
• By using angles and base line lengths, triangles are solved trigonometrically and the positions of stations (vertices) are calculated
OBJECTIVE

• To establish relative and absolute positions (Horizontal and Vertical) of a number of stations accurately
Important Formulas in Trigonometry

Sine Law:
\[
\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \theta}
\]

Cosine
\[
c^2 = a^2 + b^2 - 2ab \cos \theta
\]
\[
b^2 = a^2 + c^2 - 2ac \cos \beta
\]
\[
a^2 = c^2 + b^2 - 2bc \cos \alpha
\]
Other Important Trigonometric Identities

\[ \sin (x + y) = \sin x \cos y + \cos x \sin y \]

\[ \sin (x - y) = \sin x \cos y - \cos x \sin y \]

\[ \cos (x + y) = \cos x \cos y - \sin x \sin y \]

\[ \cos (x - y) = \cos x \cos y + \sin x \sin y \]
Example:

Given the polygon shown in Figure 1 which is a part of a triangulation system. The straight lines BE and AD are equal in length and point O is located at their midpoint. The azimuth of line BC is 255 degrees. If the angle $\theta = 70$ degrees and length AB is equal to 500 meters, determine the length of CD. Note that $\alpha = 90$ degrees.
Solution:

Consider Δ ABO,

\[
\frac{500}{\sin 40^\circ} = \frac{BO}{\sin 70^\circ}; \text{ Therefore } BO = 730.951 \text{ m}
\]

Since BE and AD are equal in length and point O are located in their midpoints,

\[
BE = AD = 2 \times BO = 2(730.951) = 1,461.902 \text{ m}
\]
BO = OD = AO = EO = 730.951 m

Consider \( \triangle BOC \),

\[ OC = 730.951 \tan 35^\circ = 511.817 \text{ m} \]

Consider \( \triangle ODC \), and by inspection, \( \beta = 50^\circ \)

And by cosine law,

\[ DC^2 = 511.817^2 + 730.951^2 - 2(511.817)(730.951) \cos 50^\circ \]

\[ DC = 561.512 \text{ m} \quad \text{ANS.} \]
Azimuth of $AB = \theta = \theta_{AB}$
Azimuth of $AC = \theta + \angle 1 = \theta_{AC}$
Azimuth of $BC = \theta + 180^\circ - \angle 2 = \theta_{BC}$
Azimuth of $BD = \theta + 180^\circ - (\angle 2 + \angle 4) = \theta_{BD}$
Azimuth of $CD = \theta - \angle 2 + \angle 5 = \theta_{CD}$

Latitude of $AB = l_{AB} \cos \theta_{AB} = L_{AB}$
Departure of $AB = l_{AB} \sin \theta_{AB} = D_{AB}$
Latitude of $AC = l_{AC} \cos \theta_{AC} = L_{AC}$
Departure of $AC = l_{AC} \sin \theta_{AC} = D_{AC}$
Latitude of $BD = l_{BD} \cos \theta_{BD} = L_{BD}$
Departure of $BD = l_{BD} \sin \theta_{BD} = L_{BD}$
Objectives of triangulation

✓ to establish accurate control for plane and geodetic surveys of large areas, by terrestrial methods,

✓ to establish accurate control for photogrammetric surveys of large areas

✓ to assist in the determination of the size and shape of the earth by making observations for latitude, longitude and gravity, and

✓ to determine accurate locations of points in engineering works such as:
  o Fixing centre line and abutments of long bridges over large rivers.
  o Fixing centre line, terminal points, and shafts for long tunnels.
  o Transferring the control points across wide sea channels, large water bodies, etc.
  o Detection of crustal movements, etc.
  o Finding the direction of the movement of clouds.
CLASSIFICATION

• First order (primary)- to determine the shape and size of the earth, to cover a vast area like a country

• Second order (secondary)- network within first order triangulation, for a region/province

• Third order (tertiary)- within second order triangulation, for detailed engineering and location surveys
# CLASSIFICATION

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Characteristics</th>
<th>First-order triangulation</th>
<th>Second-order triangulation</th>
<th>Third-order triangulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Length of base lines</td>
<td>8 to 12 km</td>
<td>2 to 5 km</td>
<td>100 to 500 m</td>
</tr>
<tr>
<td>2.</td>
<td>Lengths of sides</td>
<td>16 to 150 km</td>
<td>10 to 25 km</td>
<td>2 to 10 km</td>
</tr>
<tr>
<td>3.</td>
<td>Average triangular error (after correction for spherical excess)</td>
<td>less than 1”</td>
<td>3”</td>
<td>12”</td>
</tr>
<tr>
<td>4.</td>
<td>Maximum station closure</td>
<td>not more than 3”</td>
<td>8”</td>
<td>15”</td>
</tr>
<tr>
<td>5.</td>
<td>Actual error of base</td>
<td>1 in 50,000</td>
<td>1 in 25,000</td>
<td>1 in 10,000</td>
</tr>
<tr>
<td>6.</td>
<td>Probable error of base</td>
<td>1 in 10,000,000</td>
<td>1 in 500,000</td>
<td>1 in 250,000</td>
</tr>
<tr>
<td>7.</td>
<td>Discrepancy between two measures (k is distance in kilometre)</td>
<td>$5\sqrt{k}$ mm</td>
<td>$10\sqrt{k}$ mm</td>
<td>$25\sqrt{k}$ mm</td>
</tr>
<tr>
<td>8.</td>
<td>Probable error of the computed distances</td>
<td>1 in 50,000 to 1 in 250,000</td>
<td>1 in 20,000 to 1 in 50,000</td>
<td>1 in 5,000 to 1 in 20,000</td>
</tr>
<tr>
<td>9.</td>
<td>Probable error in astronomical azimuth</td>
<td>0.5”</td>
<td>5”</td>
<td>10”</td>
</tr>
</tbody>
</table>
TRIANGULATION FIGURES

1. Single chain of triangles
2. Double chain of triangles
3. Braced quadrilaterals
4. Centered triangles and polygons
5. A combination of above systems.

Basic triangulation figures
TRIANGULATION LAYOUTS

Single chain of triangles

narrow strip of terrain
TRIANGULATION LAYOUTS

Double chain of triangles

Larger width area
TRIANGULATION LAYOUTS

Braced quadrilaterals
TRIANGULATION LAYOUTS

Centered triangles and polygons
PRIMARY TRIANGULATION FOR LARGE COUNTRIES

GRID IRON SYSTEM

CENTRAL SYSTEM

For Primary Triangulation
A Diagram
of the Location of the Treaty of Paris and Other Events
made with Spain, and the Indian Tribes, in 1492-1494.
Drawn by J.J. Trumbull.
CRITERIA FOR SELECTION OF THE LAYOUT OF TRIANGLES

• Simple triangles should be preferably equilateral.
• Braced quadrilaterals should be preferably approximate squares.
• Centered polygons should be regular.
• The arrangement should be such that the computations can be done through two or more independent routes.
• The arrangement should be such that at least one route and preferably two routes form wellconditioned triangles.
• No angle of the figure, opposite a known side should be small, whichever end of the series is used for computation.
• Angles of simple triangles should not be less than 45°, and in the case of quadrilaterals, no angle should be less than 30°. In the case of centered polygons, no angle should be less than 40°.
• The sides of the figures should be of comparable lengths. Very long lines and very short lines should be avoided.
• The layout should be such that it requires least work to achieve maximum progress.
• As far as possible, complex figures should not involve more than 12 conditions.
WELL CONDITIONED TRIANGLE

• any error in angular measurement has a minimum effect upon the computed lengths
• To ensure that two sides of any triangle are equally affected, these should, therefore, be equal in length.
• This condition suggests that all the triangles must, therefore, be isosceles
• best shape of an isosceles triangle is that triangle whose base angles are $56^\circ14'$ each
• EQUILATERAL TRIANGLE
• having an angle less than 30° or more than 120° should not be considered.
STRENGTH OF FIGURE

• factor to be considered in establishing a triangulation system to maintain the computations within a desired degree of precision

• U.S. Coast and Geodetic Surveys
  – computations in triangulation involve use of angles of triangle and length of one known side.
  – other two sides are computed by sine law
  – a given change in the angles, the sine of small angles change more rapidly than those of large angles
  – Angle less than 30° should not be used

\[ L^2 = \frac{4}{3} d^2 R \]
square of the probable error ($L^2$) that would occur in the sixth place of the logarithm of any side, if the computations are carried from a known side through a single chain of triangles after the net has been adjusted for the side and angle conditions.

$$L^2 = \frac{4}{3} d^2 R$$

d is the probable error of an observed direction in seconds of arc, and $R$ is a term which represents the shape of figure.

$$R = \frac{D - C}{D} \sum (\delta_A^2 + \delta_A \delta_B + \delta_B^2)$$

$D$ = the number of directions observed excluding the known side of the figure,

$\delta A, \delta B, \delta C$ = the difference per second in the sixth place of logarithm of the sine of the distance angles $A$, $B$ and $C$, respectively. (Distance angle is the angle in a triangle opposite to a side), and

$C$ = the number of geometric conditions for side and angle to be satisfied in each figure. It is given by

$$C = (n' - S' + 1) + (n - 2S + 3)$$
\[ C = (n' - S' + 1) + (n - 2S + 3) \]

\( n \) = the total number of lines including the known side in a figure,
\( n' \) = the number of lines observed in both directions including the known side,
\( S \) = the total number of stations, and
\( S' \) = the number of stations occupied.
TRIANGULATION SURVEY

FIELD WORK
- Reconnaissance
- Erection of signals/towers
- Measurement of baseline
- Measurement of horizontal Angles
- Measurement of Vertical Angles
- Astronomical observations

TRIANGULATION

COMPUTATIONS
- Adjustment of angles
- Computation of sides
- Computation of latitude, departure and azimuths
- Computation of independent Coordinates
Field work

- Reconnaissance
- Erection of signals and towers
- Measurement of base line
- Measurement of horizontal angles
- Measurement of vertical angles
- Astronomical observations to determine the azimuth of the lines.
RECONNAISSANCE

• Examination of terrain to be surveyed.
• Selection of suitable sites for measurement of base lines.
• Selection of suitable positions for triangulation stations.
• Determination of intervisibility of triangulation stations.
• Selection of conspicuous well-defined natural points to be used as intersected points.
• Collection of miscellaneous information regarding:
  ✓ Access to various triangulation stations
  ✓ Transport facilities
  ✓ Availability of food, water, etc.
  ✓ Availability of labour
  ✓ Camping ground.
Instruments

• Small theodolite and sextant for measurement of angles.
• Prismatic compass for measurement of bearings.
• Steel tape.
• Aneroid barometer for ascertaining elevations.
• Heliotropes for ascertaining intervisibility.
• Binocular.
• Drawing instruments and material.
• Guyed ladders, creepers, ropes, etc., for climbing trees.
SIGNALS AND TOWERS

- **signal** is a device erected to define the exact position of a triangulation station so that it can be observed from other stations.

- **tower** is a structure over a station to support the instrument and the observer, and is provided when the station or the signal, or both are to be elevated.

**NON LUMINOUS SIGNALS –**

Pole signal, target signal, pole and brush signal, beacons

**LUMINOUS SIGNALS –**

Sun signals – Heliotrope

Night signals - Oil lamps, electric lamps, acetylene lamps, Magnesium lamps
Criteria for selection of triangulation stations

• should be intervisible. For this purpose the station points should be on the highest ground such as hill tops, house tops, etc.
• easily accessible with instruments.
• form well-conditioned triangles.
• located that the lengths of sights are neither too small nor too long.
• at commanding positions so as to serve as control for subsidiary triangulation, and for possible extension of the main triangulation scheme.
• useful for providing intersected points and also for detail survey.
• In wooded country, the stations should be selected such that the cost of clearing and cutting, and building towers, is minimum.
• Grazing line of sights should be avoided, and no line of sight should pass over the industrial areas to avoid irregular atmospheric refraction.
Station mark

• should be permanently marked on the ground so that the theodolite and signal may be centred accurately over them

• Guidelines
STATION MARK
NON- LUMINOUS SIGNALS

Pole signal  Target signal  Pole & brush signal  Beacon
LUMINOUS SIGNALS

SUN SIGNALS - HELIOTROPE
LUMINOUS SIGNALS

NIGHT SIGNALS
TOWERS

Required in flat areas to elevate instrument, observer, reflectors
TOWERS

Bilby tower- construction  Completed tower
TOWERS

Truck mounted observation tower

Portable mast-for lights/reflectors
Survey tower with signal pole and tin cone
At end of Epping Base Line
MEASUREMENT OF BASELINE

• Most important part of triangulation
• Aligned and measured with great accuracy
• Forms the basis of computations of triangulation system
• Equipment- standardized tapes, Hunter’s short base, tacheometric measurements, EDM
The accuracy of an entire triangulation system depends on that attained in the measurement of the base line.

**Selection of site for base line**

1. The site should be fairly level
2. The site should be free from obstructions
3. The ground should be firm and smooth.
4. The two extremities of the base line should be inter visible.
5. The site should be such that well-conditioned triangles can be obtained.
6. The site should be such that a minimum length of the base line as specified, is available.
BASE NET

- Class 1 triangulation chains
- Class 2 triangulation network
- Base lines and bases
- Astronomically determined points
MEASUREMENT OF BASELINE

(a) 18th and 19th Century — by wooden bars

(b) First Half 20th Century — by invar wire in catenary

(c) Today — by electromagnetic distance measuring

26. Equipment for Distance Measurement
CORRECTION TO BASE LINE MEASUREMENT

- TAPE CORRECTION
  1. Correction to absolute length
  2. Correction to temperature
  3. Correction for pull or tension
  4. Correction for sag.
  5. Correction for slope
  6. Correction for alignment
  7. Reduction to mean seal level

Learn in sem. III so refer book
## TAPE CORRECTION

<table>
<thead>
<tr>
<th>Sr. No</th>
<th>Correction</th>
<th>Nature of correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Correction to absolute length</td>
<td>+ ve OR − ve -Cumulative</td>
</tr>
<tr>
<td>2</td>
<td>Correction to temperature</td>
<td>+ ve OR − ve -Cumulative</td>
</tr>
<tr>
<td>3</td>
<td>Correction for pull or tension</td>
<td>+ ve OR − ve -Cumulative</td>
</tr>
<tr>
<td>4</td>
<td>Correction for sag.</td>
<td>− ve -Cumulative</td>
</tr>
<tr>
<td>5</td>
<td>Correction for slope</td>
<td>Always negative</td>
</tr>
<tr>
<td>6</td>
<td>Correction for alignment</td>
<td>Always negative</td>
</tr>
<tr>
<td>7</td>
<td>Reduction to mean seal level</td>
<td>Negative</td>
</tr>
</tbody>
</table>
MEASUREMENT OF HORIZONTAL ANGLES

- Optical/digital theodolites for primary and secondary triangulation
- Transit theodolites for tertiary triangulation
- Method of repetition in tertiary and secondary triangulation
- Method of reiteration in primary triangulation
DETERMINATION OF AZIMUTH OF A LINE

• Azimuth/true bearing of any line from a station (Laplace station) is determined
• Methods- observations to star, by hour angle of star or Sun, observation to circumpolar star, knowing the magnetic declination at the place
REDUCTION TO CENTRE

• To secure well conditioned triangles/intervisibility, objects such as chimneys, towers, are selected as triangulation stations
• These can be sighted from other stations, but it is not possible to occupy these stations for taking observations
• In such cases a satellite station is established and observations are taken and later reduced to what they would have been if the main station was occupied
Satellite station

To secure well-conditioned triangles or to have good visibility, objects such as chimneys, church spires, flat poles, towers, lighthouse, etc., are selected as triangulation stations.

Such stations can be sighted from other stations but it is not possible to occupy the station directly below such excellent targets for making the observations by setting up the instrument over the station point.
Satellite station/Eccentric station

Also, signals are frequently blown out of position, and angles read on them have to be corrected to the true position of the triangulation station. Thus, there are two types of problems:

1. When the instrument is not set up over the true station, and
2. When the target is out of position.

Such subsidiary stations are called as satellite or eccentric or false stations.
Different position of Satellite station and Reduction to center
REDUCTION TO CENTRE

A, B and C – Triangulation stations
S – satellite station for C
Observations are made from A, B and S

Measured quantities are

\[ \angle BAC = \theta_A \]
\[ \angle ABC = \theta_B \]
\[ \angle ASB = \theta \]
\[ \angle BSC = \gamma \]

Eccentric distance \( SC = d \)

Distance AB is known from preceding triangle
REDUCTION TO CENTRE

Let

\[ \angle SAC = \alpha \]
\[ \angle SBC = \beta \]
\[ \angle ACB = \phi \]
\[ AB = c \]
\[ AC = b \]
\[ BC = a \]

As a first approximation in \( \triangle ABC \) the \( \angle ACB \) may be taken as

\[
= 180^\circ - (\angle BAC + \angle ABC)
\]

or

\[
\phi = 180^\circ - (\theta_A + \theta_B)
\]
REDUCTION TO CENTRE

In triangle ABC

\[
\frac{c}{\sin \phi} = \frac{a}{\sin \theta_A} = \frac{b}{\sin \theta_B}
\]

\[
a = \frac{c \cdot \sin \theta_A}{\sin \phi}
\]

\[
b = \frac{c \cdot \sin \theta_B}{\sin \phi}
\]
REDUCTION TO CENTRE

From triangles SAC and SBC

\[
\frac{d}{\sin \alpha} = \frac{b}{\sin(\theta + \gamma)}
\]

\[
\frac{d}{\sin \beta} = \frac{a}{\sin \gamma}
\]

\[
\sin \alpha = \frac{d \sin(\theta + \gamma)}{b}
\]

\[
\sin \beta = \frac{d \sin \gamma}{a}
\]
REDUCTION TO CENTRE

\[ \alpha = \frac{d \sin(\theta + \gamma)}{b \sin l''} \]

\[ = \frac{d \sin(\theta + \gamma)}{b} \times 206265 \text{ seconds} \]

\[ \beta = \frac{d \sin \gamma}{a} \times 206265 \text{ seconds} \]

\[ \theta, \gamma, d, b \text{ and } a \text{ are known quantities, therefore, the values of } \alpha \text{ and } \beta \text{ can be computed.} \]
REDUCTION TO CENTRE

$\angle AOB = \theta + \alpha = \phi + \beta$

$\phi = \theta + \alpha - \beta$

$\Phi$ is the required angle which was not measured at triangulation station C
REDUCTION TO CENTRE

\[ \phi = \theta - \alpha + \beta \]

S is towards right of C

\[ \phi = \theta - \alpha - \beta \]

S is inside triangle ABC
REDUCTION TO CENTRE

\[ \phi = \theta + \alpha + \beta \]

S is outside triangle ABC
COMPUTATIONS

Adjustment of observed angles
  Station adjustment
  Figure adjustment

Computation of lengths of sides
Computation of azimuths of all sides
Computation of latitudes and departures
Computations of independent coordinates
CO-ORDINATES

Refer example problem

Reference Azimuth Point
(observed backsight from
Point "A")

Occupied Point "A"
Known:
Northing \((N_A)\)
Easting \((E_A)\)

Point "B"
Computed:
Northing \((N_B)\)
Easting \((E_B)\)

Azimuth or Bearing from \(A \rightarrow B = \alpha\)
Distance from \(A \rightarrow B = s\)

\[
\text{Northing (} N_B \text{)} = \text{Northing (} N_A \text{)} + s \cdot \cos \alpha
\]
\[
\text{Easting (} E_B \text{)} = \text{Easting (} E_A \text{)} + s \cdot \sin \alpha
\]
COMPUTING DEPARTURES & LATITUDES

- Compute by: \( \text{Dep} = L \sin \alpha \quad \text{Lat} = L \cos \alpha \)
- Where: \( \alpha = \text{azimuth} \quad L = \text{length of line} \)

[Diagram showing the calculation of Departures (Dep) and Latitudes (Lat) with respect to the azimuth (\( \alpha \)) and the length of the line (L).]
COMPUTATION OF INDEPENDENT COORDINATES

• Assuming the coordinates of one station, the coordinates of other stations can be computed