Module-1

Oscillations and Waves & Shock Waves

Blowup of the syllabus: (RBT Levels L1, L2, L3)

Free Oscillations: Definition of SHM, derivation of equation for SHM, Mechanical and electrical simple harmonic oscillators (mass suspended to spring oscillator), complex notation and phasor representation of simple harmonic motion. Equation of motion for free oscillations, Natural frequency of oscillations.

Damped and forced oscillations: Theory of damped oscillations: over damping, critical & under damping, quality factor. Theory of forced oscillations and resonance, Sharpness of resonance. One example for mechanical resonance.

Shock waves: Mach number, Properties of Shock waves, control volume. Laws of conservation of mass, energy and momentum. Construction and working of Reddy shock tube, applications of shock waves. Numerical problems

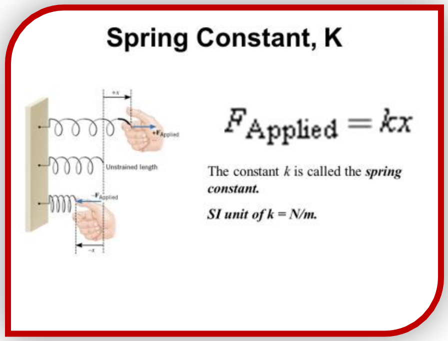
1.1 FREE OSCILLATIONS

Oscillations and vibrations play a more significant role in our lives than we realize. When you strike a bell, the metal vibrates, creating a sound wave. All musical instruments are based on some method to force the air around the instrument to oscillate. Oscillations from the swing of a pendulum in a clock to the vibrations of a quartz crystal are used as timing devices. When you heat a substance, some of the energy you supply goes into oscillations of the atoms. Most forms of wave motion involve the oscillatory motion of the substance through which the wave is moving. Despite the enormous variety of systems that oscillate, they have many features in common with the simple system of a mass on a spring. The harmonic oscillators have close analogy in many other fields; mechanical example of a weight on a spring, oscillations of charge flowing back and forth in an electrical circuit, vibrations of a tuning fork, vibrations of electrons in an atom generating light waves, oscillation of electrons in an antenna etc.,

SIMPLE HARMONIC MOTION

A mass is said to be performing Simple Harmonic Motion when the mass is the restoring force is proportional to the displacement. The restoring force is directed opposite to displacement.

Restoring force α – displacement



F = -k x

Here k is the proportionality constant known as spring constant. It represents the amount of restoring force produced per unit elongation and is a relative measure of stiffness of the material.

$$F_{Restoring} = -kx$$

$$m\frac{d^2x}{dt^2} = -kx$$

$$Let\ \omega_o^2 = \frac{k}{m}$$

$$\frac{d^2x}{dt^2} + \omega_o^2 x = 0$$

Here ω_o is angular velocity = 2.π.f

f is the natural frequency $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

The Solution is of the form $x(t) = A \cos\omega_o t + B \sin\omega_o t$.

This can also be expressed as $x(t) = C \cos(\omega_o t - \varphi)$ where $C = \sqrt{A^2 + B^2}$ $\tan\varphi = B/A$

Mechanical Simple Harmonic Oscillator:

We consider a mechanical spring which resists compression / elongation to be elastic. At the lower end of the spring, a body of mass m is attached. Mass of the spring is neglected .When the body is pulled down by a certain distance x and then released, it undergoes SHM. **When there are no external forces, the oscillations are said to be free oscillations.** The mass oscillates with its natural frequency.

The motion of a mass m attached to a spring follows a linear differential equation.

$$F_{Restoring} = -kx$$

From Newton's second law, the equation of motion is written as

$$\frac{d^2x}{dt^2} + kx = 0$$

This is a second order homogeneous linear differential equation.

Auxiliary equation is $\left(D^2 + \omega^2\right)x = 0$

Roots are D =+iω and D =-iω

The general solution is
$$\begin{aligned} x &= Ae^{i\omega t} + Be^{-i\omega t} \\ &= A(\cos\omega t + i\sin\omega t) + B(\cos\omega t - i\sin\omega t) \\ &= (A+B)\cos\omega t + i(A-B)\sin\omega t \\ &= C\cos\omega t + D\sin\omega t \end{aligned}$$

This may also be expressed as $x = A\cos(\omega t - \phi)$

where $A = \sqrt{C^2 + D^2}$ and $\varphi = \tan^{-1}(D/C)$

and C= A Cosφ and D = A Sinφ

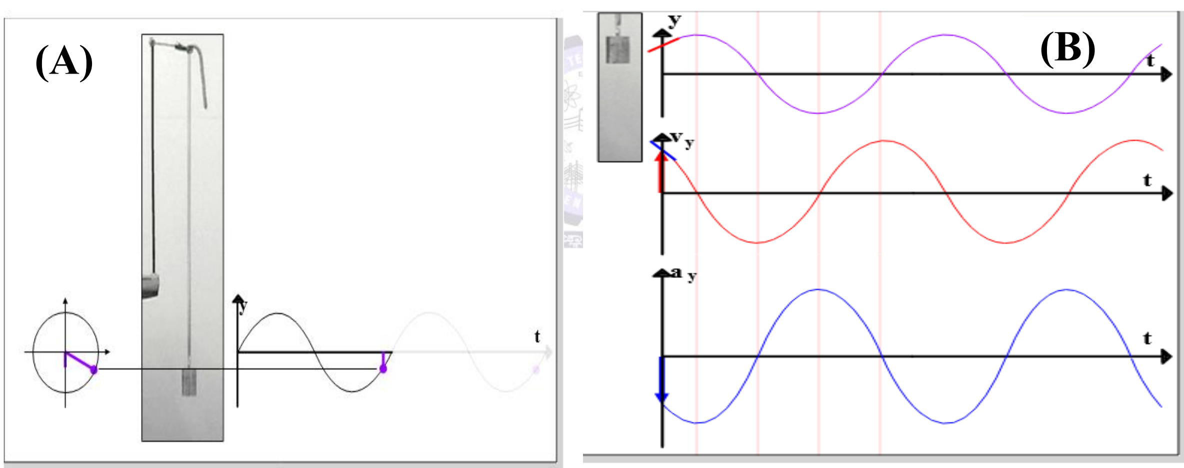


Fig. 1.1.(A) SHM as a projection of uniform circular motion and (B) Displacement, velocity and acceleration graphs for SHM.

{Reference: http://www.animations.physics.unsw.edu.au/jw/SHM.htm}

Velocity of particle $V_{Max} = \frac{dx}{dt} = -\omega A$

Acceleration of particle $= \frac{dv}{dt} = -\omega^2 x$

$$angular\ \ velocity\ \ \omega = \sqrt{\frac{k}{m}}$$

$$Period\ = 2\pi\sqrt{\frac{m}{k}}$$

$$Phase = \omega t$$

$$natural\ \ frequency = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Complex Notation

Complex numbers are a convenient tool to mathematically analyze sinusoidal functions. It can be used to represent amplitude and phase of a periodically varying function.

Rectangular form : $z = x + jy$

Polar form : $z = r\angle\theta$

Exponential form : $z = re^{j\phi}$

Phasors are Time Independent complex quantities used to represent periodically varying parameters.

Ex- Alternating current is represented as $I(t) = \hat{I}e^{i\omega t}$

Alternating voltage is represented as $V(t) = \hat{V}e^{i\omega t}$

Here $\hat{I}$ and $\hat{V}$ are phasors

A periodically force is expressed in phasor form as

$F = \widehat{F}\cos\omega t = \text{Re}\,al\ \ part\ \ of\ \ F_o(\cos\omega t + i\sin\omega t)$. Here $\widehat{F}$ is a phasor

Ex: Mechanical

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = \frac{F_o}{m}\cos\omega t$$

$$\frac{d^2(x_r + ix_i)}{dt^2} + \frac{k(x_r + ix_{i)}}{m} = \frac{F_r + iF_i}{m}$$

Electrical: Phasor representation of Impedance in LCR circuit

$$z = R + j(\omega L - \frac{1}{\omega C})$$

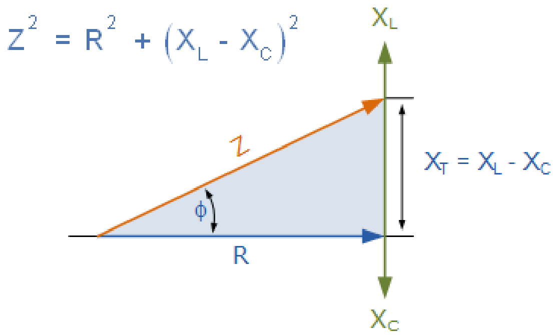


Fig.1.2. Phasor representation of impedance in LCR circuit.

Expression for Spring Constant for Series Combination

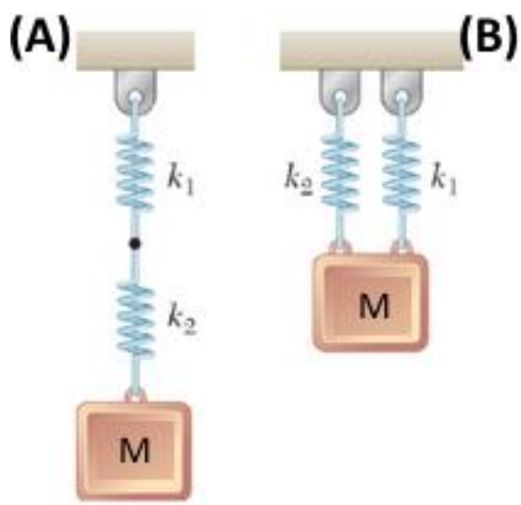


Fig.1.3. (A) Series combination and (B) Parallel combination of two springs having spring constant k_1 and k_2.

Consider a load suspended through two springs with spring constants k_1 and k_2 in series combination as shown in Fig.1.3 (A). Both the springs experience same stretching force. Let Δx_1 and Δx_2 be their elongation.

Total elongation is given by

$$\Delta X = \Delta X_1 + \Delta X_2 = \frac{F}{k_1} + \frac{F}{k_2}$$

$$\frac{F}{k_{eqv}} = \frac{F}{k_1} + \frac{F}{k_2}$$

$$\frac{1}{k_{eqv}} = \frac{1}{k_1} + \frac{1}{k_2}$$

Expression for Spring Constant for Parallel Combination

Consider a load suspended through two springs with spring constants k_1 and k_2 in parallel combination as Fig.1.3 (B). The two individual springs both elongate by x but experience the load non-uniformly. Total load across the two springs is given by

$$F = F_1 + F_2$$
$$k_{eqv}.\Delta X = k_1.\Delta X + k_2.\Delta X$$
$$k_{eqv} = k_1 + k_2$$

Free Oscillations

The oscillations are said to be free oscillations when there are no external forces. The object oscillates with natural frequency.

Ex: E**LECTRICAL OSCILLATIONS- LC Oscillations**

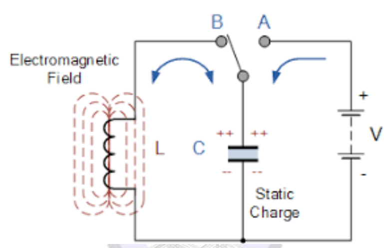


Fig.1.4. LC oscillator circuit.

If the capacitor is initially charged and the switch is then closed, we find that the charge on the capacitor oscillates.

Voltage across capacitor = q/C

Voltage across Inductor = $L\dfrac{d^2q}{dt^2}$

For the above circuit, $q/C + L\dfrac{d^2q}{dt^2} = 0$

$$\frac{d^2q}{dt^2} + \frac{q}{LC} = 0$$

Comparing this with $\dfrac{d^2x}{dt^2} + \omega^2 x = 0$; the solution may be written as q = A cosωt + B sinωt where

$$\omega = \frac{1}{\sqrt{LC}}$$

SIMILARITIES BETWEEN MECHANICAL OSCILLATIONS AND ELECTRICAL OSCILLATIONS

Variable	Mechanical Property	Electrical Property
Independent Variable	Time	Time
Dependent variable	Position(x)	Charge(q)
Inertia	mass	Inductance
Resistance	Drag coefficient	Resistance (R/L)
Stiffness	k	1/C
Resonant frequency	$\omega_o = \sqrt{k/m}$	$\omega_o = \frac{1}{\sqrt{LC}}$
Energy	Potential energy = $\frac{1}{2}kx^2$ Kinetic energy = $\frac{1}{2}mv^2$	$\frac{1}{2}CV^2$ $\frac{1}{2}LI^2$

Damped Oscillations

Mechanical Case:

In a damped harmonic oscillator, the amplitude decreases gradually due to losses such as friction, impedance etc. The oscillations of a mass kept in water, charge oscillations in a LCR circuit are examples of damped oscillations. Let us assume that in addition to the elastic force F = -kx, there is a force that is opposed to the velocity, F = b v where b is a constant known as resistive coefficient and it depends on the medium, shape of the body.

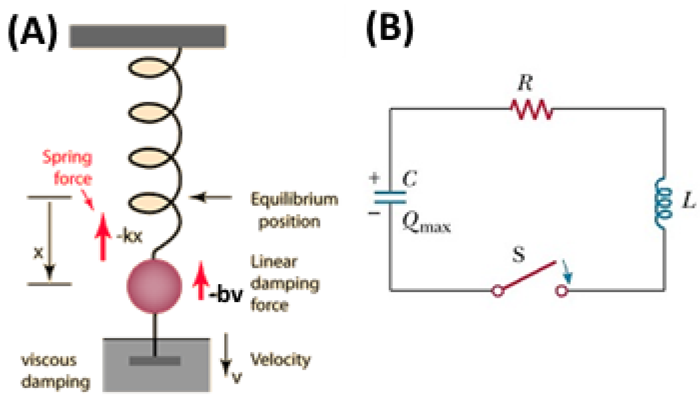


Fig.1.5. (A) Spring oscillation under damping created by viscous liquid. (B) Equivalent LCR circuit in series

For the oscillating mass in a medium with resistive coefficient b, the equation of motion is given by

$$m\frac{d^2x}{dt^2} + kx + b\frac{dx}{dt} = 0$$

This is a homogeneous, linear differential equation of second order.

The auxiliary equation is $D^2 + \frac{b}{m}D + \frac{k}{m} = 0$

The roots are $D_1 = -\frac{b}{2m} + \frac{1}{2m}\sqrt{b^2 - 4mk}$ and $D_2 = -\frac{b}{2m} - \frac{1}{2m}\sqrt{b^2 - 4mk}$

The solution can be derived as $x(t) = Ce^{-\left(\frac{b}{2m} - \frac{1}{2m}\sqrt{b^2-4mk}\right)t} + De^{-\left(\frac{b}{2m} + \frac{1}{2m}\sqrt{b^2-4mk}\right)t}$

This can be expressed as x(t) = $Ae^{-\frac{b}{2m}t}\cos(\omega t - \phi)$ where $\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$

$$A = \sqrt{C^2 + D^2} \qquad \phi = \tan^{-1}(D/C)$$

Here, the term $Ae^{-\frac{b}{2m}t}$ represents the decreasing amplitude and (ωt-ϕ) represents phase.

Case 1: $b^2 > 4mk$ OVER DAMPING

Case 1: $b^2 < 4mk$ UNDER DAMPING

Case 1: $b^2 = 4mk$ CRITICAL DAMPING

Under damped: $b^2 > 4mk$

When the retarding force is less than k. A, the system oscillates with decreasing amplitude

Critically damped: $b^2 = 4mk$

When $\frac{b}{2m} = \omega_o$, the system does not oscillate

Over damped: $b^2 > 4mk$

When the retarding force is greater than k.A, $\frac{b}{2m} > \omega_o$

Over damping takes away the energy of the system and oscillations stop.

Electrical case:

For the above circuit, $\frac{q}{c}+R\frac{dq}{dt}+L\frac{d^2q}{dt^2}=0$

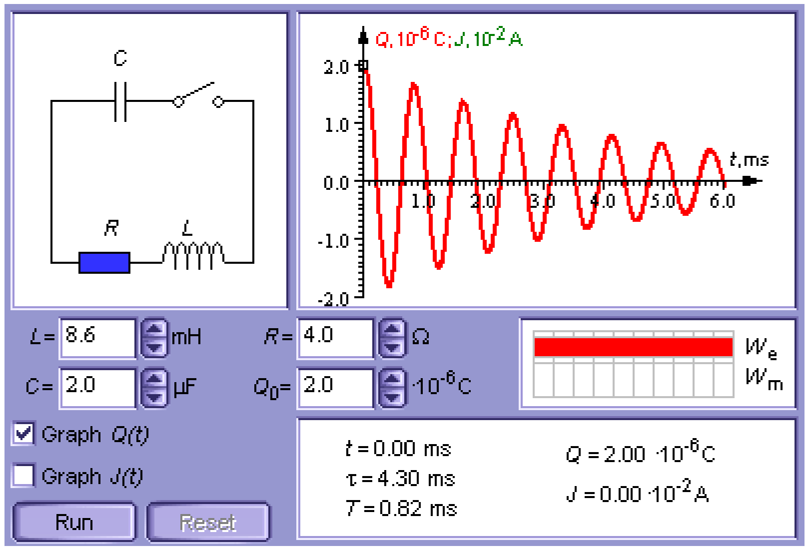


Fig.1.6. Damped Oscillations of LCR circuit as measured.

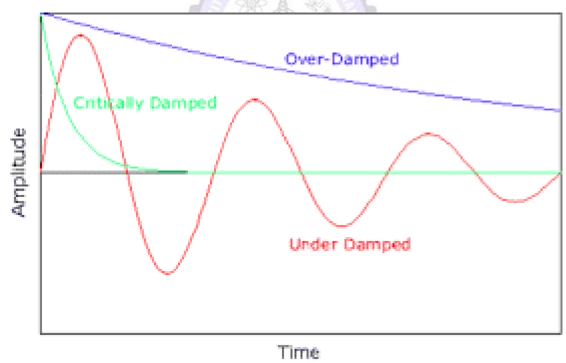
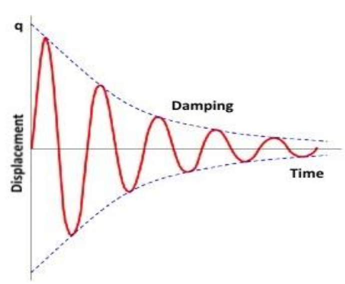


Fig. 1.7. Various kinds of damping oscillations and their die out with respect to time.

The solution is of the form $q=q_o e^{-\frac{Rt}{2L}}\cos\omega t$ where $\omega=\left[\frac{1}{LC}-\left(\frac{R}{2L}\right)^2\right]^{\frac{1}{2}}$



FORCED OSCILLATIONS

Mechanical:

Oscillations that result when an external oscillating force is applied to the particle subject to SHM. For example, a child on a swing can be kept in motion by appropriately timed "pushes." The amplitude of motion remains constant if the energy input per cycle of motion exactly equals the decrease in mechanical energy in each cycle that results from resistive forces. Vibrations of tuning fork placed on a resonating box make the walls of the box and the air inside oscillate.

Let $F = F_o \cos\omega_f t$ be the oscillating applied force

The equation of motion is given by

$$F = ma = -kx + bv + F_o \cos\omega_f t$$

$$m\frac{d^2x}{dt^2} + b.\frac{dx}{dt} + kx = F_o \cos\omega_f t \quad \text{.......(1)}$$

This equation is non-homogeneous. The complimentary function is given by

$$x_c(t) = C \cos\omega_f t + D\sin\omega_f t \quad \text{.......(2)}$$

This can also be expressed as $x_c(t) = A\cos(\omega_o t - \varphi)$ where $A = \sqrt{C^2 + D^2}$ $\tan\varphi = D/C$ ......(3)

$$x_c^{\bullet}(t) = -C\omega \sin\omega_f t + D\omega \cos\omega_f t$$

$$x_c^{\bullet\bullet}(t) = -C\omega^2 \cos\omega_f t - D\omega^2 \sin\omega_f t$$

Substituting in (1)

$$\left[(k - m\omega_f{}^2)C + Db\omega_f\right]\cos\omega_f t + \left[-bC\omega_f + \left(k - m\omega_f{}^2\right)D\right]\sin\omega_f t = F_o \cos\omega_f t$$

By equating coefficients of the sine and cosine terms on both sides

$$\left(k - m\omega_f{}^2\right)a + Db\omega_f = F_o$$

$$-bC\omega_f + \left(k - m\omega_f{}^2\right)d = 0$$

Solving for c and d

$$C = F_o \frac{k - m\omega_f{}^2}{\left(k - m\omega_f{}^2\right)^2 + b^2\omega_f{}^2} \qquad D = F_o \frac{b\omega_f}{\left(k - m\omega_f{}^2\right)^2 + b^2\omega_f{}^2}$$

Substituting for $\omega_o = \sqrt{k/m}$

$$C = F_o \frac{m(\omega_o^2 - \omega_f^{\ 2})}{m^2\left(\omega_o^2 - \omega_f^{\ 2}\right)^2 + b^2\omega_f^{\ 2}} \qquad D = F_o \frac{b\omega_f}{m^2\left(\omega_o^2 - \omega_f^{\ 2}\right)^2 + b^2\omega_f^{\ 2}}$$

The general solution is

$$x(t) = F_o \frac{m(\omega_o^2 - \omega_f^{\ 2})}{m^2\left(\omega_o^2 - \omega_f^{\ 2}\right)^2 + b^2\omega_f^{\ 2}} \cos\omega_f t + F_o \frac{b\omega_f}{m^2\left(\omega_o^2 - \omega_f^{\ 2}\right)^2 + b^2\omega_f^{\ 2}} \sin\omega_f t$$

Using (3) , the solution can also be expressed as

$$x(t) = A\cos(\omega_o t - \varphi)$$

where amplitude $A = \sqrt{C^2 + D^2}$ and phase $\varphi = \tan^{-1} D/C = \tan^{-1}\left(\frac{b\omega_f}{m(\omega_o^2 - \omega_f^2}\right)$

(Note: Refer an alternative method in APPENDIX)

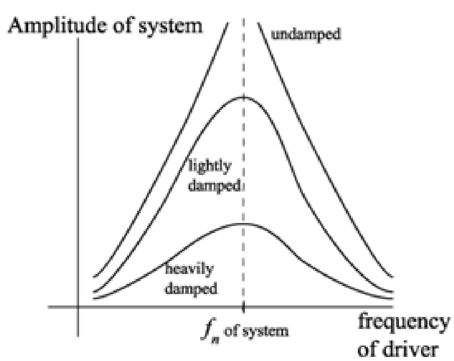
Case 1: $\boldsymbol{\omega_f < \omega_o}$, $A \approx \frac{F_0}{k\left(1 - \left(\frac{\omega_f}{\omega_0}\right)^2\right)} \Rightarrow$ A = Fo/k - Spring constant controls the oscillations

Case 2: $\boldsymbol{\omega_f = \omega_o}$, b = 0 (undamped) A $\Rightarrow \infty$ **: RESONANCE CONDITION**

When the frequency of the applied force is same as natural frequency of the oscillator, resonance occurs. Maximum transfer of energy takes place.

Case 3: $\boldsymbol{\omega_f > \omega_o}$, $A \approx \frac{F_0}{m\omega^2 \cos\omega t}$ Driving force controls the oscillations

Note: Phase lag between the applied force and displacement increases from 0 to π/2 as driving frequency increases from 0 to ω_o



Forced oscillations : Electrical:

$$\frac{q}{c} + R\frac{dq}{dt} + L\frac{d^2q}{dt^2} = V_o e^{iwt} = \widehat{V}$$

$$\widehat{q} = q_o e^{iw\,t}$$

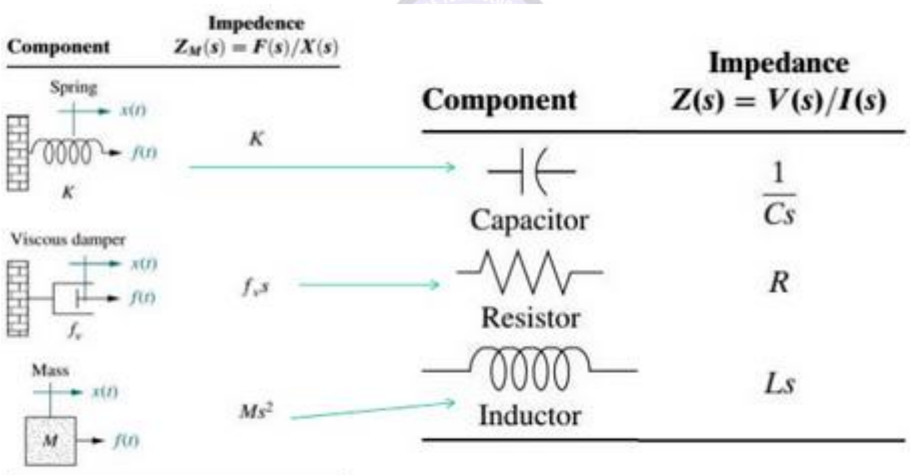
$$\left[L(iw)^2 + R(iw) + \frac{1}{c}\right] q_o e^{iw\,t} = V_o e^{iw\,t}$$

$$\widehat{q} = \frac{\widehat{V}}{L\left(i\omega^2 - R(i\omega) + \frac{1}{C}\right)}$$

$$\widehat{I} = \frac{d\widehat{q}}{dt} = iw\widehat{q}$$

$$\widehat{V} = [R + i(wL - \frac{1}{wC})]\widehat{I} = \widehat{I}\widehat{Z}$$

Electrical Mechanical Analogy



FORCED OSCILLATIONS –Alternative method

Let F = F_o Cosω_ft be the oscillating applied force

The equation of motion is given by

$$F = ma = -kx + bv + F_o \sin \omega_f t$$

$$m\frac{d^2x}{dt^2} + b.\frac{dx}{dt} + kx = F_o \sin \omega_f t$$

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F_o}{m}\sin \omega_f t$$

$$Let \;\; \frac{b}{m} = 2k; \frac{k}{m} = \omega^2; \frac{F_o}{m} = F$$

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + \omega_o{}^2 x = F \sin \omega_f t \qquad \text{.....(1)}$$

Let one particular solution be $y = a.\sin(\omega_f t - \phi)$

$$\frac{dy}{dt} = \omega_f a.\cos(\omega_f t - \phi)$$

$$\frac{d^2y}{dt^2} = -\omega_f^2 a.\sin(\omega_f t - \phi)$$

Also

$$F \sin \omega_f t = F.\sin(\omega_f t - \phi + \phi)$$

$$= F \sin(\omega_f t - \phi)\cos\phi + F\cos(\omega_f t - \phi)\sin\phi$$

Substituting in (1)

$$-\omega_f^2 a.\sin(\omega_f t - \phi) + 2ka\omega_f \cos(\omega_f t - \phi) + \omega_o^2 a \sin(\omega_f t - \phi) = F\sin(\omega_f t - \phi)\cos\phi + F\cos(\omega_f t - \phi)\sin\phi$$

Comparing coefficients of $\sin(\omega_f t - \phi)$ *and* $\cos(\omega_f t - \phi)$ on both sides

$$a(.\omega_o^2 - \omega_f^2) = F\cos\phi$$

$$2ka\omega_f = F\sin\phi$$

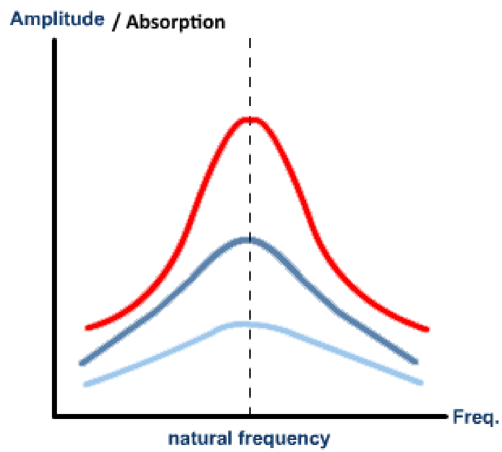
$$\therefore F^2 = a^2(\omega_o^2 - \omega_f^2)^2 + 4k^2a^2\omega_f^2$$

$$a = \frac{F}{\sqrt{(\omega_o^2 - \omega_f^2)^2 + 4k^2\omega_f^2}} \qquad \tan\phi = \frac{2k\omega_f}{\omega_o^2 - \omega_f^2}$$

EXAMPLES OF RESONANCE

1. Oscillations of the stretched string kept under the influence of oscillating magnetic field caused by oscillating current. The string vibrates with maximum amplitude when the applied frequency matches with Natural frequency of the string.

2. Sodium chloride crystal has alternately Sodium and Chloride ions. If an electric field is applied on the crystal, the charges would oscillate back and forth. The natural frequency is in Infrared range.

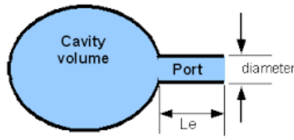


3. LC circuit in a radio tuner is tuned to radio station frequency (say 91.5 MHz radio city) matches with Natural frequency $\left(f = \frac{1}{2\pi\sqrt{LC}}\right)$ for amplification of signals.

4. Helmholtz resonator – It is used to analyze complex note. It consists of a hollow sphere of thin glass or brass with an opening through a narrow neck. It is filled with air. The opening receives exciting sound waves and the ears are kept close to the neck. When air is pushed into the sphere and released, the pressure will drive it out. The volume of air in the container behaves as a mass on a spring which is pulled down and released. Compressed air tends to move out and creates low pressure inside. The air will oscillate into and out of the container at its natural frequency given by the expression

$$f = \frac{V}{2\pi}\sqrt{\frac{A}{lv}}$$

V is the velocity of sound, l the length, A the area of the opening, v the volume of the resonator



Quality factor

It is customary to describe the amount of damping with a quantity called the quality factor (Q). It is defined as the number of cycles required for the energy to fall off by a factor of 535. (The origin of this numerical factor is $e^{2\pi}$). A mechanical device that can vibrate for many oscillations before it loses a significant fraction of its energy would be considered a high-quality device. Let W be the amount of work done by friction in the first cycle of oscillation, i.e., the amount of energy lost to heat. Let the original energy be E.

$$\frac{1}{Q} = \frac{1}{2\pi}\left(\frac{W}{E}\right)$$

The Full Width at Half Maxima (FWHM) of a resonance is related to its Q and its resonant frequency fres by the equation

FWHM = frequency Q

Shock waves

Shock waves: Fluid dynamics is the study of fluids at rest or in motion. It has traditionally been applied in such areas in the design of canal, dam systems, pumps, compressors in refrigerators, air conditioning systems, aerodynamics of automobiles and air planes, ocean phenomena such as tornadoes, hurricanes, tsunami, blood flow, air circulation in our body, lubrication in rotating MP3 disc player etc.

Fluid is a substance that deforms continuously under the application of tangential stress. Liquids and gases are the forms that fluids can take.
Uniform flow: Velocity remains constant.
Steady flow: Velocity at each point in the flow remains constant with time.
Stream lines : Lines drawn parallel to the direction of flow of fluid.
Newtonian fluid: Fluids where shear stress is proportional to deformation.
Laminar flow: Fluid particles move in smooth layers.

Turbulent flow: Fluid particles rapidly mix as they move due to random velocity of fluctuation.

Drag: It is the component of the force on body acting opposite to the direction of motion. Consider a ball flying through the air, in addition to gravity, the ball experiences aerodynamic drag of the air. It is due to pressure build up in the front of the ball as it pushes the air out of the way. Now look at a dust particle falling under gravity at a terminal velocity of 1cm/s, it experiences viscous drag (due to viscosity of air) rather than aerodynamic drag.

Shock: What we perceive as sound generally consists of pressure pulses that move through air. When air undergoes large and rapid compression (following an explosion/the release of engine gases in to an exhaust pipe/when an air craft or a bullet flies at supersonic velocity) a thin wave of large pressure change is produced. This discontinuity is pressure propagates as a wave known as shock wave. A shock wave develops when the flow is supersonic.

Shock tube: A long cylinder is portioned with a cellophane film to give a pressure difference between the two sections. When the partition is ruptured, shock wave develops. The shock wave in this case is at right angles to the flow and is called a normal shock wave.

Mach number: M = Velocity of fluid/velocity of sound.

Subsonic speed	$V_{object} < V_{sound}$	Mach number <1
Sonic speed	$V_{object} = V_{sound}$	Mach number = 1
Supersonic speed	$V_{object} > V_{sound}$	Mach number >1
Transonic		Mach number 0.8 -1.2
Hypersonic		Mach number >5

Reynolds Number:

To compare viscous and aerodynamic drag, Reynold's number is used.

Reynold's number $R_Y = \frac{\rho v L}{\mu}$

Here ρ is fluid density, μ is viscosity, v is velocity and L is size scale of the flow.

Large Reynold's number indicates high aerodynamic drag.

Mach number is the ratio of velocity of fluid causing the shock wave generation to the velocity of sound in the medium. It represents the compressibility nature of the medium.

Ultrasonic wave: Sound waves of frequency greater than 20,000Hz.

Acoustic waves: A longitudinal wave that consists of a sequence of pressure pulses propagating in a medium. The speed of an acoustic wave in a material medium is determined by the temperature, pressure, and elastic properties of the medium.

Subsonic waves: These are sound waves with Mach number less than 1. Velocity of the object is less than velocity of sound.

Ex: Low intensity shock waves produced during the motion of ordinary aircrafts.

Supersonic waves: These are shock waves with Mach number greater than 1. Velocity of the object is greater than velocity of sound.

Ex: shock waves produced during the motion of jet planes, bullets etc.

Transonic waves: These are shock waves with Mach number less than 1in the range 0.8 to 1.2

Hypersonic waves: These are shock waves with Mach number greater than 5.

Mach angle: Shock waves propagate as a cone. The semi vertical angle of the cone of shock waves is known as Mach angle (μ). $\mu = \sin^{-1}(1/M)$

Control Volume : It is a volume through which the fluid flows. The boundary of this volume is the physical boundary of the region through which the fluid flows. The equation of continuity, energy flow, variation in pressure and volume are determined with respect to this region.

Important expressions used in fluid dynamics:

Ideal gas equation: $pv = n RT$ or $p = \rho RT$

Here p is pressure , v is volume, ρ is density, T is absolute temperature

Equation of continuity/Law of conservation of mass: ρVA = constant

Here V is velocity of the fluid, A is are of cross section

Law of conservation of energy / Bernoullie's equation :

$$\frac{p}{\rho}+\frac{V^2}{2}+gh=\text{Constant}$$

$$\frac{\rho V^2}{2}+p+\rho gh=\text{Constant}$$

$$\frac{k}{k-1}RT+\frac{1}{2}u^2=\text{constant}$$

$$\frac{k}{k-1}\frac{p}{\rho}+\frac{u^2}{2}=\text{constant}$$

k is the ratio of specific heat at constant pressure to specific heat at constant volume

Law of Conservation of Momentum

$$p_1+\rho_1 u_1^2=p_2+\rho_2 u_2^2$$

Derivation of Rankine –Hugoniot equation ***(Ref: Fluid Mechanics – Nakayama)***

This equation relates pressure, density, temperature ahead and behind a shock wave.

Consider a shock tube with partition separating two regions as shown in the figure.

Continuity equation $\rho_1 u_1=\rho_2 u_2$ since area A is constant in this case …………(1)

Equation for Energy conservation $\frac{k}{k-1}\frac{p_1}{\rho_1}+\frac{u_1^2}{2}=\frac{k}{k-1}\frac{p_2}{\rho_2}+\frac{u_2^2}{2}$ ………………(2)

Equation for momentum conservation $p_1+\rho_1 u_1^2=p_2+\rho_2 u_2^2$ ………………(3)

From (2)

$$u_1^2 - u_2^2 = \frac{2k}{k-1}\left(\frac{p_2}{\rho_2} - \frac{p_1}{\rho_1}\right)$$

From (1) and (3) …….(4)

$$u_1^2 = \frac{p_2 - p_1\rho_2}{\rho_2 - \rho_1\rho_2} - \qquad u_2^2 = \frac{p_2 - p_1\rho_1}{\rho_2 - \rho_1\rho_2}$$

$$\frac{\rho_2}{\rho_1} = \frac{[(k+1)/(k-1)](p_2/p_1)+1}{(k+1/k-1)+p_2/p_1} = \frac{u_1}{u_2}$$

$$\text{since } p = \rho RT$$

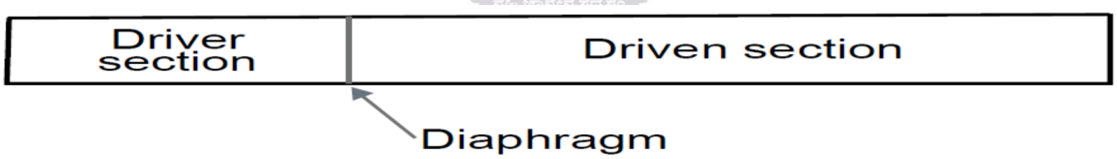
From (4)

$$\frac{T_2}{T_1} = \frac{(k+1/k-1)+p_2/p_1}{(k+1/k-1)+p_1/p_2}$$

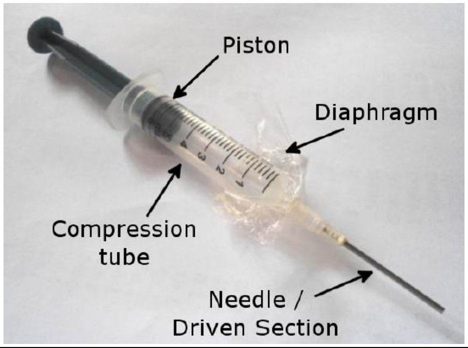
Reddy shock tube:

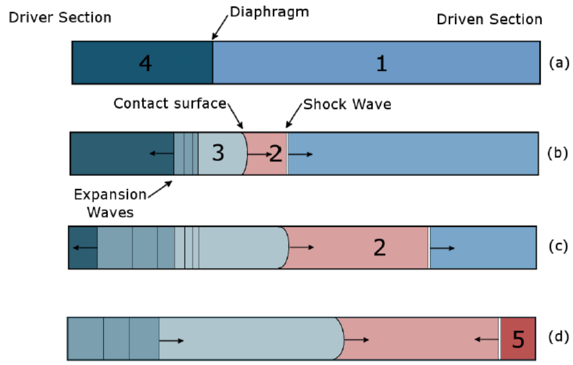
A shock tube is a device used to study the changes in pressure & temperature which occur due to the propagation of a shock wave. A shock wave may be generated by an explosion caused by the buildup of high pressure which causes diaphragm to burst.

It is a hand driven open ended shock tube. It was conceived with a medical syringe. A plastic sheet placed between the plastic syringe part and the needle part constitutes the diaphragm.



- A high pressure (driver) and a low pressure (driven) side separated by a diaphragm.
- When diaphragm ruptures, a shock wave is formed that propagates along the driven section.
- Shock strength is decided by driver to driven pressure ratio, and type of gases used.



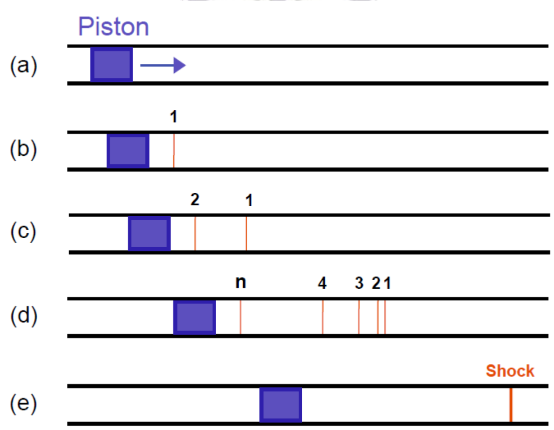


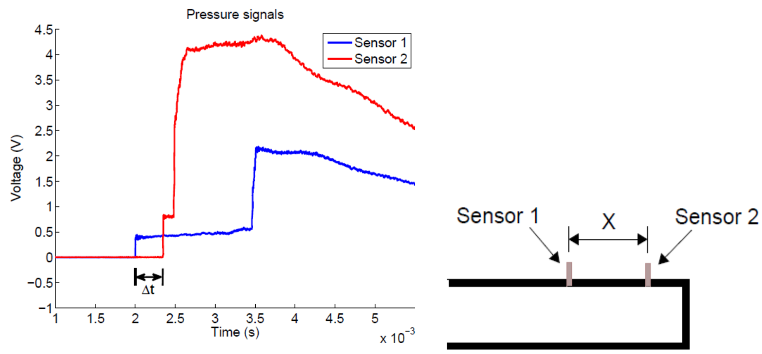
Working:

- The piston is initially at rest and accelerated to final velocity V in a short time t.
- The piston compresses the air in the compression tube. At high pressure, the diaphragm ruptures and the shock wave is set up. For a shock wave to form, $V_{piston} > V_{sound}$.

  Formation of shock wave:

  As the piston gains speed, compression waves are set up. Such compression waves increase in number. As the piston travels a distance, all the compression waves coalesce and a single shock wave is formed. This wave ruptures the diaphragm.





$$U_s = \frac{X}{\Delta t}$$

<u>Mach number</u> = Velocity of the shock wave U_s/Velocity of sound

<u>Uses:</u>

- Aerodynamics – hypersonic shock tunnels, scramjet engines.
- High temperature chemical kinetics – ignition delay
- Rejuvenating depleted bore wells
- Material studies – effect of sudden impact pressure, blast protection materials
- Investigation of traumatic brain injuries
- Needle-less drug delivery
- Wood preservation