



Module-2

ELASTIC PROPERTIES OF MATERIALS

One of the most fundamental question that an engineer has to know is how a material behaves under stress, and when does it break. Ultimately, it is the answer to those two questions which would steer the development of new materials, and determine their survival in various environmental and physical conditions.

Elasticity is an elegant and fascinating subject that deals with determination of the stress, strain and displacement distribution in an elastic solid under the influences of external forces. Following the assumptions of linear, small deformation theory, the formulation establishes a mathematical model providing solutions to problems that have applications in many engineering and scientific fields. Civil engineering applications include stress and deflection analysis of structures like rods, beams, plates, shells, soil, rock, concrete, asphalt. Mechanical engineering uses elasticity in numerous problems of thermal stress analysis, fracture mechanics, fatigue, design of machine elements. Materials engineering uses elasticity to determine the stress fields of crystalline solids, dislocations, micro structures. Applications in Aeronautical engineering include stress fluctuations, fracture, fatigue analysis in aerostructures. The subject also provides the basis for study of material behavior in plasticity, viscoelasticity.

Definitions

Stress: Restoring force per unit area

Strain: Ratio of change in dimension to original dimension

Hooke's Law: For sufficiently small stresses, strain is proportional to stress; the constant of proportionality known as modulus of elasticity depends on the material being deformed and on the nature of the deformation.

Elasticity: The property of materials which undergo deformation under stress and regain their original dimension.

Ex: Spider web, Steel, Graphene

Linear strain (α) - It is the increase per unit length per unit tension along the force

Lateral strain (β) - It is the lateral contraction per unit length per unit tension perpendicular to force



The **elastic limit** of a substance is defined as the maximum stress that can be applied to the substance before it becomes permanently deformed and does not return to its initial length.

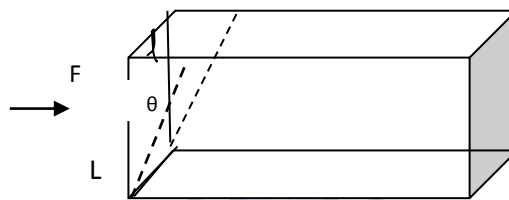
Young's Modulus of Elasticity = Longitudinal stress / Linear Strain = $\frac{FL}{Al}$

Longitudinal stress or tensile stress is applied along the length and hence causes change in length.

Linear strain is the ratio of change in length to original length

Rigidity Modulus of Elasticity = Tangential stress / shear Strain = $\frac{F}{A\theta}$

Shearing stress is applied tangential to a surface. As a result, one surface is displaced with respect to another fixed surface. The ratio of displacement to perpendicular distance between the two surfaces is known as shearing strain.



Shearing strain $\theta = \frac{l}{L}$ when θ is small

Bulk Modulus of elasticity = Normal stress / Volume Strain = $\frac{FV}{Av}$

Application of normal (compressive) stress causes change in volume. Volume strain is the ratio of change in volume to original volume.

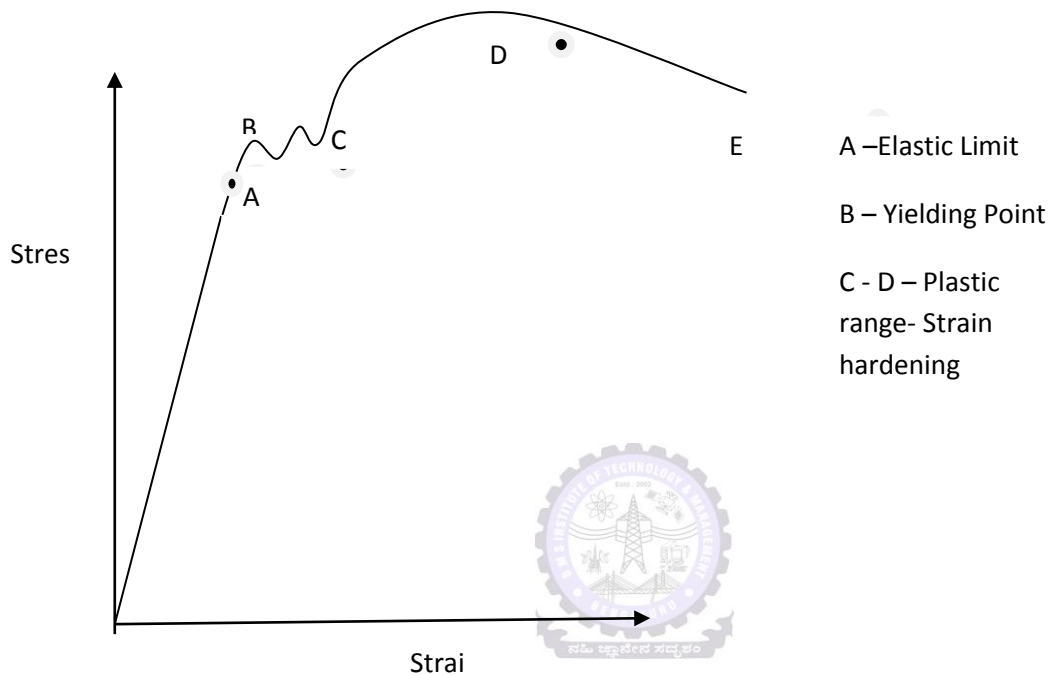
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Plasticity: The property of materials which undergo deformation under stress and do not regain their original dimension.

θ



HOOKE’S LAW



Stress is proportional to strain at smaller magnitudes. As the stress is increased to large magnitudes strain increases more rapidly and the linear relationship between stress and strain ceases to hold. This is referred as elastic limit (A). After the yielding point (B), the strain randomly increases. This may lead to strain softening in some materials. C onwards the material attains permanent status (Plastic) and is known as strain hardening region. After D, the material breaks.

Effect of stress – Temperature – Annealing – Impurities on Elasticity

Effect of stress:

1) Elastic fatigue

Elastic properties of a body repeatedly subjected to stress show random variation.

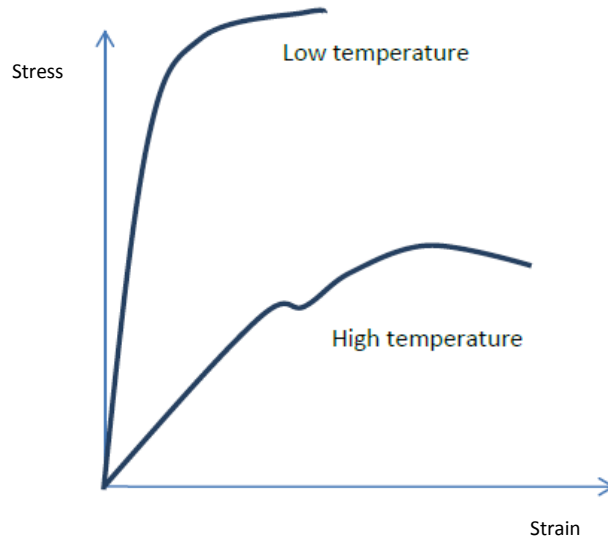
Ex: Piston and connecting rods in a locomotive are subjected to repeated tensions and compressions during each cycle. Their elastic properties randomly fluctuate. It may break under a stress less than elastic limit.

2. Annealing : Annealing operation involves heating and gradual cooling. The crystal grains form a uniform orientation forming larger domains. This causes decrease in elastic



properties. Operations like hammering, rolling break up the crystal grains resulting in increase of elastic properties.

3. Temperature : Inter molecular forces decreases with rise in temperature. Hence the elasticity decreases with rise in temperature. (But the elasticity of invar steel (alloy) does not change with change of temperature). Carbon filament which is highly elastic at ordinary temperature, becomes plastic when heated.



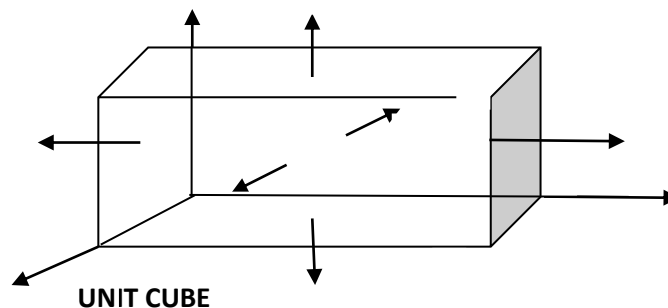
4. Impurities : Presence of impurities alters elasticity. It can increase or decrease depending on the nature of impurities. Carbon is added in minute quantities to molten Iron to increase its elastic property.

FACTOR OF SAFETY

To avoid permanent deformation due to maximum stress, the engineering tools are to be used within the elastic limit with a working stress.

$$\text{Factor of safety} = \text{Breaking stress} / \text{Working stress}$$

RELATION BETWEEN BULK MODULUS (K) - α - β



Study material

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Let stresses T_X , T_Y and T_Z act perpendicular to faces of a unit cube as shown in the figure. Let α be the increase per unit length per unit tension (linear strain) along the force, β be the lateral contraction (lateral strain) per unit length per unit tension perpendicular to force.

Elongation produced along X axis = $T_X \cdot \alpha \cdot 1$

Contraction produced along X axis = $(T_Y \cdot \beta + T_Z \cdot \beta) \cdot 1$

Length of AB = $1 + T_X \alpha - T_Y \beta - T_Z \beta$

Length of BE = $1 + T_Y \alpha - T_X \beta - T_Z \beta$

Length of BC = $1 + T_Z \alpha - T_Y \beta - T_X \beta$

Volume of cube = $(1 + T_X \alpha - T_Y \beta - T_Z \beta) \times (1 + T_Y \alpha - T_X \beta - T_Z \beta) \times (1 + T_Z \alpha - T_Y \beta - T_X \beta)$

$= 1 + T_X \alpha + T_Y \alpha + T_Z \alpha - T_X \beta - T_Y \beta - T_Z \beta - T_X \beta - T_Y \beta - T_Z \beta$
 $= 1 + (\alpha - 2\beta)(T_X + T_Y + T_Z)$ neglecting terms containing $\alpha \cdot \beta, \alpha^2, \beta^2$

$= 1 + (\alpha - 2\beta)(3T)$ if $T_X = T_Y = T_Z$

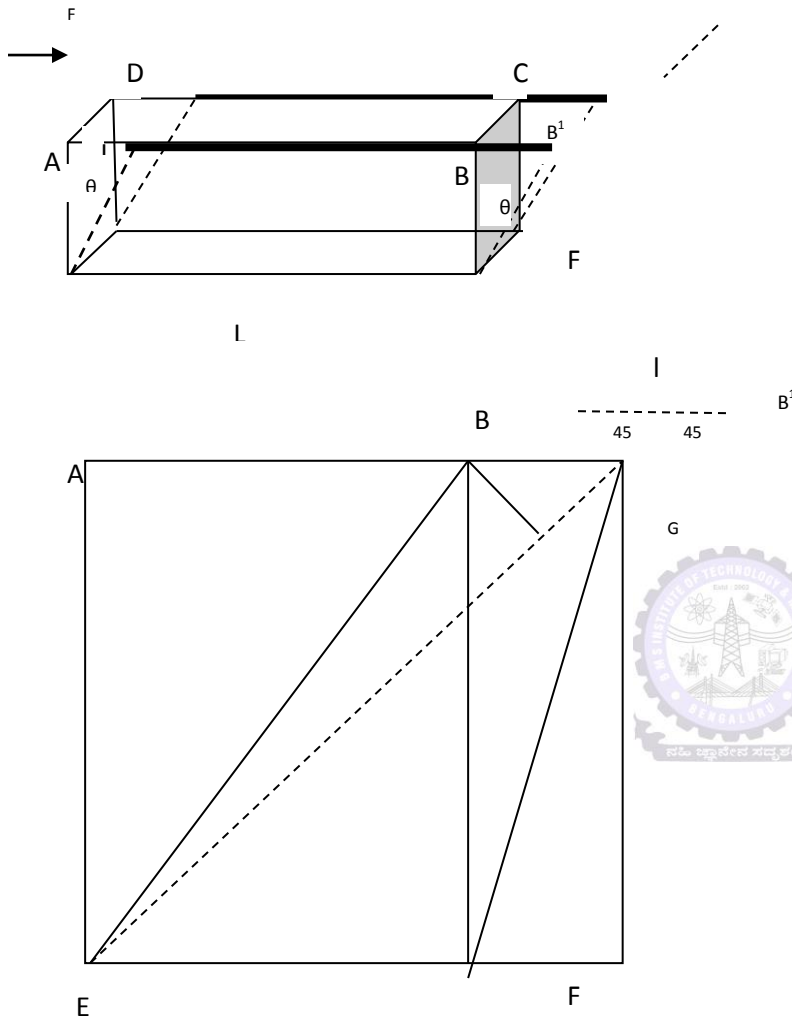
Increase in volume = $1 + (\alpha - 2\beta)(3T) - 1$

If Inward pressure is applied, **reduction in volume** = $1 + (\alpha - 2\beta)(3P)$

$$\text{Bulk Modulus} = \frac{P}{3P(\alpha - 2\beta)} = \frac{1}{3(\alpha - 2\beta)}$$



RIGIDITY MODULUS



Let the face ABCD of a cube of side L be sheared by a Force F through an angle θ .

$$\text{Shearing stress} = \frac{F}{L^2} = T$$

$$\text{Shearing Strain} = \frac{l}{L} = \theta$$

$$\text{Rigidity Modulus} = \frac{T}{\theta}$$

Shearing stress along AB is equivalent to expansive stress along EB and compressive stress along AF. Let α be the longitudinal expansive strain per unit Stress per unit length and β be the lateral compressive strain per unit stress per unit length respectively.

$$\text{Elongation along EB} = EB \cdot \alpha \cdot T.$$

$$\text{Compression along AF} = AF \cdot \beta \cdot T.$$

$$\text{Net extension } EB' = L \cdot \sqrt{2} \cdot T (\alpha + \beta)$$

Also, from right angled triangle $BB'G$,

$$\text{Elongation } EB' = \frac{l}{\sqrt{2}}$$

$$L \cdot \sqrt{2} \cdot T (\alpha + \beta) = \frac{l}{\sqrt{2}}$$

$$\frac{T}{\frac{l}{L}} = \frac{1}{2(\alpha + \beta)}$$

$$n = \frac{1}{2(\alpha + \beta)}$$



RELATION BETWEEN ELASTIC CONSTANTS

$$\alpha - 2\beta = \frac{1}{3K} \dots\dots(1)$$

$$\alpha + \beta = \frac{1}{2n} \dots\dots\dots(2)$$

$$(2) - (1)$$

$$3\beta = \frac{1}{2n} - \frac{1}{3K}$$

$$\beta = \frac{3K - 2n}{18nK}$$

$$2 \cdot (2) + (1)$$

$$\alpha = \frac{3K + n}{9Kn}$$

$$Y = \frac{1}{\alpha} = \frac{\sigma}{\beta}$$

$$\frac{9}{Y} = \frac{3}{n} + \frac{1}{K}$$

POISSON RATIO

When a material is stretched, the increase in its length (α) is accompanied by decrease in cross section (lateral strain β). Within the elastic limit, the lateral strain is proportional to longitudinal strain and the ratio between them is a constant for a material known as Poisson ratio.



$$\sigma = \frac{\beta}{\alpha}$$

RELATION BETWEEN K - n - σ

$$K = \frac{1}{3(\alpha - 2\beta)}$$

$$n = \frac{1}{2(\alpha + \beta)}$$

$$K = \frac{1}{3\alpha(1 - 2\sigma)}$$

$$n = \frac{1}{2\alpha(1 + \sigma)}$$

$$K = \frac{Y}{3(1 - 2\sigma)}$$

$$n = \frac{Y}{2(1 + \sigma)}$$

$$\sigma = \frac{3K - 2n}{6K + 2n}$$

LIMITS OF σ

$$3(1 - 2\sigma) = 2n(1 + \sigma)$$

1. If σ be a positive quantity, $(1 - 2\sigma)$ should be positive

$$2\sigma < 1$$

$$\sigma < 0.5$$

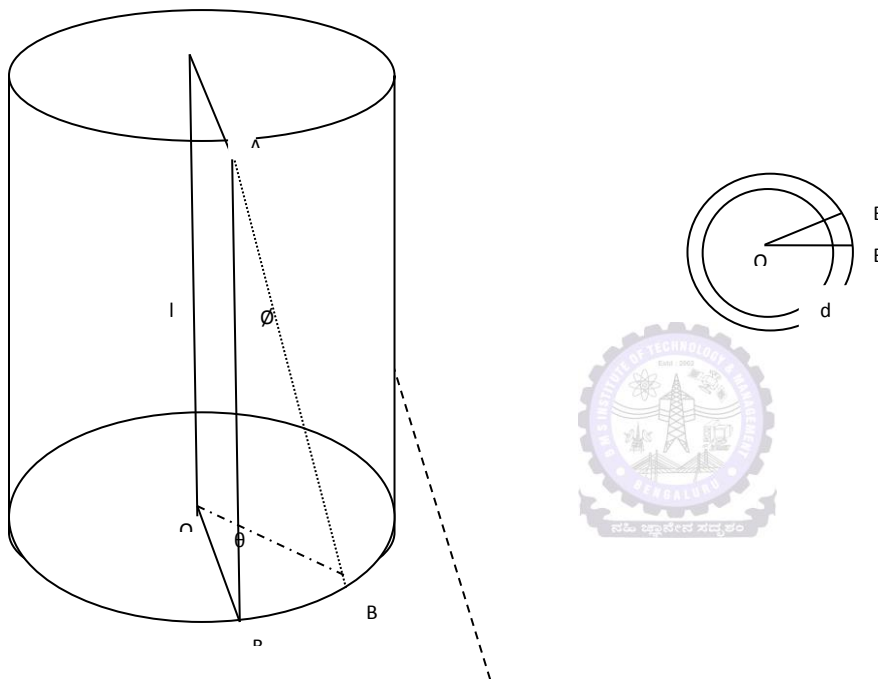
When $\sigma = 0.5$, the material is said to be incompressible



2. If σ be a negative quantity, $(1 + \sigma)$ should be positive
 $\sigma < -1$

Resilience: Capacity to resist a heavy stress without acquiring permanent elongation.

COUPLE PER UNIT TWIST OF A SOLID CYLINDER



Consider a cylindrical rod of rigidity modulus n , length l , radius r fixed at one end and twisted at the other end through an angle θ by a couple. Imagine the cylinder to be made of large number of coaxial cylinders of increasing radius. Consider a cylinder of radius x and thickness dx . For a given couple, the displacement at its rim is maximum. On twisting, the point B shifts to B^1 .

$$BB^1 = l\phi = x\theta$$

$$\phi = \frac{x\theta}{l}$$

$$n = \frac{F}{\phi} = \frac{nx\theta}{l}$$

This force is acting on the area $2.\pi x.dx$



$$\text{Total force } F = \frac{nx\theta}{l} 2\pi x \cdot dx$$

$$\text{Moment of force along } OO^1 = \text{couple} = \frac{nx^3\theta}{l} 2\pi \cdot dx$$

$$\text{Total twisting couple} = \int_0^R \frac{nx^3\theta}{l} 2\pi \cdot dx = \frac{2\pi n\theta}{l} \frac{R^4}{4}$$

$$\text{Couple per unit twist} = \frac{\pi n R^4}{l \cdot 2}$$

TORSIONAL PENDULUM

A heavy object suspended from end of a fine wire rotating about an axis constitutes a torsional pendulum. Let θ be the twisting angle. The restoring couple set up in it is equal to $\frac{\pi n\theta R^4}{l \cdot 2}$. This produces angular acceleration.

If I is the moment of Inertia of the object



$$\text{Torque} = I \cdot \frac{d\omega}{dt} = -\frac{\pi n\theta R^4}{2l}$$

$$\frac{d\omega}{dt} = -\frac{\pi n r^4 \theta}{2I} = -\frac{C}{I} \theta \quad [\text{Angular Acceleration } \alpha - \text{angular}$$

displacement]

C is the couple per unit twist.

The motion is simple harmonic as shown by the expression torque =

$$\tau = I \frac{d^2\theta}{dt^2} = -C\theta$$

$$F = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\omega = \sqrt{\frac{k}{m}}$$



BENDING MOMENT

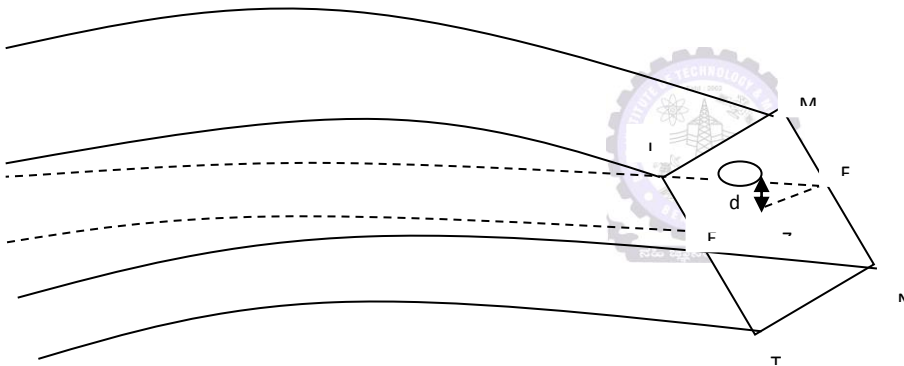
In the above figure, ABCD is a beam fixed at AD and loaded at B. EF is neutral axis. Whereas the load tends to bend the beam, an equal and opposite reactional force W^1 will be acting upwards along pp^1 . These two forces constitute a couple and the moment of this couple is called bending moment.

Let the beam be bent in the form of circular arc subtending angle θ at the centre of curvature O. Let a^1b^1 be an element at a distance Z from the neutral axis.

$$a^1b^1 = (R + Z)\theta$$

$$\text{Increase in length of the filament} = a^1b^1 - ab = (R + Z)\theta - R\theta$$

$$\text{Strain} = \frac{Z\theta}{R\theta} = \frac{Z}{R}$$



Let LMNT be the rectangular cross section perpendicular to length. EF is the neutral surface. The restoring force on upper half acts inwards and outwards on the lower half.

Consider a small area da at a distance z from the Neutral surface.

$$\text{Strain produced in the filament} = \frac{Z}{R}$$

$$\text{Force on area } da = Y \cdot da \cdot \frac{Z}{R}$$

$$\text{Moment of this force about the neutral surface} = F \cdot Z = Y \cdot da \cdot \frac{Z^2}{R}$$



Total moment of forces in LMNT $M = Y \cdot \frac{\sum Z^2 \cdot da}{R} = \frac{Y}{R} I$

Here I is Geometrical Moment of Inertia.

For rectangular cross section

$$\text{area} = b \times d, \quad k = \frac{d^2}{12}$$

$$I = ak^2 = \frac{bd^3}{12}$$

$$M = \frac{Ybd^3}{12R}$$

For Circular cross section

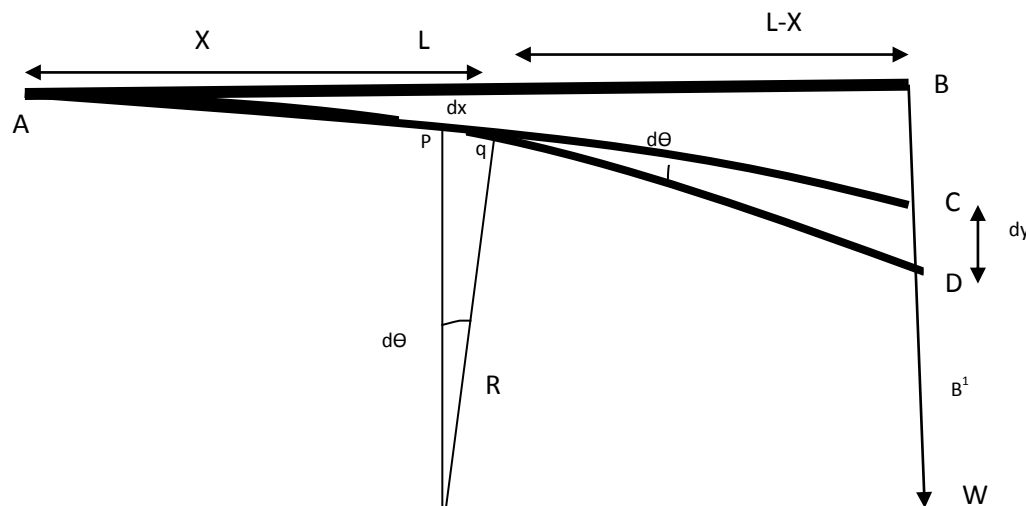
$$\text{area} = \pi R^2, \quad k = \frac{r^2}{4}$$

$$I = ak^2 = \frac{\pi R^4}{4}$$

$$M = \frac{Y\pi r^4}{4R}$$

CANTILEVER

It is a beam fixed horizontally at one end and loaded at the other.





Let AB be the neutral axis of the cantilever of length L fixed at A and loaded at B. Consider a section P of the beam at a distance x from A.

$$\text{Bending moment} = W \cdot PC = W(L-X) = Y \frac{I}{R} = Ya \frac{k^2}{R}$$

Here R is the radius of curvature of neutral axis at P. The moment of the load increases towards the point A, the radius of curvature is different at different points and decreases towards A. For a point Q at a distance dx from P, it is same as at P.

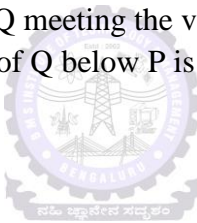
$$PQ = dx = R \cdot d\theta$$

$$\text{Bending moment } W(L-X) = Y \frac{ak^2 d\theta}{dx}$$

Draw tangents to the neutral axis at P and Q meeting the vertical line at C and D. The angle subtended by them is dθ. The depression of Q below P is given by

$$dy = (L-X)d\theta = W \frac{(L-X)^2}{Yak^2} \cdot dx$$

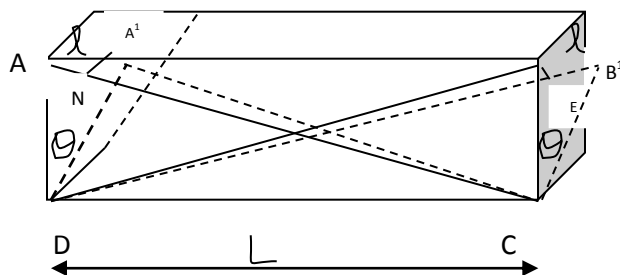
Total depression BB¹ of the loaded end



$$\int dy = \int_0^L W \frac{(L-X)^2}{Yak^2} \cdot dx = W \frac{L^3}{3YI}$$

Relation between Shear strain – Longitudinal strain – Compression stain

COROLLARY 1: A shear of θ is equivalent to an elongation strain θ/2 and compression strain θ/2 at right angles to each other.





From the triangle BEB^1 , $\hat{B}B^1E = 45^\circ$

$$\cos 45 = \frac{EB^1}{BB^1} \Rightarrow EB^1 = \frac{l}{\sqrt{2}}$$

$$DE = \sqrt{2}L$$

Extension strain along DE = $\frac{EB^1}{DE} = \frac{l}{2L} = \frac{\theta}{2}$ where is the shearing strain

From the triangle AA^1N , $AA^1N = 45^\circ$

$$\cos 45 = \frac{AN}{AA^1} \Rightarrow AN = \frac{l}{\sqrt{2}}$$

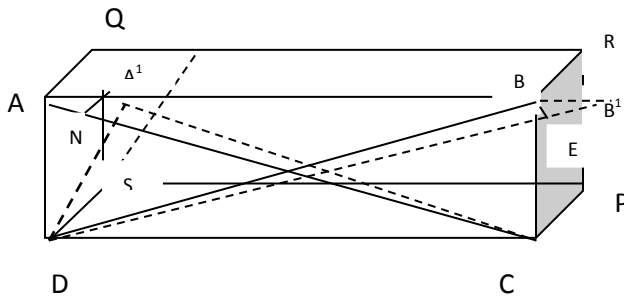
$$CN = \sqrt{2}L$$

Compression strain along AC = $\frac{AN}{CN} = \frac{l}{2L} = \frac{\theta}{2}$

Elongation strain + Compression strain = $\theta/2 + \theta/2 = \theta$

COROLLARY 2

A shearing stress of F is equivalent to an tensile stress $F/2$ and compression stress $F/2$ at right angles to each other.[A tangential force of F is equivalent to a tensile force $\frac{F}{\sqrt{2}}$ and compression force $\frac{F}{\sqrt{2}}$ at right angles to each other]



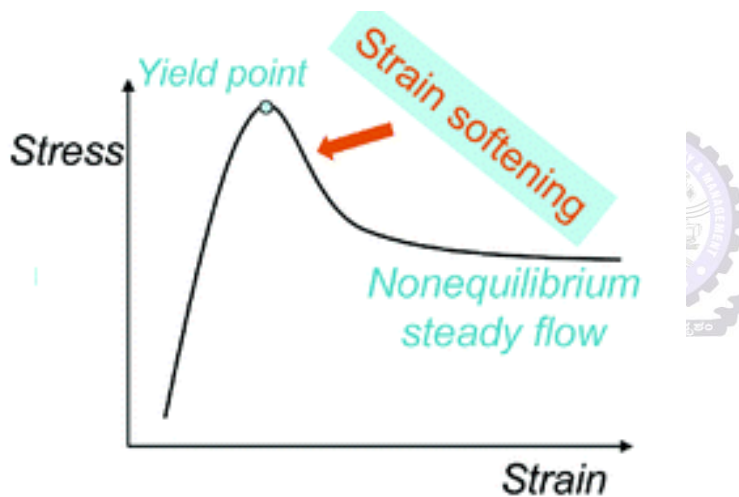


Tensile stress along DB perpendicular to the surface AQPC $= \frac{F \cos 45}{L.L\sqrt{2}} = \frac{F/\sqrt{2}}{L.L\sqrt{2}}$

Compression stress along AC perpendicular to the surface DBRS $= \frac{F \cos 45}{L.L\sqrt{2}} = \frac{F/\sqrt{2}}{L.L\sqrt{2}}$

Stain softening

Strain softening is defined as the region in which the stress in the material is decreasing with an increase in strain. This observed in certain materials after yielding point as represented in the diagram. It causes deterioration of material strength with increasing strain, which is a phenomenon typically observed in damaged quasi brittle materials, including fiber reinforced composites and concrete. It is primarily a consequence of brittleness and heterogeneity of the material.



Strain Hardening

When a material is strained beyond the yield point, more and more stress is required to produce additional plastic deformation and the material becomes stronger and more difficult to deform. This is known as Strain Hardening. The material is permanently deformed increasing its resistance to further deformation. Strain hardening reduces ductility and increases brittleness.

A material that does not show any strain hardening is said to be perfectly plastic.

The strain hardening exponent (coefficient) is given by the expression $\sigma = K \epsilon^n$.



σ is the applied stress

ϵ is strain

n is the strain hardening coefficient

K is the strength coefficient (elasticity) . It is a measure of the ability of a metal to strain harden.

The value of n lies between 0.1 and 0.5 for most metals.

A material with a higher value of n has a greater elasticity than a material with a low value of n .

Applications of beams

Beams are an integral part of Civil engineering structural elements (bridges, dams, multistoreid buildings), measuring devices (Tunneling microscopes), automobile frames, aircraft components, machine frames. They are designed to withstand heavy load. Cantilever beam, simple support beam (connecting beam to beam), roller beam are generally observed in heavy structures.

Applications of Torsional Pendulum:

1. The working of Torsion pendulum clocks is based on torsional oscillation.
2. The freely decaying oscillation of Torsion pendulum in medium (like polymers),helps to determine their characteristic properties.
3. Determination of frictional forces between solid surfaces and flowing liquid environments using forced torsion pendulums.
4. Torsion springs are used in torsion pendulum clocks.

Clothes Pins. The working of clothes pins is facilitated by the torsion springs. These springs provide an excellent clamping action.

Automotive: Torsion springs are known for providing even tension, along with smooth and frictionless motion. These springs are widely used in the automotive industry for various parts such as a vehicle suspension system, chassis, automotive valves, clutches, and gear shifters.

Medical Equipment: In the medical industry, the torsion springs are used in medical immobilization devices, hospital beds, several dental applications, wheelchair lifts and many more.

Door Hinges: These springs are widely used in different types of door hinges. These springs allow the door to come back to its original position.